

Heavy-flavor particles: a tool to access the properties of the hot matter produced in the Big and Little Bang

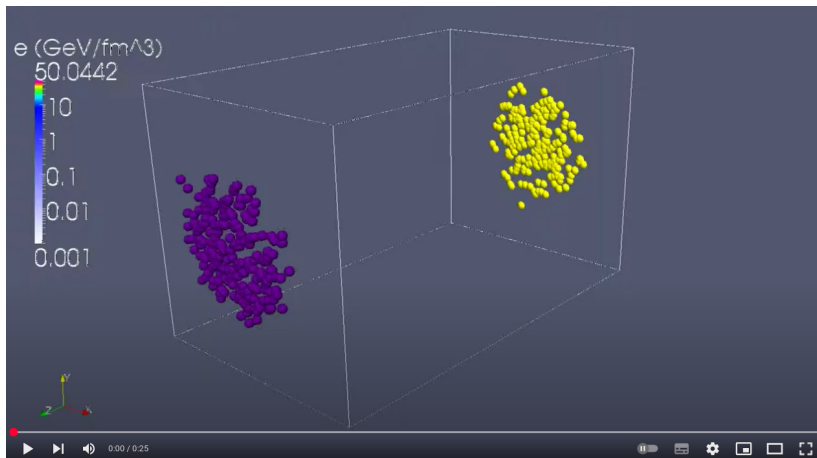
Andrea Beraudo

INFN - Sezione di Torino

Cortona, 1-3 October 2025

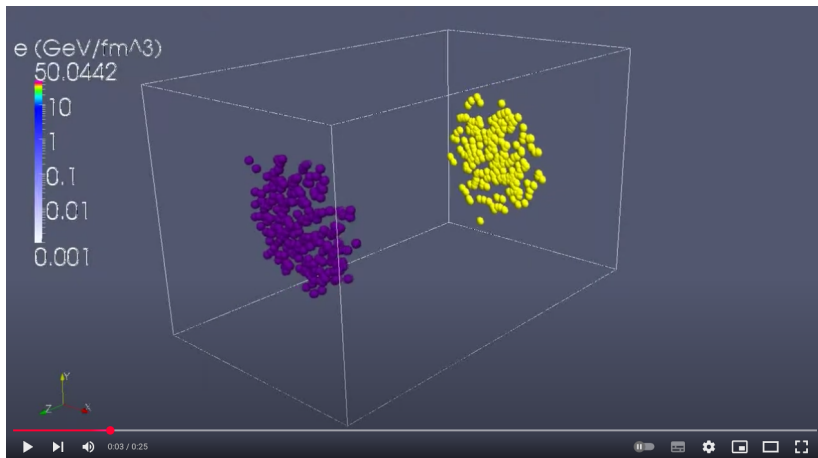


Simulating a HIC



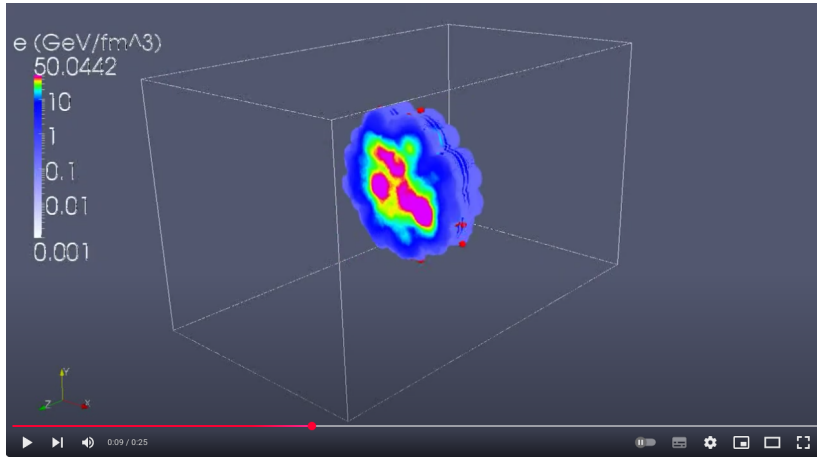
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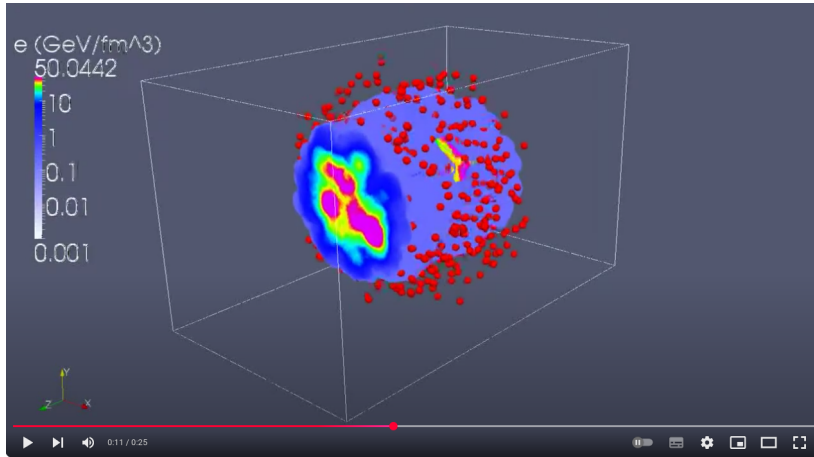
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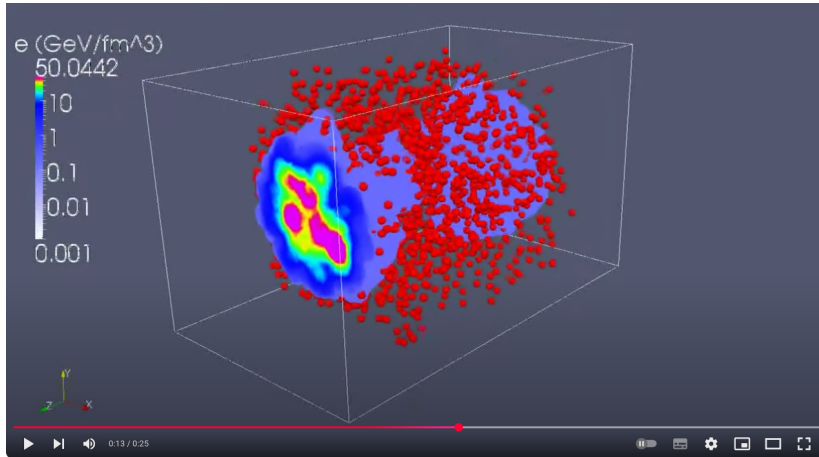
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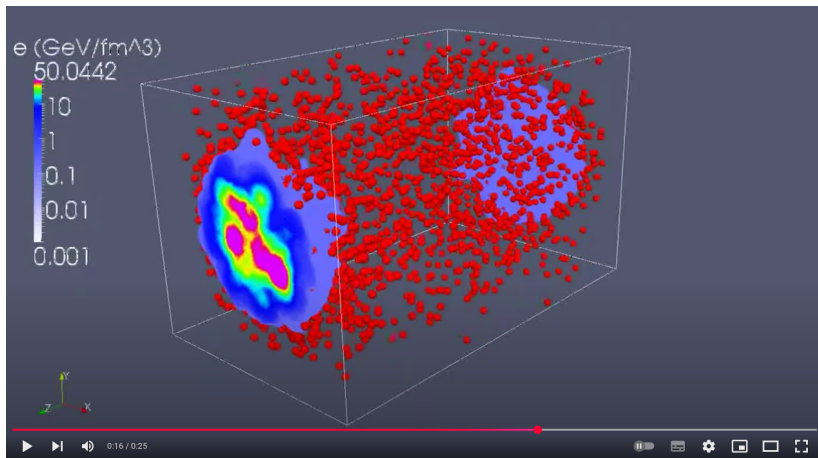
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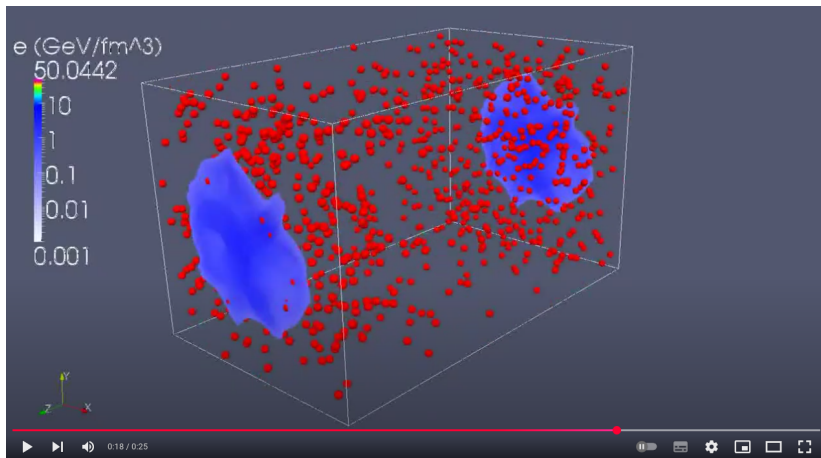
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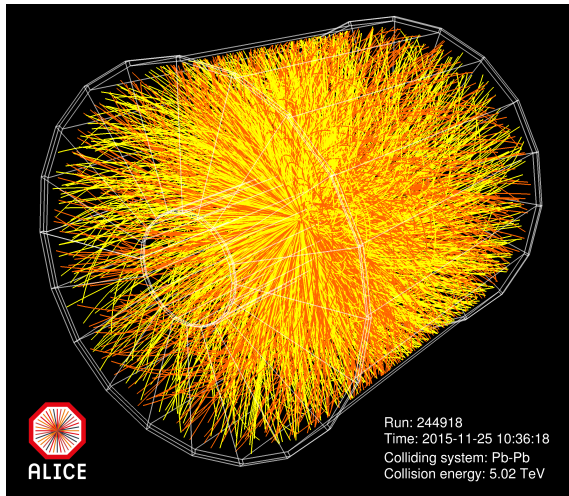
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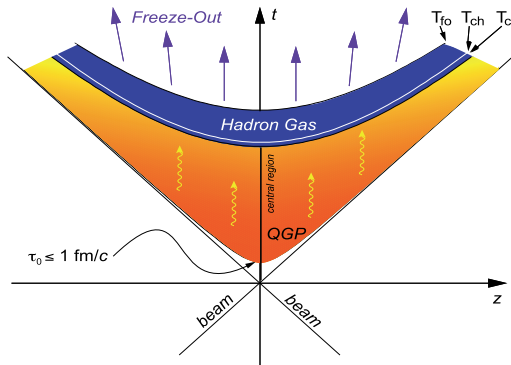


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An event as seen by the detectors

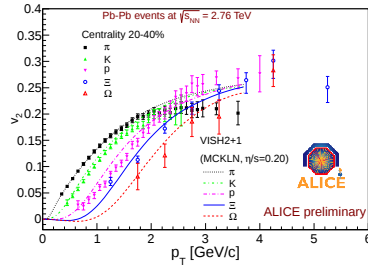
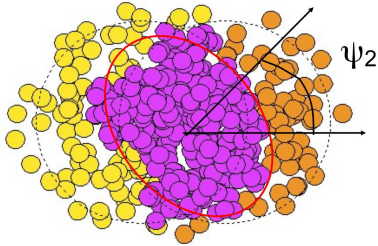


Getting information on what happened before



- **Soft probes** (low- p_T hadrons): **collective behavior** of the *medium*;
- **Hard probes** (high- p_T particles, **heavy quarks**, quarkonia): produced in *hard* $pQCD$ processes in the initial stage, allow to perform a **tomography of the medium**

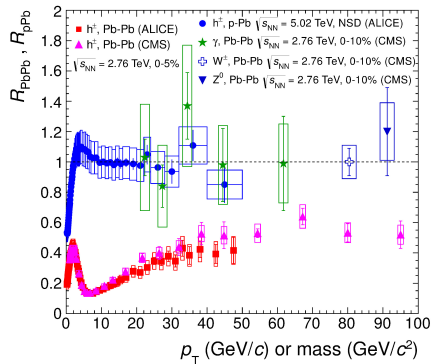
A medium displaying a collective behavior



Anisotropic azimuthal distribution of hadrons as a **response to pressure gradients** quantified by the *Fourier coefficients* v_n

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi} \left(1 + 2 \sum_n v_n \cos[n(\phi - \psi_n)] + \dots \right)$$
$$v_n \equiv \langle \cos[n(\phi - \psi_n)] \rangle$$

A medium inducing energy-loss to colored probes



Medium-induced suppression of high-momentum hadrons and jets quantified through the *nuclear modification factor*

$$R_{AA} \equiv \frac{(dN^h/dp_T)^{AA}}{\langle N_{\text{coll}} \rangle (dN^h/dp_T)^{pp}}$$

HF in HIC's: what do we want to learn? A bit of history...

Ultramikroskopie für Kolloide

Nach SIEDENTOPF und ZSIGMONDY.



2. Ausgabe 1903. Telegrafendresse: ZIESSOWEN JENA.

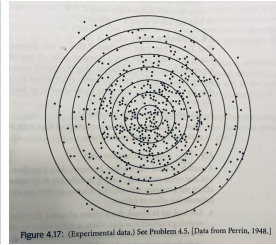
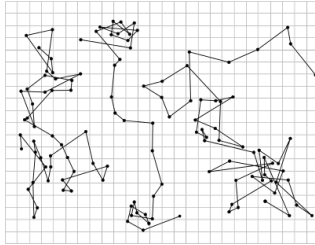


Figure 4.17: (Experimental data.) See Problem 4.5. [Data from Perrin, 1948.]

Einstein (1905) and Perrin (1909) study of Brownian motion: from the random walk of small grains ($a \sim 0.5\mu\text{m}$) in water one extracts the **diffusion coefficient**

$$\langle x^2 \rangle_{t \rightarrow \infty} \sim 2D_s t$$

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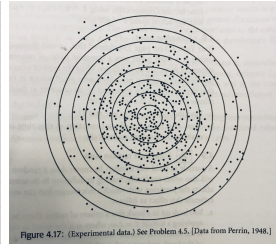
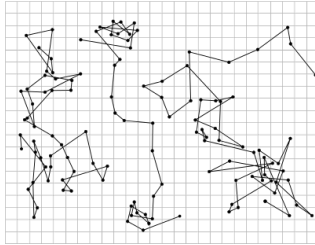


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and estimates the **Avogadro number** (proof of the *granular structure of matter*):

$$\mathcal{N}_A K_B \equiv \mathcal{R} \quad \longrightarrow \quad \mathcal{N}_A = \frac{\mathcal{R} T}{6\pi a \eta D_s}$$

Perrin obtained the values $\mathcal{N}_A \approx 5.5 - 7.2 \cdot 10^{23}$.

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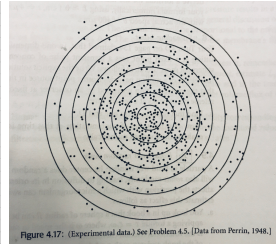
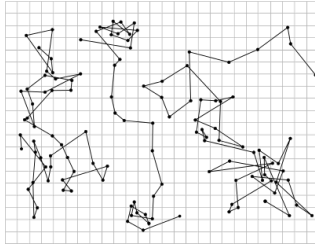


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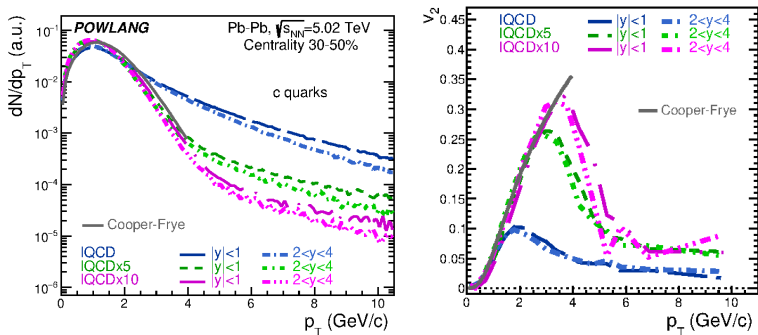
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Perrin obtained the values $\mathcal{N}_A \approx 5.5 - 7.2 \cdot 10^{23}$. We would like to **derive HQ transport coefficients in the QGP** with a comparable precision and accuracy!

We do not have a microscope!

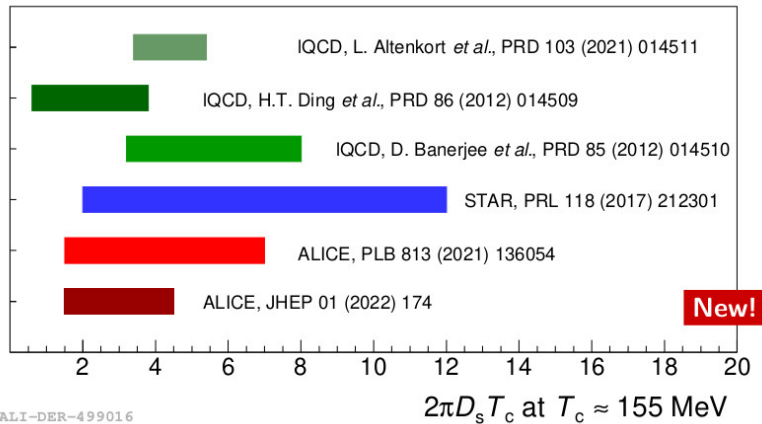


Transport coefficients can be accessed indirectly, comparing transport predictions with different values of **momentum broadening**

$$\kappa = \frac{2T^2}{D_s}$$

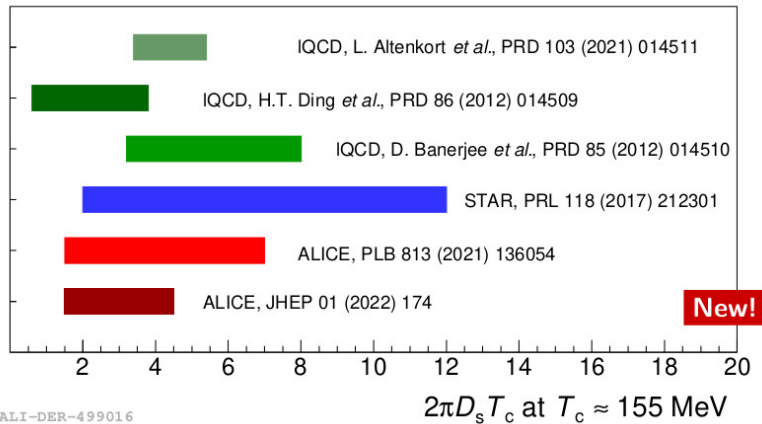
with experimental results for momentum (left) and angular (right) HF particle distributions (figure from [A.B. et al., JHEP 05 \(2021\) 279](#))

Where do we stand?



Still far from accuracy and precision of Perrin result for $\mathcal{N}_A$...

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Why such large systematic uncertainties?

HQ transport: the relativistic Langevin equation

HQ diffusion through the fireball simulated via the relativistic Langevin equation (A.B. et al., Nucl.Phys. A831 (2009) 59)

$$\frac{\Delta p^i}{\Delta t} = - \underbrace{\eta_D(\mathbf{p}) p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \rangle = 0 \quad \langle \xi^i(\mathbf{p}_t) \xi^j(\mathbf{p}_{t'}) \rangle = b^{ij}(\mathbf{p}) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_{\parallel}(\mathbf{p}) \hat{p}^i \hat{p}^j + \kappa_{\perp}(\mathbf{p}) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

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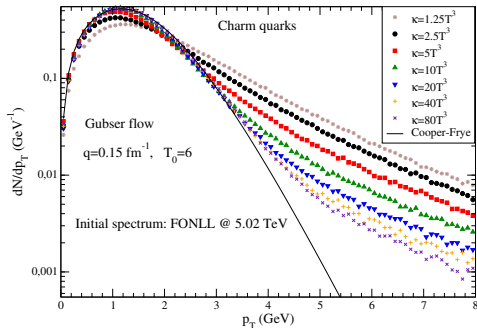
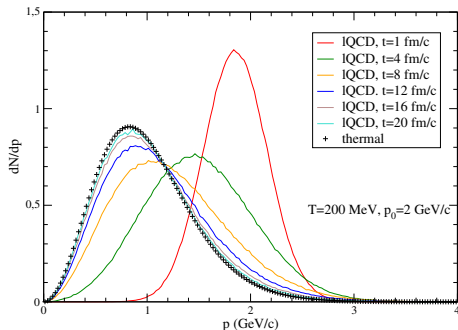
Transport coefficients describe the HQ-medium coupling

- **Momentum diffusion** $\kappa_{\perp} \equiv \frac{1}{2} \frac{\langle \Delta p_{\perp}^2 \rangle}{\Delta t}$ and $\kappa_{\parallel} \equiv \frac{\langle \Delta p_{\parallel}^2 \rangle}{\Delta t}$;
- **Friction** term (dependent on the **discretization scheme**!)

$$\eta_D^{\text{Ito}}(\mathbf{p}) = \frac{\kappa_{\parallel}(\mathbf{p})}{2TE_p} - \frac{1}{E_p^2} \left[(1 - v^2) \frac{\partial \kappa_{\parallel}(\mathbf{p})}{\partial v^2} + \frac{d-1}{2} \frac{\kappa_{\parallel}(\mathbf{p}) - \kappa_{\perp}(\mathbf{p})}{v^2} \right]$$

fixed in order to assure approach to equilibrium (**Einstein relation**)

Asymptotic approach to thermalization

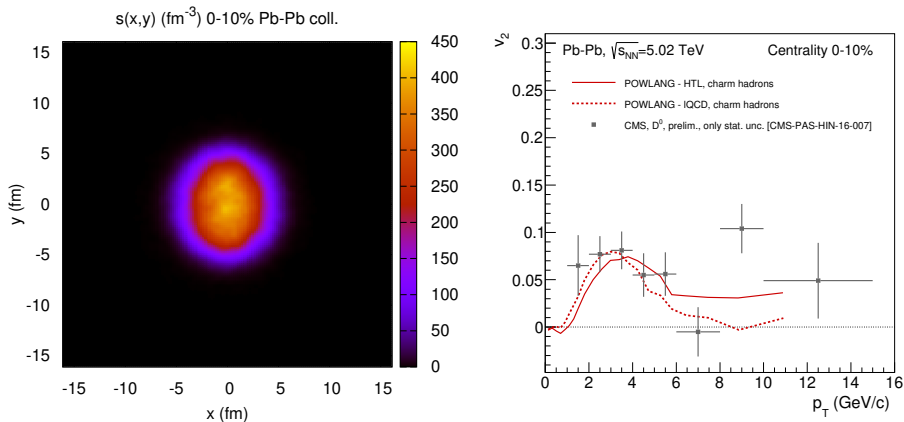


Validation of the model (figures adapted from [Federica Capellino master thesis](#)):

- Left panel: evolution in a static medium
- Right panel: decoupling from an expanding medium at $T_{FO} = 160$ MeV

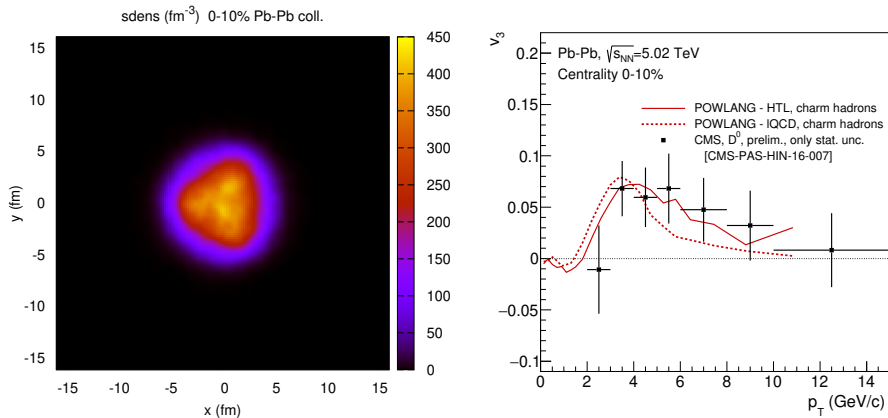
For late times or very large transport coefficients HQ's approach local kinetic equilibrium with the medium. For an expanding medium high- p_T tail remains off equilibrium.

Some results: D -meson v_2 and v_3 in Pb-Pb



Transport calculations carried out in [JHEP 1802 \(2018\) 043](#), with hydrodynamic background calculated via the [ECHO-QGP code](#) ([EPJC 73 \(2013\) 2524](#)) starting from [EBE](#) Glauber Monte-Carlo initial conditions: $v_2 \neq 0$ in central collisions, $v_3 \neq 0$

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HQ momentum diffusion: lattice-QCD

From the **non-relativistic limit** of the Langevin equation one gets

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t) \xi^j(t') \rangle = \delta^{ij} \delta(t - t') \kappa$$

$$\text{hence} \quad \kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\text{HQ}} = \frac{1}{3} \int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t) F^i(0) \rangle_{\text{HQ}}}_{\equiv D^>(t)}$$

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Lattice-QCD simulations provide *Euclidean* ($t = -i\tau$) **electric-field** ($M = \infty$) **correlator**

$$D_E(\tau) = - \frac{\langle \text{Re Tr}[U(\beta, \tau) g \mathbf{E}^i(\tau, \mathbf{0}) U(\tau, 0) g \mathbf{E}^i(0, \mathbf{0})] \rangle}{\langle \text{Re Tr}[U(\beta, 0)] \rangle}$$

How to proceed?

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How to proceed? κ comes from the $\omega \rightarrow 0$ limit of the FT of $D^>$. In a thermal ensemble $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega}) D^>(\omega)$, so that

$$\kappa \equiv \lim_{\omega \rightarrow 0} \frac{D^>(\omega)}{3} = \lim_{\omega \rightarrow 0} \frac{1}{3} \frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \underset{\omega \rightarrow 0}{\sim} \frac{1}{3} \frac{T}{\omega} \sigma(\omega)$$

HQ momentum diffusion: systematic uncertainties

The spectral density $\sigma(\omega)$ has to be extracted from the euclidean correlator

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

An ill-posed problem! $D_E(\tau)$ known for a **limited set** (~ 20) **of** points, while one wishes to obtain a **fine scan of** the the spectral function $\sigma(\omega_j)$. A direct χ^2 -fit is not applicable.

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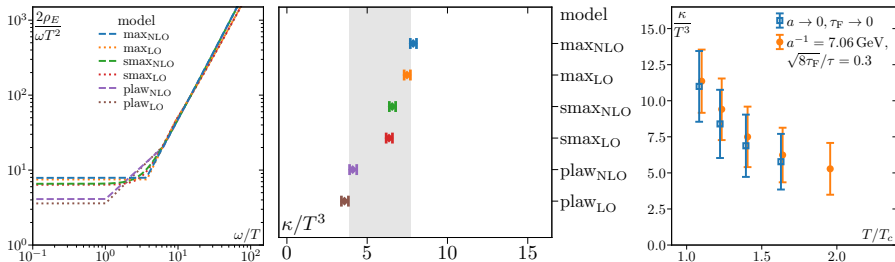
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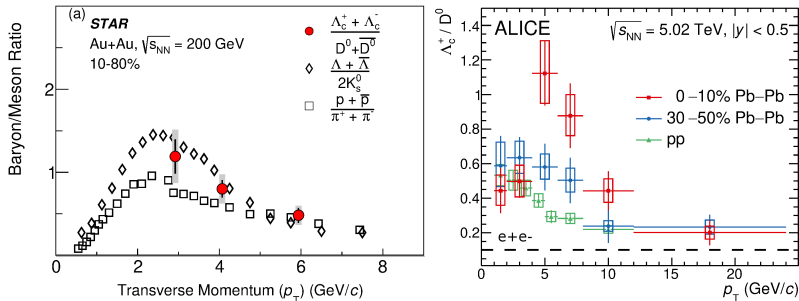
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- Bayesian techniques (Maximum Entropy Method)
- Theory-guided ansatz for the behaviour of $\sigma(\omega)$ to constrain its functional form (new results for $N_f=2+1$ [HotQCD, PRL 130 \(2023\) 23, 231902](#))

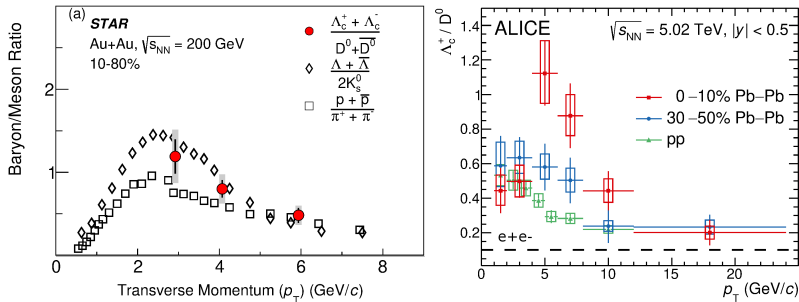


Systematic uncertainties II: hadronization



Strong **enhancement of charmed baryon/meson ratio**, incompatible with hadronization models tuned to reproduce e^+e^- data

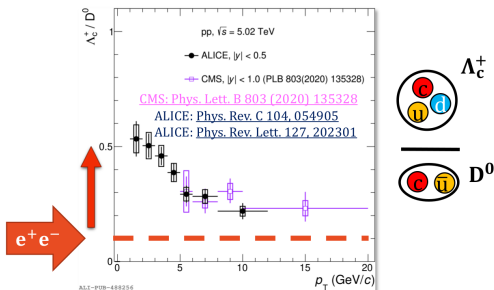
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- pattern similar to light hadrons
- baryon **enhancement observed also in pp collisions**: is a dense medium formed also there?
Breaking of factorization description in pp collisions

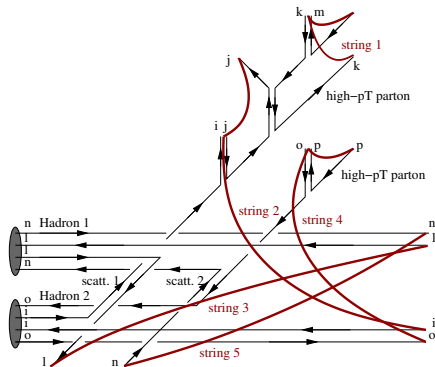
$$d\sigma_h \neq \sum_{a,b,X} f_a(x_1) f_b(x_2) \otimes d\hat{\sigma}_{ab \rightarrow c\bar{c}X} \otimes D_{c \rightarrow h_c}(z)$$

Hadronization models: common features

Grouping colored partons into color-singlet structures: strings (PYTHIA), clusters (HERWIG), hadrons/resonances (coalescence/recombination).

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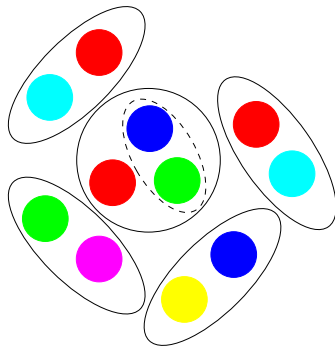
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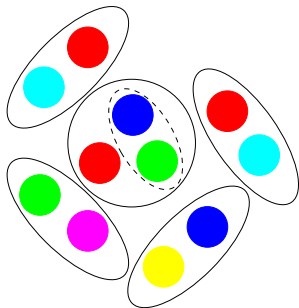


- in “elementary collisions” (what is elementary?): from the hard process, shower stage, underlying event and beam remnants;
- in heavy-ion collisions (only?): from the hot medium produced in the collision.
NB Involved partons closer in space in this case and this has deep consequence!

Local Color Neutralization (LCN): basic ideas

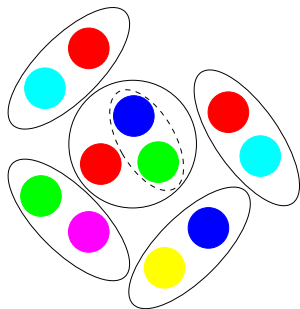
Both in AA and pp collisions a **big/small deconfined fireball** is formed. Around the QCD crossover temperature quarks undergoes **recombination with the closest opposite color-charge** (antiquark or **diquark**, favoring baryon production).

- Why? screening of color-interaction, **minimization of energy stored in confining potential**
- Implication: recombination of particles *from the same fluid cell* → **Space-Momentum Correlation** (SMC), recombined partons tend to share a common collective velocity



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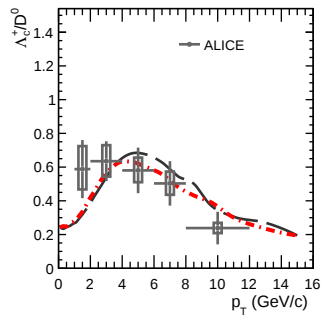
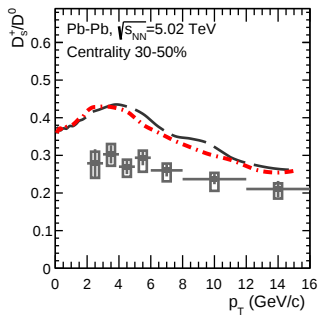
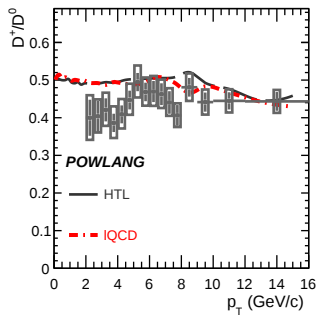


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Color-singlet structures are thus formed, eventually undergoing **decay into the final hadrons**: $2 \rightarrow 1 \rightarrow N$ process, usually a **charmed hadron plus a very soft particle**

- Exact four-momentum conservation;
- No direct bound-state formation, hence no need to worry about overlap between the final hadron and the parent parton wave-functions

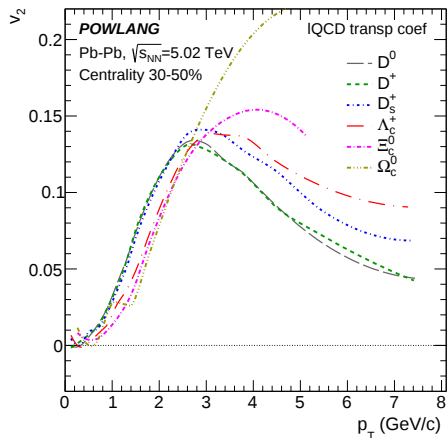
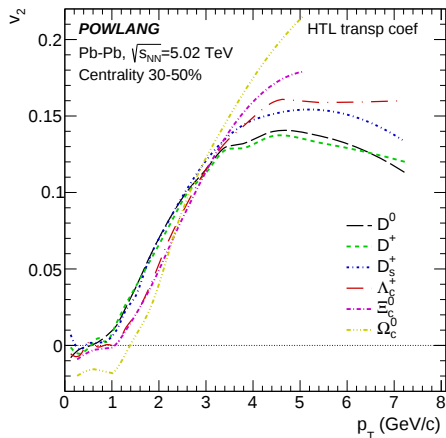
LCN in AA collisions: charmed-hadron ratios



- Enhanced HF baryon-to-meson ratios up to intermediate p_T nicely reproduced, thanks to formation of *small invariant-mass charm+diquark clusters*¹
- Smooth approach to e^+e^- limit ($\Lambda_c^+/D^0 \approx 0.1$) at high p_T : high- M_c clusters fragmented as Lund strings, as in the vacuum

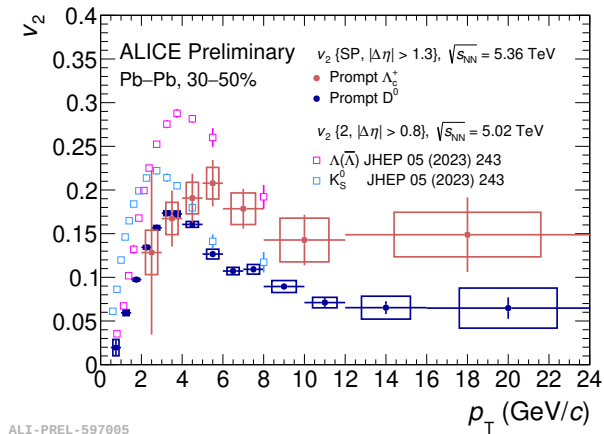
¹A.B. et al., EPJC 82 (2022) 7, 607

LCN in AA collisions: elliptic flow



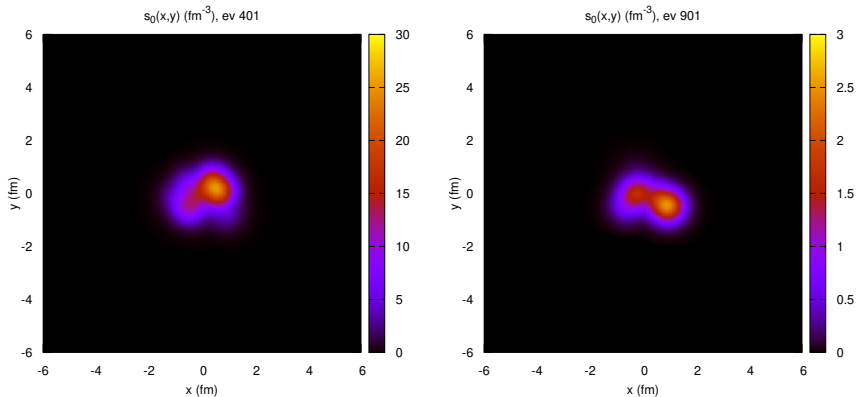
Mass ordering of the v_2 coefficient!

LCN in AA collisions: elliptic flow



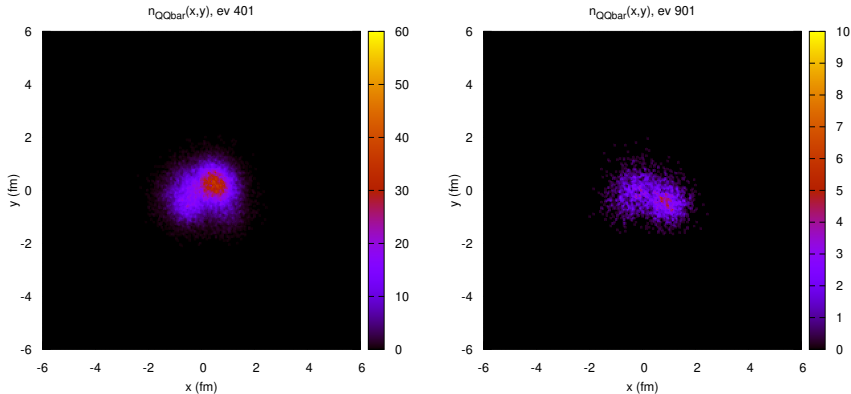
Mass ordering of the v_2 coefficient!

Addressing pp collisions...



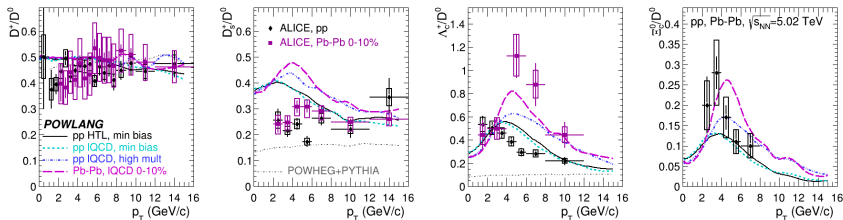
$Q\bar{Q}$ production biased towards hot spots of highest multiplicity events

Addressing pp collisions...



$Q\bar{Q}$ production biased towards hot spots of highest multiplicity events

Results in pp: particle ratios

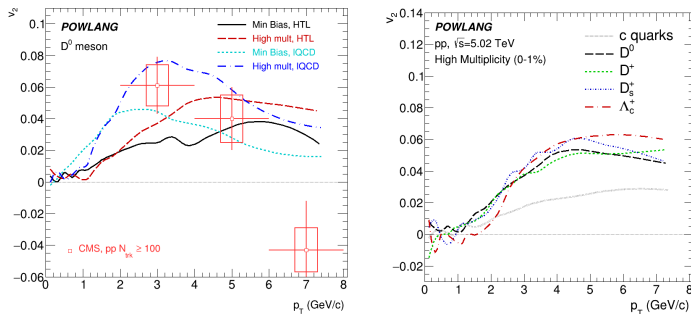


First results for particle ratios²:

- POWHEG+PYTHIA standalone strongly underpredicts baryon-to-meson ratio
- Enhancement of charmed baryon-to-meson ratio qualitatively reproduced if propagation+hadronization in a small QGP droplet is included
- Multiplicity dependence of radial-flow peak position (just a reshuffling of the momentum, without affecting the yields): $\langle u_{\perp} \rangle_{pp}^{mb} \approx 0.33$, $\langle u_{\perp} \rangle_{pp}^{hm} \approx 0.53$, $\langle u_{\perp} \rangle_{PbPb}^{0-10\%} \approx 0.66$

²In collaboration with D. Pablos, A. De Pace, F. Prino et al., PRD 109 (2024) 1, L011501

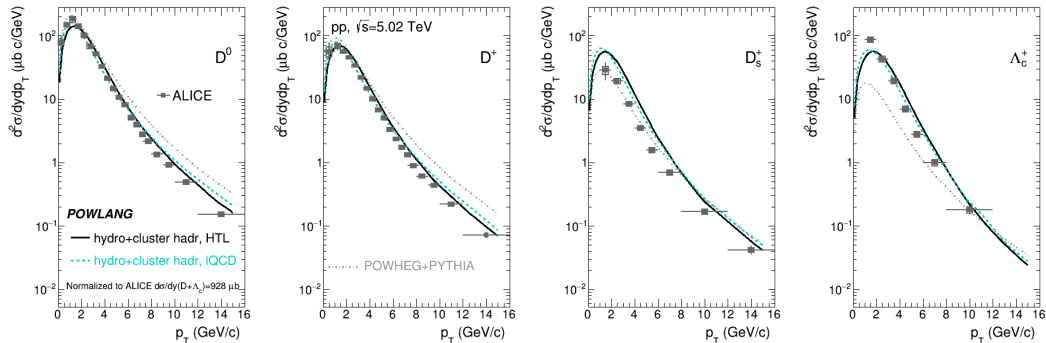
Results in pp: elliptic flow



Response to **initial elliptic eccentricity** ($\langle \epsilon_2 \rangle^{\text{mb}} \approx \langle \epsilon_2 \rangle^{\text{mh}} \approx 0.31$) $\longrightarrow$ **non-vanishing v_2** coefficient

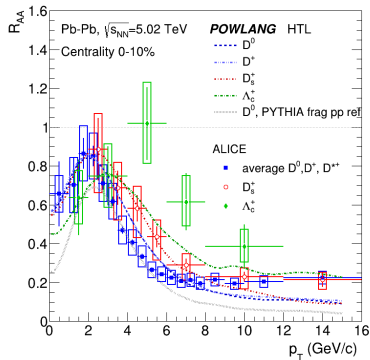
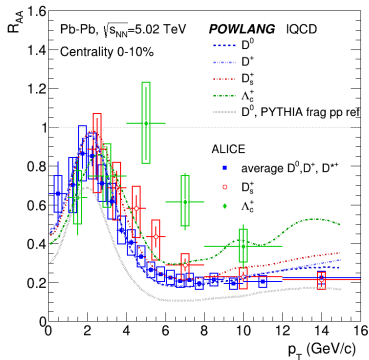
- Differences between minimum-bias and high-multiplicity results only due to longer time spent in the fireball ($\langle \tau_H \rangle^{\text{mb}} \approx 1.95$ fm/c vs $\langle \tau_H \rangle^{\text{hm}} \approx 2.92$ fm/c)
- Mass ordering at low p_T ($M_{qq} > M_q$)
- **Sizable fraction of v_2 acquired at hadronization**

Relevance to quantify nuclear effects



- Slope of the spectra in pp collisions better described including medium effects

Relevance to quantify nuclear effects



- Slope of the spectra in pp collisions better described including medium effects
- Inclusion of medium effects in minimum-bias pp benchmark fundamental to better describe charmed hadron R_{AA} , both the radial-flow peak and the species dependence

To summarize

- What we **learnt**: A rich set of experimental data shows evidence of at least *partial kinetic equilibration of charm* in heavy-ion collisions and of the *breaking of universality of the hadronization process*;
- What is only **partially known**: the transport coefficients
 - **lattice-QCD calculations** getting better (e.g. from $N_f = 0$ to $N_f = 2 + 1$), BUT *unavoidable systematic uncertainties*;
 - **theory-to-data comparison** (e.g. Bayesian analysis) has to focus on *kinematic windows where transport equations are reliable* (e.g. beauty at low- p_T): just a matter of time to improve;
 - systematic uncertainty from **hadronization**: if recombination occurs, *same flow of HF hadrons with smaller partonic transport coefficients*. *Modified hadrochemistry* (integrated yields) allows one to quantify the relevance of recombination, both in pp and AA.
- The same **Local Color-Neutralization (LCN) model** developed to describe medium-modification of HF hadronization in AA collisions has been applied to the pp case. Consistent description of several HF observables: shape of the p_T -distributions, enhanced baryon-to-meson ratio, charmed-hadron R_{AA} and non-vanishing v_2 coefficient,