Relativistic corrections and three-nucleon forces in neutron star matter

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Introduction and Motivations

The equation of state (EOS) of cold nuclear matter in beta-equilibrium is necessary to solve the Tolman–Oppenheimer–Volkoff (TOV) equations and to compute macroscopic properties of **Neutron Stars** (NS) such as mass, radius, and tidal deformability.

- To describe non-equilibrium and finite-temperature phenomena—such as neutrino emissivity, transport coefficients, and weak response—knowledge beyond the EOS is needed.
- The Correlated Basis Function (**CBF**) **effective interaction** offers a consistent framework for these calculations. This approach is rooted in non-relativistic nuclear many-body theory (**NMBT**). It was first introduced by Cowell and Pandharipande [1], then further developed and refined in recent years [2][3].
- Within the NMBT formalism, the role of **relativistic corrections** has long been a subject of debate. Their importance for neutron star physics, especially in light of recent multi-messenger observations, has been recently addressed.

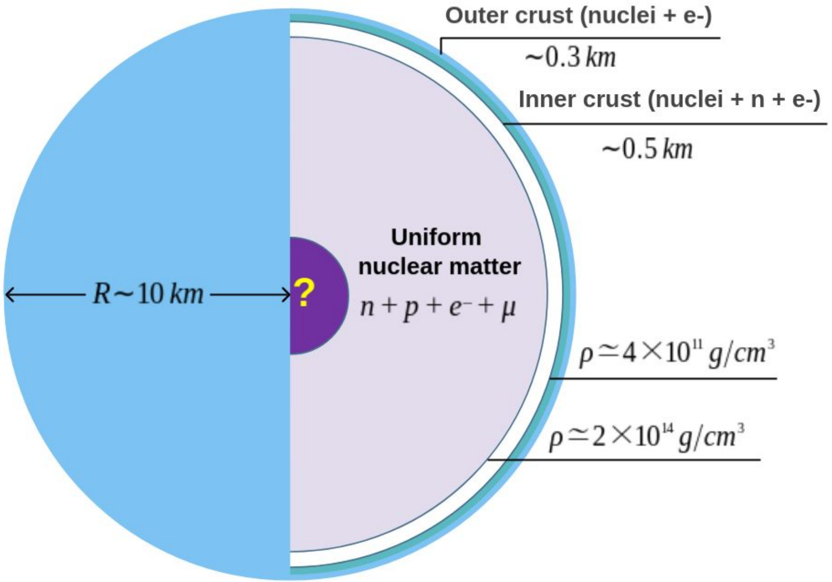
In this presentation, we will discuss the role of relativistic corrections and present the results of a study aimed at incorporating such corrections into the CBF effective interaction formalism.

[1] S. T. Cowell and V. R. Pandharipande, Phys. Rev. C 67(2003) 035504
[2] O. Benhar and A. Lovato,Phys. Rev. C 96 (2017) 054301
[3] O. Benhar, A. Lovato, and G. Camelio,The Astrophysical Journal 939 no. 1,(Nov, 2022) 52

Neutron Stars

Neutron Stars (NSs) are extremely compact objects, with masses as large as two solar masses and with radii of about ten kilometers.

In the NS interior, matter reaches extreme conditions, impossible for Earth-based experiments.



In the innermost region

$$\rho > \rho_0$$

$$\rho_0 = 2.67 \times 10^{14}\, g/cm^3\, (0.16\, fm^{-3})$$

$$T \sim 10^9\, K << T_F$$

$$T_F \sim 10^{12}\, K$$

NSs provide a unique opportunity to investigate the properties of nuclear matter at high density, low temperature and large neutron excess.

Nuclear Dynamics

Non-relativistic nuclear many body theory (NMBT). Nuclear matter is described as an infinite system of point-like nucleons, interacting through nucleon-nucleon (NN) and three-nucleon (NNN) potentials.

$$\mathcal{H} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

The equation of state of cold nuclear matter can be carried out by computing the ground state energy of the system

$$E_0 = \langle \psi_0 | \mathcal{H} | \psi_0 \rangle$$

The calculation of such expectation value is not trivial. The **strong repulsive core** of NN interactions forbid a treatment in standard perturbation theory ➜ More sophisticated approaches, such as Variational Chain Summation techniques, G-Matrix perturbation theory or Monte Carlo methods are required.

In this work we focus on phenomenological Hamiltonians and the Correlated Basis Function (CBF) variational approach, that will be used to define a density dependent effective interaction.

Nucleon-Nucleon Potential

Realistic phenomenological NN potentials can be obtained by fitting the large body of **data coming from two-nucleon systems**, both in bound and scattering states. The **Argonne V18 (AV18)** potential is the perfect example of this kind of models.

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O_{ij}^p$$

Where:

$$O_{ij}^{p\leq 6} = [1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}] \otimes [1, \vec{\tau}_i \cdot \vec{\tau}_j]$$

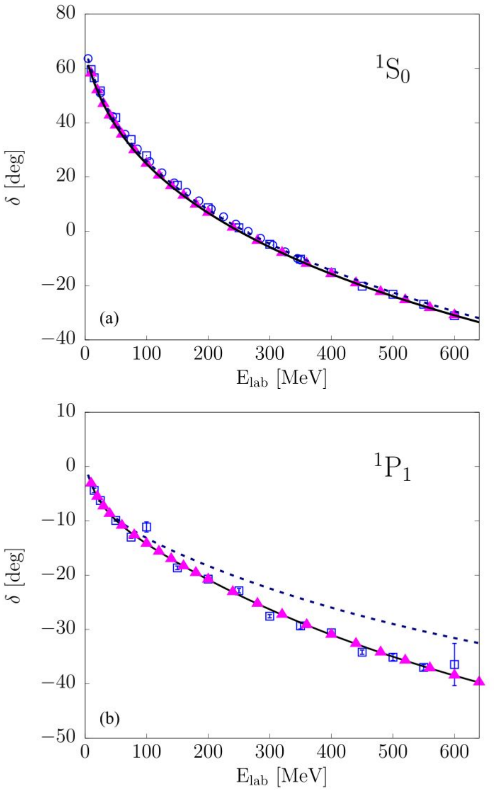
$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) - (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

$$O_{ij}^{p=7,8} = (\vec{L} \cdot \vec{S}) \otimes [1, \vec{\tau}_i \cdot \vec{\tau}_j]$$

The operators with p=9, … , 18 account for small corrections related to other non-static terms and small violations of charge symmetry and charge independence.

Very good description of NN scattering data up to energies of 600 MeV in the laboratory frame
➔ scattering in degenerate matter up to 4 ϱ_0
➔ reliable at NS densities.

A simplified version of this potential is represented by the **Argonne V6' (AV6P) model** ➔ Reprojection of the full AV18 on the first six operators.
➔ Quite accurate in the context of neutron star matter.

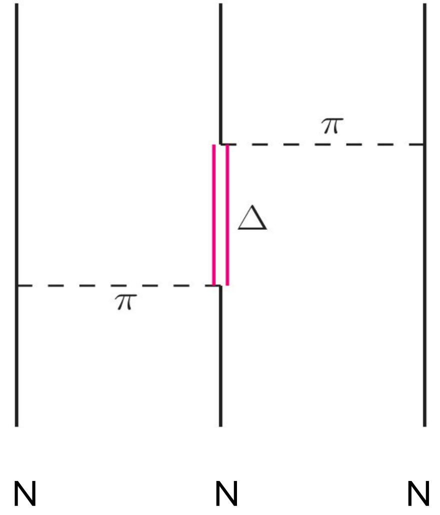


Three-Nucleon Potential

Three-nucleon interactions must be introduced in order to account for the internal structure of nucleons.

We take as reference the **Urbana IX** (UIX) phenomenological model, often used in conjunction with the Argonne potentials. It comprises two terms.

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$



The **coupling constants** involved in the definition of this potential are adjusted in order to independently reproduce the binding energies of ^3H and ^4He, and the correct value of the nuclear saturation density respectively.

Relativistic Boost Corrections

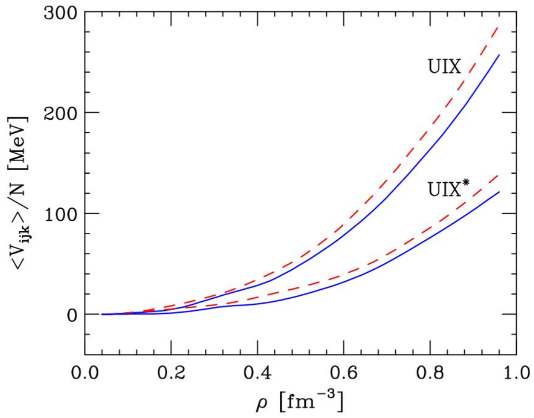
- NN potentials are largely determined by a fit to NN scattering data → The NN potential is defined in the rest frame of two interacting nucleons.
- To consistently describe NN interactions in a locally inertial frame associated with a NS, the NN potential must be boosted to a frame in which the total momentum of the interacting pair is different from zero.

The Hamiltonian can be modified by introducing a **boost-correction** term, which depends on the total momentum of the interacting pair

$$\mathcal{H}^* = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} (v_{ij} + \delta v(\vec{P}_{ij})) + \sum_{i<j<k} V^*_{ijk}$$

$$\delta v_{ij}(\mathbf{P}_{ij}) = -\frac{\mathbf{P}_{ij}^2}{8m^2} v_{ij} + \frac{1}{8m^2} \mathbf{P}_{ij} \cdot \mathbf{r}_{ij} \mathbf{P}_{ij} \cdot \boldsymbol{\nabla} v_{ij}.$$

Calculations of the properties of light nuclei have shown that **the presence of the boost interaction accounts for the 37% of the the repulsive contribution of the NNN interaction.**



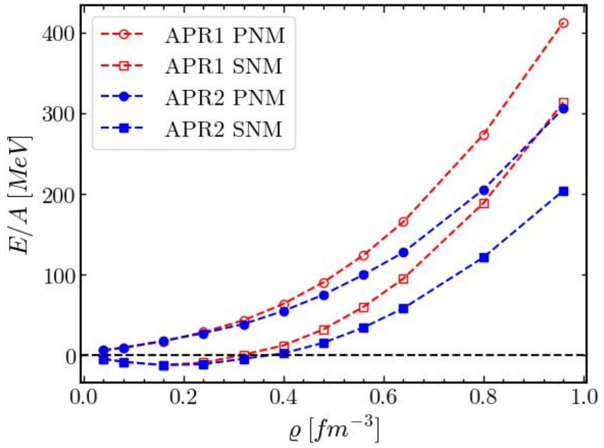
L. Forest, V. R. Pandharipande, and A. Arriaga, Phys. Rev. C 60, 014002 (1999)

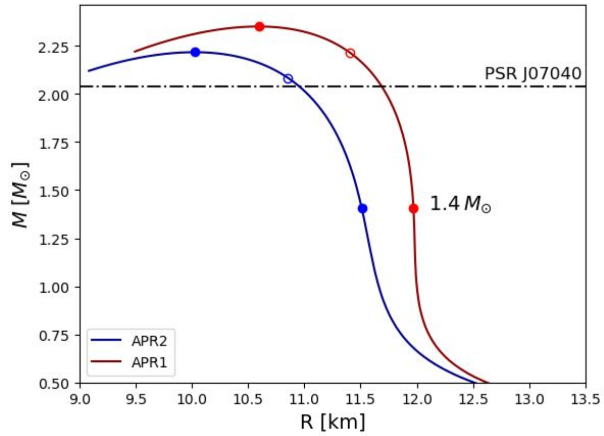
The APR equations of state

The two EOS models proposed by Akmal, Pandharipande and Revenhall, **extensively used for NS applications** and known as the **APR** models, incorporate all the concepts introduced so far. The two models, referred to as APR1 and APR2, differ by the presence of relativistic corrections.

- APR1: Phenomenological Hamiltonian Based on the AV18+UIX combination of nuclear potentials
- APR2: AV18+δv+UIX* Hamiltonian. The inclusion of relativistic corrections is done by adding the contribution δv+(UIX*-UIX) at first order in perturbation theory to the APR1 variational energies.
- The EOS is computed within the variational framework with a correlated basis function and FHNC/SOC summation techniques.

The *relativistic* APR2 model results in a *softer* EOS. Since boost interactions are repulsive, the softening has to be ascribed to the lower three-nucleon repulsion, which provides the dominant contribution at high density.





A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998)

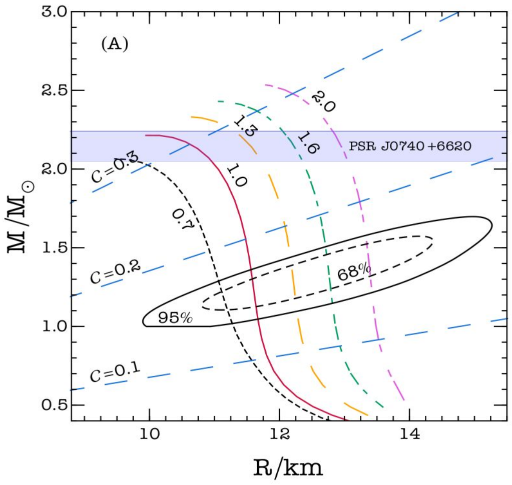
Constraining three-body forces with multimessenger data

In recent works we addressed the possibility of inferring quantitative information about three-nucleon forces at high density. To this purpose we have generated **a set of APR-like EOSs** by re-parametrizing the expectation value of the NNN repulsion according to:

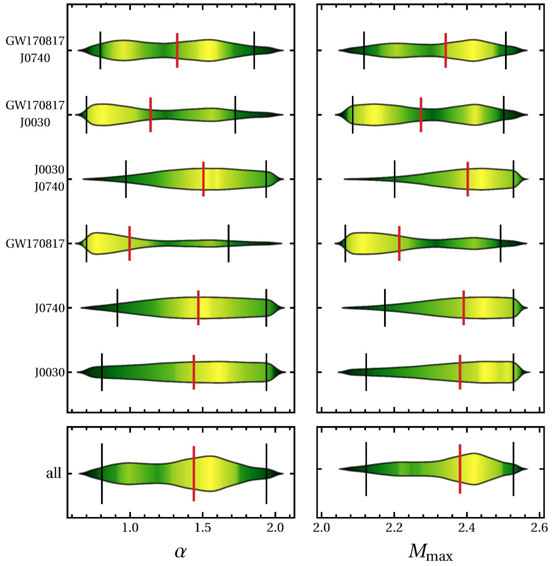
$$\langle V^R_{ijk} \rangle \rightarrow \alpha \langle V^R_{ijk} \rangle$$

with α=1 we recover the APR2 EOS.

NICER PSR J0030+0451



Bayesian parameter estimation



The multimessenger posterior appears to be dominated by the EM measurements yielding $\alpha \simeq 1.4$.

More repulsive NNN forces
➔ smaller relativistic contributions.

A. Maselli, A. Sabatucci and O. Benhar, Phys. Rev. C 103, 065804, 2021
A. Sabatucci, O. Benhar, A. Maselli and C. Pacilio Phys. Rev. D **106**, 083010 – 2022

CBF Effective Interaction

A convenient framework to perform perturbative calculations of several nuclear matter properties, can be obtained by defining a density dependent **effective interaction**.

$$\langle\Psi_0|\mathcal{H}|\Psi_0\rangle = T_F + \langle\Phi_0|\sum_{i<j} v_{ij}^{\text{eff}}|\Phi_0\rangle$$

The analytic expression of the effective interaction is derived by performing a **cluster expansion** of the left and side and truncating at the lowest order. Then the correlation parameters are determined by enforcing the above identity.

$$\langle\Psi_0|\mathcal{H}|\Psi_0\rangle = T_F + (\Delta E)|_{2b} + \ldots$$

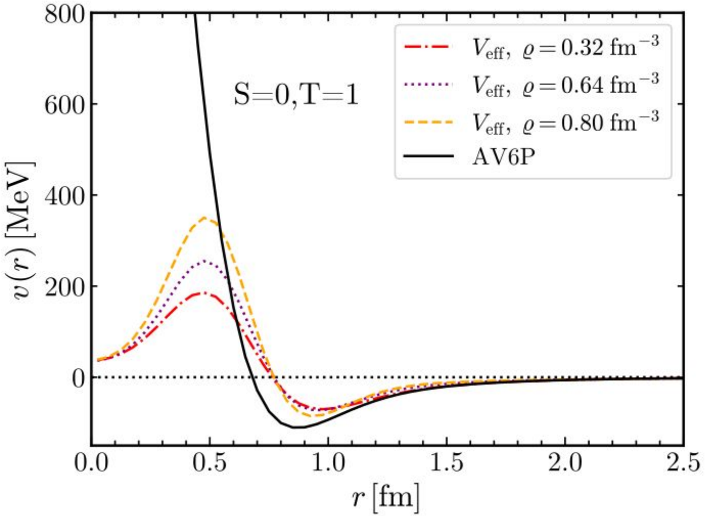
$$|\Psi_0\rangle = \mathcal{F}|\Phi_0\rangle$$

$$\mathcal{F} = \mathcal{S}\prod_{i<j} F_{ij}$$

$$F_{ij} = \sum_p f^p(r_{ij})O_{ij}^p$$

$$v_{ij}^{\text{eff}} = F_{ij}v_{ij}F_{ij} - \frac{1}{m}\nabla F_{ij}\cdot\nabla F_{ij}$$

$$v_{ij}^{\text{eff}} = \sum_p v^{\text{eff},p}(r_{ij})O_{ij}^p$$



The effective Interaction does not have the strong repulsive core typical of bare NN interactions.

CBF Effective Interaction

With this procedure we obtain an effective Hamiltonian that can be treated in perturbation theory with respect to the Fermi gas basis and that reproduces the ground-state energies per particle, computed with accurate many-body methods, at first order in perturbation theory.

State of the art models include three-nucleon forces in their definition, and have been extensively used to compute several matter properties of nuclear matter.

$$\langle V_{\rm eff} \rangle = \langle \mathcal{F}^\dagger [T, \mathcal{F}] \rangle|_{2b} + \langle \mathcal{F}^\dagger V_{NN} \mathcal{F} \rangle|_{2b} + \langle \mathcal{F}^\dagger V_{NNN} \mathcal{F} \rangle|_{3b}.$$

Applications

- Compute the properties of matter at arbitrary temperature and proton fraction
- Compute single particle properties, such as the single particle spectrum and nucleon effective mass
- Compute transport properties
- Compute nuclear matter response to weak interactions
- Compute Neutrino emissivity and mean free path relevant for Neutron Star cooling

Some references

O. Benhar and M. Valli, Phys. Rev. Lett. 99, 232501 (2007).
A. Lovato, C. Losa, and O. Benhar, Nucl. Phys. A 901, 22 (2013)
O. Benhar, A. Cipollone, and A. Loreti, Phys. Rev. C 87, 014601 (2013)
A. Lovato, O. Benhar, S. Gandolfi, and C. Losa, Phys. Rev. C 89, 025804 (2014)
A. Mecca, A. Lovato, O. Benhar, and A. Polls, Phys. Rev. C 91, 034325 (2015).
A. Mecca, A. Lovato, O. Benhar, and A. Polls, Phys. Rev. C 93, 035802 (2016).
O. Benhar and A. Lovato, Phys. Rev. C 96, 054301 (2017)
L. Tonetto and O. Benhar, Phys. Rev. D 106, 103020 (2022).

Relativistic corrections to the CBF Effective interaction

PHYSICAL REVIEW C **110**, 055801 (2024)

Relativistic corrections to the correlated basis function effective nuclear Hamiltonian

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In our last paper, we addressed the inclusion of relativistic corrections in the CBF effective interaction (CBF-EI) formalism.

We have defined our CBF-EIs by reproducing the variational energies of the APR EOSs, which are the only models in the literature based on NMBT and the CBF variational approach, and which also account for boost corrections. We remark that the difference between the two APR models can be expressed as

$$\left(\frac{E}{A}\right)_{\text{APR2}} = \left(\frac{E}{A}\right)_{\text{APR1}} + \frac{1}{A}\langle \mathcal{F}^{\dagger}[\delta V + (\gamma - 1)V^{R}]\mathcal{F}\rangle$$

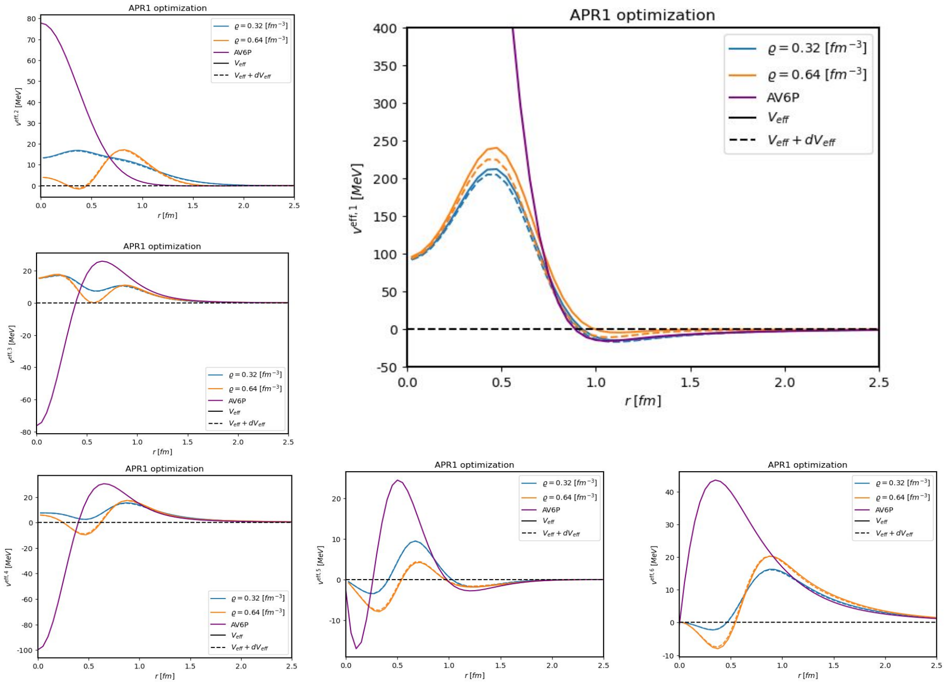
We have considered the effect of boosts in the EI both perturbatively and at the operatorial level.

Relativistic corrections to CBF-EI

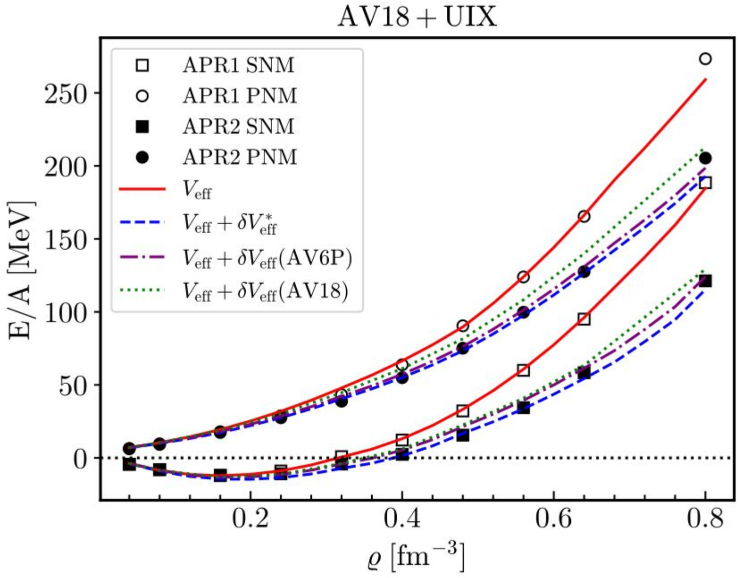
$$V_{\text{eff}} \to V_{\text{eff}} + \delta V_{\text{eff}}$$

$$\langle \delta V_{\text{eff}} \rangle = \langle \mathcal{F}^\dagger \delta V \mathcal{F} \rangle|_{2b} + (\gamma - 1)\langle \mathcal{F}^\dagger V^R \mathcal{F} \rangle|_{3b}$$

$$\langle \delta V^*_{\text{eff}} \rangle = (\gamma - 1)\langle \mathcal{F}^\dagger V^R \mathcal{F} \rangle|_{3b}$$



Remarkably, the energies computed with the effective interaction plus a perturbative correction are in very good agreement with the APR2 variational energies. This provides valuable insight into the reliability of the formalism. Furthermore, it is clear that the dominant contribution comes from the rescaling of the three-nucleon repulsion, whereas the contribution associated with relativistic boosts appears to be marginal.



Boost correction dependence on the NN potential

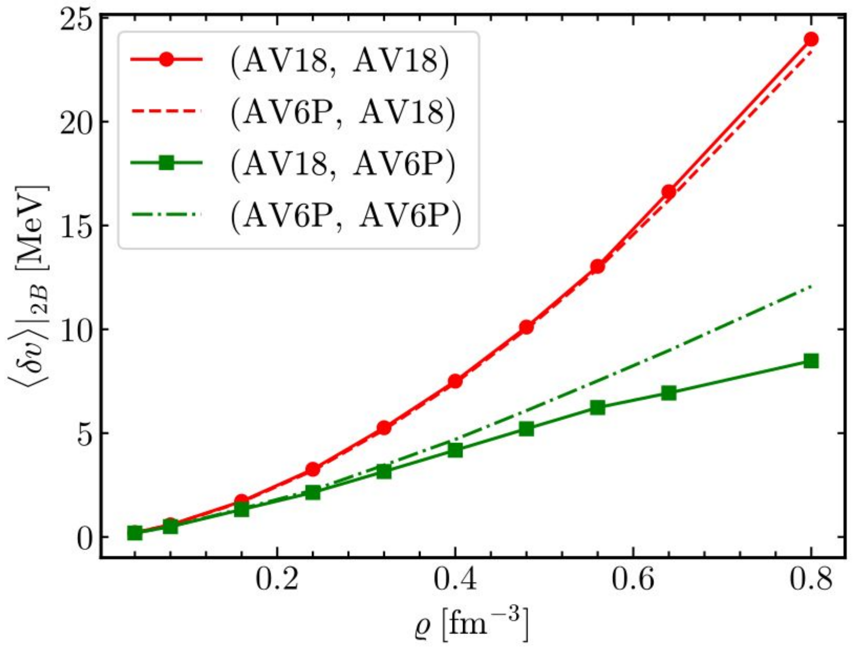
In the original work of Akmal *et al.* ➔ only the static components of the AV18 potential was included in calculation of the boost interaction.

In our calculations ➔ consistently used the AV6P potential.

We found appreciable discrepancies between the boost corrections computed with this two different models.

Lower boost contribution ➔ main relativistic effect on the high density EOS is associated with the softening of NNN repulsion.

In particular, it could be argued that the smaller relativistic boosts associated with AV6P may entail a stiffer NNN repulsion. But this statement should be taken carefully.



Summary

- The treatment of relativistic corrections is crucial in relation of their strong interplay with NNN forces.
- The work discussed represents a first step toward the development of an effective interaction that incorporates relativistic corrections.
- Boost corrections strongly depend on the NN potential, and appreciable differences have been found between the first six components of AV18 (used in the APR calculations) and AV6P (used in our calculations) at the two-body cluster level.
- Neutron Star observations seem to point towards more repulsive NNN forces. At first glance, this observation appears to align with lower boost corrections.

Future Perspectives

- Apply this effective interaction to compute finite temperature and single particle properties (ongoing).
- Derive a more consistent effective interaction model by using new target values → new calculations with a boost corrected AV6P+UIX (UIX*) Hamiltonian and state-of-the art many-body methods.
- Better understanding of the interplay between relativistic boost corrections and NNN forces in a broader density range, from nuclei to neutron stars
- Formal improvements are also possible. Three-body cluster contributions of the boost interaction may be significant. Therefore a fully quantitative assessment of their role will certainly be needed.

Thank you!

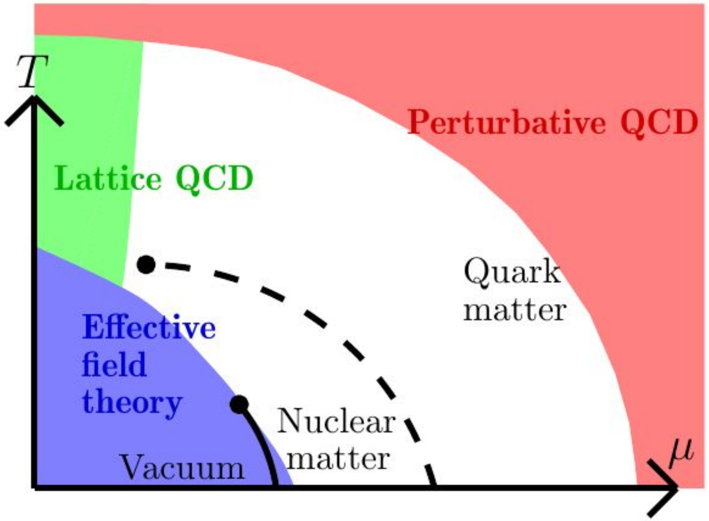
Backup

Outline

- Introduction and motivations
- Framework
  - Non-relativistic Nuclear Many-Body Theory
  - Phenomenological Hamiltonians
  - Variational Principle
  - Relativistic Corrections
- The APR equation of state
- The correlated basis function (CBF) effective interaction
- Relativistic corrections to the CBF effective interaction
  - Results and discussion
- Summary and Perspectives

QCD Phase Diagram

Quantum Chromodynamics (QCD) is well established as the fundamental theory of strong interactions...



Matti Jarvinen, arXiv:2110.08281 [hep-ph]

...however, because of its non-perturbative nature at low energy scales and the occurrence of color confinement, a QCD description of **dense and cold nuclear matter** presents both conceptual and technical difficulties.

Neutron Stars

High compactness ⟹ *General Relativity*.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Solving the Einstein equations leads to the Tolman Oppenheimer and Volkov (TOV) equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$T^{\mu\nu} = u^\mu u^\nu(\epsilon + P) + g_{\mu\nu}P$$

TOV →

$$\begin{cases} \dfrac{dM(r)}{dr} = 4\pi r^2\epsilon(r) \\ \dfrac{dP}{dr} = -\dfrac{[\epsilon(r) + P(r)]\,[M(r) + 4\pi r^3 P(r)]}{r\,[r - 2M(r)]} \end{cases}$$

In order to solve TOV equations we have to specify the Equation Of State (EOS) of neutron star matter

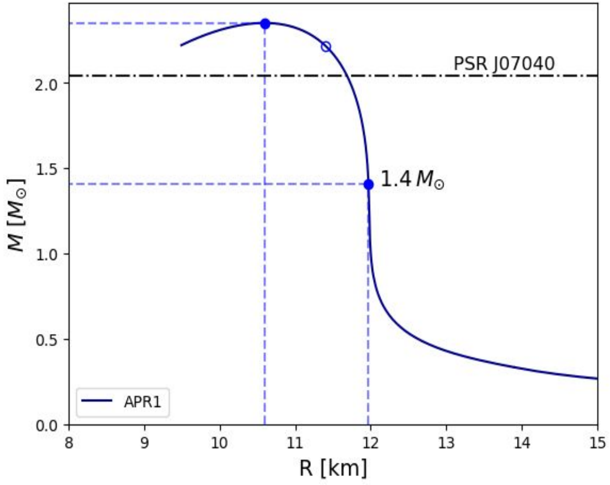
$$P = P(\epsilon)$$

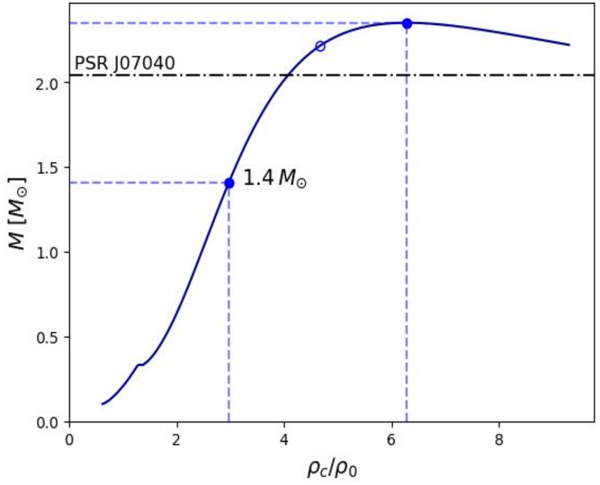
Neutron Stars

Non-interacting nucleons → $M_{\max} \sim 0.8\, M_\odot$

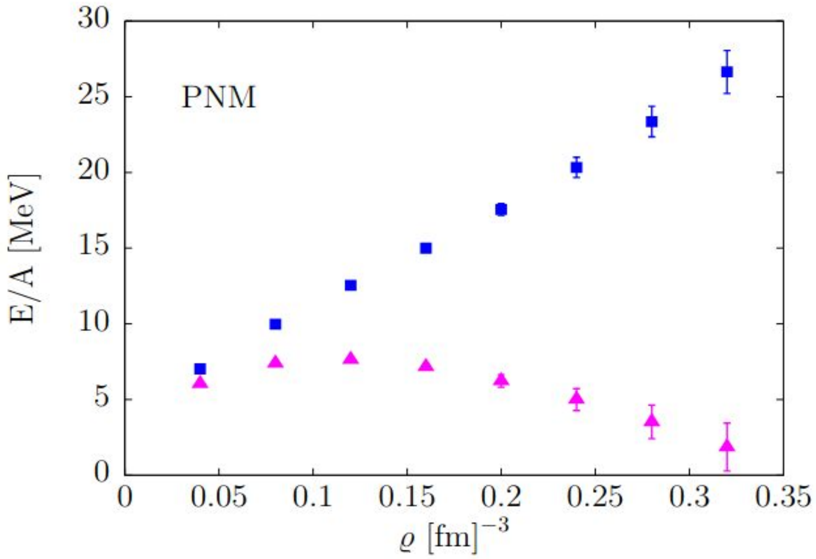
BUT we observe → $M \sim 1.4\, M_\odot$

Therefore we have to keep into account **interactions between nucleons!**





PNM calculations of AV6P vs AV18T



Variational Principle and Correlated Basis Function (CBF)

One way to circumvent the non-perturbative nature of the NN interaction consists in exploiting the variational principle and short range correlations

$$E_V = \frac{\langle \Psi_T | \mathcal{H} | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq \langle \Psi_0 | \mathcal{H} | \Psi_0 \rangle$$

We can define a trial ground state as

$$|\Psi_T\rangle = \mathcal{F} |\Phi_0\rangle$$

with $$\mathcal{F} = \mathcal{S} \prod_{i<j} F_{ij}$$

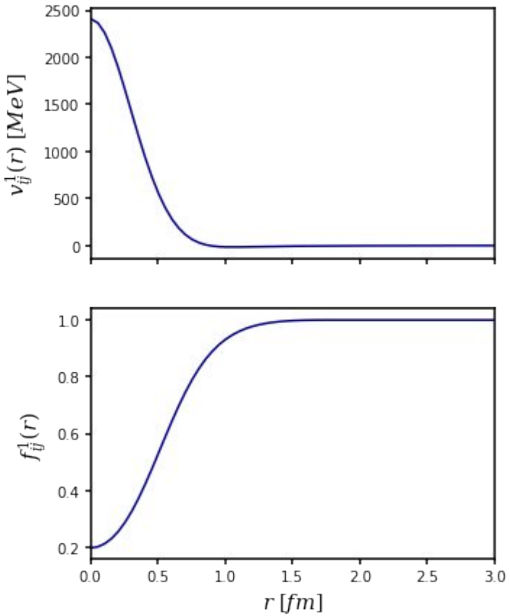
and $$F_{ij} = \sum_p f^p(r_{ij}) O^p_{ij}$$

The "f ^{p}" functions are pair correlation functions whose radial shape is determined by the minimization of the trial ground state.

$$\frac{\delta E_V}{\delta f^p} = 0.$$

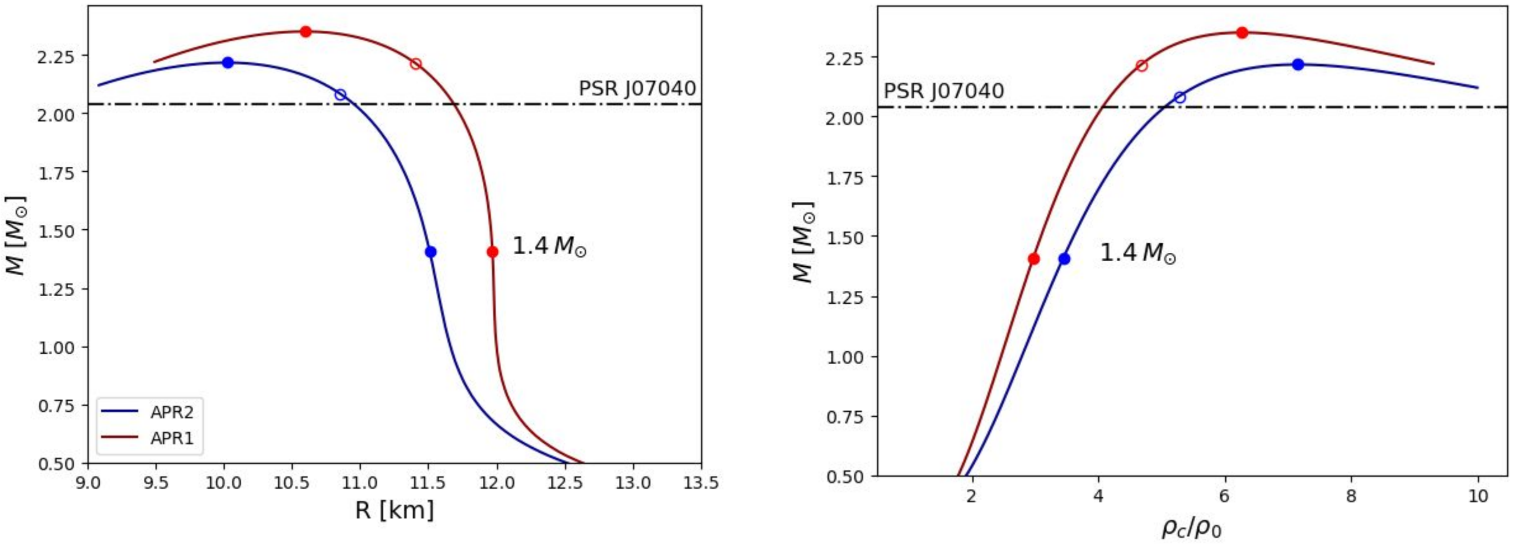
$$f^p(r \geq d^p) = \delta_{1p}$$

$$\left. \frac{df^p(r)}{dr} \right|_{d^p} = 0.$$



The APR equations of state

Mass-Radius and Mass-Central Density diagrams for the two APR models



The relativistic APR2 model results in a softer EOS. Since boost interactions are repulsive it is clear that the softening has to be ascribed to the lower three-nucleon repulsion, which provides the dominant contribution at high density.

Intermediate Summary

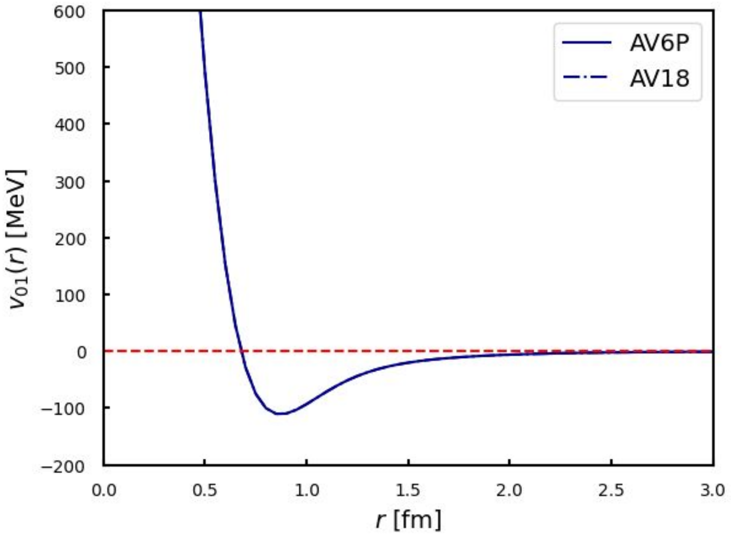
- Nuclear Many Body theory attempts to describe every aspect of nuclear matter starting from a microscopic dynamics reproducing the available nuclear data.
- Effective Interaction formalism allows to perform perturbative calculations in nuclear matter with respect to the Fermi gas basis.
- We have defined an effective interaction reproducing the APR energies per nucleon.
- We have developed a framework to also include boost corrections in the analytic expression of the effective interaction.
- Boost corrections strongly affect the NNN repulsion which in turn provides the dominant contribution to the EOS at high density.

NN Interaction

$$v_{ij} = \sum_p v^p(r_{ij}) O^p_{ij}$$

NN interaction determined by reproducing nucleon-nucleon scattering and bound states.

$$O^p_{ij} = 1,\ (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j),\ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ S_{ij},\ S_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)$$



The labels in the plot refer to two NN potentials of the Argonne family, respectively the **Argonne v6**' (AV6P) and **Argonne v18** (AV18) potentials. These two models differ by their operatorial structure but are designed to be identical in the S=0, T=1 channel, as we can see from the plot where they are perfectly overlapped.

The strong repulsive core doesn't allow for a treatment in standard perturbation theory.

Given $$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij}$$ we CAN'T compute

$$E_0 = T_F + \Delta E$$

with

$$\Delta E = \langle \Phi_0 | \sum_{i<j} v_{ij} | \Phi_0 \rangle$$

$$T_F = \langle \Phi_0 | \sum_i \frac{p_i^2}{2m} | \Phi_0 \rangle$$

where $|\mathbf{\Phi}_0 >$ is the ground state of a non-interacting Fermi gas.

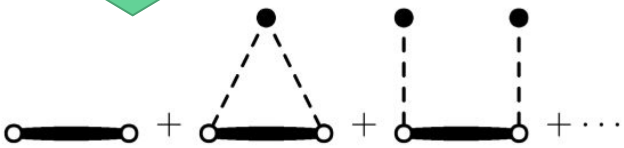
Cluster Expansion (Just to understand how it works)

Given the short range nature of nuclear correlations, it is possible to write the expectation values over full correlated states as a sum of terms involving an increasing number of correlated bodies (clusters).

$$\mathcal{F} \equiv F = \prod_{i<j} f_{ij} \quad \longrightarrow \quad \langle \Psi_T | v_{12} | \Psi_T \rangle = \langle \Phi_0 | \prod_{i<j} f_{ij} v_{12} \prod_{i<j} f_{ij} | \Phi_0 \rangle = \langle \Phi_0 | f_{12} v_{12} f_{12} \prod_{i<j \neq 1,2} f_{ij}^2 | \Phi_0 \rangle$$

$$f_{ij}^2 \equiv 1 + h_{ij} \quad \longrightarrow \quad \prod_{i<j} f_{ij}^2 = \prod_{i<j} (1 + h_{ij}) = 1 + \sum_{i<j} h_{ij} + \sum_{i<j<k} (h_{ij} h_{jk} + h_{ij} h_{jk} h_{ki}) + \dots$$

$$\langle \Phi_0 | F v_{12} F | \Phi_0 \rangle =$$



$$\langle \Phi_0 | F v_{12} F | \Phi_0 \rangle = \Delta E|_{2b} + \Delta E|_{3b} + \dots$$

Selected classes of diagrams can be summed to all orders in the cluster expansion by means of a set of integral equations known as Fermi Hyper-Netted Chain (FHNC) equations.

Relativistic Boost Corrections

Leslie L. Foldy.Phys. Rev., 122:275–288, 1961;
J. L. Forest et al Phys. Rev. C 52, 568 – 1995.

Boost corrections could be derived by imposing **relativistic covariance** on our system.

Relativistic covariance can be implemented by requiring the Hilbert space of our theory to be a representation of the Poincaré group. By *imposing the commutation relations of the* ***Poincaré algebra***, and performing an *expansion in powers of 1/m* we can carry out the explicit expression of the boost interaction.

$$
\begin{aligned}
&[P^i, P^j] = [H, P^i] = [J^i, H] = 0, \\
&[K^i, H] = iP^i, \qquad [J^i, J^j] = i\epsilon_{ijk}J^k, \\
&[K^i, P^j] = i\delta_{ij}H, \quad [J^i, K^i] = i\epsilon_{ijk}K^k, \\
&[J^i, P^j] = i\epsilon_{ijk}P^k, \quad [K^i, K^j] = -i\epsilon_{ijk}J^k.
\end{aligned}
$$

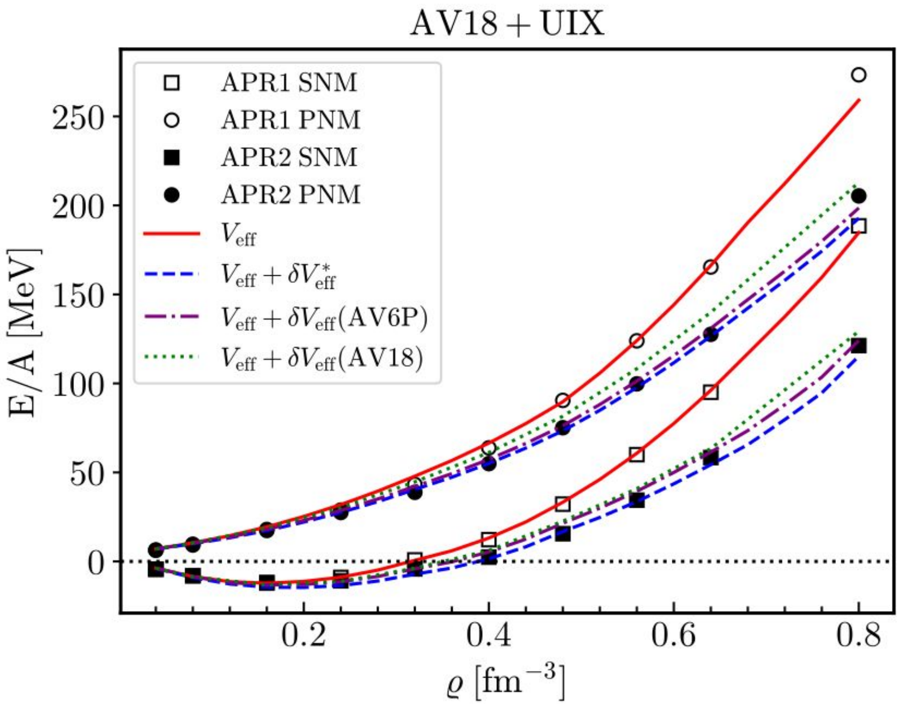
$$
\delta v_{ij}(\mathbf{P}_{ij}, \mathbf{r}_{ij}) = -\frac{\mathbf{P}_{ij}^2}{8m^2} v_{ij} + \frac{1}{8m^2}\left[(\mathbf{P}_{ij}\cdot\mathbf{r}_{ij})(\mathbf{P}_{ij}\cdot\boldsymbol{\nabla}), v_{ij}\right] + \frac{1}{8m^2}\left[(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\times(\mathbf{P}_{ij}\cdot\boldsymbol{\nabla}), v_{ij}\right]
$$

Perturbative relativistic corrections

$$\delta V_{\text{eff}} = \langle \mathcal{F}^\dagger [\delta V + (\gamma - 1) V^R] \mathcal{F} \rangle$$

$$\delta V^*_{\text{eff}} = (\gamma - 1) \langle \mathcal{F}^\dagger V^R \mathcal{F} \rangle$$

Remarkably, the energies computed with the effective interaction plus a perturbative correction, i.e. without re-optimizing the correlation parameters, are in very good agreement with the APR2 variational energies. This is a good insight about the reliability of the formalism. Furthermore, it clearly appears that the dominant contribution comes from the rescaling of three-nucleon repulsion, whereas, the one associated with relativistic boosts, appears to be marginal.



Relativistic correction to CBF Effective interaction

The effective interaction without boost corrections and including also a contribution from three-nucleon forces can be expressed as

$$\langle\Phi_0|V_{\rm eff}|\Phi_0\rangle = \langle\Psi_0|T - T_F|\Psi_0\rangle|_{2b} + \langle\Psi_0|V_{\rm NN}|\Psi_0\rangle|_{2b} + \langle\Psi_0|V_{\rm NNN}|\Psi_0\rangle|_{3b}$$

where T_F is the energy of a non-interacting Fermi gas—$T_F = \langle\Phi_0|T|\Phi_0\rangle$—and

$$V_{\rm eff} = \sum_{i<j} v_{ij}^{\rm eff}, \ \text{with} \ v_{ij}^{\rm eff} = \sum_{p=1}^{6} v^{{\rm eff},p}(r_{ij})O_{ij}^p,$$

$$T = \sum_i \frac{p_i^2}{2m}, \ V_{\rm NN} = \sum_{i<j} v_{ij}, \ \text{and} \ V_{\rm NNN} = \sum_{i<j<k} V_{ijk}.$$

The boost contribution to the effective interaction will be accounted for by considering

$$V_{\rm eff} \to V_{\rm eff} + \delta V_{\rm eff}$$

with

$$\delta V_{\rm eff} = \sum_{i<j} \delta v_{ij}^{\rm eff}(k_F), \ \text{and} \ \delta v_{ij}^{\rm eff}(k_F) \equiv \sum_{p=1}^{6} \delta v^{{\rm eff},p}(k_F, r_{ij})O_{ij}^p.$$

Relativistic correction to CBF Effective interaction

The correction δV_{eff} can be derived by considering a two-body cluster of the boost and a three-body cluster of the modified three-nucleon force.

$$\langle\Phi_0|\delta V_{\text{eff}}|\Phi_0\rangle = \langle\Psi_0|\delta V|\Psi_0\rangle|_{2b} + (\gamma - 1)\langle\Psi_0|V^R_{NNN}|\Psi_0\rangle|_{3b}$$

with

$$\delta V = \sum_{i<j} \delta v_{ij}(\mathbf{P}_{ij})$$

and

$$\delta v_{ij}(\mathbf{P}_{ij}) = -\frac{\mathbf{P}^2_{ij}}{8m^2} v_{ij} + \frac{1}{8m^2}\mathbf{P}_{ij}\cdot\mathbf{r}_{ij}\mathbf{P}_{ij}\cdot\boldsymbol{\nabla} v_{ij}.$$

We have considered different cases:

i) The effective correlation parameters are computed by optimizing the V_{eff} interaction over non-relativistic target values (APR1 EOS) and then the correction δV_{eff} is added perturbatively without changing the correlation parameters again. We will refer to this effective interaction as $V_{\text{eff}} + \delta V_{\text{eff}}$.

ii) We have calculated the correlation parameters for the full relativistic effective interaction by reproducing the relativistic calculations (APR2), i.e. by imposing the identity

$$E_{\text{APR2}} = T_F + \langle\Psi_0|V_{\text{eff}} + \delta V_{\text{eff}}|\Psi_0\rangle.$$

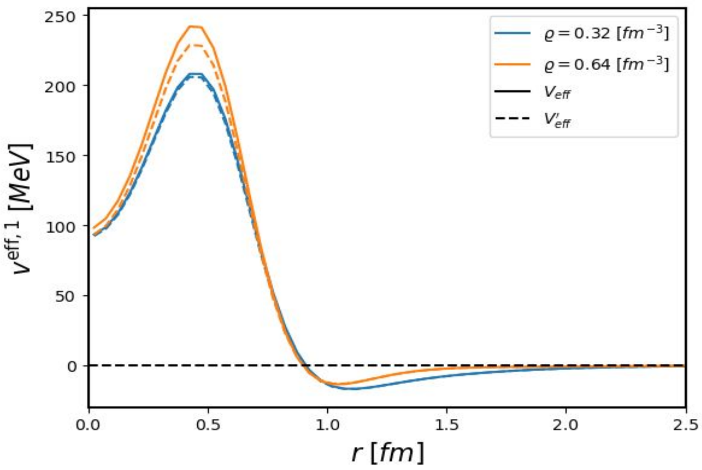
We will refer to this effective interaction as V'_{eff}.

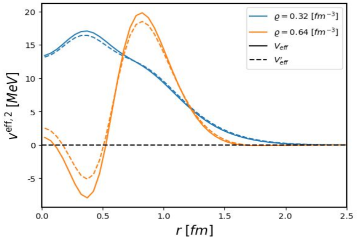
Operatorial Relativistic Corrections

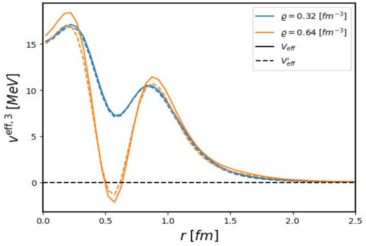
$$\langle V_{\rm eff} \rangle = \langle \mathcal{F}^\dagger [T, \mathcal{F}] \rangle|_{2b} + \langle \mathcal{F}^\dagger V_{NN} \mathcal{F} \rangle|_{2b} + \langle \mathcal{F}^\dagger V^*_{NNN} \mathcal{F} \rangle|_{3b},$$

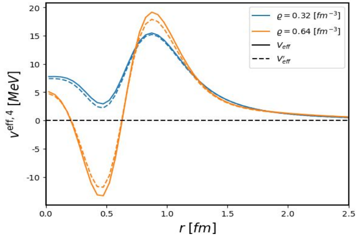
$$\langle V'_{\rm eff} \rangle = \langle \mathcal{F}^\dagger [T, \mathcal{F}] \rangle|_{2b} + \langle \mathcal{F}^\dagger (V_{NN} + \delta V) \mathcal{F} \rangle|_{2} + \langle \mathcal{F}^\dagger V^*_{NNN} \mathcal{F} \rangle|_{3b}$$

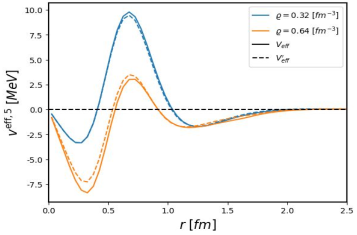
- Assess the impact of boost corrections on the functional form of the effective interaction.
- The effect seems to be small.

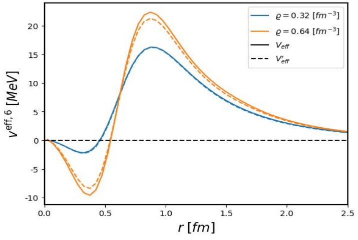


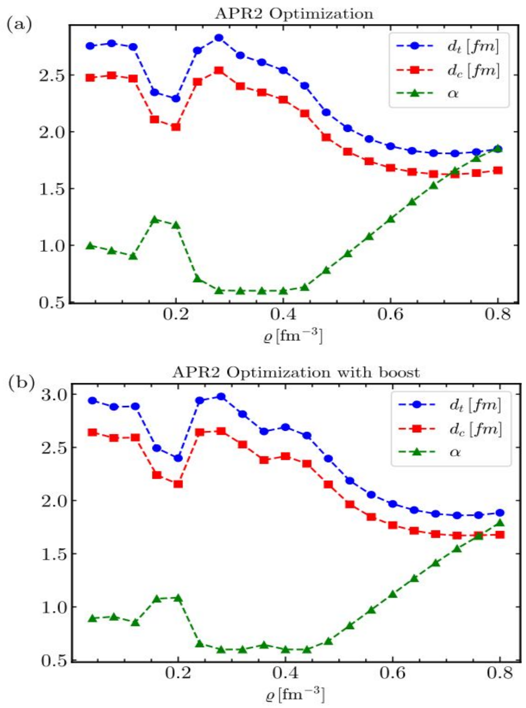








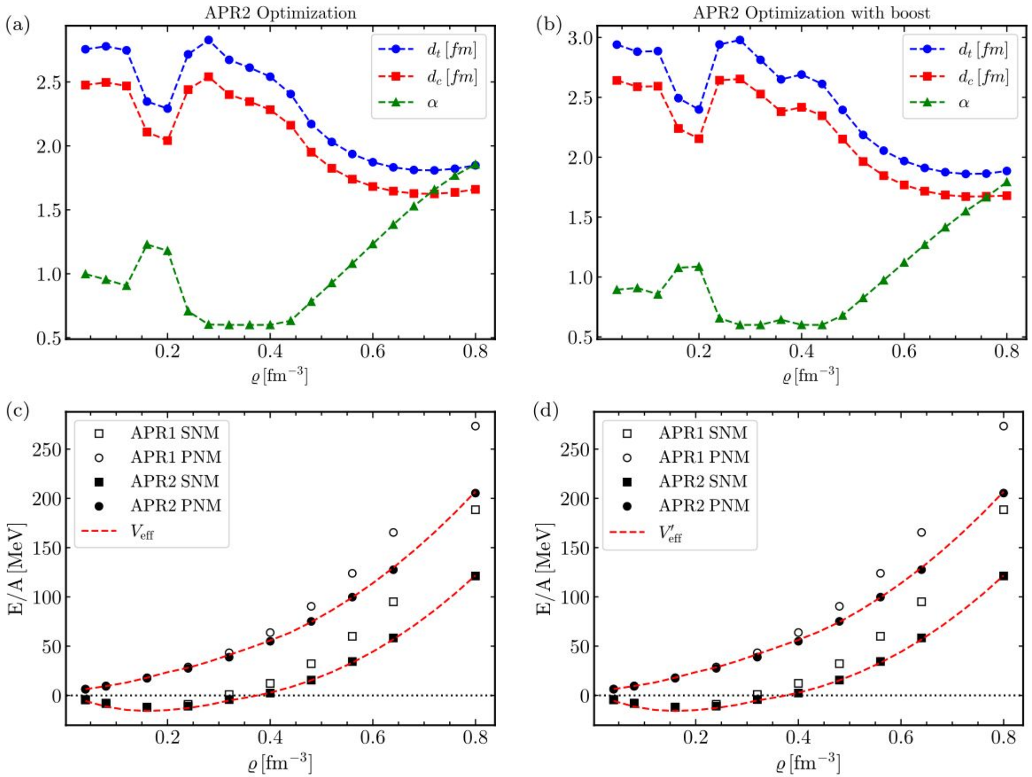


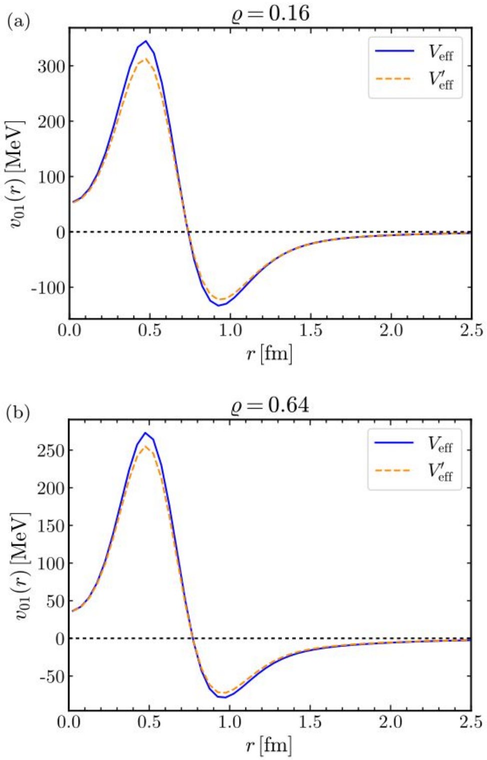


Operatorial Relativistic Corrections

$$\langle V_{\text{eff}} \rangle = \langle \mathcal{F}^\dagger [T, \mathcal{F}] \rangle|_{2b} + \langle \mathcal{F}^\dagger V_{NN} \mathcal{F} \rangle|_{2b} + \langle \mathcal{F}^\dagger V^*_{NNN} \mathcal{F} \rangle|_{3b},$$

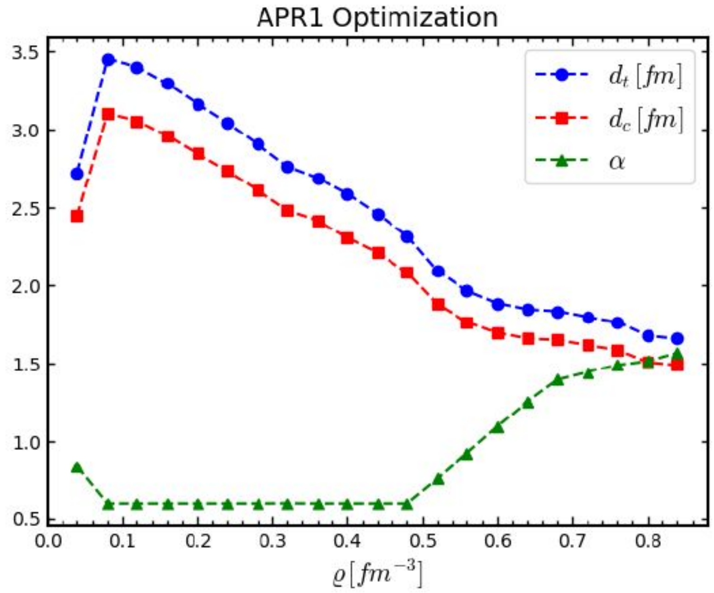
$$\langle V'_{\text{eff}} \rangle = \langle \mathcal{F}^\dagger [T, \mathcal{F}] \rangle|_{2b} + \langle \mathcal{F}^\dagger (V_{NN} + \delta V) \mathcal{F} \rangle|_{2} + \langle \mathcal{F}^\dagger V^*_{NNN} \mathcal{F} \rangle|_{3b}$$

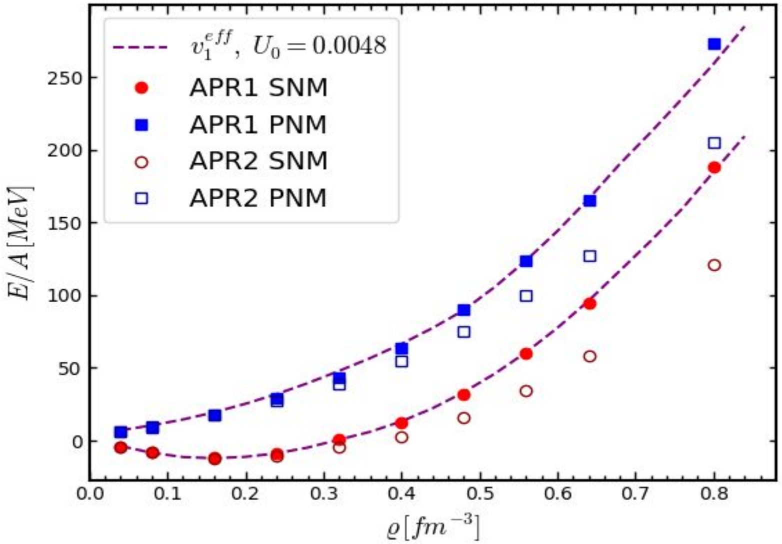




Effective Interaction with NNN force

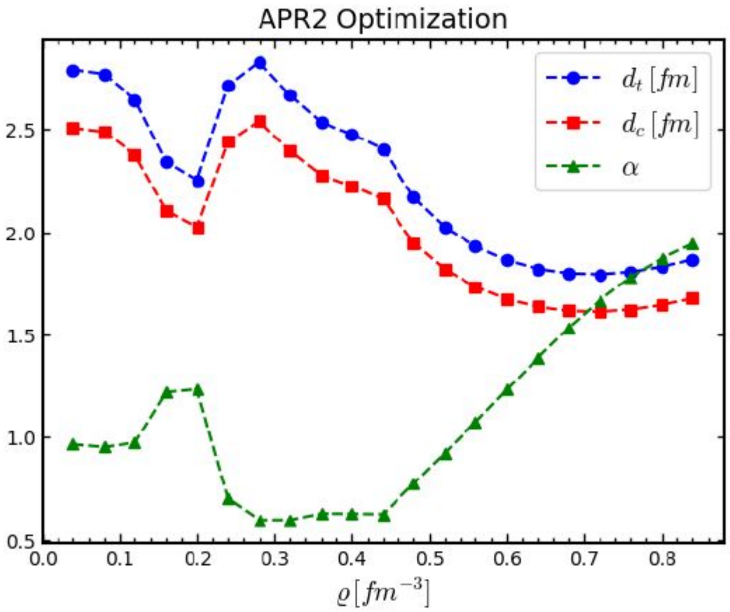
$$\frac{\varrho}{2}\int d\boldsymbol{r}_{12}\,\mathrm{CTr}\left[v_1^{\mathrm{eff}}\left(1-\hat{P}^{\sigma\tau}_{12}l^2_{12}\right)\right] \equiv \langle T-T_F\rangle|_{2b} + \langle v_{NN}\rangle|_{2b} + \langle V_{NNN}\rangle|_{3b}$$

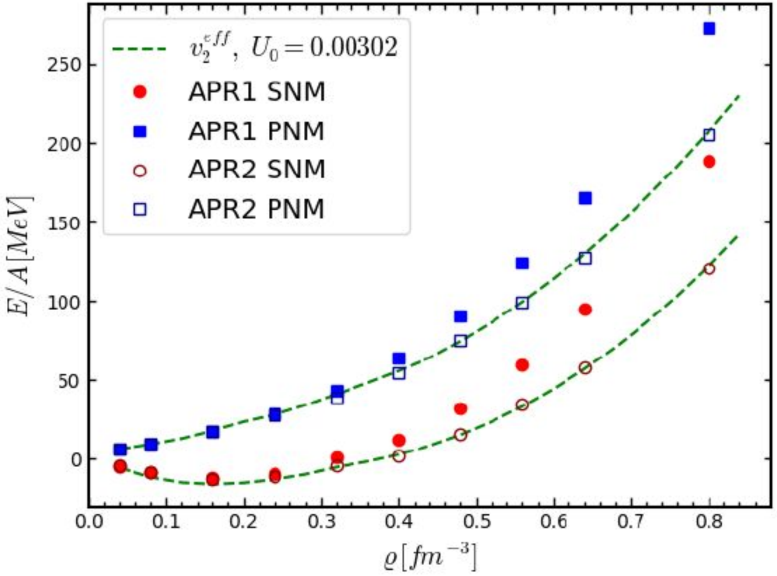




Effective Interaction with NNN force

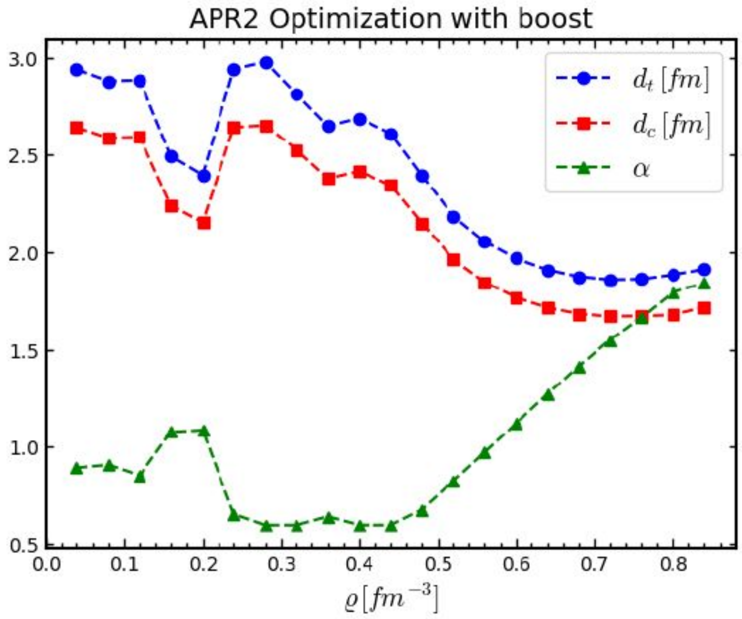
$$\frac{\varrho}{2}\int d\boldsymbol{r}_{12}\,\mathrm{CTr}\left[v_2^{\mathrm{eff}}\left(1-\hat{P}_{12}^{\sigma\tau}l_{12}^2\right)\right] \equiv \langle T - T_F\rangle|_{2b} + \langle v_{NN}\rangle|_{2b} + \langle V_{NNN}^*\rangle|_{3b}$$

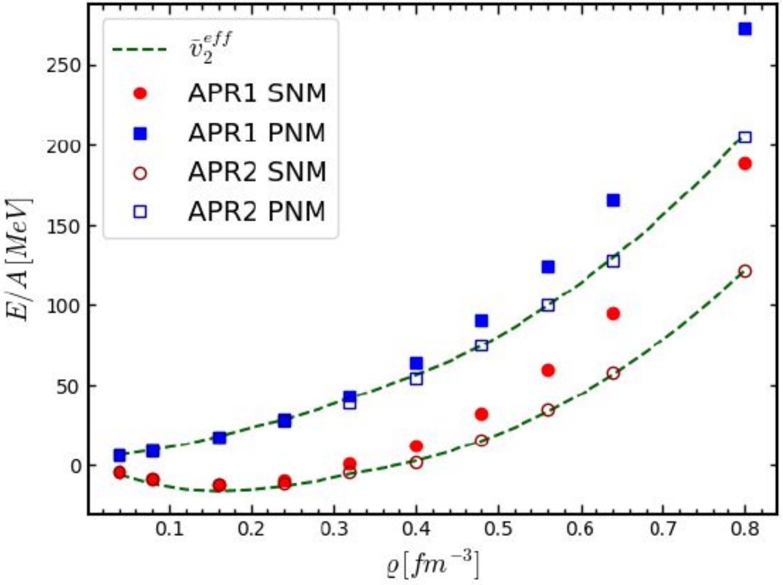




Effective Interaction with NNN force

$$\frac{\varrho}{2}\int d\boldsymbol{r}_{12}\,\mathrm{CTr}\left[\tilde{v}_2^{\mathrm{eff}}\left(1-\hat{P}_{12}^{\sigma\tau}l_{12}^2\right)\right]\equiv\langle T-T_F\rangle|_{2b}+\langle v_{NN}+\delta v\rangle|_{2b}+\langle V_{NNN}^*\rangle|_{3b}$$





Nucleon-Nucleon Potential

- Observation of deuteron only in the state with S=1,T=0 ➙ strong spin-isospin dependence
- Non-central charge distribution in atomic nuclei ➙ non central interactions
- Saturation of central density ➙ short range repulsion
- Binding energy per nucleon nearly constant with increasing mass number ➙ short range interaction

$$P_0 = \frac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \quad P_1 = \frac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

$$v_{NN} = \sum_{S,T} [v_{TS}(r) + \delta_{S1} v_{tT}(r) S_{12}] \, P_S \Pi_T$$

$$S_{12} = \frac{3}{r^2}(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

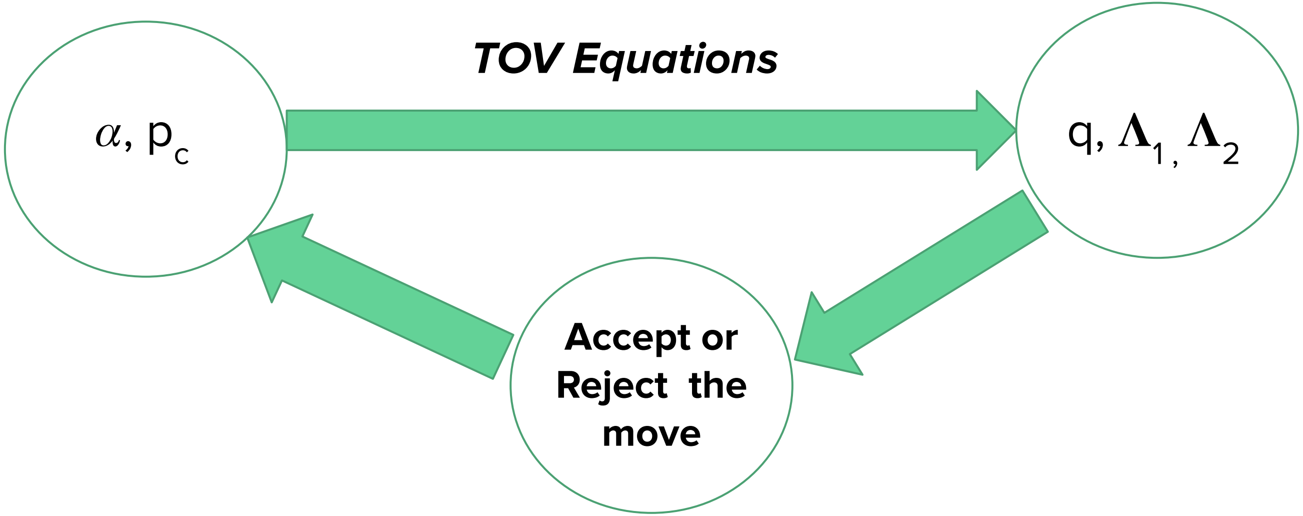
⇩

$$v_{ij} = \sum_{p=1}^{6} v^p(r_{ij}) O^p_{ij}$$

$$\{O^p_{ij}\} = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}) \otimes (1, \vec{\tau}_i \cdot \vec{\tau}_j).$$

Sampling The Posterior

$$\mathcal{P}(\alpha, p_c^{(1)} | O_{\mathrm{GW}}) \propto \mathcal{P}_0(\alpha, p_c^{(1)}, p_c^{(2)}) \mathcal{L}_{\mathrm{GW}}(q, \Lambda_1, \Lambda_2)$$



Bayesian Inference Framework

We have made Bayesian inference on α employing the following dataset:

- Gravitational Wave (GW) observation of the binary system GW170817 made by the LIGO-Virgo collaboration (LVC)
- The mass and radius provided by the spectroscopic observation of the millisecond pulsar PSR J0030+0451 performed by the NICER satellite.
- The mass and radius provided by the heavy pulsar PSR J0740+6620.

The posterior distribution defined through Bayes Theorem

$$\mathcal{P}(\theta|O) \propto \mathcal{P}_0(\theta) \prod_{i=1}^{n} \mathcal{L}(O^{(i)}|D(\theta))$$

Is sampled with Markov Chain Monte Carlo (MCMC) numerical simulations using the *emcee* algorithm[3].

[3] D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, Astron. Soc. Pac. 125, 306 (2013)

Extension to Future GW detections

We extended our study by repeating the **same analysis** but with a set of **simulated data** in order to investigate the following scenarios

- Increasing number of observations
- New generation of GW detectors

We simulated **30 binary neutron star events** for two different observatories:

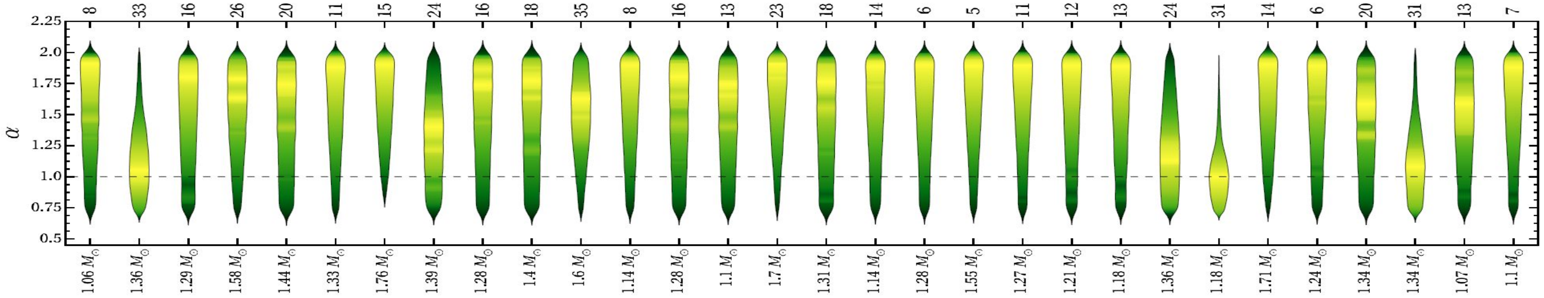
- **LIGO Hanford, LIGO Livingston, and Virgo detectors** at design sensitivity
- The future third-generation interferometer **Einstein Telescope**

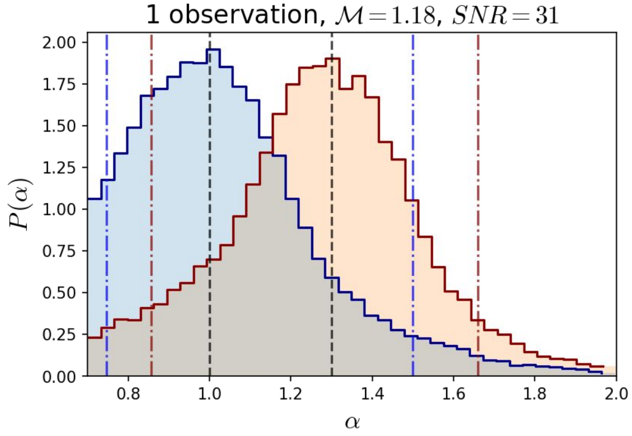
We have generated two different sets of 30 binaries by using EOSs associated with two different values of α for each observatory.

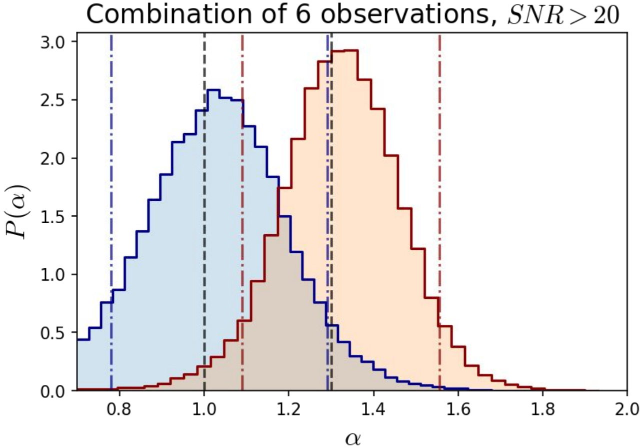
- The injected values of α are α=1.0 and α=1.3
- Sky location and inclination uniformly distributed over the sky.
- We assumed the chirp mass of each event to be measured with infinitesimal precision.

A. Sabatucci, O. Benhar, A. Maselli and C. Pacilio Phys. Rev. D **106**, 083010 – **Published 14 October, 2022**

Mocked Data: LIGO/Virgo

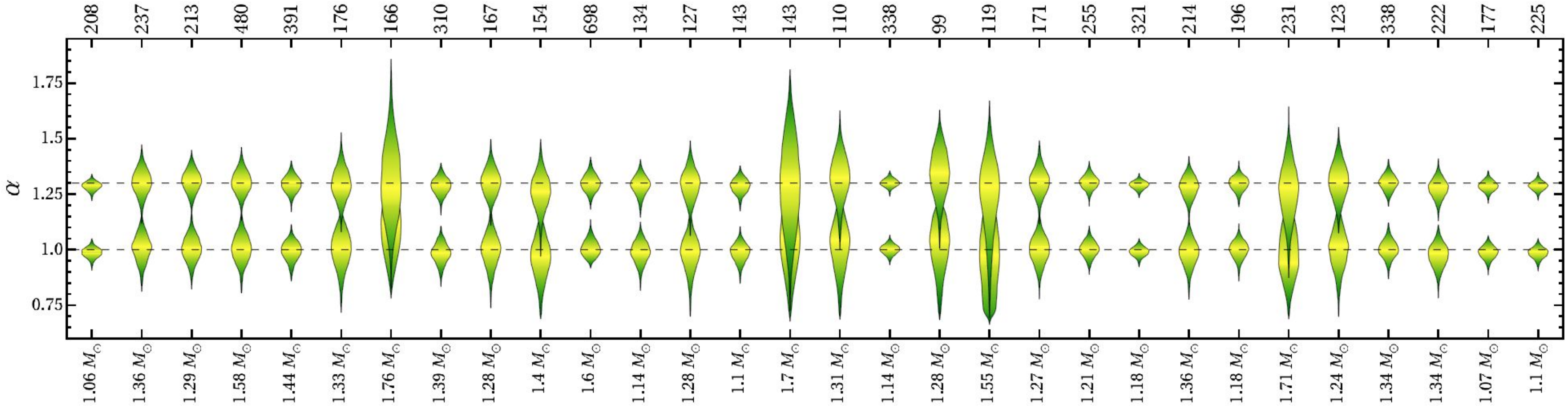






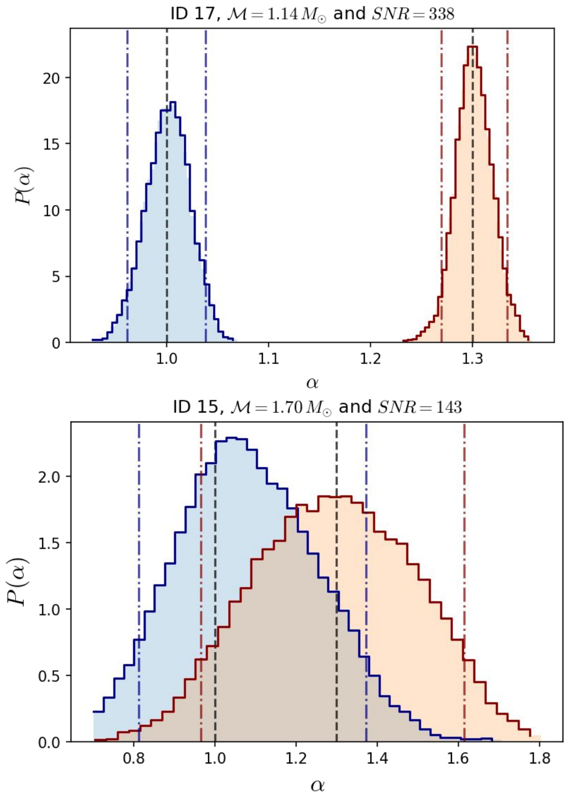
Mocked Data: Einstein Telescope

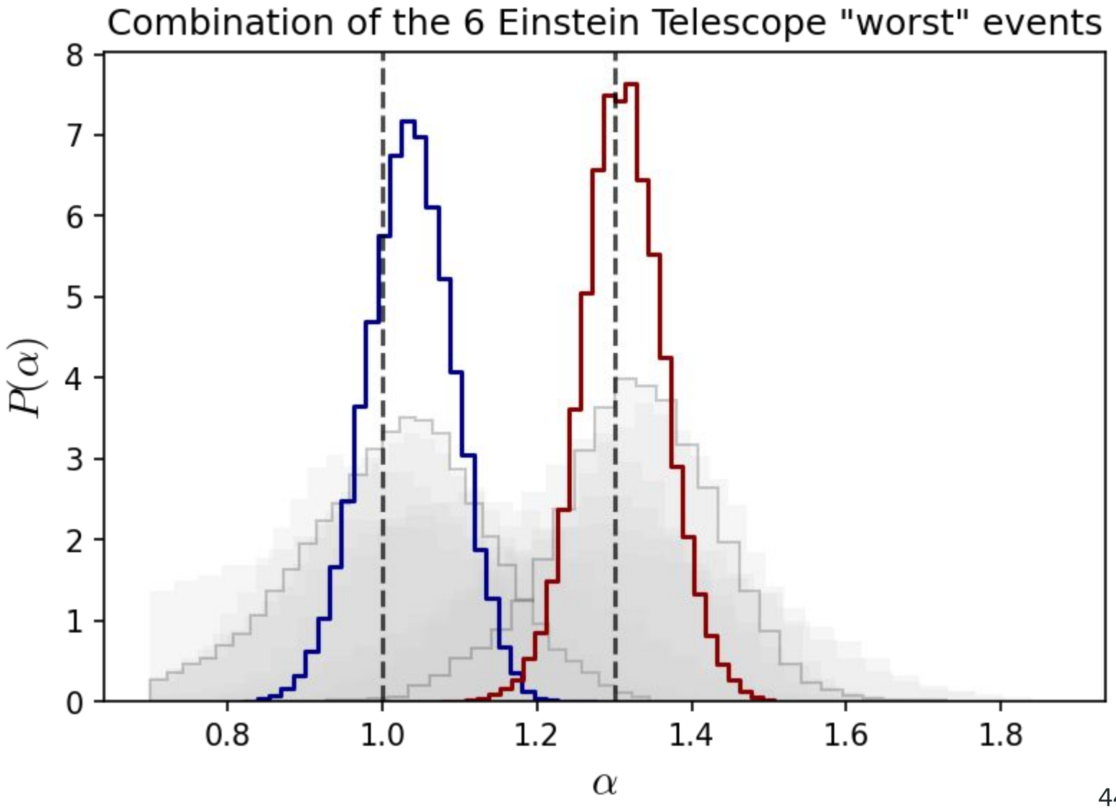
Violin plot of the marginal posterior of α for the 30 ET events. On the bottom and top axes are reported the chirp mass and the signal-to-noise ratio (SNR) for each event respectively.



This analysis has been approved by the ET collaboration.

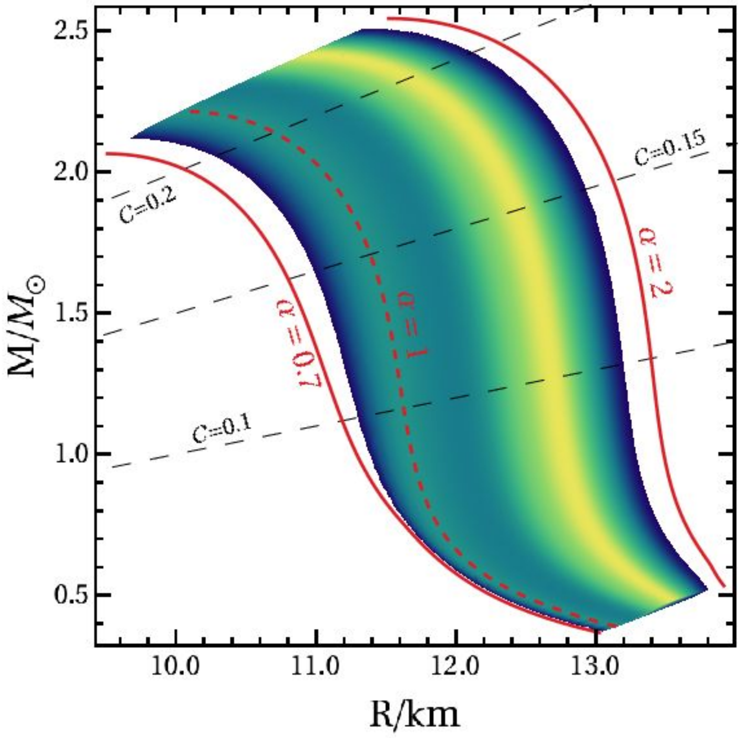
Mocked Data: Einstein Telescope



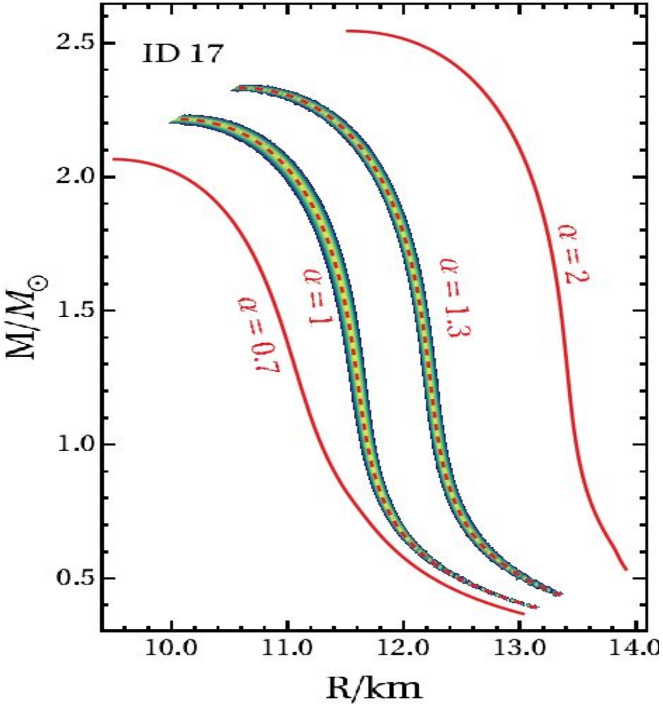


Comparison between current and future constraints

Current multimessenger data
(LIGO/Virgo + NICER)

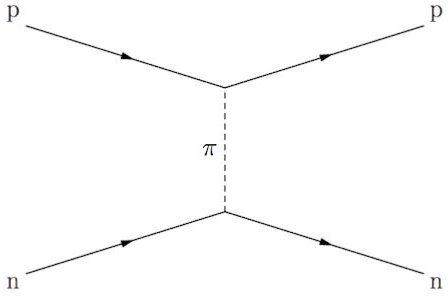


Expected Einstein Telescope constraints



Derivation of the One-Pion-Exchange (OPE) Potential

$$\mathcal{L} = \bar{\psi}^N (i\gamma^\mu \partial_\mu - m)\psi^N + \mathcal{L}_\Pi + \mathcal{L}_I$$



$$\mathcal{L}_I = ig(\bar{\psi}_i^N \gamma^5 \psi_j^N)(T_{ij}^a \pi^a)$$

$$i\mathcal{M} = -g^2\, \bar{u}(p_{2'}, s_{2'})\gamma^5 u(p_2, s_2) \frac{1}{k^2 - m_\pi^2} \bar{u}(p_{1'}, s_{1'})\gamma^5 u(p_1, s_1) \langle T_1^a \rangle \langle T_2^a \rangle .$$

$$S_{fi} = \sqrt{\frac{m}{E_1}}\sqrt{\frac{m}{E_2}}\sqrt{\frac{m}{E_{1'}}}\sqrt{\frac{m}{E_{2'}}}(2\pi)^4 \delta^{(4)}(p_{2'} + p_{1'} - p_2 - p_1)\ [\mathcal{M} - \mathcal{M}'] ,$$

$$S_{fi} \approx -i\frac{g^2}{4m^2}(2\pi)^4 \delta^{(4)}(p_{1'} + p_{2'} - p_1 - p_2) \cdot \langle T_1^a T_2^a \rangle\, \chi_{1'}^\dagger \chi_{2'}^\dagger \frac{-(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})}{|\vec{k}|^2 + m_\pi^2} \chi_2 \chi_1$$

$$S_{fi} = -i(2\pi)^4 \delta^{(4)}(p_{1'} + p_{2'} - p_1 - p_2)\langle v^\pi(\vec{k})\rangle$$

$$v^\pi(\vec{k}) = -\left(\frac{f_\pi}{m_\pi}\right)^2 \frac{(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})}{\vec{k}^2 + m_\pi^2} T_1^a T_2^a \longrightarrow v_\pi(\vec{r}) = \frac{g^2}{4m^2} T_1^a T_2^a \,(\vec{\sigma}_1 \cdot \nabla)\,(\vec{\sigma}_2 \cdot \nabla)\, \frac{e^{-m_\pi r}}{r}$$

CBF Perturbation Theory

Now that we have found an approximated ground state thanks to the variational principle we can compute perturbative corrections by defining a new set of states as

$$|\Psi_n\rangle = \frac{\mathcal{F}|\Phi_n\rangle}{\sqrt{\langle\Phi_n|\mathcal{F}^\dagger\mathcal{F}|\Phi_n\rangle}}$$

Now we can split the Hamiltonian according to

$$\mathcal{H} = H_0 + H_1$$

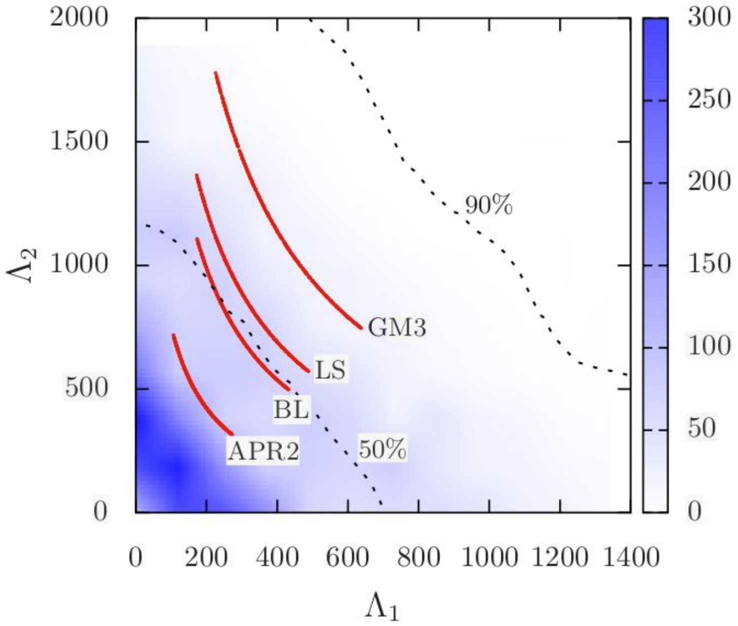
with

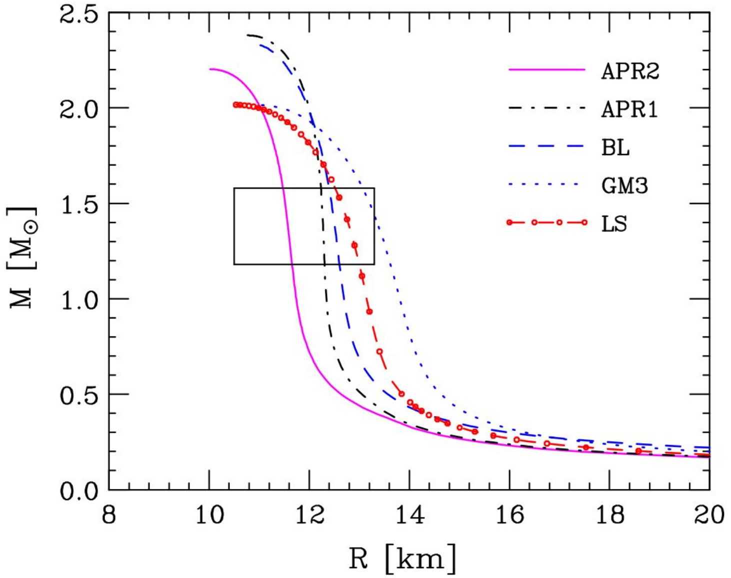
$$\begin{aligned} \langle\Psi_m|H_0|\Psi_n\rangle &= \delta_{mn}\langle\Psi_m|\mathcal{H}|\Psi_n\rangle \\ \langle\Psi_m|H_1|\Psi_n\rangle &= (1-\delta_{mn})\langle\Psi_m|\mathcal{H}|\Psi_n\rangle \end{aligned}$$

If we made a good choice for the trial ground state the off diagonal terms will be small and the H_1 term can be safely treated in perturbation theory.

Boost Corrections to NN Potential

There is a significant difference in stellar observables calculated with and without boost corrections. This discrepancy is mainly due to the modification that the inclusion of the boost corrections induces on the repulsive contribution of the NNN potential, which becomes dominant at large densities.





[4]A. Sabatucci and O. Benhar, Phys. Rev. C 1091, 0545807(2020).
[5]O. Benhar and A. Lovato, Phys. Rev. C 96:054301, Nov 2017.

Details of the Parametrization

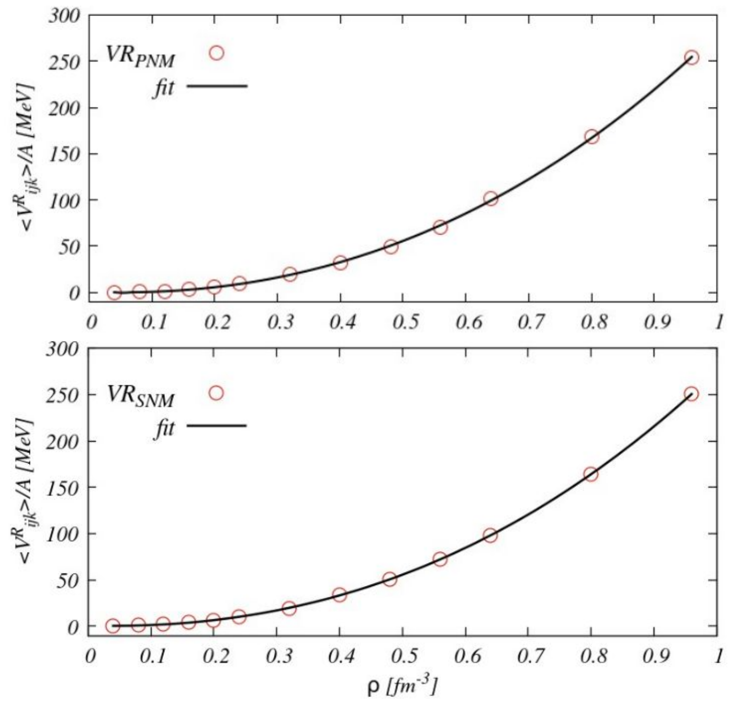
$$\epsilon(\varrho,x) = \left[\frac{1}{2m} + f(\varrho,x)\right]\tau_p + \left[\frac{1}{2m} + f(\varrho,1-x)\right]\tau_n + g(\varrho,x)$$

$$g(\varrho,x) = g(\varrho,1/2)[1-(1-2x)^2] + g(\varrho,0)(1-2x)^2,$$

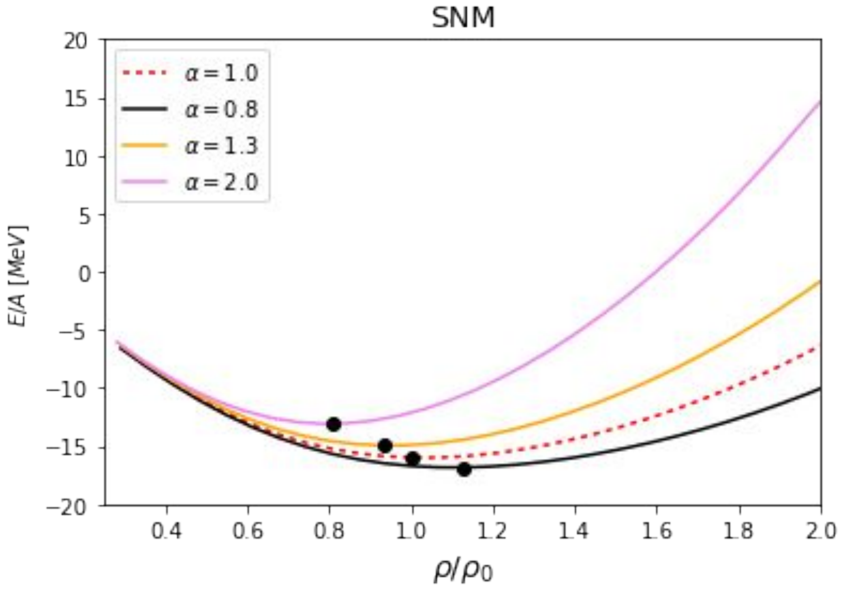
$$\langle V^R\rangle(\rho) = a_1 + a_2\,\rho + a_3\,\rho^2 + a_4\,\rho^3.$$

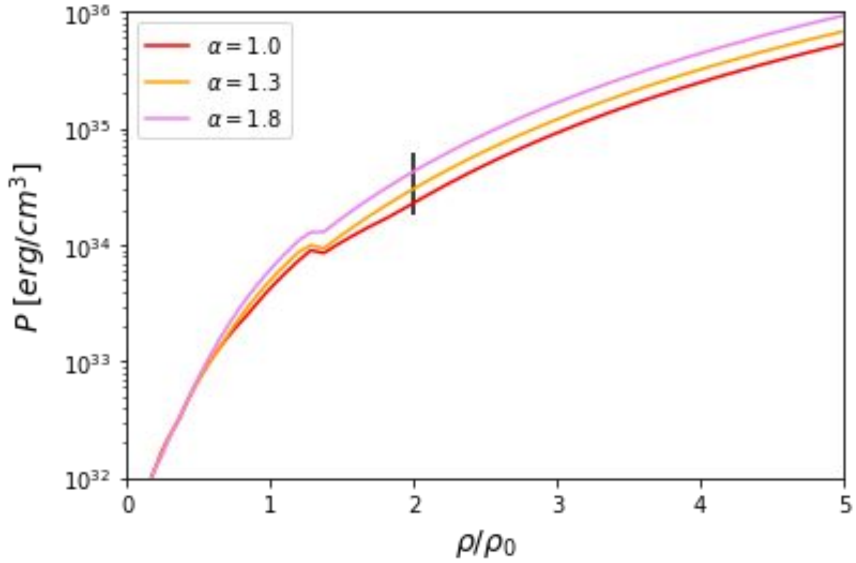
$$g'_A(\rho) = g_A(\rho) + (\alpha - 1)v^R_A(\rho)$$

with $A = SNM, PNM$ and $v_R = \rho\langle V^R\rangle$

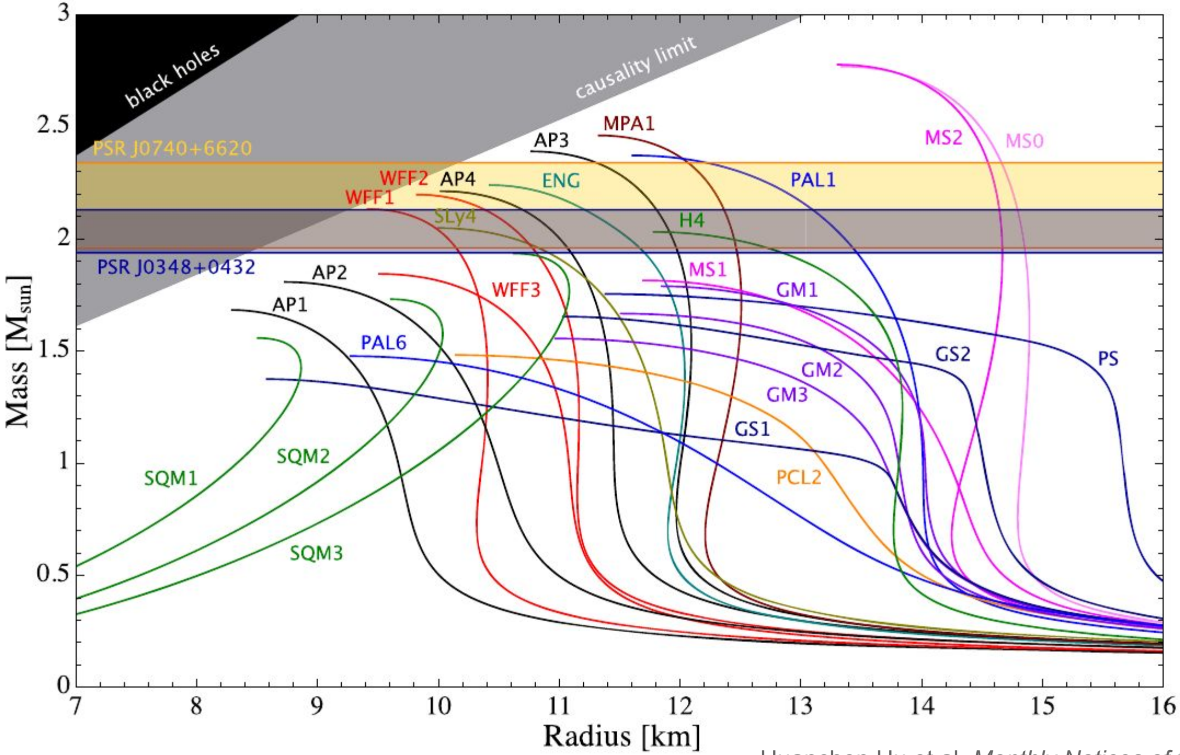


Resulting EOSs





How can we choose between different EOS models?

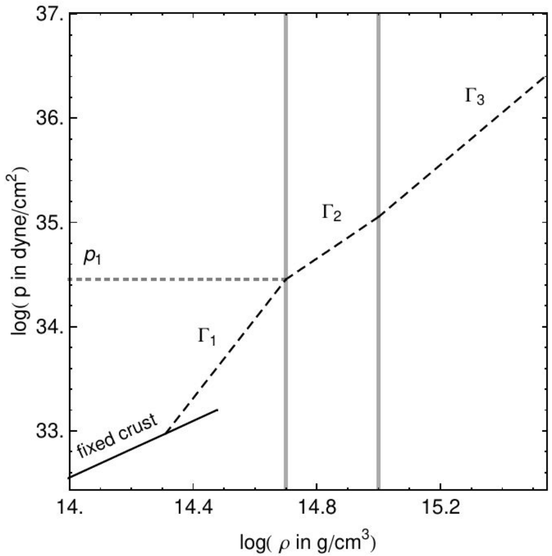


Huanchen Hu et al, *Monthly Notices of the Royal Astronomical Society*, Volume 497, Issue 32020, Pages 3118–3130

Bayesian Inference on Astrophysical Data

A large set of different EOSs can be described by a unified parametric model.
If we have a parametric EOS we can infer the probability distribution of its parameters from astrophysical data by means of Bayes theorem

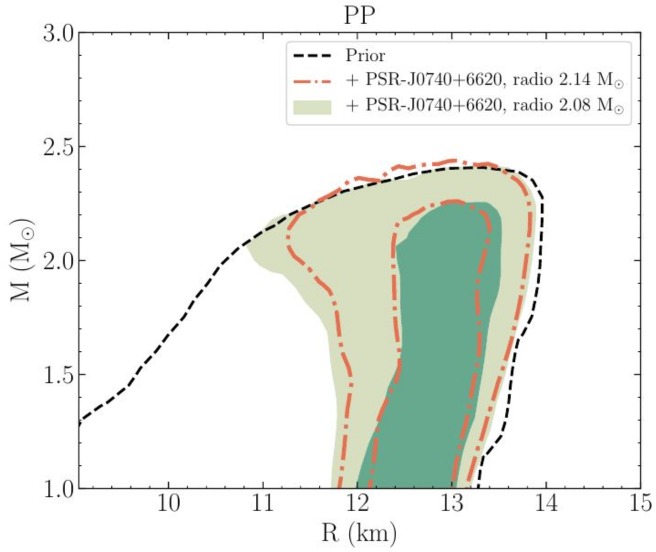
$$\mathcal{P}(\theta|O) \propto \mathcal{P}_0(\theta) \prod_{i=1}^{n} \mathcal{L}(O^{(i)}|D(\theta))$$



J. S. Read et al Phys. Rev. D 79, 124032 (2009)

$$\mathcal{L}_{EM}(M(\theta), R(\theta))$$

$$\vec{\theta} = (\Gamma_1, \Gamma_2, \Gamma_3, p_1)$$



G. Raaijmakers *et al* *ApJL* 918 L29 (2021)

Chiral Potentials

Chiral EFT is a **low-energy** effective theory of QCD, in which nucleons and pions are chosen as the relevant degrees of freedom. This effective field theory is constrained to be symmetric under the group

$$SU(2)_L \otimes SU(2)_R \,.$$

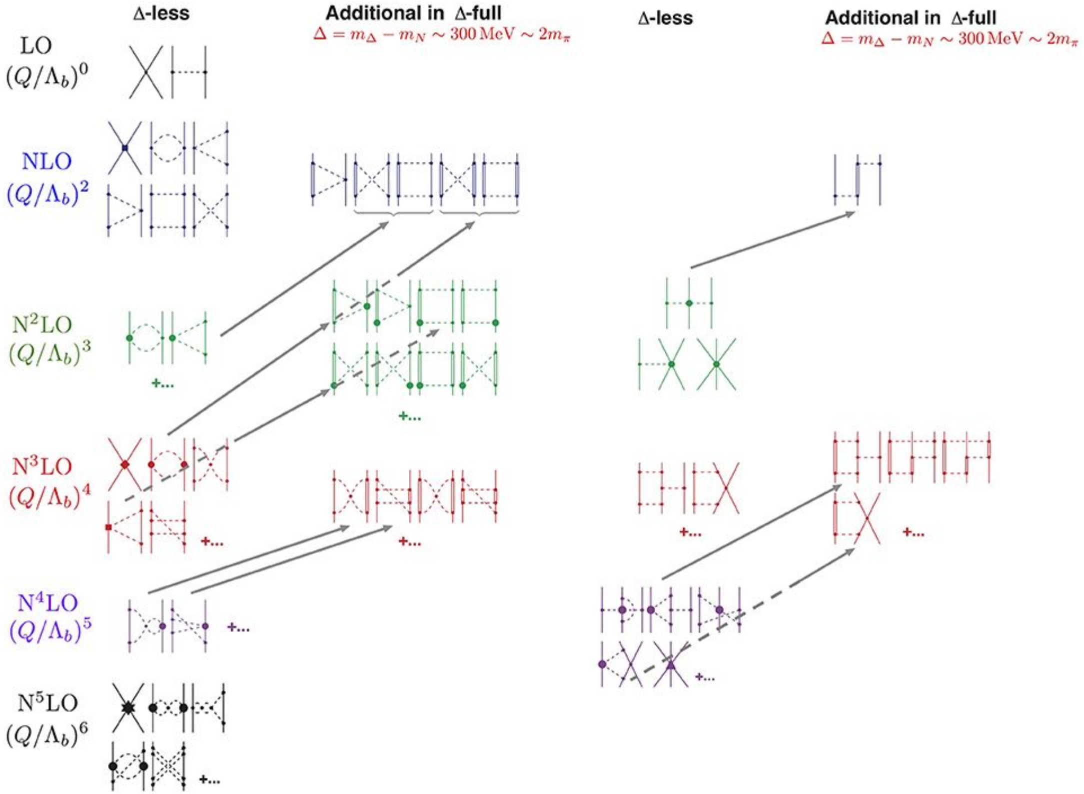
This is an approximated symmetry of the QCD lagrangian, that turns out to be a good approximation of the real theory in the light quark sector. This symmetry is **spontaneously broken** and the pions are its Goldstone bosons.

The starting point in chiral EFT is to write the most general Lagrangian in terms of the chosen degrees of freedom. This Lagrangian contains an infinite number of terms and must be truncated using a given power-counting scheme.

This approach was first proposed by Weinberg in 1990. The ***Interaction is expanded in powers*** of the typical p over the breakdown scale, p/Λ_b .

$$\mathcal{L} = \mathcal{L}^{(0)} + \left(\frac{p}{\Lambda_b}\right)^2 \mathcal{L}^{(2)} + \left(\frac{p}{\Lambda_b}\right)^3 \mathcal{L}^{(3)} + \ldots$$

Chiral Potentials

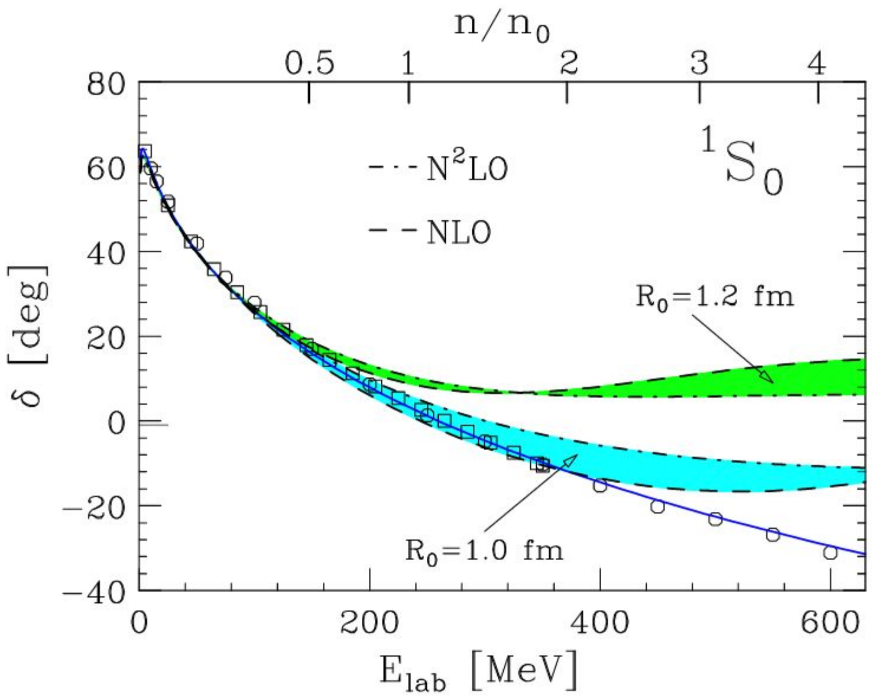


Chiral contributions to NN and NNN interactions based on Weinberg power counting.

The main advantage of the chiral EFTs is that they give a way to **systematically derive two- and many body-interactions in a consistent fashion**.

Maria Piarulli and Indigo Tews, Front. Phys., 30 January 2020

Comparison between Chiral and AV18 potentials



Neutron-proton scattering phase shifts as a function of the kinetic energy of the beam particle in the laboratory frame (bottom axis). The corresponding density is given in the top axis.

The kinetic energy in the lab frame is related to the particle density through:

$$E_{lab} = 2E_{cm} = \frac{2}{m}(3\pi^2\rho)^{2/3}$$

From this plot clearly appears that the AV18 potential yields an accurate description of the data up to energies of about 600 MeV corresponding to 4 times the nuclear saturation density. Conversely *chiral potentials seem to be limited up to twice the nuclear saturation density.*

O. Benhar, arXiv:1903.11353 [nucl-th] (2019).