THE HYPERON-NUCLEON INTERACTION IN LOW-ENERGY EFFECTIVE FIELD THEORY

Margherita Sagina (University of Pisa, INFN - Pisa)

margherita.sagina@phd.unipi.it

In collaboration with

Laura Elisa Marcucci (University of Pisa, INFN - Pisa)

Alex Gnech (ODU, Jefferson Lab)

OVERVIEW

1 Motivations

2 Potential model derivation

OVERVIEW

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2 Potential model derivation → depends on constants (LECs)

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fitted to experimental cross section data

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2 Potential model derivation → depends on constants (LECs)

3 Cross section calculation

4 Fitting procedure

fitted to experimental cross section data

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4 Fitting procedure

5 Preliminary results

6 Outlook

depends on constants (LECs)

fitted to experimental cross section data

Aim of the project

Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

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- Consistent with a well tested theory (QCD)
- EFT $\rightarrow$ perturbative expansion allows theoretical error estimation

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Simple model

Easy to handle numerically

(Also suitable computationally expensive tasks)

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Many applications to different research fields

- Hyperon puzzle in Neutron Stars
- Hypernuclei studies

POTENTIAL MODEL DERIVATION

Pionless Effective Field Theory

In $\not{\pi}$EFT even **pions** can be considered **high-energy dof** $\Rightarrow$ interaction described only by contact terms

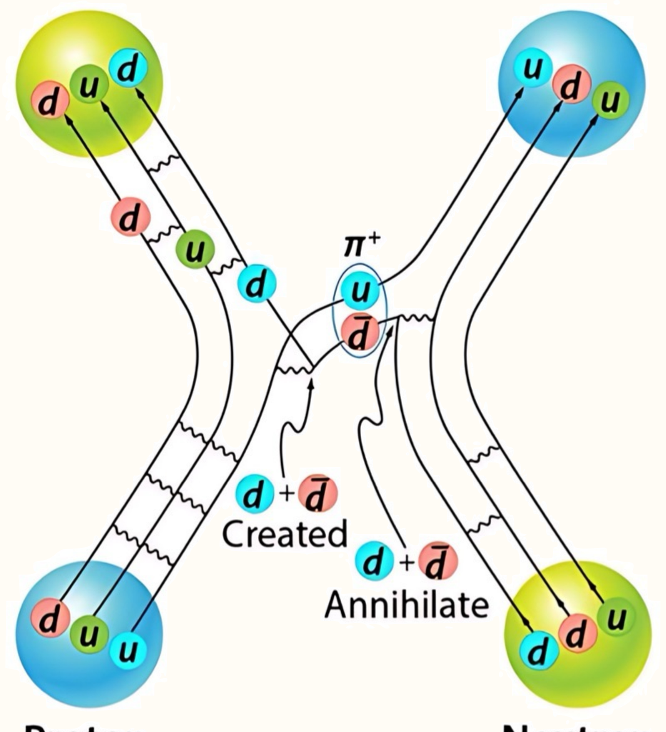
QCD
quarks, gluons
$\gtrsim$ 1 GeV

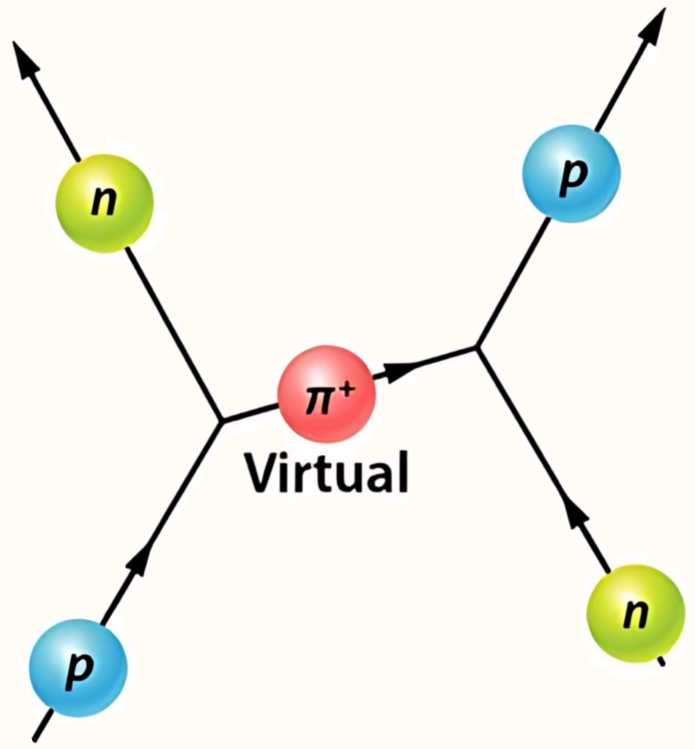
$\Rightarrow$

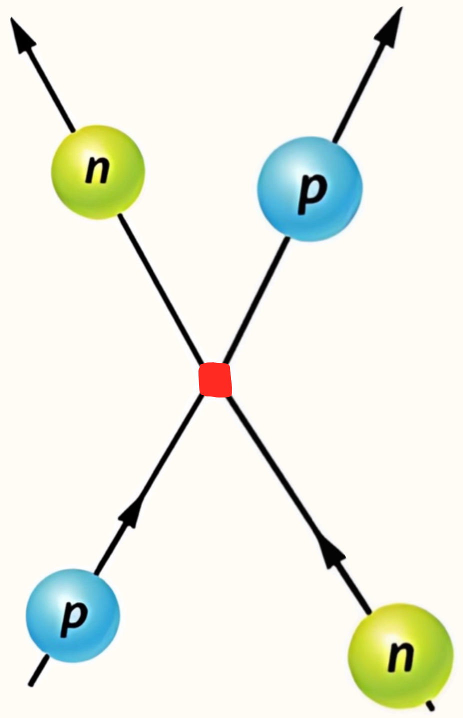
χEFT
nucleons, pions
$\sim$ 100 MeV

$\Rightarrow$

$\not{\pi}$EFT
nucleons
$\sim$ 10 MeV



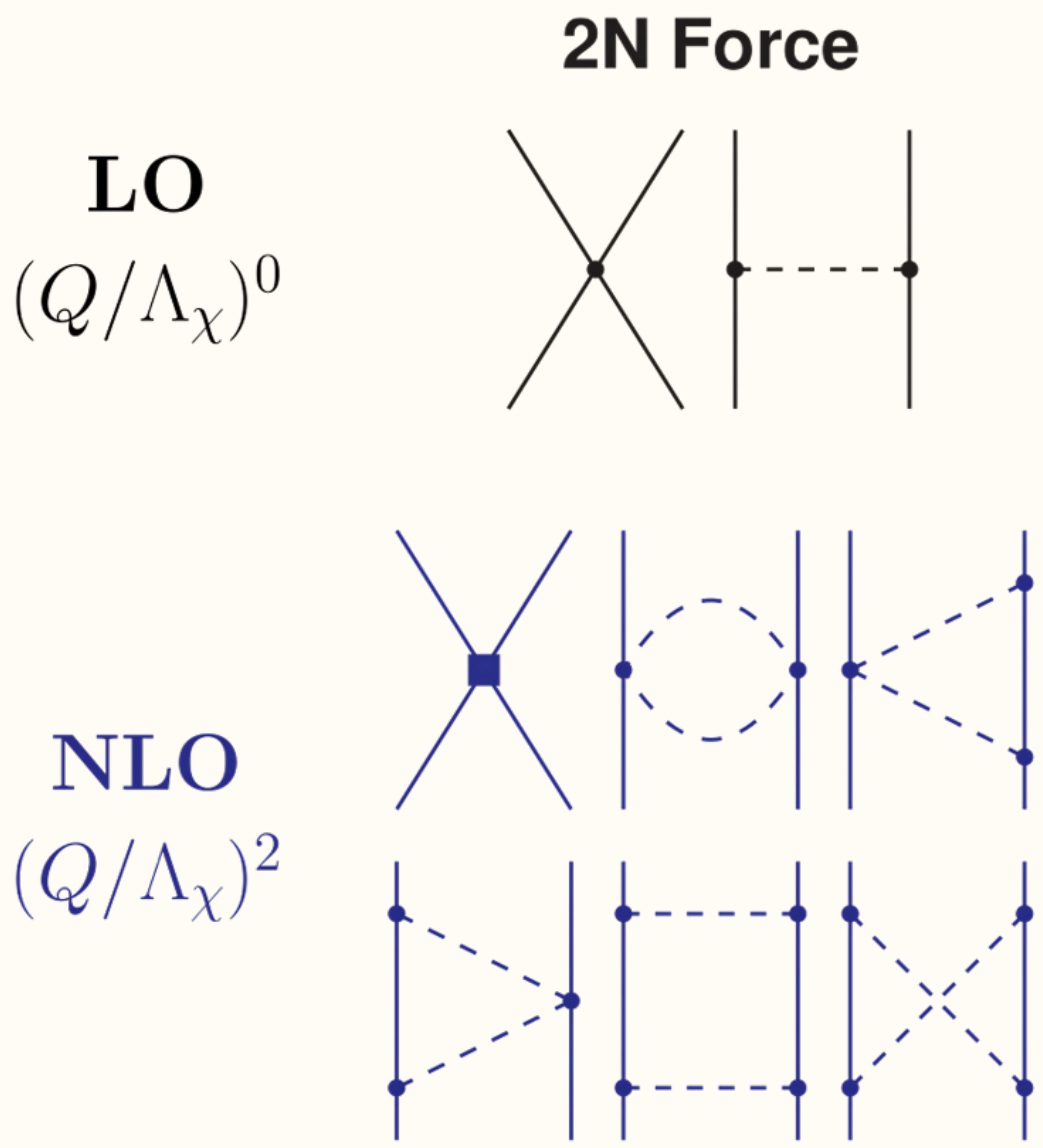




Adapted from APS/Alan Stonebraker

POTENTIAL MODEL DERIVATION

“Weinberg-ized” Pionless Effective Field Theory



- Keep same power counting as χEFT

POTENTIAL MODEL DERIVATION

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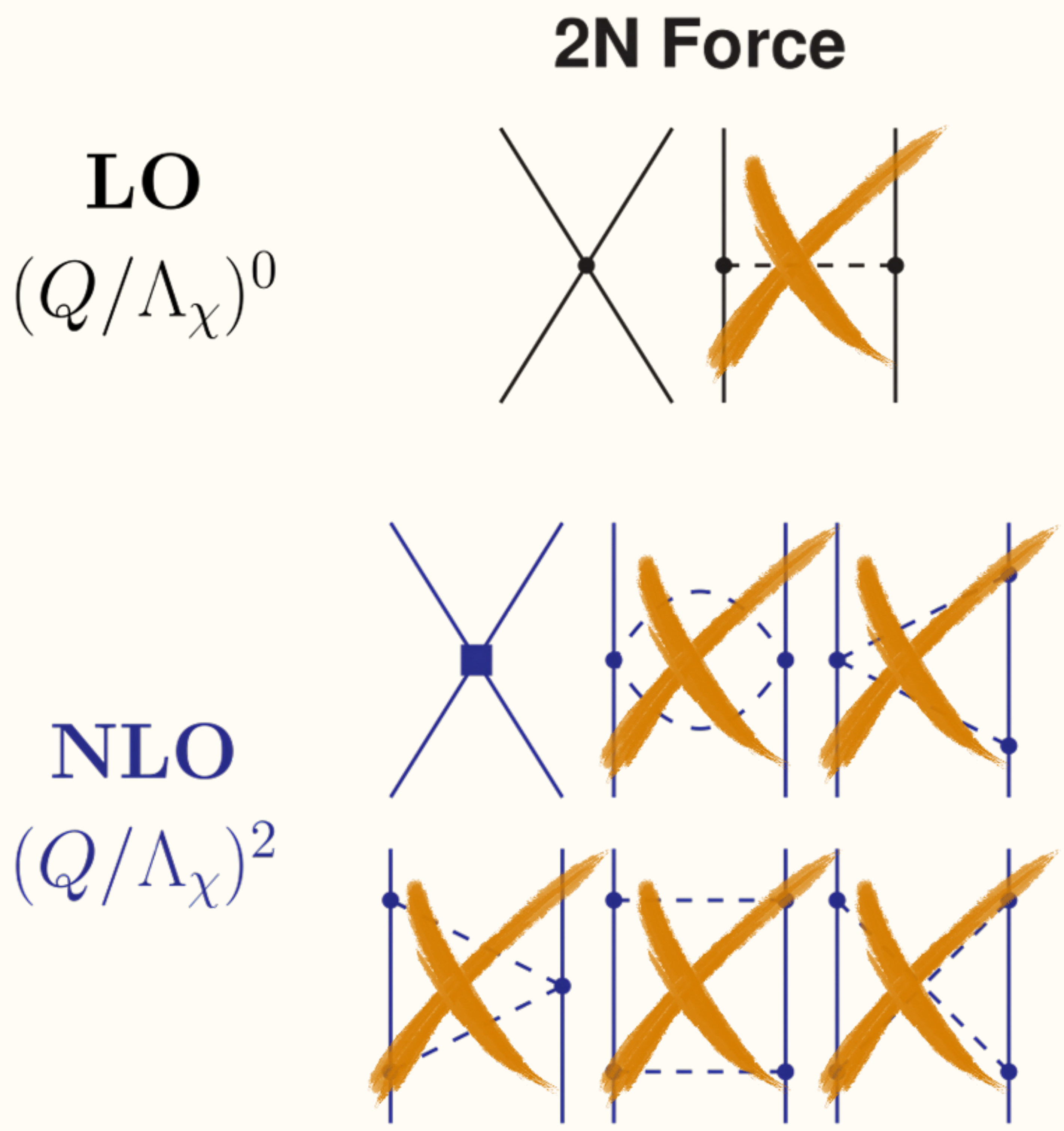

- Keep same power counting as χEFT
- Remove diagrams that involve pion exchanges → only contact terms
- Approach used in *Schiavilla et al. (2021)*

POTENTIAL MODEL DERIVATION

Hyperon-nucleon potential model – Momentum space

Literature:

- *Haidenbauer et al. (2023)*: YN interaction in χEFT up to N2LO, momentum space
- *Schiavilla et al. (2021)*: NN interaction in contact EFT up to N3LO, coordinate space

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$\downarrow$

- Hyperon-nucleon interaction in momentum space, up to NLO, only contact terms:

$$
\begin{aligned}
V_{\Lambda N}^{\mathrm{LO}} &= C_S + C_T\,(\sigma_\Lambda \cdot \sigma_N)\,, \\
V_{\Lambda N}^{\mathrm{NLO}} &= C_1 \mathbf{q}^2 + C_2 \mathbf{q}^2 (\sigma_\Lambda \cdot \sigma_N) + iC_3 \mathbf{S} \cdot (\mathbf{k} \times \mathbf{q}) \\
&\quad + C_4 S_{\Lambda N}(\mathbf{q}) + iC_5 \mathbf{D} \cdot (\mathbf{k} \times \mathbf{q})
\end{aligned}
$$

$\downarrow$

7 **LECs**

$$
\begin{aligned}
\mathbf{q} &= \mathbf{p}' - \mathbf{p}\,, \\
\mathbf{k} &= (\mathbf{p} + \mathbf{p}')/2 \\
\mathbf{S} &= (\sigma_\Lambda + \sigma_N)/2\,, \\
\mathbf{D} &= (\sigma_\Lambda - \sigma_N)/2 \\
S_{\Lambda N}(\mathbf{q}) &= 3\,\sigma_\Lambda \cdot \mathbf{q}\,\sigma_N \cdot \mathbf{q} - q^2 \sigma_\Lambda \cdot \sigma_N\,.
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POTENTIAL MODEL DERIVATION

Hyperon-nucleon potential model — Coordinate space

- To regularize the interaction $\longrightarrow$ multiply $V_{\Lambda N}$ by a regulator function $\tilde{F}(k)$, as done in *Schiavilla et al. (2021)*

$$\tilde{F}(k) = \exp\left(-\frac{R_0{}^2 k^2}{4}\right)$$

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$$\Lambda_0\,[\mathrm{MeV}] = \frac{2 \cdot \hbar c}{R_0\,[\mathrm{fm}]}$$

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Fourier Transform

$$F(r) = \frac{1}{\pi^{3/2}{R_0}^3}\exp\left(-\frac{r^2}{{R_0}^2}\right)$$

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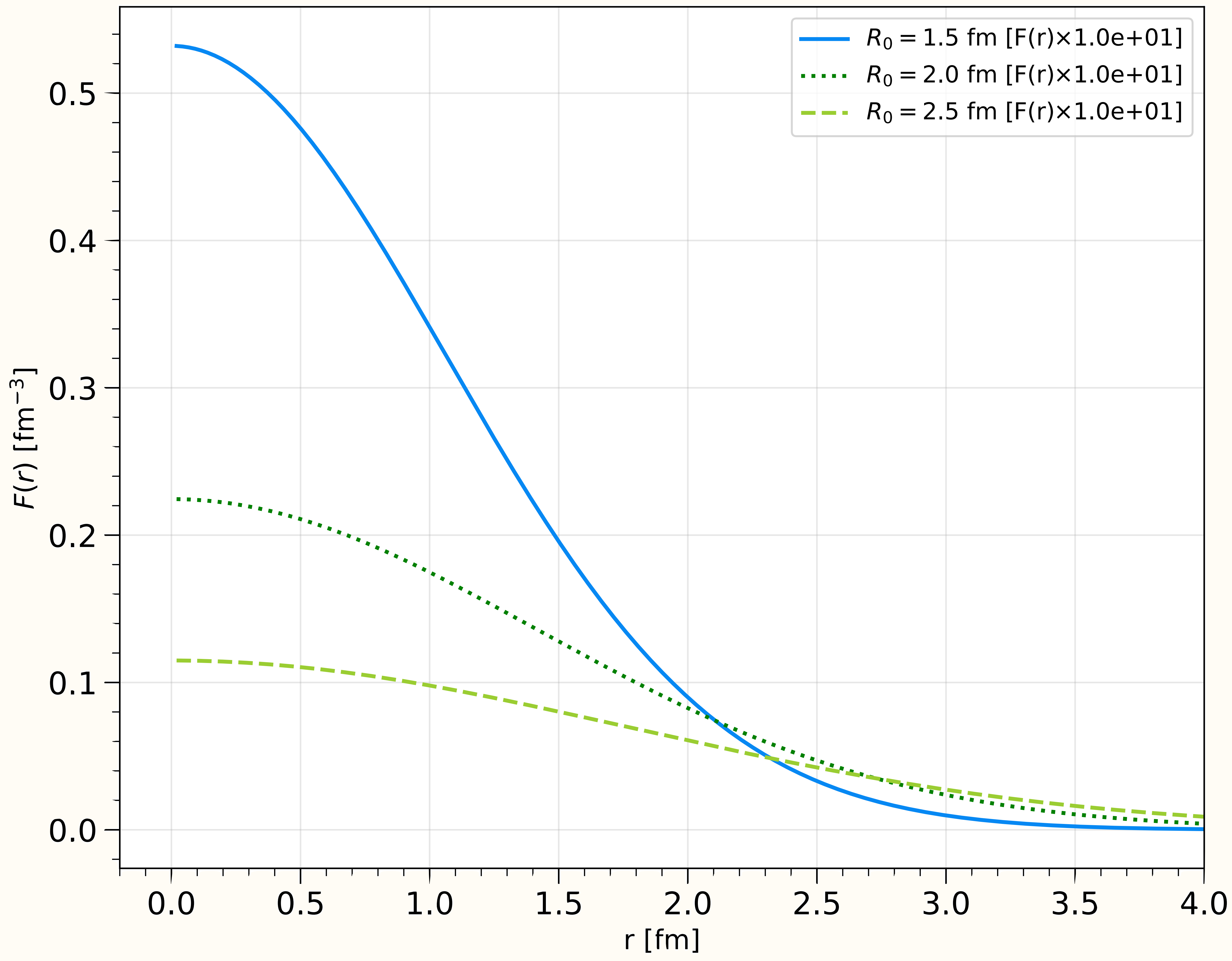
- Investigated cutoff parameter values $R_0 \in [1.5, 2.5]$ fm

$\Lambda_0 \in [158, 263]$ MeV



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- Fourier transform of the regularized potential in momentum space to obtain $V_{\Lambda N}$ in coordinate space

$$V_{\Lambda N}^{\mathrm{LO}} = \left[C_S + C_T\,(\sigma_\Lambda \cdot \sigma_N)\right] F(r)\,,$$

$$V_{\Lambda N}^{\mathrm{NLO}} = \sum_i \mathrm{v_i}\,(\mathbf{r})\,\mathcal{O}_i\,, \text{ with } \mathcal{O}_i = \mathbf{1}, \sigma_\Lambda \cdot \sigma_N, S_{\Lambda N}(\hat{\mathbf{r}}), \mathbf{L}\cdot\mathbf{S}, \mathbf{L}\cdot\mathbf{D}$$

Radial functions containing combinations of $F(r), F'(r), F''(r)$ and LECs $(C_1, \cdots, C_5)$

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↓

fixed through fitting procedure to experimental data

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Available experimental data to perform the fit: Λp elastic scattering cross section

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- Decompose wave function in **short-range (core)** and **long-range (asymptotic)**

$$\psi_c^\alpha(q,r) = \sum_{i=1}^{M} d_{\alpha,i}(q) f_i(r)$$

$$\psi_a^\alpha(q,\mathbf{r}) = \sum_{\beta} \left[\delta_{\alpha\beta}\,\Omega_\beta^R(q,\mathbf{r}) + R_{\alpha\beta}\,\Omega_\beta^I(q,\mathbf{r})\right]$$

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$$[R_{\alpha\beta}(q)] = R_{\alpha\beta}(q) - \frac{2\mu}{\hbar^2}\langle\psi_\beta(q,\mathbf{r})\,|\,H-E\,|\,\psi_\alpha(q,\mathbf{r})\rangle \longrightarrow d_{\alpha,i}(q), R_{\alpha\beta}(q)$$

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- Rewrite asymptotic wave function as sum of **plane wave** and **outgoing spherical wave**

$$\psi_\alpha^a(q,\mathbf{r}) = -2i\left[\Omega_\alpha^R(q,\mathbf{r}) + \sum_\beta T_{\alpha\beta}\,\Omega_\alpha^+(q,\mathbf{r})\right], \qquad \Omega_\alpha^+(q,\mathbf{r}) = \Omega_\alpha^I(q,\mathbf{r}) + i\Omega_\alpha^R(q,\mathbf{r})$$

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$$T_{\alpha\beta} = i(R_{\alpha\beta} + i\delta_{\alpha\beta})^{-1}R_{\alpha\beta}$$

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Total unpolarised cross section

$$\boxed{\sigma = \frac{4\pi}{q^2}\sum_{J,\alpha,\beta}\left|T_{\alpha\beta}\right|^2 \frac{(2J+1)}{(2s_\Lambda+1)(2s_N+1)}}$$

$$T_{\alpha\beta} = i(R_{\alpha\beta} + i\delta_{\alpha\beta})^{-1}R_{\alpha\beta}$$

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- First attempt: $\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2}$

Cross section experimental data from *Radboud University, NN online archive*

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  $\boxed{\times}$ Does not work: $V_{\Lambda N} = 0$ at LO when $S = 1$

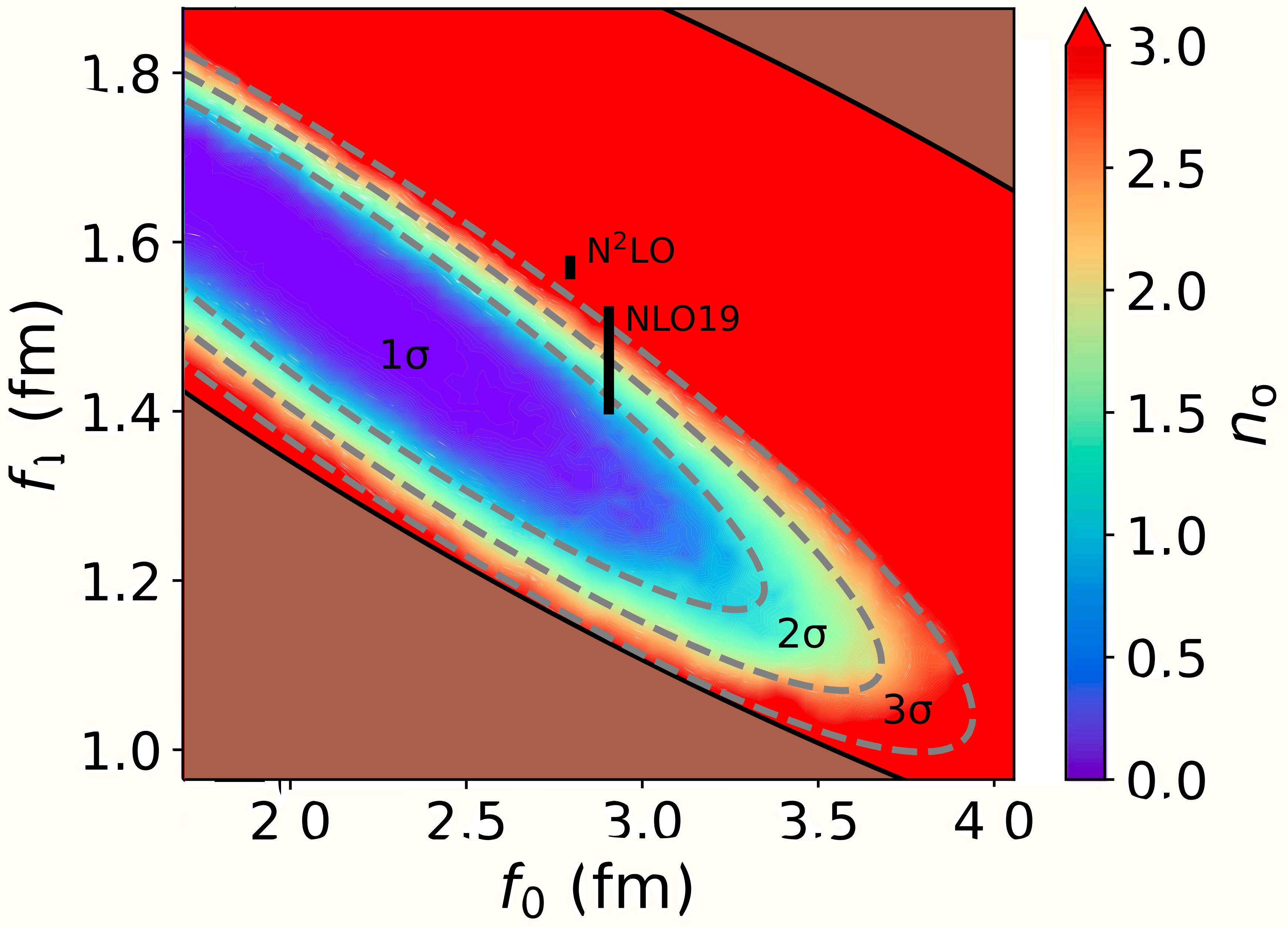
- Second attempt: **constraint on scattering length**

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th}(C_S, C_T, C_1, \ldots, C_5) - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

$\boxed{\checkmark}$ Solves issue in $S = 1$ case

“Experimental” data for Λp scattering length from *Mihaylov et al. (2024)*

- No experimental data available for scattering lengths



Adapted from Mihaylov et al. (2024)

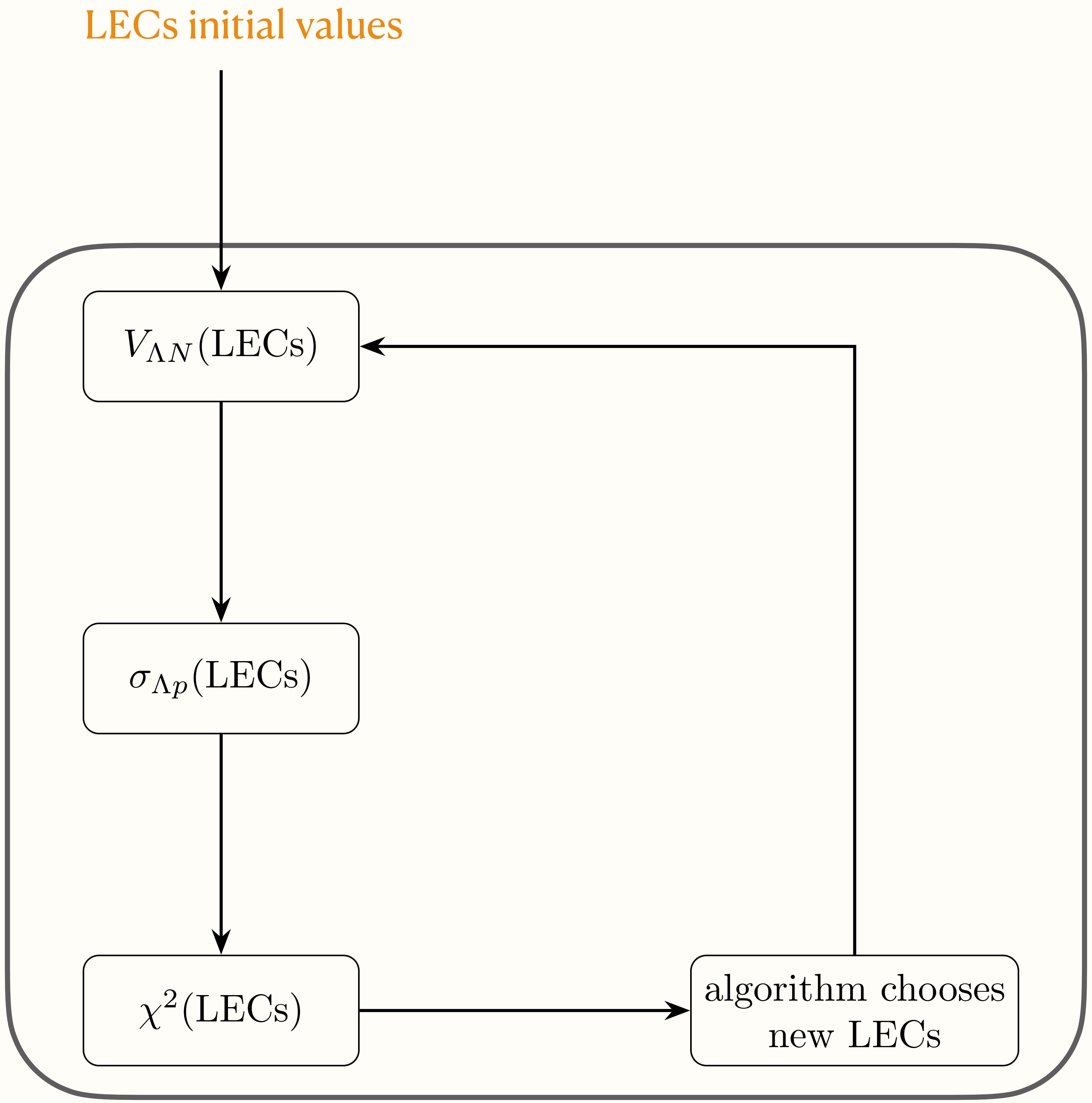
Mihaylov D.L, Haidenbauer J., Mantovani Sarti V., (2024)

- First combined analysis low-energy $p\Lambda$ scattering

  cross section + correlation data (ALICE)

- Best set of scattering lengths

  $f_0, f_1 = (2.1, 1.56)$ to $(3.34, 1.18)$ fm.

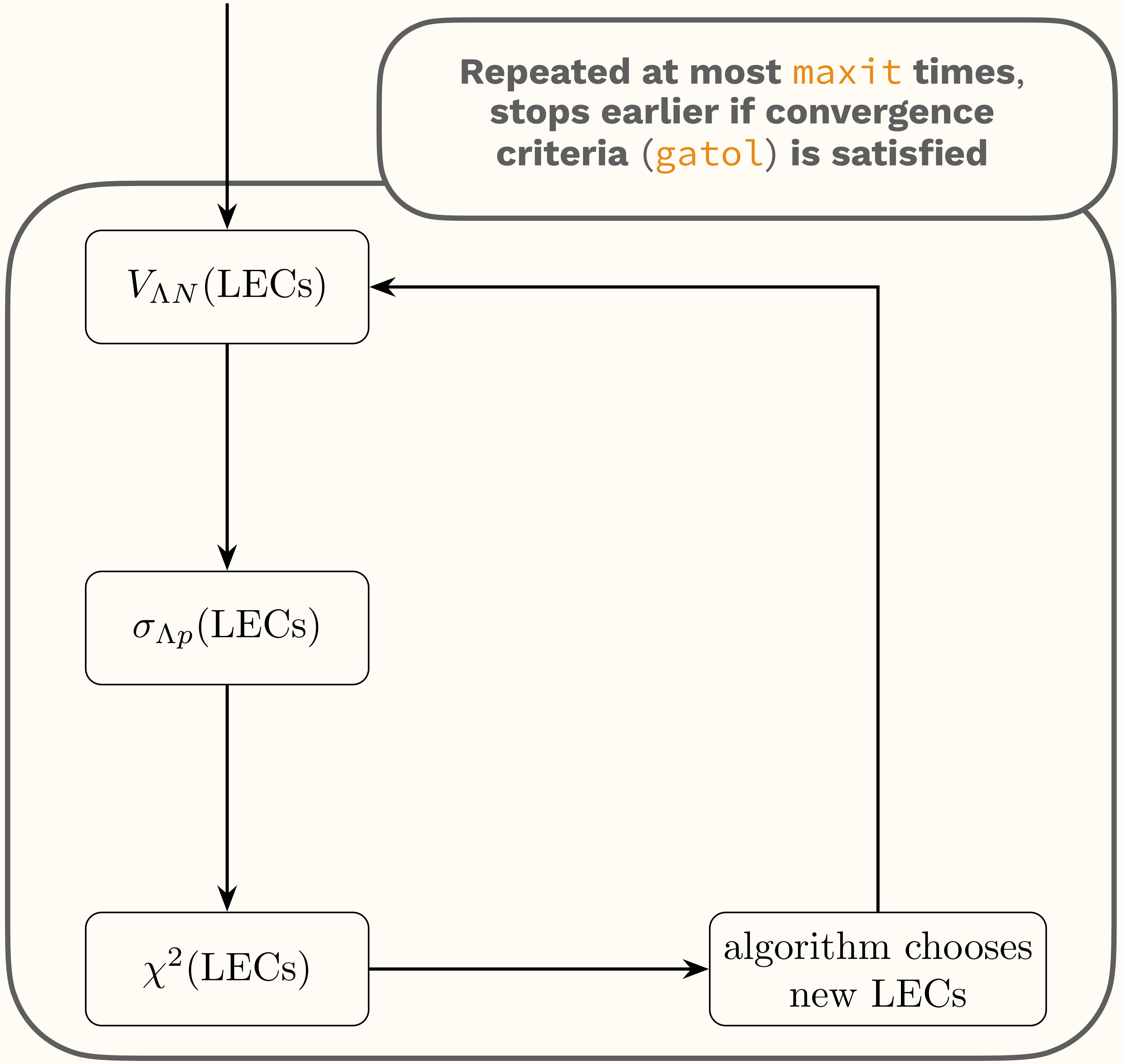


- TAOPOUNDERS, from PETSc + MPI to parallelise
- Adjustable parameters of the fitting procedure:
  - Cutoff parameter R_0
  - **Grid for initial values** of the LECs (`max, min, step`)

FITTING PROCEDURE

Minimization algorithm

LECs initial values



- TAOPOUNDERS, from PETSc + MPI to parallelise
- Adjustable parameters of the fitting procedure:
  - Cutoff parameter R_0
  - Grid for initial values of the LECs (`max, min, step`)
  - **Maximum number of calls** to optimization algorithm (`maxit`)
  - Parameter to define the threshold for **converged optimization** (`gatol`)
- Chosen values for algorithm parameters:
  - $R_0 \in [1.5, 2.5]$ fm
  - `gatol` $= 10^{-8}$
  - `maxit` $= 2 \times 10^3$

FITTING PROCEDURE

LO vs NLO fitting strategy

LO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 15$ MeV
  - $J = 0, 1$
  - Only positive parity
- All LECs chosen on a grid $\longrightarrow$ 2 LECs

NLO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 80$ MeV
  - $J = 0, 1$
  - Both parities
- LO LECs (i.e. C_s, C_T) fixed at LO best results $\longrightarrow$ 7 LECs
  NLO LECs (i.e. $C_1, \ldots, C_5$) chosen on a grid

FITTING PROCEDURE
LO vs NLO fitting strategy

LO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 15$ MeV
  - $J = 0, 1$
  - Only positive parity
- All LECs chosen on a grid $\longrightarrow$ **2 LECs**
- Parameters for grid of initial values:
  - min $= -15$
  - max $= 15$ $\Rightarrow$ ~ 900 points
  - step $= 1$

NLO

- Total angular momentum, energy and parity constraints:
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- LO LECs (i.e. C_s, C_T) fixed at LO best results $\longrightarrow$ **7 LECs**
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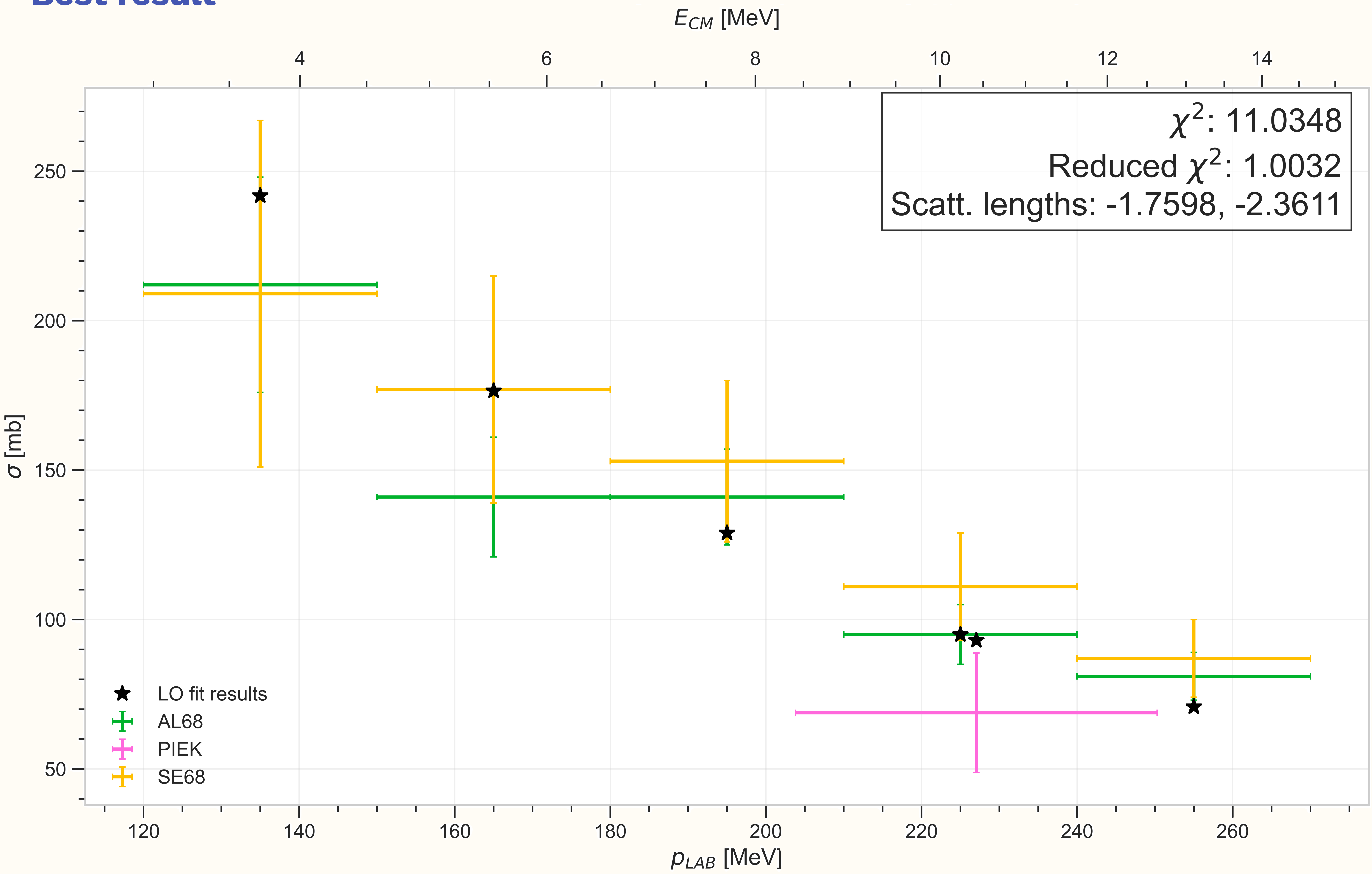
- Parameters for grid of initial values:
  - min $= -15$
  - max $= 15$ $\Rightarrow$ $\sim 10^5$ points
  - step $= 3$

PRELIMINARY RESULTS

Leading order – Best result

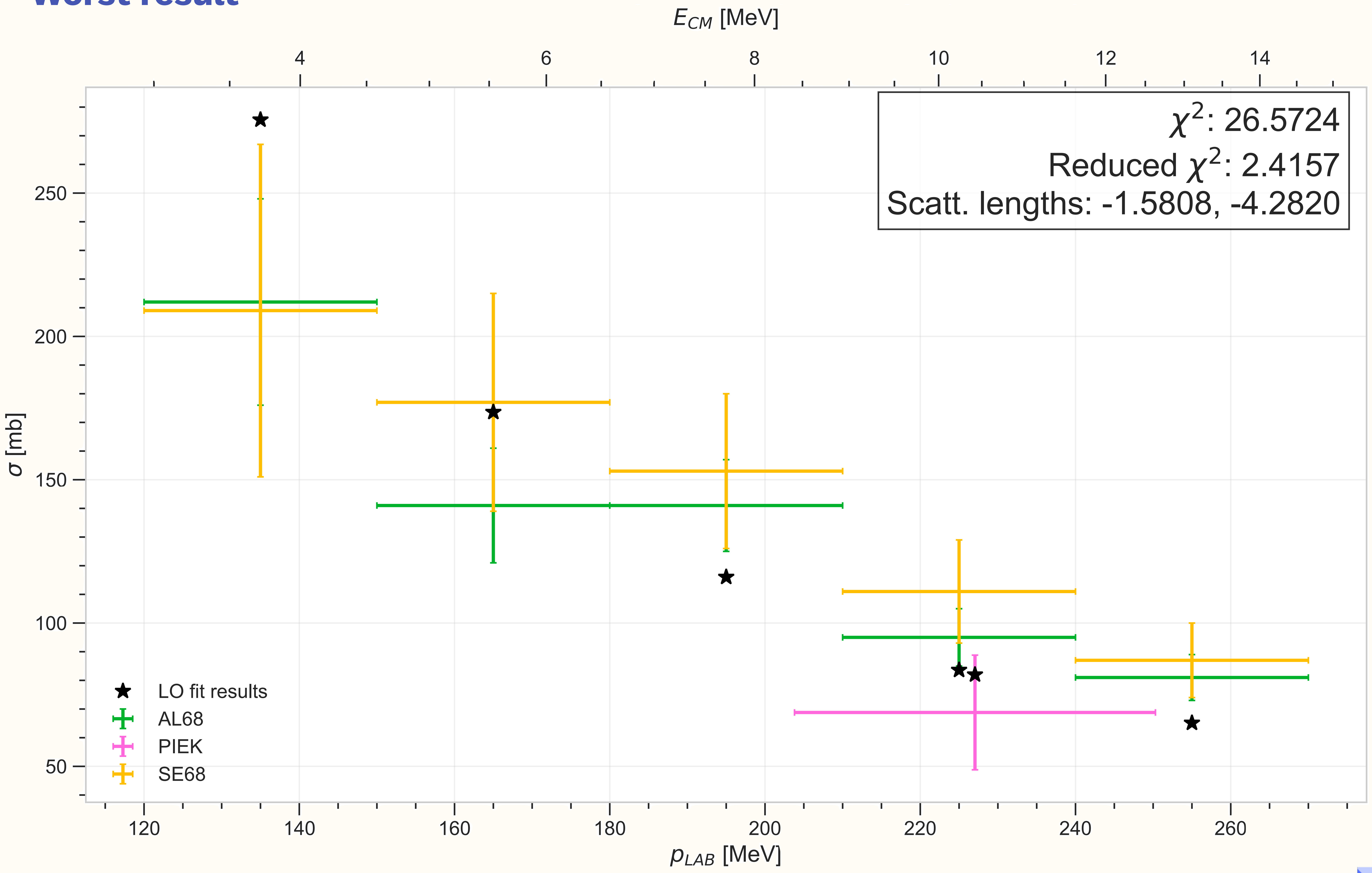
$R_0 = 1.5\ \mathrm{fm}$



PRELIMINARY RESULTS

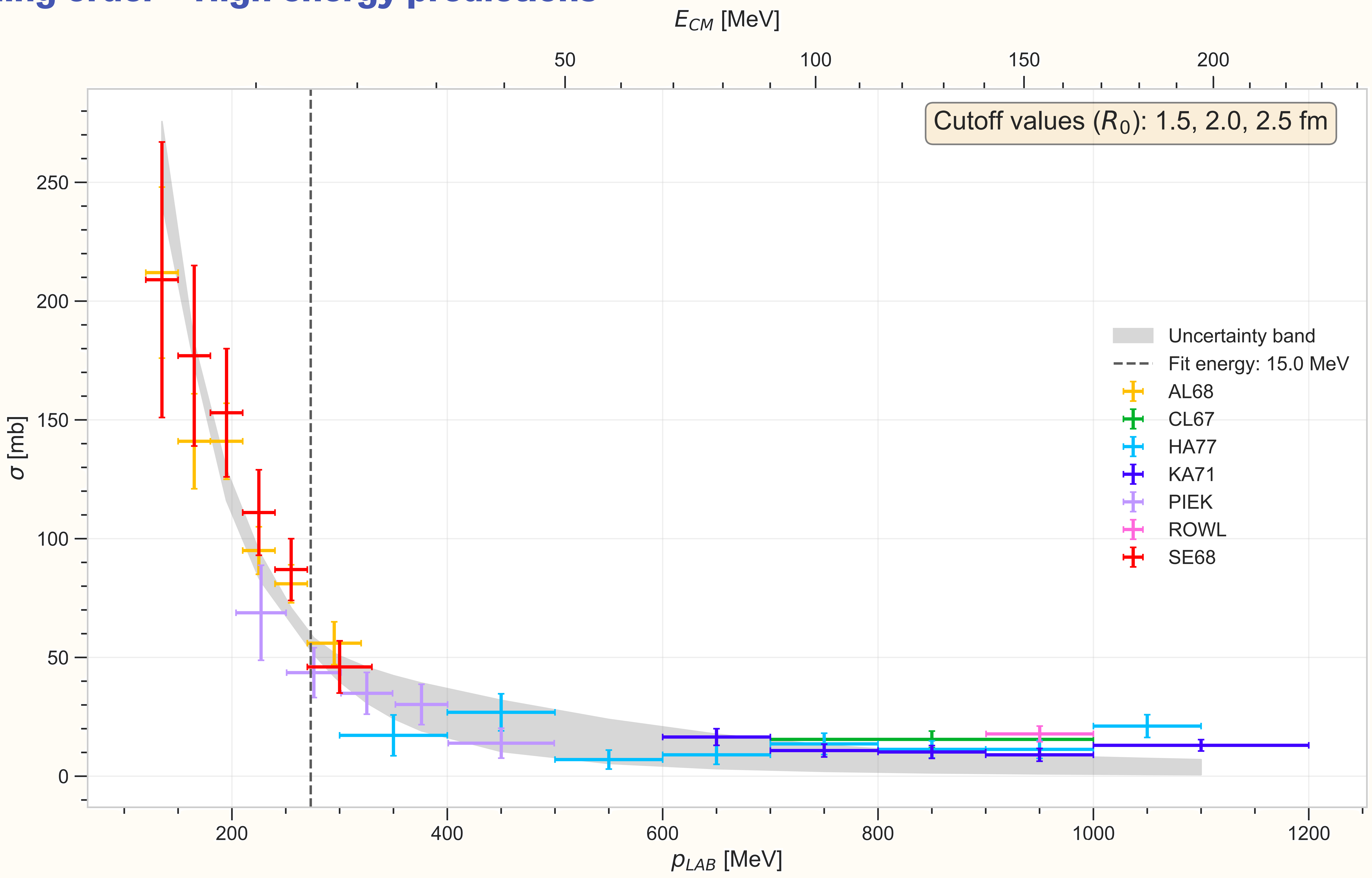
Leading order - Worst result

$R_0 = 2.5\ \mathrm{fm}$



PRELIMINARY RESULTS

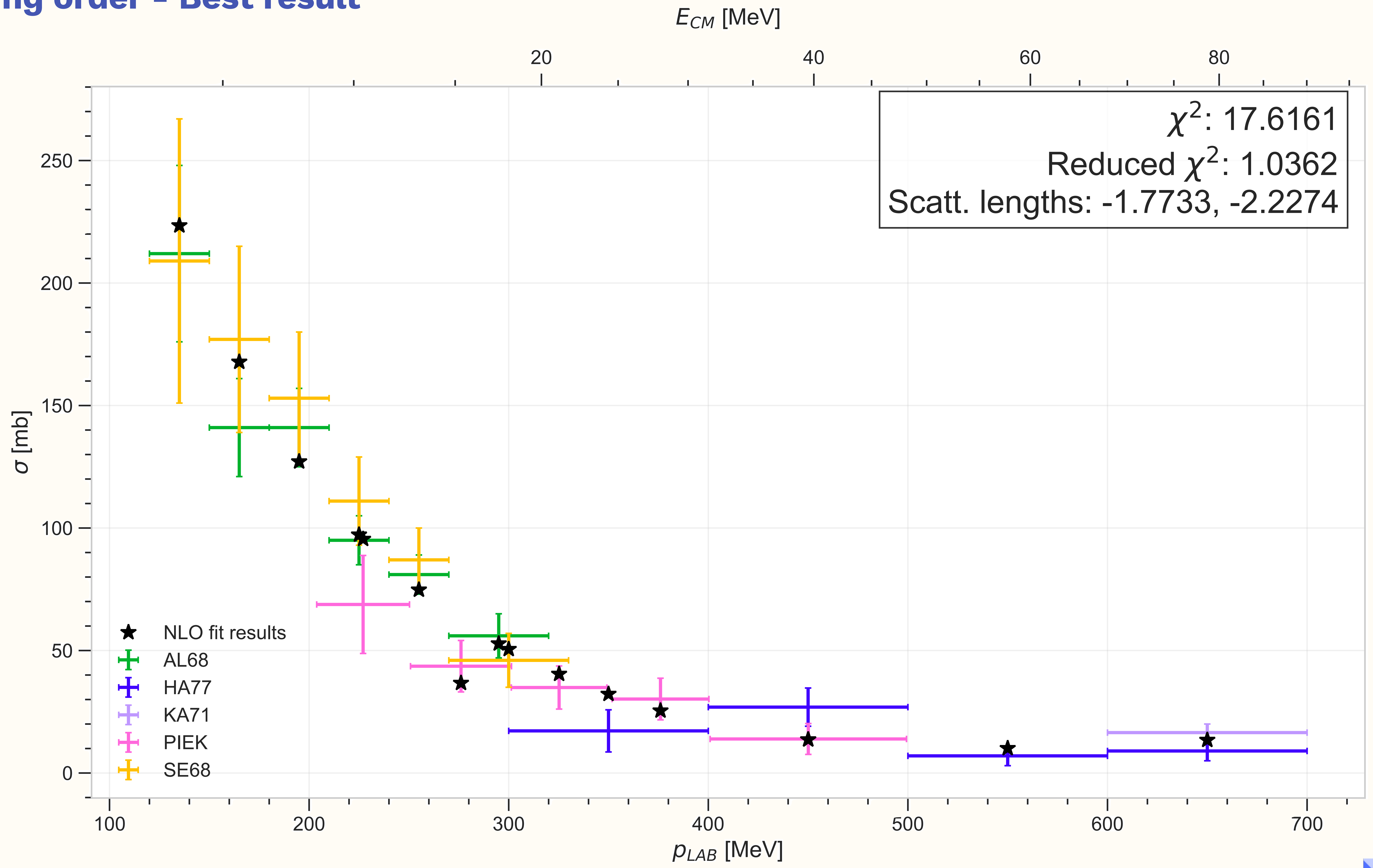
Leading order – High energy predictions



PRELIMINARY RESULTS

Next to leading order – Best result

$R_0 = 1.5\ \mathrm{fm}$



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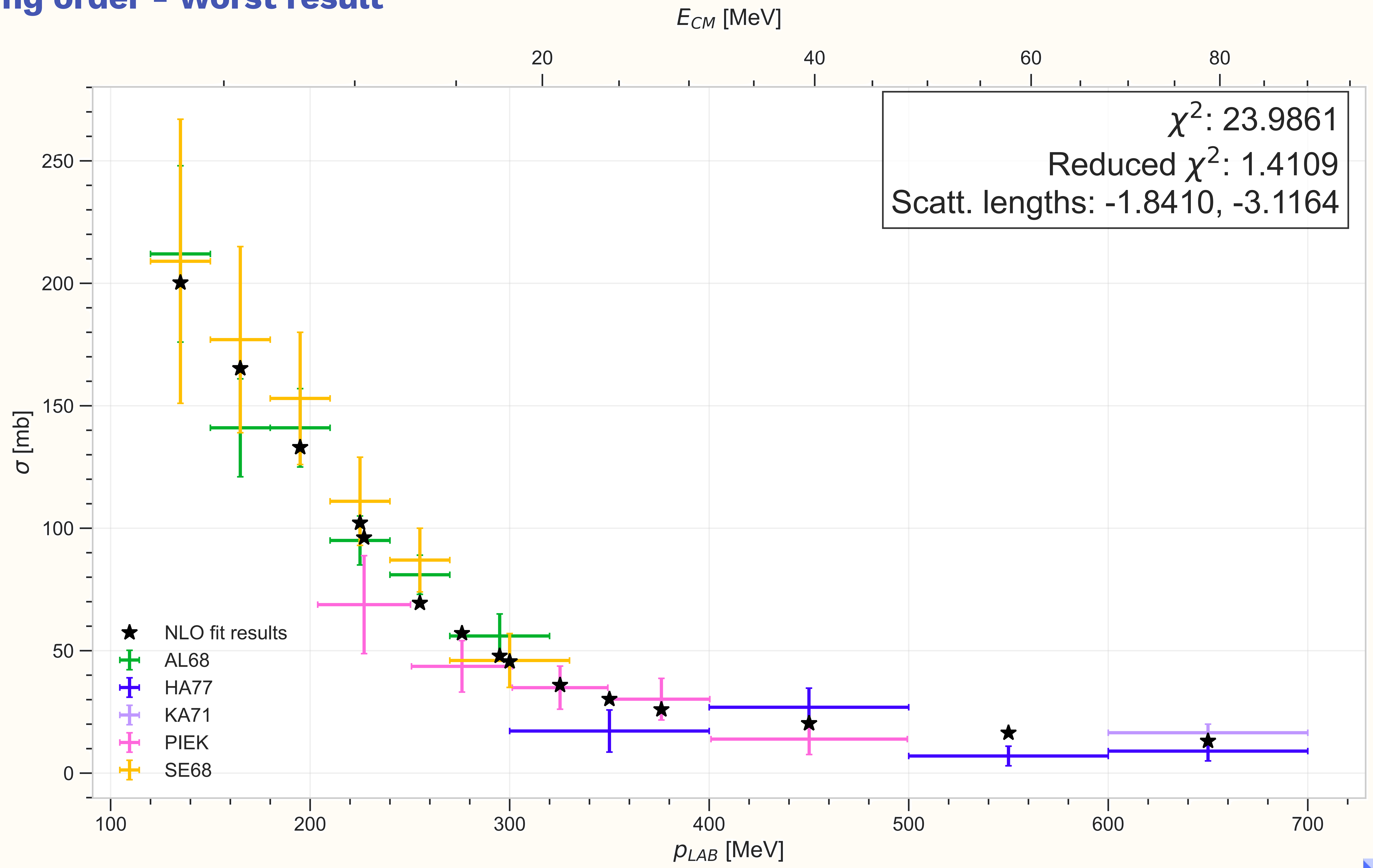

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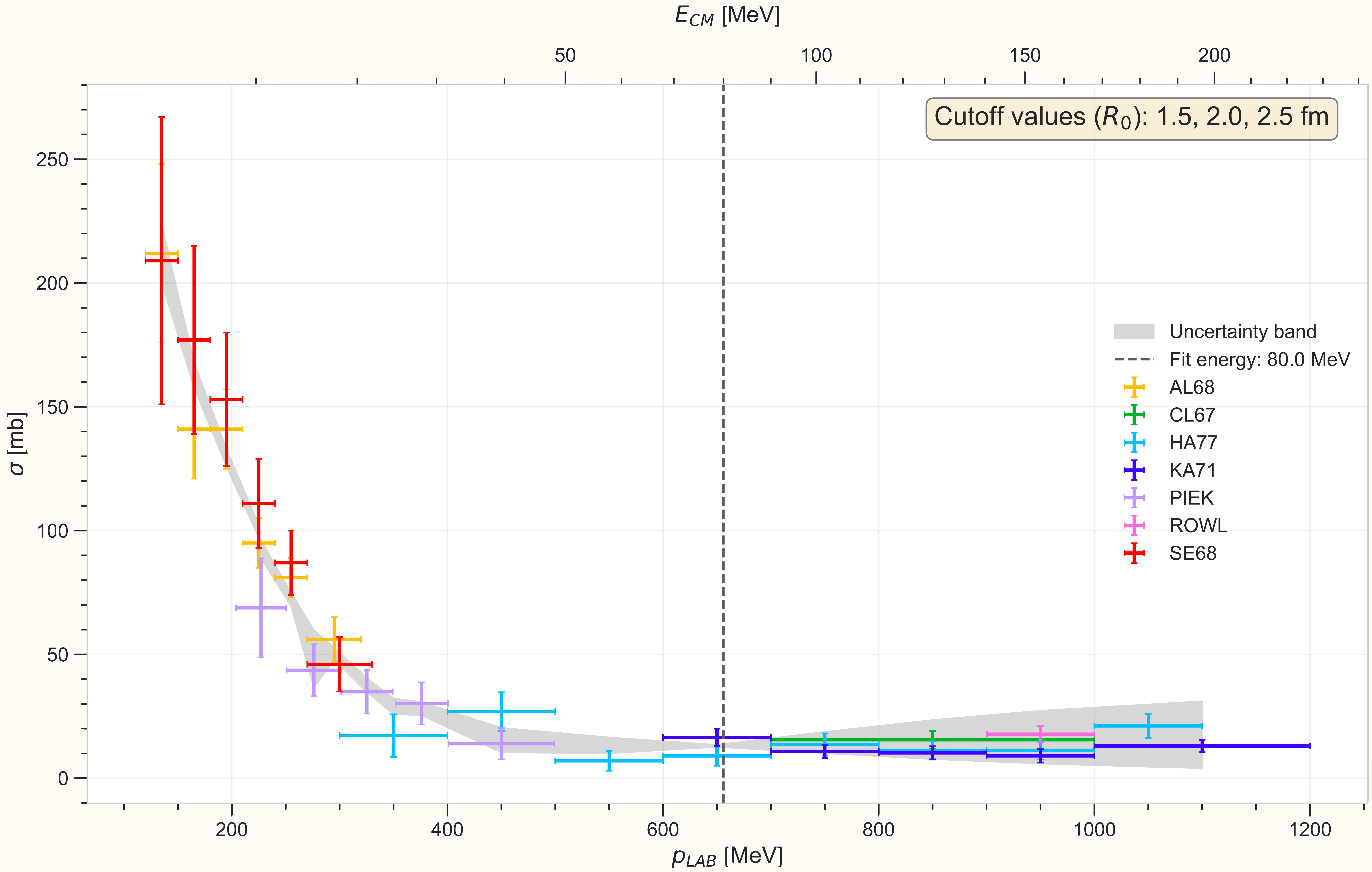

CONCLUSIONS

In **summary**:

- Developed a local contact potential model for the ΛN interaction up to NLO
  - Sophisticated fitting procedure
  - Compatibility with scattering data and scattering lengths
- Further analysis on different values of the cutoff

Future developments

- Develop a local χEFT potential model, with $\Lambda N - \Sigma N$ coupling
- Three-body forces (YNN, YYN, YYY)
- Hypernuclei studies and $pp\Lambda$ correlation functions
- Studies on NS EoS

THANK YOU FOR YOUR ATTENTION !

Margherita Sagina (University of Pisa, INFN - Pisa)
margherita.sagina@phd.unipi.it

POTENTIAL MODEL DERIVATION

Chiral Effective Field Theory

- Interaction described by most general Lagrangian that respects symmetries of QCD

- Cutoff scale (Λ_χ) → defines range of applicability of the theory

↓

High energy effects included in contact terms
that depend on low energy constants (LECs)

↘

LECs determined through fit to
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$\downarrow$

High energy effects included in contact terms that depend on low energy constants (LECs)

$\searrow$ LECs determined through fit to experimental data

- Lagrangian expanded in powers of $Q/\Lambda_\chi < 1$

  $\Rightarrow$ Organization in leading and sub-leading terms (LO, NLO, ...)

QCD

$\Lambda_\chi \sim 1\,\text{GeV}$

χEFT

$Q \sim 100\,\text{MeV}$

FITTING PROCEDURE

Defining energy limits

- Partial wave projection of the potential, in momentum space

$$V(^1S_0) = 4\pi(C_S - 3C_T) + \pi(4C_1 + C_2 - 12C_3 - 3C_4 - 4C_6 - C_7)(p^2 + p'^2)\,,$$

$$V(^3S_1) = 4\pi(\underbrace{C_S + C_T}_{\text{LO LECs}}) + \pi\frac{3}{2}(12C_1 + 3C_2 + 12C_3 + 3C_4 + 4C_6 + C_7)(p^2 + p'^2)\,.$$

- Semi classical approach to compute p_{LAB} threshold for LO $\quad \ell\,\hbar = p_{LAB}\,b$

- $\ell = 1$ to exclude P-waves and higher-order partial waves
- $b \sim 1$ fm, $\hbar c = 197.33$ MeV fm

$\Big\}\quad p_{LAB} \sim 200$ MeV $\Rightarrow E_{CM} \sim 15$ MeV