Semi-inclusive neutrino-nucleus scattering within the relativistic distorted wave approach

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SEZIONE DI LECCE

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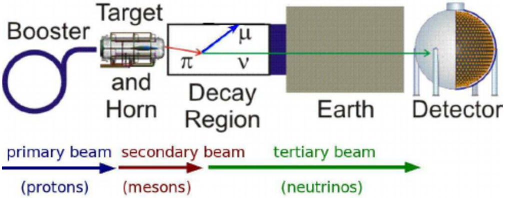
What do we need neutrino-nucleus scattering for?

$$P_{\alpha\to\beta} = \sin^2 2\theta \sin^2 \frac{L\Delta m^2}{4E_\nu}$$

To measure neutrino oscillation parameters we need to know the energy of the neutrinos



Typical targets: ^{12}C, ^{16}O and ^{40}Ar

Type of Uncertainty	$\nu_e/\bar{\nu}_e$ Candidate Relative Uncertainty (%)
Super-K Detector Model	1.5
Pion Final State Interaction and Rescattering Model	1.6
Neutrino Production and Interaction Model Constrained by ND280 Data	2.7
Electron Neutrino and Antineutrino Interaction Model	3.0
Nucleon Removal Energy in Interaction Model	3.7
Modeling of Neutral Current Interactions with Single γ Production	1.5
Modeling of Other Neutral Current Interactions	0.2
Total Systematic Uncertainty	6.0

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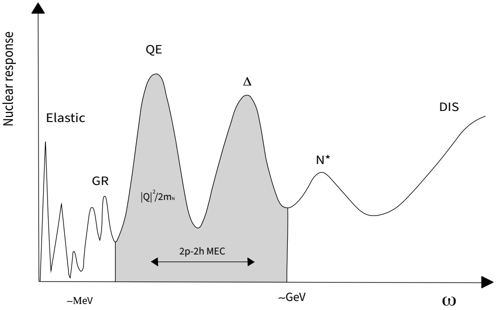
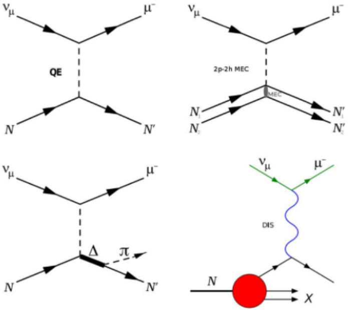
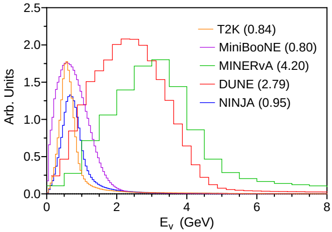
Charged-current neutrino-nucleus interactions





The energy distribution of the neutrinos in the beam is very broad compared with almost monochromatic beams used in electron scattering



Charged-current neutrino-nucleus interactions

The experimental signal is a combination of all different processes occurring inside the nucleus

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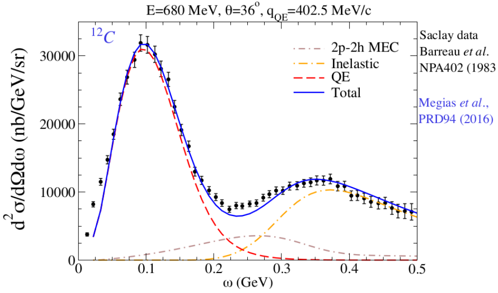

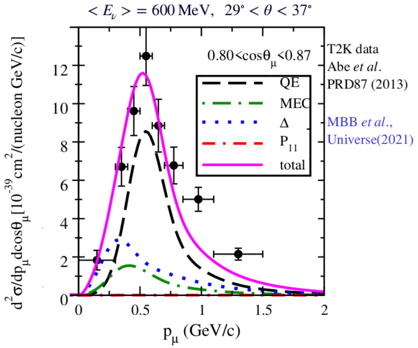

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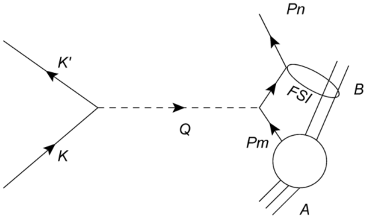
Inclusive vs semi-inclusive scattering

One proton knockout process in the IA



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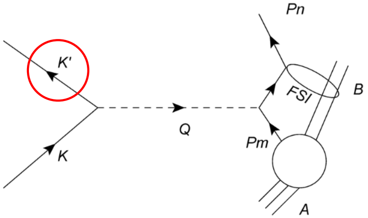
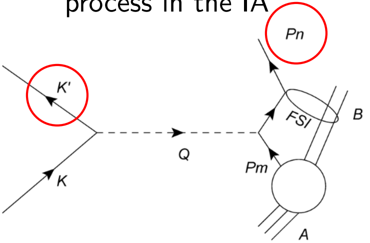


$\longrightarrow$ Inclusive process: only the final lepton k' is detected

Inclusive vs semi-inclusive scattering

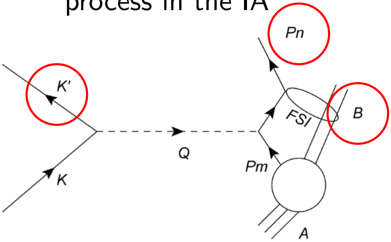
One proton knockout process in the IA



$\longrightarrow$ Semi-inclusive process: one or more particles are detected in coincidence with the final lepton (k' and p_N)

Inclusive vs semi-inclusive scattering

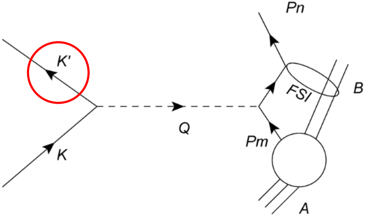
One proton knockout process in the IA



$\longrightarrow$ Exclusive process: the complete final system is known, including the residual nucleus (possible for electron but not for neutrino scattering)

Inclusive vs semi-inclusive scattering

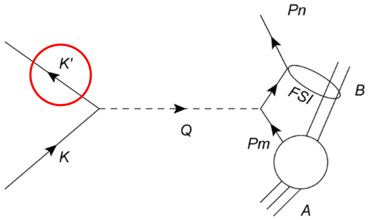
One proton knockout process in the IA



$\longrightarrow$ Until not many years ago, the majority of the experimental and theoretical work in neutrino reactions have focused on inclusive reactions.

Inclusive vs semi-inclusive scattering

One proton knockout process in the IA



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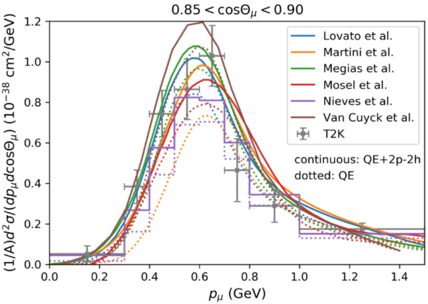
A. *Branca et al.*, Symmetry **2021**, 13, 1625



A good agreement between theory and experiment for this kind of reactions can be achieved using very different approaches.

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Semi-inclusive neutrino-nucleus formalism

$$\left\langle \frac{d^6\sigma}{dk'\, d\Omega_{k'}\, dp_N d\Omega_N^L} \right\rangle = \int dk\, P\,(k) \times K \times L_{\mu\nu} H^{\mu\nu} \longrightarrow$$

The leptonic tensor $L_{\mu\nu}$: function of initial and final leptons kinematics.

Semi-inclusive neutrino-nucleus formalism

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The hadronic tensor $H^{\mu\nu}$: contains the information about nuclear dynamics

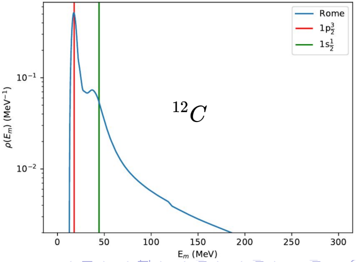
$$H^{\mu\nu}_{\kappa} = \rho_\kappa (E_m) \times \sum_{m_j, s_N} \left[J_{\kappa, m_j, s_N} (Q, p_N) \right]^* J_{\kappa, m_j, s_N} (Q, p_N)$$

$$J^{\mu}_{\kappa, m_j, s_N} = \int d\mathbf{r} \overline{\Psi}_{s_N} (\mathbf{p_N}, \mathbf{r}) \left(F_1 \gamma^\mu + \frac{iF_2}{2m_N} \sigma^{\mu\nu} Q_\nu + G_A \gamma^\mu \gamma^5 + \frac{G_P}{2m_N} Q^\mu \gamma^5 \right) \Psi^{m_j}_{\kappa} (\mathbf{r})$$

- Scattered W.F.
- CC2 operator
- Bound W.F.

Description of the initial nuclear state:

- Pure shell model: missing energy profile is given by a Dirac delta per shell
- Realistic model, *i.e.* Benhar spectral function: short- and long-range correlations included



Scattered nucleon description: RPWIA and RDWIA

Regarding the scattered nucleon, we consider different models:

- Relativistic plane-wave impulse approximation (RPWIA): The ejected nucleon is considered a plane-wave (no FSI).

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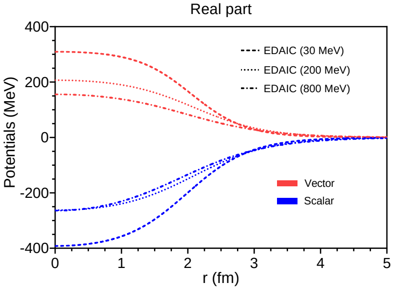
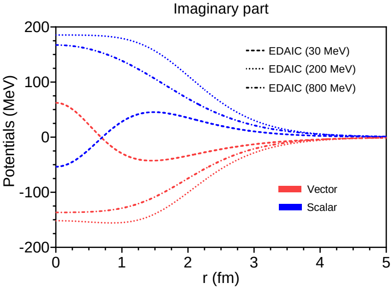
- Relativistic optical potential (ROP): The scattered nucleon travels under the influence of a phenomenological relativistic optical potential fitted to elastic proton-nucleus scattering data.





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$$\Psi_{s_N}(\mathbf{r},\mathbf{p_N}) = 4\pi\sqrt{\frac{E_N+m_N}{2E_N}}\sum_{\kappa,m_l,m_j} e^{-i\delta^*_\kappa} i^l \langle l\, m_l\, 1/2\, s_N | j\, m_j\rangle\, Y^*_{l\,m_l}(\Omega_N)\,\psi^{m_j}_\kappa(\mathbf{r},\mathbf{p_N})$$

$$\Psi^{m_j}_\kappa(\mathbf{r}) = \begin{pmatrix} g_\kappa(r)\Phi^{m_j}_\kappa(\hat{\mathbf{r}}) \\ i f_\kappa(r)\Phi^{m_j}_{-\kappa}(\hat{\mathbf{r}}) \end{pmatrix}$$

$$\frac{df_\kappa}{dr} = \frac{\kappa-1}{r} f_\kappa - [E_N - m_N - S(r) - V(r)]\, g_\kappa$$

$$\frac{dg_\kappa}{dr} = -\frac{\kappa+1}{r} g_\kappa + [E_N + m_N + S(r) - V(r)]\, f_\kappa .$$

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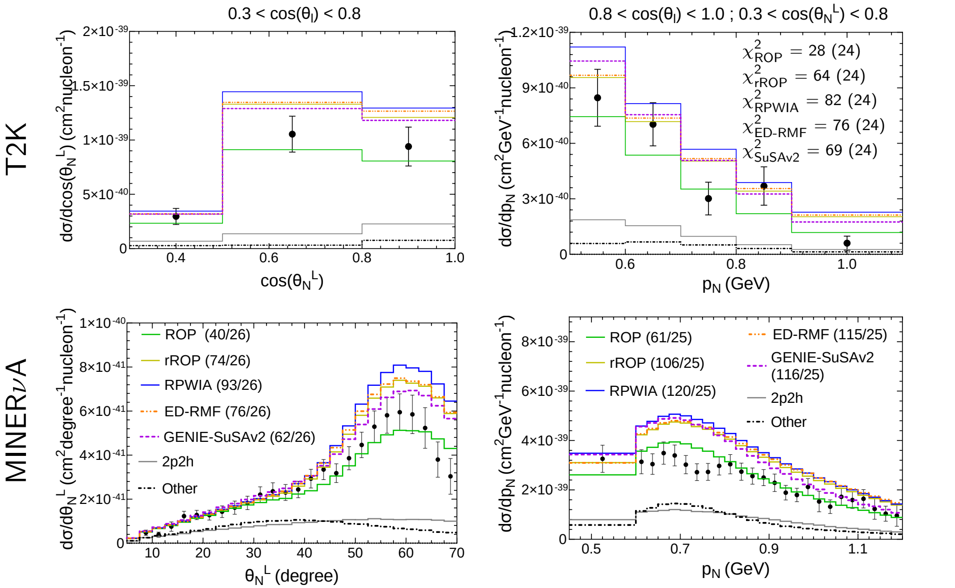
Cross sections vs proton kinematics: T2K and MINERνA



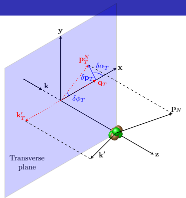
Cross sections vs transverse kinematic imbalances

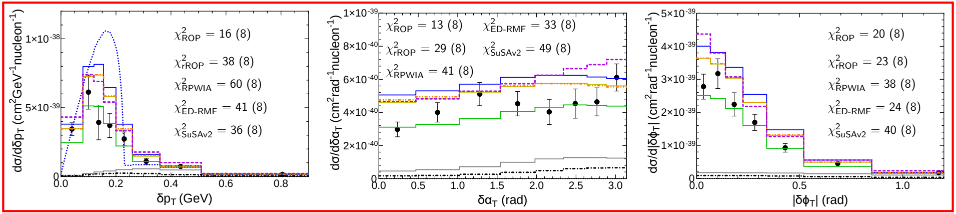
$$\delta p_T = \left| \mathbf{k}'_\mathbf{T} + \mathbf{p}^\mathbf{N}_\mathbf{T} \right|$$ ⟶ Peaked distribution at zero

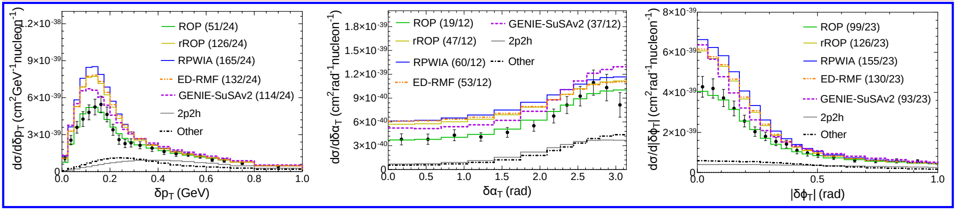
$$\delta \alpha_T = \arccos\left(\frac{-\mathbf{k}'_\mathbf{T} \cdot \delta \mathbf{p_T}}{k'_T \delta p_T} \right)$$ ⟶ Undefined, flat distribution

$$\delta \phi_T = \arccos\left(\frac{-\mathbf{k}'_\mathbf{T} \cdot \delta \mathbf{p}^\mathbf{N}_\mathbf{T}}{k'_T p^N_T} \right)$$ ⟶ Peaked distribution at zero

- T2K (red)
- MINERνA (blue)







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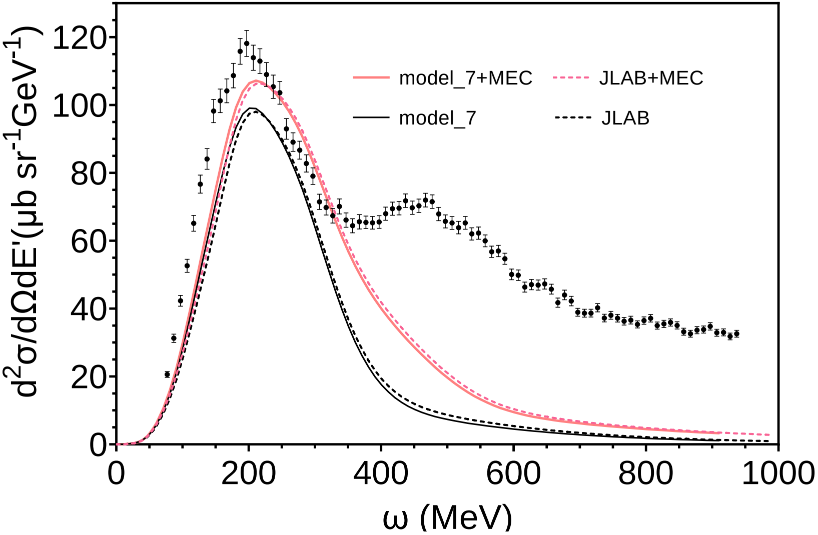
Summary

- Experimental and theoretical efforts to measure and describe semi-inclusive cross sections to help constrain nuclear models for oscillation experiments.
- The ED-RMF and ROP models have been successfully applied in the past to the study of inclusive and exclusive electron scattering, therefore they could be good candidates to be used for neutrino scattering.
- We have described several ways to include FSI in our theoretical model which improves in general the agreement with experimental data. Variables that measure correlations between both particles in the final state allow us to discriminate between nuclear models and separate contributions from different channels.

Thanks for your attention!

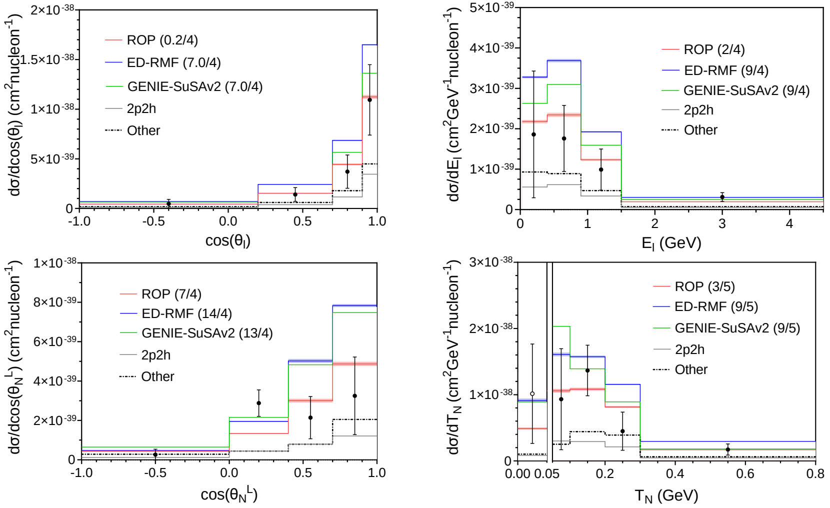
Backup slides

JLAB ^{40}Ar(e,e') $E_i = 2.222$ GeV, $\theta_e = 15.541°$

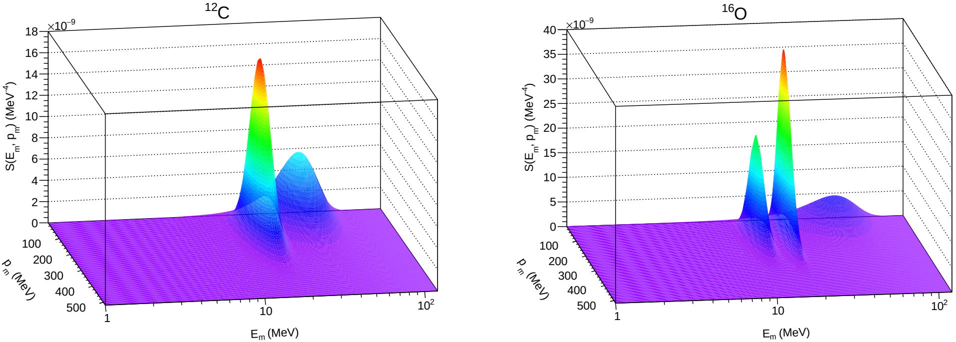


MicroBooNE 1eCC0πNp: electron and proton kinematics

1eCC0πNp	k' (GeV)	$\cos\theta_l$	p_N (GeV)
	> 0.0305	-	> 0.3105

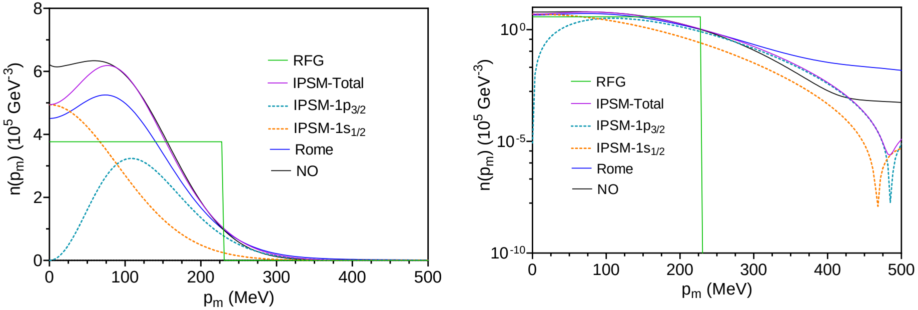


^{12}C and ^{16}O Benhar SF



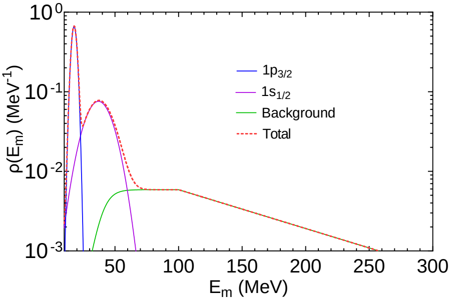
3-D plot of the carbon (left) and oxygen (right) spectral functions calculated using the Rome approach, i.e. the single-particle contribution extracted from the analysis of $(e, e'p)$ experimental data and using the LDA for the correlated part.

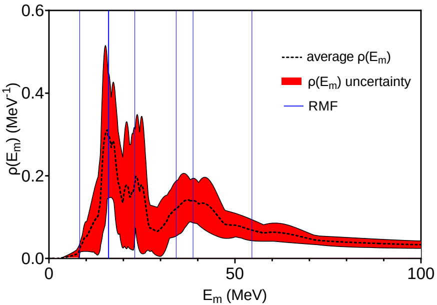
Momentum distributions of different ^{12}C nuclear models



Momentum distribution $n(p_m)$ calculated by integrating the spectral function of different nuclear models of ^{12}C. The value of the Fermi momentum for ^{12}C is $k_F = 0.228$ GeV and the individual shells that are included in the IPSM are shown separately. The distributions are shown in linear (left) and semi-logarithmic (right) scales to expose the differences between the models in the low- and high-missing momentum zones.

Missing energy profiles $\rho(E_m)$ for ^{12}C and ^{40}Ar





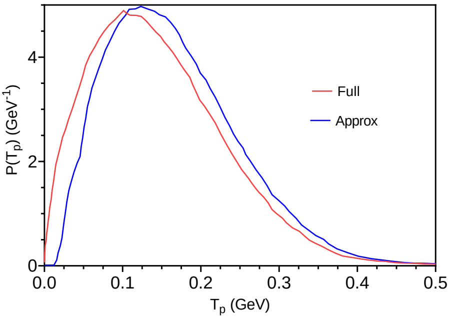
κ	E_κ (MeV)	σ_κ (MeV)	n_κ
$1s_{1/2}$	37.0	10.0	1.9
$1p_{3/2}$	17.8	2.0	3.3

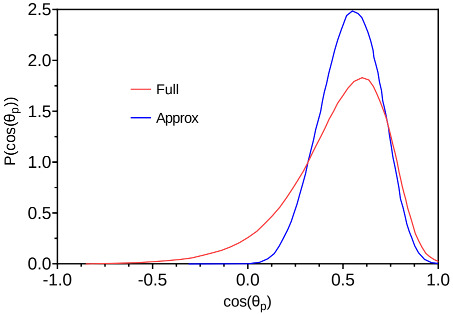
α	E_κ (MeV)	σ_κ (MeV)	S_κ
$1s_{1/2}$	55 ± 6	30 ± 15	0.9 ± 0.15
$1p_{3/2}$	39 ± 4	12 ± 6	0.9 ± 0.15
$1p_{1/2}$	34 ± 3	12 ± 6	0.9 ± 0.15
$1d_{5/2}$	23 ± 2	5 ± 3	0.75 ± 0.15
$2s_{1/2}$	16.1 ± 1.6	5 ± 3	0.75 ± 0.15
$1d_{3/2}$	16.0 ± 1.6	5 ± 3	0.75 ± 0.15
$1f_{7/2}$	9.869 ± 0.005	5 ± 3	0.75 ± 0.15

Final nucleon kinematics generation in GENIE-SuSAv2 model

- Draw initial nucleon momentum $\mathbf{p_m}$ from a Fermi gas momentum distribution.
- Compute $E_m^2 = p_m^2 + m_N^2$.
- $E_N = E_m + \omega - E_b\left(q\right)$
- $p_N^2 = E_N^2 - m_N^2$; $|\mathbf{p_m} + \mathbf{q}| \neq p_N = \sqrt{E_N^2 - m_N^2}$; $\mathbf{p_N} = \frac{p_N}{|\mathbf{p_m}+\mathbf{q}|}\left(\mathbf{p_m} + \mathbf{q}\right)$
- Give the residual momentum to the remnant.

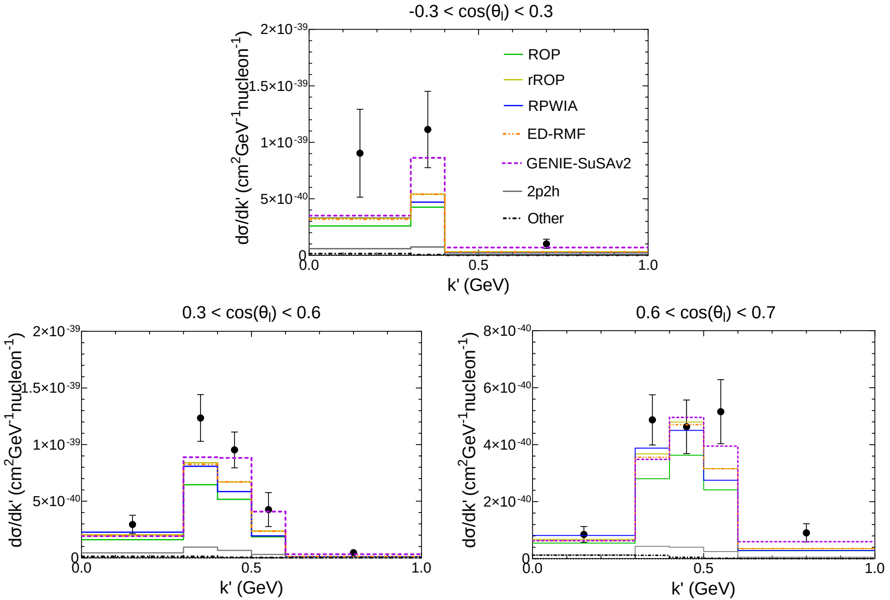
Final nucleon kinematics generation in GENIE-SuSAv2 model





Proton kinetic energy (left) and scattering angle (right) distributions for the process $e + A \to e' + p + B$. The electron beam energy is fixed to 1.159 GeV and the outgoing electron phase-space is limited to $17^\circ < \theta_{e'} < 40^\circ$ and $E_{e'} > 0.4$ GeV. The red lines show the unfactorized RDWIA predictions obtained with ED-RMF potential for carbon (named "Full") while the blue lines show the equivalent distributions obtained after applying the GENIE algorithm and before adding GENIE cascade FSI (named "Approx"). Figure from **A. Nikolakopoulos *et. al.* 2302.12182**

T2K Semi-inclusive cross sections: $p_N < 0.5$ GeV



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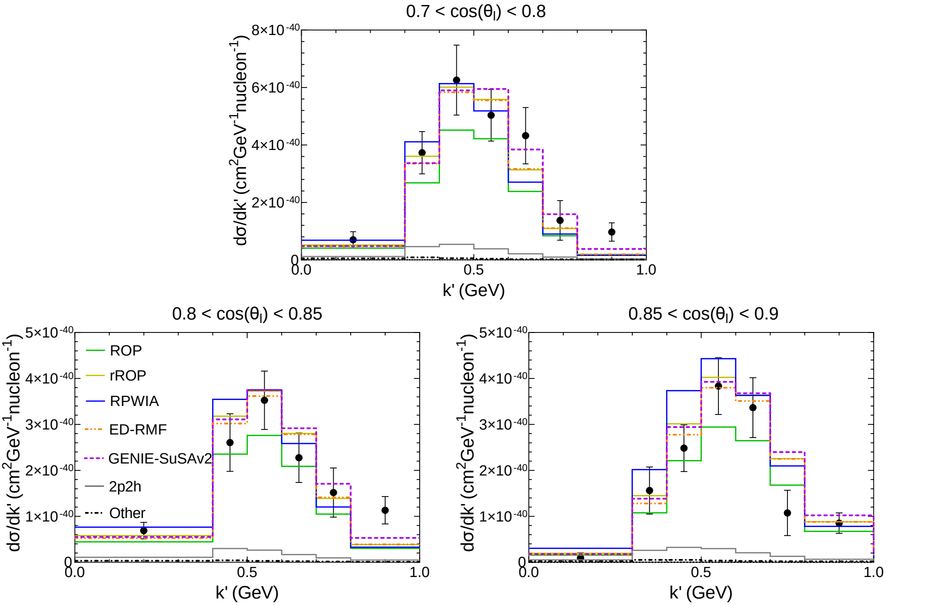

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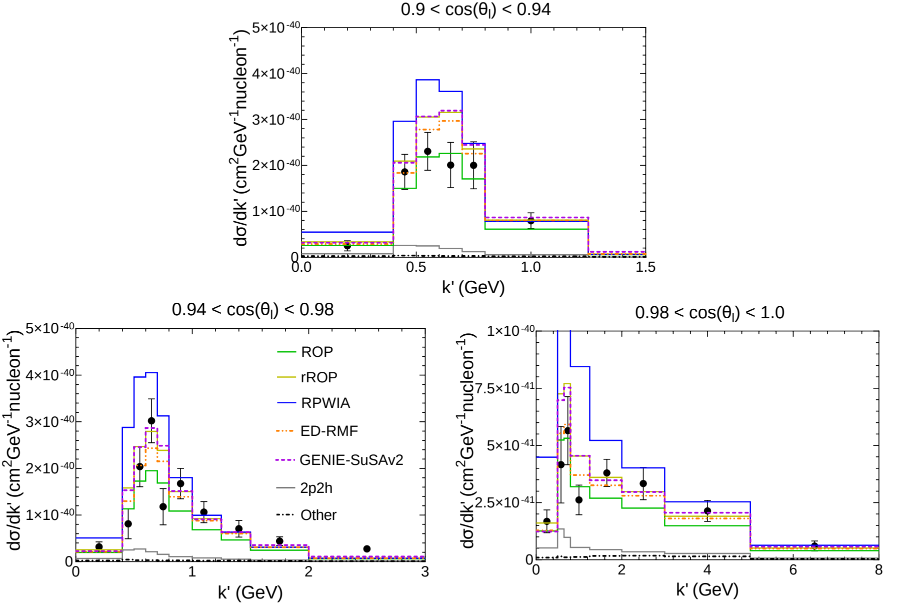
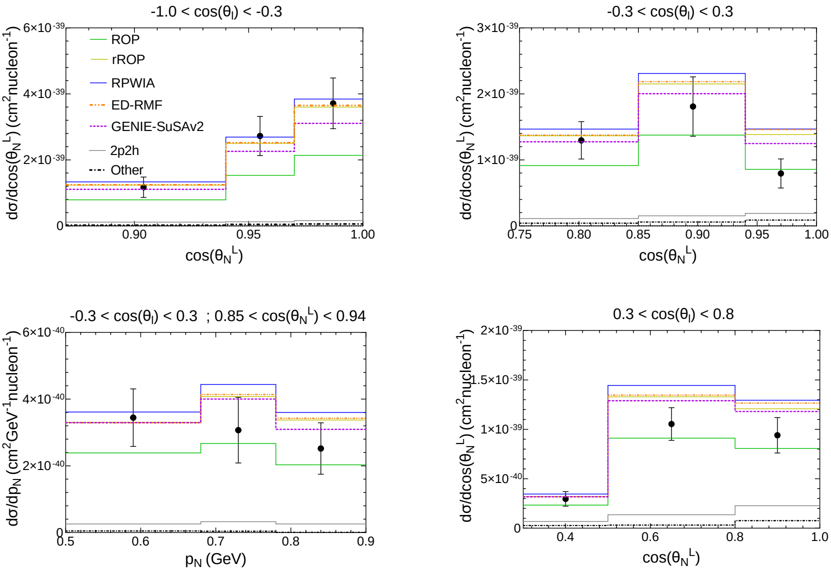


T2K semi-inclusive cross sections: $p_N > 0.5$ GeV



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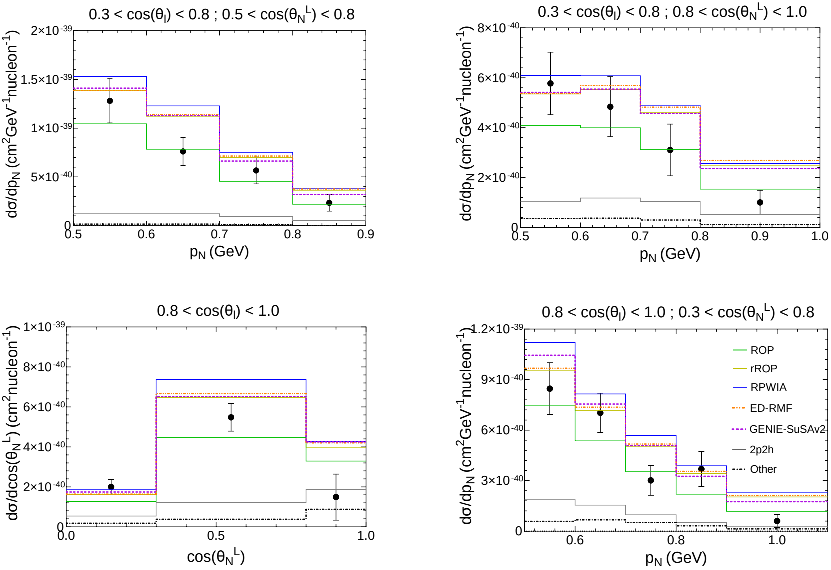

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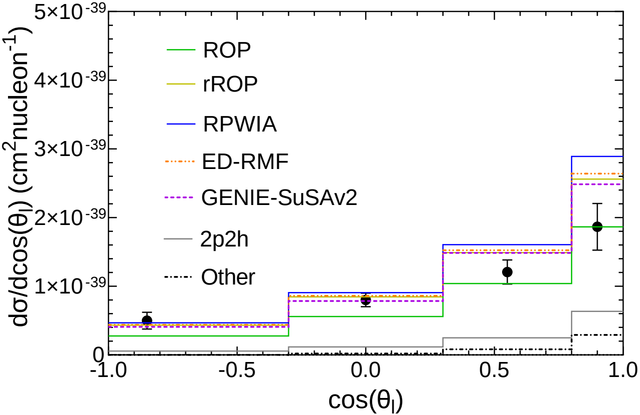
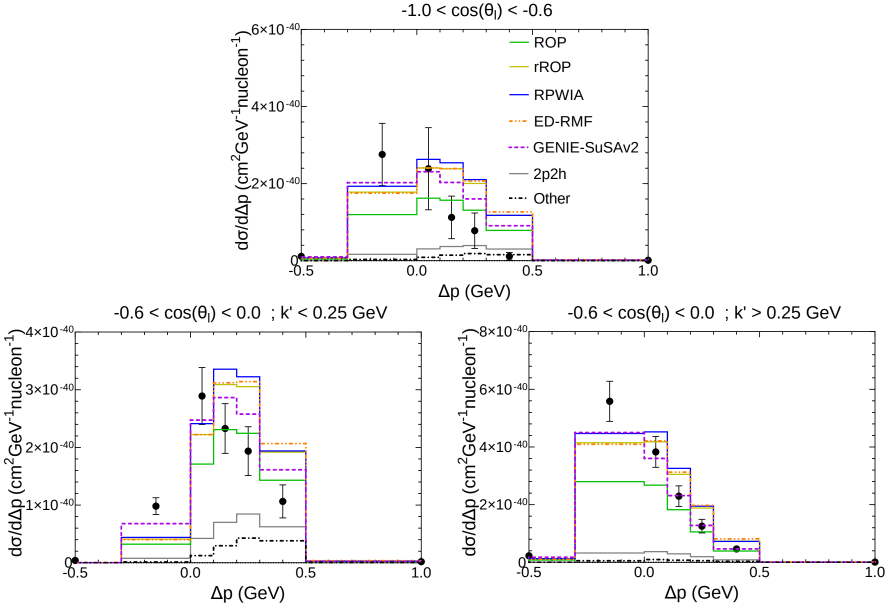
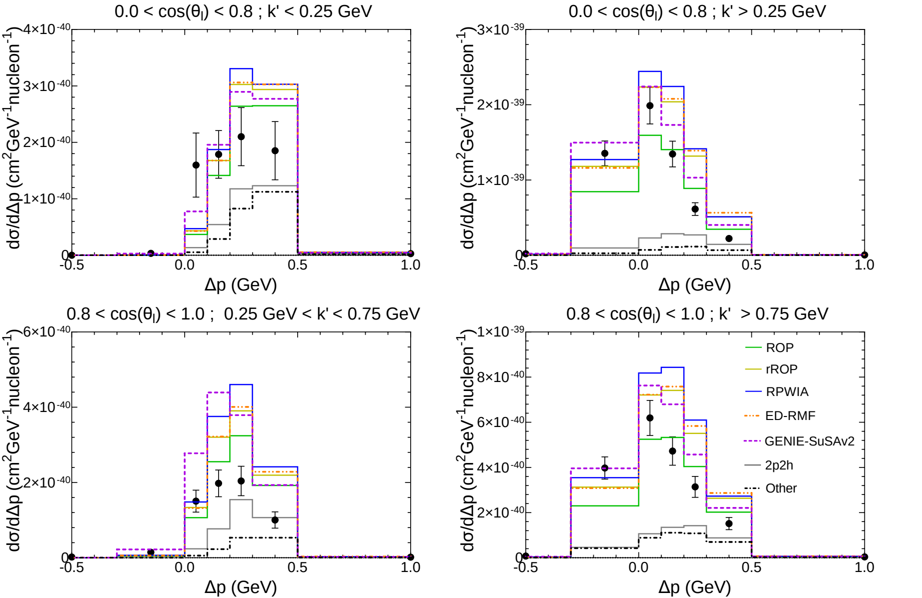


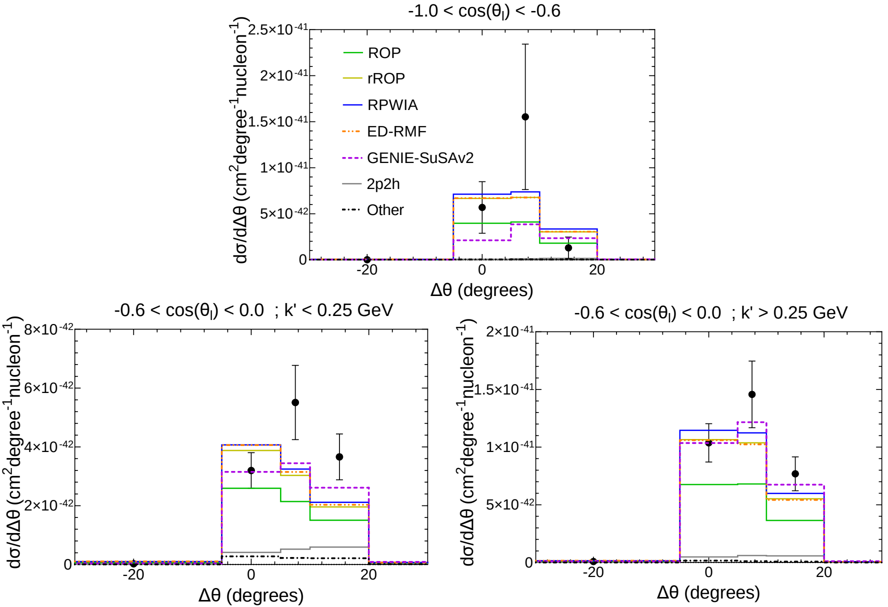
T2K semi-inclusive cross sections: Δp



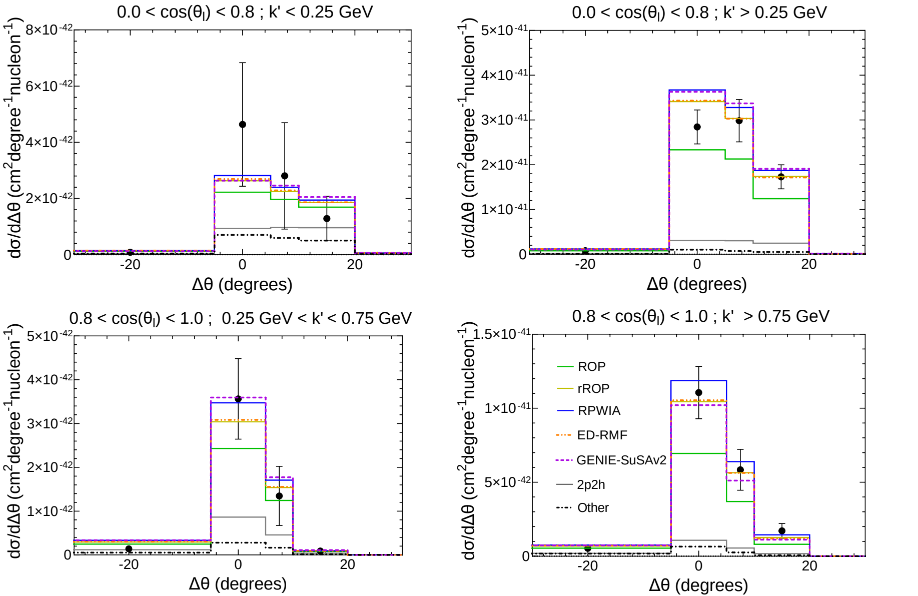
T2K semi-inclusive cross sections: Δp



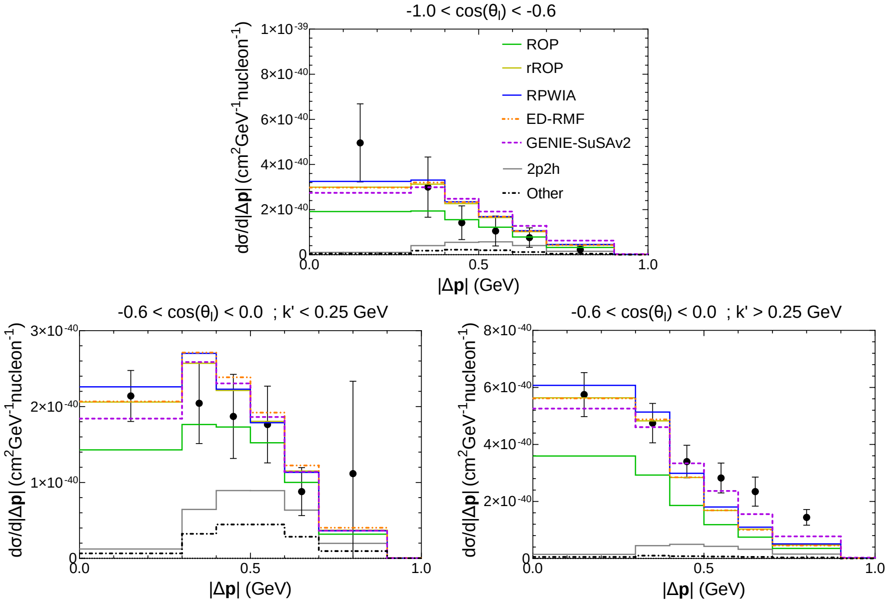
T2K semi-inclusive cross sections: $\Delta\theta$



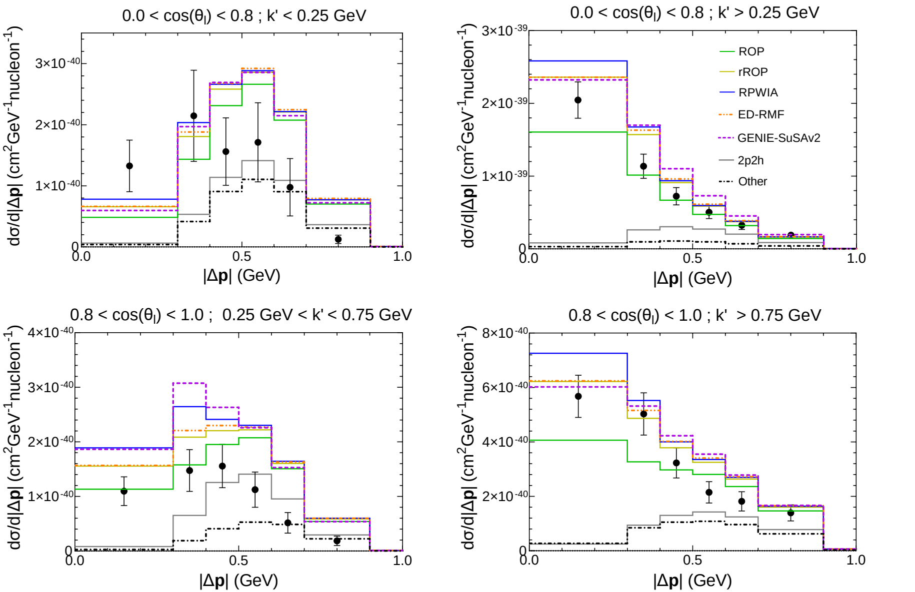
T2K semi-inclusive cross sections: $\Delta\theta$

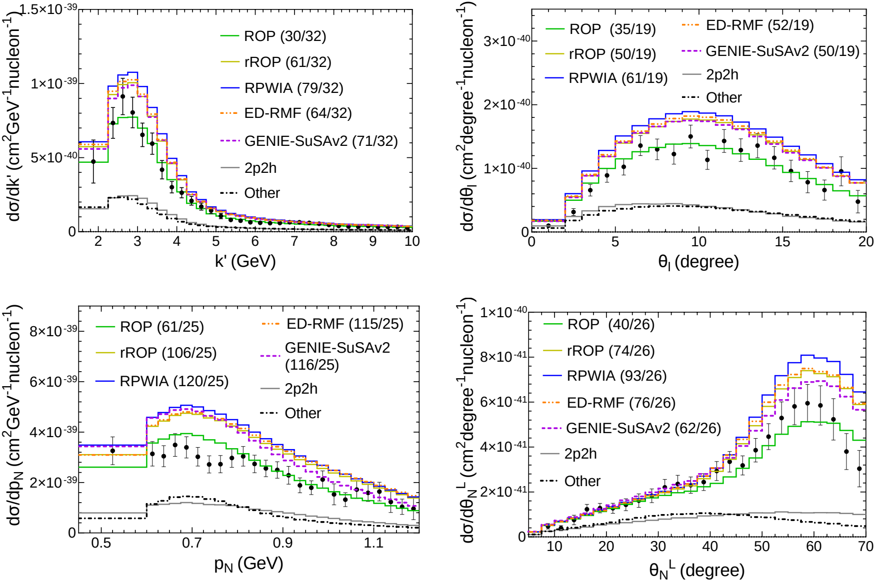


T2K semi-inclusive cross sections: |Δp|



T2K semi-inclusive cross sections: $|\Delta \mathbf{p}|$

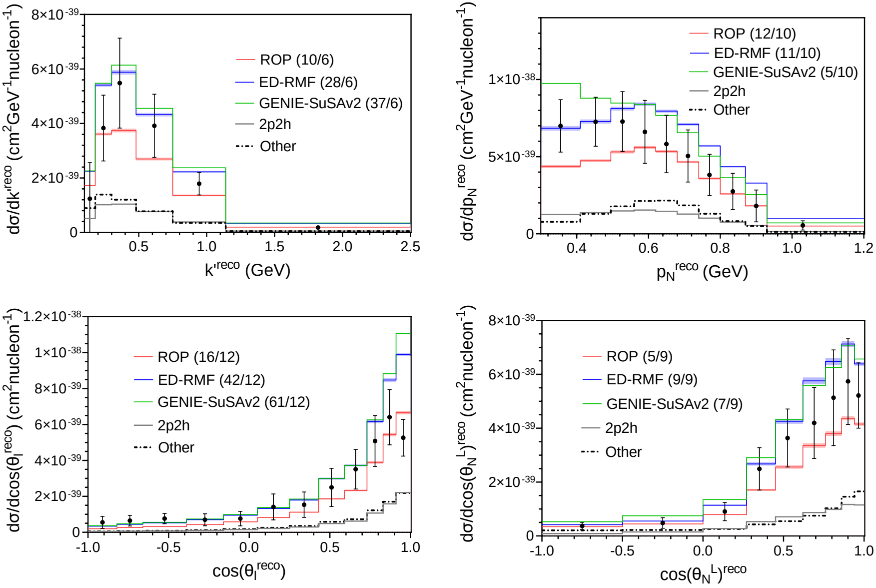



ROP (30/32)
rROP (61/32)
RPWIA (79/32)
ED-RMF (64/32)
GENIE-SuSAv2 (71/32)
2p2h
Other
dσ/dk' (cm²GeV⁻¹nucleon⁻¹)
k' (GeV)
ROP (35/19)
rROP (50/19)
RPWIA (61/19)
ED-RMF (52/19)
GENIE-SuSAv2 (50/19)
2p2h
Other
dσ/dθ_l (cm²degree⁻¹nucleon⁻¹)
θ_l (degree)
ROP (61/25)
rROP (106/25)
RPWIA (120/25)
ED-RMF (115/25)
GENIE-SuSAv2 (116/25)
2p2h
Other
dσ/dp_N (cm²GeV⁻¹nucleon⁻¹)
p_N (GeV)
ROP (40/26)
rROP (74/26)
RPWIA (93/26)
ED-RMF (76/26)
GENIE-SuSAv2 (62/26)
2p2h
Other
dσ/dθ_N^L (cm²degree⁻¹nucleon⁻¹)
θ_N^L (degree)

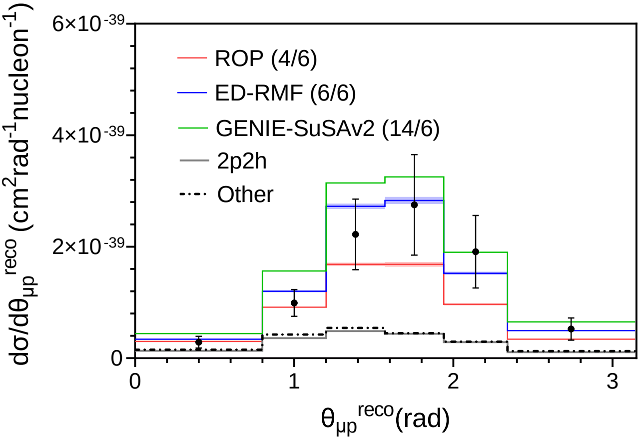
MicroBooNE kinematic cuts

1μCC0πNp	k' (GeV)	$\cos\theta_l$	p_N (GeV)	$\cos\theta_N^L$	ϕ_N^L ($^\circ$)	$\theta_{\mu p}$ ($^\circ$)	δp_T (GeV)
	$>$ 0.1	-	0.3-1.2	-	-	-	-
1eCC0πNp							
	$>$ 0.0305	-	$>$ 0.3105	-	-	-	-
1μCC0π1p							
	0.1-1.5	-0.65-0.95	0.3-1.0	$>$ 0.15	145-215	35-145	$<$ 0.35

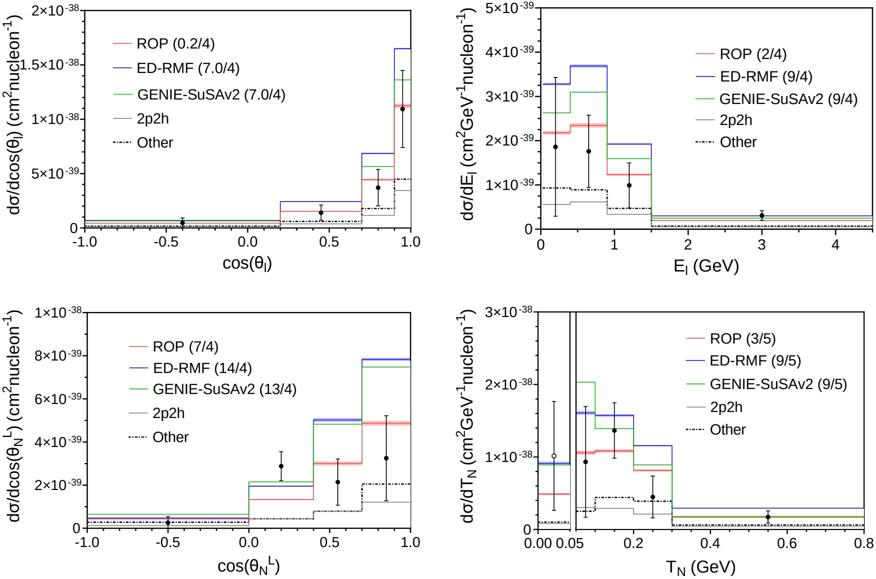
MicroBooNE semi-inclusive cross sections: $1\mu CC0\pi Np$



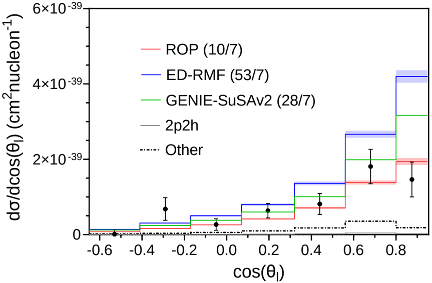
MicroBooNE semi-inclusive cross sections: $1\mu CC0\pi Np$

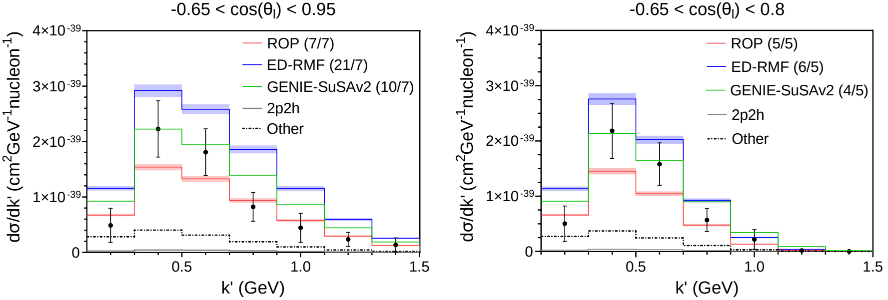


MicroBooNE semi-inclusive cross sections: $1eCC0\pi Np$

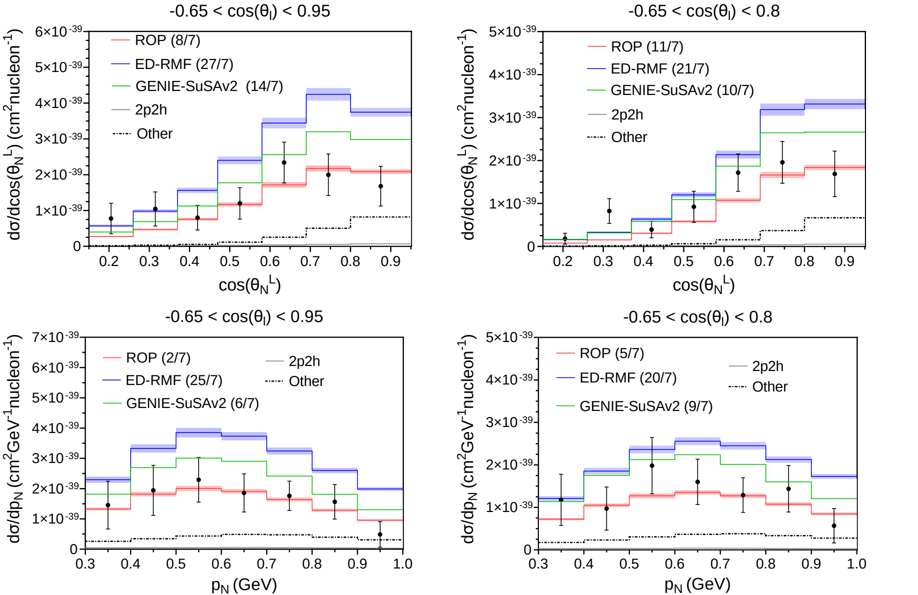


MicroBooNE semi-inclusive cross sections: $1\mu CC0\pi 1p$

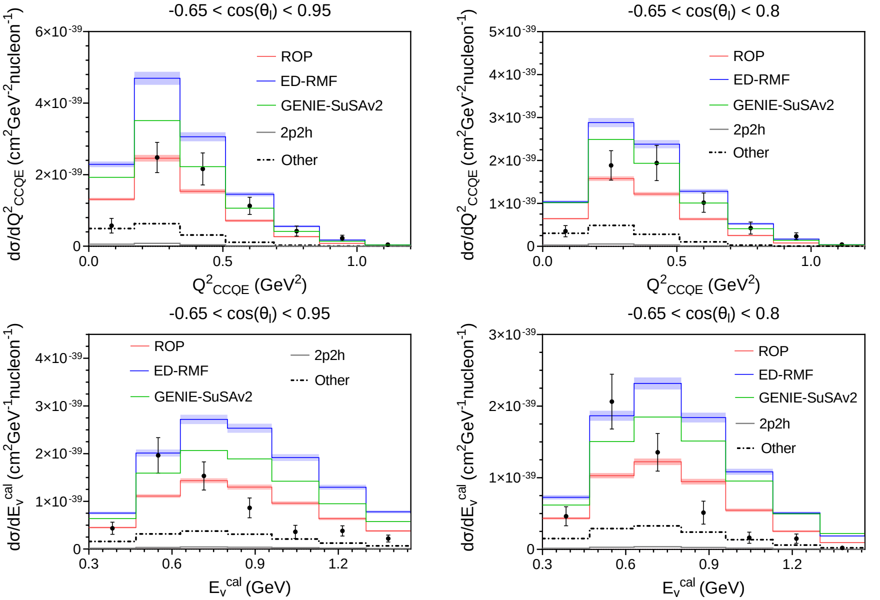




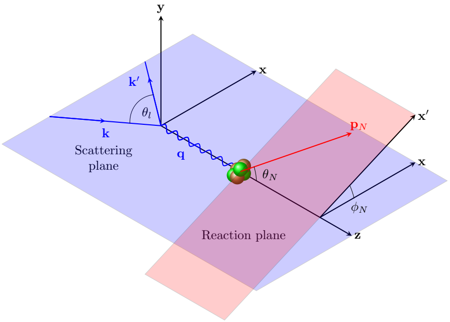
MicroBooNE semi-inclusive cross sections: $1\mu CC0\pi 1p$

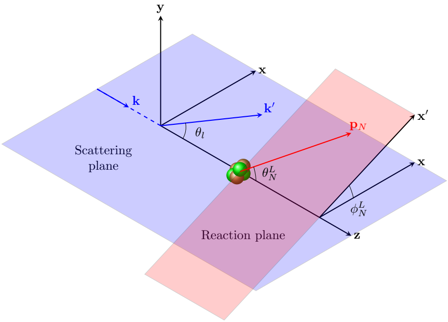


MicroBooNE semi-inclusive cross sections: $1\mu CC0\pi 1p$



Reference systems





T2K χ^2 table

	rROP	ROP	RPWIA	ED-RMF	GENIE-SuSAv2
$1\mu CC0\pi 0p$ (59)	232	127	1172	180	209
$1\mu CC0\pi Np$ (24)	64	28	82	76	69
Δp (49)	666	373	756	773	366
$\Delta\theta$ (35)	1170	466	1285	1379	159
$\Delta\theta^*$ (33)	129	92	152	146	123
$\|\Delta \mathbf{p}\|$ (49)	348	290	357	376	336
δp_T (8)	38	16	60	41	36
$\delta\alpha_T$ (8)	29	13	41	33	49
$\delta\phi_T$ (8)	23	20	38	24	40

RMF parameters

Parameter	HS	NL1	NL2	NLSH
m (MeV)	939	938	938	939
m_σ (MeV)	520	492.25	504.89	526.059
m_ω (MeV)	783	795.359	780	783
m_ρ (MeV)	770	763	763	763
g_σ	10.47	10.138	9.111	10.444
g_ω	13.8	13.285	11.493	12.945
g_ρ	4.035	4.976	5.507	4.383
g_2 (fm^{-1})	0	-12.172	-2.304	-6.9099
g_3	0	-36.265	13.783	-15.8337
ρ_0 (fm^{-3})	0.148	0.151	0.146	0.146
E/A (MeV)	-15.731	-16.426	-17.018	-16.346
K (MeV)	546.3	211.11	399.2	355.4
m^*/m	0.541	0.573	0.670	0.597
J (MeV)	34.9	43.5	45.1	36.1