UNIVERSITÀ DI PAVIA

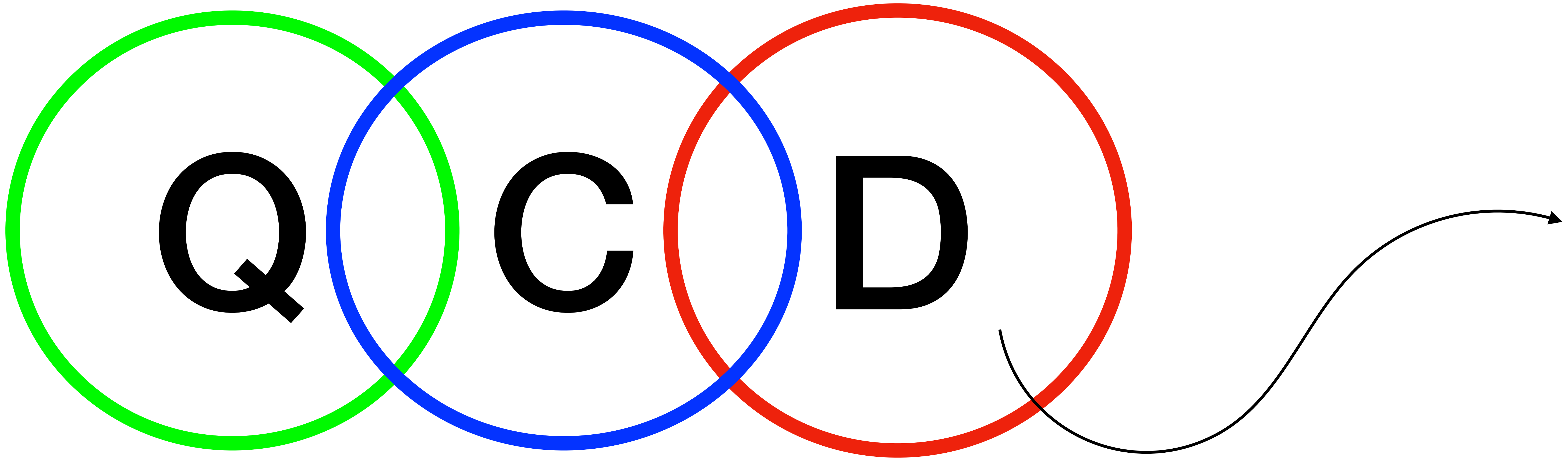
INFN Istituto Nazionale di Fisica Nucleare

Extraction of di-hadron fragmentation functions at NNLO

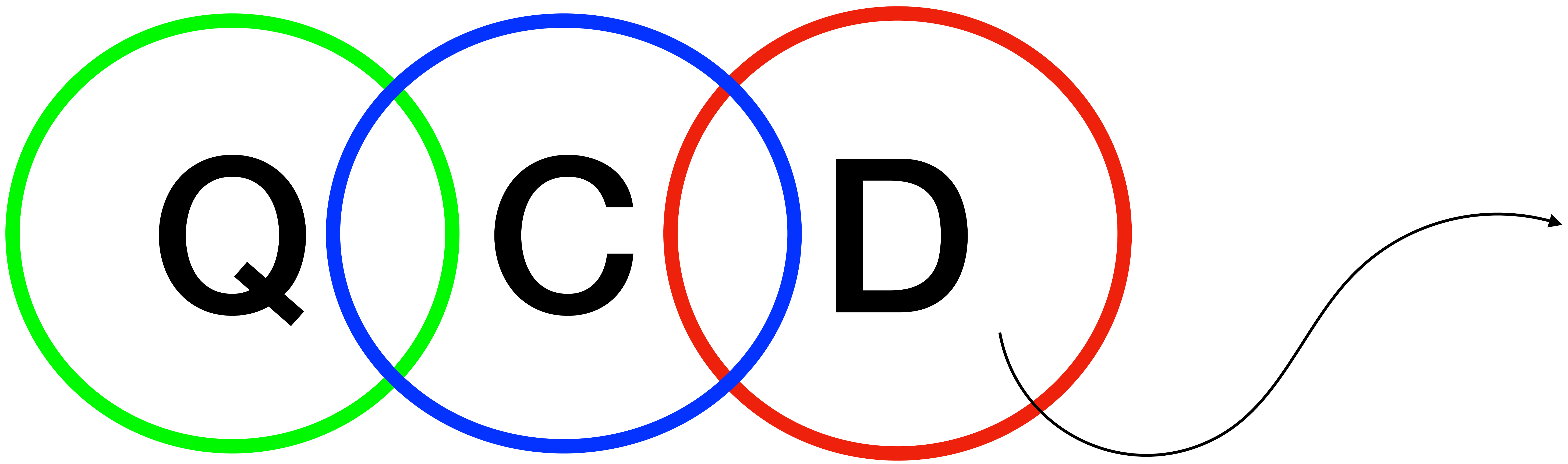
PhD candidate:
Luca Polano

Supervisor:
Marco Radici
Alessandro Bacchetta

TNPI2025, Cortona



Quarks & gluons

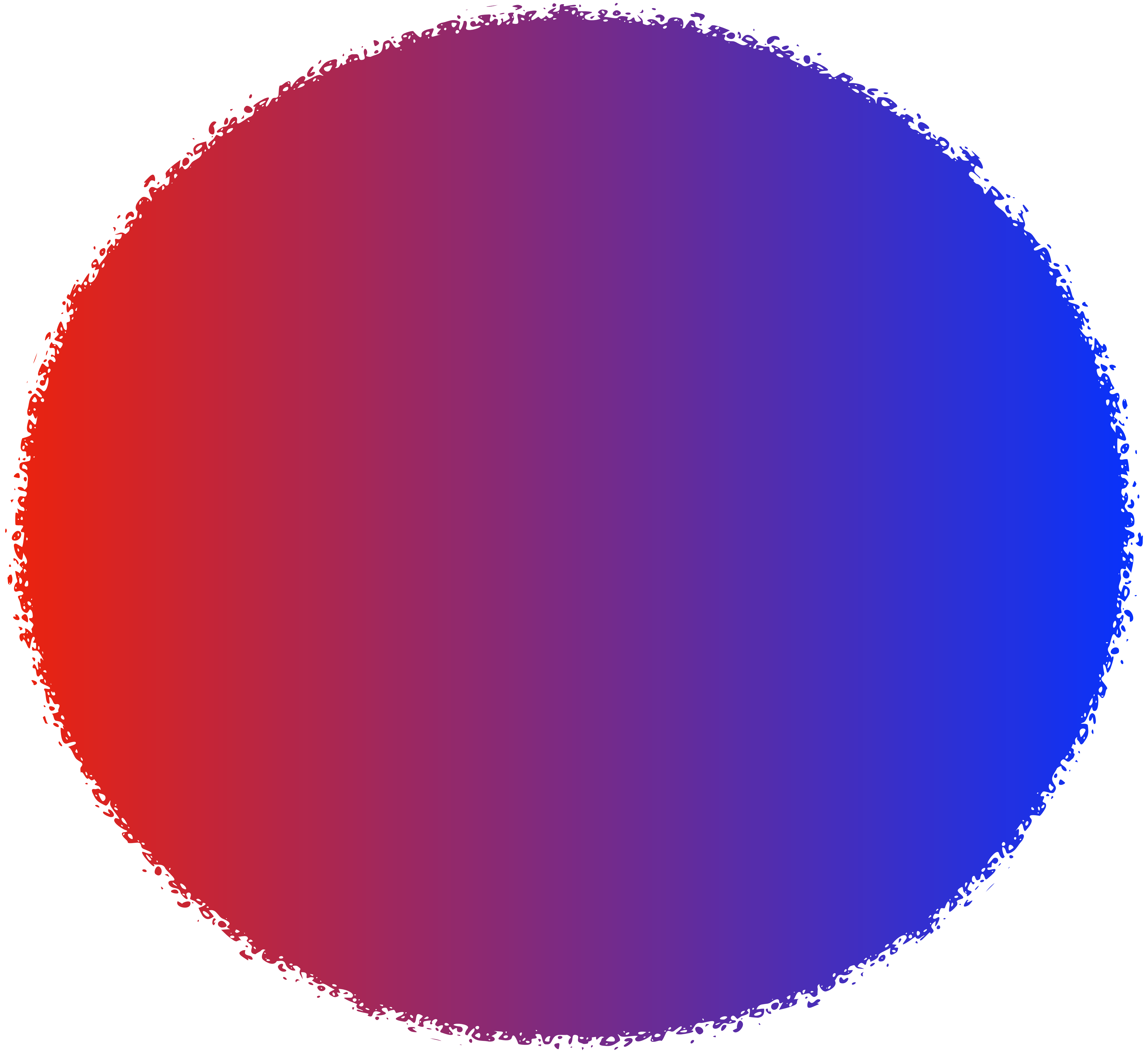


Quarks & gluons

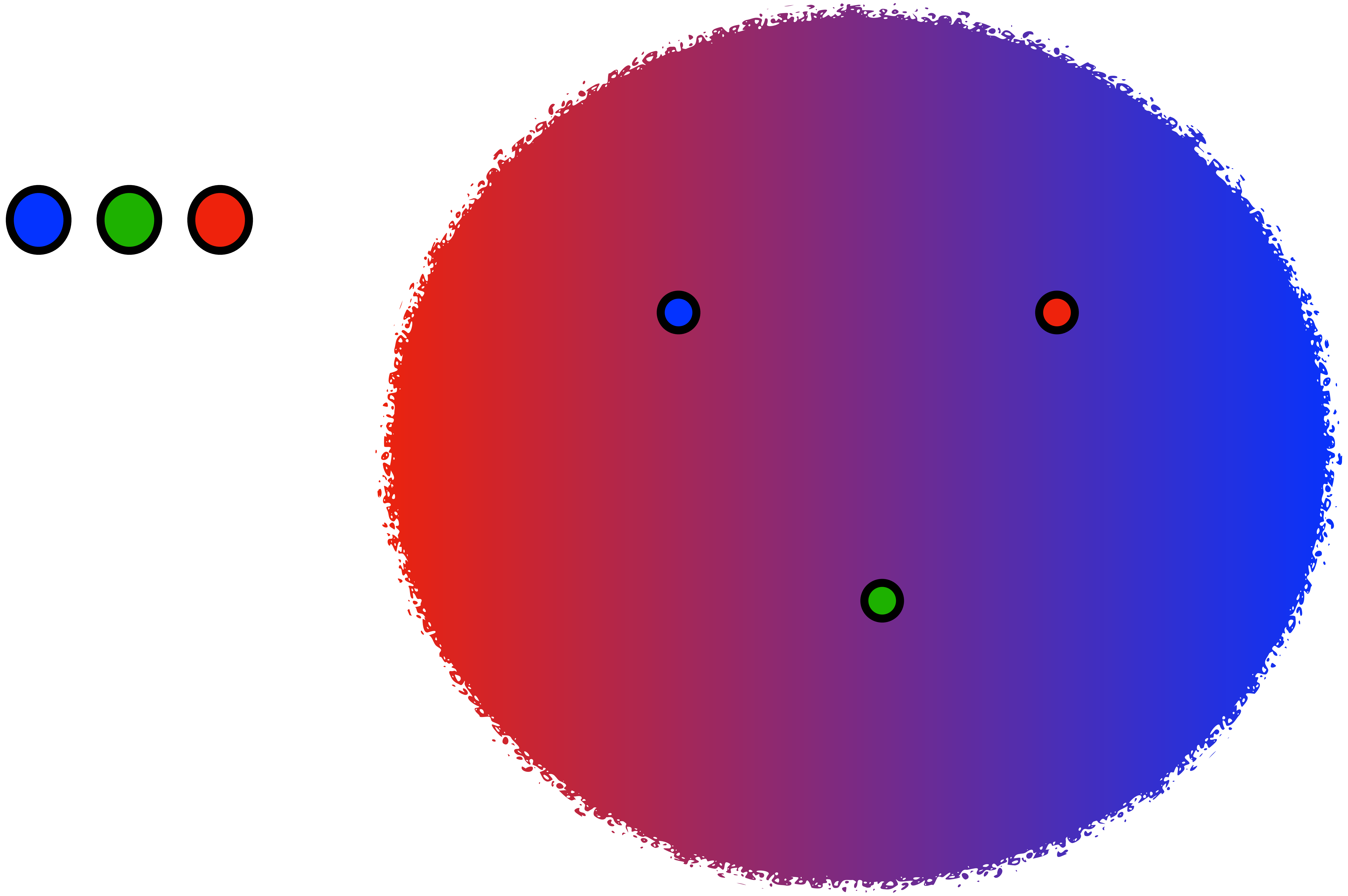
High energies

Low, $\Lambda_{QCD} \sim$ 200 MeV

At $\sim \Lambda_{QCD} \sim 200$ **MeV**



At $\sim \Lambda_{QCD} \sim 200$ MeV



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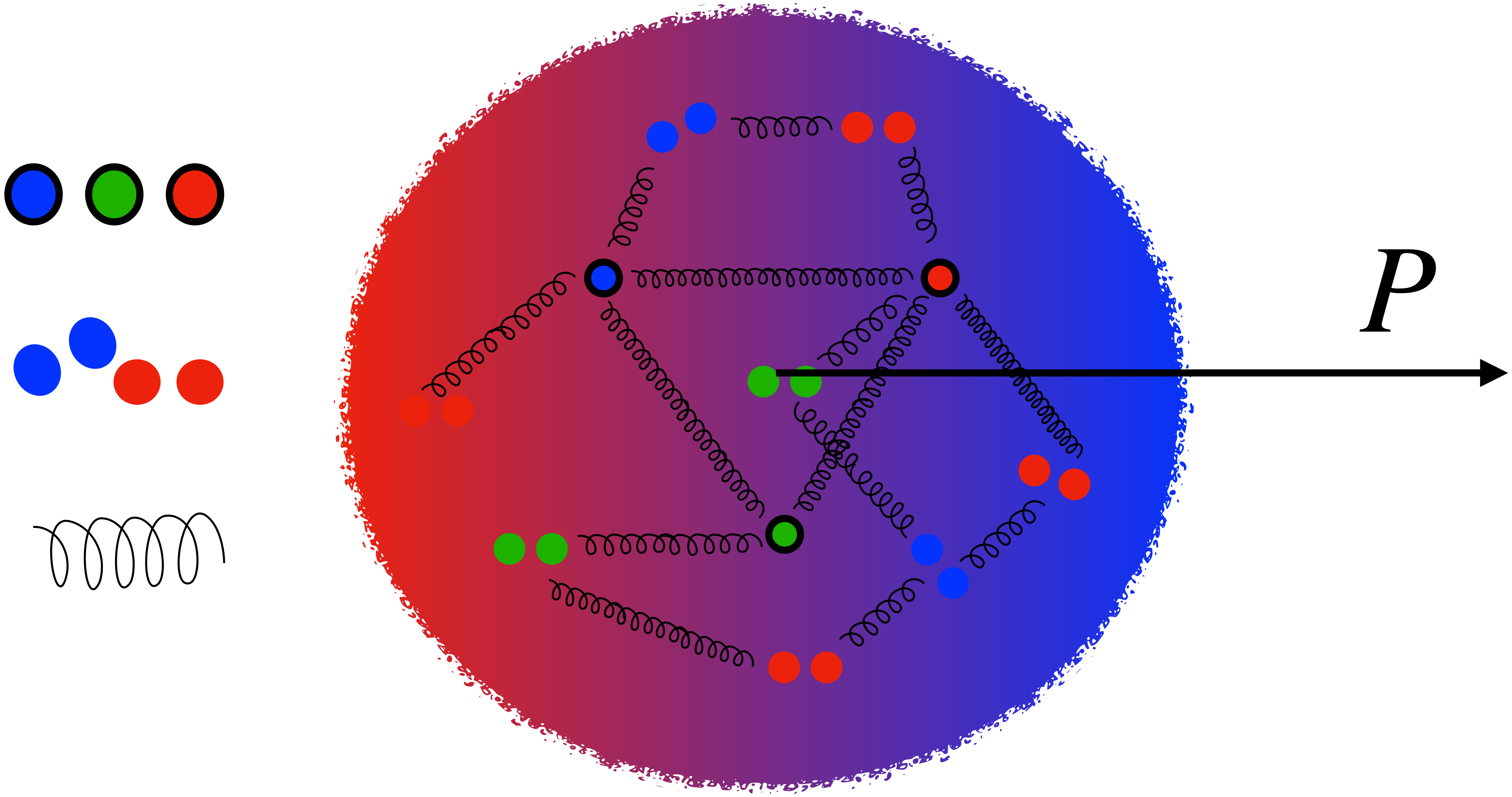

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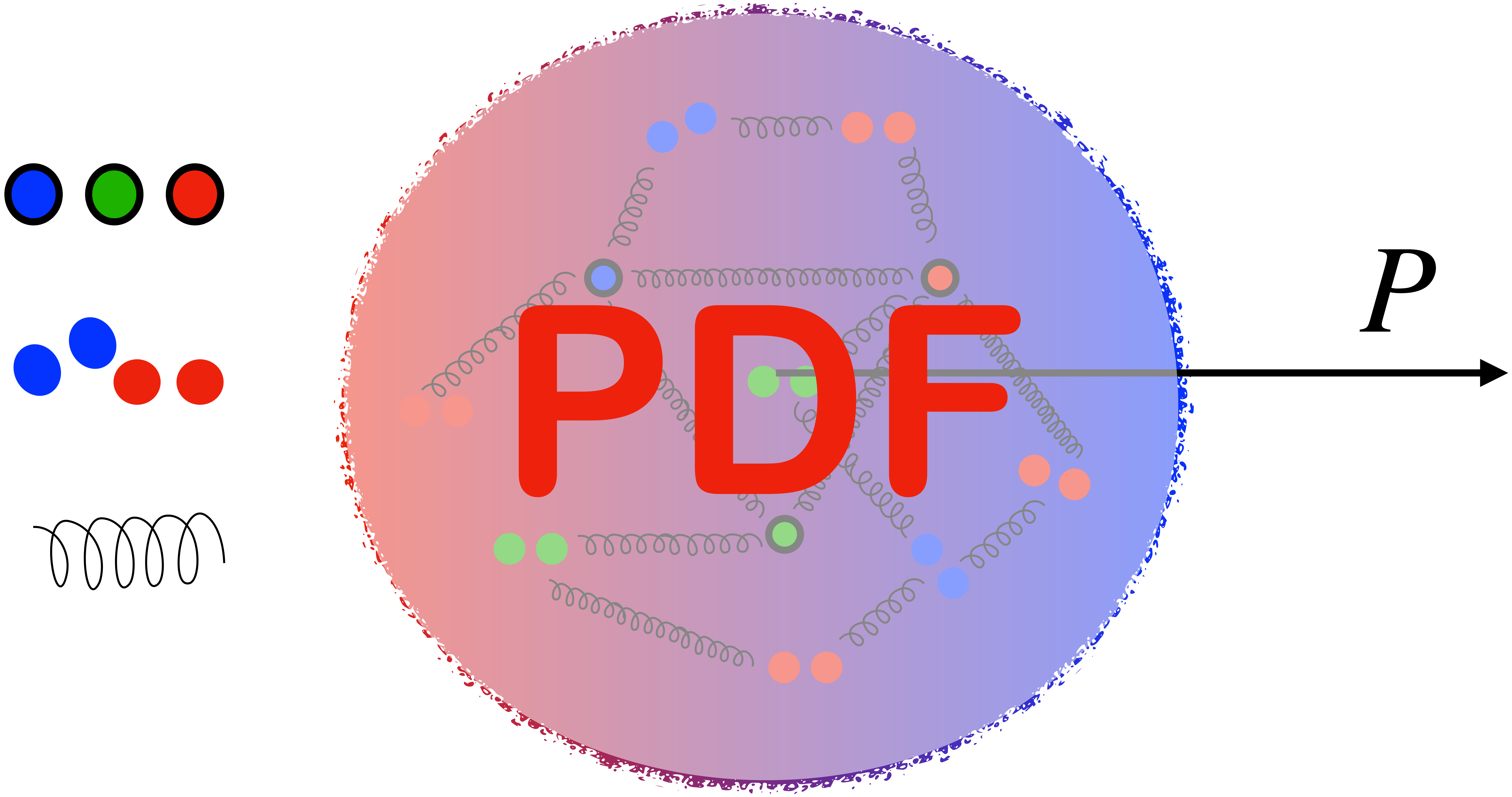
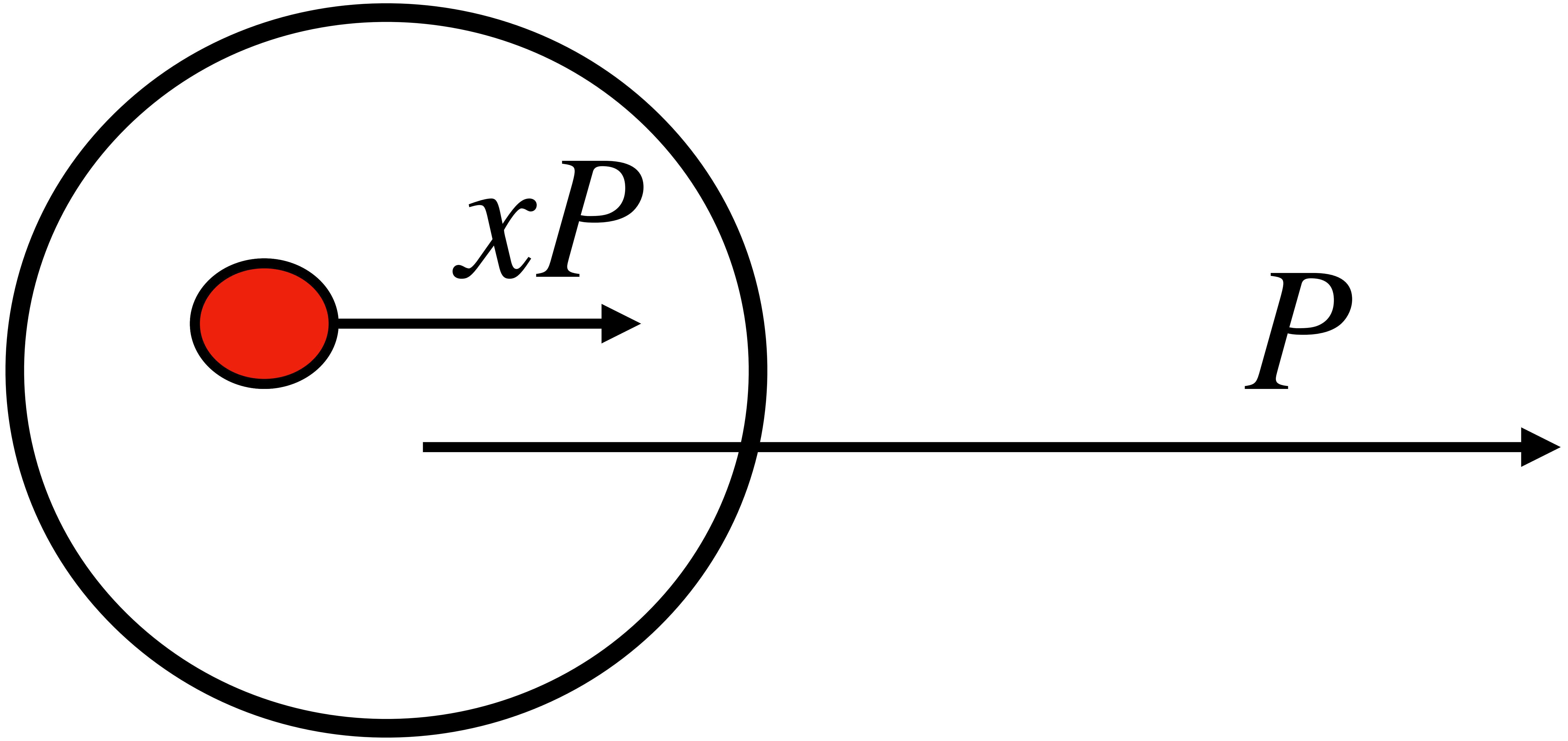


At $\sim \Lambda_{QCD} \sim 200$ MeV

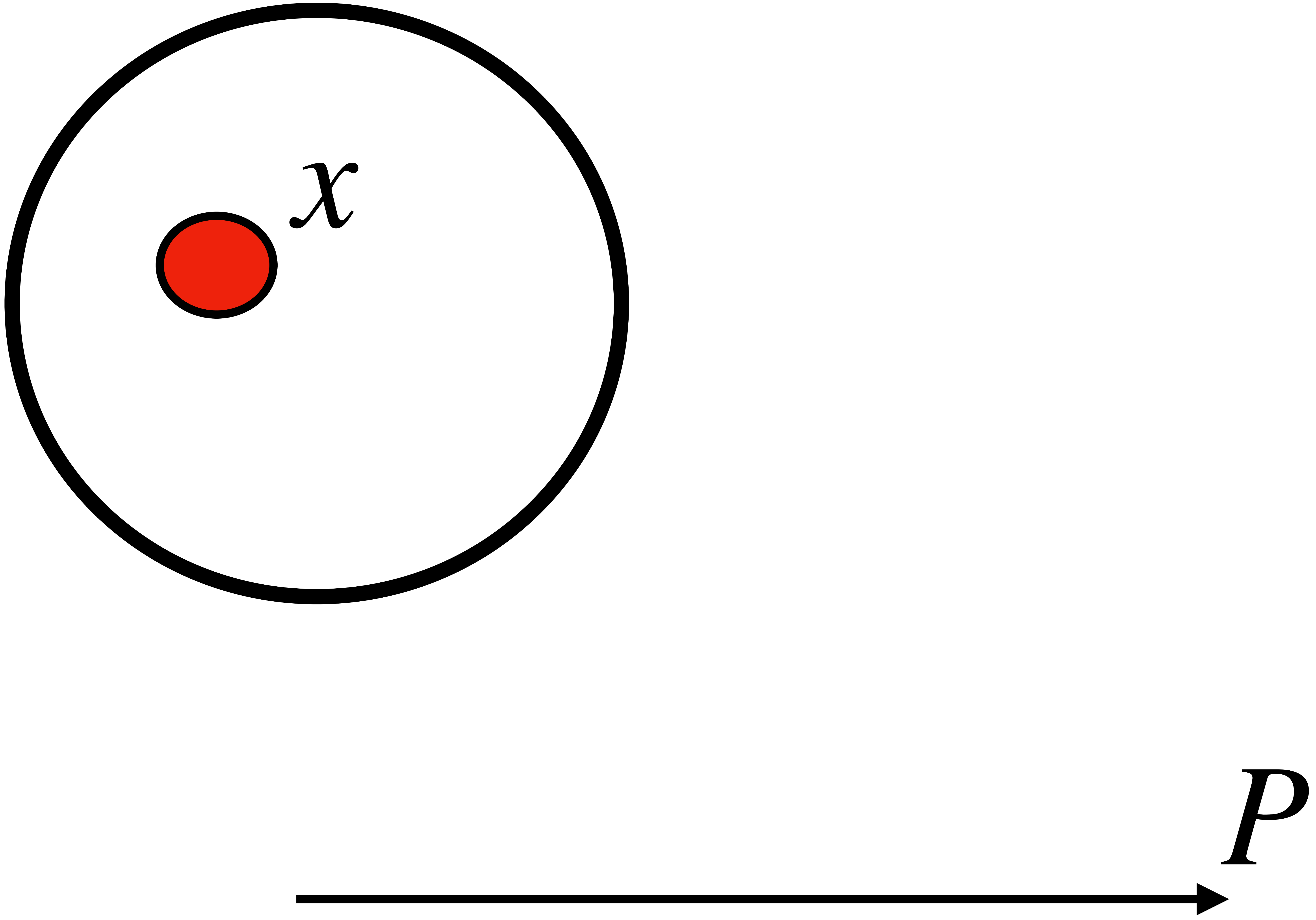
PDF



At $\sim \Lambda_{QCD} \sim 200$ MeV

PDF

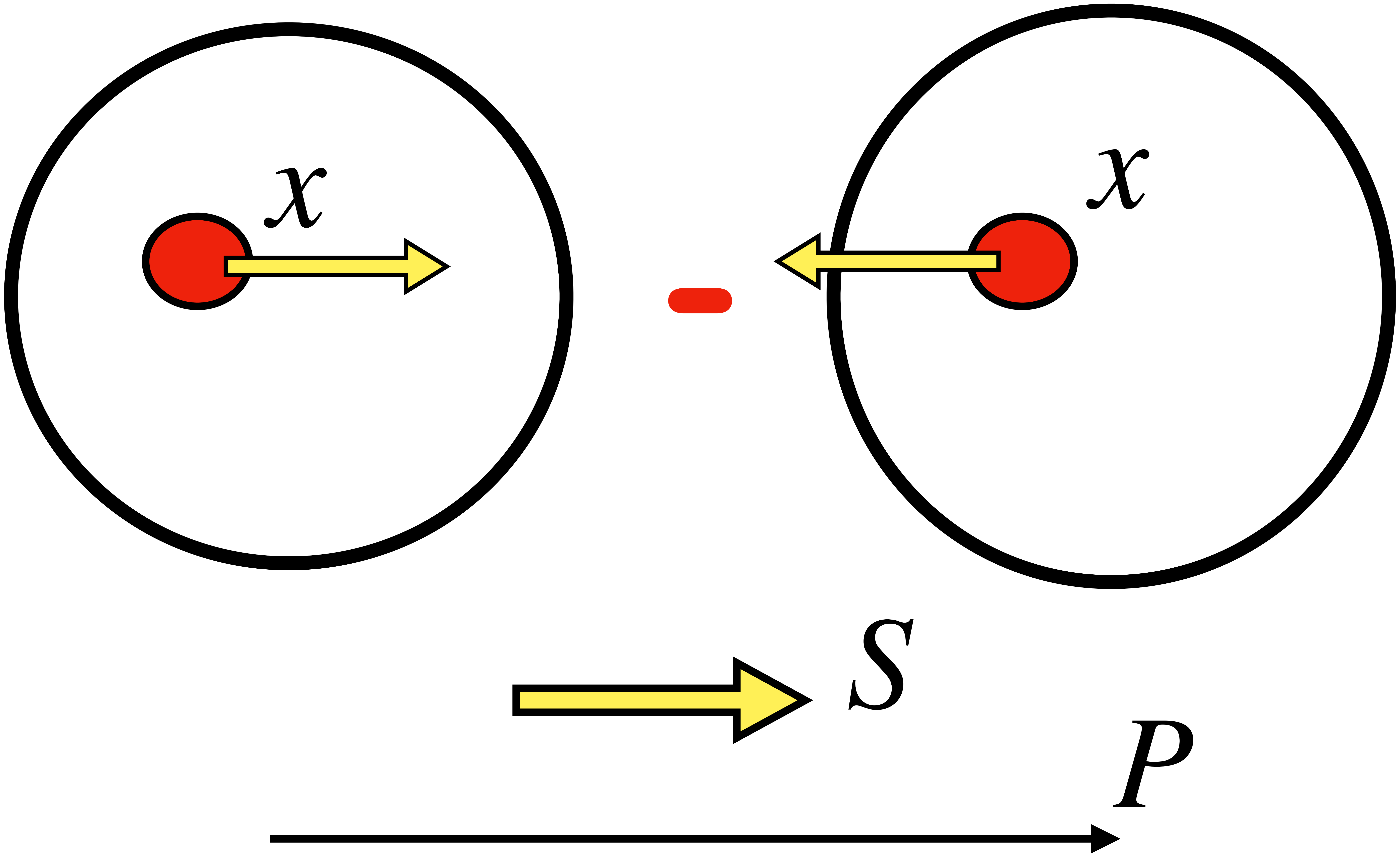
f_1



At $\sim \Lambda_{QCD} \sim 200$ MeV

PDF

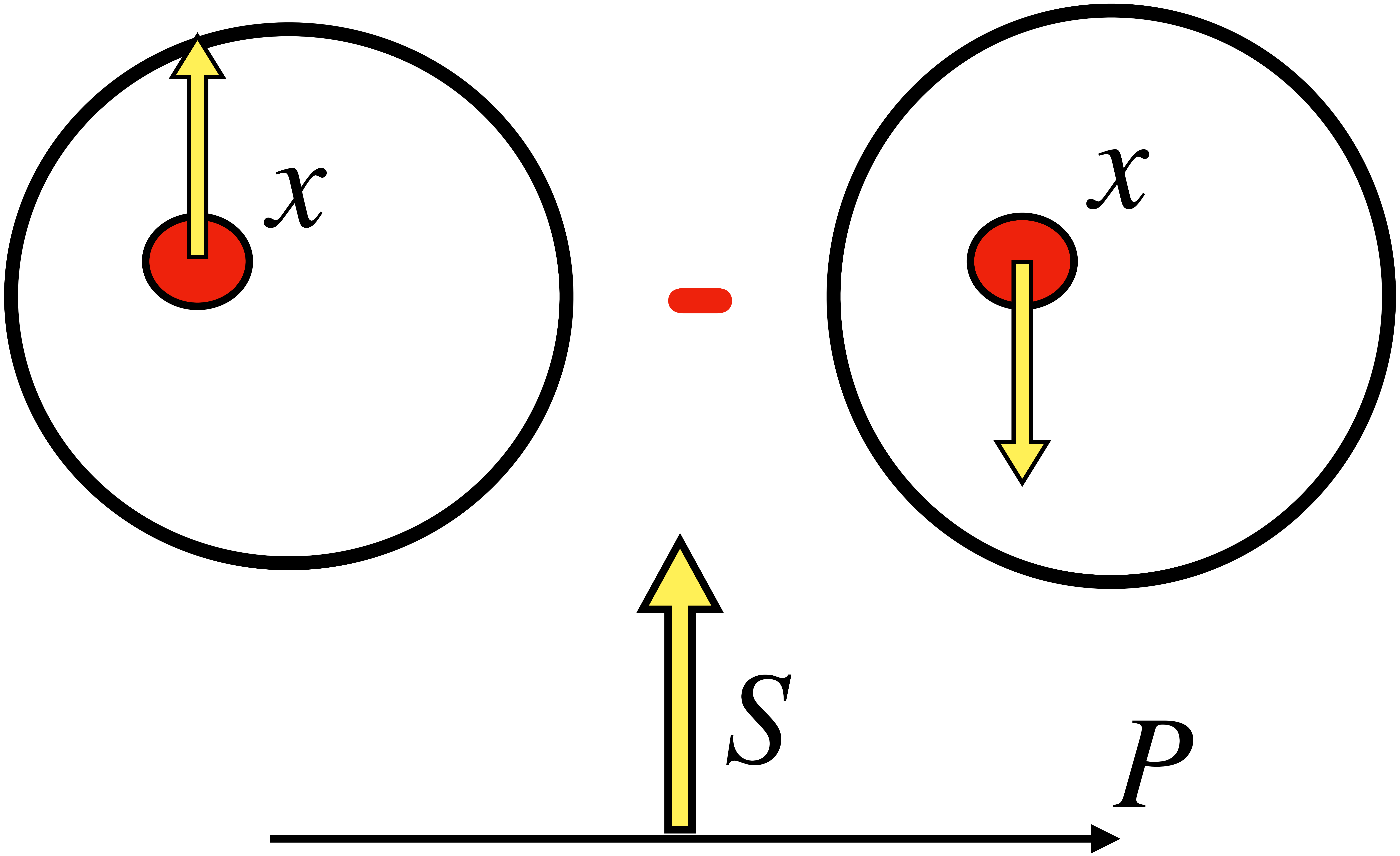
g_1



At $\sim \Lambda_{QCD} \sim 200$ MeV

PDF

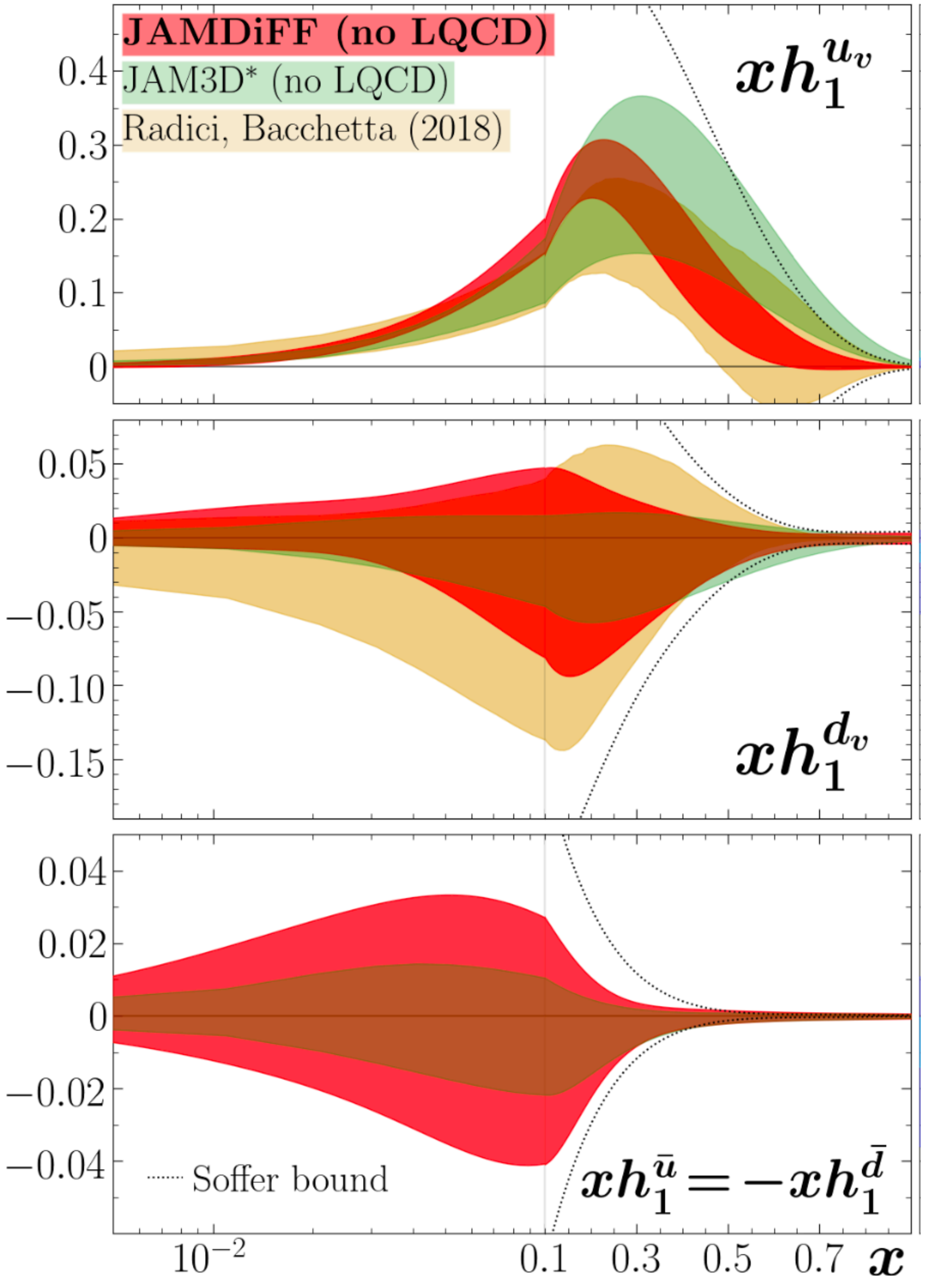
h_1



Why caring about transversity?

- Have a better knowledge of the proton structure

N.Sato et al, Phys. Rev. D 109, (2024) 034024



Why caring about transversity?

- Have a better knowledge of the proton structure

- Can be used to investigate physics beyond the Standard Model

Chiral-odd structures do not appear in the SM Tree level Lagrangian

<u>Neutrino β-decay</u>

$$g_T = \delta u - \delta d$$

$$\delta q = \int_0^1 dx\, h_1^q(x)$$

<u>constraints on CP violation</u>

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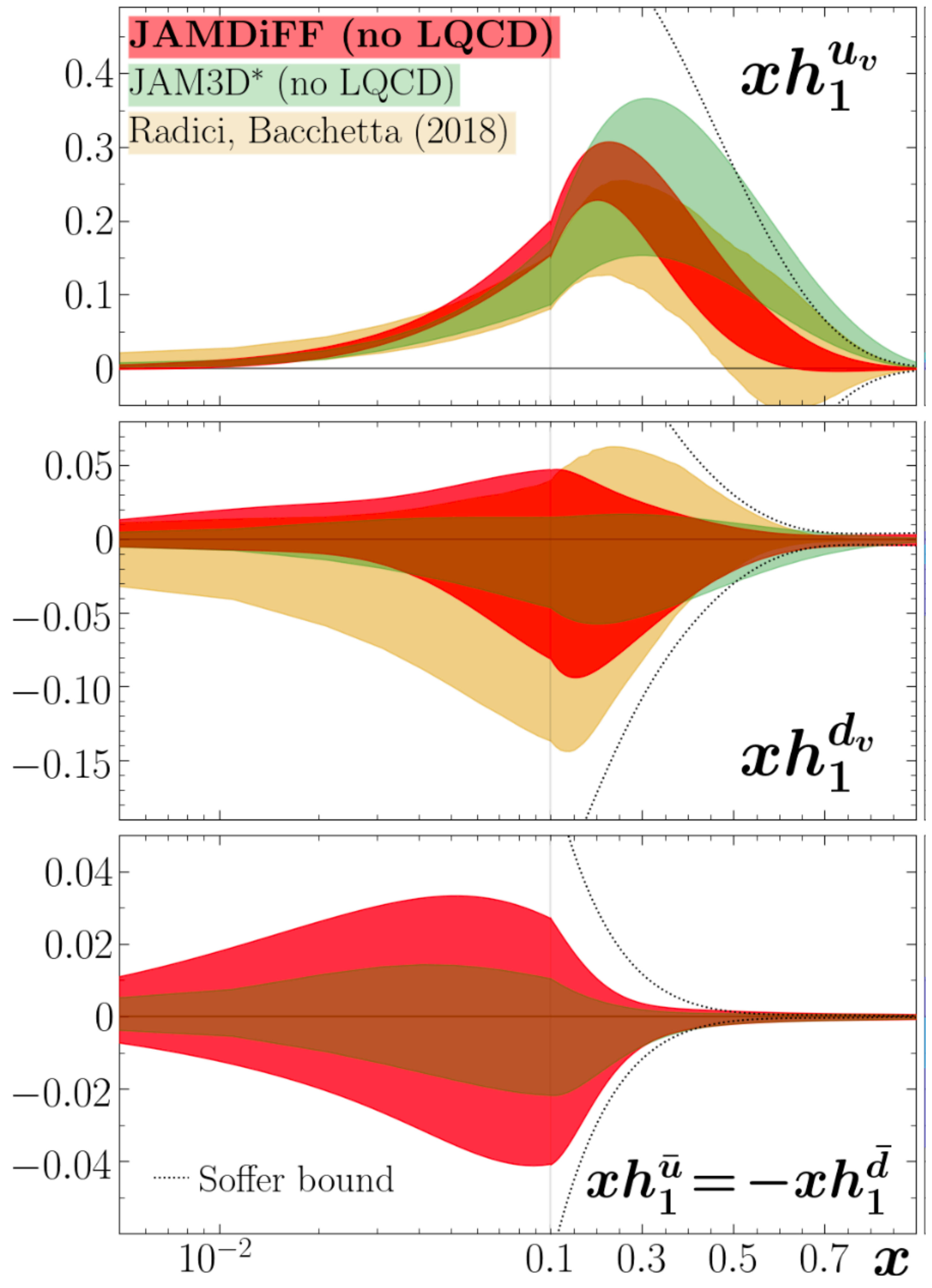

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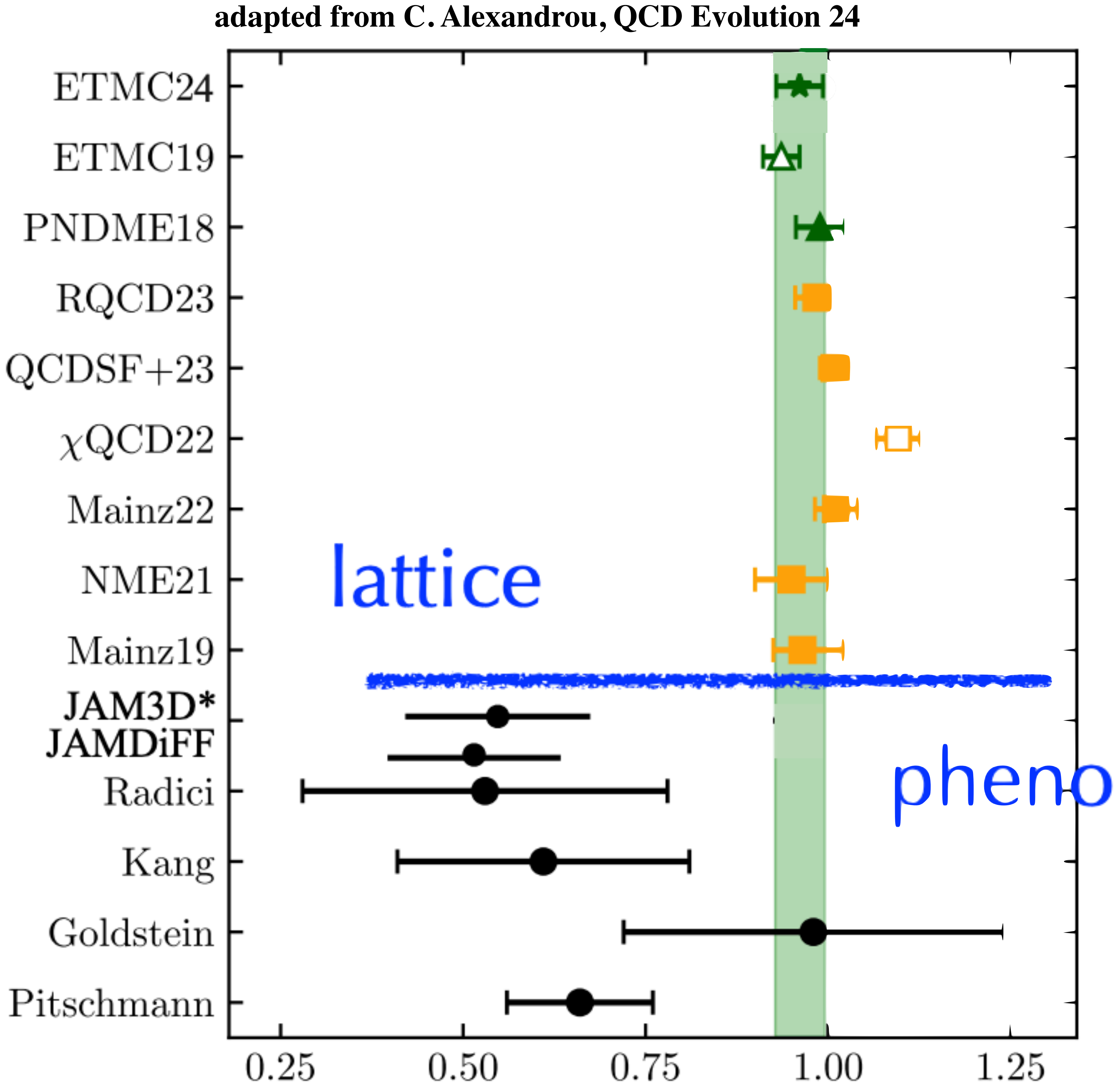
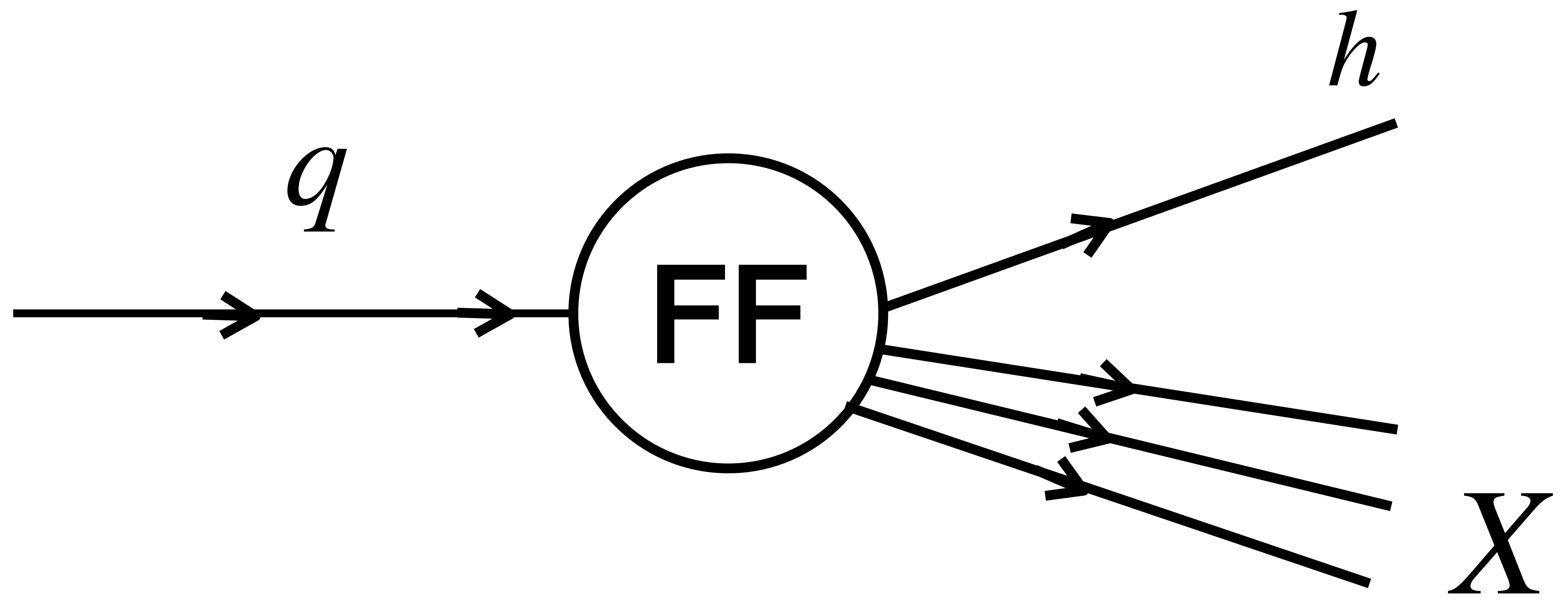
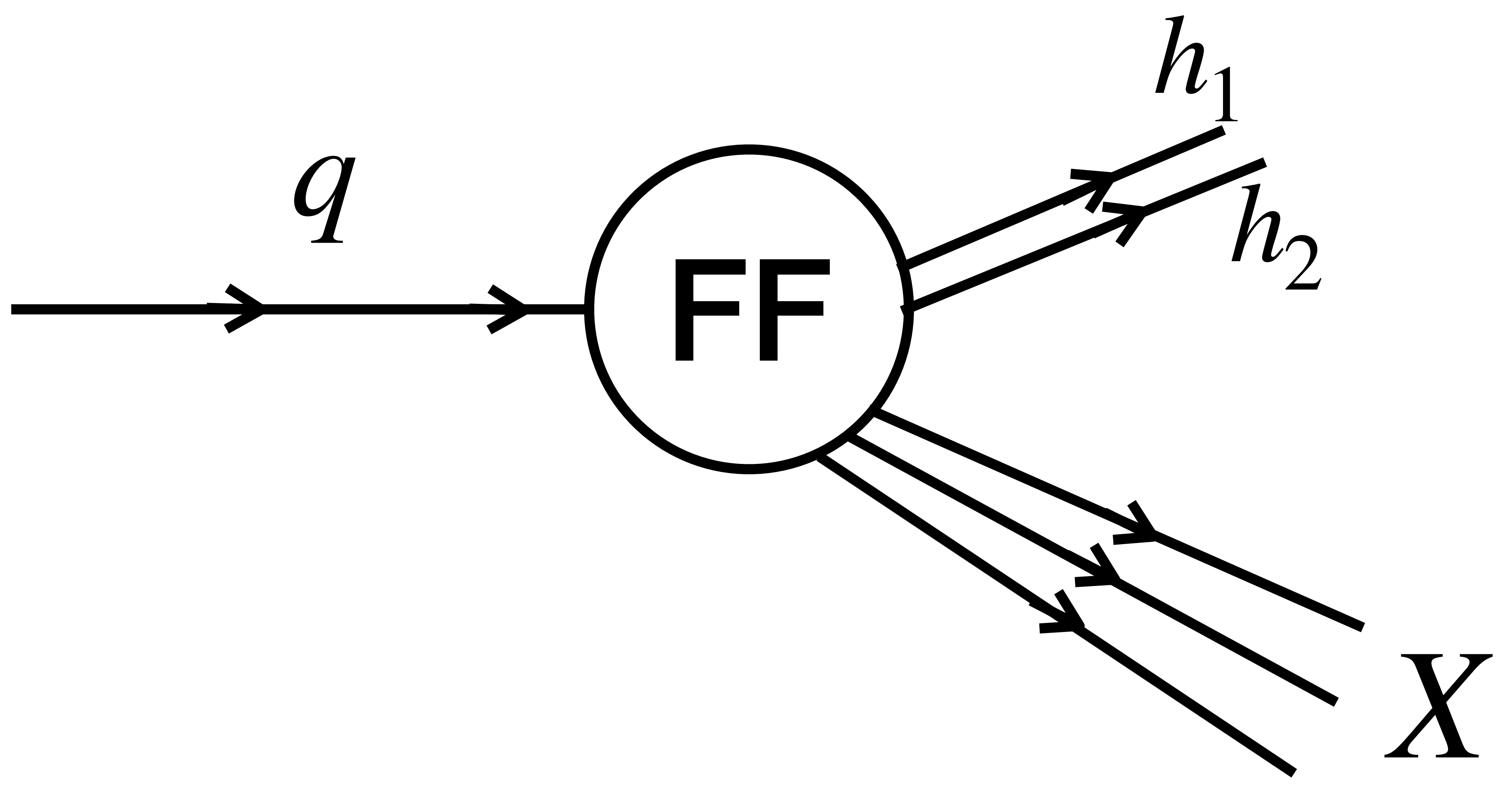
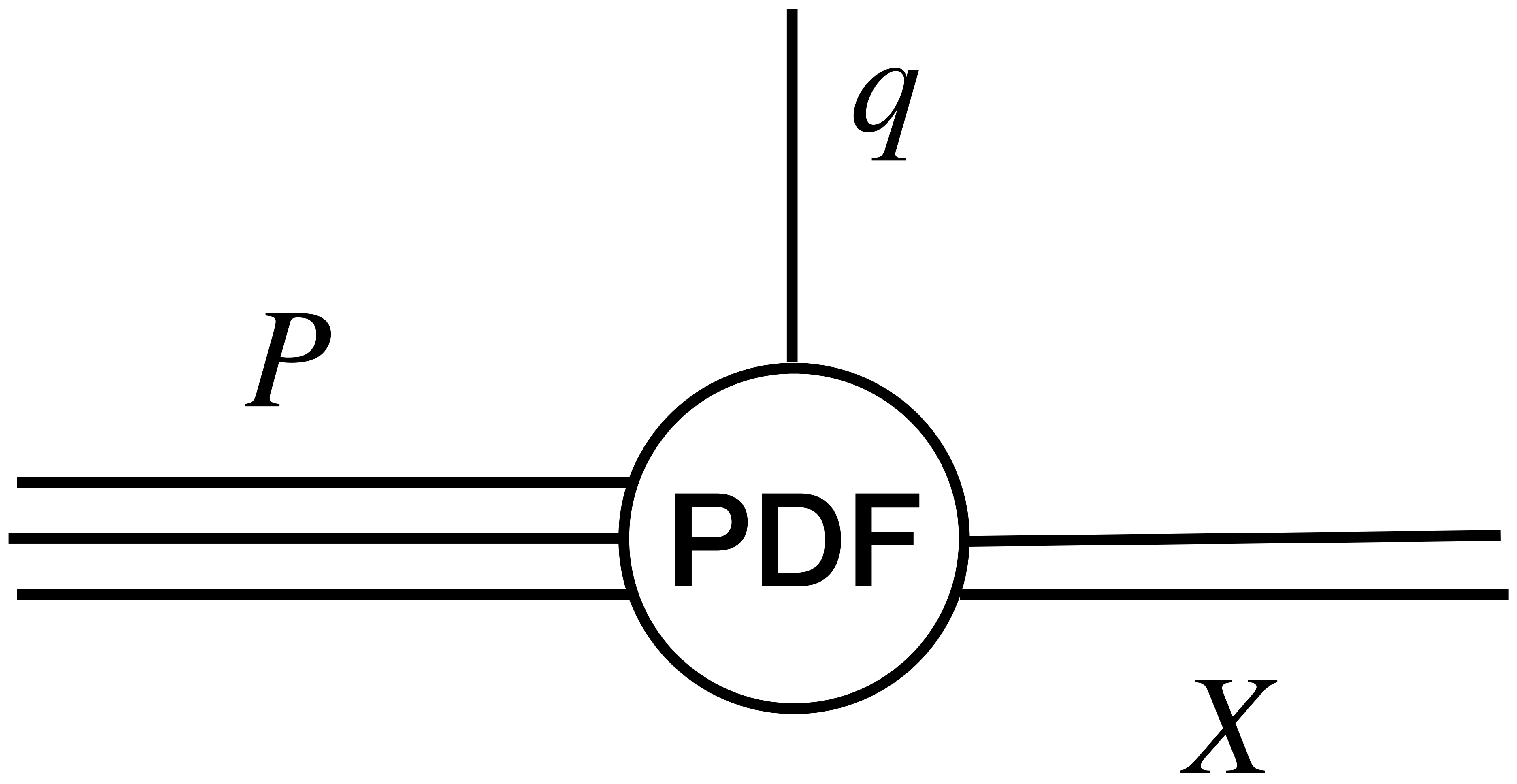
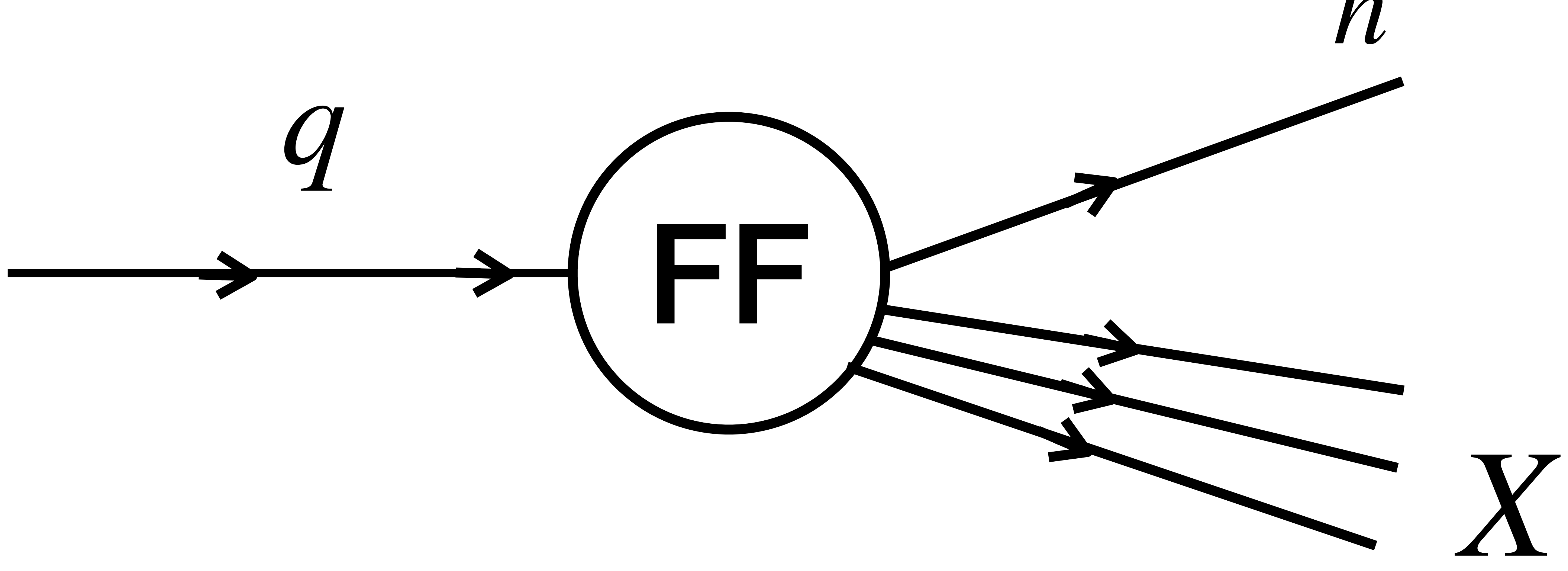
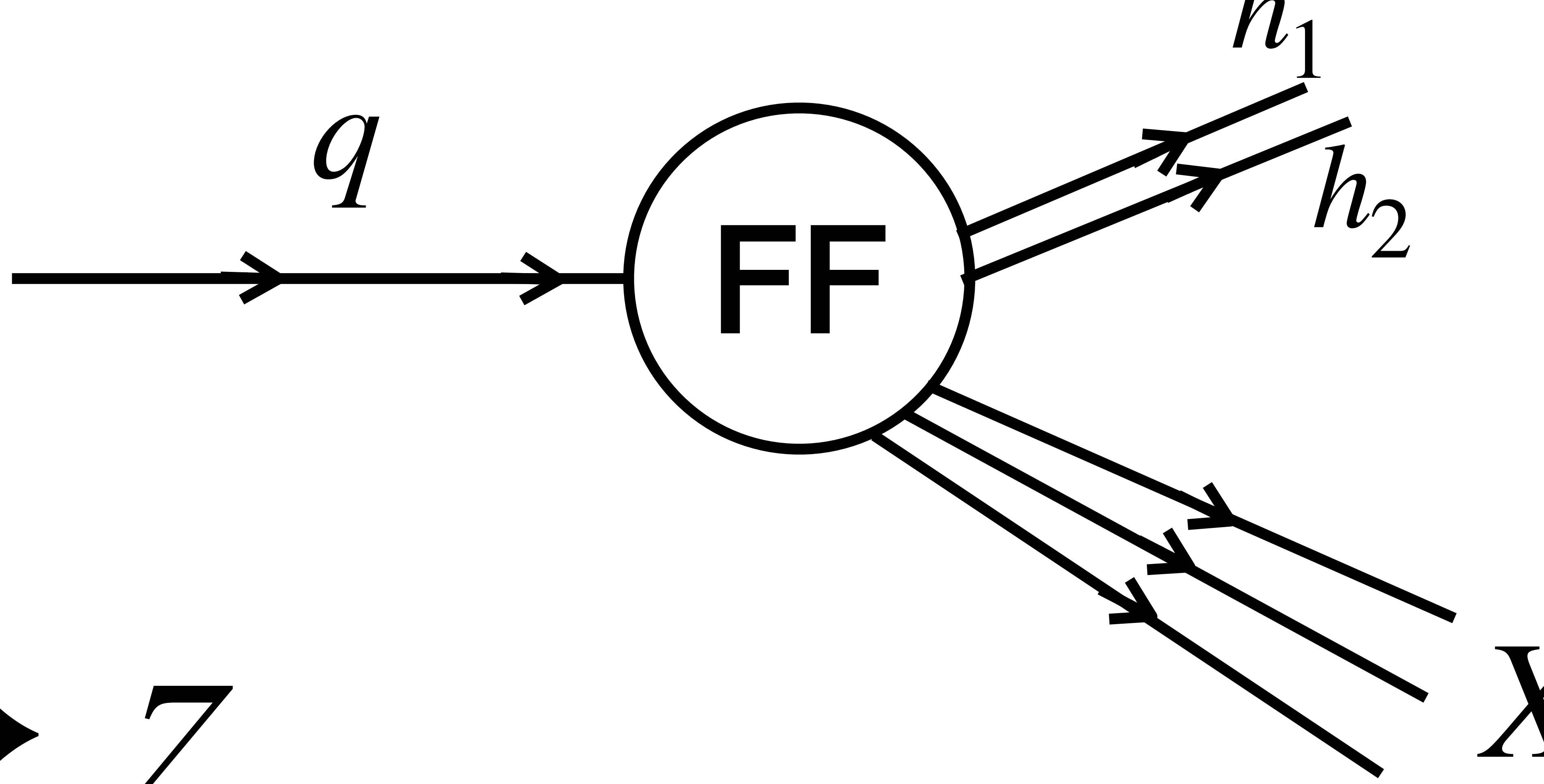



h
q
FF
X

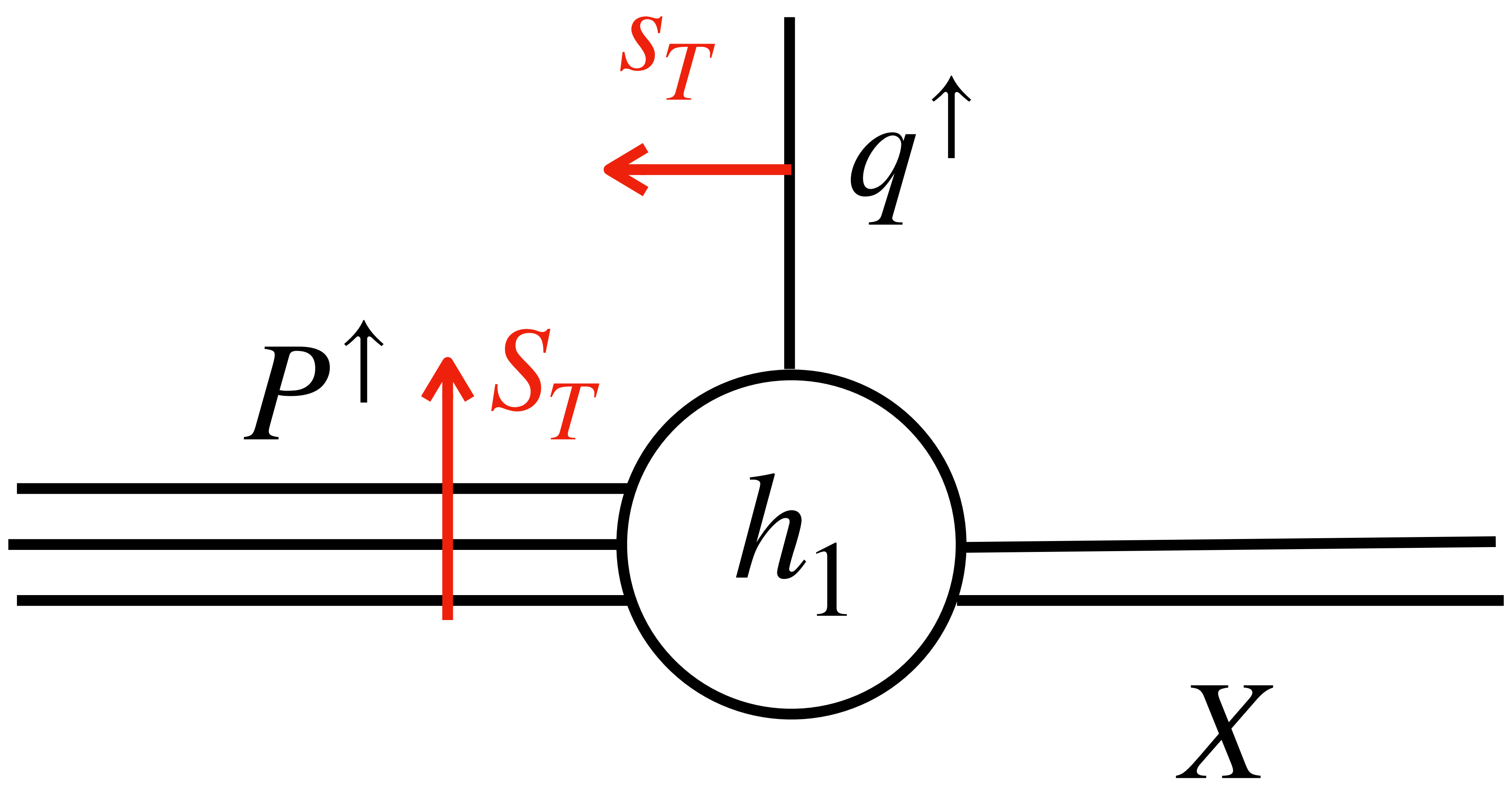

h1
h2
q
FF
X

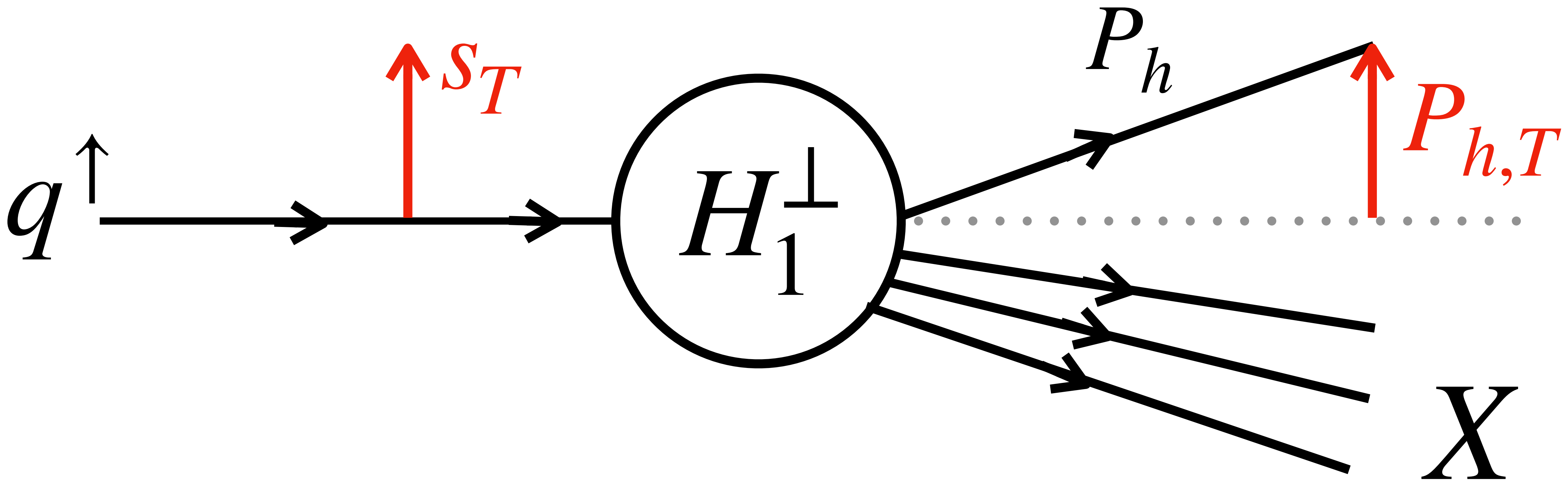


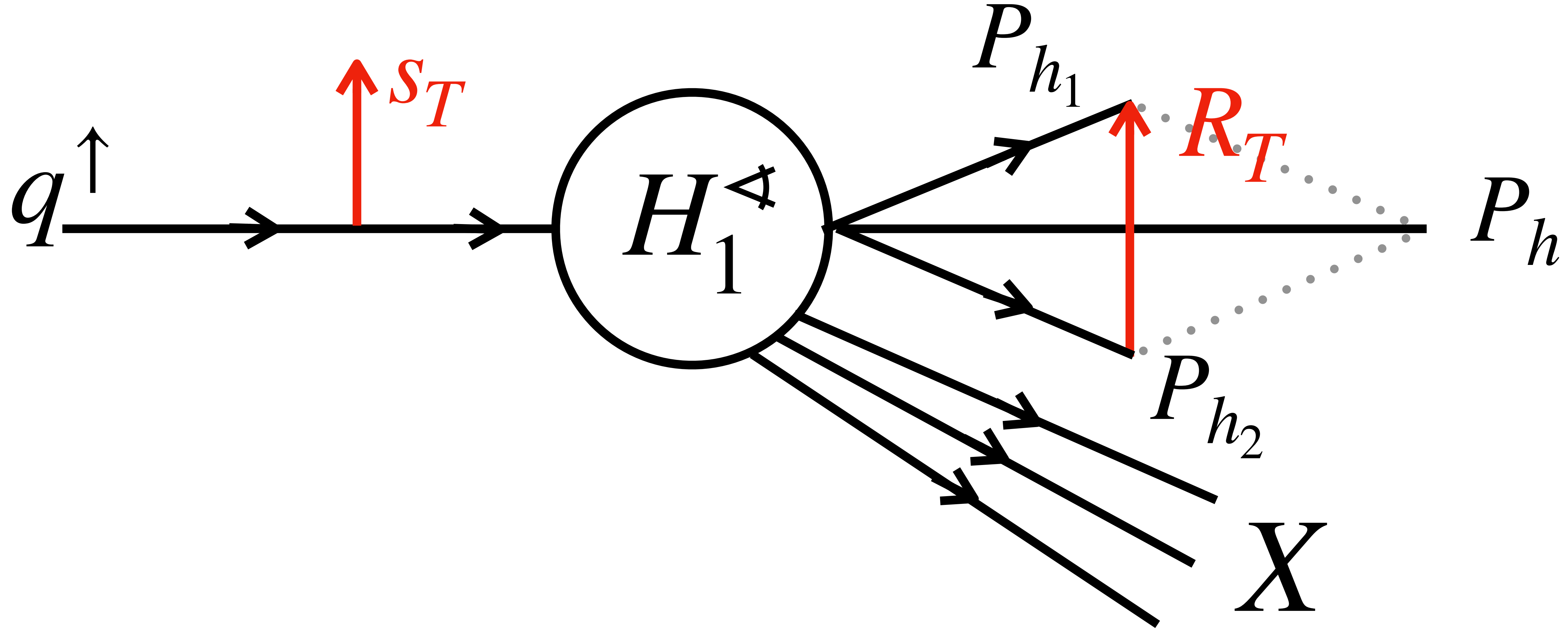


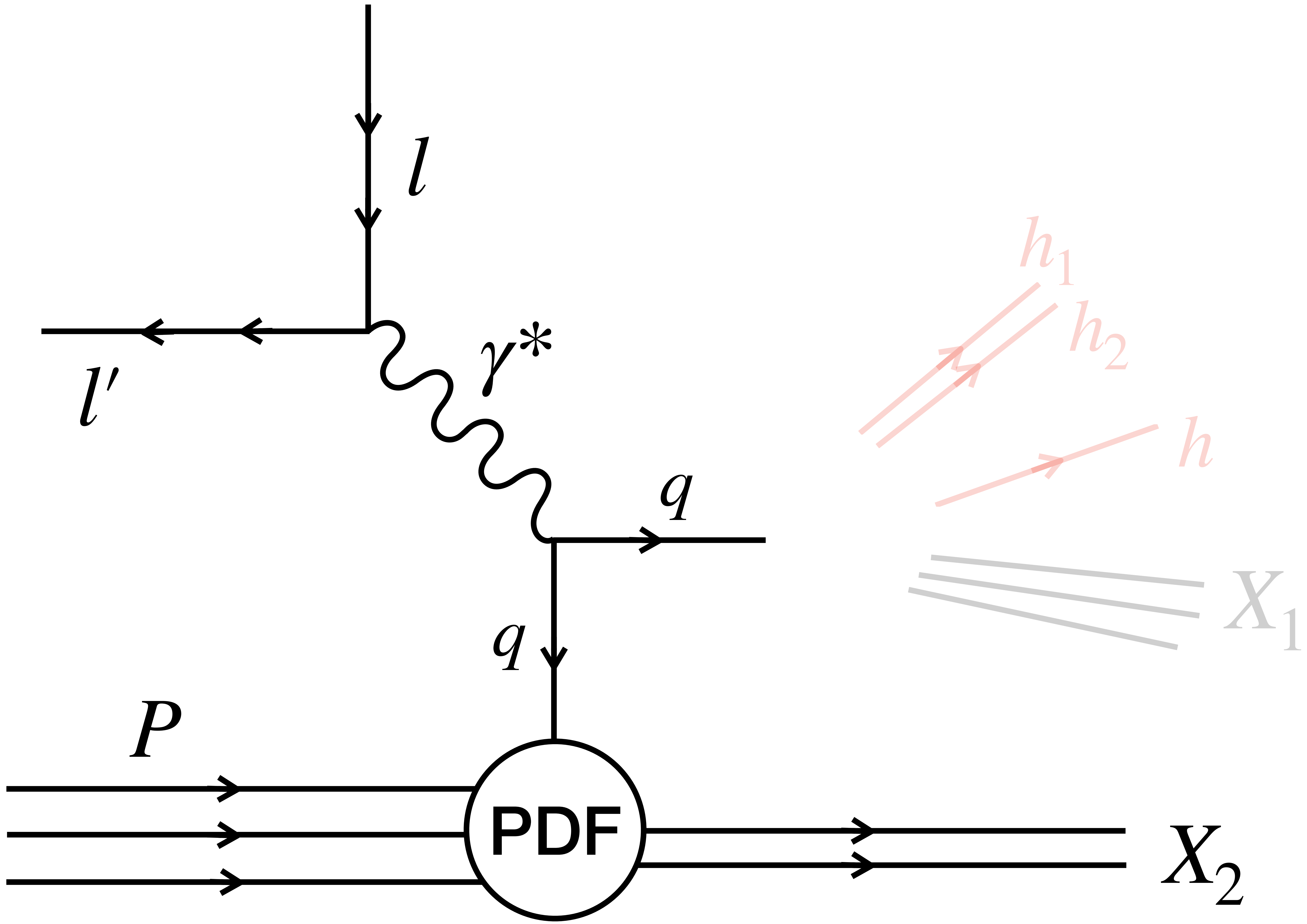


$$x \leftrightarrow z$$

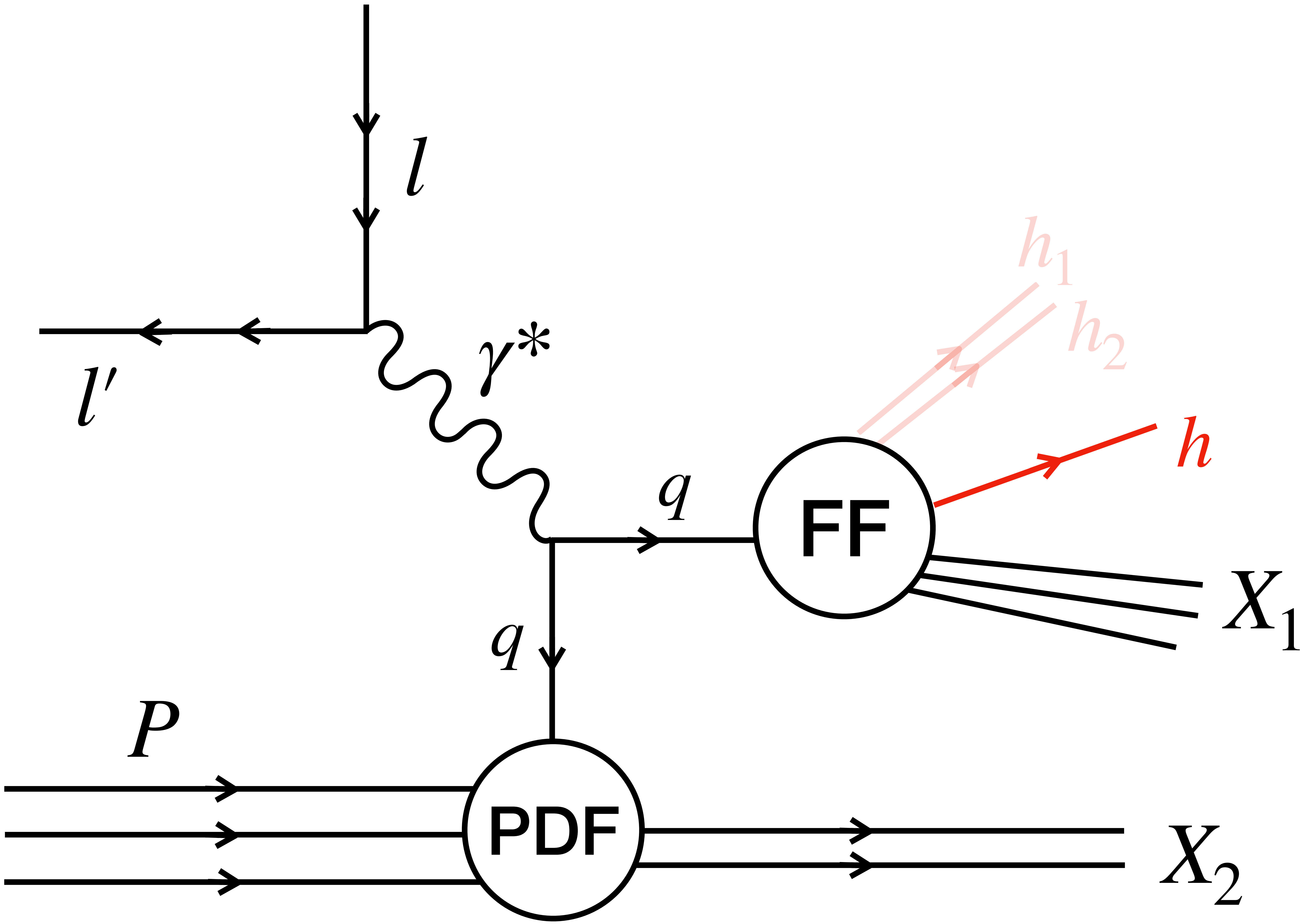

s_T
$q^\uparrow$
$P^\uparrow$
S_T
h_1
X


$q^\uparrow$
s_T
$H_1^\perp$
P_h
$P_{h,T}$
X

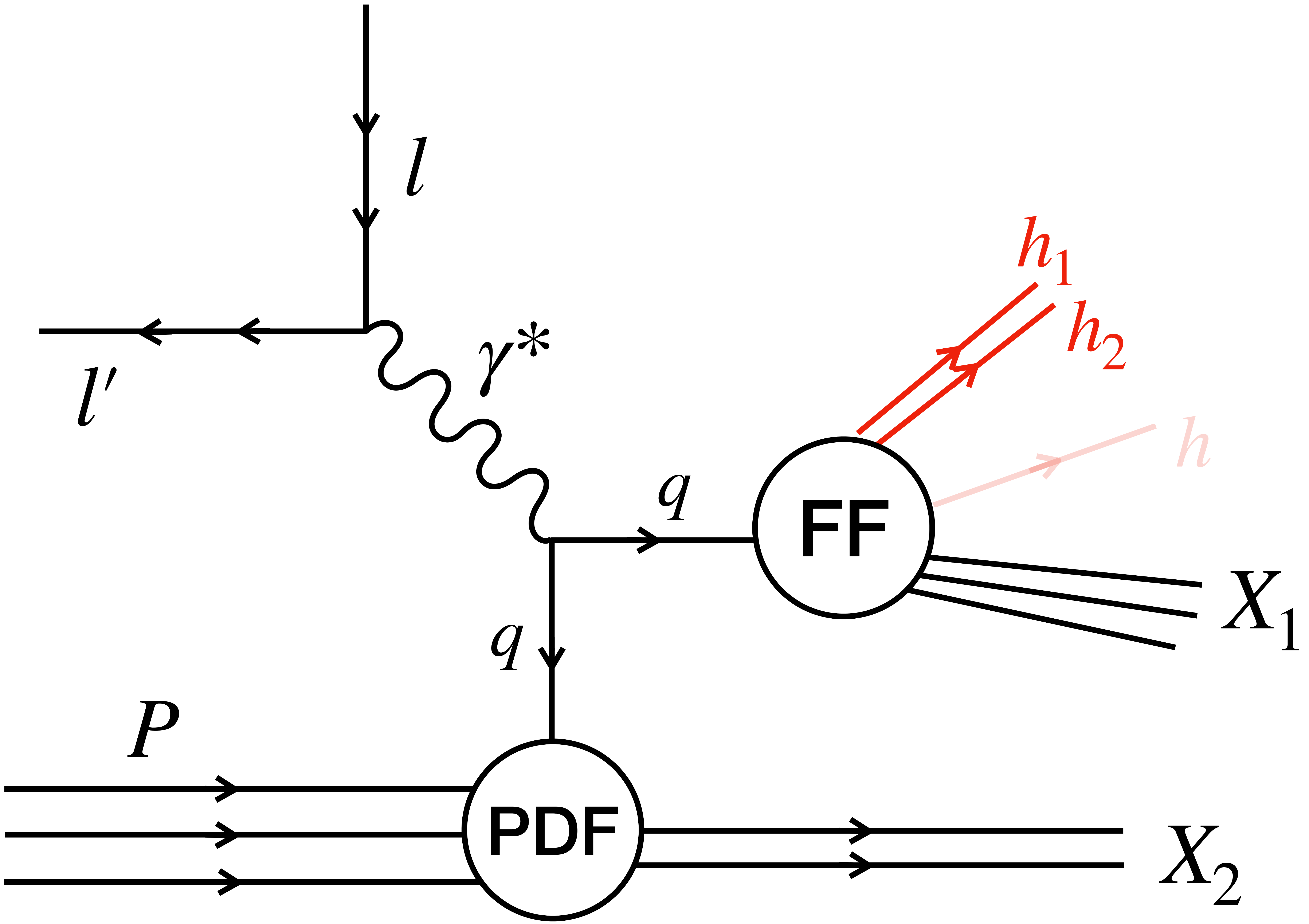

$q^\uparrow$
s_T
$H_1^\sphericalangle$
P_{h_1}
R_T
P_h
P_{h_2}
X



l
l'
γ^*
h_1
h_2
h
q
q
X_1
P
PDF
X_2

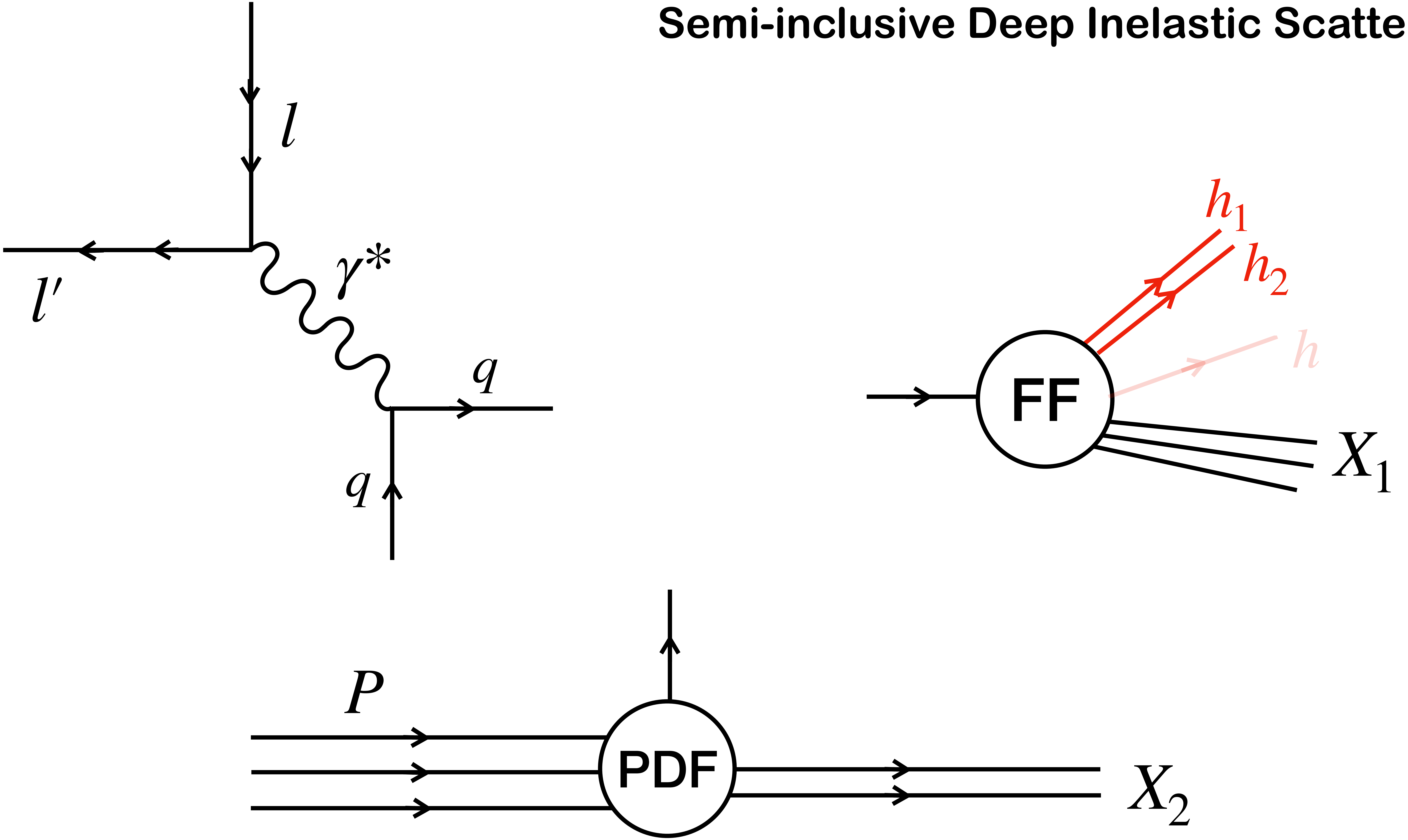


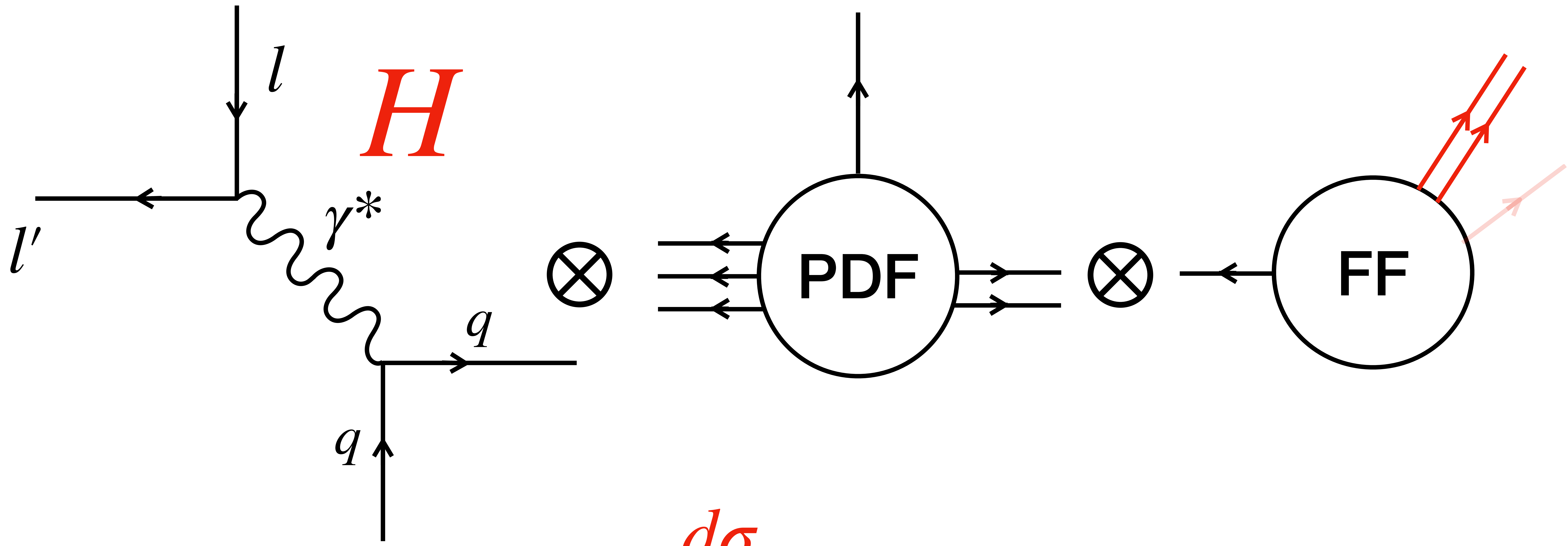
l
l'
γ^*
q
q
FF
h_1
h_2
h
X_1
P
PDF
X_2



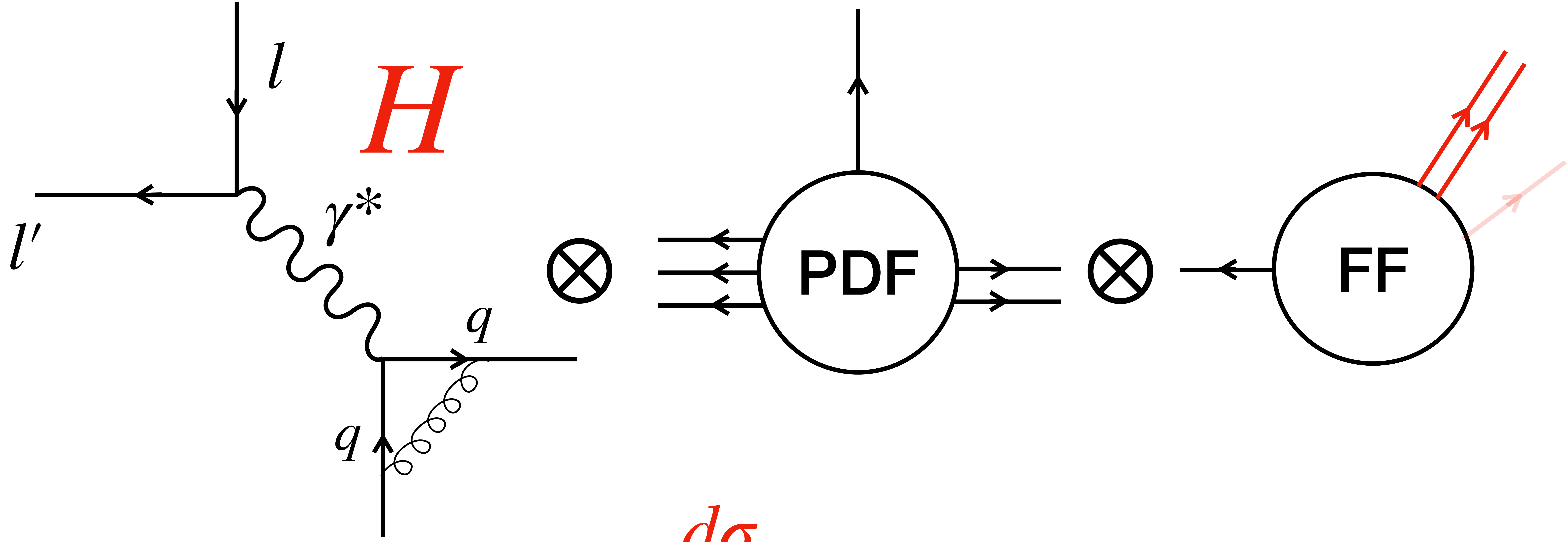
l
l′
γ*
q
q
FF
PDF
P
h1
h2
h
X1
X2

Semi-inclusive Deep Inelastic Scattering





$$\frac{d\sigma}{dx dQ^2 \ldots} = H \otimes PDF \otimes FF$$

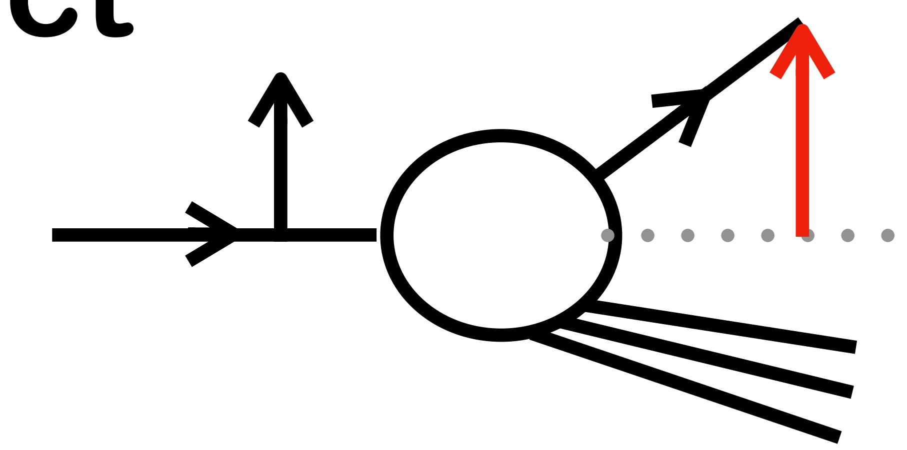


Ex.: NLO

$$\frac{d\sigma}{dxdQ^2\ldots} = H \otimes PDF \otimes FF$$

Alternative method to extract trasversity PDF

Collins effect



$$A_{UT}^{h} \sim \frac{\sum_q e_q^2 \cdot C[h_1^q(x_B, k_T) H_1^{\perp,q}(z, p_\perp)]}{\sum_q e_q^2 C[f_1^q(x_B, k_T) D_1^q(z, p_\perp)]}$$

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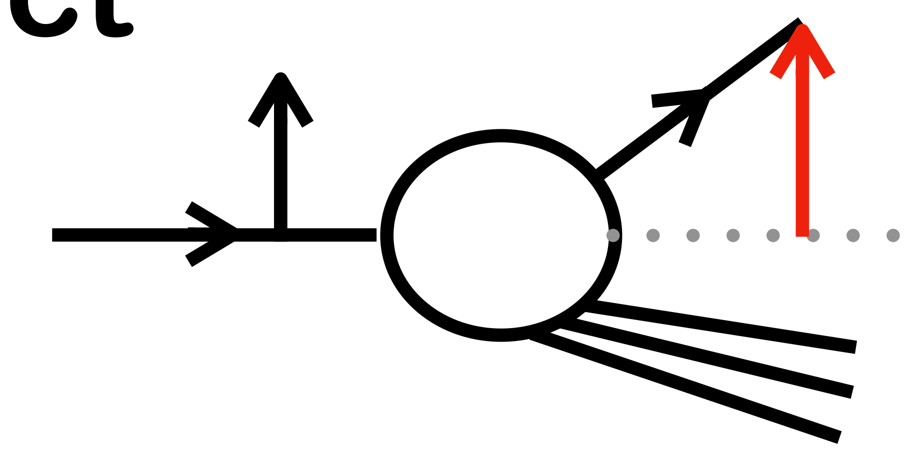

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$C[]$ **convolution over**

$$k_T, p_\perp$$

SIDIS only

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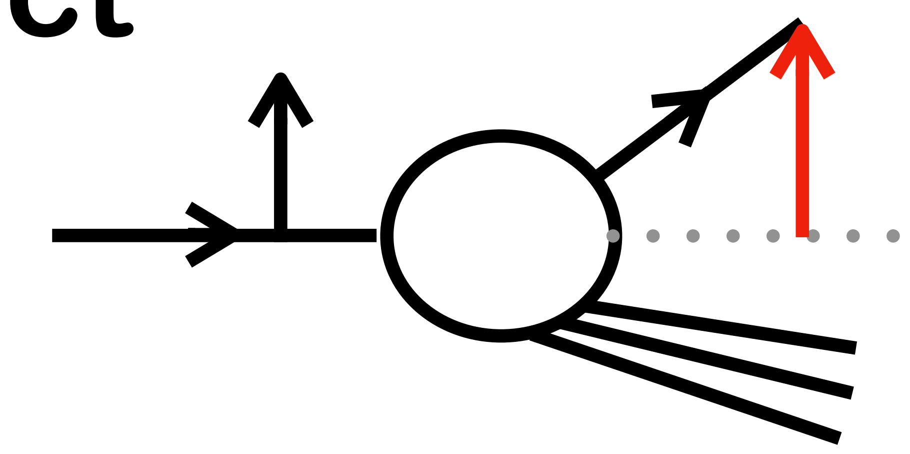

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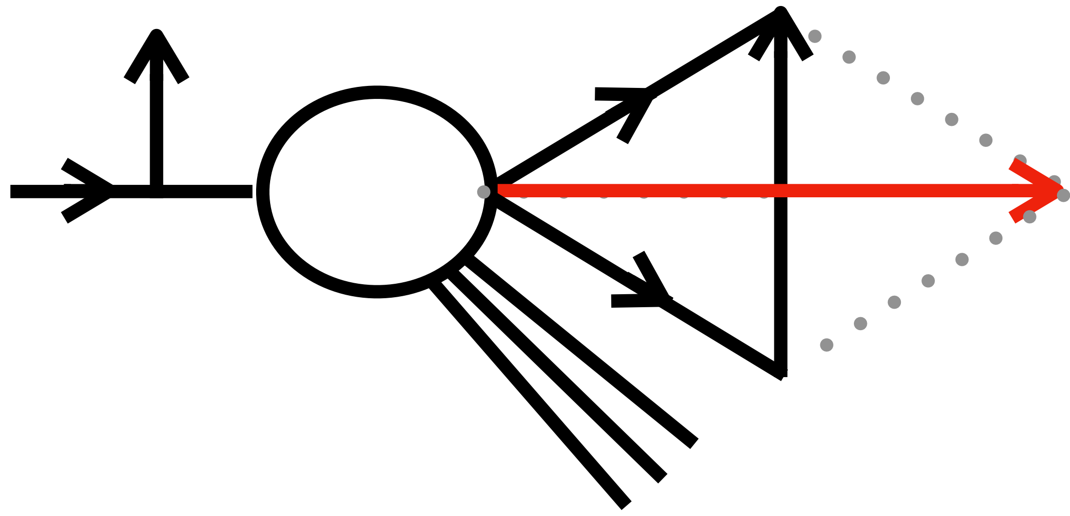
Di-hadron



$$A_{UT}^{hh} \sim \frac{\sum_q e_q^2 \cdot h_1^q(x_B) \cdot H_1^{<,q}(z, M_h)}{\sum_q e_q^2 f_1^q(x_B) \cdot D_1^q(z, M_h)}$$

Alternative method to extract trasversity PDF

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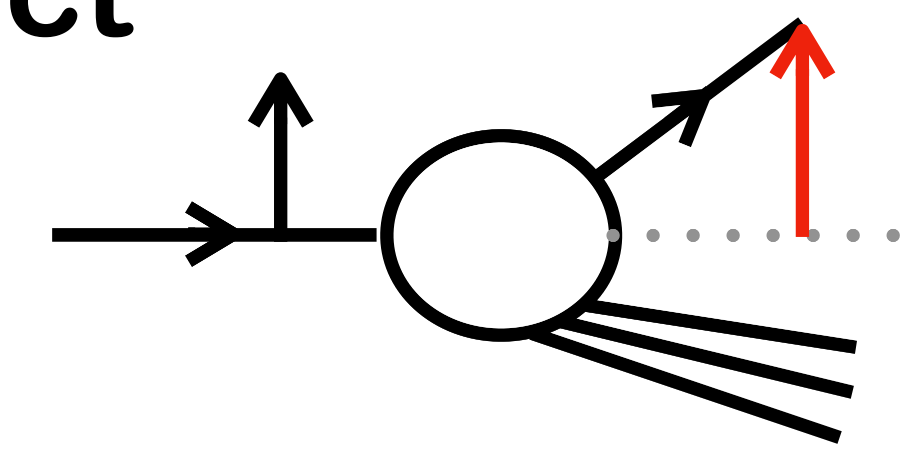

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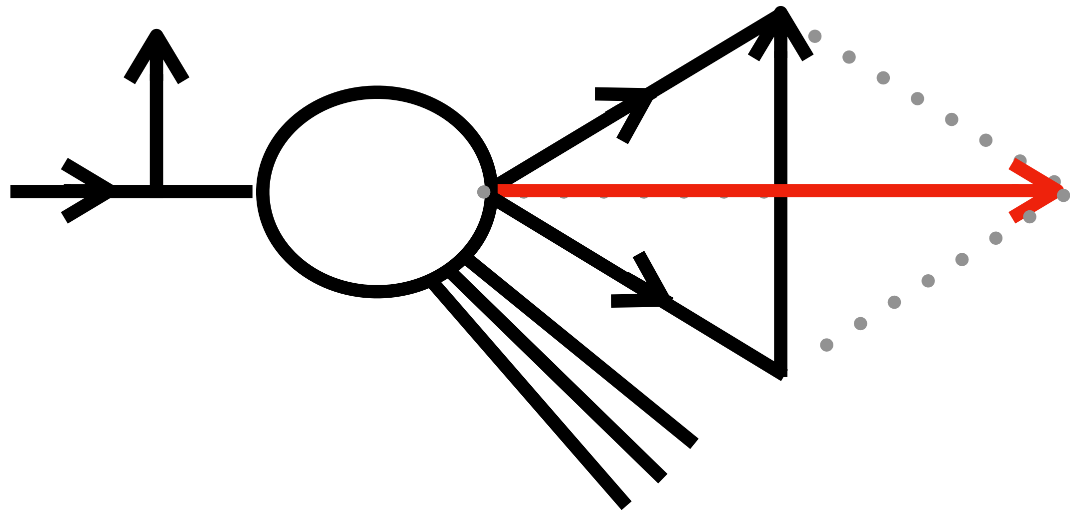
Di-hadron



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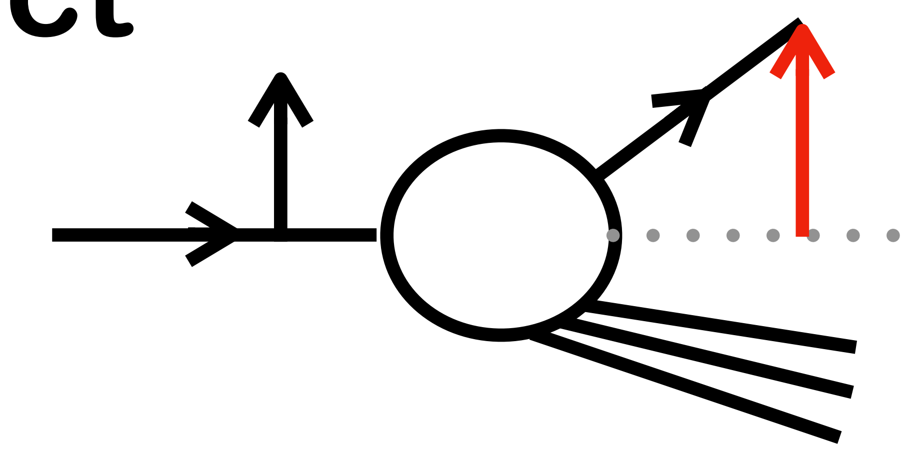
$$M_h \ll Q^2$$

Alternative method to extract trasversity PDF

Collins effect



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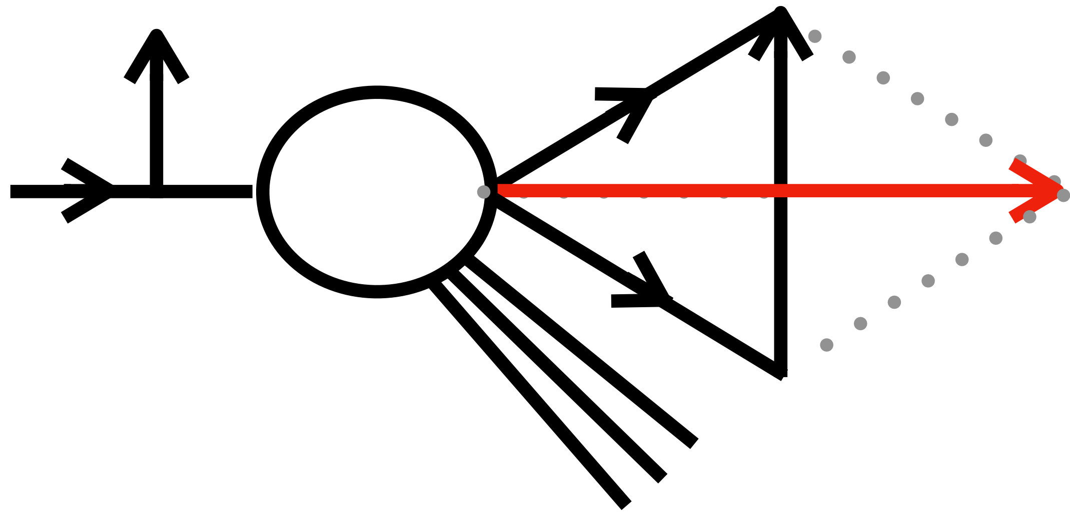
Di-hadron



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$$M_h \ll Q^2$$

Collinear and ordinary product

$h_1 H_1^{<,q}$ **also in** $pp^\uparrow$ **collisions**

2 measured hadrons + X

$$A_{UT}^{hh} \sim \frac{\sum_q e_q^2 \cdot h_1^q(x_B) \cdot H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2 \cdot f_1^q(x_B) \cdot D_1^q(z, M_h)}$$

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How to acces them?

2 measured hadrons + X

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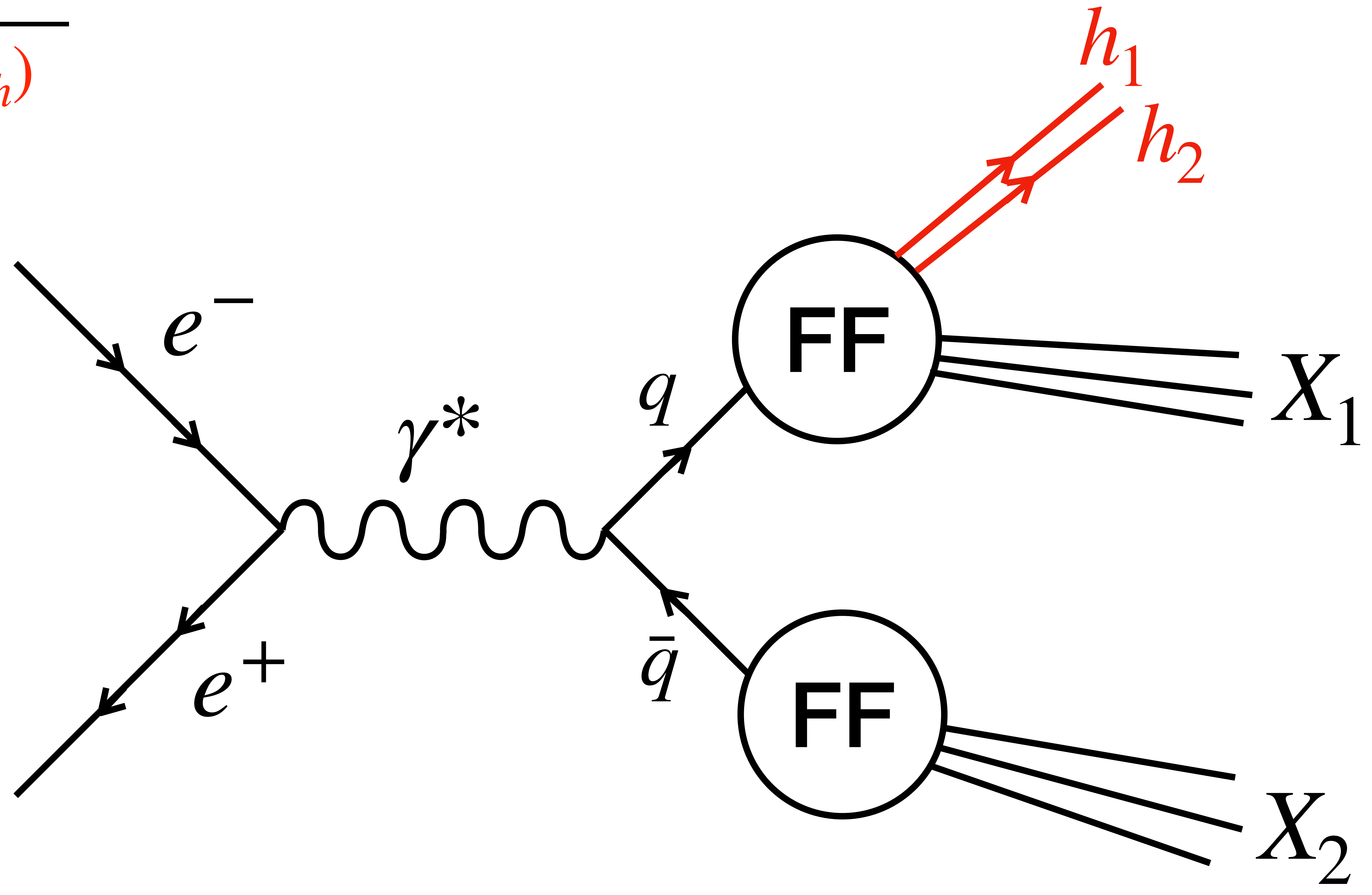

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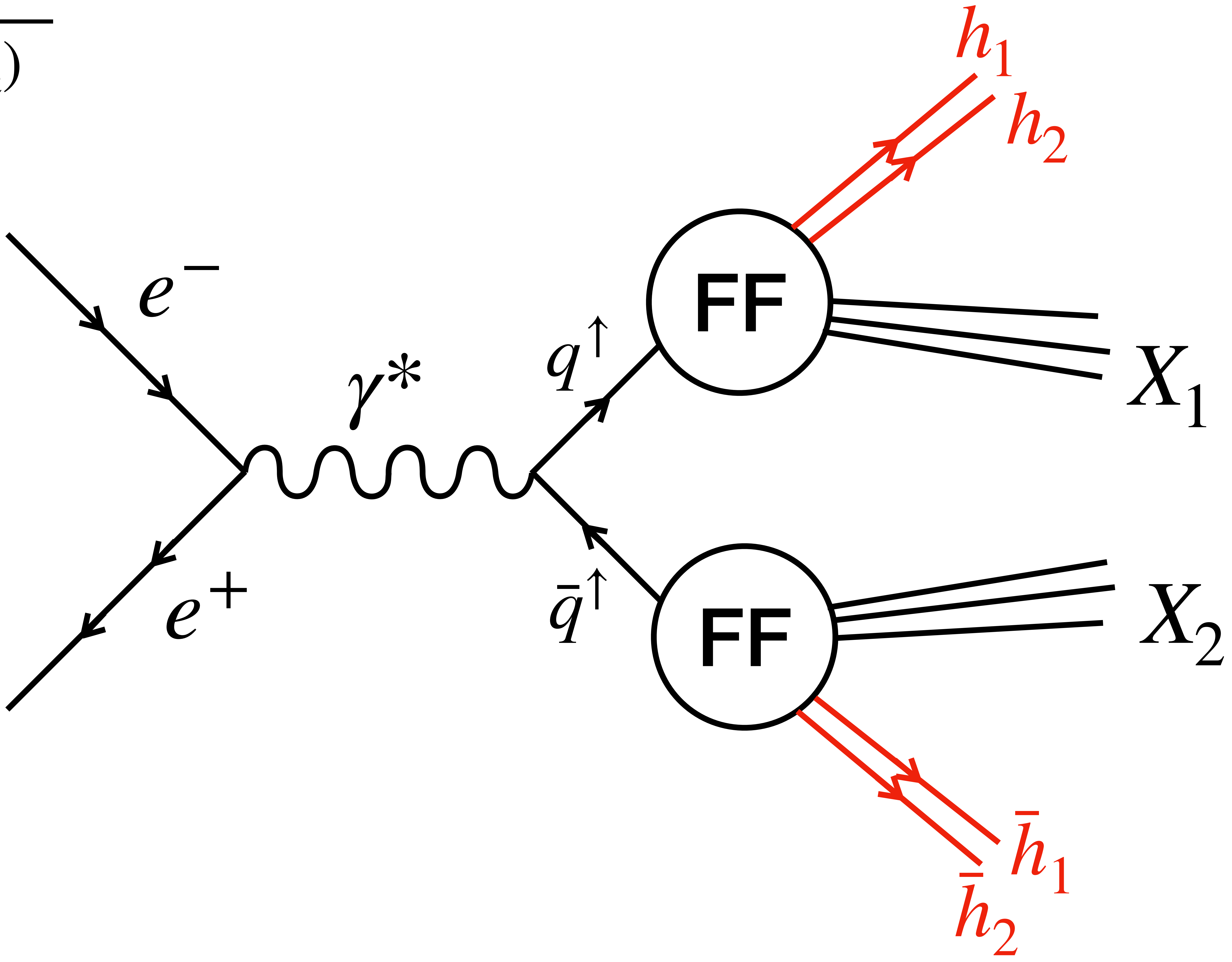

Extraction of the unpolarised D_1

GOALS of the D_1 extraction

- **2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV**

GOALS of the D_1 extraction

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- **Use of Monte-Carlo simulation for flavor separation only**

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$$\left.\frac{d\sigma}{dz dM_h dQ^2}\right|_{BELLE} = \frac{4\pi\alpha^2}{Q^2} \sum_q e_q^2 D_1^q(z, Mh, Q)$$

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$$R_{MC}^q(z, M_h, Q^2) = \frac{d\sigma_{MC}^q}{dz\,dM_h\,dQ^2} \Bigg/ \frac{d\sigma_{MC}}{dz\,dM_h\,dQ^2}$$

GOALS of the D_1 extraction

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$$\left.\frac{d\sigma}{dz dM_h dQ^2}\right|_{BELLE} = \frac{4\pi\alpha^2}{Q^2}\sum_q e_q^2 D_1^q(z, Mh, Q)$$

$$R^q_{MC}(z, M_h, Q^2) = \frac{d\sigma^q_{MC}}{dz\, dM_h\, dQ^2} \Big/ \frac{d\sigma_{MC}}{dz\, dM_h\, dQ^2}$$

$$\frac{d\sigma^q}{dz dM_h dQ^2} = \left.\frac{d\sigma}{dz dM_h dQ^2}\right|_{BELLE} R^q_{MC}(z, M_h; Q^2)$$

GOALS of the D_1 extraction

- **2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV**
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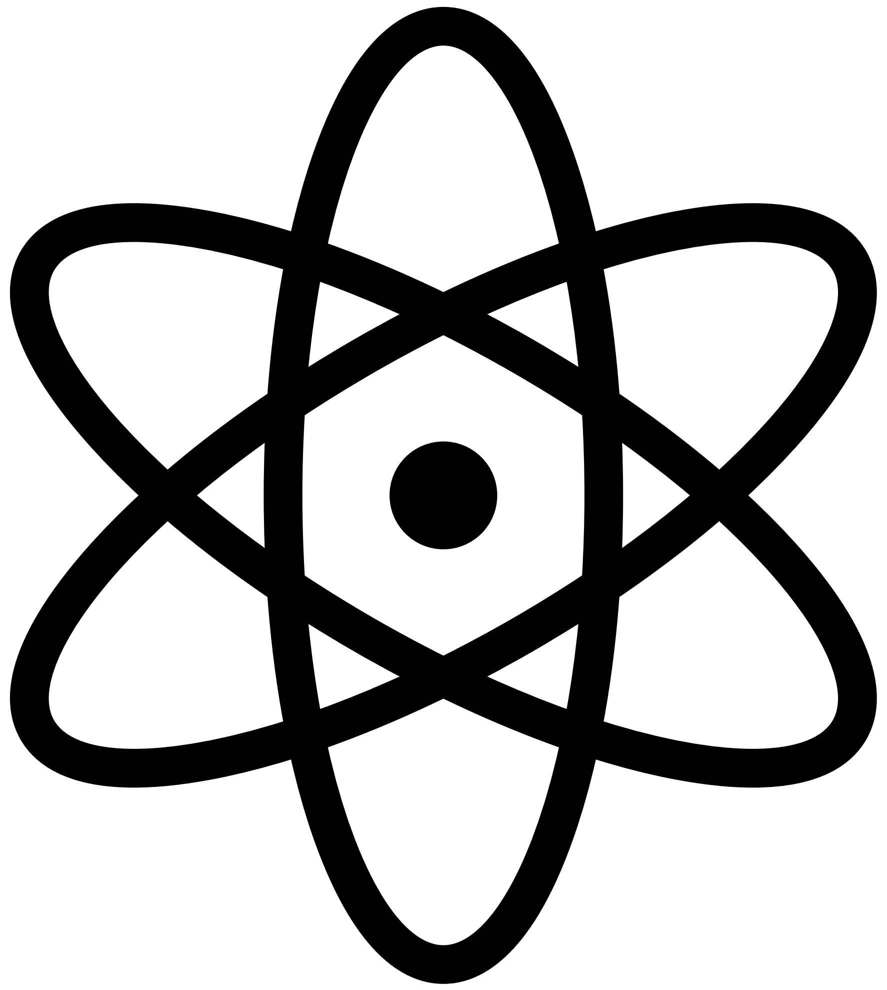
- **Extend the analysis up to NNLO**

GOALS of the D_1 extraction

- 2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV
- Use of Monte-Carlo simulation for flavor separation only
- Extend the analysis up to NNLO
- Explore a Neural Network parameterisation

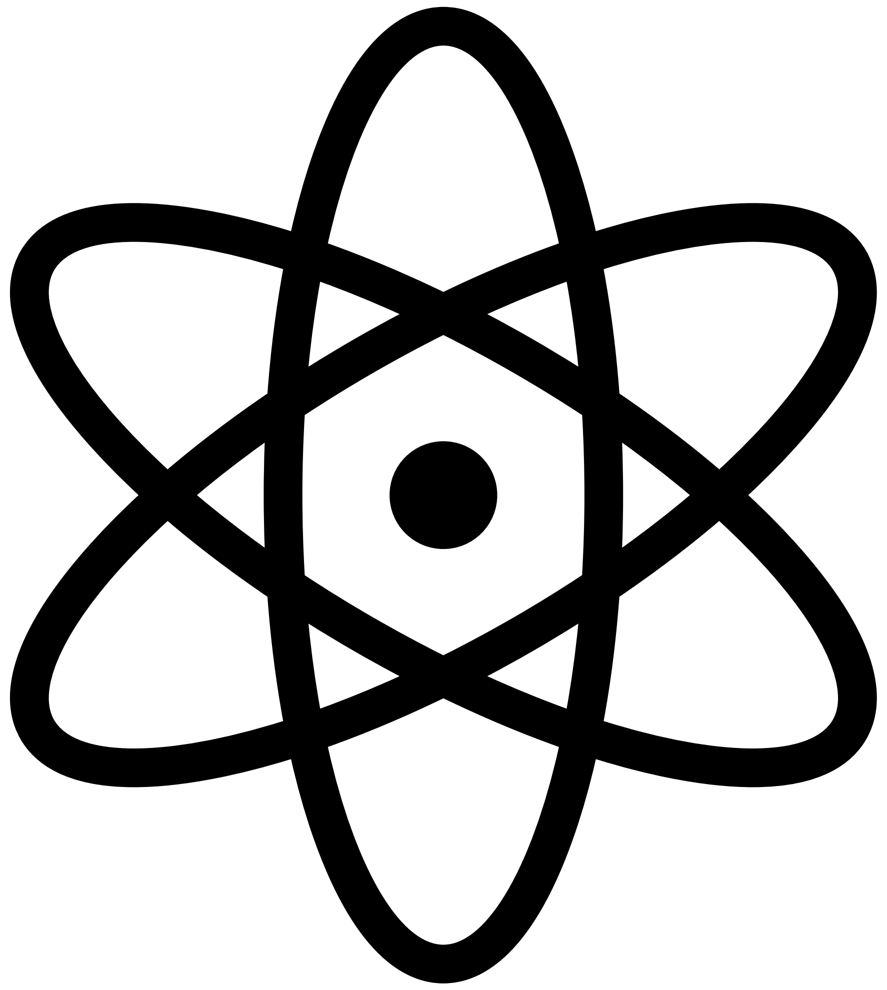
PHYSICS INFORMED

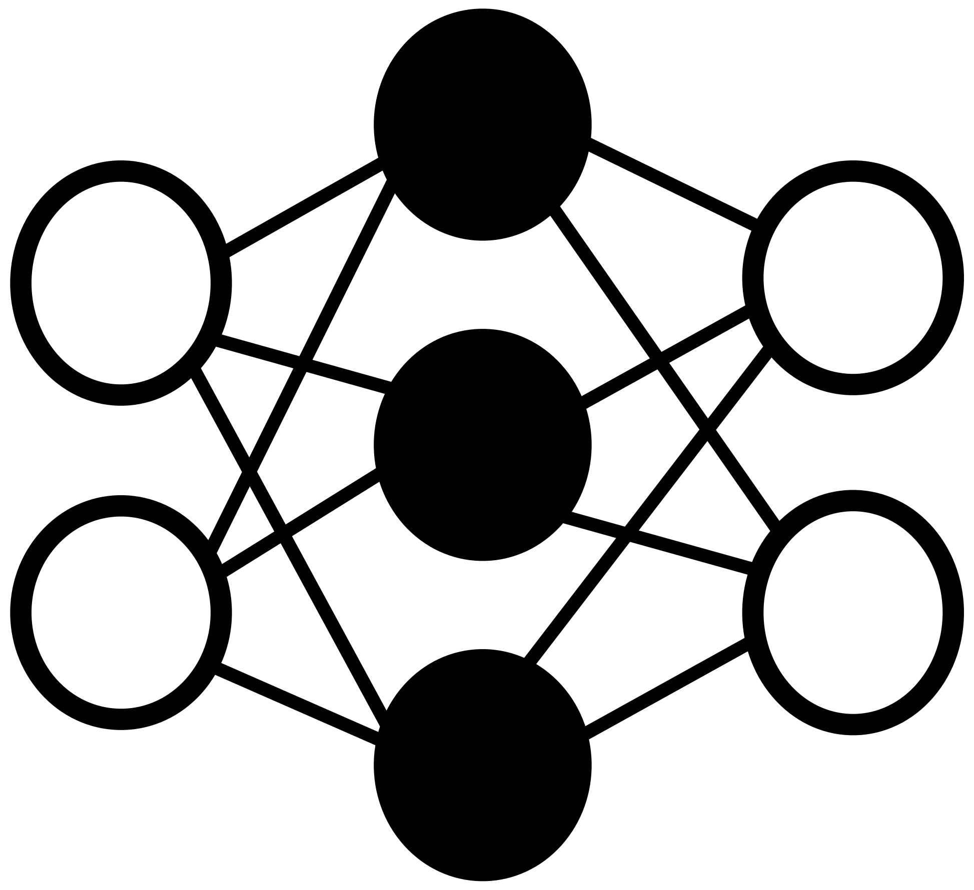


71 par

PHYSICS INFORMED

NEURAL NETWORK



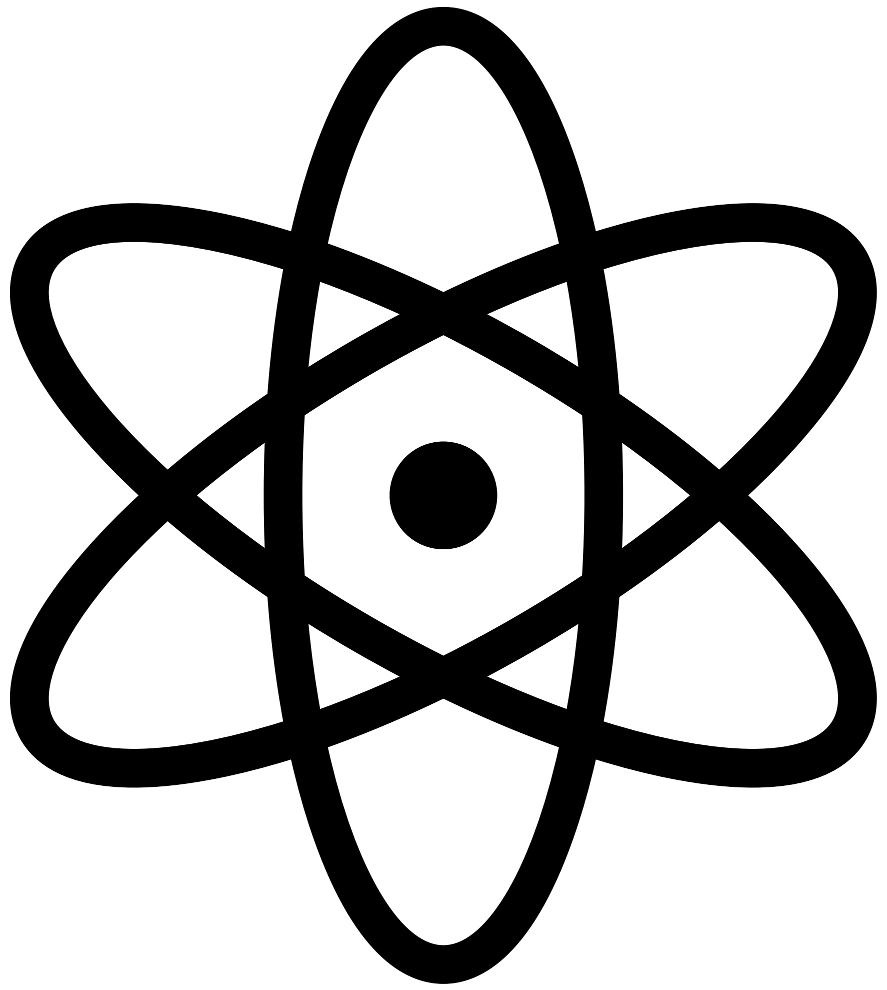


71 par

NN architecture

[2,25,5] ⤳ 205 par

PHYSICS INFORMED



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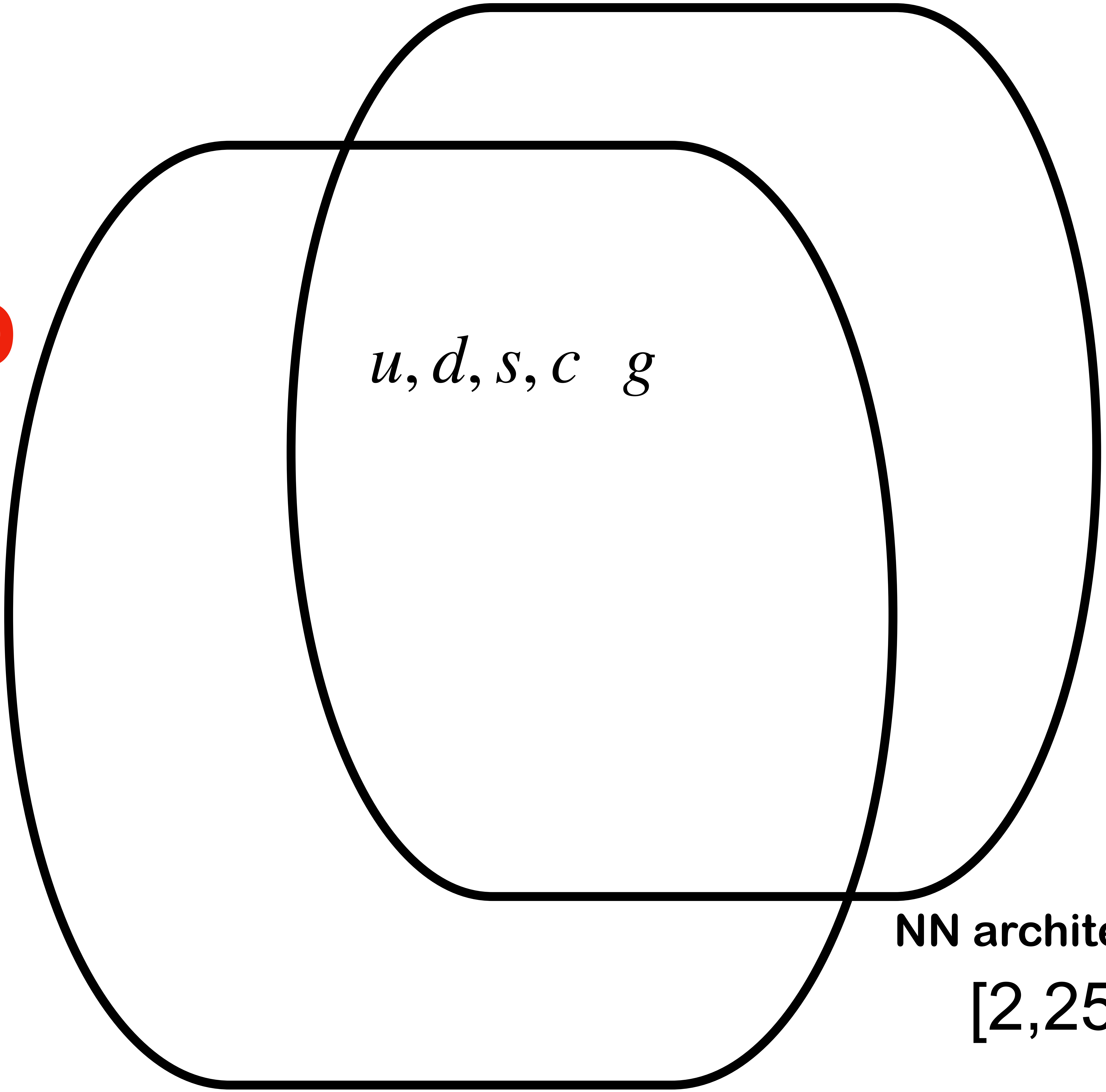
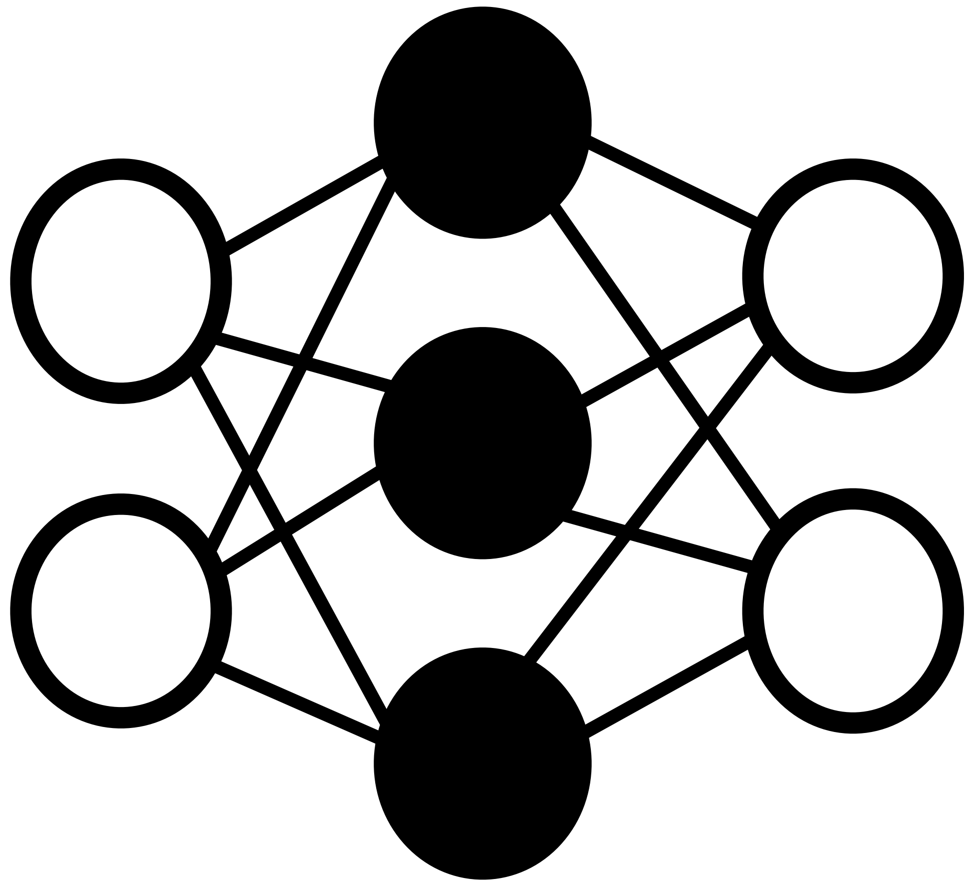


NEURAL NETWORK



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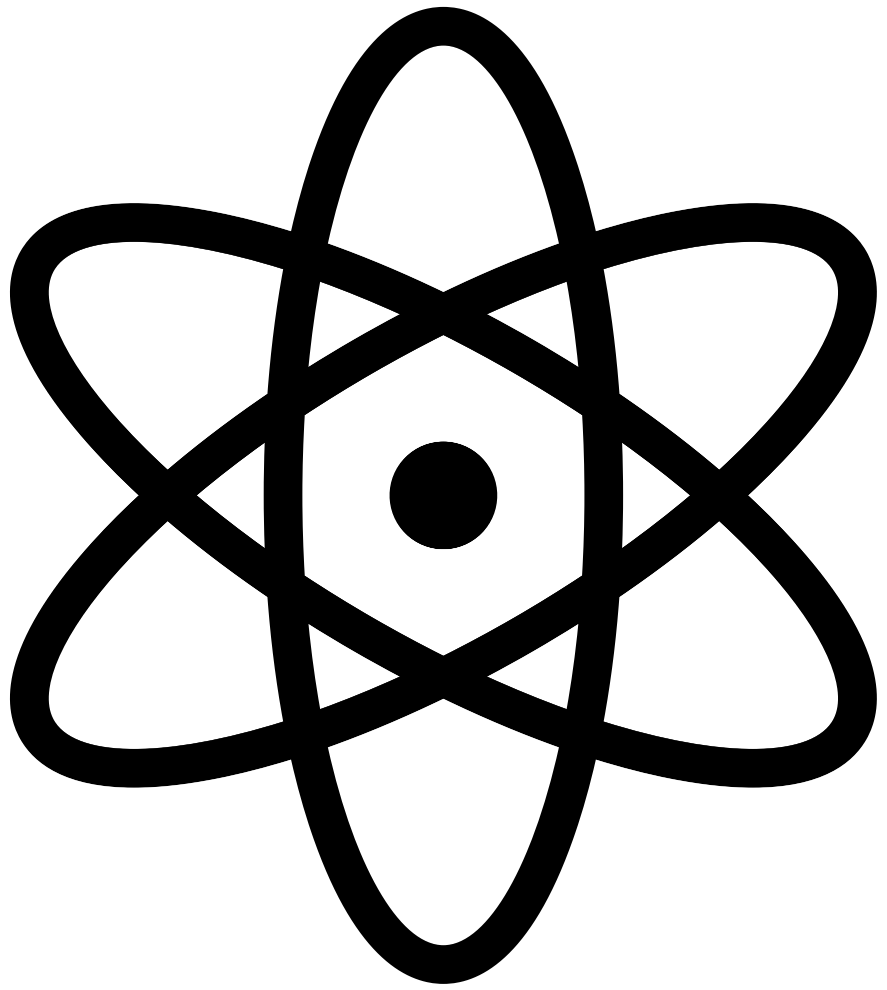
PHYSICS INFORMED



71 par

$Q_0 = 1$ GeV

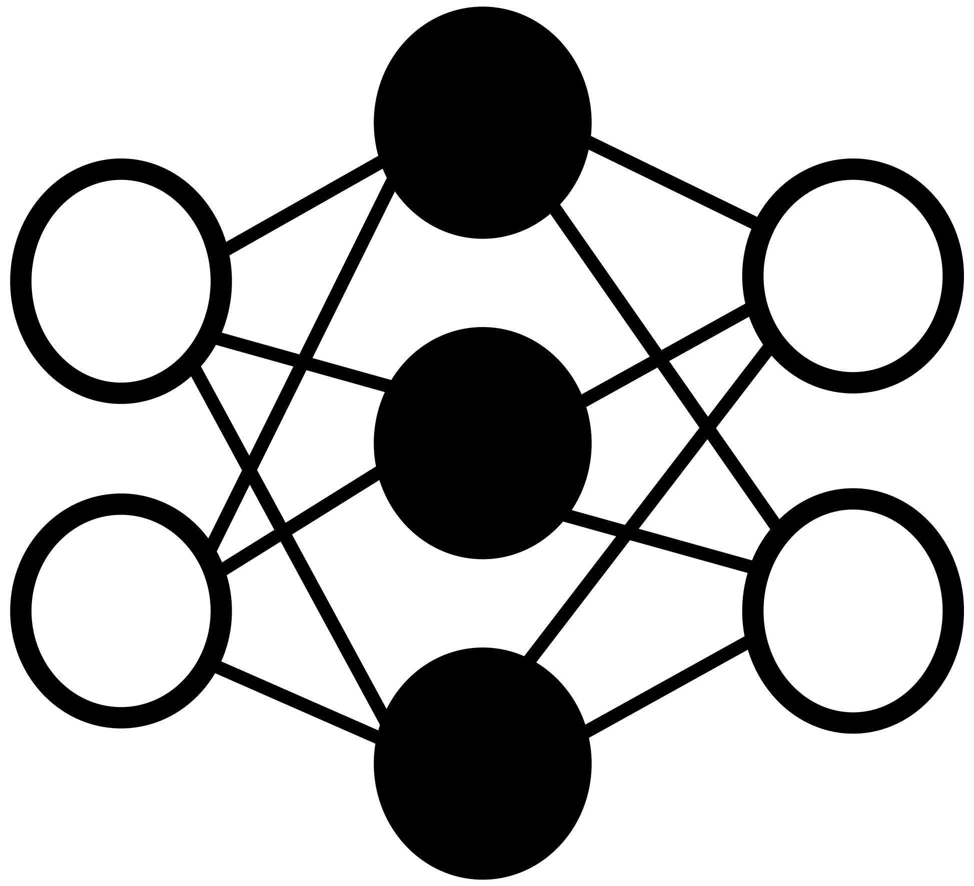
$u, d, s, c \quad g$

$D_1^q = D_1^{\bar{q}}$

$D_1^q = 0$ for $M_h < 2M_\pi$

$D_1^q \to 0 \; z \to 1$

NEURAL NETWORK



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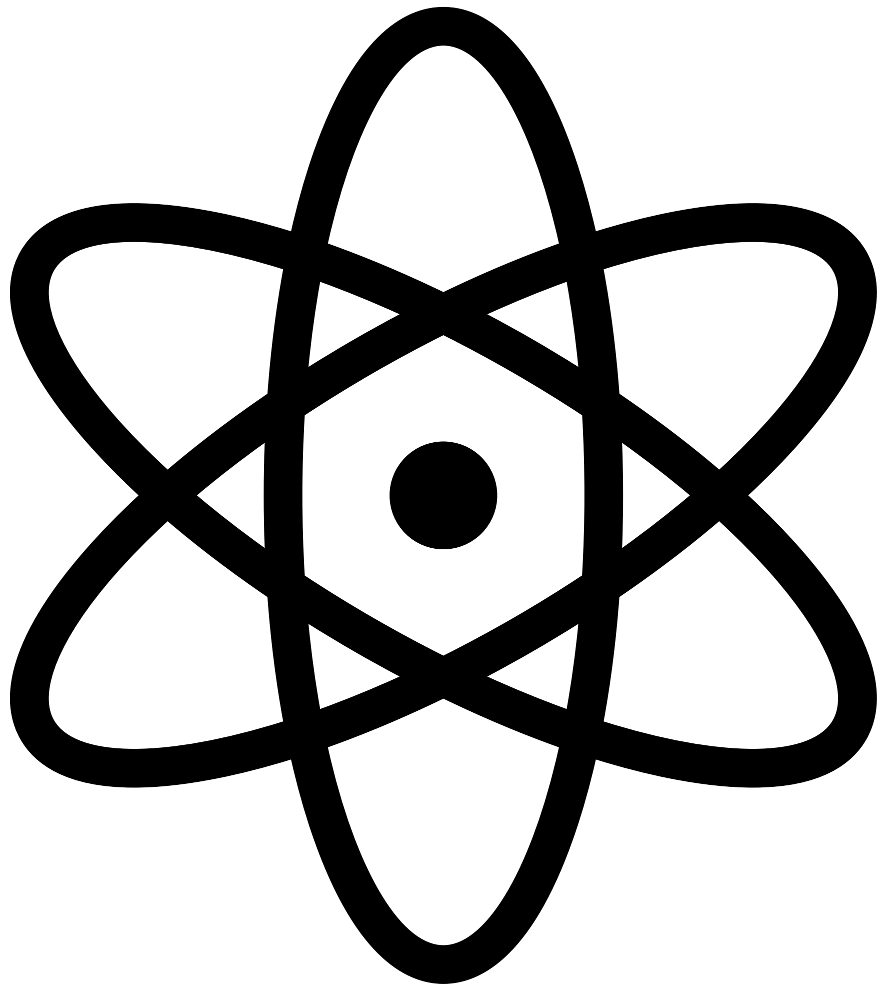

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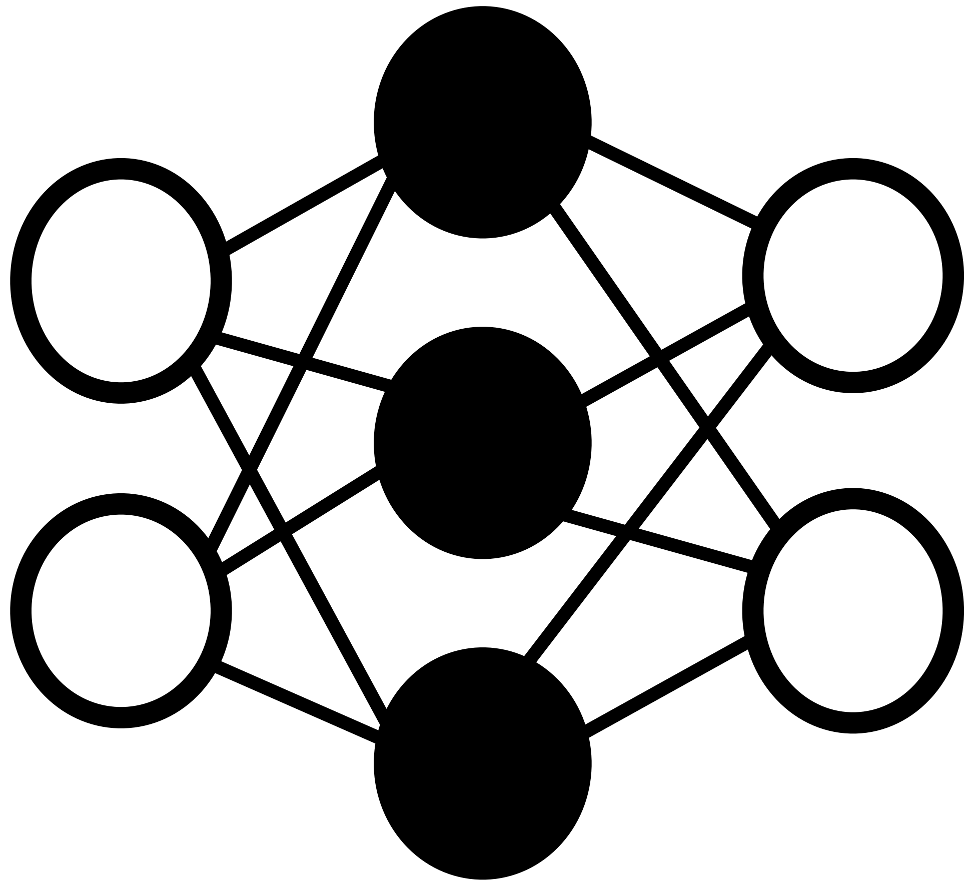
$D_1^q = 0$ for $M_h < 2M_\pi$

$D_1^q \to 0 \; z \to 1$

$D_1^u = D_1^d$

$D_1^g = N z^\alpha (1-z)^\beta D_1^u$

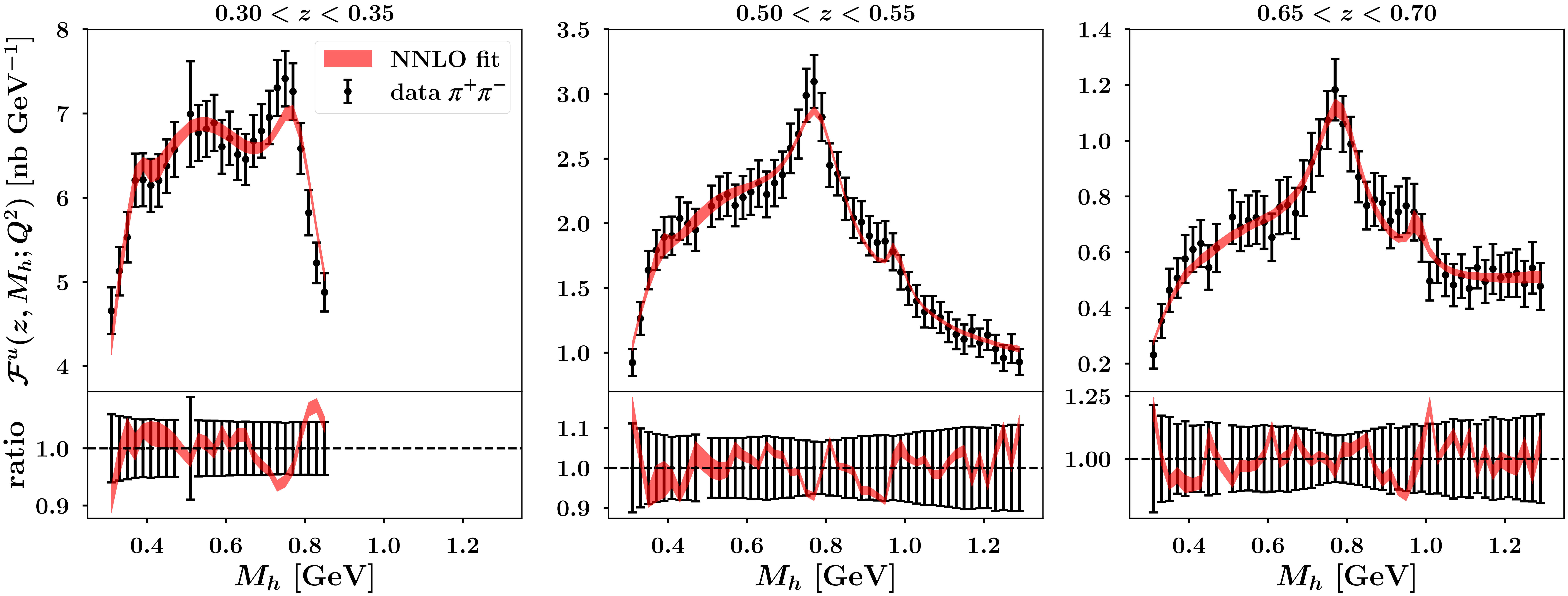
NEURAL NETWORK



NN architecture

[2,25,5] ⤳ 205 par

Predictions: Physics informed



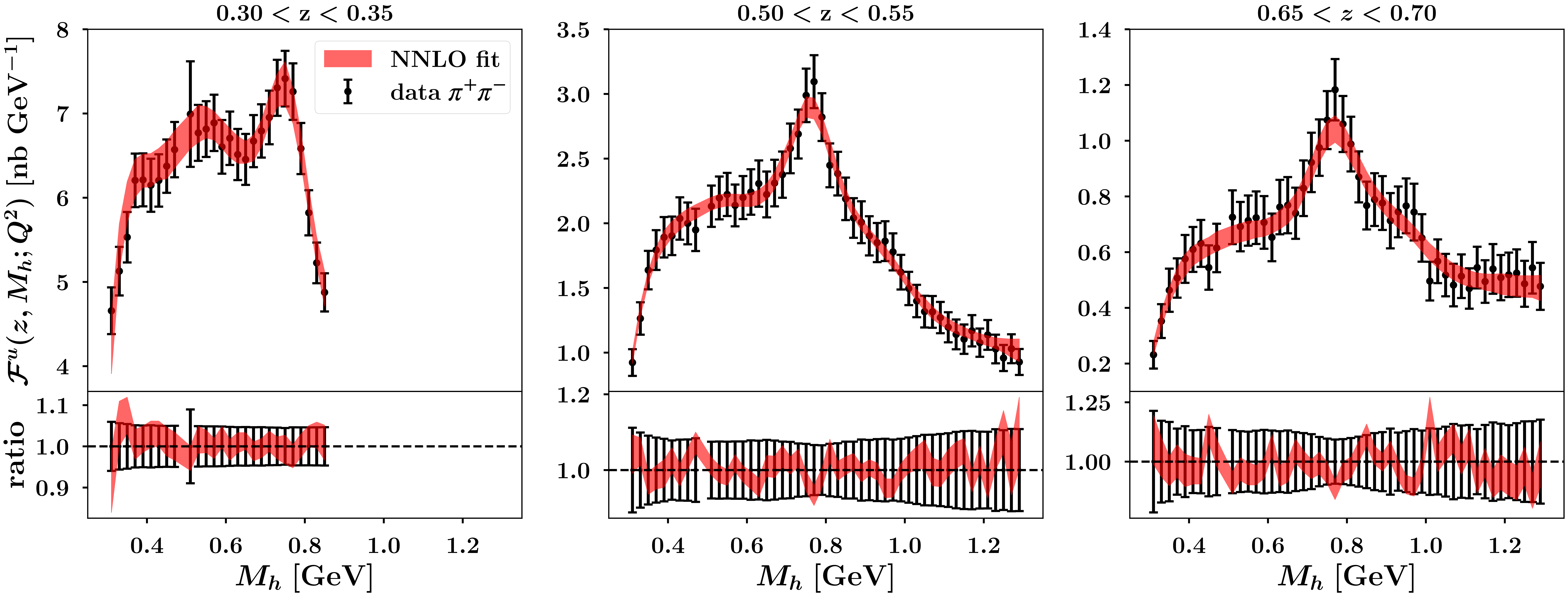
NNLO

100 replicas

$\chi_u^2 = 0.350$ $\chi_s^2 = 0.850$ $\chi_{tot}^2 = 0.648$

$\chi_d^2 = 0.591$ $\chi_c^2 = 0.802$

Predictions: Neural Network



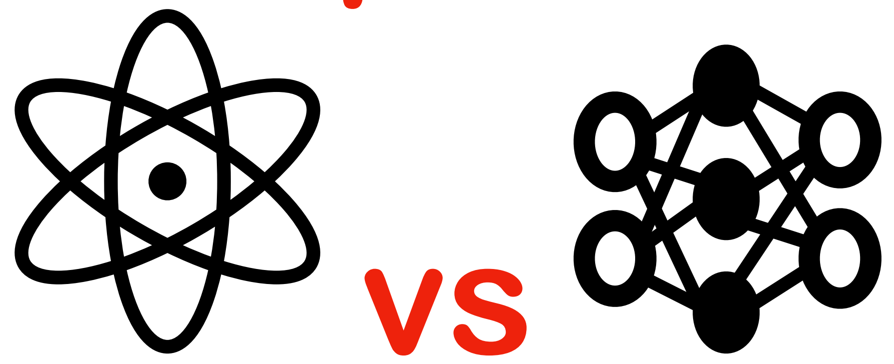
NNLO

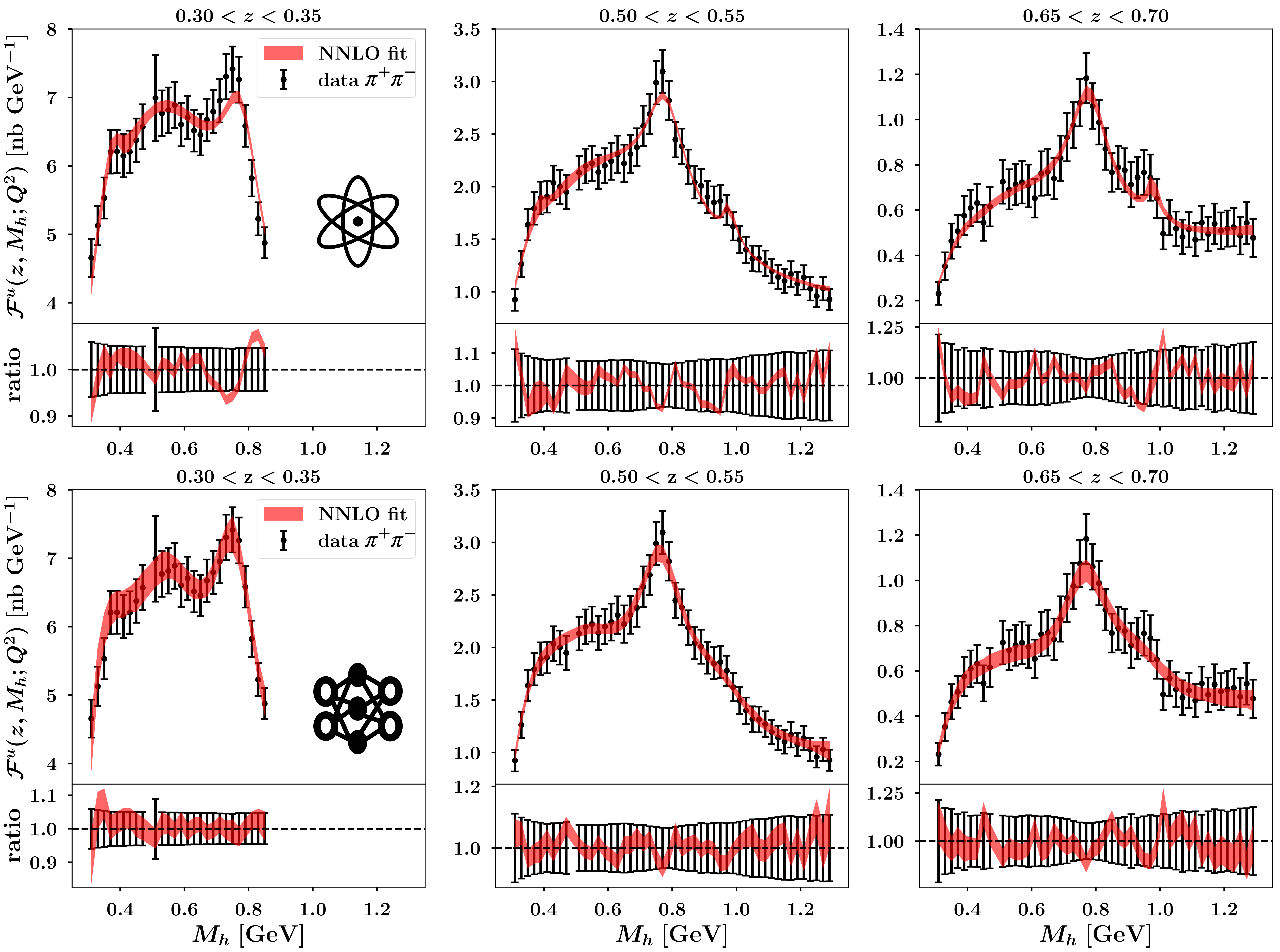
100 replicas

$\chi^2_u = 0.214$ $\chi^2_s = 0.793$ $\chi^2_{tot} = 0.537$

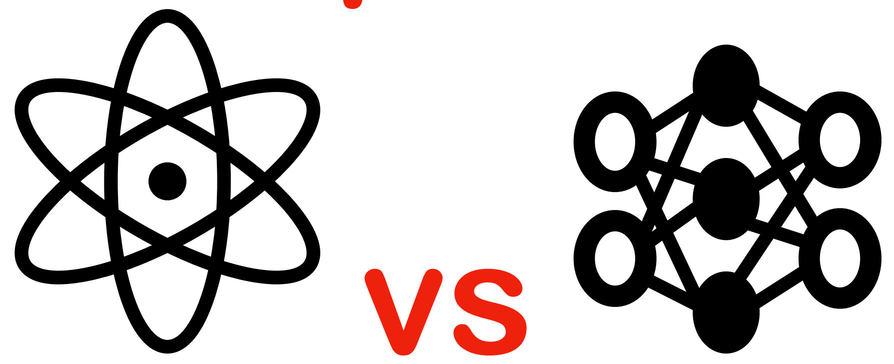
$\chi^2_d = 0.534$ $\chi^2_c = 0.607$

Comparison

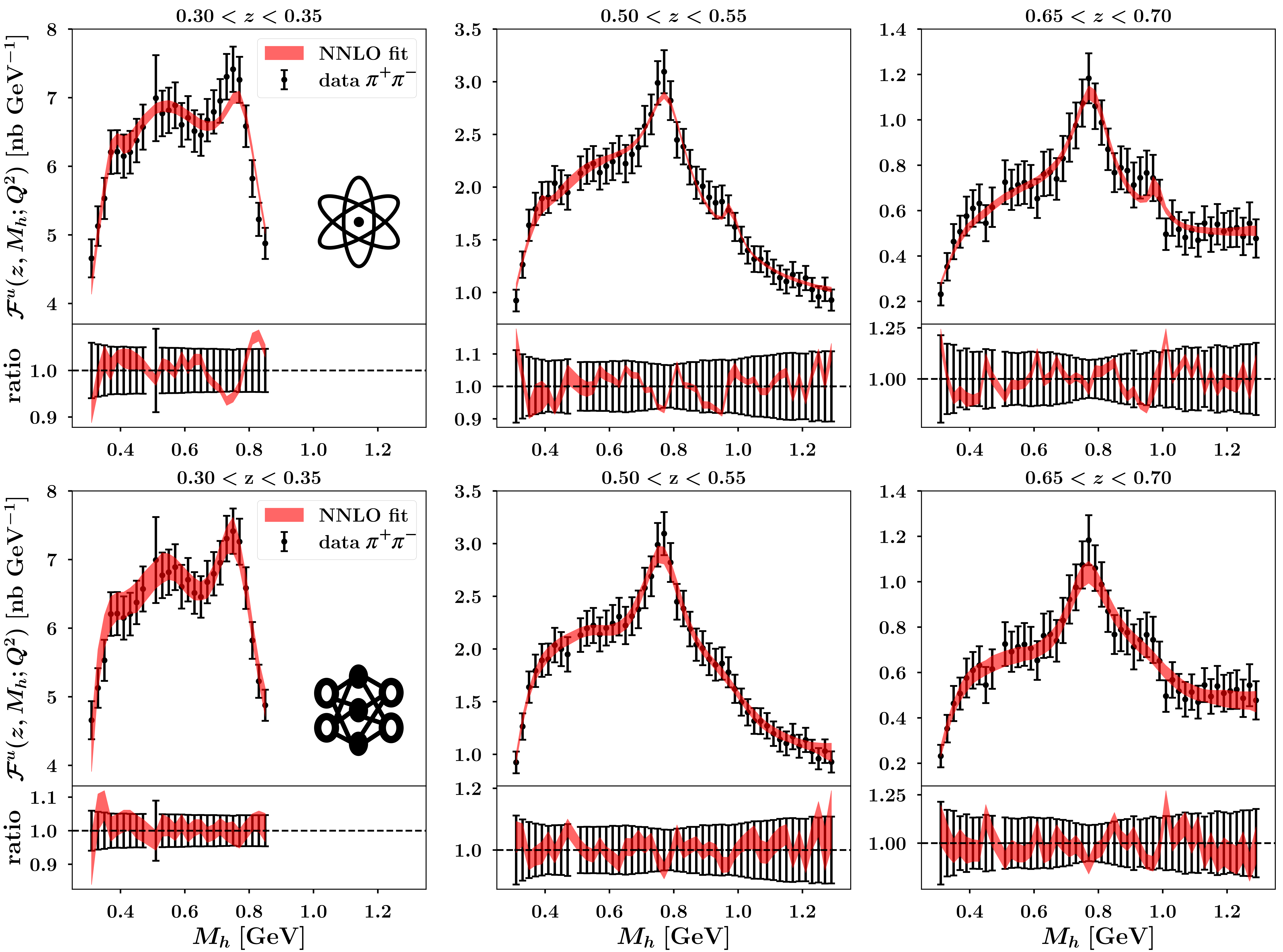

vs


0.30 < z < 0.35
0.50 < z < 0.55
0.65 < z < 0.70
NNLO fit
data $\pi^+\pi^-$
$\mathcal{F}^u(z, M_h; Q^2)$ [nb GeV^{-1}]
ratio
M_h [GeV]

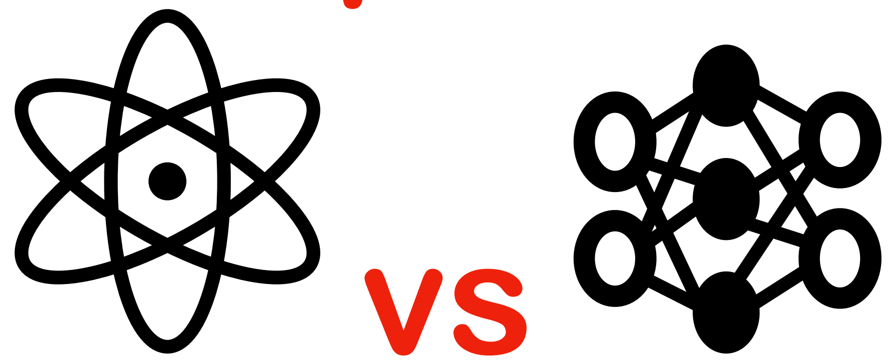
Comparison



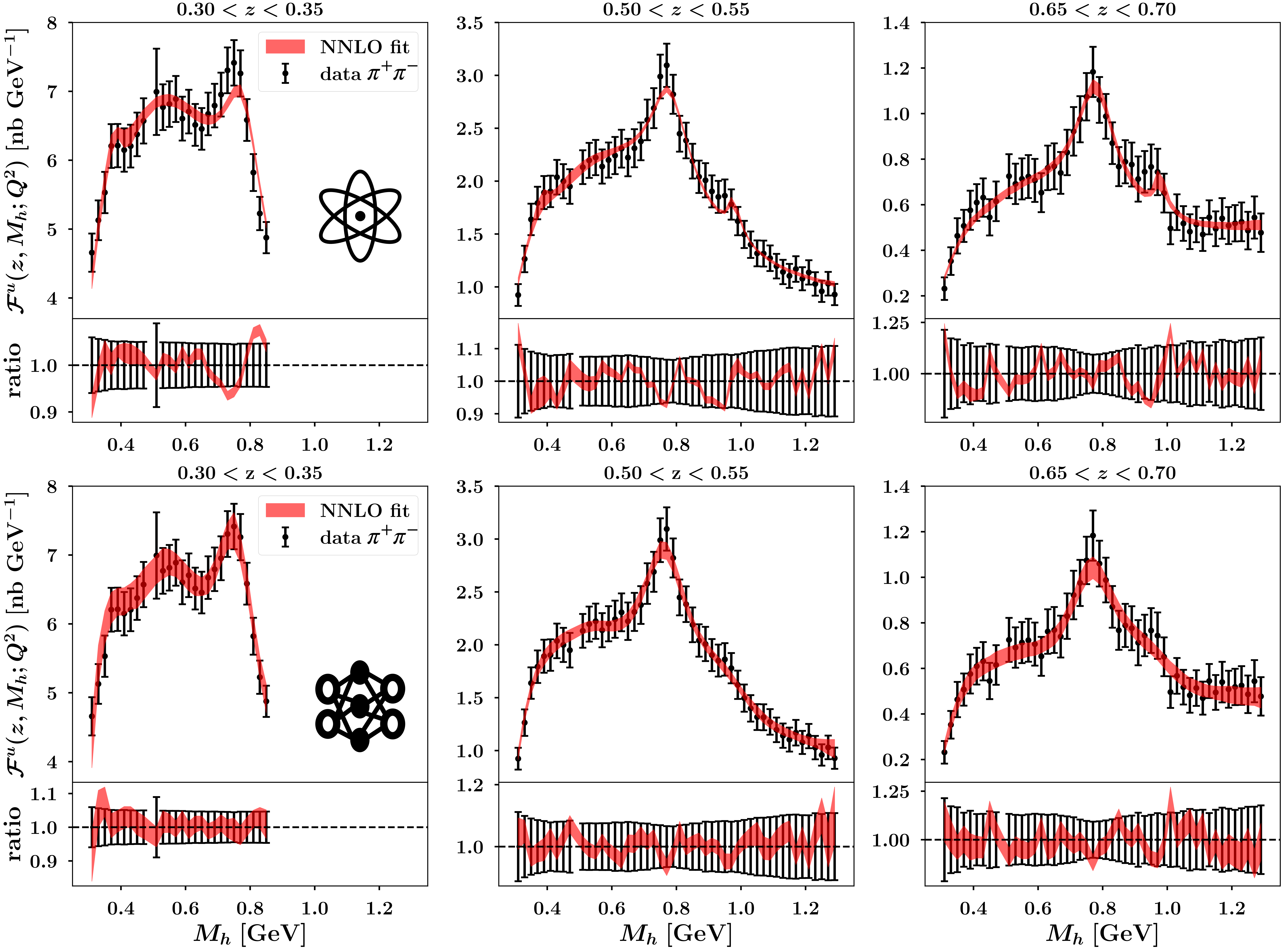
- **NN gives a better χ^2**



Comparison

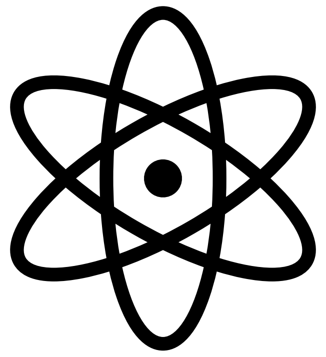


- NN gives a better χ^2
- NN has less sensitivity on resonances



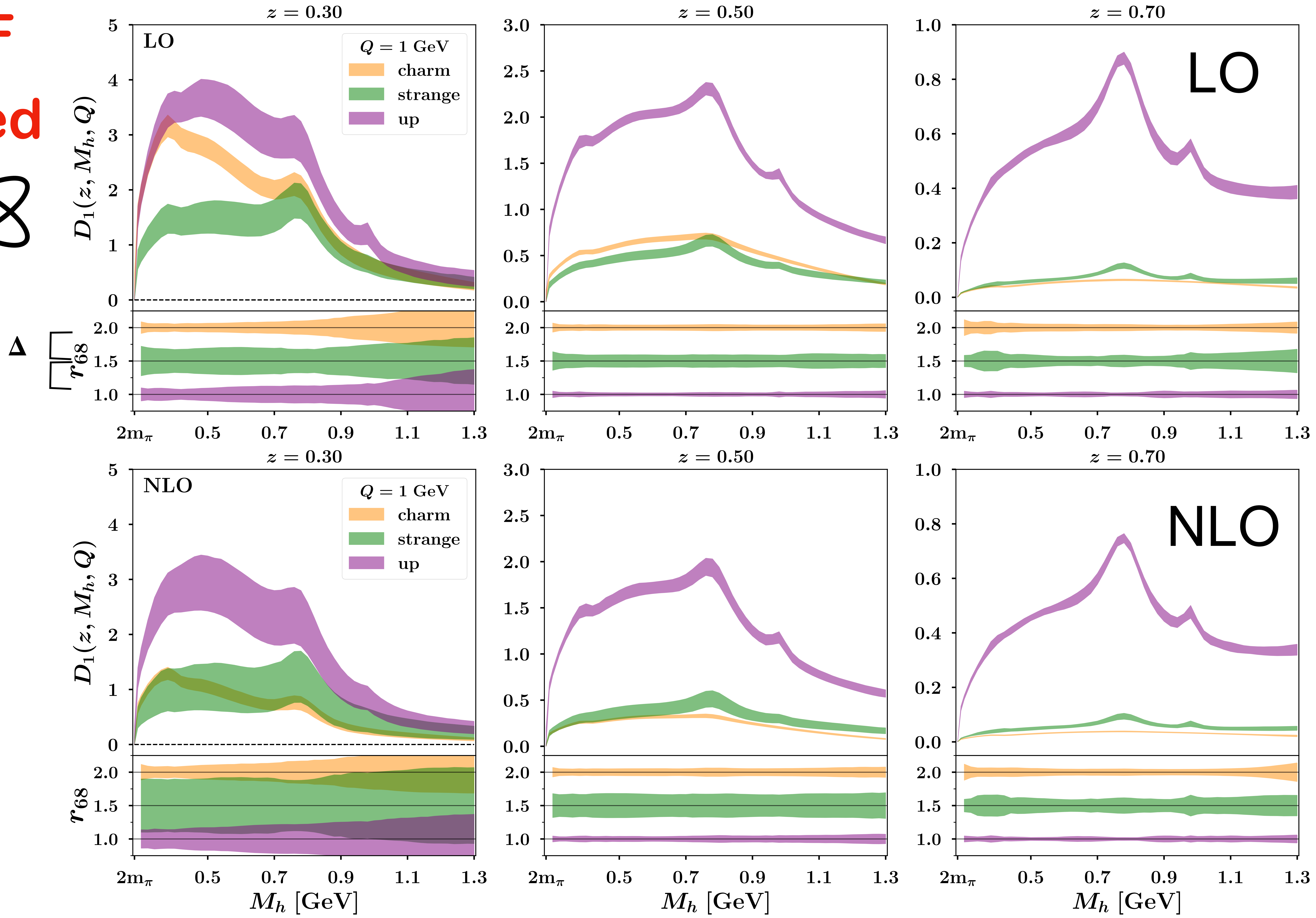
DI-HADRON FF

Physics informed



u=d

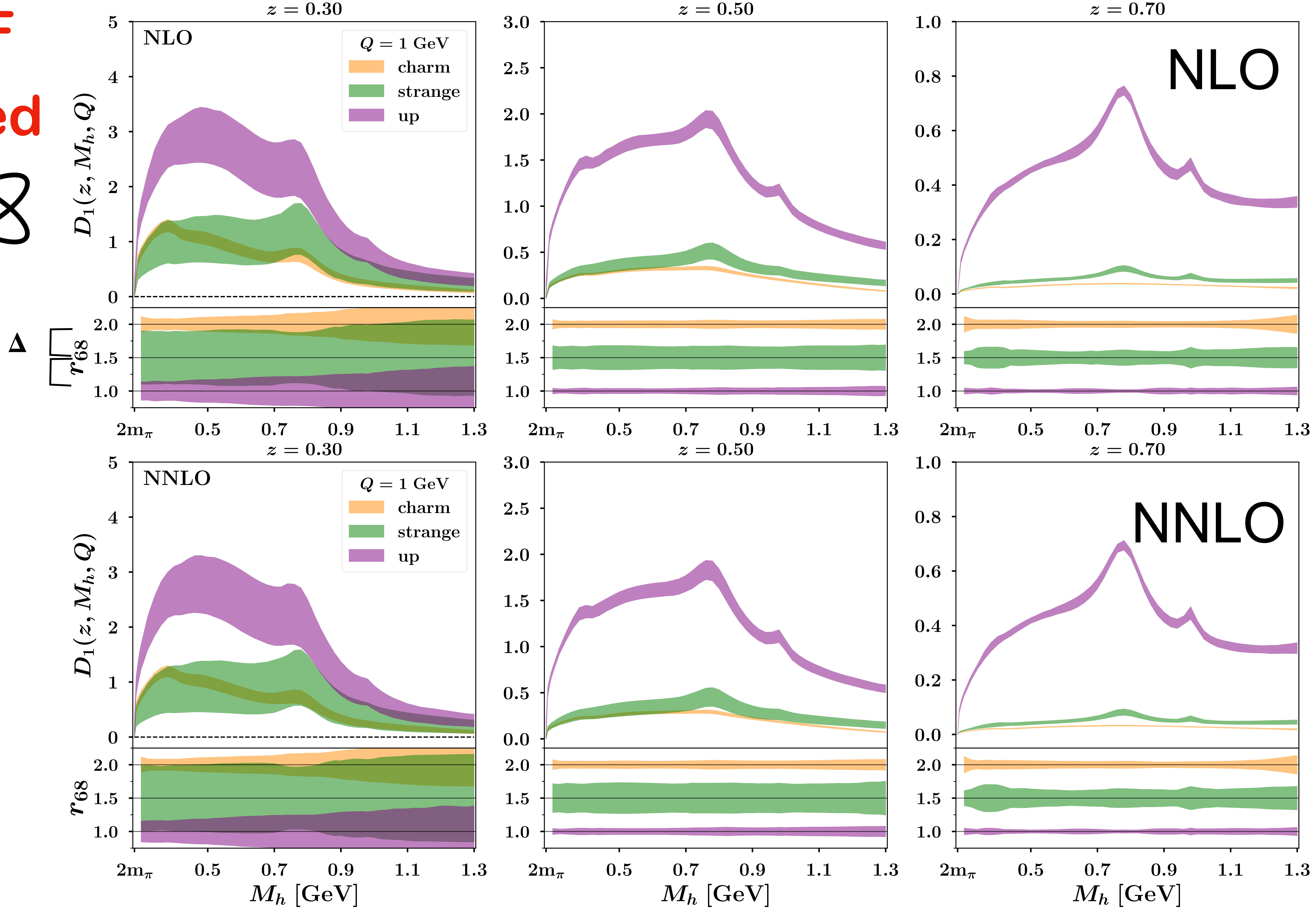
ratio offset Δ = 0.5



DI-HADRON FF
Physics informed

u=d

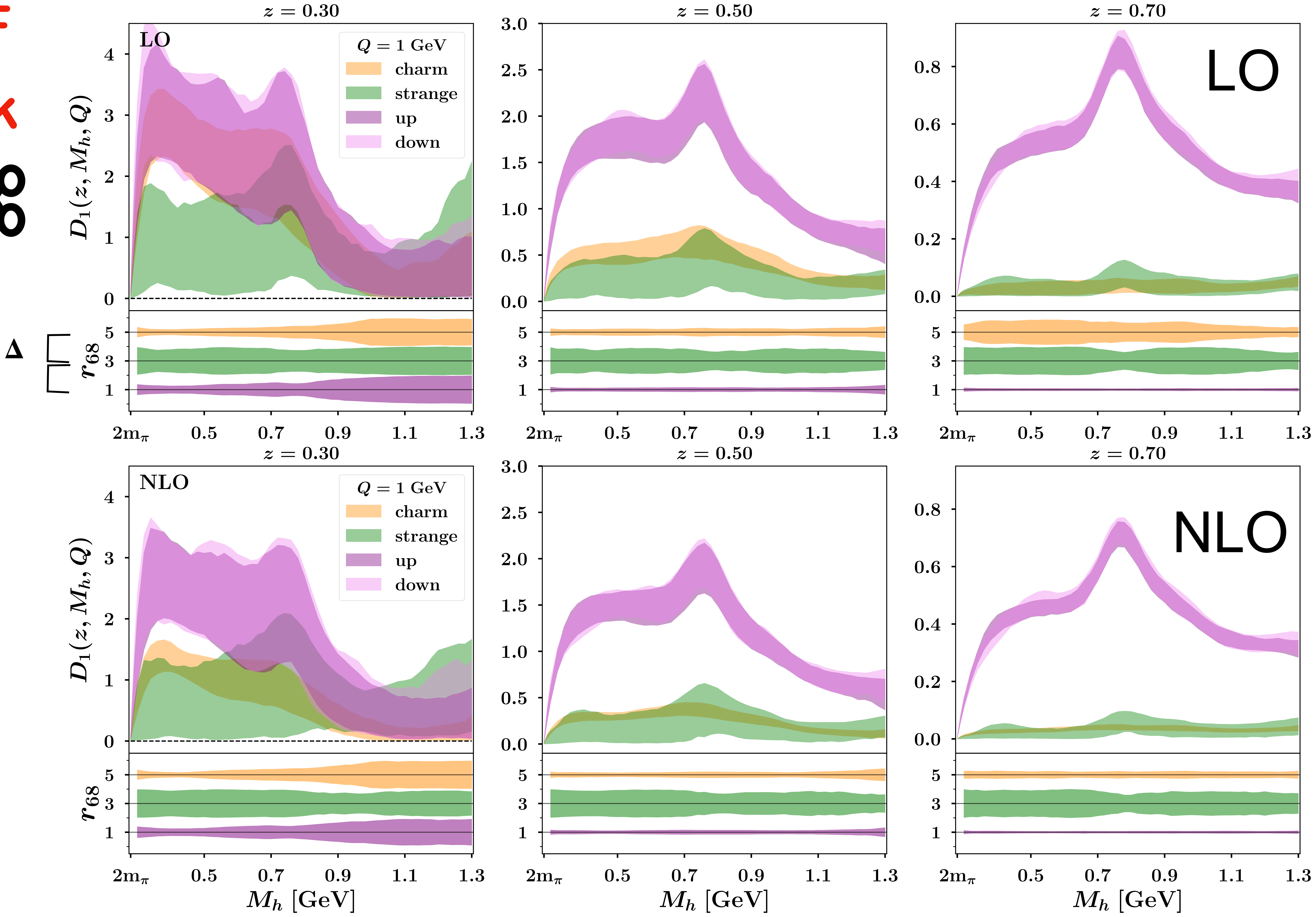
ratio offset **Δ** = 0.5



DI-HADRON FF
Neural Network

u ≠ d

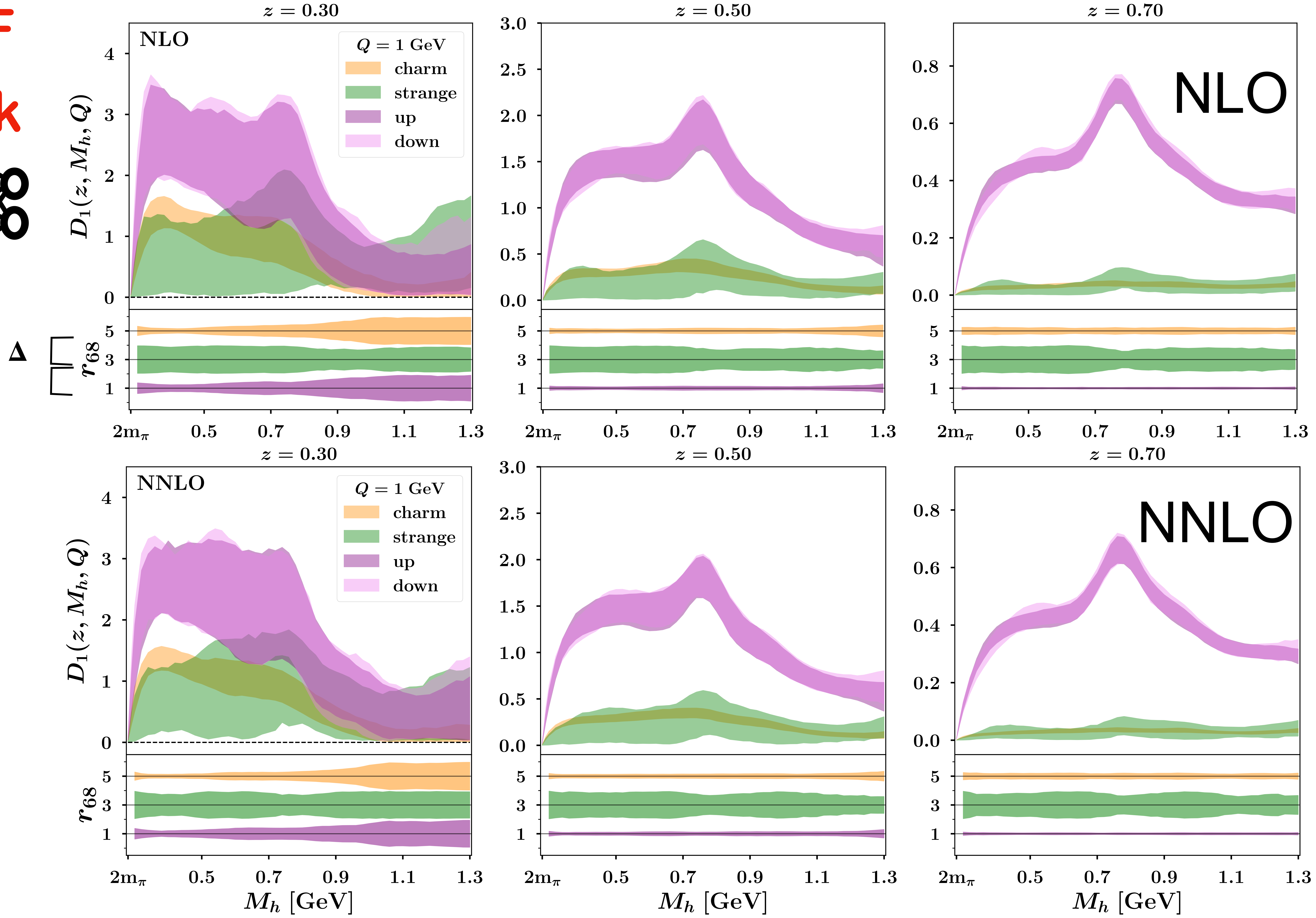
ratio offset **Δ** = 2.5



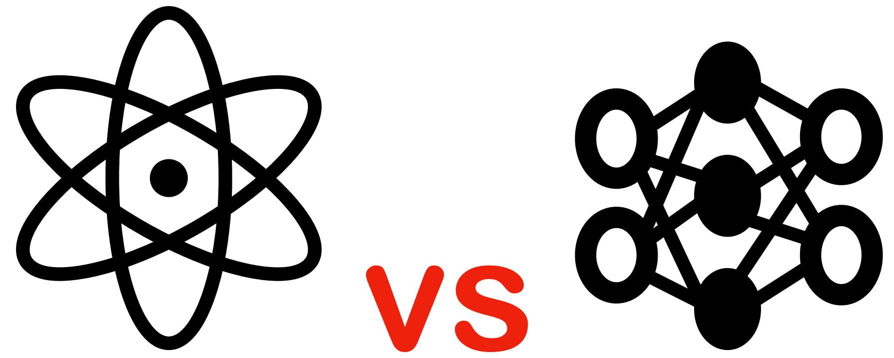
DI-HADRON FF
Neural Network

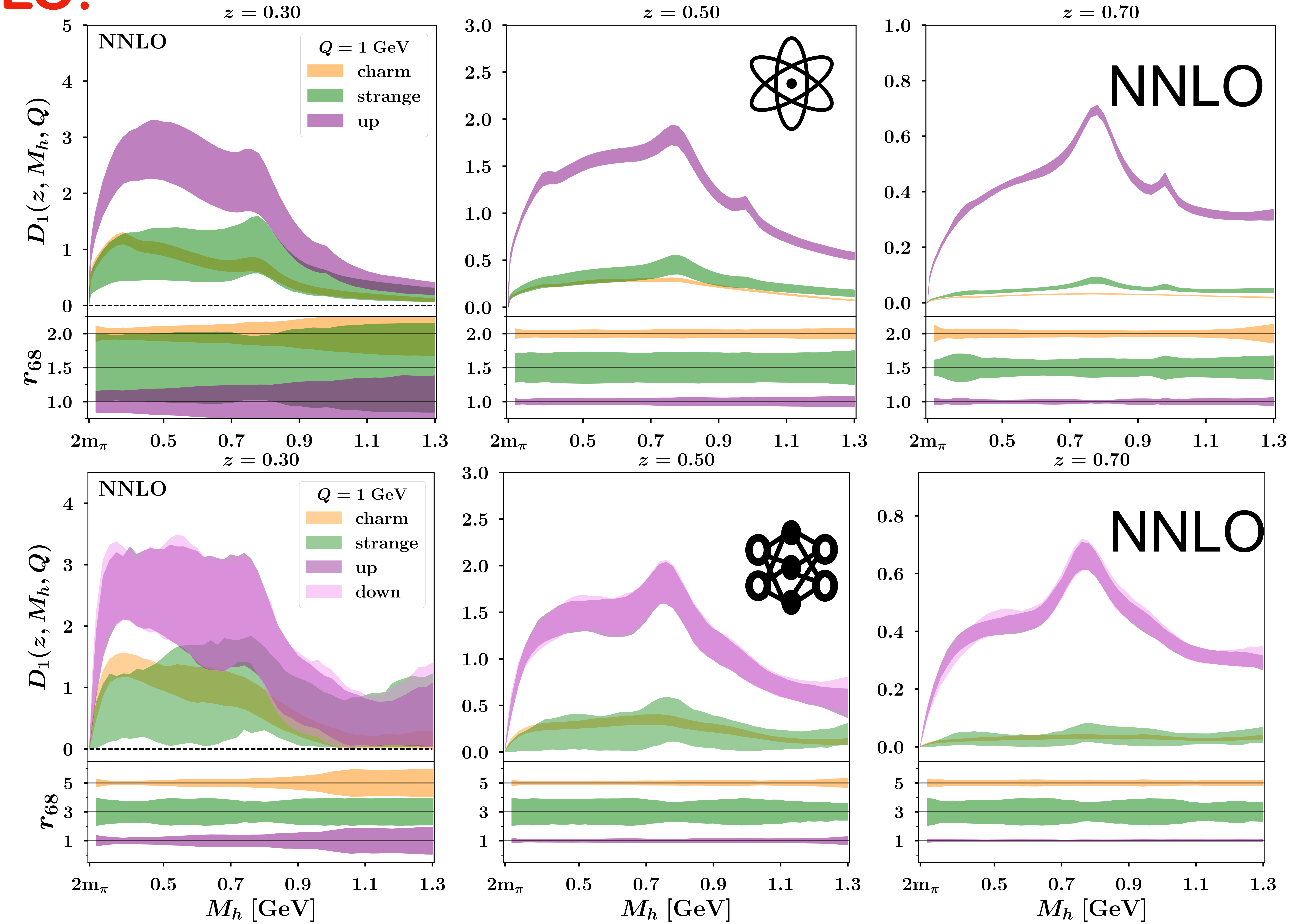
$\mathbf{u \neq d}$

ratio offset Δ = 2.5



Comparison at NNLO:

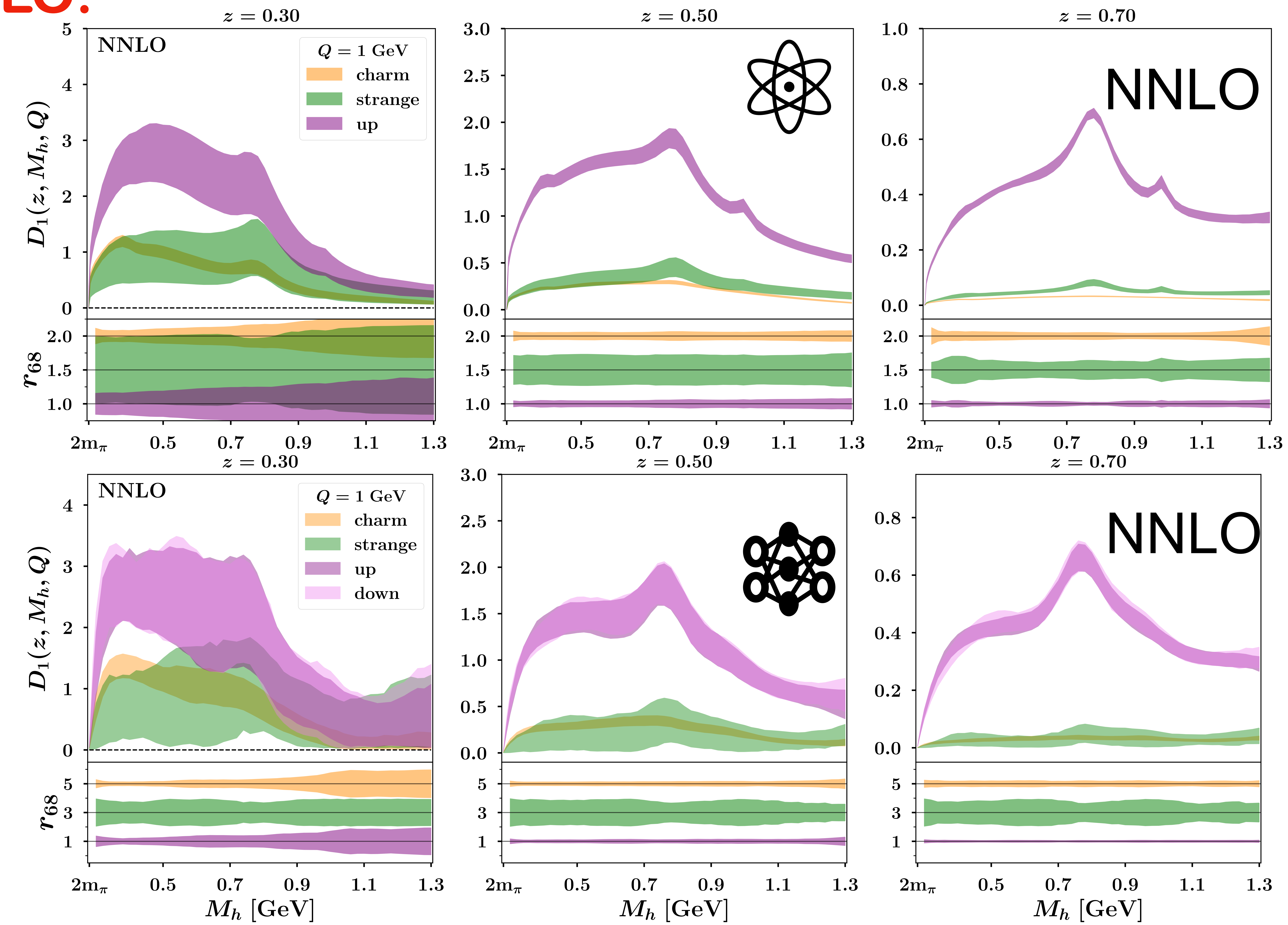




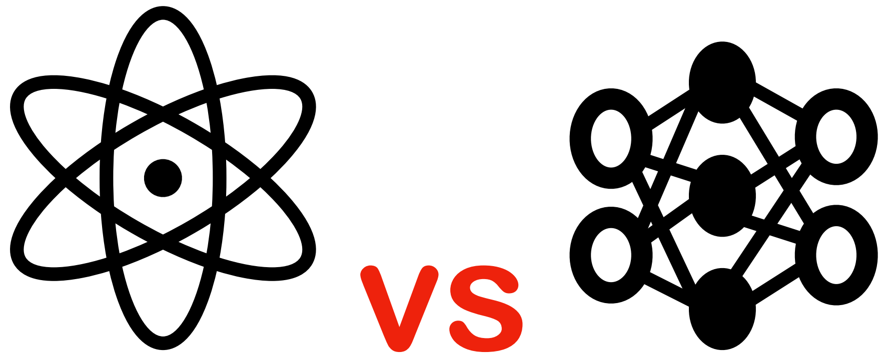
Comparison at NNLO:

vs

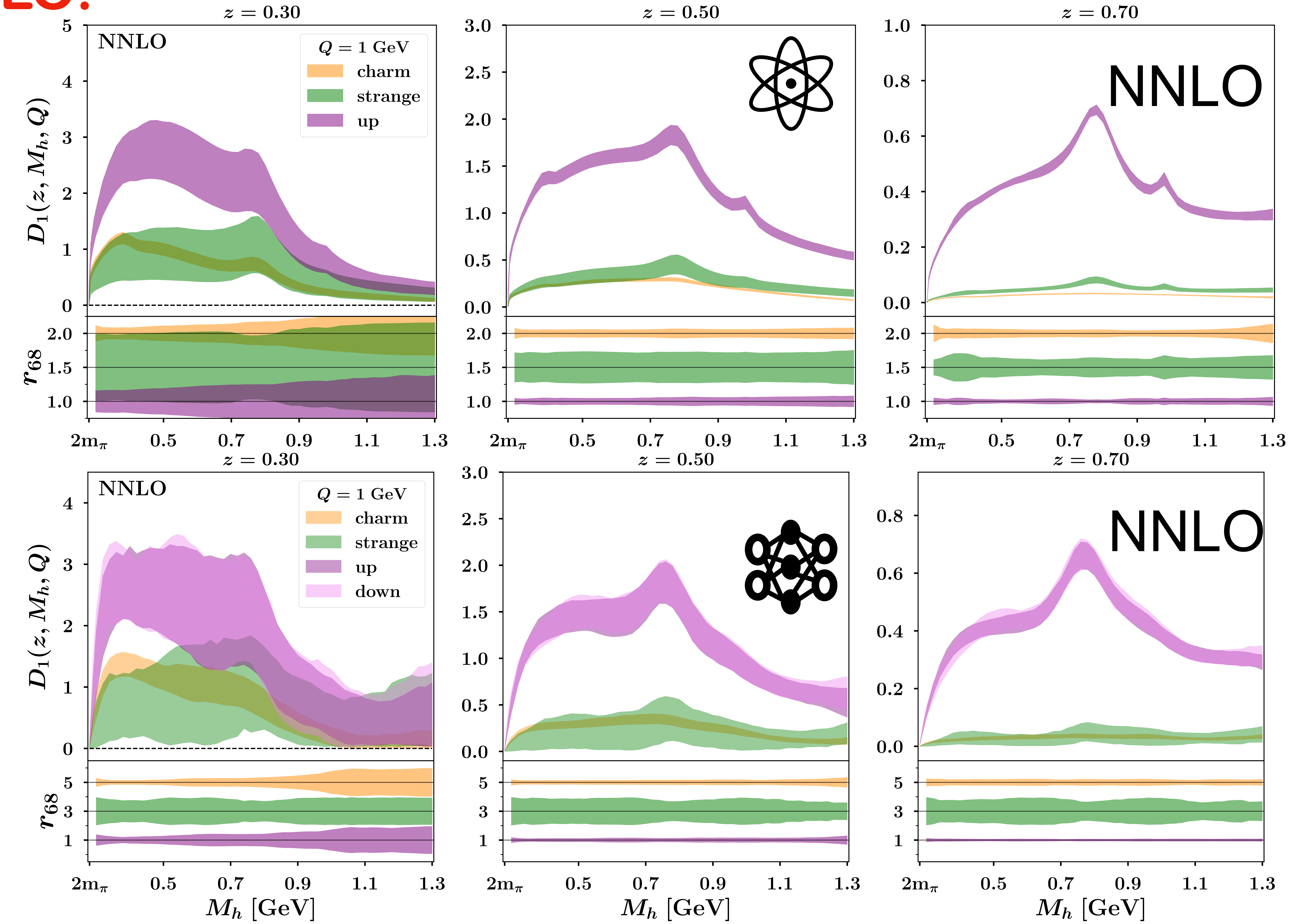
- **Similar features and compatible results**



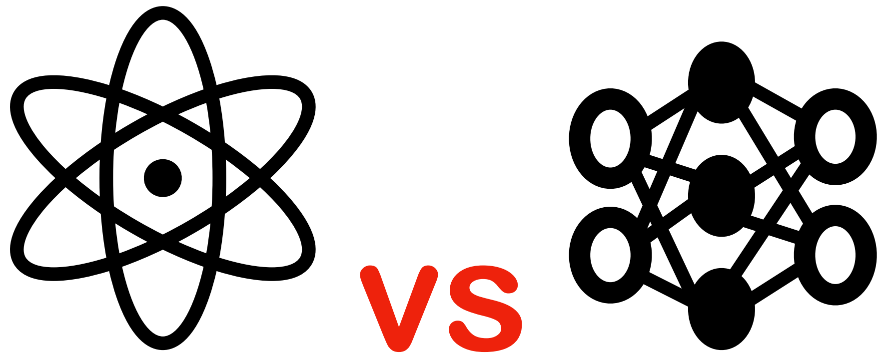
Comparison at NNLO:



- Similar features and compatible results
- NN has wider uncertainty bands



Comparison at NNLO:



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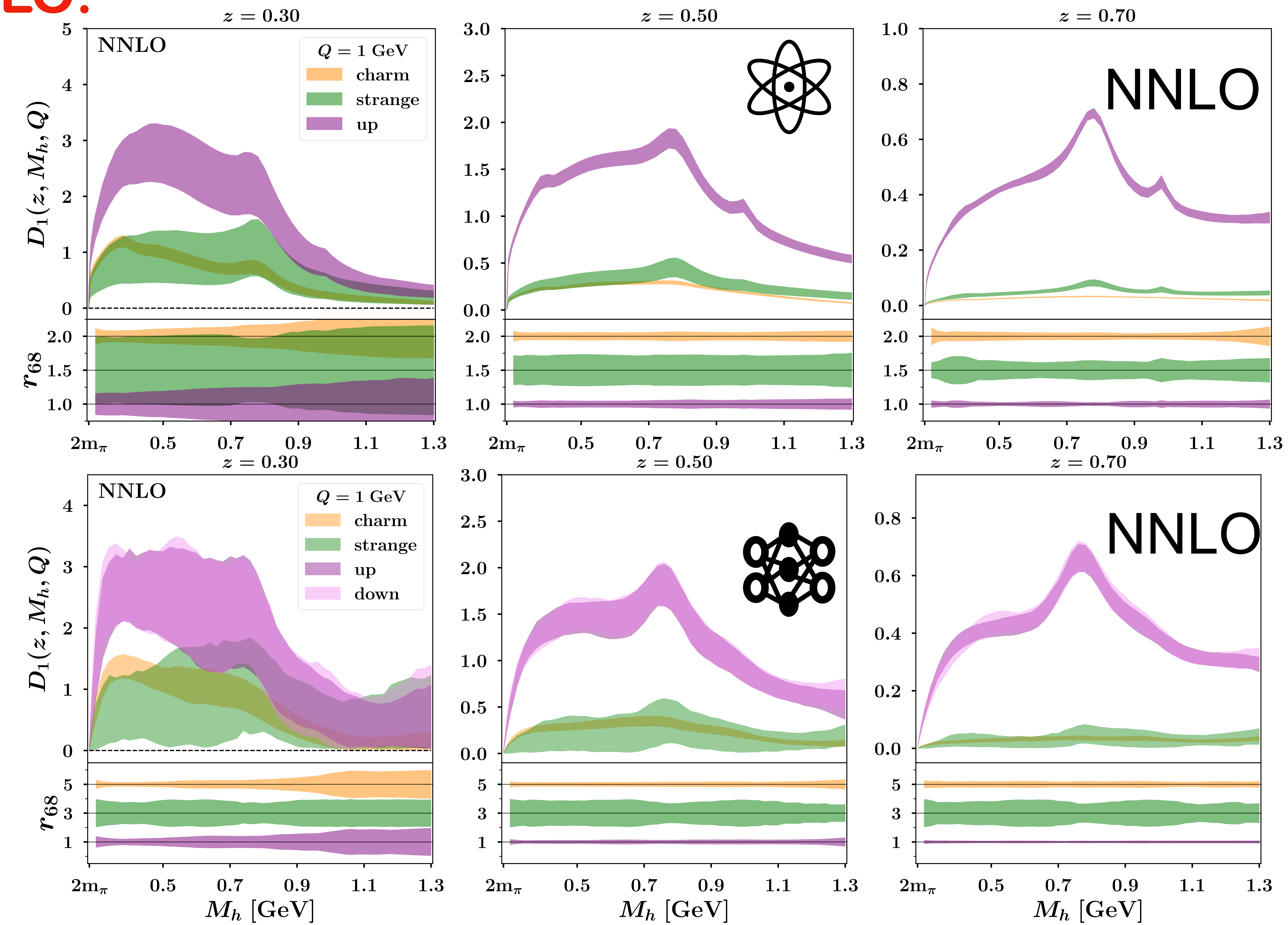

Comparison at NNLO:

vs

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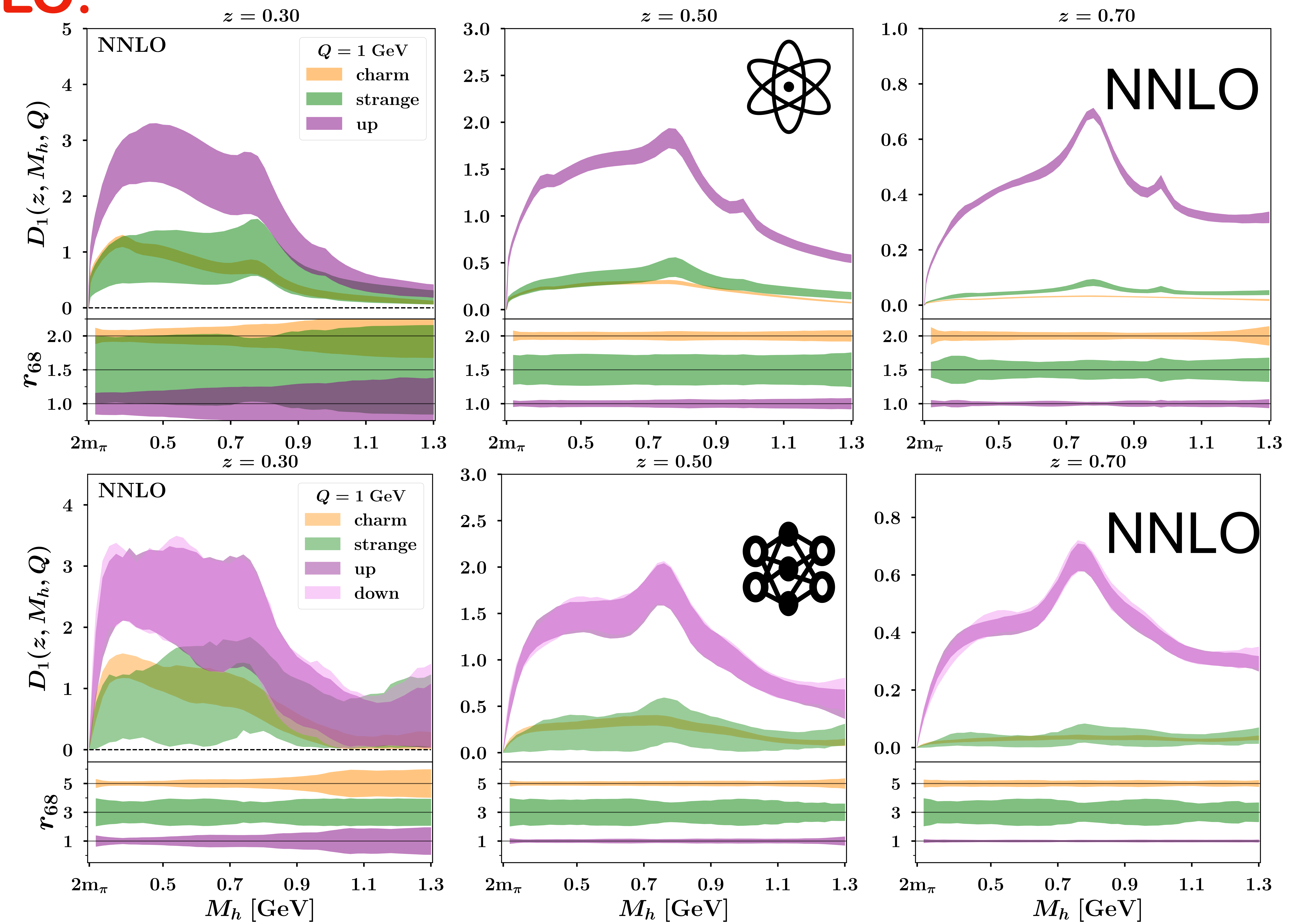
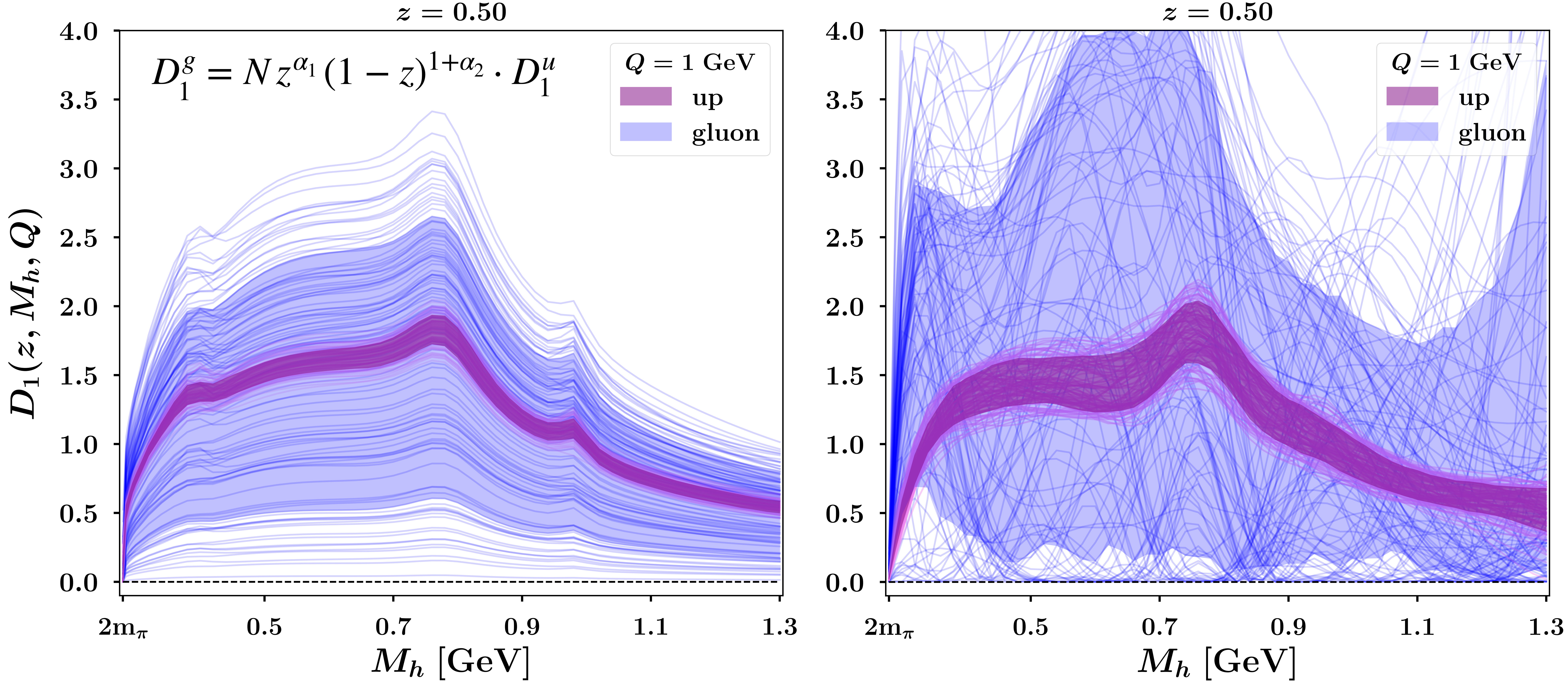
- Gluon?



Gluon and up bands at NNLO



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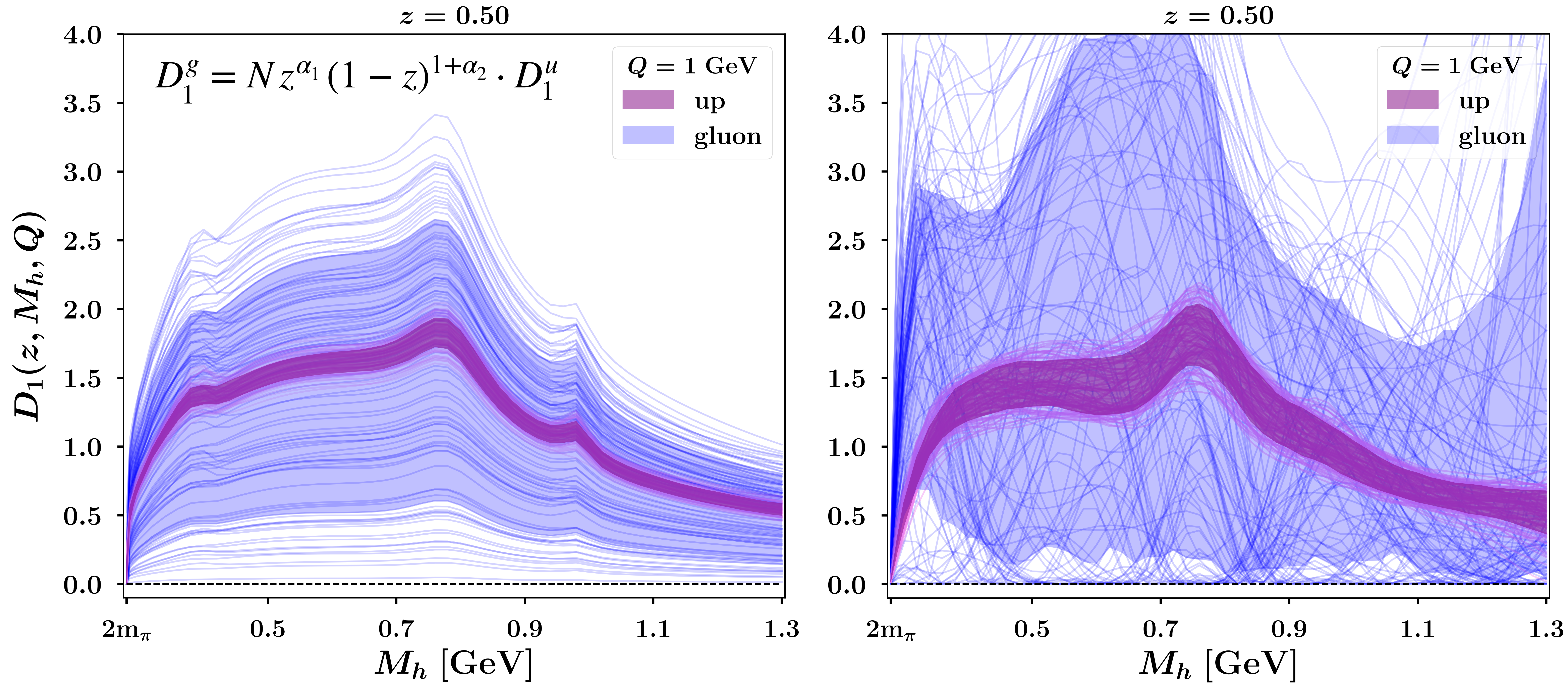

- NN has no bias, shows a random behaviour → gluon unconstrained

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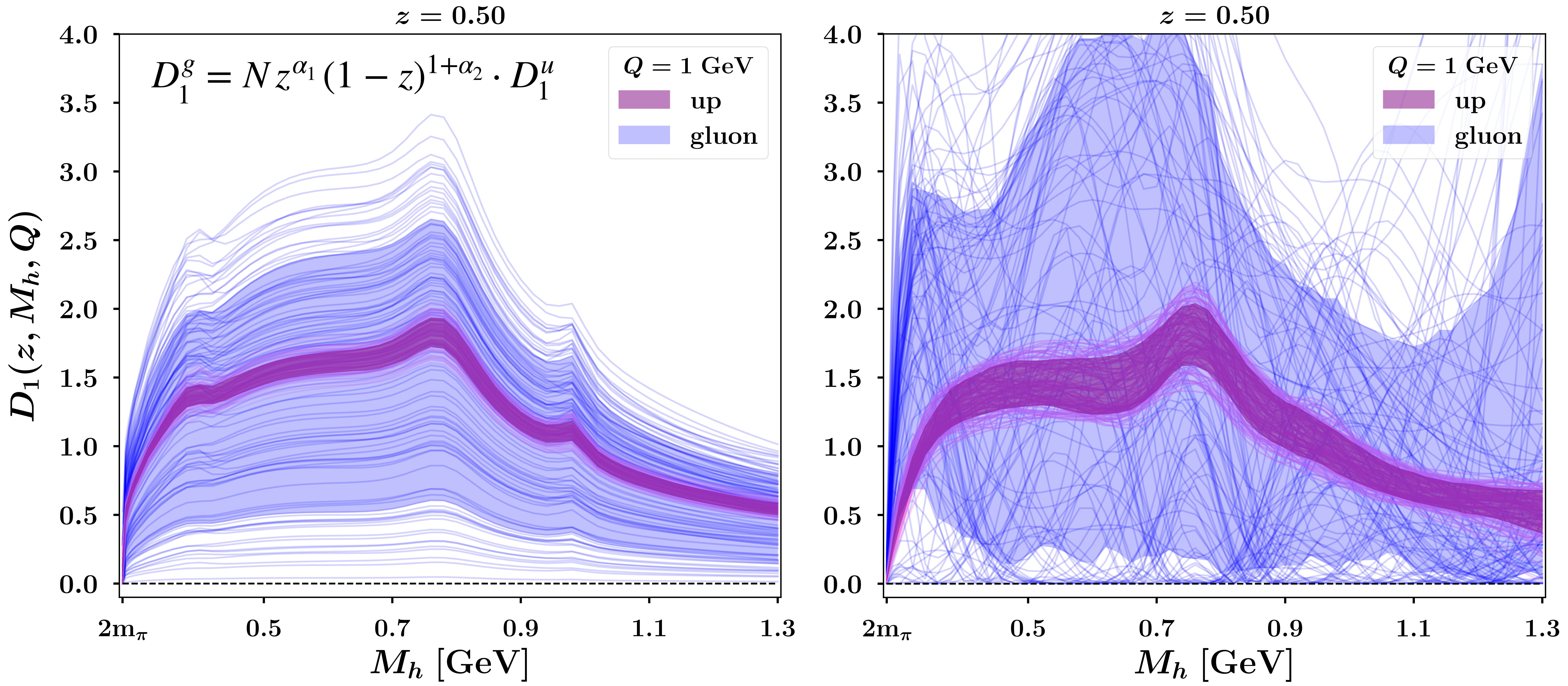

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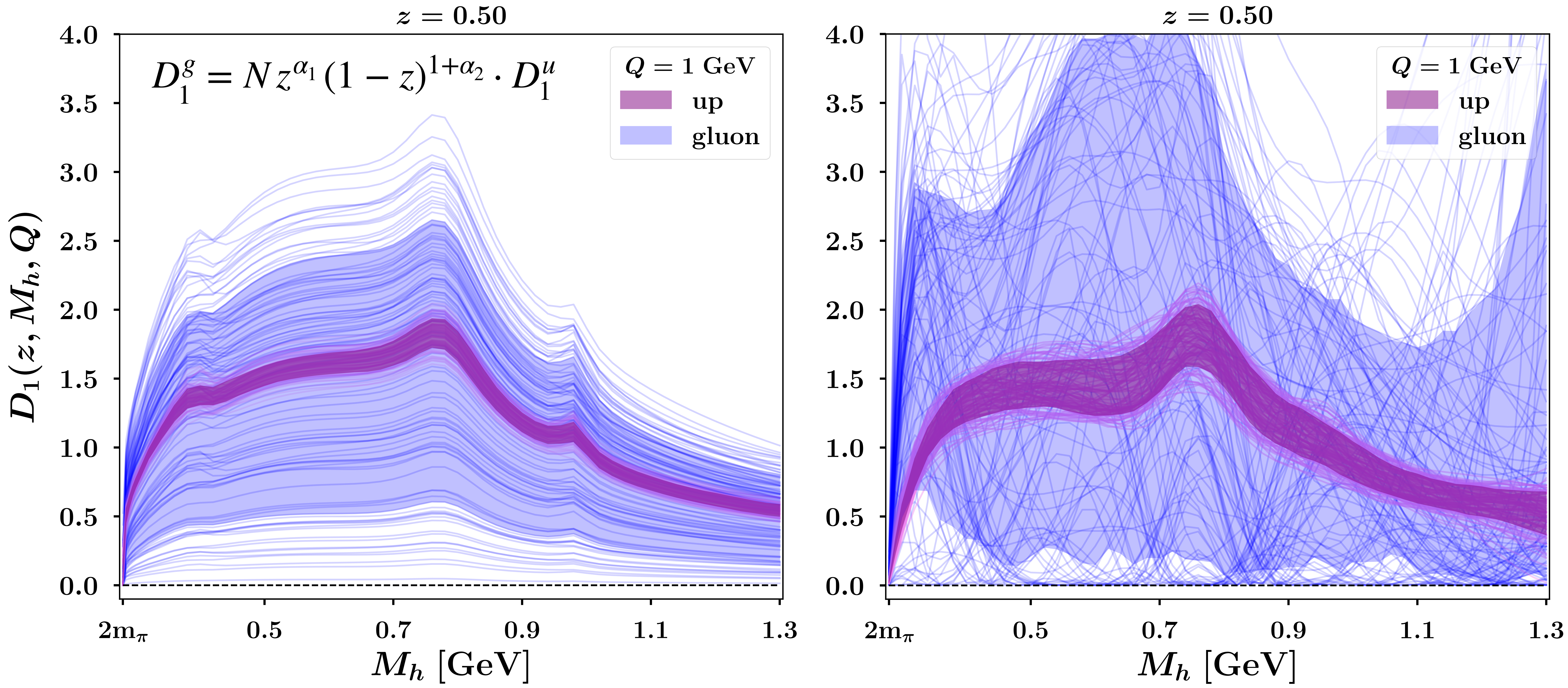

- **NN has no bias, shows a random behaviour → gluon unconstrained**
- **Physics informed is more constrained, but the biases come from justified physical requirements**
- **Physics informed description can be more realistic than the NN one**

CONCLUSIONS AND OUTLOOKS

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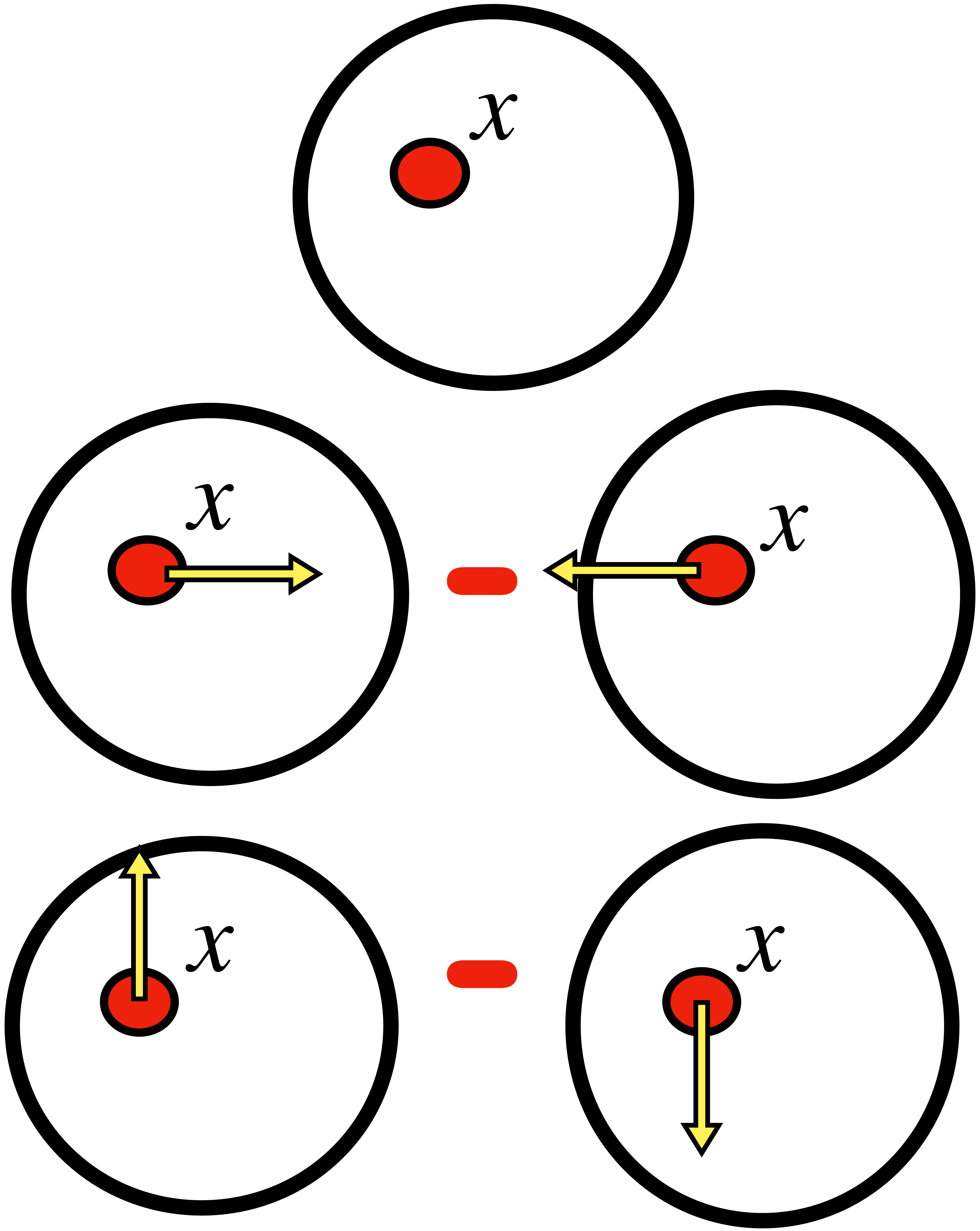
- Use the D_1 to extract $H_1^{\sphericalangle}$ and then h_1

PDF



f_1

g_1

h_1

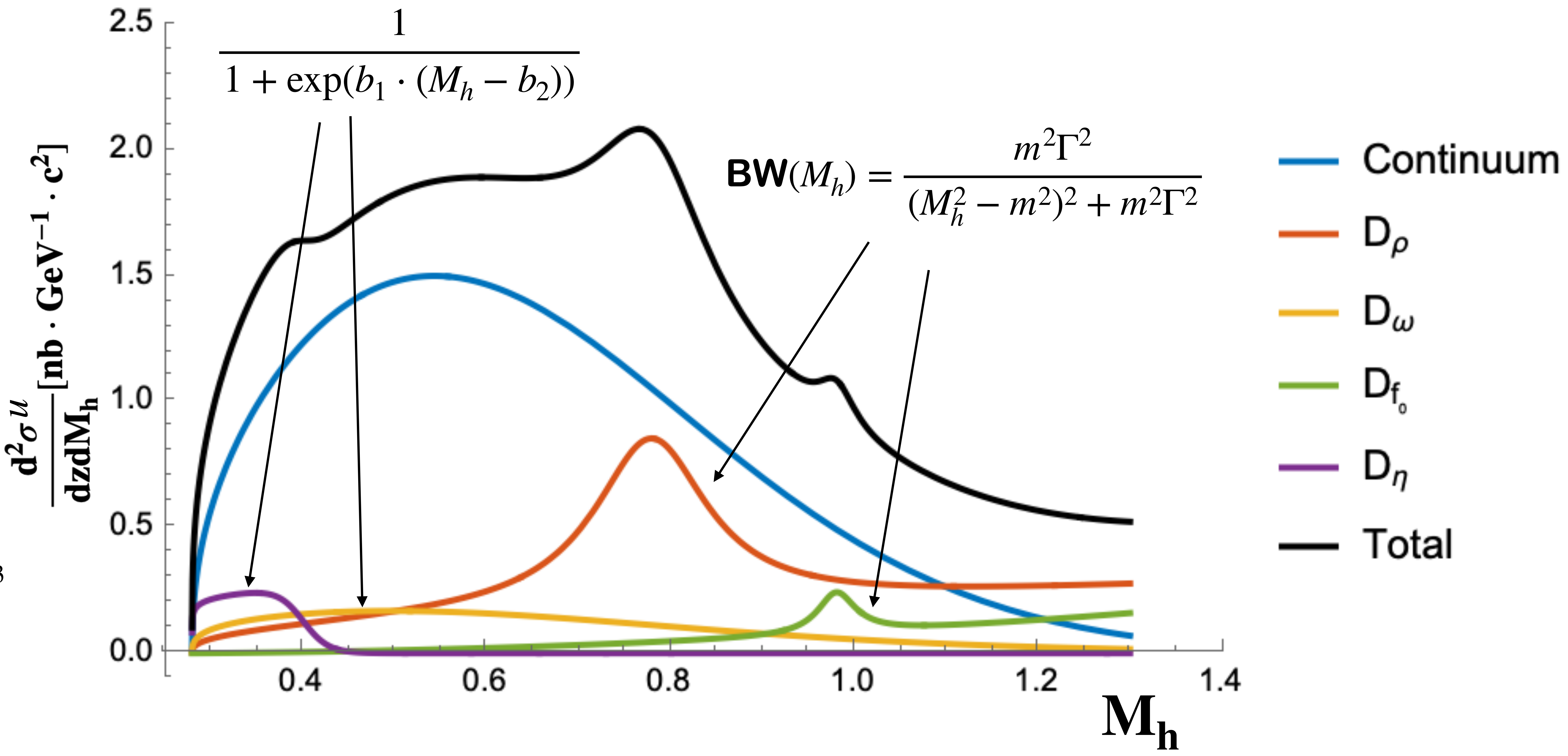
PHYSICS INFORMED

Example for the up quark parameterisation

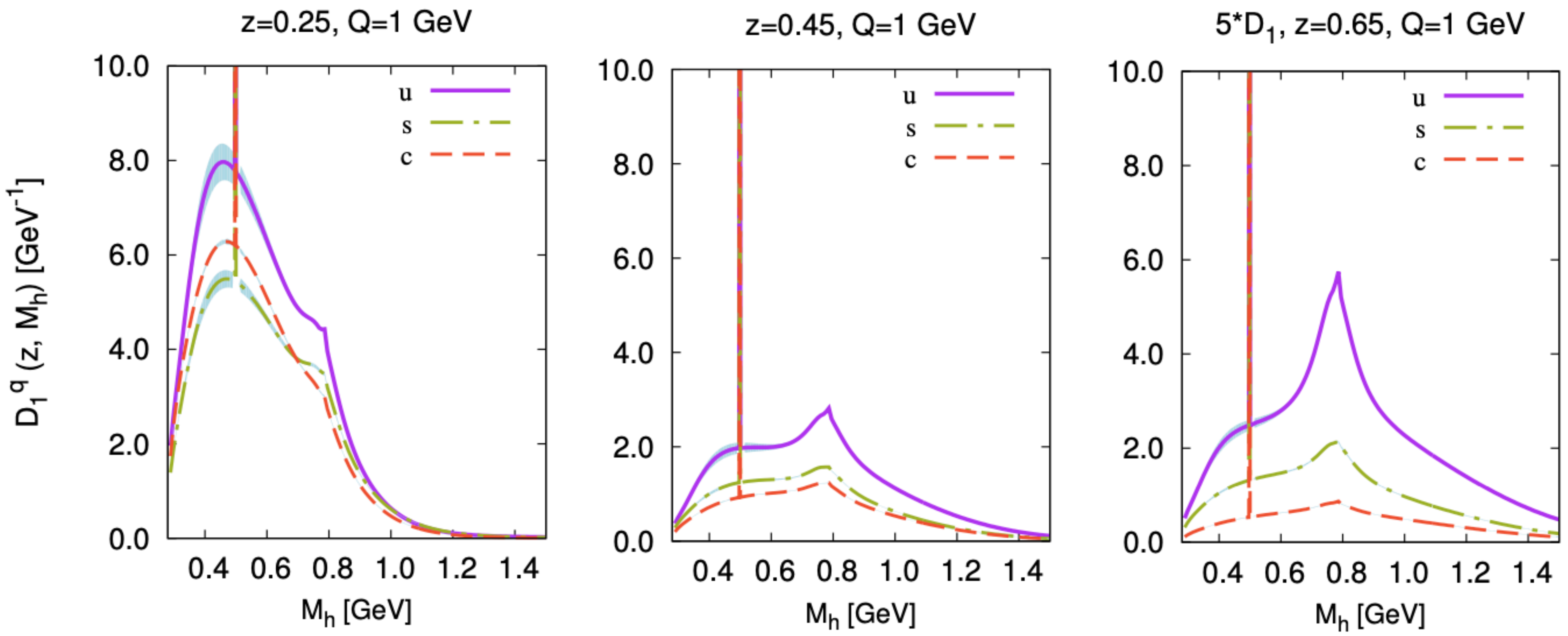
$$\mathbf{R}(M_h) = \frac{1}{2}\sqrt{M_h^2 - 4\,m_\pi^2}$$

$$z^\alpha(1-z)^\beta$$

$$\mathbf{P}(a_1, a_2, a_3, a_4, a_5; z) = \frac{a_1}{z} + a_2 + a_3 \cdot z \; + a_4 \cdot z^2 + a_5 \cdot z^3$$



2012 extraction by Pavia group



A.Bacchetta, M.Radici, A.Bianconi, A.Courtoy, Phys. Rev. D 85, (2012) 114023

Fit of MonteCarlo simulation

79 free parameters

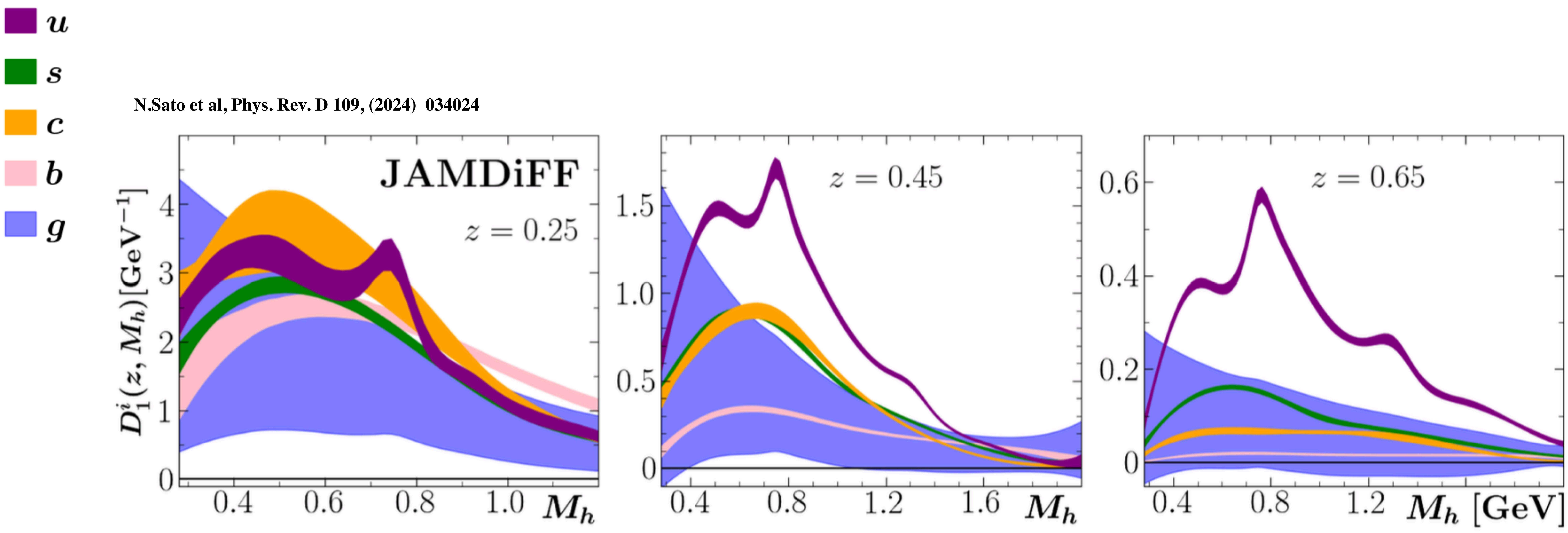
LO

2017 BELLE data of $e^+e^- \to \pi^+\pi^- X$ at $\sqrt{S} = 10.58$ GeV

Latest extraction:

+ MonteCarlo simulation **LO**

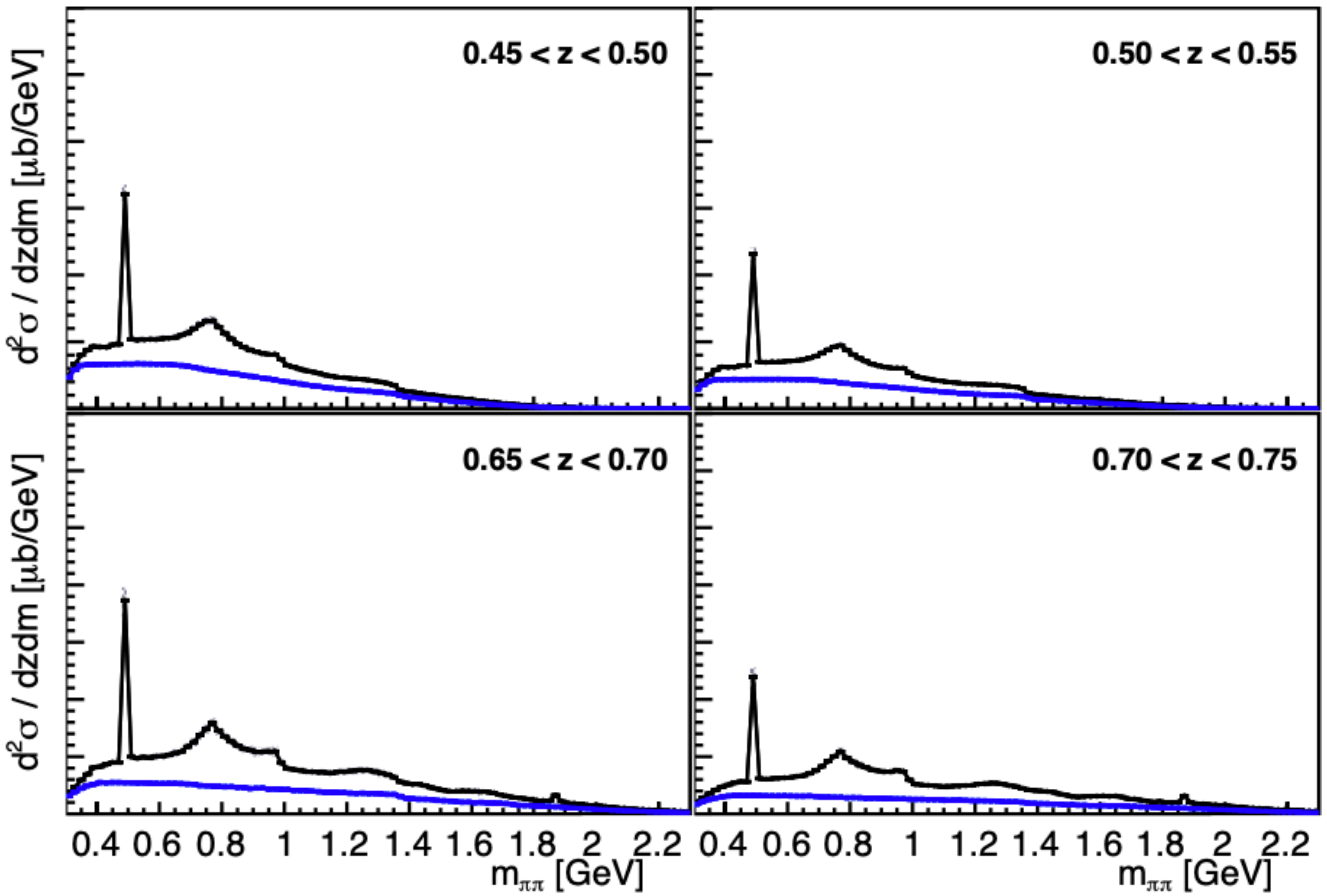
N.Sato et al, Phys. Rev. D 109, (2024) 034024



Flavour analysis

Physical review D 96 (2017)
R.Siedl et al

$$\frac{d\sigma}{dzdM_h dQ^2} = \frac{4\pi\alpha^2}{Q^2}\sum_q e_q^2 D_1^q(z, Mh, Q) =$$



Flavour analysis

$$\frac{d\sigma}{dzdM_hdQ^2} = \frac{4\pi\alpha^2}{Q^2}\sum_q e_q^2 D_1^q(z, Mh, Q) = \frac{4\pi\alpha^2}{Q^2}\mathbf{D(z, Mh, Q)}$$

Need of extra information → **Monte Carlo**

$$\mathbf{R_u^{MC}} = \frac{\mathbf{D^u_{MC}(z, M_h, Q)}}{\mathbf{D_{MC}(z, M_h, Q)}}$$

→ **Build 4 pseudo-dataset**

$$\mathscr{F}^u(z, M_h; Q^2) = \frac{4\pi\alpha^2}{Q^2}\mathbf{D(z, M_h, Q)} \times \frac{\mathbf{D^u_{MC}(z, M_h, Q)}}{\mathbf{D_{MC}(z, M_h, Q)}}$$

$$\mathscr{F}^q$$

$$q = u, d, s, c$$

DI-HADRON FF

Physics informed

u=d

gluon → assumptions

$$D_1^g(z, M_h; Q^2) = N\, z^{\alpha_1} (1-z)^{1+\alpha_2} \cdot D_1^u(z, M_h; Q^2)$$

N random unif. in (0,2)

α_1, α_2 random unif. in (0,1)