Neutron Star Modeling: “Shortcuts” or Precision?

Two ways to build neutron stars models.

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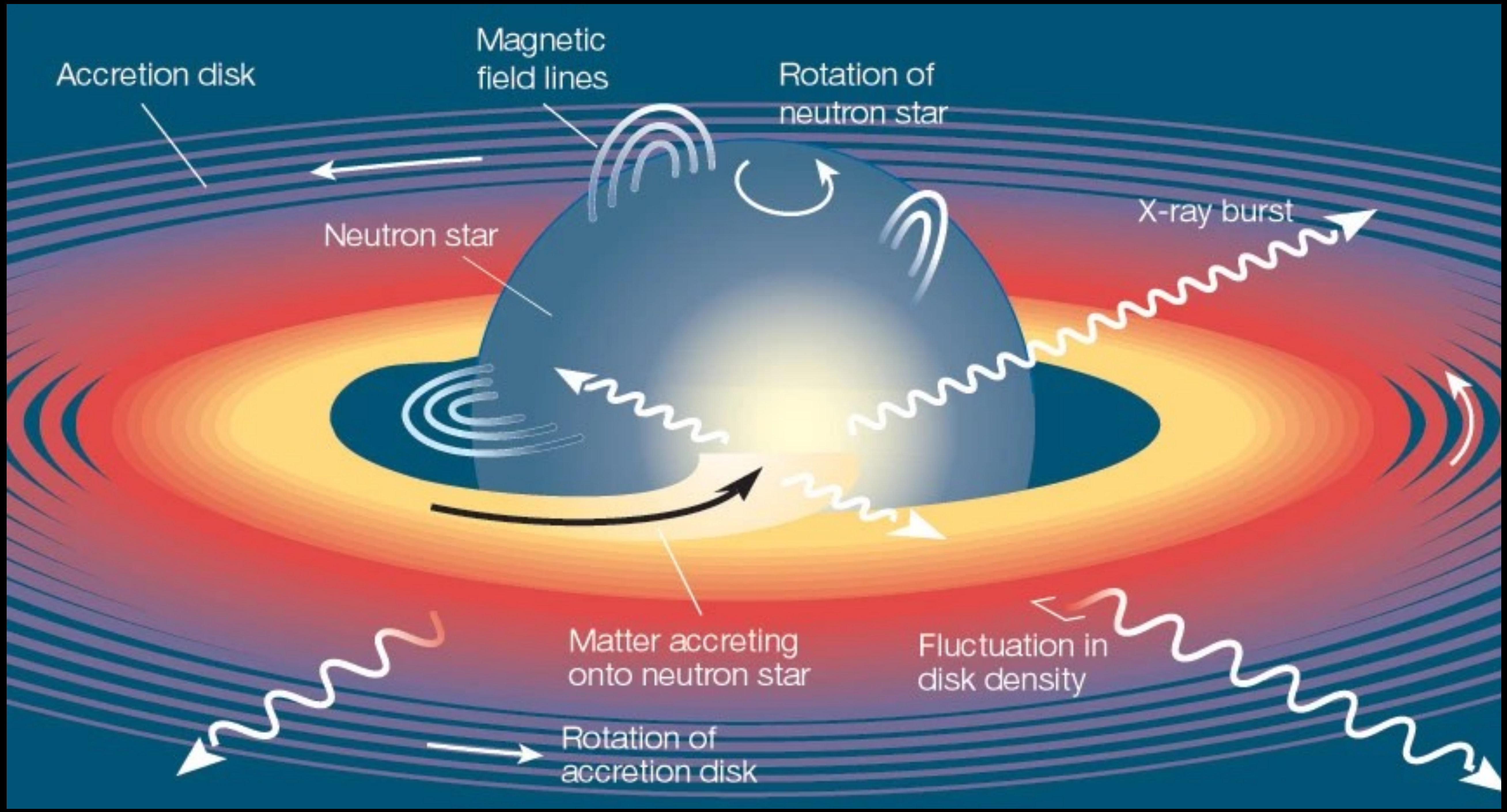
Neutron stars [1] provide a unique environment to probe properties of dense nuclear matter.

They represent the result of the collapse of a red supergiant.

↓

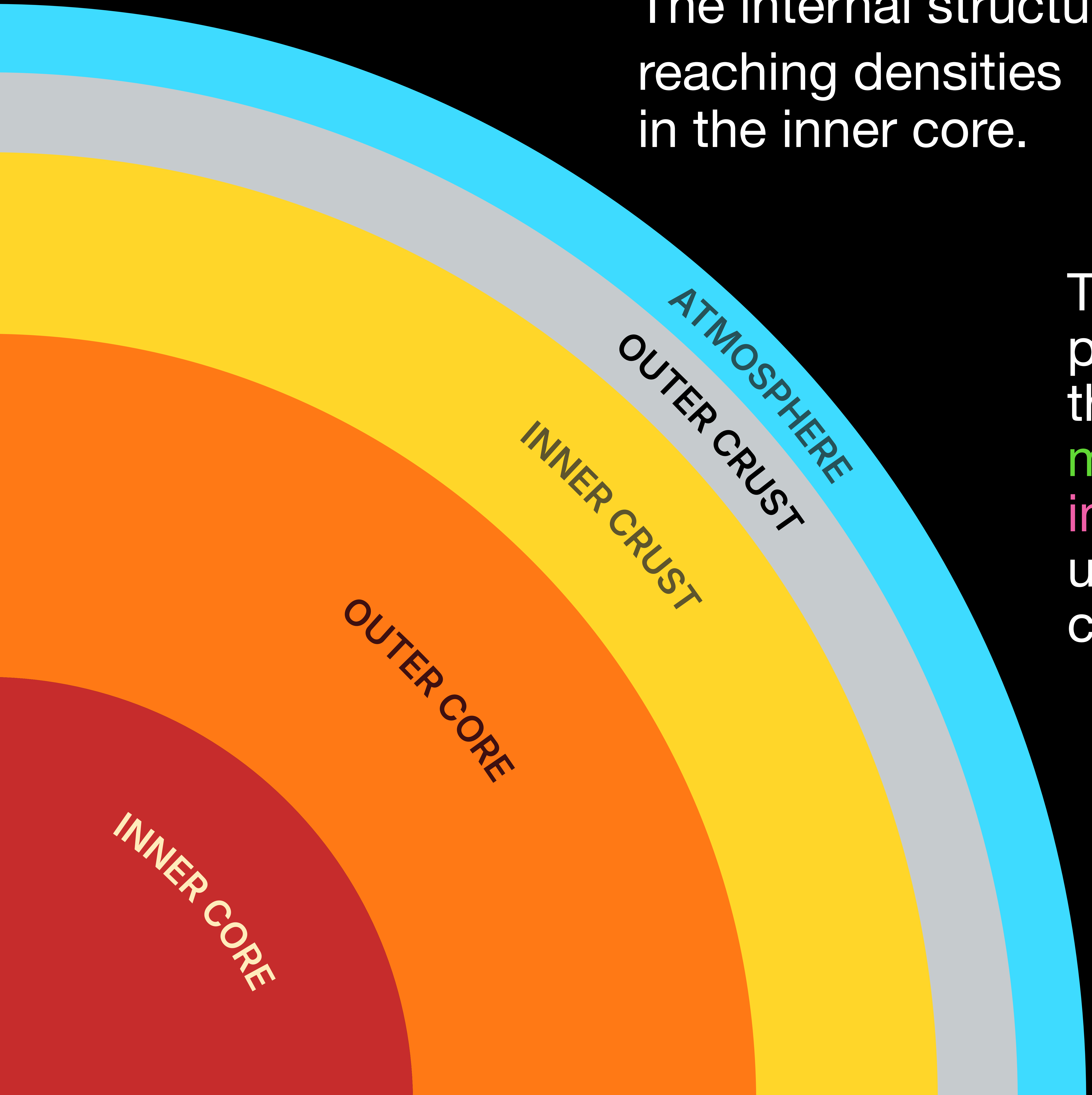
The collapse process is triggered by:

1. Electron captures on nuclei, with production of neutrinos
2. Photodisintegration of iron into alpha particles



→ Both mechanisms decrease pressure support until gravity dominates, driving catastrophic collapse on millisecond timescales.

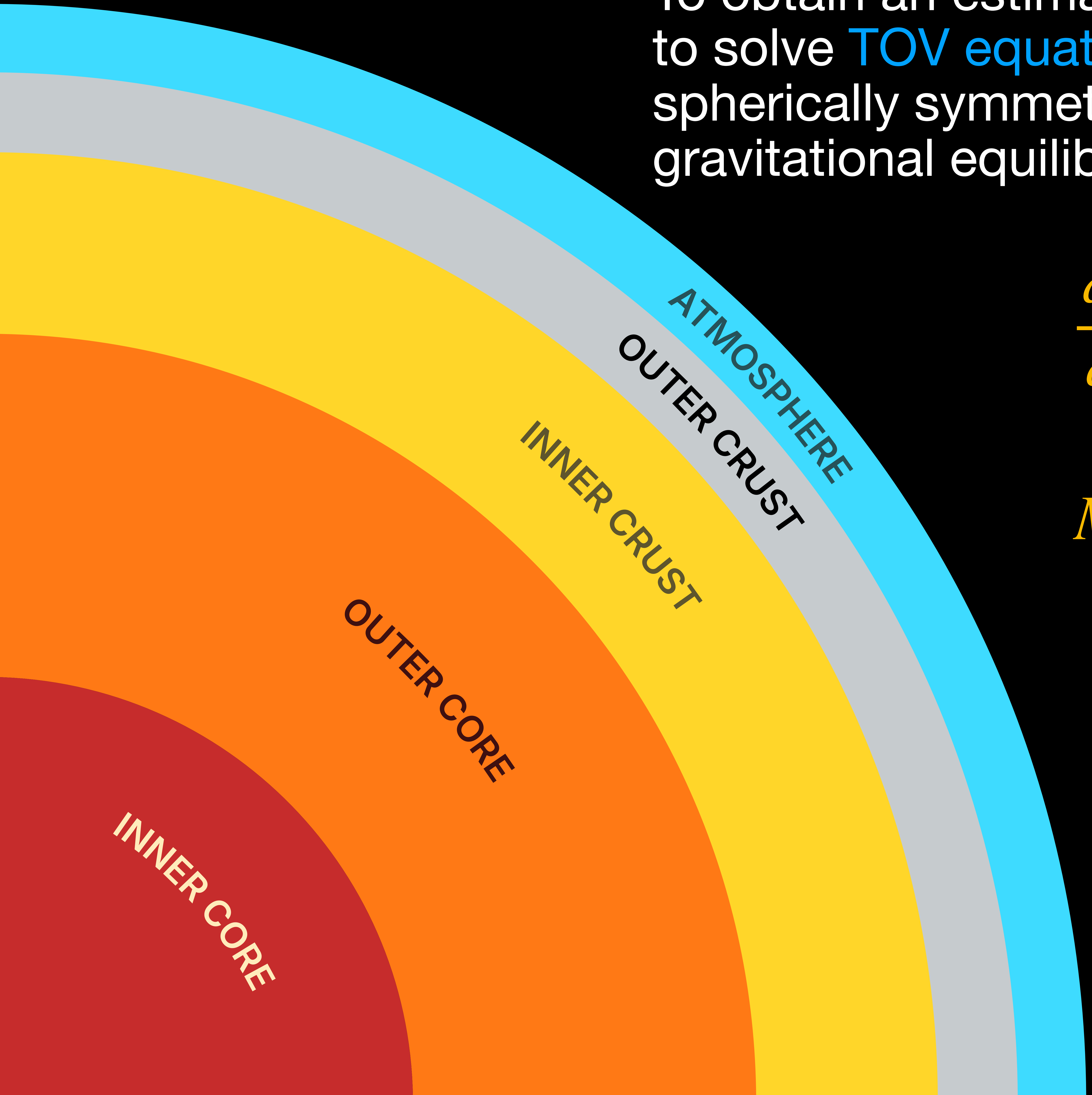
The internal structure is layered, reaching densities $\sim 10^{15}$ g/cm^3 in the inner core.



The core represents the most mysterious part of the neutron star. Theories suggest that it may be composed of exotic matter, such as quarks, or an intermediate phase that leads to unexpected behavior, which may be crucial to understanding the universe.

The different density regions allow us to phenomenologically parameterize the equation of state, obtaining estimates on astrophysical observables.

To obtain an estimation of mass-radius relation, we have to solve TOV equations, describing the structure of a spherically symmetric body of isotropic material in static gravitational equilibrium.



$$\frac{dp}{dr} = -\frac{\left[\varrho(r) + p(r)\right]\left[M(r) + 4\pi r^3 p(r)\right]}{r\left[r - 2M(r)\right]}$$

$$M(r) = 4\pi \int_0^r dr'\, \varepsilon(r') r'^{\,2}$$

Once we have the M-R relation, we can calculate other useful observables:

k_2, tidal deformability, moment of inertia and I-Love-Q relations.

The growing number of astrophysical observations from gravitational waves (e.g., GW170817), pulsar timing (e.g. PSR J9740+6620) and X-ray measurements (e.g. NICER) have imposed increasingly stringent constraints on the mass-radius relation and tidal deformability of neutron stars.

***GW170817*:**

$M = 2.73 \pm 0.03\, M_\odot$

$R = 10.8 \pm 2.2$ km

PRS J0740+6620:

$M = 2.08 \pm 0.07\, M_\odot$

$R = 12.92 \pm 1.61$ km

***NICER*:**

$M = 1.44 \pm 0.15\, M_\odot$

$R = 13.02 \pm 1.15$ km

On average, observations tell us that

$$1.4\, M_\odot \lesssim M \lesssim 2.5\, M_\odot$$

$$10\,\text{km} \lesssim R \lesssim 13\text{km}$$

The goal of my work is to connect nuclear physics to astrophysics, finding the best way to describe the neutron star's equation of state in agreement with astrophysical observations.

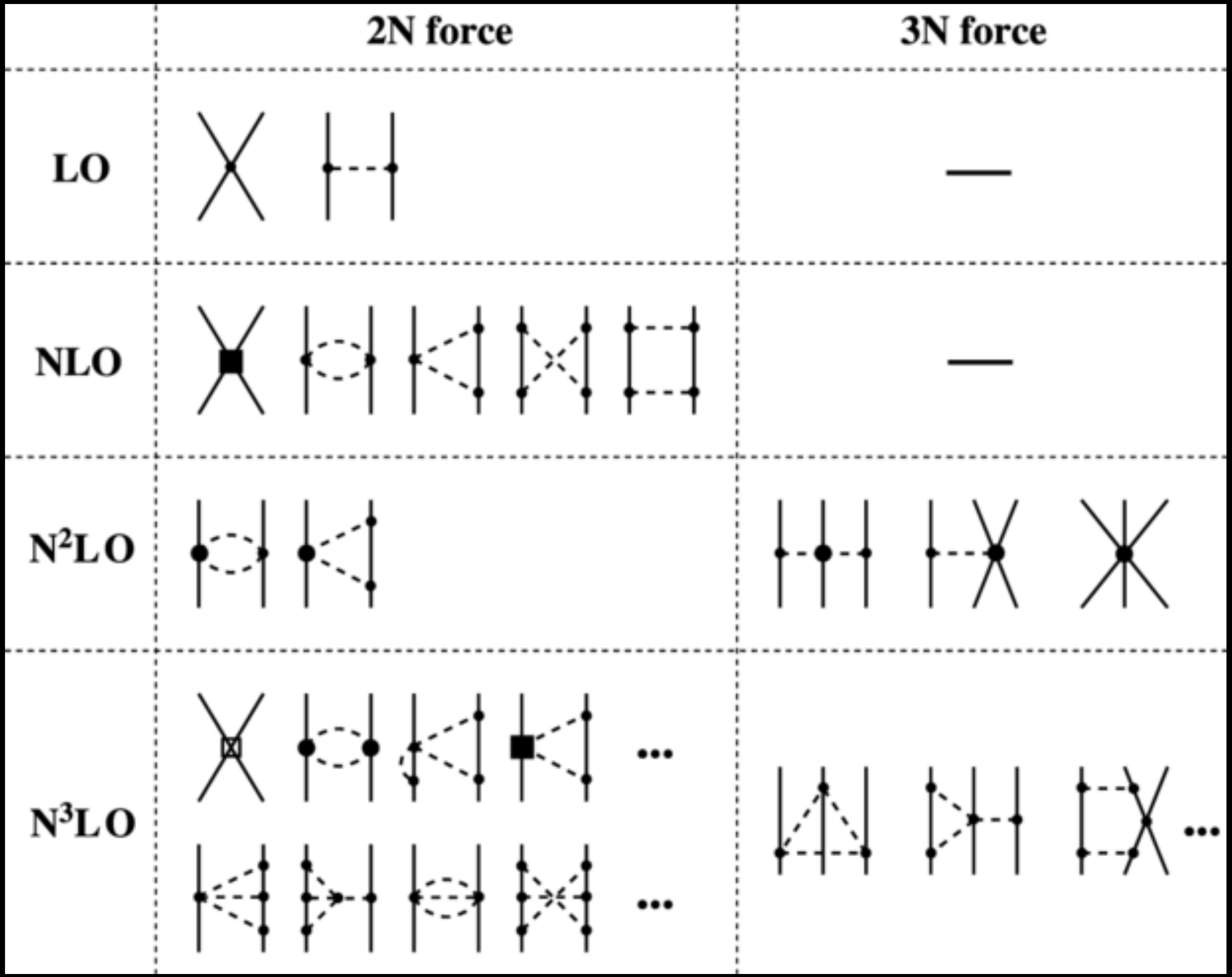
The use of theories based on quantum chromodynamics, and therefore quantum field theory, offers a wide range of possibilities to describe the neutron star equation of state and calculate observables .

Using standard observables, we can set limits on Beyond SM physics (like axion couplings or Pauli Exclusion Principle violation) which, if too large, would contradict the observed nature of neutron stars.

For a phenomenological description of neutron star EoS, χEFT is a good starting point.

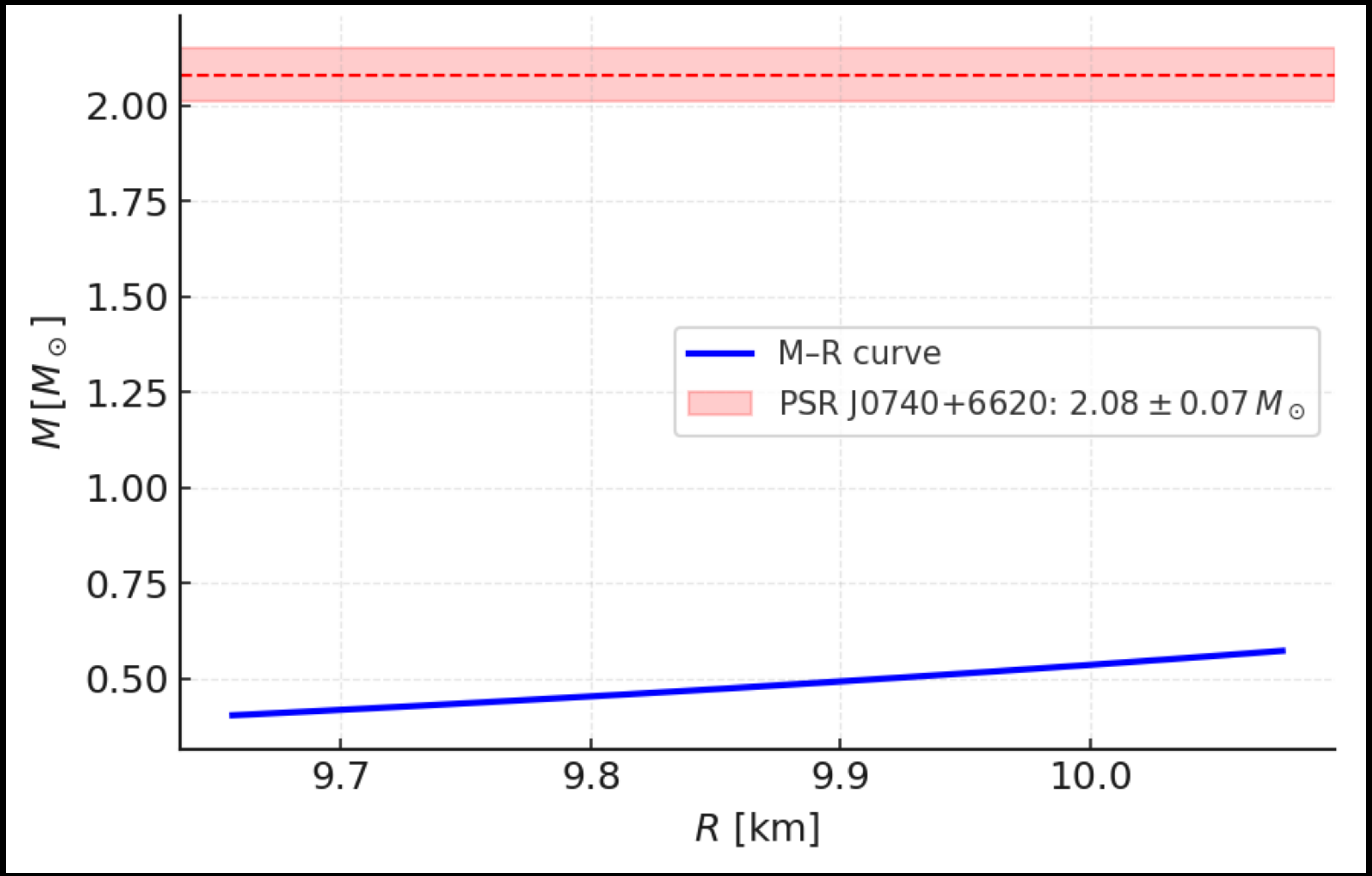
Here, the χEFT based EoS is calculated though Many Body Perturbation Theory, allowing us to predict properties of nuclear matter.

The use of χEFT to describe neutron stars presupposes the inclusion of three-body interactions, in order to reproduce the correct pressure profile.



The calculations presented here use a chiral potential N3LO NN + N2LO 3N 450.

Despite the inclusion of the 3-body contribution, it is easily shown that χEFT alone is not sufficient for the description at extreme densities (core).

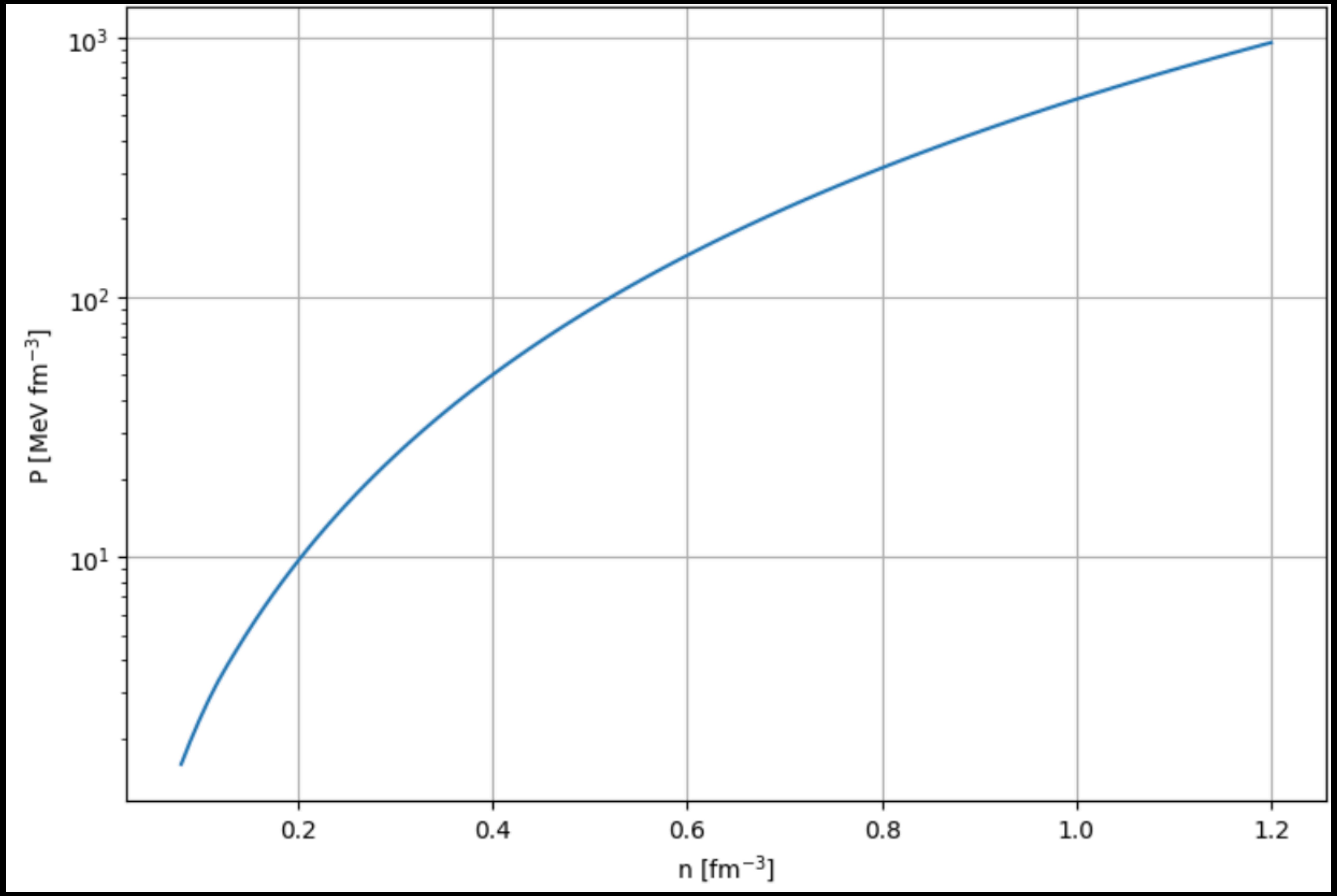


Result obtained using only N3LO NN + N2LO 3N 450 Chiral Potential.

The equation of state considered in this plot is the one for asymmetric nuclear matter, using only χEFT.

Solving the TOV equations results in a mass-radius relation that is clearly insufficient to match the masses estimated from observational data.

The starting point of phenomenological polytropic approach is the use of EoS for symmetric nuclear matter ($\beta = 0$) and pure neutron matter ($\beta = 1$), where β is the asymmetry parameter [2, 3].



$P(n)$ plot for asymmetric nuclear matter after β-stabilization process

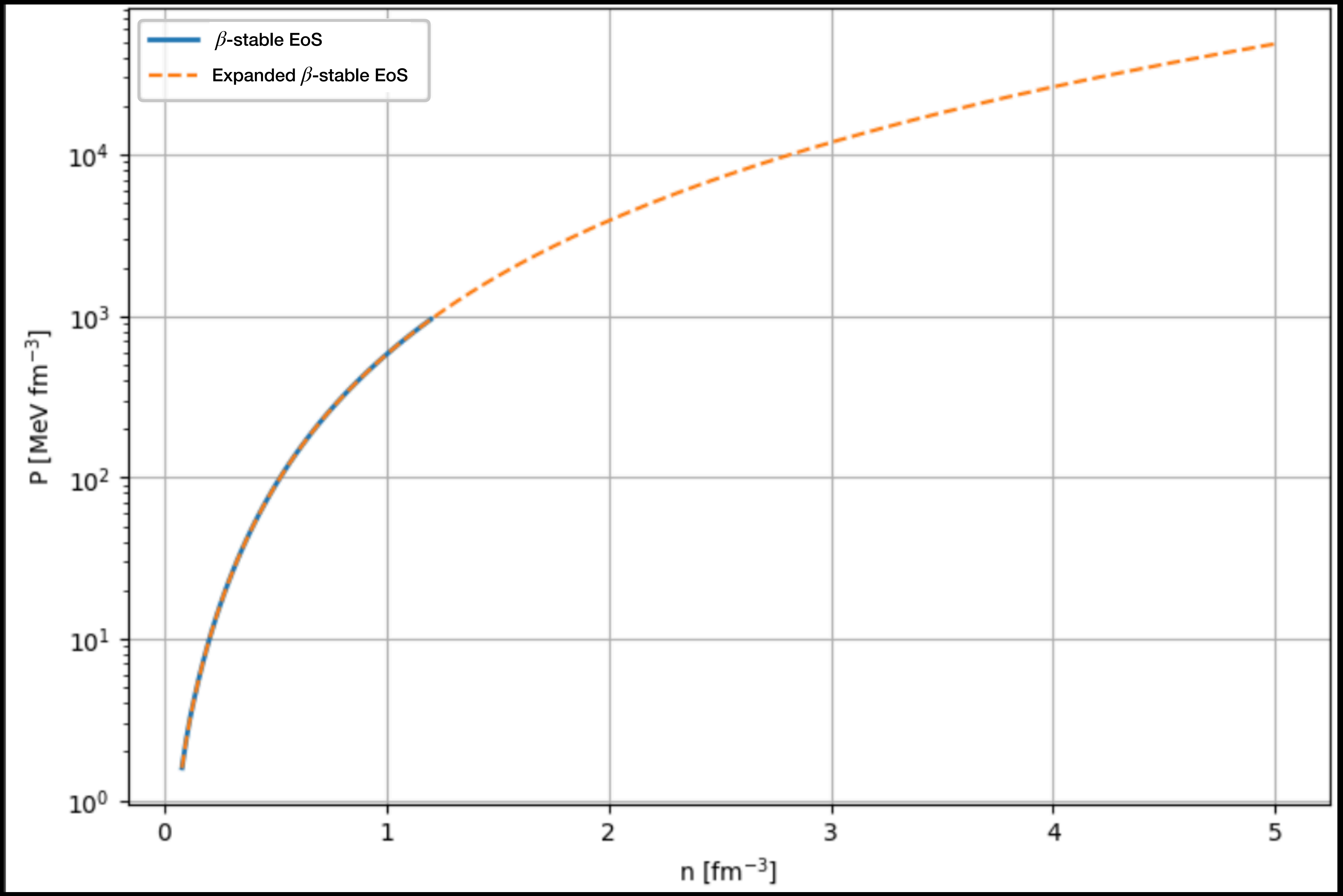
They are combined to obtain an equation of state for asymmetric nuclear matter ($0 \leq \beta \leq 1$). The process of β-stabilization is fundamental. The composition of β-stable matter is calculated solving the equation for chemical equilibrium in neutrino-free matter a given total nucleon density n

$$\mu_n - \mu_p = \mu_e, \quad \mu_e = \mu_\mu$$

and for charge neutrality

$$n_p = n_e + n_\mu \,.$$

e^- and μ are treated as relativistic ideal Fermi gases.



$P(n)$ plot for extended EoS

The β-stable EoS is expanded through a phenomenological parametrized polytropic approach, resulting in an extended high-density EoS, constrained by the causality condition $\frac{v_s}{c} < 1$. Pressure in polytropic form is expressed as

$$P = (\Gamma - 1) b n^{\Gamma}$$

where $K = (\Gamma - 1)\frac{b}{(a/c^2)^{\Gamma}}$ and a, b, Γ are the parameterization coefficients for the β-stable EoS.

Combining parameterized form and causality constraint allows us to extend the EoS by using the parameters a and b obtained from the parameterization process.

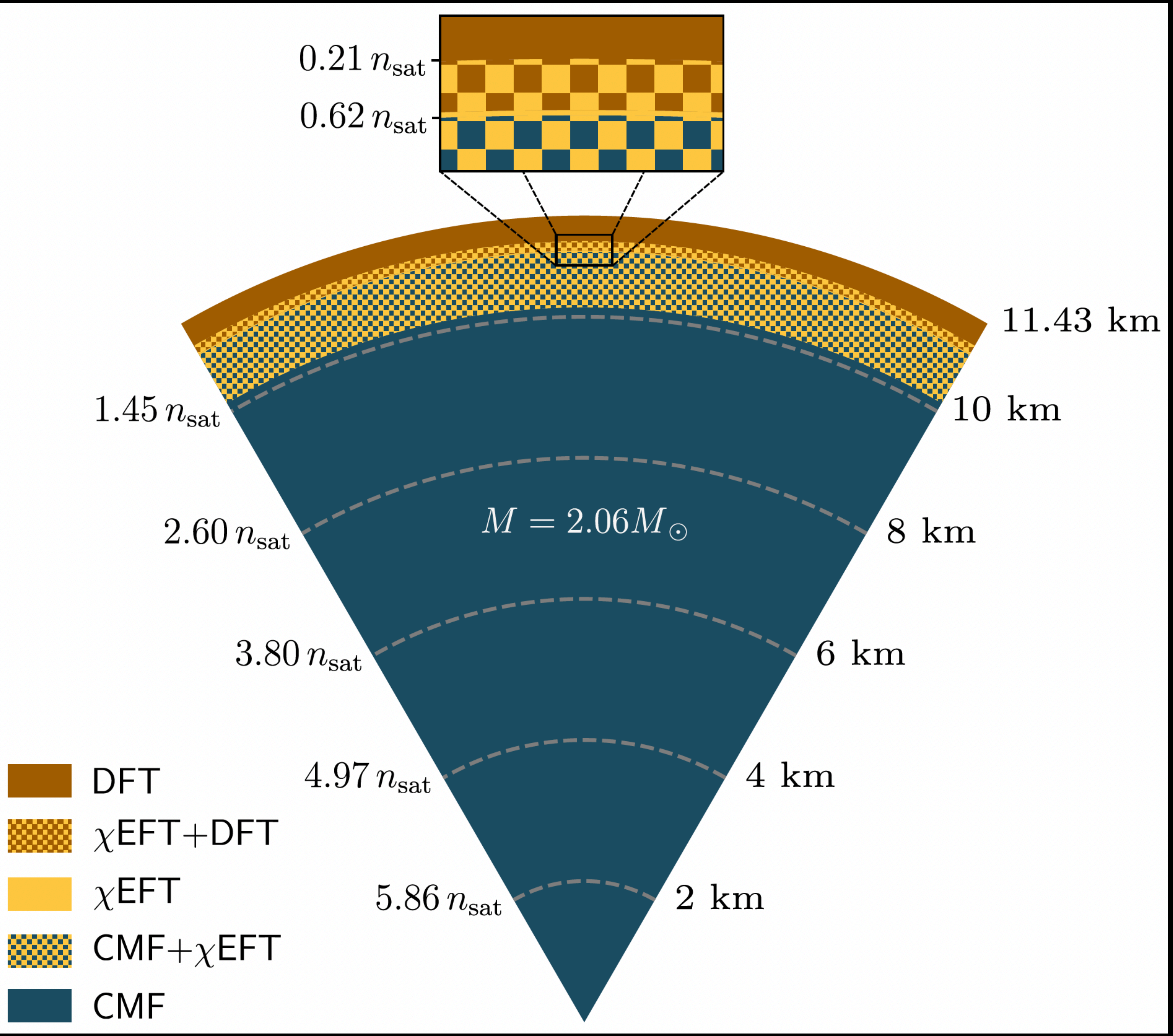
The Γ parameters are calculated such that the extended EoS must reproduce a function that is, at least, of class C^1.

MUSES CE [4] is a composable workflow management system that performs calculations of the EoS by using algorithms spanning three different models:

1. Crust Density Functional theory, at low densities
2. χEFT, around saturation density
3. Chiral Mean Field model, beyond saturation density

The complete description of a neutron star requires three additional modules:

A. Lepton module, to ensure β-equilibrium and charge neutrality
B. Synthesis module, to obtain a single EoS
C. QLIMR module, a full-general-relativity solver, to obtain neutron star observables



Both approaches are based on QCD, but they have substantial differences.

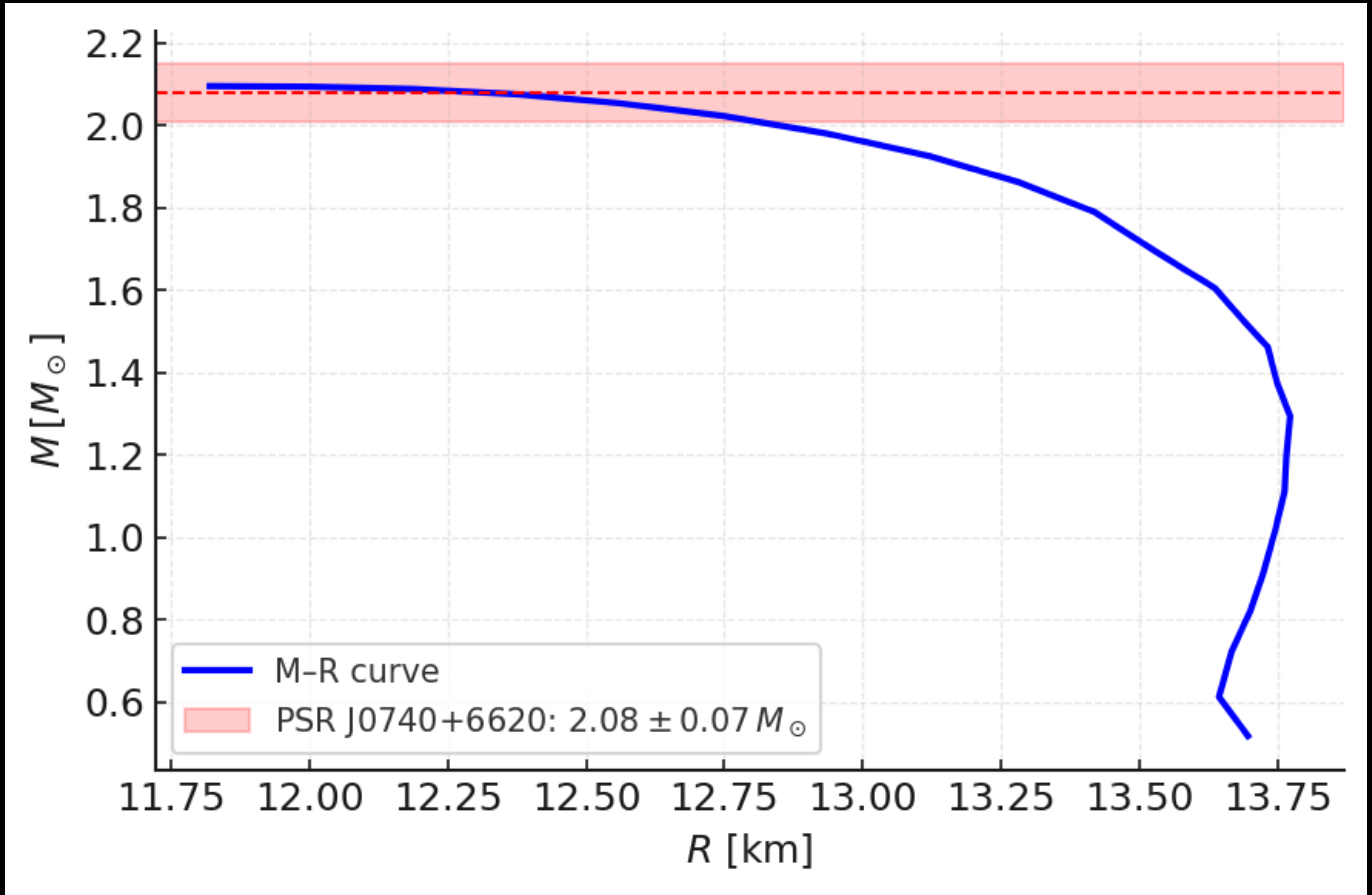
The phenomenological approach is computationally less expensive, and the equation of state is calculated from nuclear interactions.

MUSES CE combines three different theories for the three different density regions. This leads to a significantly greater computational effort.

Polytropic expansion at low and high densities does not take into account the various types of additional interactions present in the core and outer crust.

Each layer is described with the theory that best fits its material conditions, taking into account the different types of interactions in the layer under consideration.

In the following slides I plot the preliminary M-R curve obtained through MUSES CE and the polytropic approach, compared to the PSR J0740+6620 measured mass.

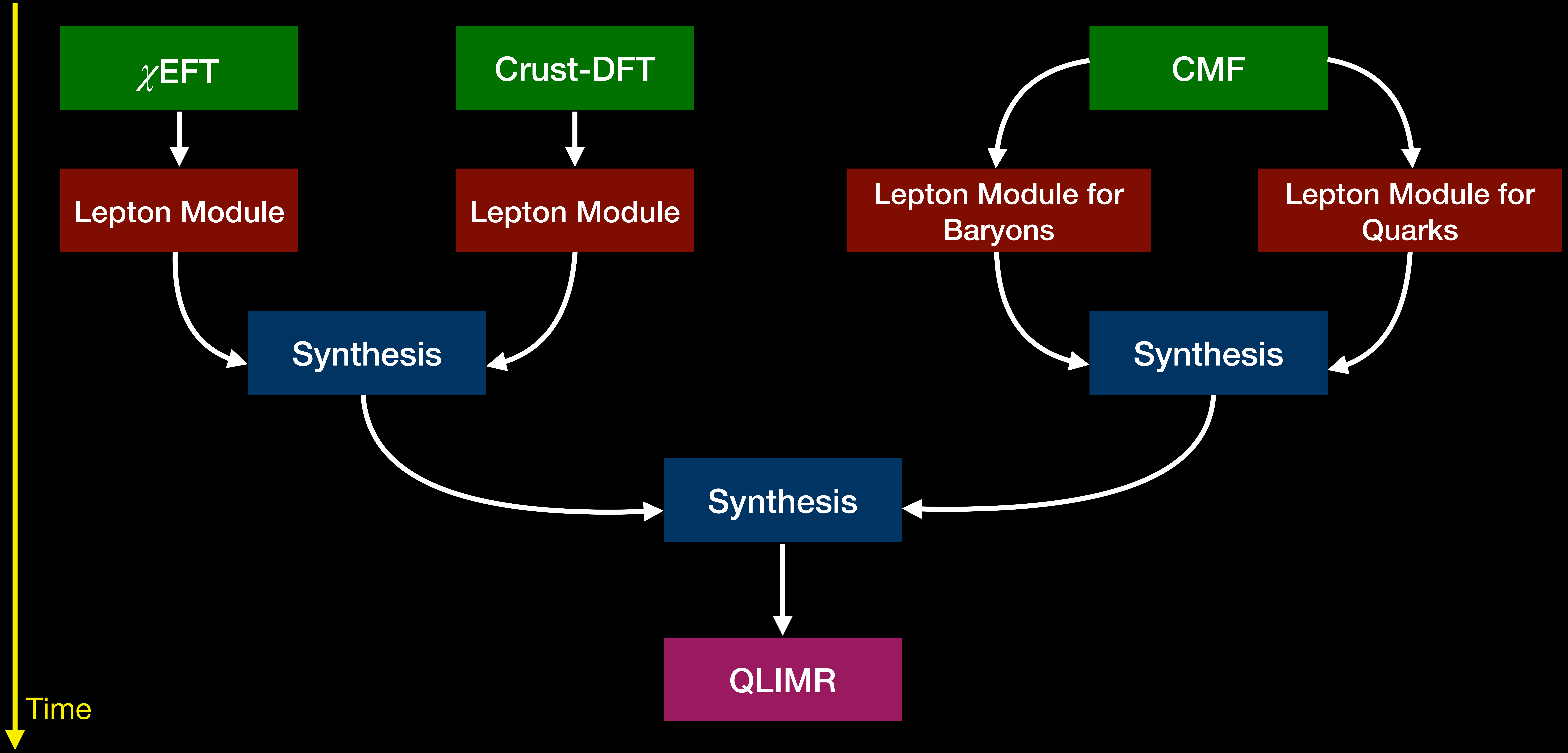


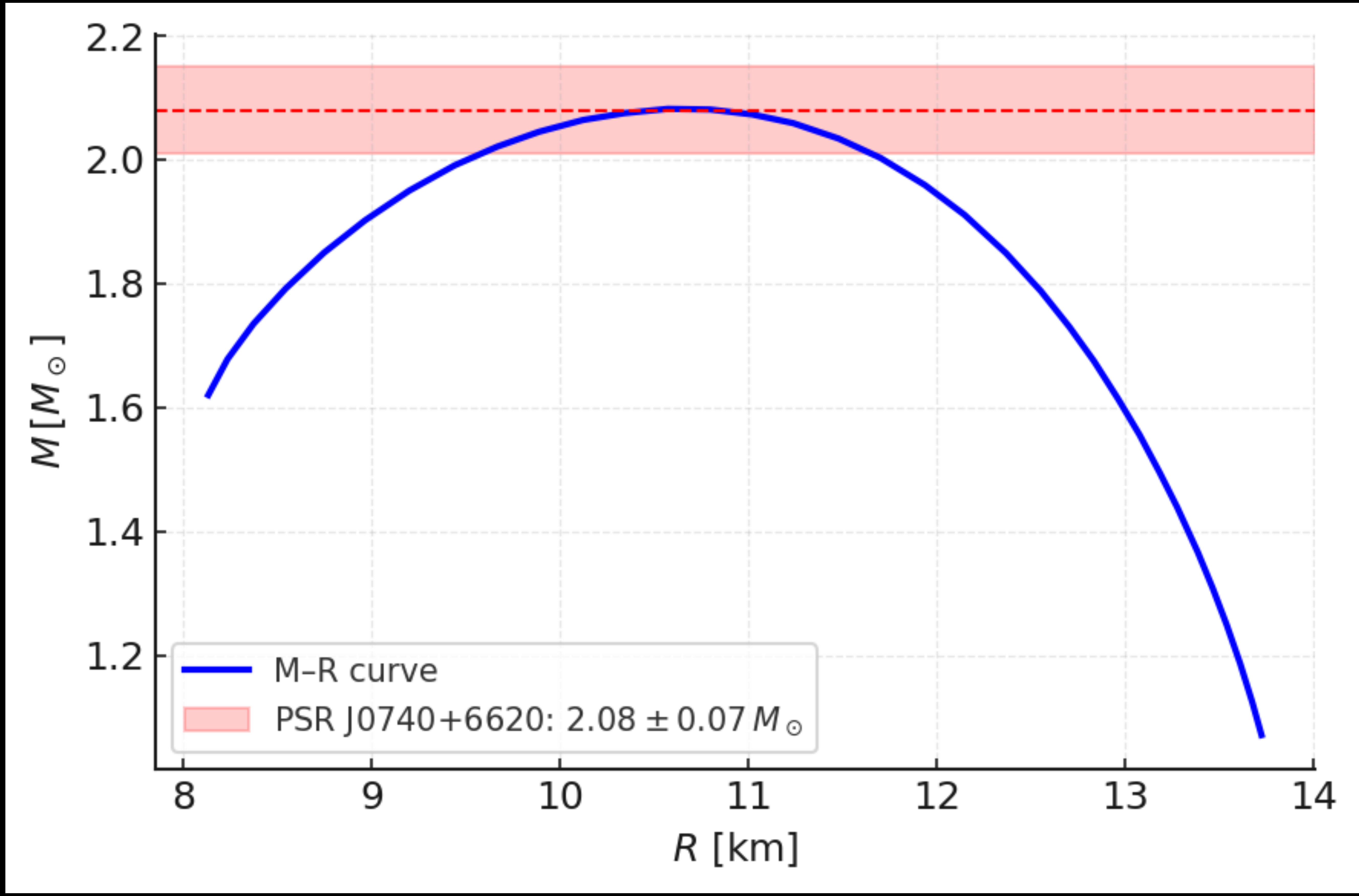
Preliminary result obtained through MUSES CE using N3LO NN + N2LO 3N 450 Chiral Potential.

The plot shows a preliminary result [5] obtained using MUSES CE standard workflow.

the plot shows a compatibility with the PSR J0740+6620 observational data and, in general, with the constraints $1.4\,M_\odot \lesssim M \lesssim 2.5\,M_\odot$ and $10\,\text{km} \lesssim R \lesssim 13\text{km}$

The workflow used for calculations in MUSES CE is non-linear, in contrast to the phenomenological case. This allows for optimization of computation time by running multiple processes simultaneously.





Preliminary result obtained through polytropic approach using N3LO NN + N2LO 3N 450 Chiral Potential.

The plot shows a preliminary result obtained using the polytropic phenomenological approach.

A difference in the curve is clearly noticeable, given the small dataset used for the calculation. Later versions of the code, currently under development, will improve the results for the M-R curve and consequently more precise calculation of k_2, Λ and moment of inertia.

Despite that, the plot shows a small compatibility with the PSR J0740+6620 observational data and, in general, with the constraints $1.4\, M_\odot \lesssim M \lesssim 2.5\, M_\odot$ and $10\,\text{km} \lesssim R \lesssim 13\text{km}$.

In conclusion, it is essential to highlight some aspects resulting from this study

To reproduce the correct pressure profile it is necessary to include the contribution of three-body interactions to the EoS

The complete EoS of the neutron star must include “beyond-χEFT" contributions

The study of astrophysical observables and neutron star EoS is essential to the study of limits to BSM physics

A parametrized polytropic χEFT approach reproduces results consistent with astrophysical observations, with less computational effort.
This model can be used to study astrophysical characteristics that are not directly dependent on the star's microphysics.

MUSES CE reproduces results consistent with astrophysical observations, but the computational effort is significantly greater
This tool can also be used to study the microphysical characteristics of the neutron star, but the complexity of the code, especially for the description of the core, requires attention and in-depth knowledge

THANK YOU ALL!!

[1] Schaffner-Bielich, Jürgen. *Compact star physics*. Cambridge University Press, 2020.
[2] Bombaci, Ignazio, and Domenico Logoteta. "Equation of state of dense nuclear matter and neutron star structure from nuclear chiral interactions." *Astronomy & Astrophysics* 609 (2018): A128.
[3] Sammarruca, Francesca, and Tomiwa Ajagbonna. "General features of the stellar matter equation of state from microscopic theory, new maximum-mass constraints, and causality." *arXiv preprint arXiv:2501.00668* (2024).
[4] Pelicer, Mateus Reinke, et al. "Building neutron stars with the MUSES calculation engine." *Physical Review D* 111.10 (2025): 103037.
[5] T. A. Manning, Muses calculation engine, 2, 2025.

Moment of inertia I is a global property strongly tied to the EoS. In the slowly rotating configuration (Hartle-Thorne approximation) it is expressed as:

$$I = \frac{8\pi}{3} \int_0^R dr\, r^4 \frac{[\varepsilon(r) + p(r)]}{\sqrt{1 - \frac{2GM(r)}{rc^2}}} \frac{\bar{\omega}(r)}{\Omega}$$

The Love number k_2 depends on the stellar compactness parameter and on the internal structure determined by the EoS. It's related to the dimensionless tidal deformability Λ, that can be constrained directly from inspiral waveforms if binary neutron star mergers:

$$\Lambda = \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$$

These quantities, together with the quadrupole moment, can be combined into the I-Love-Q relations, which turns out to be EoS-insensitive.

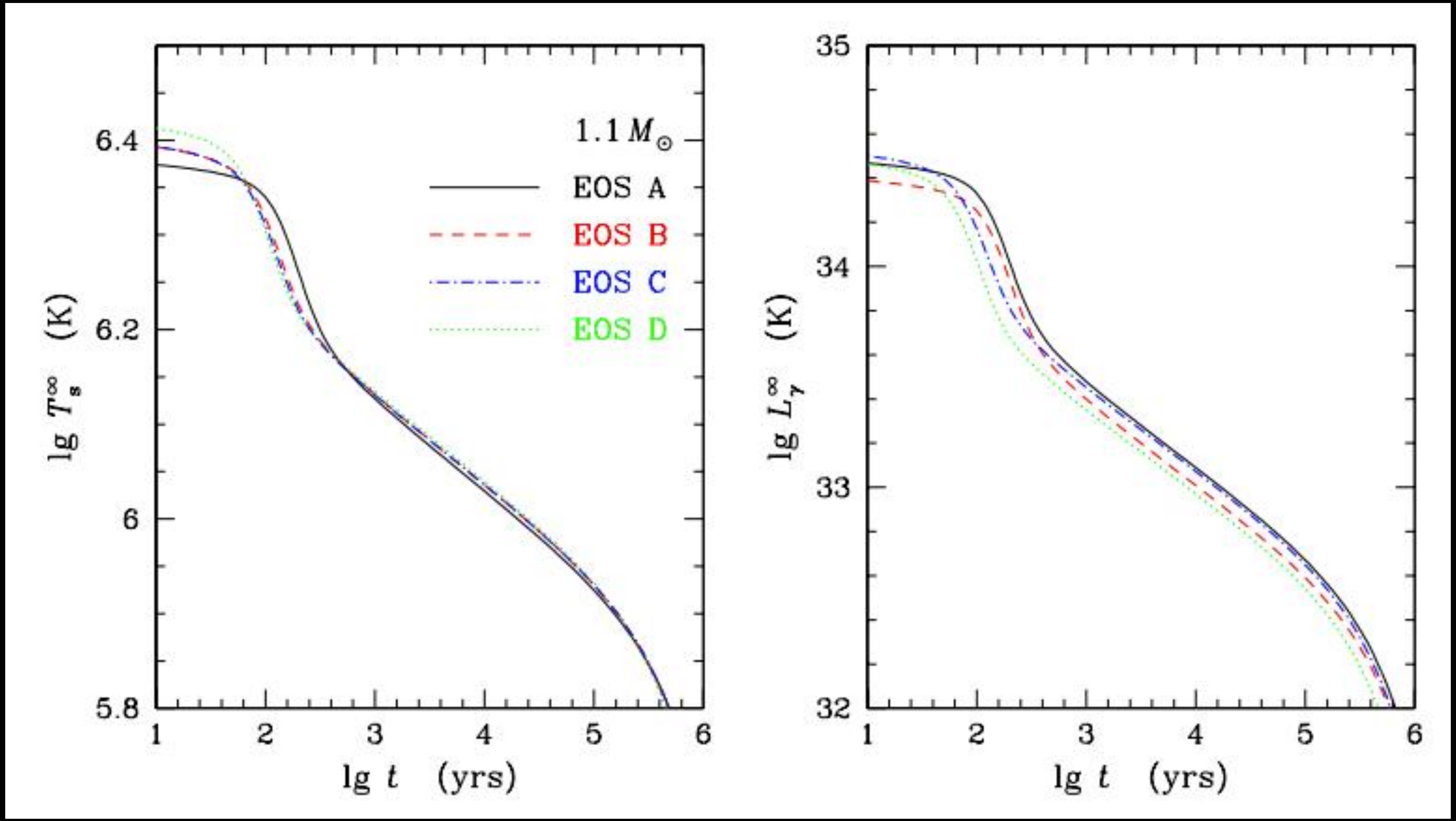
Pauli Exclusion Principle is a cornerstone of quantum physics. Despite this, there is still interest in researching possible small violations of this principle.

Probability of PEP violations are conventionally expressed using $\delta^2 = \frac{\beta^2}{2}$, where β represents a parameter introduced in the operators $a, a^\dagger$, where for $\beta \to 0$ we return to the usual Fermi representation.

Thanks to their extreme conditions, neutron stars are a great astrophysical arena to test quantum foundation principles. In this framework, a small violation of PEP is modeled as a controlled softening of the degeneracy contribution to the pressure

The combined study of the equation of state with astrophysical observables can be used to place bounds on the violation parameter β

It is possible to study the relation between the neutron star's EoS and the axion-nucleon coupling through the cooling curve, i.e. the evolution of the star.



Evolution of $1.1\,M_\odot$ neutron star for 4 different equation of state, calculated in *Yakovlev, Dima G., and C. J. Pethick. "**Neutron star cooling.**" Annu. Rev. Astron. Astrophys. 42.1 (2004): 169-210.*

A correct phenomenological description based on χEFT presupposes the inclusion of 3-body N2LO interactions.

The EoS for symmetric nuclear matter $(\beta = 0)$ and pure neutron matter $(\beta = 1)$ can be combined to obtain an equation of state for asymmetric nuclear matter $(0 \leq \beta \leq 1)$.

The process of β-stabilization is fundamental, and it's realized by studying thermodynamic relations.

The β-stable EoS is expanded through a phenomenological parametrized polytropic approach, resulting in an extended high-density EoS, constrained by the causality condition:

$$\frac{v_s}{c} < 1$$