Application of covariance matrix in TMD effects

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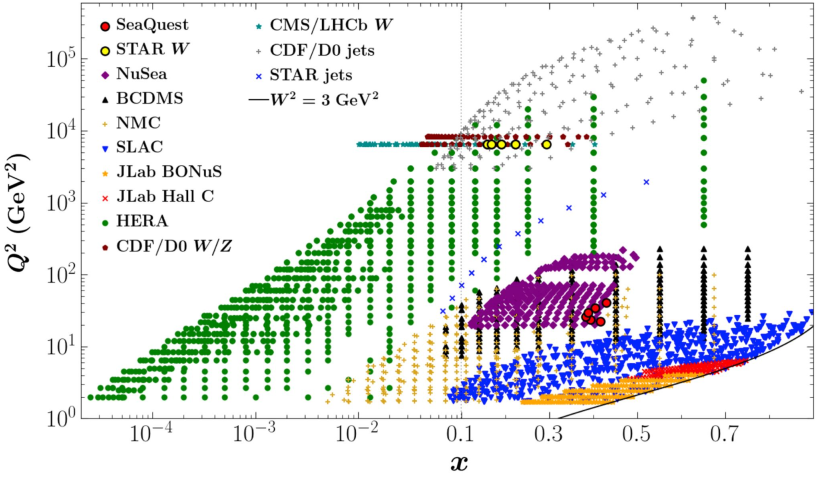
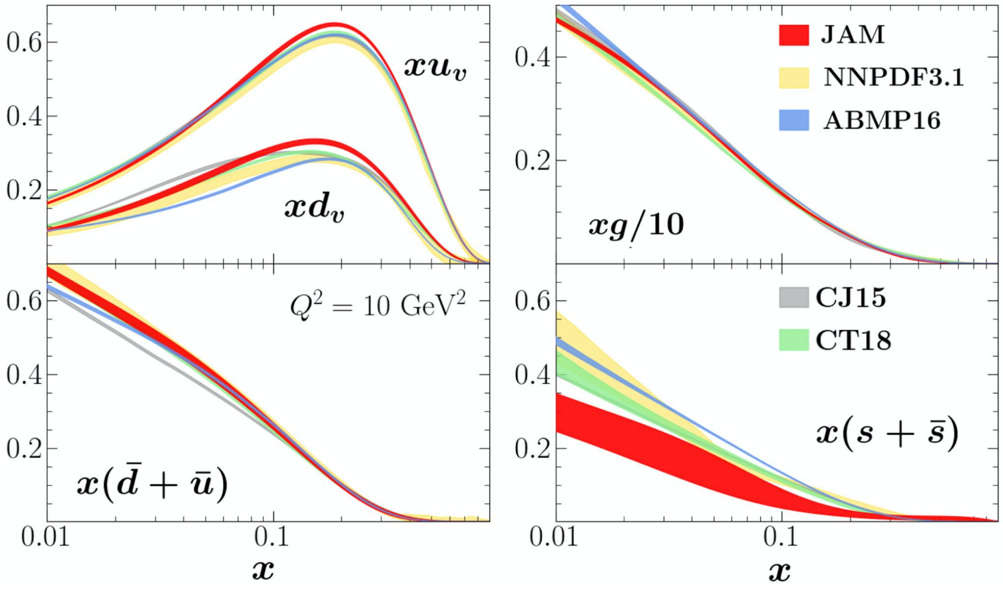
TNPI 2025, Oct 3, 2025

Collinear structures

Most well-known are the collinear parton distribution functions (PDFs)

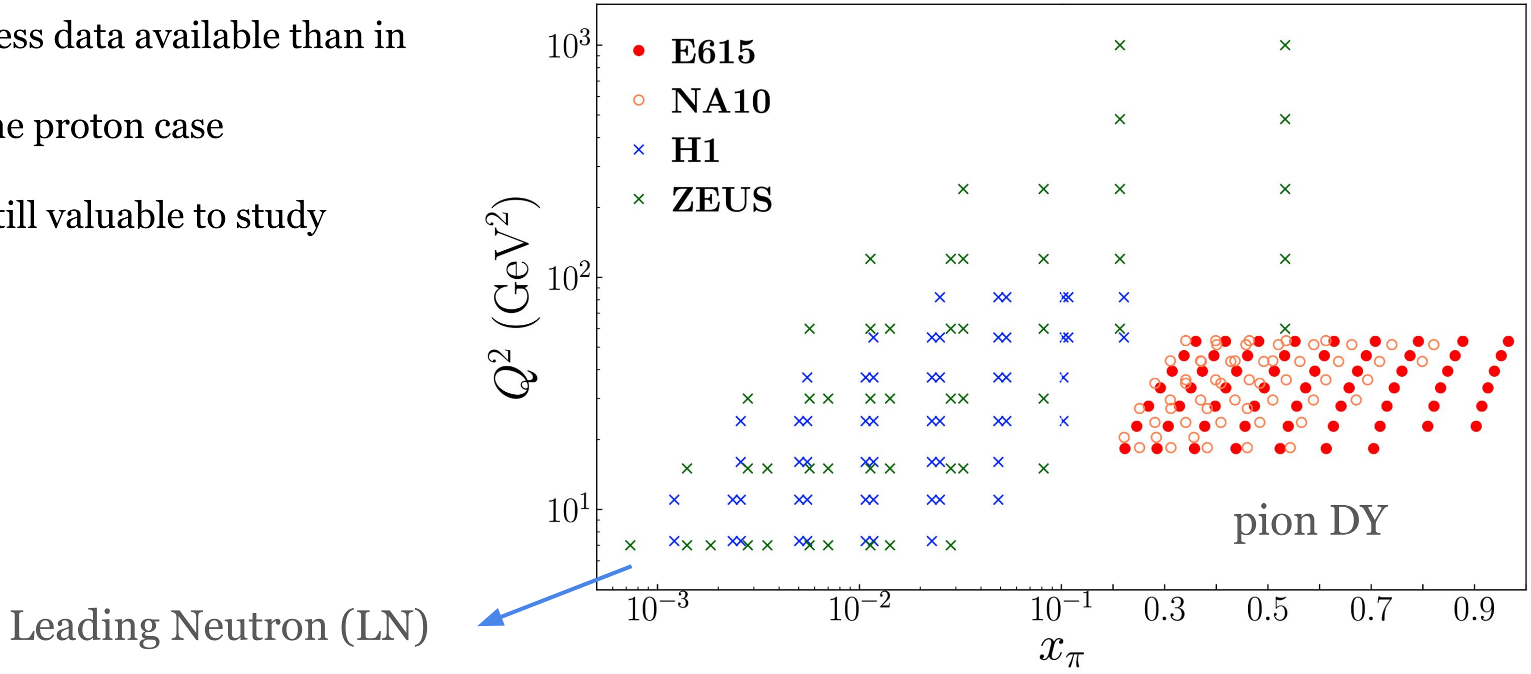




Cocuzza et al: [2109:00677]

Datasets for studying pion structure

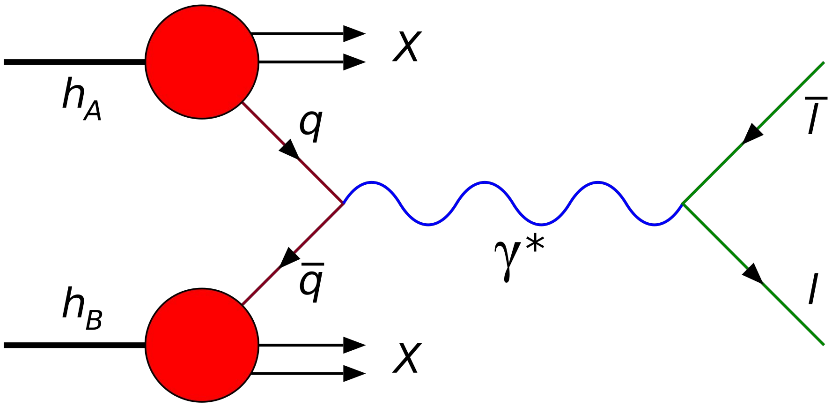
- Less data available than in the proton case
- Still valuable to study



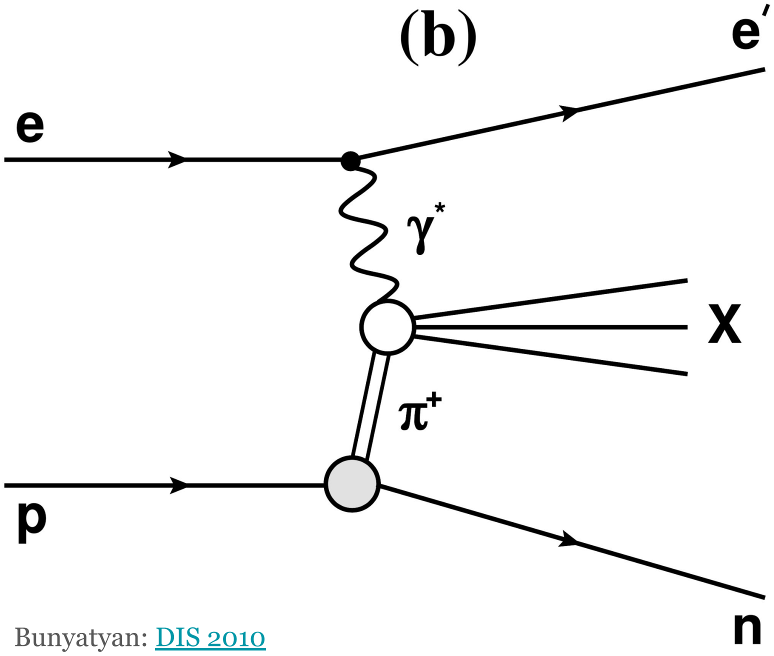
Barry: thesis

Processes to study pion structure

- Drell-Yan (DY) and leading neutron(LN)



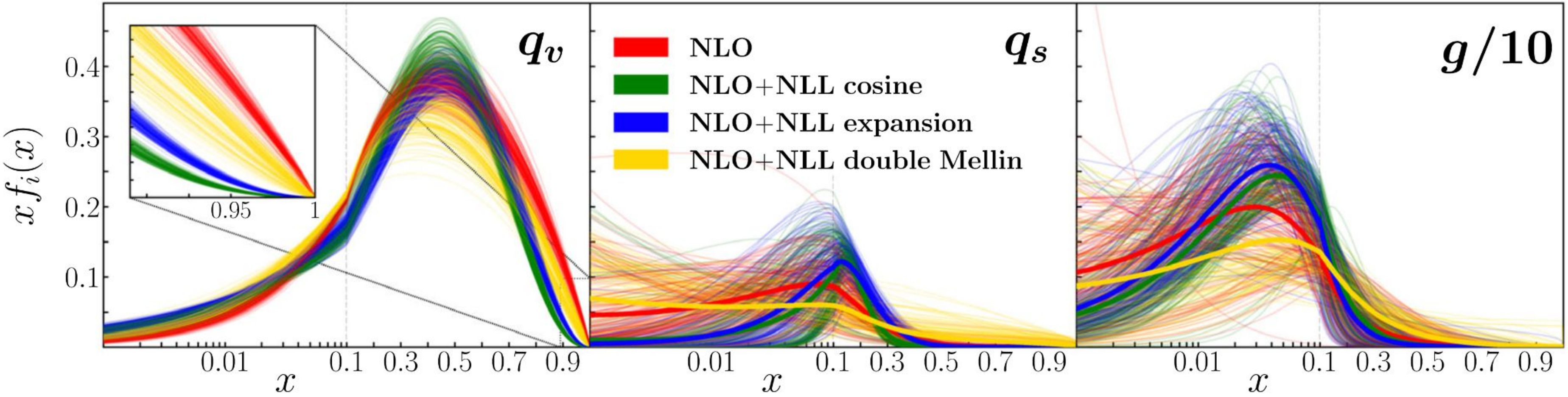
Wikipedia



Bunyatyan: DIS 2010

Collinear pion PDFs

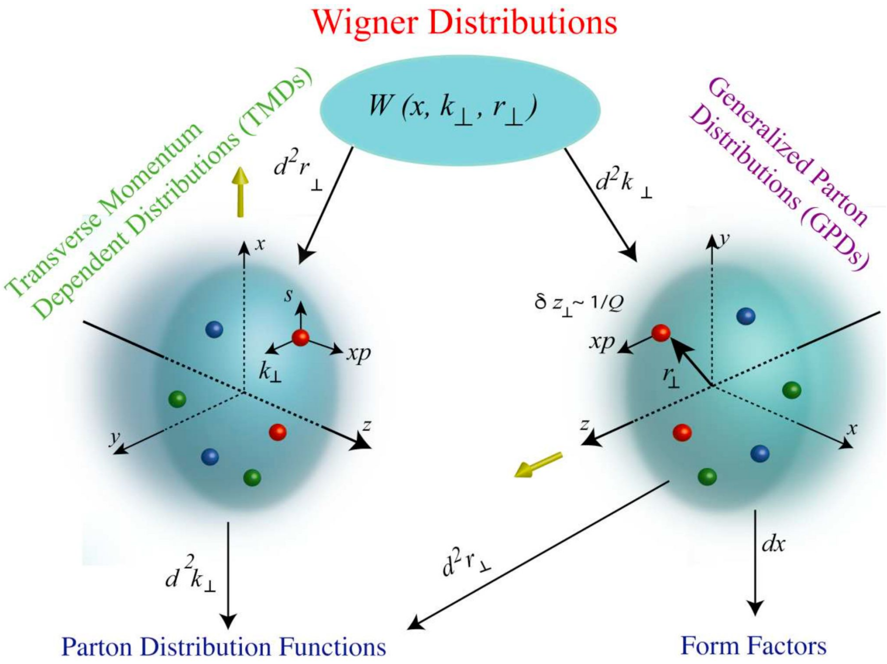
- Collinear pion PDFs are analyzed for valence, sea quarks and gluon



Barry, Ji, Sato & Melnitchouk: [2108.05822](2108.05822)

Structures in the transverse direction

- TMD (transverse momentum dependent) distributions
- GPDs (generalized parton distributions)



TMD Handbook

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration

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Martha Constantinou
William Detmold
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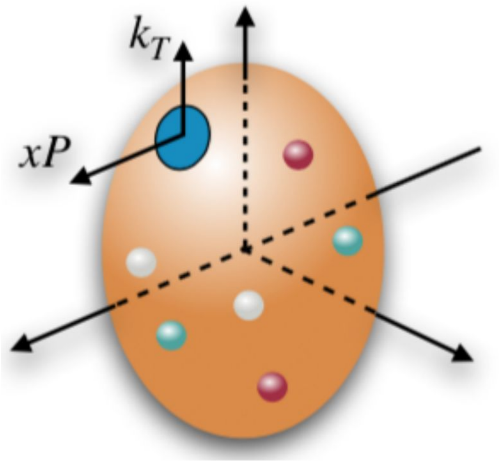
3D structure in momentum space

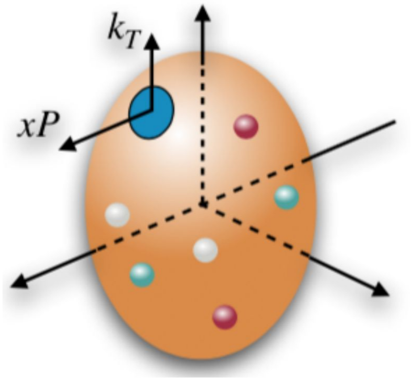
TMD (transverse momentum dependent) distributions:

- longitudinal momentum fraction
- transverse momentum k_T of quarks in the hadron



3D structure in momentum space

TMD (transverse momentum dependent) distributions:



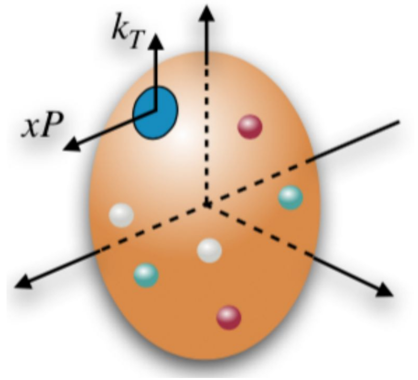
- longitudinal momentum fraction
- transverse momentum k_T of quarks in the hadron
- b_T is conjugate to the intrinsic transverse momentum k_T
- one can learn about coordinate space correlation of quarks

$$\widetilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \,\mathrm{Tr}\Big(\Big\langle \mathcal{N} \Big| \overline{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \Big| \mathcal{N} \Big\rangle\Big)$$

Factorization of low-q_T Drell-Yan

The Drell-Yan q_T-dependent cross section can be factorized in the low q_T region into:

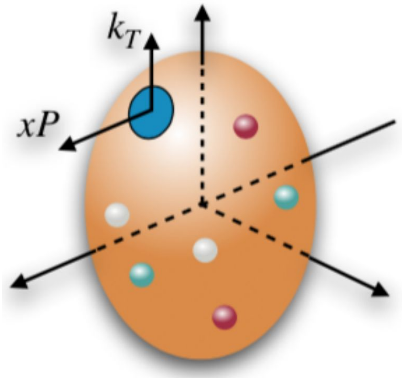
- Hard part H that describes partonice scattering amplitude
- TMD distributions that describe the structures of pion and proton/nucleus



$$\frac{d^3\sigma}{d\tau\, dy\, dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau s^2} \sum_q H_{q\bar{q}}^{\mathrm{DY}}\left(Q^2,\mu\right) \int d^2\boldsymbol{b}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, \widetilde{f}_{q/\pi}\left(x_\pi, b_T, \mu, Q^2\right) \widetilde{f}_{q/A}\left(x_A, b_T, \mu, Q^2\right)$$

TMD PDFs with the b_* prescription

b_* prescription is applied to smoothly join low and high b_T regions



- $b_* \equiv b_T / \sqrt{1 + b_T^2 / b_{\text{max}}^2}$
- $g_{q/N}$: intrinsic non-perturbative structure of the TMD
- g_K: universal non-perturbative Collins-Soper kernel
- S_{pert}: perturbative evolution of the TMD

$$\tilde{f}_{q/\mathcal{N}}\left(x, b_T, \mu = Q, Q^2\right) = (C \otimes f)_{q/\mathcal{N}}(x, b_*) \times \exp\left(-g_{q/\mathcal{N}}(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) - S_{\text{pert.}}(b_*, Q_0, Q, \mu = Q)\right)$$

Nuclear TMD correction: previous approach

We model the nuclear TMD PDFs as:

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

And further modify the $g_{q/N/A}$ as [Alrashed et al: 2107.12401]:

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}}\left(1 + a_N\left(A^{1/3} - 1\right)\right)$$

We have also assumed/used:

- Bound protons and neutrons follow TMD factorization
- Isospin symmetry so that $u/p/A \leftrightarrow d/n/A$

Nuclear TMD correction: new approach

We still model the nuclear TMD PDFs as:

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

We now introduce the nuclear covariance matrix:

$$\frac{1}{\sqrt{N_{\mathrm{rep.}}}}\Delta_{i,n} = \frac{1}{\sqrt{N_{\mathrm{rep.}}}}\left(T_{i,n}^{\mathrm{nuc.}} - T_{i,n}^{\mathrm{vac.}}\right)$$

$N_{\mathrm{rep.}}$: number of replicas

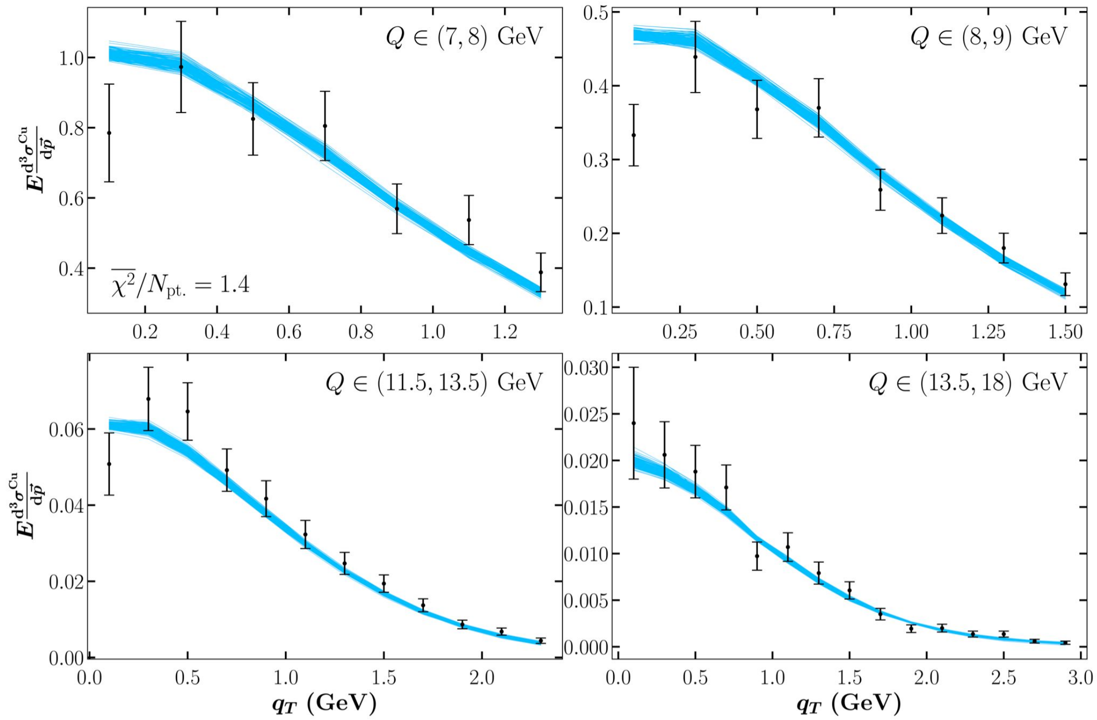
$T_{i,n}^{\mathrm{nuc.}}$: theory value **with** nuclear TMD NP kernel

$T_{i,n}^{\mathrm{vac.}}$: theory value **without** nuclear TMD NP kernel

All data points are connected, across different datasets!

Data-theory agreement

dataset	N_{pt}	χ^2/N_{pt}
E288 (pPt)	30	1.5
E288 (pPt)	39	1.1
E288 (pPt)	62	0.9
E605 (pCu)	42	1.4
E772 (pD)	51	2.6
E866 (Fe/Be)	7	3.0
E866 (W/Be)	7	2.9
E615 (πW)	40	1.6
E537 (πW)	27	1.3
total	**1370**	**0.90**



Data-theory agreement

dataset	N_{pt}	χ^2/N_{pt}
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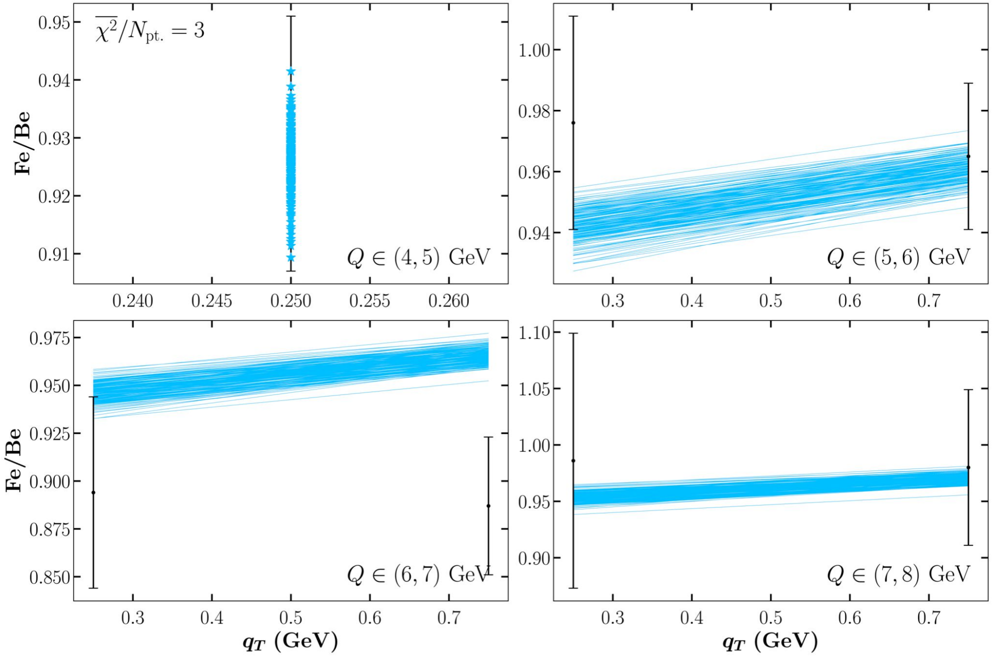
E615 (πW)	40	1.6
E537 (πW)	27	1.3
total	**305**	**1.6**



Description is actually good, large χ^2 comes from penalty terms that connect data points across datasets

Data-theory agreement: compare to baseline

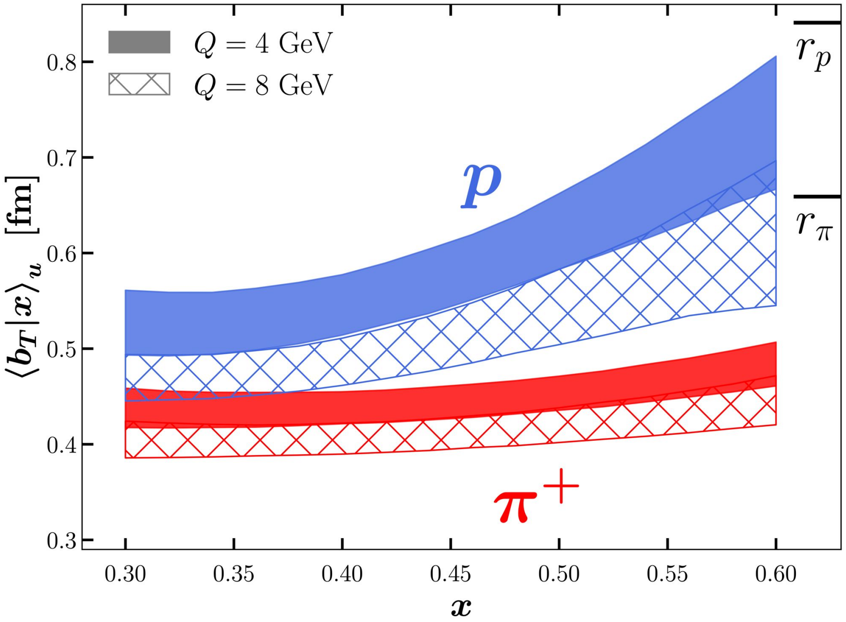
dataset	N_{pt}	χ^2/N_{pt} (baseline)	χ^2/N_{pt}
E288 (pPt)	30	1.1	1.5
E288 (pPt)	39	1.0	1.1
E288 (pPt)	62	0.8	0.9
E605 (pCu)	42	1.2	1.4
E772 (pD)	51	2.5	2.6
E866 (Fe/Be)	7	1.1	3.0
E866 (W/Be)	7	1.0	2.9
E615 (πW)	40	1.4	1.6
E537 (πW)	27	1.0	1.3
total	**305**	**1.3**	**1.6**

Average b_T as a function of x

$$\widetilde{f}_{q/\mathcal{N}}\left(b_T|x;Q,Q^2\right) \equiv \frac{\widetilde{f}_{q/\mathcal{N}}\left(b_T,x;Q,Q^2\right)}{\int \mathrm{d}^2\boldsymbol{b}_T\,\widetilde{f}_{q/\mathcal{N}}\left(b_T,x;Q,Q^2\right)}$$

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2),$$

- Nominal charge radius from PDG are marked
- As $x \to 1$, transverse motion phase space becomes narrower, hence b_T increases
- As Q increases, more gluons are radiated and k_T becomes larger, hence b_T decreases

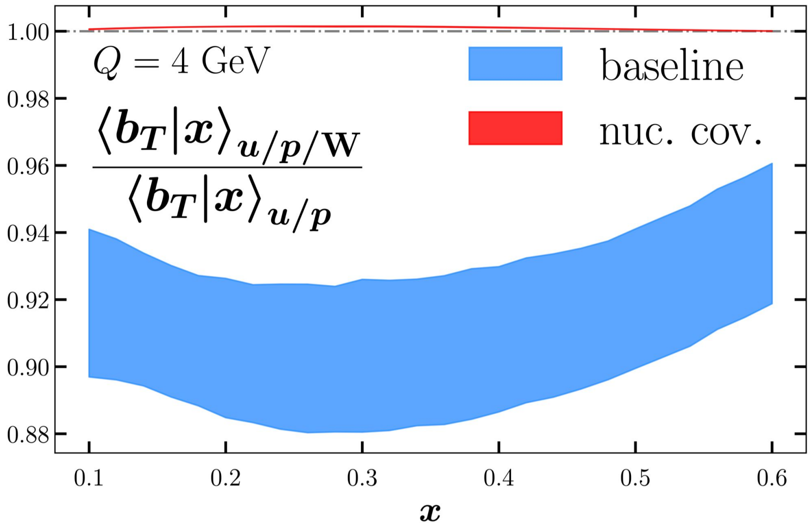


Nuclear modified average b_T

$$\widetilde{f}_{q/\mathcal{N}}\left(b_T|x;Q,Q^2\right) \equiv \frac{\widetilde{f}_{q/\mathcal{N}}\left(b_T,x;Q,Q^2\right)}{\int \mathrm{d}^2\boldsymbol{b}_T\, \widetilde{f}_{q/\mathcal{N}}\left(b_T,x;Q,Q^2\right)}$$

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2),$$

- Broadening of b_T (hence smaller b_T) is observed in the baseline
- With nuclear covariance matrix, no suppression is observed and uncertainty is vanishing (a_N = 0)



Summary

- We have explored a new approach for quantifying nuclear correction in TMD PDFs
- Studying the TMD distributions in pion is as important as studying the proton
- In the future, lattice datasets can be included in the analysis
- Future tagged experiments at EIC, JLab 22 GeV and AMBER at CERN can provide flavor separation

Thank you for your attention!