Nuclear structure calculations for $0\nu\beta\beta$: an overlook on present and future perspectives

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NUCSYS

The strongly correlated nuclear system

Abstract

The purpose of the present project is to bring together theorists who study the nuclear systems from different perspectives with overlapping expertise in the methodologies, interaction models and possible applications, ranging from fundamental symmetries and Physics beyond the Standard Model (BSM) to applied research in nuclear medicine.
The research team is articulated in 6 independent units which collaborate, producing in some cases joint publications, providing a favorable environment for the growth of PhD students and young post-docs.
The interactions of nucleons among themselves and with electroweak probes as derived from the Effective Field Theory (EFT) paradigm build the common ground of the initiative. The possibility of solving without approximations the dynamical equations for few-nucleon systems makes them the ideal laboratory to implement and refine the above interactions. Furthermore, since most of the experiments to probe BSM physics, from long-baseline neutrino experiments to neutrinoless double beta decay and dark matter searches, involve medium-heavy nuclei, it is also essential to deal with the complexities arising in heavier systems, using effective theories such as Hartree-Fock and Random Phase Approximation (RPA) methods or the nuclear shell model. Thus, this project constitutes a bridge between few-body and many-body nuclear systems, envisaging also benchmarks among different approaches.

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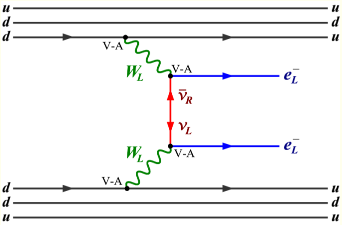
The neutrinoless double β-decay

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, since its detection would correspond to a violation of the conservation of the leptonic number, and may provide more informations on the nature of the neutrinos and its effective mass



- The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element $M^{0\nu}$, which relates the parent and grand-daughter wave functions via the decay operator.

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu} \left|M^{0\nu}\right|^2 \left|g_A^2 \frac{\langle m_\nu \rangle}{m_e}\right|^2$$

- The calculation of $M^{0\nu}$ links $\left[T^{0\nu}_{1/2}\right]^{-1}$ to the neutrino effective mass $\langle m_\nu \rangle = \mid \sum_k m_k U^2_{ek} \mid$ (light-neutrino exchange)

The structure of $M^{0\nu}$

Within the light-neutrino exchange, the matrix elements $M^{0\nu}_{\alpha}$ are defined in terms of:

- the 1-body transition density matrix elements between the parent (i), daughter (k), grand-daughter (f) nuclei;
- the matrix elements of the Gamow-Teller (GT), Fermi (F), and tensor (T) decay-operators:

$$M^{0\nu}_{\alpha} = \sum_{k} \sum_{j_p j_{p'} j_n j_{n'}} \langle f | a^{\dagger}_{p} a_{n} | k \rangle \langle k | a^{\dagger}_{p'} a_{n'} | i \rangle \left\langle j_p j_{p'} \mid \tau^{-}_{1} \tau^{-}_{2} \Theta^{k}_{\alpha} \mid j_n j_{n'} \right\rangle$$

with $\alpha = (GT,\ F,\ T)$

$$\begin{aligned} \Theta^{GT}_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \\ \Theta^{F}_{12} &= H_F(r) \\ \Theta^{T}_{12} &= [3 (\vec{\sigma}_1 \cdot \hat{r}) (\vec{\sigma}_1 \cdot \hat{r}) \\ &\quad - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_T(r) \end{aligned}$$

The neutrino potentials H_{α} depend on the energy of the initial, final, and intermediate states:

$$H_{\alpha}(r) = \frac{2R}{\pi} \int_0^{\infty} \frac{j_{\alpha}(qr) h_{\alpha}(q^2) q dq}{q + E_k - (E_i + E_f)/2}$$

The structure of $M^{0\nu}$: the closure approximation

- Only few nuclear models allow the inclusion of a number of intermediate k states, large enough to provide converging results.
- Then, for the most part, nuclear structure calculations resort to the closure approximation by replacing the intermediate-states energies with an average value.
- This approximation leads to shift from the product of 1-body transition-density to 2-body transition-density matrix elements, and simplifies the expression of the neutrino potentials $H_\alpha(r)$

$$E_k - (E_i + E_f)/2 \rightarrow \langle E \rangle$$

$$\sum_k \langle f | a^\dagger_p a_n | k \rangle \langle k | a^\dagger_{p'} a_{n'} | i \rangle = \langle f | a^\dagger_p a_n a^\dagger_{p'} a_{n'} | i \rangle$$

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty \frac{j_\alpha(qr) h_\alpha(q^2) q dq}{q + < E >}$$

This approximation works since $q \approx$ 100-200 MeV and model-space excitation energies $E_{exc} \approx 10$ MeV

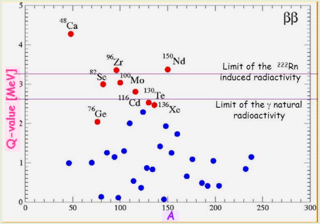
In *Phys. Rev. C* **88**, *064312 (2013)* the impact of the approximation has been evaluated as being within 10% of the exact value

The structure of $M^{0\nu}$: the closure approximation

Bottom line: the closure-approximation allows to write the $0\nu\beta\beta$ nuclear matrix element $M^{0\nu}$ in straightforward form:

$$M_{\alpha}^{0\nu} = \sum_{j_n j_{n'} j_p j_{p'}} \langle f | a_p^{\dagger} a_n a_{p'}^{\dagger} a_{n'} | i \rangle \langle j_p j_{p'} \mid \tau_1^- \tau_2^- \Theta_{\alpha} \mid j_n j_{n'} \rangle$$

The above expression underlines that to calculate $M^{0\nu}$ one needs to compute the matrix elements of the $0\nu\beta\beta$ decay operator, as well as the wave functions of the parent and grand-daughter nuclei.



However, all candidates of experimental interest lie in a mass region where "exact" solutions of the nuclear eigenvalue problem cannot be obtained

Theoretical nuclear structure calculations

The study of the many-body Schrödinger equation, for system with $A > 4$, needs the introduction of truncations and approximations, and follows two main approaches:

Mean-field and collective models

- Energy Density Functional (EDF)
- Quasiparticle Random-Phase Approximation (QRPA)
- Interacting Boson Model IBM

Microscopic approaches

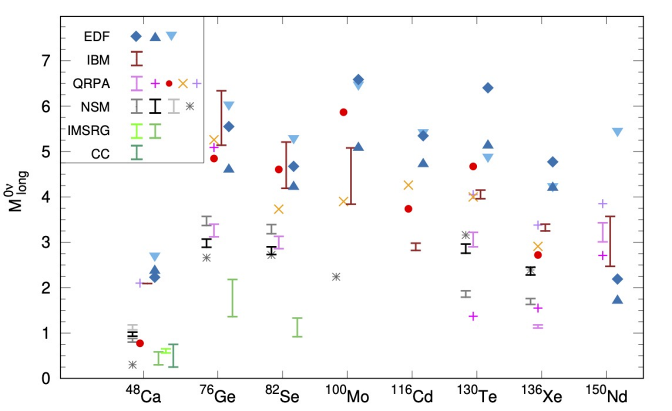
- *ab initio* methods
  - No-Core Shell Model (NCSM)
  - Coupled-Cluster Method (CCM)
  - In-Medium Similarity Renormalization Group (IMSRG)
  - Self-Consistent Green's Function approach (SCGF)
- Nuclear Shell Model

The many-body problem

- Mean-field and collective models operate a drastic cut of the nuclear degrees of freedom, the computational problem is alleviated
- Their effective Hamiltonian $H_{\rm eff}$ cannot be derived from realistic nuclear forces and depend from parameters fitted to reproduce a selection of observables
- This reduces the predictive power, that is crucial to search "new physics"

- The degrees of freedom of *ab initio* methods and SM Hamiltonians are the microscopic ones of the single nucleons (very demanding calculations)
- Consequently, they may operate starting from realistic nuclear forces
- These features enhance the predictiveness and the calculated wave functions are more reliable

Nuclear structure calculations

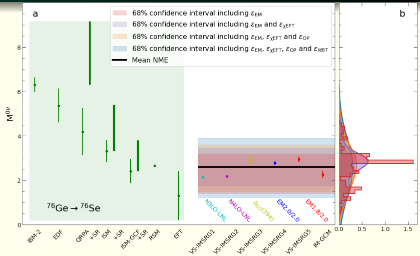


- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

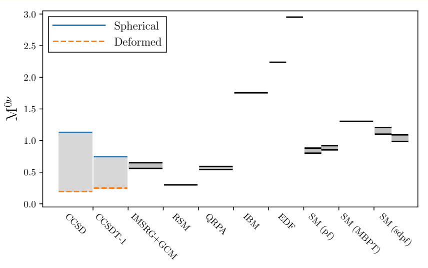
M. Agostini, G. Benato, J. A. Detwiler, J. Menéndez, and F. Vissani, Rev. Mod. Phys. 95, 025002 (2023)

Ab initio methods: β-decay in medium-mass nuclei

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to calculate $M^{0\nu}$ for the $0\nu\beta\beta$ decay of ^{48}Ca, ^{76}Ge, and ^{82}Se



In-medium SRG



Coupled-Cluster Method

- Advantage → The degrees of freedom of *all* constituent nucleons are included, the number of correlations among nucleons is enormous
- Shortcoming → Highest-degree of computational complexity, the comparison with spectroscopic data is not yet satisfactory

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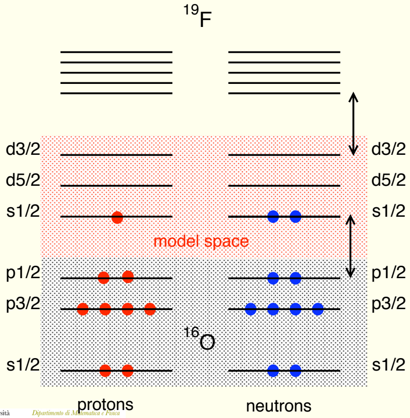
The interacting shell model

The nucleons are subject to the action of a mean field, that takes into account most of the interaction of the nuclear constituents.

Only valence nucleons interact by way of a residual two-body potential, within a reduced model space.



- Advantage → It is a microscopic and flexible model, the degrees of freedom of the valence nucleons are explicitly taken into account.
- Shortcoming → High-degree computational complexity.

Ab initio calculations of $M^{0\nu}$

- *Ab initio* methods that can be considered the most complex approach to the calculation of $M^{0\nu}$, in terms of the single-nucleons degrees of freedom of the single nucleons
- All those calculations start from a nuclear Hamiltonian that has been derived through chiral perturbative expansion of an EFT Lagrangian (ChEFT)
- This aspect, in conjuction with the property of *ab initio* methods of being ”size extensive”, allows, in principle, to estimate the theoretical error

Ab initio calculations of $M^{0\nu}$

- The computational complexity of *ab initio* methods makes the calculation of a large number of intermediate states very demanding and complicated, so the validation through the computing of $2\nu\beta\beta$ nuclear matrix elements
- The most important outcome of *ab initio* methods is that the calculated $M^{0\nu}$s are sistematically much smaller than all other nuclear structure calculations

Decay	CCM $M^{0\nu}$	VS-IMSRG $M^{0\nu}$	$T^{0\nu}_{1/2}$ (in yr)
$^{48}\mathrm{Ca}_1 \rightarrow ^{48}\mathrm{Ti}_1$	$0.25 \leq M^{0\nu} \leq 0.75$[1]	$0.58(1)$[2]	$> 2 \times 10^{29}$
$^{76}\mathrm{Ge}_1 \rightarrow ^{76}\mathrm{Se}_1$		2.60 ± 1.4[3]	$> 2 \times 10^{28}$
$^{82}\mathrm{Se}_1 \rightarrow ^{82}\mathrm{Kr}_1$		$1.24(5)$[2]	$> 2 \times 10^{28}$

1 *S. Novario et al., Phys. Rev. Lett.* **121** *182502 (2021)*

2 *A. Belley et al., Phys. Rev. Lett.* **126** *042502 (2021)*

3 *A. Belley et al., Phys. Rev. Lett.* **132** *182502 (2024)*

Realistic shell-model

The nuclear shell model is a microscopic one, then it is possible to construct, within the many-body theory, effective Hamiltonians and decay operators starting from realistic nuclear potentials

Realistic shell model (RSM)

1. Choose a realistic *NN* potential (*NNN*)
2. Renormalize its short range correlations
3. Identify the model space better tailored to study the physics problem
4. Derive the effective shell-model Hamiltonian and consistently effective shell-model operators for decay amplitudes, by way of the many-body perturbation theory
5. Calculate the observables (energies, e.m. transition probabilities, β-decay amplitudes...), using only theoretical SP energies, two-body matrix elements, and effective SM operators.

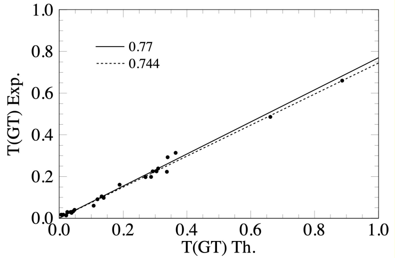
RSM calculations of $M^{0\nu}$: results

Decay	bare operator	$\Theta_{\rm eff}$	
^{48}Ca $\to$ ^{48}Ti	0.53	0.30	-40%
^{76}Ge $\to$ ^{76}Se	3.35	2.66	-20%
^{82}Se $\to$ ^{82}Kr	3.30	2.72	-20%
^{100}Mo $\to$ ^{100}Ru	3.96	2.24	-40%
^{130}Te $\to$ ^{130}Xe	3.27	3.16	-3%
^{136}Xe $\to$ ^{136}Ba	2.47	2.39	-3%

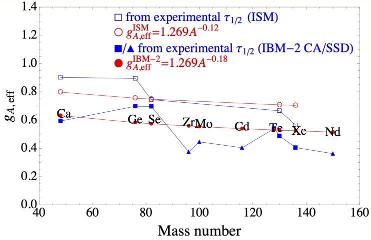
- Results obtained with the effective shell-model operator are relatively reduced with respect those with bare operator: quenching effect is much smaller than the two-neutrino double-β decay
- The impact of the effective operator is not homogeneous, strongly depending on the nuclear system under consideration
- The values of the calculated $M^{0\nu}$s for ^{48}Ca and ^{76}Ge are consistent with current *ab initio* results

The quenching of g_A

A major issue in the calculation of quantities related to spin-isospin-dependent transitions is the need to quench the axial coupling constant g_A by a factor q in order to reproduce the data.



G. Martínez Pinedo et al., Phys. Rev. C **53**, *R2602 (1996)*



J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **91**, *034304 (2015)*

This is an important question when studying $0\nu\beta\beta$ decay, in fact the need of a quenching factor q largely affects the value of the half-life $T^{0\nu}_{1/2}$, since the latter would be enlarged by a factor q^{-4}

The quenching of g_A

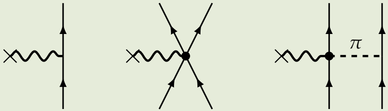
The two main sources of the need of a quenching factor q may be identified as:

Truncation of the nuclear configurations

Nuclear models operate a cut of the nuclear degrees of freedom in order to diagonalize the nuclear Hamiltonian $\Rightarrow$ effective Hamiltonians and decay operators must be considered to account for the neglected configurations in the nuclear wave function

Nucleon internal degrees of freedom

Nucleons are not point-like particles $\Rightarrow$ contributions to the free value of g_A come from two-body meson exchange currents:



- *K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. A* **226***, 282 (1974)*
- *I. S. Towner, Phys. Rep.* **155***, 263 (1987)*

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The effective operators for decay amplitudes

- Ψ_α eigenstates of the full Hamiltonian H with eigenvalues E_α
- Φ_α eigenvectors obtained diagonalizing $H_{\rm eff}$ in the model space P and corresponding to the same eigenvalues E_α

$$\Rightarrow |\Phi_\alpha\rangle = P\,|\Psi_\alpha\rangle$$

Obviously, for any decay-operator Θ:

$$\langle\Phi_\alpha|\Theta|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

We then require an effective operator $\Theta_{\rm eff}$ defined as follows

$$\Theta_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle \langle\Phi_\beta|$$

Consequently, the matrix elements of $\Theta_{\rm eff}$ are

$$\langle\Phi_\alpha|\Theta_{\rm eff}|\Phi_\beta\rangle = \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

This means that the parameters characterizing $\Theta_{\rm eff}$ are renormalized with respect to $\Theta \Rightarrow g_A^{\rm eff} = q \cdot g_A \neq g_A$

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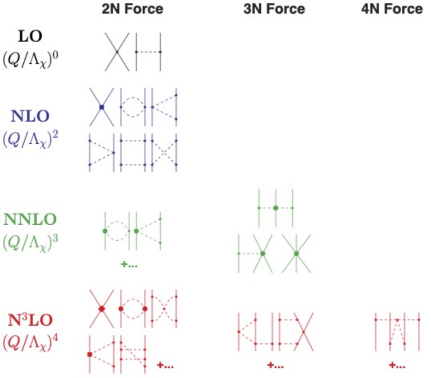
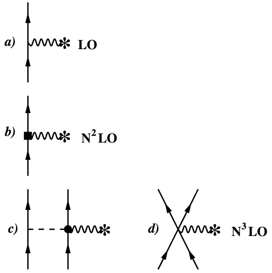
Two-body meson exchange currents

A powerful approach to the derivation of two-body currents (2BC) is to resort to effective field theories (EFT) of quantum chromodynamics.
In such a way, both nuclear potentials and 2BC may be consistently constructed, since in the EFT approach they appear as subleading corrections to the one-body Gamow-Teller (GT) operator $\sigma\tau^{\pm}$.

Nuclear Hamiltonian



Two-body currents



The impact of 2BC on the calculated β-decay properties has been investigated in terms of *ab initio* methods

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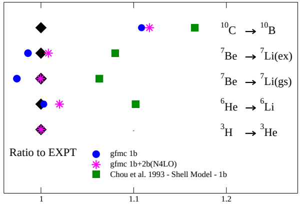
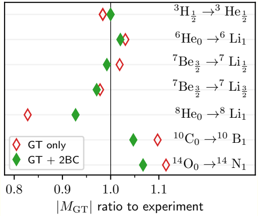
β-decay in light nuclei

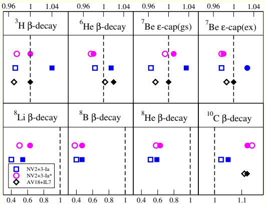
GT nuclear matrix elements of the β-decay of *p*-shell nuclei have been calculated with Green's function Monte Carlo (GFMC) and no-core shell model (NCSM) methods, including contributions from 2BC



S. Pastore et al., Phys. Rev. C **97** 022501(R) (2018)



P. Gysbers et al., Nat. Phys. **15** 428 (2019)



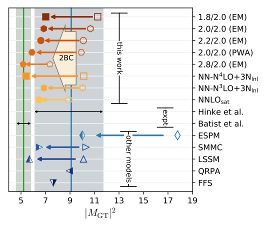
G. B. King et al., Phys. Rev. C **102** 025501 (2020)

The contribution of 2BC improves systematically the agreement between theory and experiment

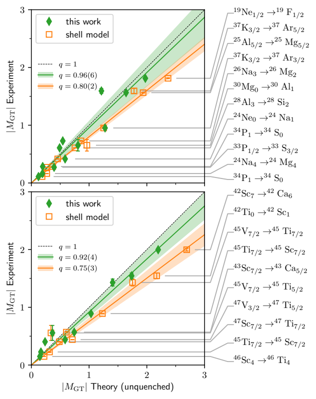
Ab initio methods: β-decay in medium-mass nuclei

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to overcome the quenching problem g_A to reproduce β-decay observables in heavier systems

P. Gysbers et al., Nat. Phys. **15** *428 (2019)*



Coupled-Cluster Method

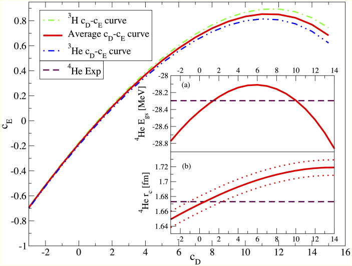


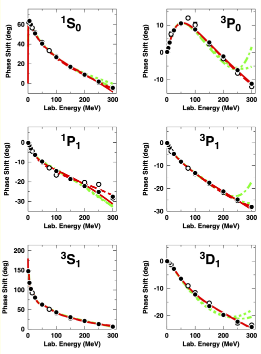
In-Medium SRG

A proper treatment of nuclear correlations and consistency between GT two-body currents and 3N forces, derived in terms of ChPT, explains the "quenching puzzle"

Our approach to the realistic shell model

- Nuclear Hamiltonian: Entem-Machleidt N^{3}LO two-body potential plus N^{2}LO three-body potential





- Axial current $\mathbf{J}_A$ calculated at N^{3}LO in ChPT: LECs c_3, c_4, c_D are consistent with the 2NF and 3NF potentials
- H_{eff} calculated at 3rd order in perturbation theory
- Effective operators are consistently derived using MBPT

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The axial current $\mathbf{J}_A$

The matrix elements of the axial current $\mathbf{J}_A$ are derived through a chiral expansion up to N^{3}LO, and employing the same LECs as in 2NF and 3NF

$$\mathbf{J}^{\rm LO}_{A,\pm} = -g_A \sum_i \boldsymbol{\sigma}_i \tau_{i,\pm} \ ,$$

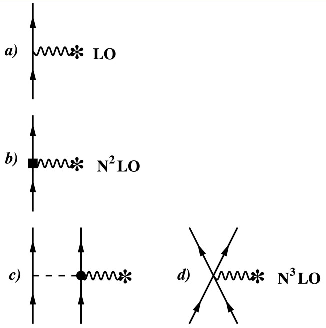
$$\mathbf{J}^{\rm N^2LO}_{A,\pm} = \frac{g_A}{2m_N^2} \sum_i \mathbf{K}_i \times (\boldsymbol{\sigma}_i \times \mathbf{K}_i)\, \tau_{i,\pm} \ ,$$

$$\mathbf{J}^{\rm N^3LO}_{A,\pm}({\rm 1PE};\mathbf{k}) = \sum_{i<j} \frac{g_A}{2f_\pi^2} \Big\{ 4c_3 \tau_{j,\pm} \mathbf{k} + (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_\pm \times \Big[\Big(c_4 + \frac{1}{4m} \boldsymbol{\sigma}_i \times \mathbf{k} - \frac{i}{2m} \mathbf{K}_i \Big) \Big] \Big\} \boldsymbol{\sigma}_j \cdot \mathbf{k} \frac{1}{\omega_k^2} \ ,$$

$$\mathbf{J}^{\rm N^3LO}_{A,\pm}({\rm CT};\mathbf{k}) = \sum_{i<j} z_0 (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_\pm (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \ ,$$

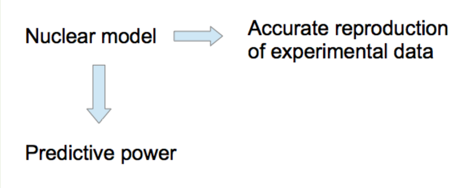
where

$$z_0 = \frac{g_A}{2f_\pi^2 m_N} \left[\frac{m_N}{4g_A \Lambda_\chi} c_D + \frac{m_N}{3}(c_3 + 2c_4) + \frac{1}{6} \right] .$$



A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C **93**, 015501 (2016)

Shell-model calculations and results



RSM calculations, starting from ChPT two- and three-body potentials and two-body meson-exchange currents for spectroscopic and spin-isospin dependent observables of ^{48}Ca, ^{76}Ge, ^{82}Se

$\Downarrow$

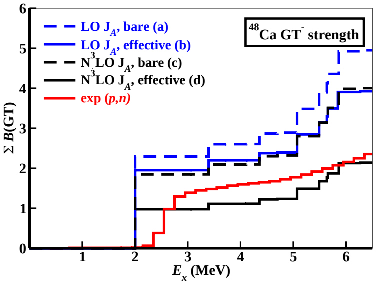
Check RSM approach calculating GT strengths and $2\nu\beta\beta$-decay

$$\left[T_{1/2}^{2\nu}\right]^{-1} = G^{2\nu}\left|M_{\mathrm{GT}}^{2\nu}\right|^2$$

where

$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+||\mathbf{J}_A||1_n^+\rangle\langle 1_n^+||\mathbf{J}_A||0_i^+\rangle}{E_n + E_0}$$

Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Total GT$^-$ strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^-)$	24.0	17.5	20.9	11.2	15.3 ± 2.2

The impact of meson-exchange currents on the GT$^-$ matrix elements is $\approx 20\%$

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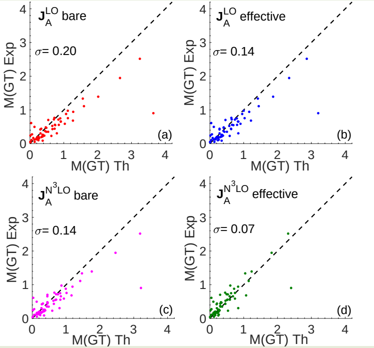
Gamow-Teller observables



GT matrix elements of 60 experimental decays of 43 $0f1p$-shell nuclei, only yrast states involved

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

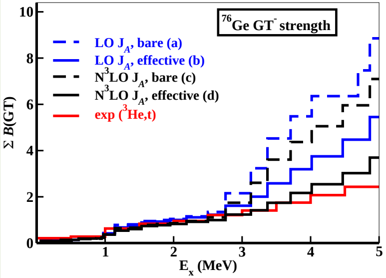
(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ $^{48}Ca \rightarrow {}^{48}Ti$

$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.057	0.048	0.033	0.019	0.042 ± 0.004
$0_1^+ \rightarrow 2_1^+$	0.131	0.102	0.097	0.057	≤ 0.023
$0_1^+ \rightarrow 0_2^+$	0.102	0.086	0.073	0.040	≤ 2.72

Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

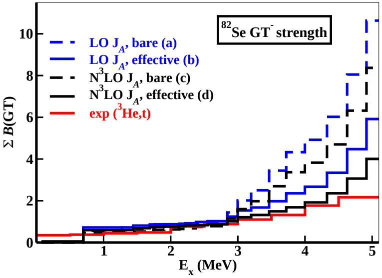
(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Total GT$^-$ strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^-)$	15.8	10.8	12.8	7.4	$\sim$

The impact of meson-exchange currents on the GT$^-$ matrix elements is $\approx 18\%$

Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Total GT$^-$ strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^-)$	19.0	11.4	14.9	7.5	$\sim$

The impact of meson-exchange currents on the GT$^-$ matrix elements is $\approx$ 20%

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Gamow-Teller observables

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ ^{76}Ge$\rightarrow$^{76}Se

$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.211	0.153	0.160	0.118	0.129 ± 0.004
$0_1^+ \rightarrow 2_1^+$	0.023	0.042	0.025	0.048	≤ 0.035
$0_1^+ \rightarrow 0_2^+$	0.009	0.086	0.016	0.063	≤ 0.089

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ ^{82}Se$\rightarrow$^{82}Kr

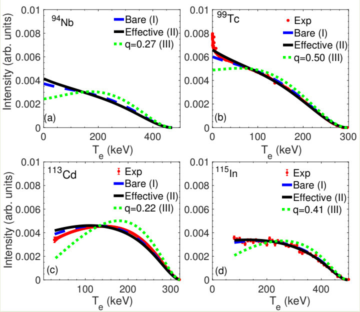
$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.173	0.123	0.136	0.095	0.103 ± 0.001
$0_1^+ \rightarrow 2_1^+$	0.003	0.006	0.008	0.033	≤ 0.020
$0_1^+ \rightarrow 0_2^+$	0.018	0.007	0.013	0.007	≤ 0.052

L. C., N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C **109***, 014301 (2024)*

Testing RSM: forbidden β-decay energy spectra

Forbidden β-decay logfts

Nucleus	bare	effective	Exp
^{94}Nb	11.30	11.58	11.95 (7)
^{99}Tc	11.580	11.876	12.325 (12)
^{113}Cd	21.902	22.493	23.127 (14)
^{115}In	21.22	21.64	22.53 (3)

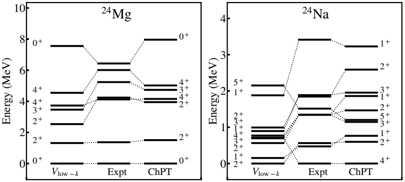


G. De Gregorio, R. Mancino, LC, N. Itaco, Phys. Rev. C **110**, *014324 (2024)*

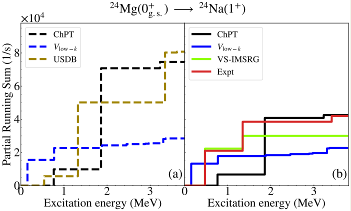
Testing RSM: muon capture in *sd* shell nuclei

The study of ordinary muon capture (OMC) is an intriguing test for nuclear structure calculations, since it is an electroweak process that occurs with exchange momenta far larger than ordinary β decays ($\approx$ 100 MeV/c), in a range that is consistent with $0\nu\beta\beta$ decay

Spectroscopy with ChPT and $V_{\text{low-}k}$ H_{eff}s



Running sums



Our extensive study of OMC has evidenced a correlation between the quality of the reproduction of the observed spectroscopy and the agreement between calculated and experimental OMC running sums

S. Lyu, G. De Gregorio, T. Fukui, N. Itaco, LC, arXiv:2508.06221[nucl-th]

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Conclusions and Outlook

- The new developments of microscopic approaches to the nuclear many-body problem are leading towards reliable calculations of the $0\nu\beta\beta$ nuclear matrix elements.
  This goal may be achieved by focusing theoretical efforts on two main tasks:
  a) improving our knowledge of nuclear forces;
  b) estimation of the theoretical error from the application of many-body methods.
- The efforts of the EFT community are also providing new aspects of our knowledge of the $0\nu\beta\beta$ decay operator
- Nuclear-structure microscopic calculations, when carried out in a fully consistent framework, have proved that the so-called "quenching puzzle" in the study of β decay processes is no longer an issue.
- More benchmark calculations with different theoretical approaches need to to be performed, in order to narrow the spread among different theoretical results

Backup slides

The calculation of $M^{0\nu}$: LO contact transition operator

Within the framework of ChEFT, there is the need to introduce a LO short-range operator, which is missing in standard calculations of $M^{0\nu}$s, to renormalize the operator and make it independent of the ultraviolet regulator

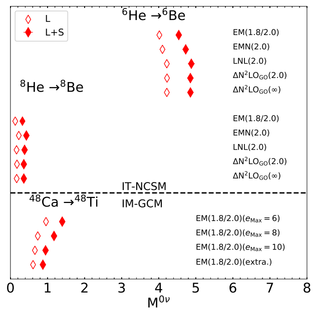
V. Cirigliano et al., Phys. Rev. Lett. **120** *202001 (2020)*

$$M^{0\nu}_{sr} = \frac{1.2A^{1/3}\,\mathrm{fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \mathbf{1} \left[\frac{4g_\nu^{NN}}{\pi} \int j_0(qr)\, f_S(p/\Lambda_S)\, q^2 dq \right] | 0_i^+ \rangle$$

The open question is the determination of the low-energy constant g_ν^{NN}

A recent attempt to fit g_ν^{NN} by computing the transition amplitude of the $nn \to ppe^- e^-$ process using nuclear NN and NNN interactions has shown that $M^{0\nu}_{sr}$ enlarges the $M^{0\nu}$ for ^{48}Ca $0\nu\beta\beta$ decay

R. Wirth et al., Phys. Rev. Lett. **127** *242502 (2021)*



The effective shell-model Hamiltonian

We start from the many-body Hamiltonian H defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \begin{array}{c} \mathcal{H} = \Omega^{-1} H \Omega \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow \Omega = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$

$$H_1^{\text{eff}}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P -$$
$$-PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\text{eff}}(\omega)$$

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The perturbative approach to the shell-model H^{eff}

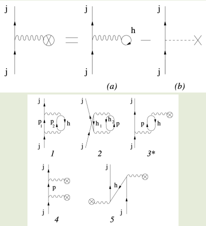
The $\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

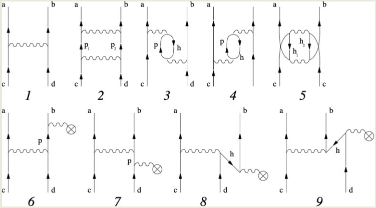
Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

$\hat{Q}$-box: 1st- & 2nd-order 1-b diagrams



$\hat{Q}$-box: 1st- & 2nd-order 2-b diagrams



The effective SM operators for decay amplitudes

The χ_n operators are defined in terms of the vertex function $\hat{\Theta}$ as:

$$
\begin{aligned}
\chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} \ , \\
\chi_1 &= (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.) \ , \\
\chi_2 &= (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q}\hat{Q} + h.c.) + \\
&\quad (\hat{\Theta}_{02} \hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11}\hat{Q} \ , \\
&\quad \ldots
\end{aligned}
$$

and

$$\hat{\Theta}(\epsilon) = P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P$$

$$\hat{\Theta}(\epsilon_1;\epsilon_2) = PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P$$

$$\hat{\Theta}_m = \frac{1}{m!} \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m} \Big|_{\epsilon=\epsilon_0}$$

$$\hat{\Theta}_{nm} = \frac{1}{n!m!} \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1;\epsilon_2) \Big|_{\epsilon_{1,2}=\epsilon_0}$$

V

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The effective SM operators for decay amplitudes

Any shell-model effective operator may be derived consistently with the $\hat{Q}$-box-plus-folded-diagram approach to $H_{\rm eff}$

It has been demonstrated that, for any bare operator Θ, a non-Hermitian effective operator $\Theta_{\rm eff}$ can be written in the following form:

$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q} + \hat{Q} \hat{Q}_2 + \cdots)(\chi_0 + \\ + \chi_1 + \chi_2 + \cdots) \ ,$$

where

$$\hat{Q}_m = \frac{1}{m!} \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \Big|_{\epsilon=\epsilon_0} \ ,$$

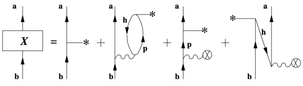
ϵ_0 being the model-space eigenvalue of the unperturbed Hamiltonian H_0

K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

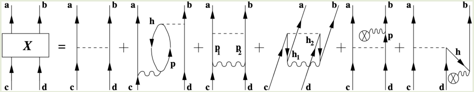
The effective SM operators for decay amplitudes

The $\hat{\Theta}$-box is then calculated perturbatively, here are diagrams up to 2nd order of the effective decay operator Θ_{eff} expansion:

One-body operator



Two-body operator



- *L. C., L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95**,
- *L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100**, *014316 (2019).*
- *L. C., A. Gargano, N. Itaco, R. Mancino, and F. Nowacki, Phys. Rev. C* **101**, *044315 (2020).*
- *L. C., N. Itaco, G. De Gregorio, A. Gargano, R. Mancino, and F. Nowacki, Phys. Rev. C* **105**, *034312 (2022).*

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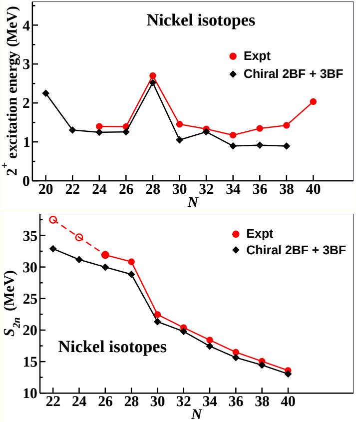
$0f1p$-shell nuclei

- Model space spanned by 4 proton and neutron orbitals $0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}$
- Effects of induced 3-body forces have been included
- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input



Y. Z. Ma, L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. R. Xu, Phys. Rev. C **100**, *034324 (2019)*

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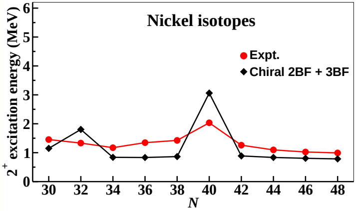
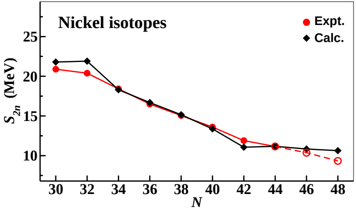
$0f1p0g$-shell nuclei

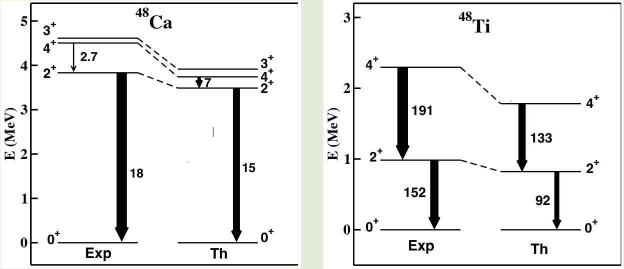
- Model space spanned by 4 proton and neutron orbitals $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- Effects of induced 3-body forces have been included
- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input





L. C., N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C **109***, 014301 (2024)*

$0f1p$-shell nuclei spectroscopic properties



Nucleus	$J_i \rightarrow J_f$	bare	effective	$B(M1)_{\text{Expt}}$
^{48}Ca				
	$3_1^+ \rightarrow 2_1^+$	0.090	0.044	0.023 ± 0.004

Nucleus	J^π	bare	effective	μ_{Expt}
^{48}Ti				
	2_1^+	0.26	0.34	$+0.78 \pm 0.04$
	4_1^+	1.0	1.1	$+2.2 \pm 0.5$

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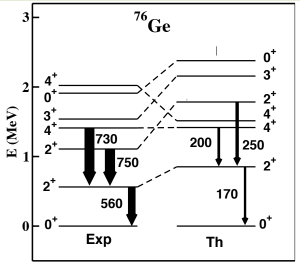
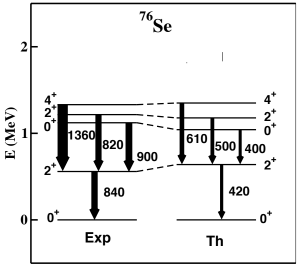
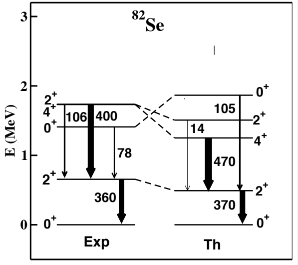
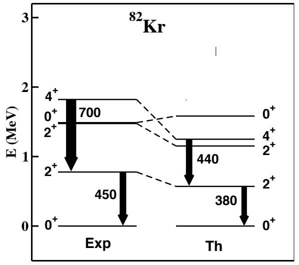
$0f_{5/2}1p0g_{9/2}$-shell nuclei spectroscopic properties

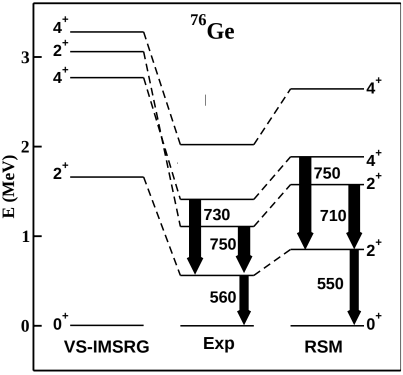


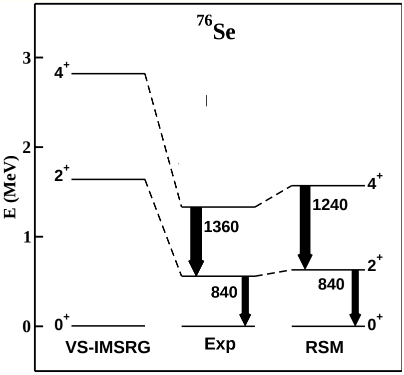




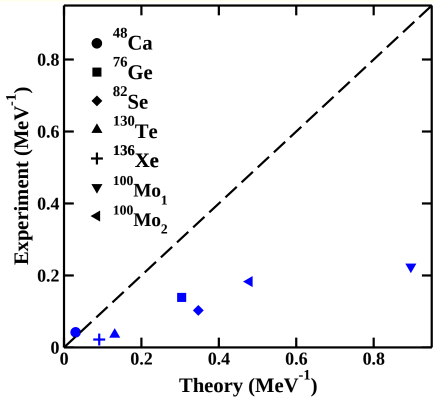


Ab initio vs NSM calculations





$2\nu\beta\beta$ nuclear matrix elements

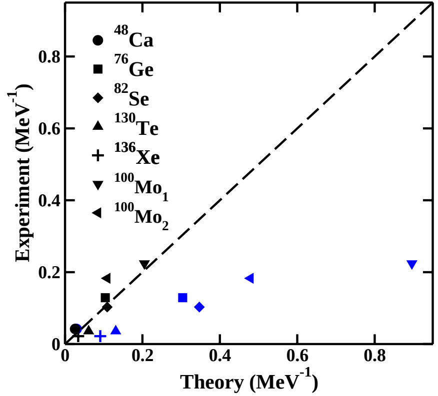


Blue symbols: bare GT operator

Decay	Expt.	Bare
$^{48}Ca_1 \rightarrow {}^{48}Ti_1$	0.042 ± 0.004	0.030
$^{76}Ge_1 \rightarrow {}^{76}Se_1$	0.129 ± 0.005	0.304
$^{82}Se_1 \rightarrow {}^{82}Kr_1$	0.103 ± 0.001	0.347
$^{130}Te_1 \rightarrow {}^{130}Xe_1$	0.036 ± 0.001	0.131
$^{136}Xe_1 \rightarrow {}^{136}Ba_1$	0.0219 ± 0.0007	0.0910
$^{100}Mo_1 \rightarrow {}^{100}Ru_1$	0.224 ± 0.002	0.896
$^{100}Mo_1 \rightarrow {}^{100}Ru_2$	0.183 ± 0.006	0.479

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

$2\nu\beta\beta$ nuclear matrix elements



Blue symbols: bare GT operator
Black symbols: effective GT operator

Decay	Expt.	Eff.
^{48}Ca$_1$ $\rightarrow$ ^{48}Ti$_1$	0.042 ± 0.004	0.026
^{76}Ge$_1$ $\rightarrow$ ^{76}Se$_1$	0.129 ± 0.005	0.104
^{82}Se$_1$ $\rightarrow$ ^{82}Kr$_1$	0.103 ± 0.001	0.109
^{130}Te$_1$ $\rightarrow$ ^{130}Xe$_1$	0.036 ± 0.001	0.061
^{136}Xe$_1$ $\rightarrow$ ^{136}Ba$_1$	0.0219 ± 0.0007	0.0341
^{100}Mo$_1$ $\rightarrow$ ^{100}Ru$_1$	0.224 ± 0.002	0.205
^{100}Mo$_1$ $\rightarrow$ ^{100}Ru$_2$	0.183 ± 0.006	0.109

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

Decay	q
^{48}Ca $\rightarrow$ ^{48}Ti	0.83
^{76}Ge $\rightarrow$ ^{76}Se	0.58
^{82}Se $\rightarrow$ ^{82}Kr	0.56
^{100}Mo $\rightarrow$ ^{100}Ru	0.48
^{130}Te $\rightarrow$ ^{130}Xe	0.68
^{136}Xe $\rightarrow$ ^{136}Ba	0.61

- *LC, L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100**, *014316 (2019).*
- *LC, N. Itaco, G. De Gregorio, A. Gargano, R. Mancino, and F. Nowacki, Phys. Rev. C* **105** *034312 (2022).*

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RSM calculations of $M^{0\nu}$

Earliest RSM calculation of $M^{0\nu}$ performed by Kuo and coworkers for ^{48}Ca decay *(Phys. Lett. B* **162** *227 (1985))*

- SM effective TBMEs and decay operator from Paris and Reid potential
- Brueckner *G*-matrix and 2nd-order MBPT
- SRC derived through the calculation of the defect function

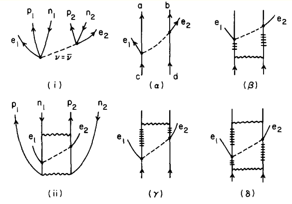


Fig. 1. Diagrams of $H^{\mathrm{eff}}_{\beta\beta}$; virtual Majorana neutrinos are represented by dotted lines, nucleon *G*-matrix interaction by wavy lines, and nucleons outside the model space P by railed lines.

$M^{0\nu}$ for ^{76}Ge, ^{82}Se, and ^{48}Ca decay by Holt and Engel *(Phys. Rev. C* **87** *064315 (2013), Phys. Rev. C* **88** *045502 (2014))*

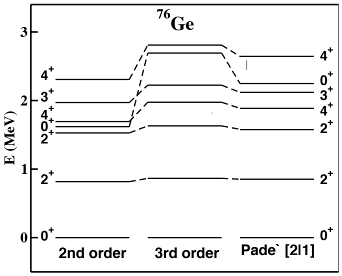
- Wave functions calculated with GCN28.50, JUN45, and GXPF1A Hamiltonians
- SM decay operator from N^{3}LO (EM) potential, $V_{\mathrm{low-k}}$ renormalization, and 3rd-order MBPT
- Jastrow-type short-range correlations

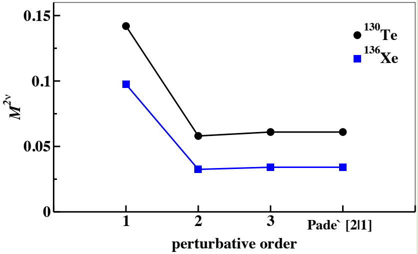
$M^{0\nu}$ for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, and ^{136}Xe decay by our collaboration *(Phys. Rev. C* **101** *044315 (2020); Phys. Rev. C* **105** *034312 (2022))*

- H_{eff} and effective decay operators from CD-Bonn potential, $V_{\mathrm{low-k}}$ renormalization, and 3rd-order MBPT
- $V_{\mathrm{low-k}}$-transformation SRC
- Three-body correlations contributions included to account for the Pauli-blocking effect

Perturbative properties

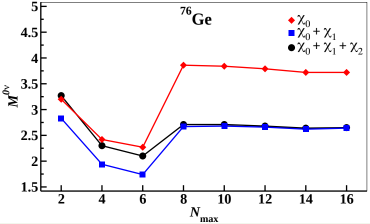
Order-by-order convergence



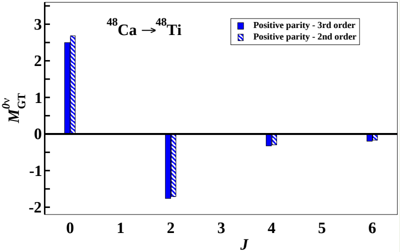


Perturbative properties of the 0ν effective operator

Convergence with respect the number of intermediate states

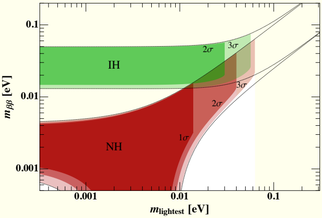


Order-by-order convergence



Contributions from pairs of decaying neutrons with given J^π to $M^{0\nu}_{GT}$ for ^{48}Ca $0\nu\beta\beta$ decay. The bars filled in blue corresponds to the results obtained with Θ_{eff} calculated a 3rd order in perturbation theory, those in dashed blue are calculated at 2nd order

The calculation of $M^{0\nu}$: results



To rule out the Inverted Hierarchy of neutrino mass spectra, the upper bound of neutrino effective mass should be $\langle m_{\beta\beta} \rangle < 18.4 \pm 1.3$ meV.

We could then evaluate the lower bound of the half lives of the decay processes, accordingly to our calculated $M^{0\nu}$

	$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$
$T^{0\nu}_{1/2}$ (in yr)	$> 2 \times 10^{28}$	$> 4 \times 10^{27}$	$> 4 \times 10^{27}$	$> 2 \times 10^{27}$	$> 4 \times 10^{27}$
	(0.3-2.8) $\times 10^{28}$	(1-6) $\times 10^{27}$	(0.4-4) $\times 10^{27}$	(0.5-11) $\times 10^{27}$	(0.9-17) $\times 10^{27}$

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