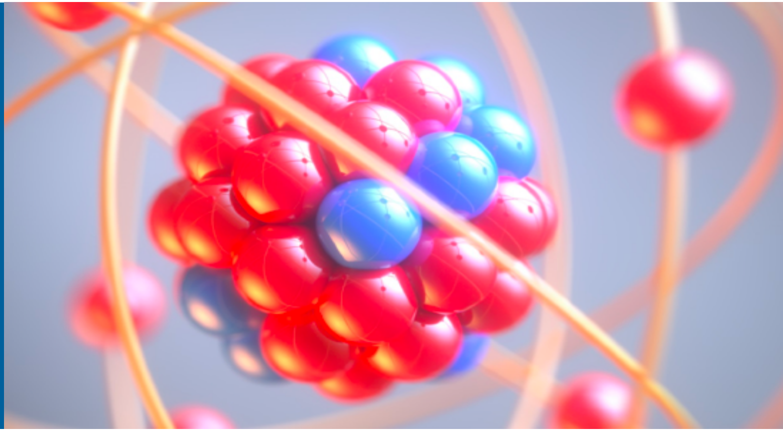
NUCLEI & HYPERNUCLEI WITH NEURAL QUANTUM STATES



ANDREA DI DONNA

XX Conference on Theoretical Nuclear Physics in Italy

October 2, 2025

“AB-INITIO” NUCLEAR THEORY

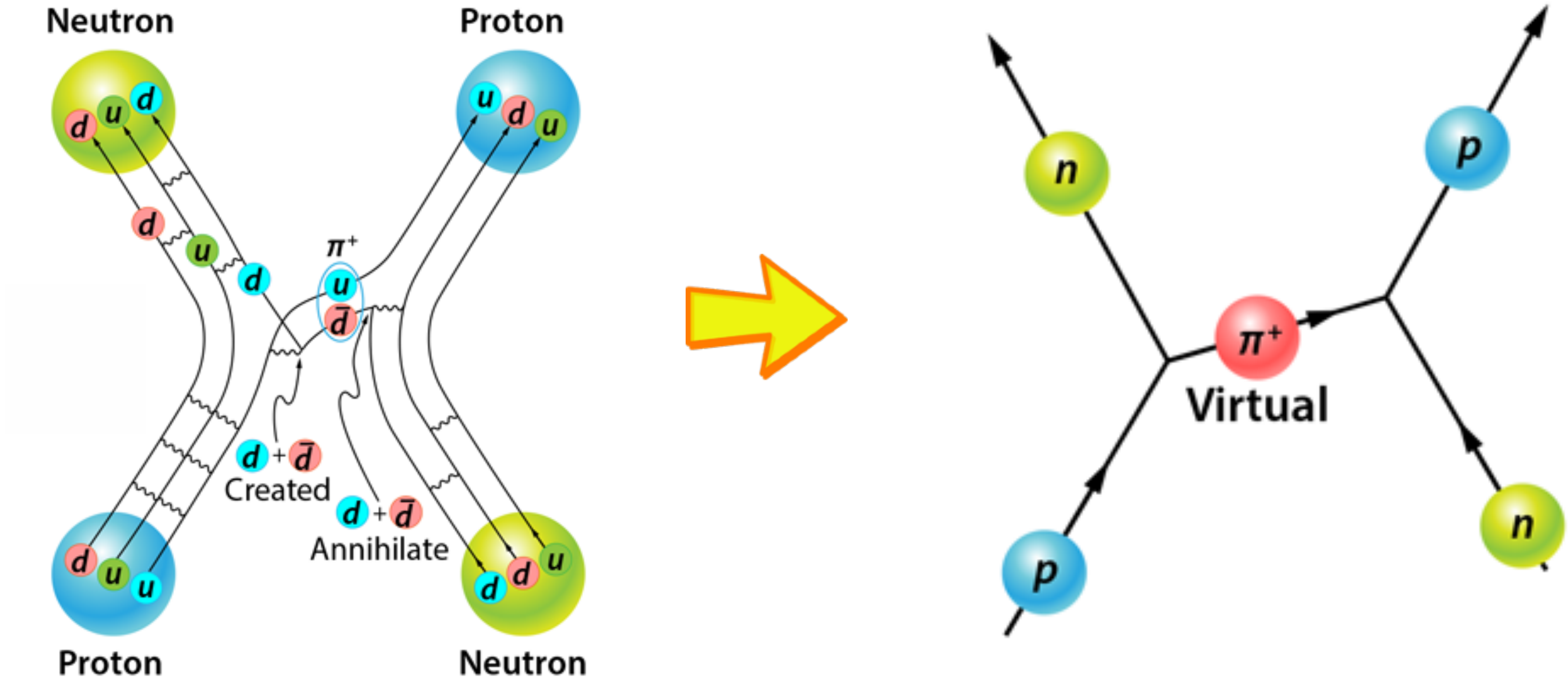
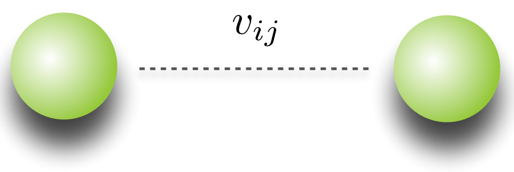


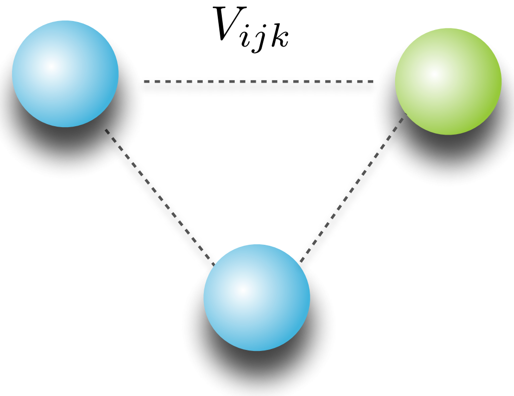
Illustration by APS / Alan Stonebraker

NUCLEAR HAMILTONIAN

Realistic nuclear Hamiltonians include two- and three-body potentials

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$





THE QUANTUM MANY-BODY PROBLEM

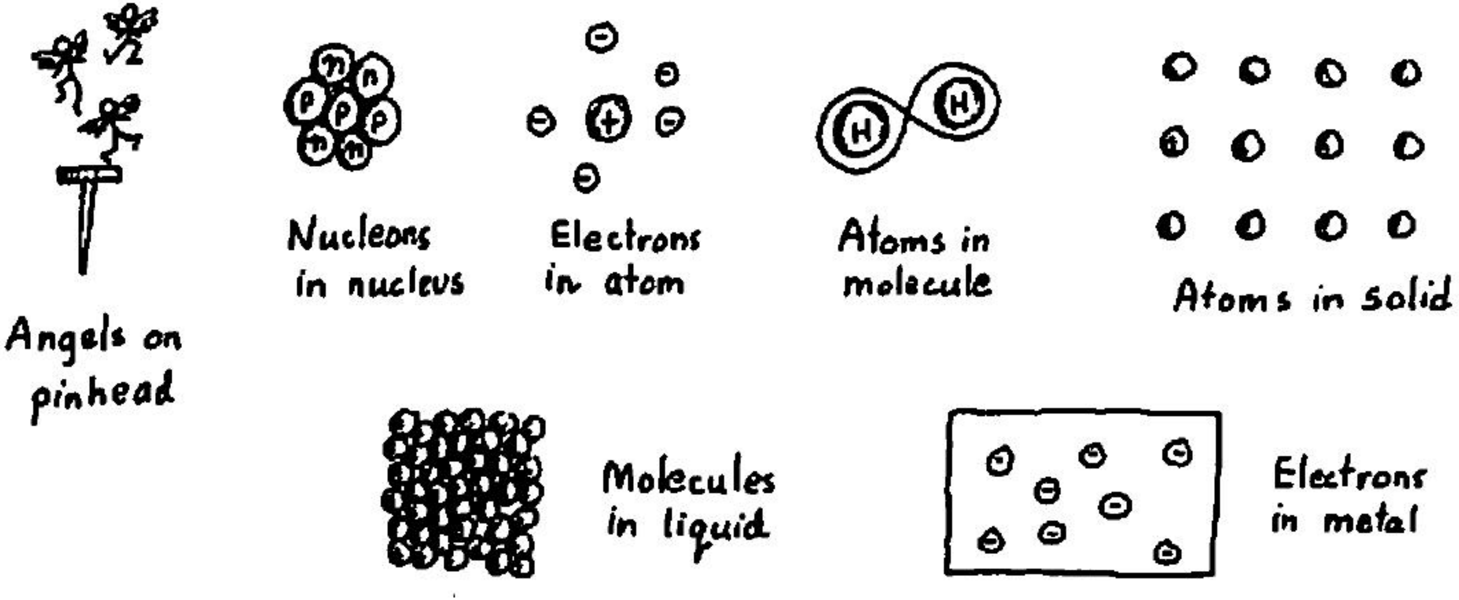
- Non relativistic many body theory aims at solving the many-body Schrödinger equation

$$H\Psi_n(x_1, \dots, x_A) = E_n\Psi_n(x_1, \dots, x_A)$$

- Nucleons are fermions, so the wave function must be anti-symmetric

$$\Psi_n(x_1, \dots x_i, \dots, x_j, \dots, x_A) = -\Psi_n(x_1, \dots x_j, \dots, x_i, \dots, x_A)$$

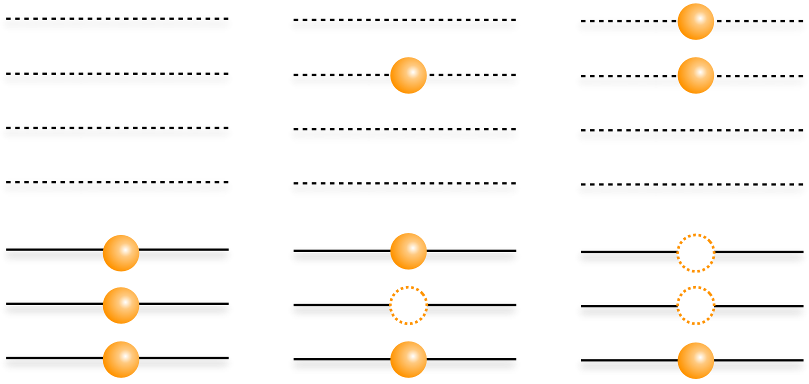
THE QUANTUM MANY-BODY PROBLEM



A guide to Feynman diagrams in the many-body problem

CURSE OF DIMENSIONALITY

$$\Psi_0(x_1, \ldots, x_A) = \sum_n c_n \Phi_n(x_1, \ldots, x_A)$$



$|\Phi_0\rangle$ $\quad$ $|\Phi_{h_2}^{p_3}\rangle$ $\quad$ $|\Phi_{h_2 h_3}^{p_3 p_4}\rangle$

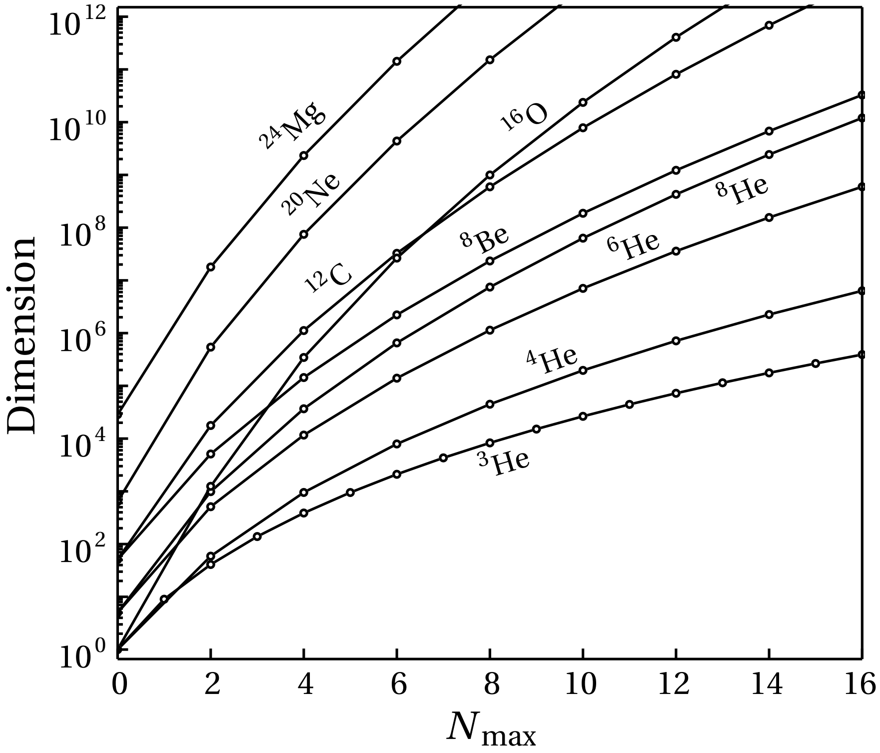
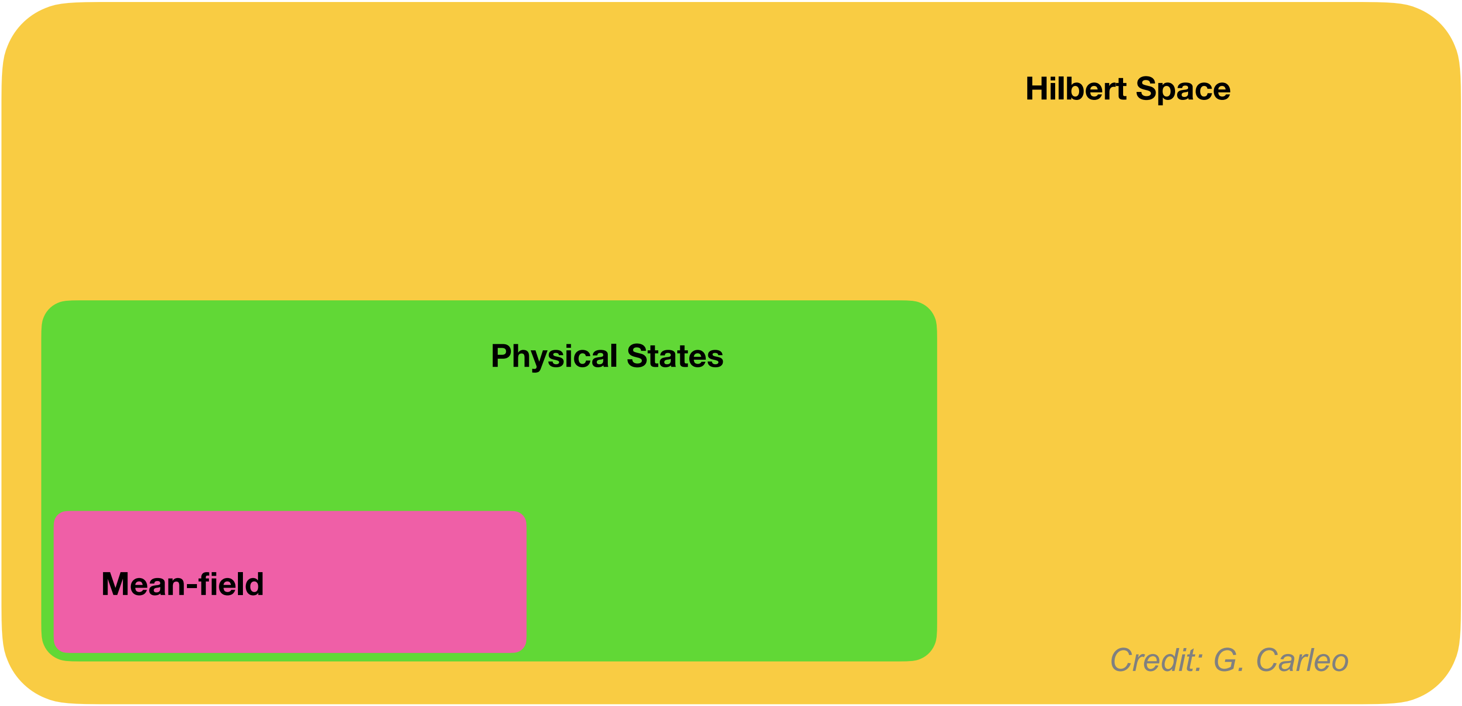


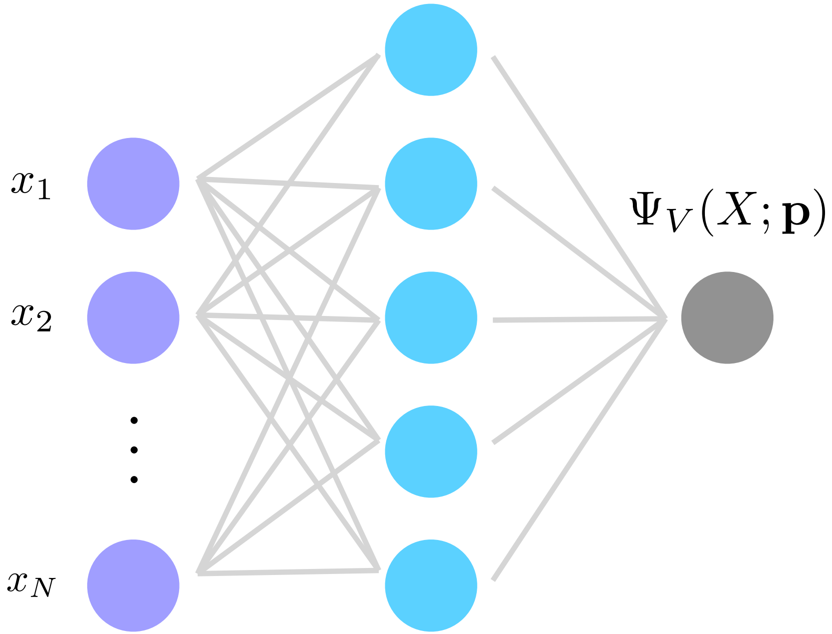
Image courtesy of Patrick Fasano

NEURAL-NETWORK QUANTUM STATES

Quantum states of physical interest have distinctive features and intrinsic structures



NEURAL-NETWORK QUANTUM STATES



$$E_V \equiv \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} > E_0$$

$$E_V \simeq \frac{1}{N} \sum_{X \in |\Psi_V(X)|^2} \frac{\langle X | H | \Psi_V \rangle}{\langle X | \Psi_V \rangle}$$

PFAFFIAN-JASTROW ANSATZ

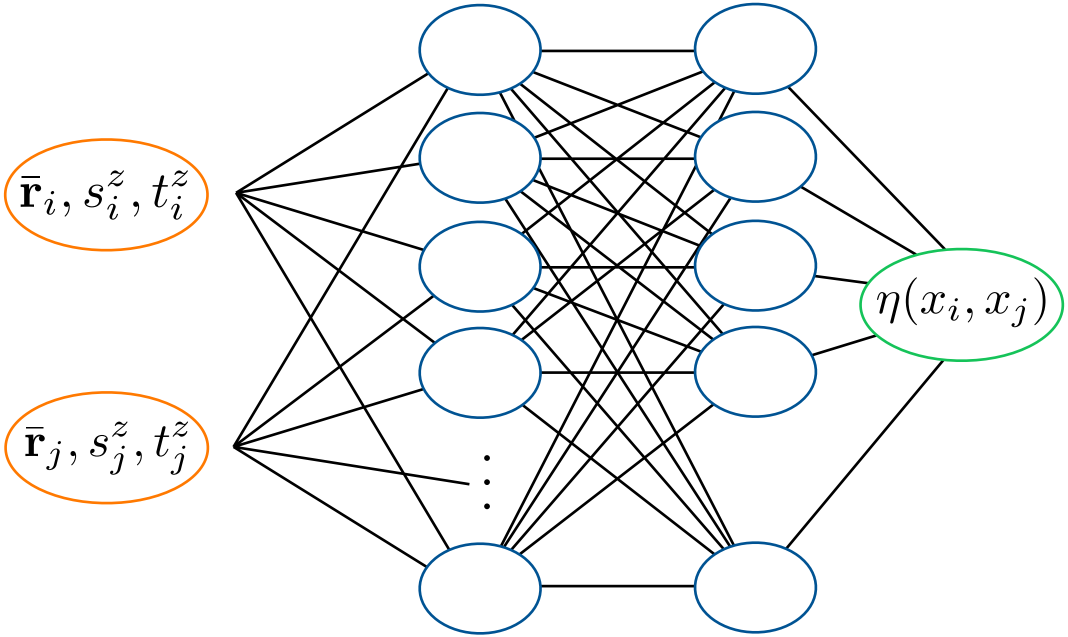
$$\Phi_{PJ}(X) = e^{J(X)} \times \text{pf}\begin{bmatrix} 0 & \phi(x_1, x_2) & \cdots & \phi(x_1, x_N) \\ \phi(x_2, x_1) & 0 & \cdots & \phi(x_2, x_N) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(x_N, x_1) & \phi(x_N, x_2) & \cdots & 0 \end{bmatrix}$$

The above matrix has to be skew-symmetric:

$$\phi(x_i, x_j) = \eta(x_i, x_j) - \eta(x_j, x_i)$$

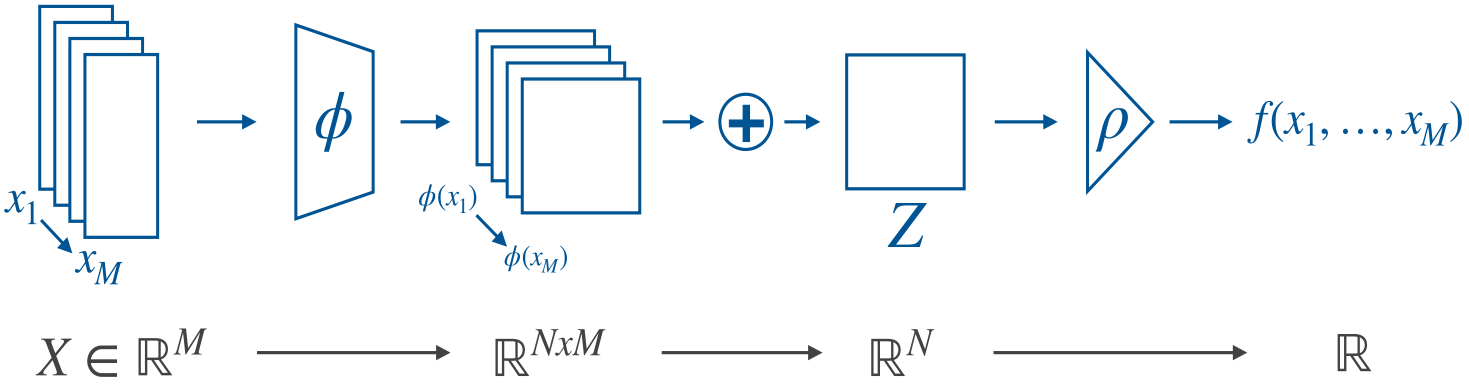
Example: $$\text{pf}\begin{bmatrix} 0 & \phi_{12} & \phi_{13} & \phi_{14} \\ -\phi_{12} & 0 & \phi_{23} & \phi_{24} \\ -\phi_{13} & -\phi_{23} & 0 & \phi_{34} \\ -\phi_{14} & -\phi_{24} & -\phi_{34} & 0 \end{bmatrix} = \phi_{12}\phi_{34} - \phi_{13}\phi_{24} + \phi_{14}\phi_{23}$$

PFAFFIAN-JASTROW ANSATZ



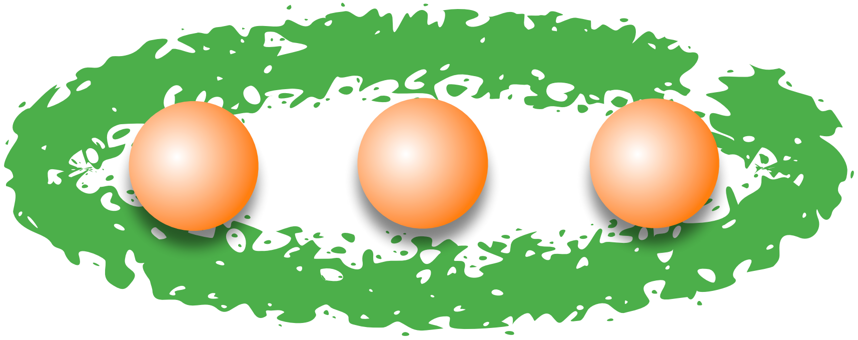
SLATER JASTROW ANSATZ

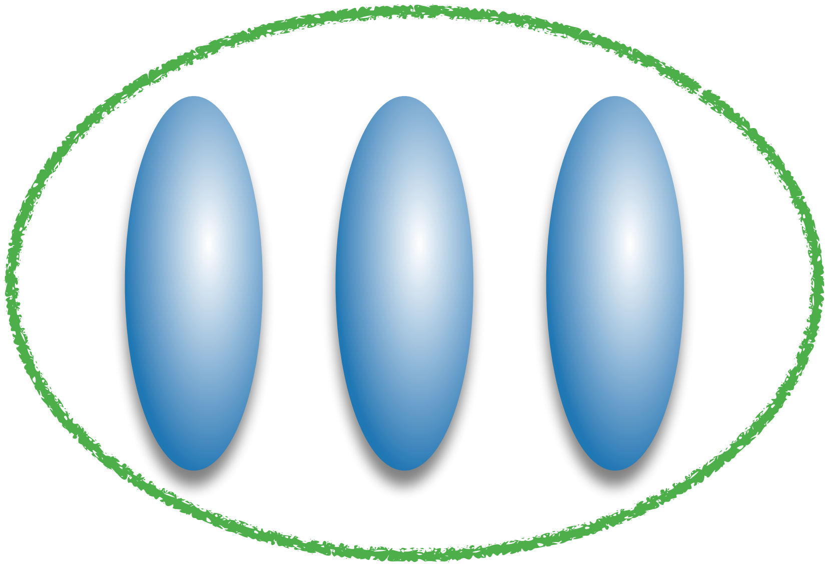
$$J(X) = \rho_F \left[\sum_i \vec{\phi}_{\mathcal{F}}(\bar{\mathbf{r}}_i, \mathbf{s}_i) \right]$$



NEURAL BACKFLOW CORRELATIONS

The nodal structure is improved with neural back-flow transformations $\mathbf{x}_i \rightarrow \phi(\mathbf{x}_i; \mathbf{x}_{j \neq i})$





Di Luo and B. K. Clark, Phys. Rev. Lett. ***122****, 226401 (2019)*

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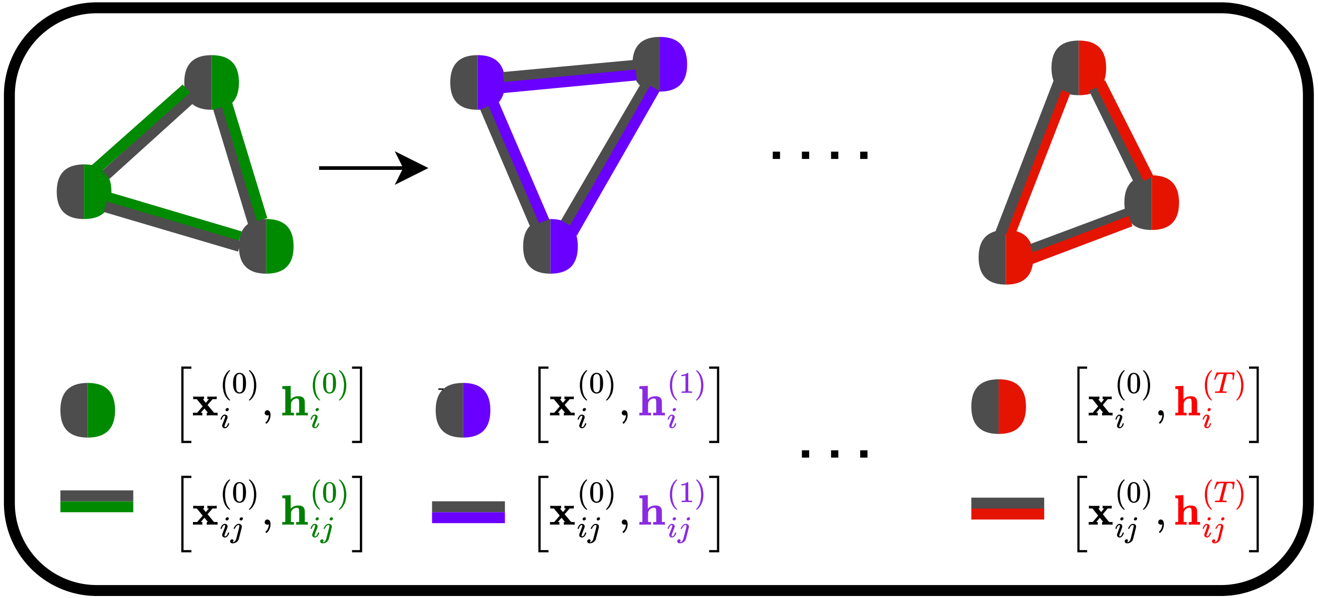


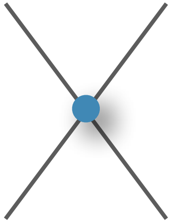
G. Pescia, et al., *Phys. Rev. B* ***110*** *(2024) 3, 035108*

“ESSENTIAL" HAMILTONIAN

Input: Hamiltonian inspired by a LO pionless-EFT expansion

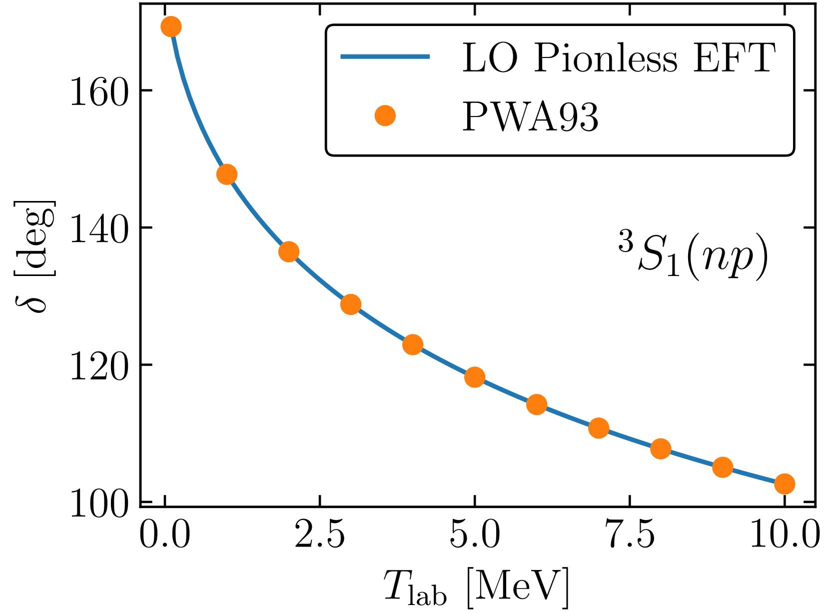
$$H_{LO} = -\sum_i \frac{\vec{\nabla}_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- NN potential fit to s-wave np scattering lengths and effective ranges



$$v_{ij}^{\mathrm{CI}} = \sum_{p=1}^{4} v^p(r_{ij}) O_{ij}^p \,,$$

$$O_{ij}^{p=1,4} = (1, \tau_{ij}, \sigma_{ij}, \sigma_{ij}\tau_{ij})$$



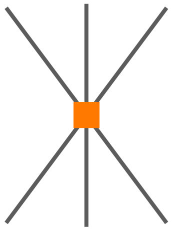
R. Schiavilla, AL, PRC **103**, 054003 (2021)

“ESSENTIAL" HAMILTONIAN

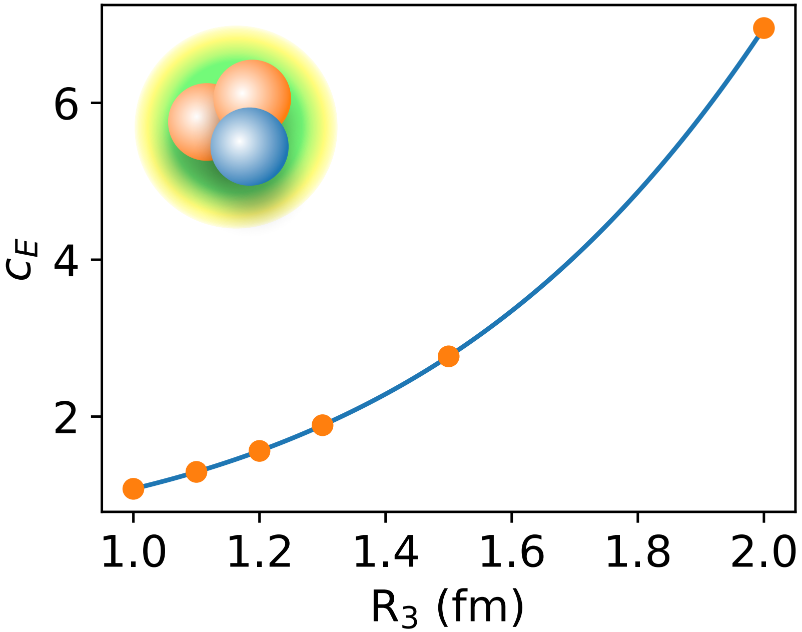
Input: Hamiltonian inspired by a LO pionless-EFT expansion

$$H_{LO} = -\sum_i \frac{\vec{\nabla}_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

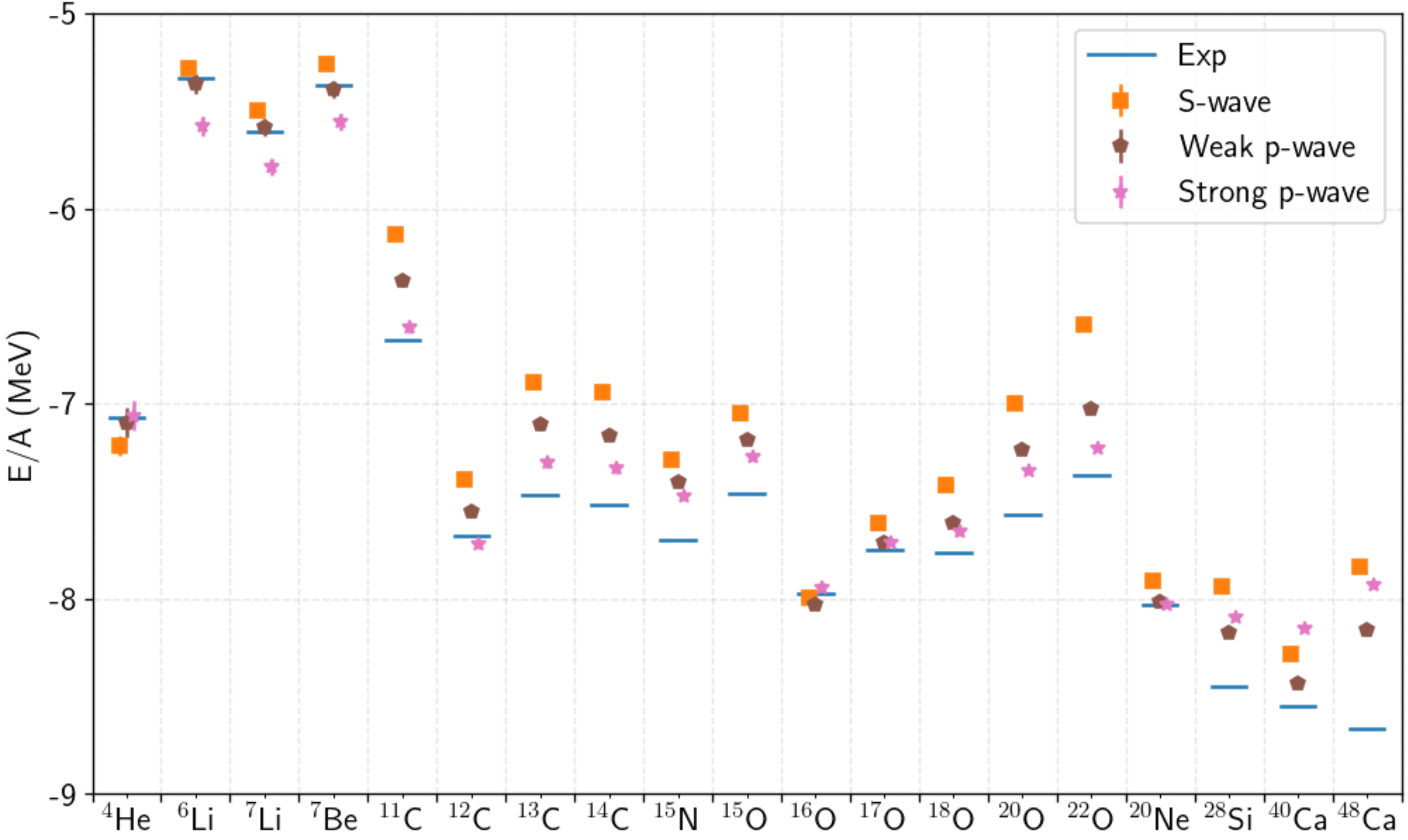
- 3NF adjusted to reproduce the energy of ^{3}H.



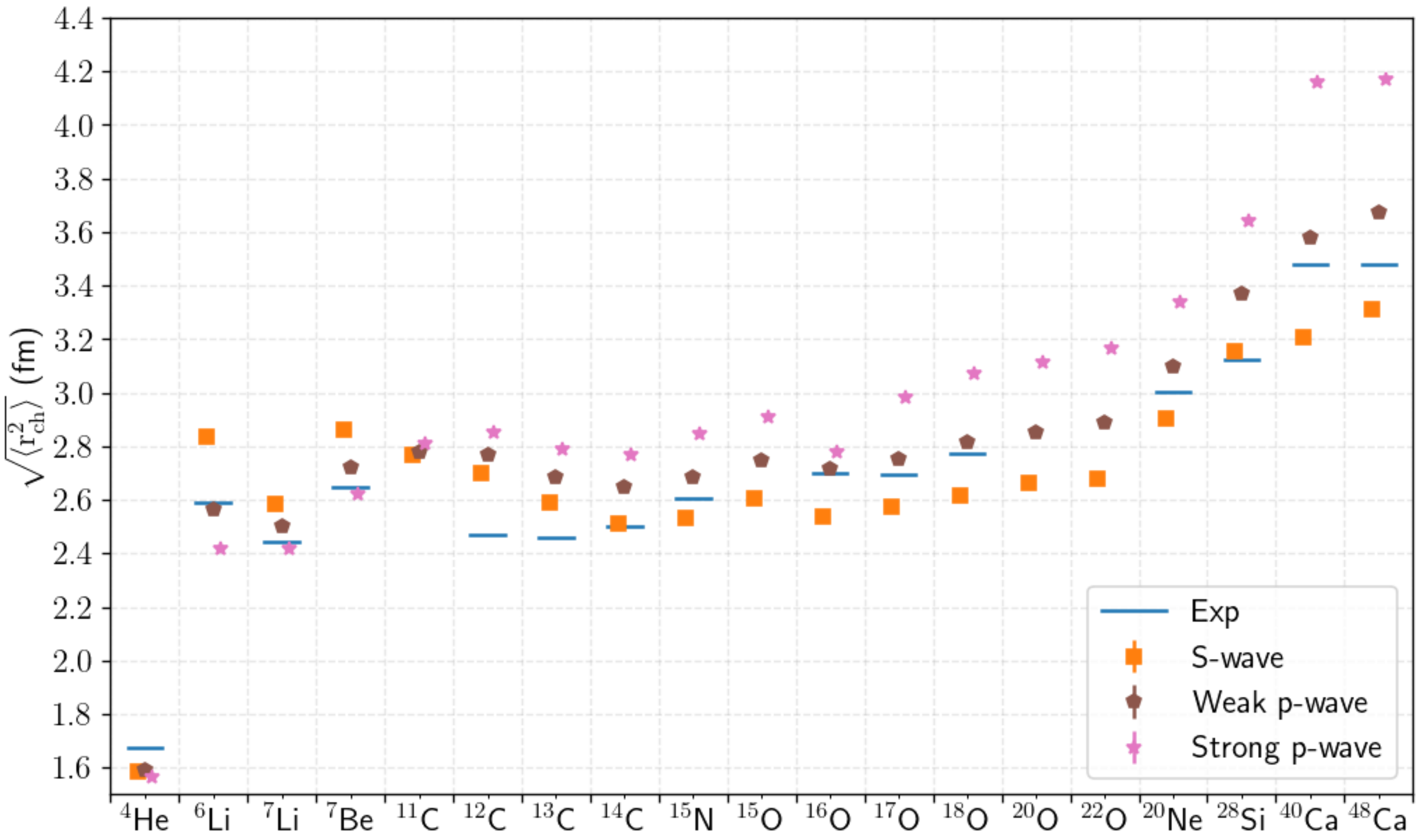
$$V_{ijk} \propto c_E \sum_{\mathrm{cyc}} e^{-(r_{ij}^2 + r_{jk}^2)/R_3^2}$$



*R. Schiavilla, AL, PRC **103**, 054003 (2021)*



E/A (MeV)
-5
-6
-7
-8
-9
Exp
S-wave
Weak p-wave
Strong p-wave
^{4}He
^{6}Li
^{7}Li
^{7}Be
^{11}C
^{12}C
^{13}C
^{14}C
^{15}N
^{15}O
^{16}O
^{17}O
^{18}O
^{20}O
^{22}O
^{20}Ne
^{28}Si
^{40}Ca
^{48}Ca



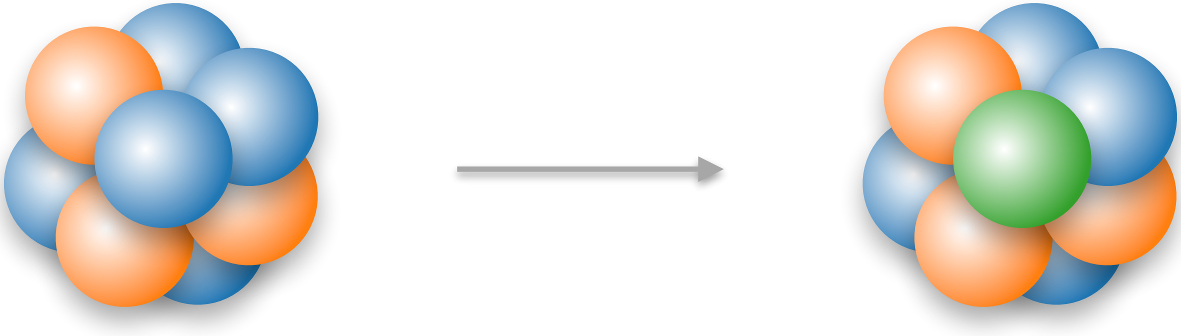
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
$\sqrt{\langle r_{ch}^2 \rangle}$ (fm)
^{4}He
^{6}Li
^{7}Li
^{7}Be
^{11}C
^{12}C
^{13}C
^{14}C
^{15}N
^{15}O
^{16}O
^{17}O
^{18}O
^{20}O
^{22}O
^{20}Ne
^{28}Si
^{40}Ca
^{48}Ca
Exp
S-wave
Weak p-wave
Strong p-wave

Andrea Di Donna, Ph.D. student at the University of Trento, in collaboration with Prof. Francesco Pederiva.

NUCLEI WITH A STRANGE PARTICLE

Hypernuclei: bound state between an ordinary nucleus with one (or more) hyperons

Our work: consider single-Λ hypernuclei to begin with



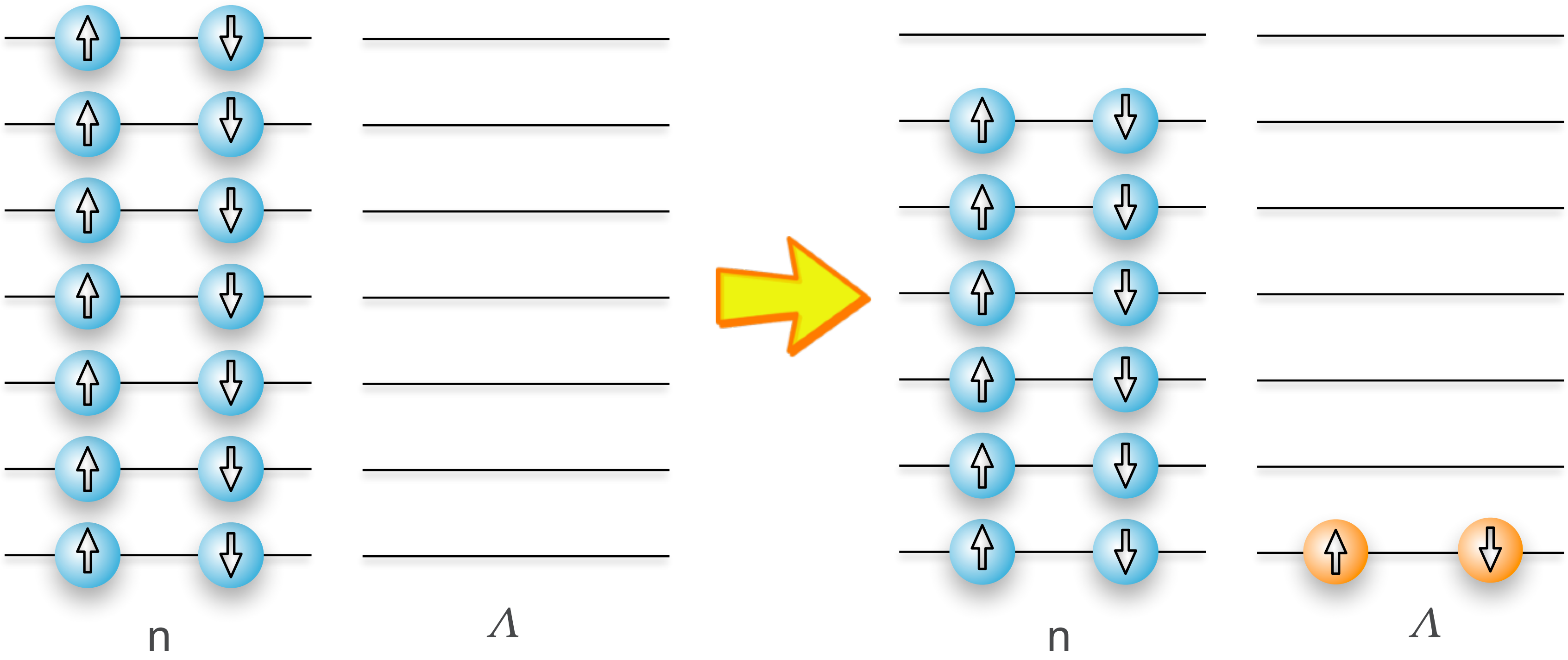
$^{12}\mathrm{C}$

6 protons, 6 neutrons

$^{12}_{\Lambda}\mathrm{C}$

6 protons, 5 neutrons, 1 lambda

THE HYPERON PUZZLE



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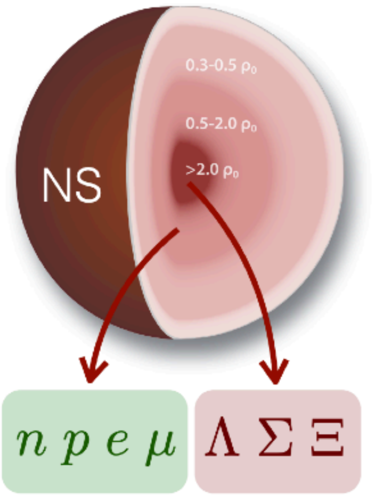
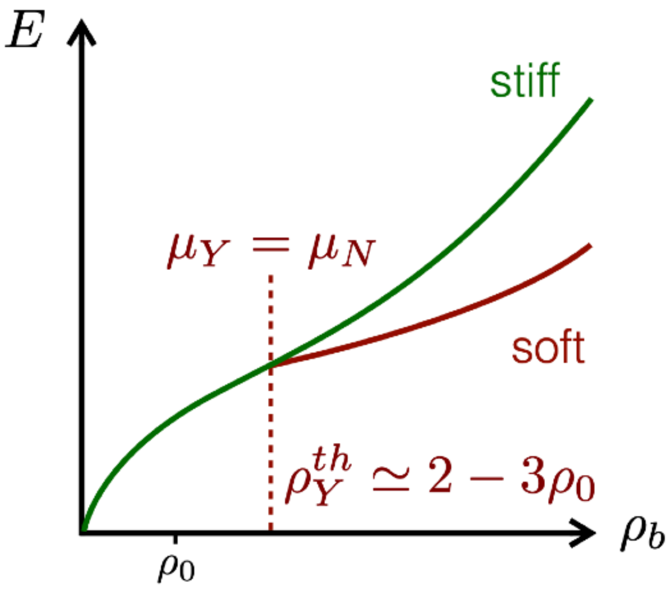
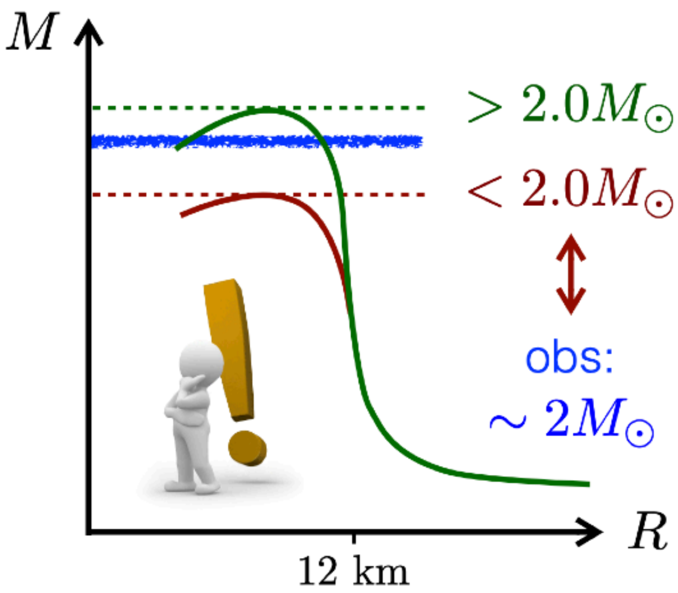
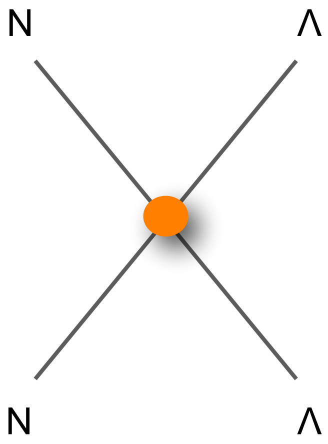






HYPERON-NUCLEON POTENTIAL

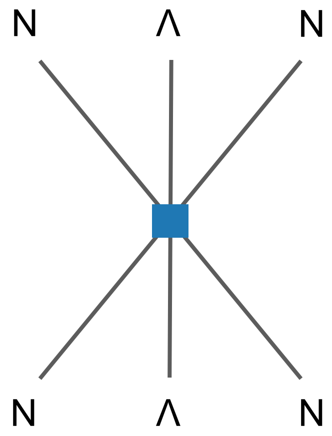


$$V_{\mathrm{N}\Lambda} = \sum_{\mathrm{IS}} \tilde{C}_{\lambda}^{\mathrm{IS}} \sum_{i} \mathcal{P}_{\mathrm{IS}}\,(i\Lambda)\,\delta_{\lambda}(\mathbf{r}_{i\Lambda})$$

- T=1/2, S=1
- T=1/2, S=0

Channel	IS	C_{IS} (MeV)	λ_{IS} (fm^{-1})	a_{exp} (fm)	r_{exp} (fm)
np	10	-31.0633	1.10117	-23.7148(43)	2.750(18)
	01	-68.3747	1.30512	5.4112(15)	1.7436(19)
pΛ	$\frac{1}{2}$0	-33.5417	1.54720	-1.80	2.80
	$\frac{1}{2}$1	-25.3115	1.41379	-1.60	3.30

HYPERON-NUCLEON-NUCLEON POTENTIAL

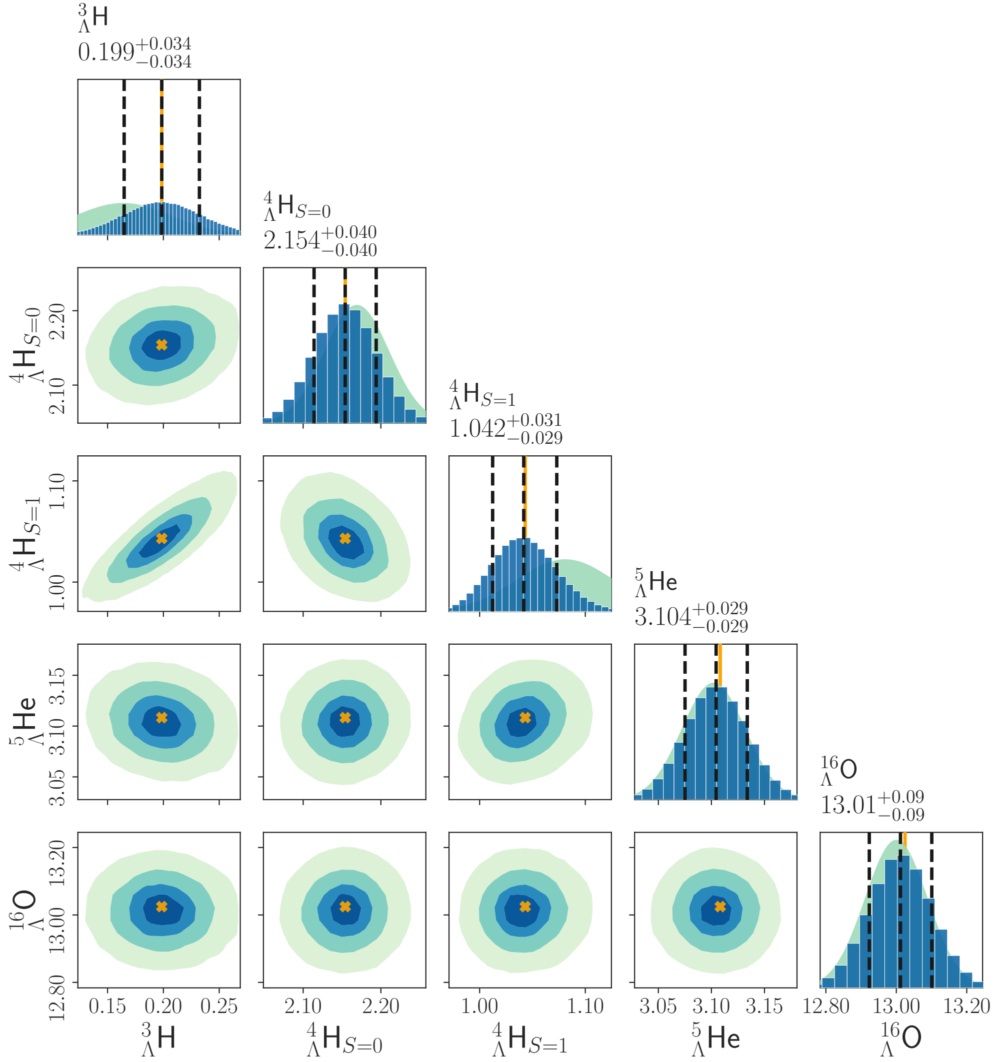


$$V_{\mathrm{NN}\Lambda} = \sum_{\mathrm{IS}} \tilde{D}_{\lambda}^{\mathrm{IS}} \sum_{i<j} \mathcal{Q}_{\mathrm{IS}}(ij\Lambda)\delta_{\lambda}(\vec{r}_{i\Lambda})\delta_{\lambda}(\vec{r}_{j\Lambda}).$$

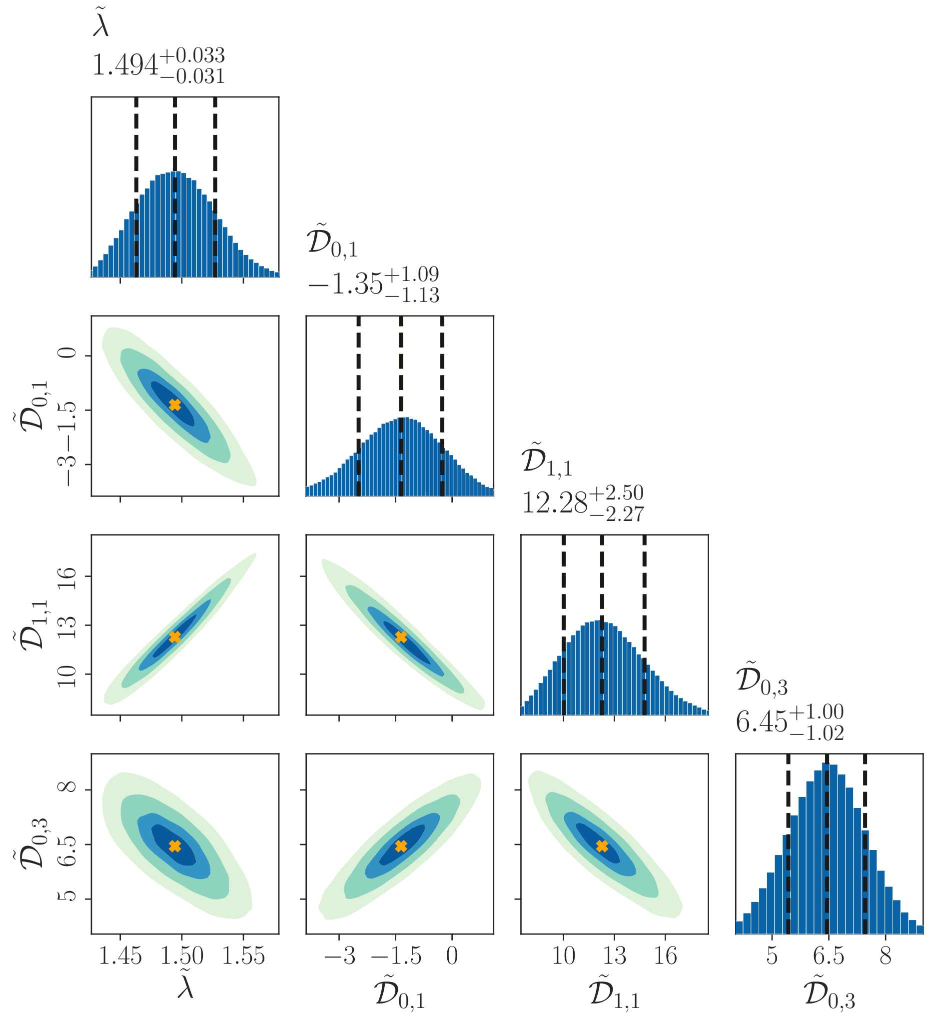
- T=0, S=1/2
- T=0, S=3/2
- T=1, S=1/2

Fit to hyper nuclear separation energies

System	B_Λ (MeV)	σ (MeV)	IS Dependence
${}^{3}_{\Lambda}\mathrm{H}$	0.164	0.043	$(0\frac{1}{2})$
${}^{4}_{\Lambda}\mathrm{H}_{S=0}$	2.169	0.042	$(0\frac{1}{2})$, $(1\frac{1}{2})$
${}^{4}_{\Lambda}\mathrm{H}_{S=1}$	1.081	0.046	$(0\frac{1}{2})$, $(1\frac{1}{2})$, $(0\frac{3}{2})$
${}^{5}_{\Lambda}\mathrm{He}$	3.102	0.030	$(0\frac{1}{2})$, $(1\frac{1}{2})$, $(0\frac{3}{2})$
${}^{16}_{\Lambda}\mathrm{O}$	13.00	0.089	$(0\frac{1}{2})$, $(1\frac{1}{2})$, $(0\frac{3}{2})$



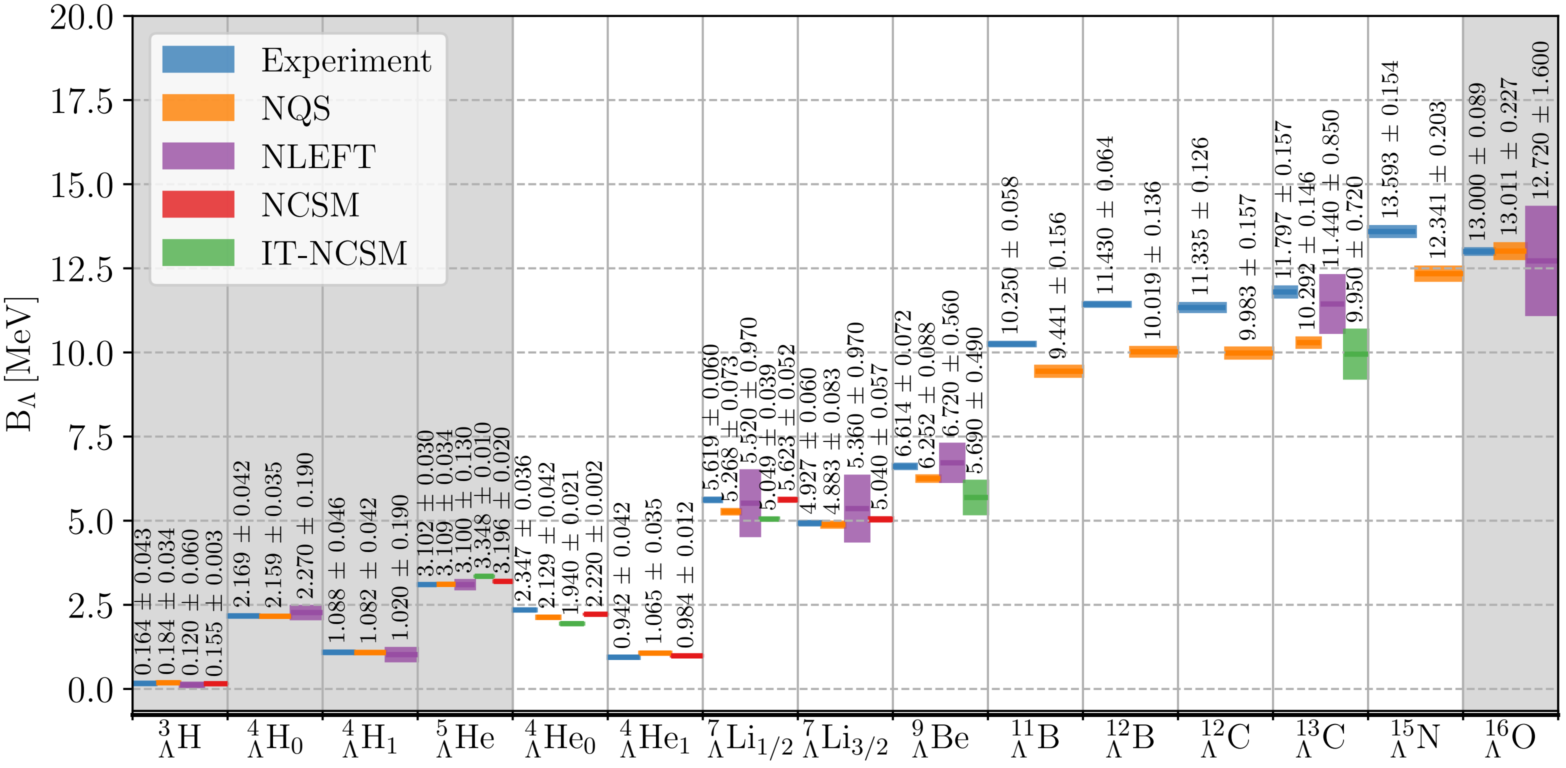
$^{3}_{\Lambda}$H
$0.199^{+0.034}_{-0.034}$
$^{4}_{\Lambda}\mathrm{H}_{S=0}$
$2.154^{+0.040}_{-0.040}$
$^{4}_{\Lambda}\mathrm{H}_{S=1}$
$1.042^{+0.031}_{-0.029}$
$^{5}_{\Lambda}$He
$3.104^{+0.029}_{-0.029}$
$^{16}_{\Lambda}$O
$13.01^{+0.09}_{-0.09}$
2.10 2.20
1.00 1.10
3.05 3.10 3.15
12.80 13.00 13.20
0.15 0.20 0.25
2.10 2.20
1.00 1.10
3.05 3.10 3.15
12.80 13.00 13.20



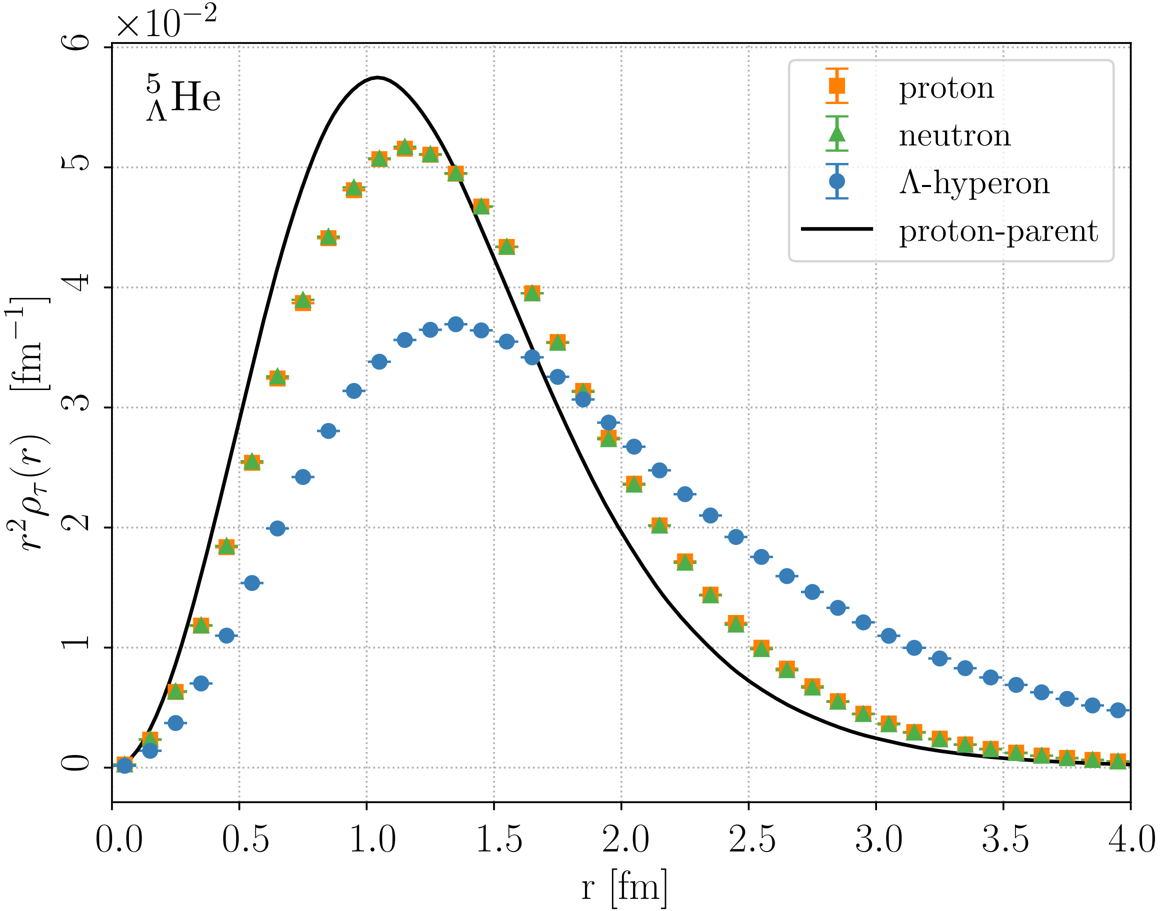
$\tilde{\lambda}$
$1.494^{+0.033}_{-0.031}$
$\tilde{\mathcal{D}}_{0,1}$
$-1.35^{+1.09}_{-1.13}$
$\tilde{\mathcal{D}}_{1,1}$
$12.28^{+2.50}_{-2.27}$
$\tilde{\mathcal{D}}_{0,3}$
$6.45^{+1.00}_{-1.02}$
1.45 1.50 1.55
−3 −1.5 0
10 13 16
5 6.5 8

ESSENTIAL HAMILTONIAN

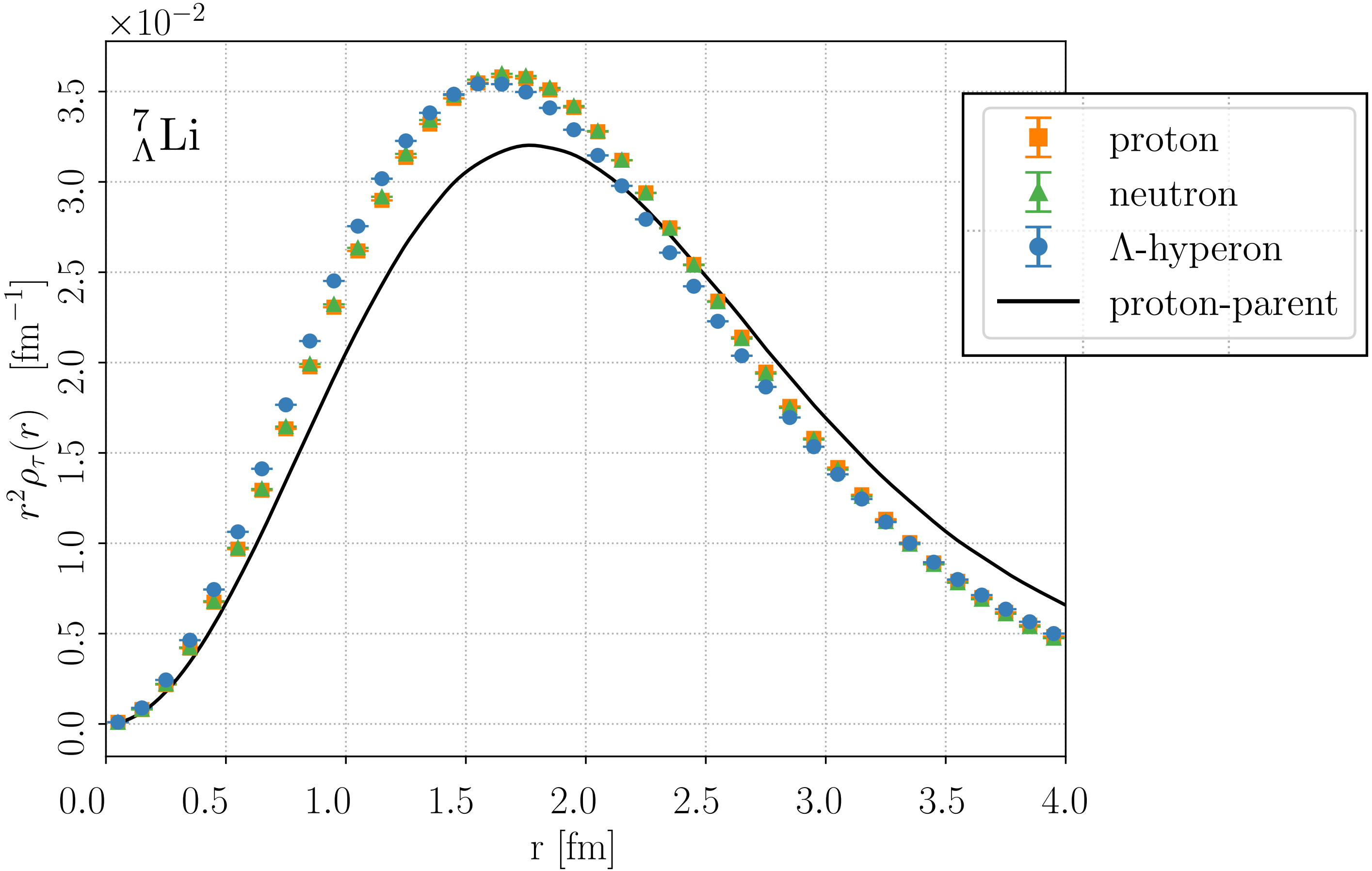
- Good agreement with experimental data, intermediate hypernuclei somewhat under-bound.



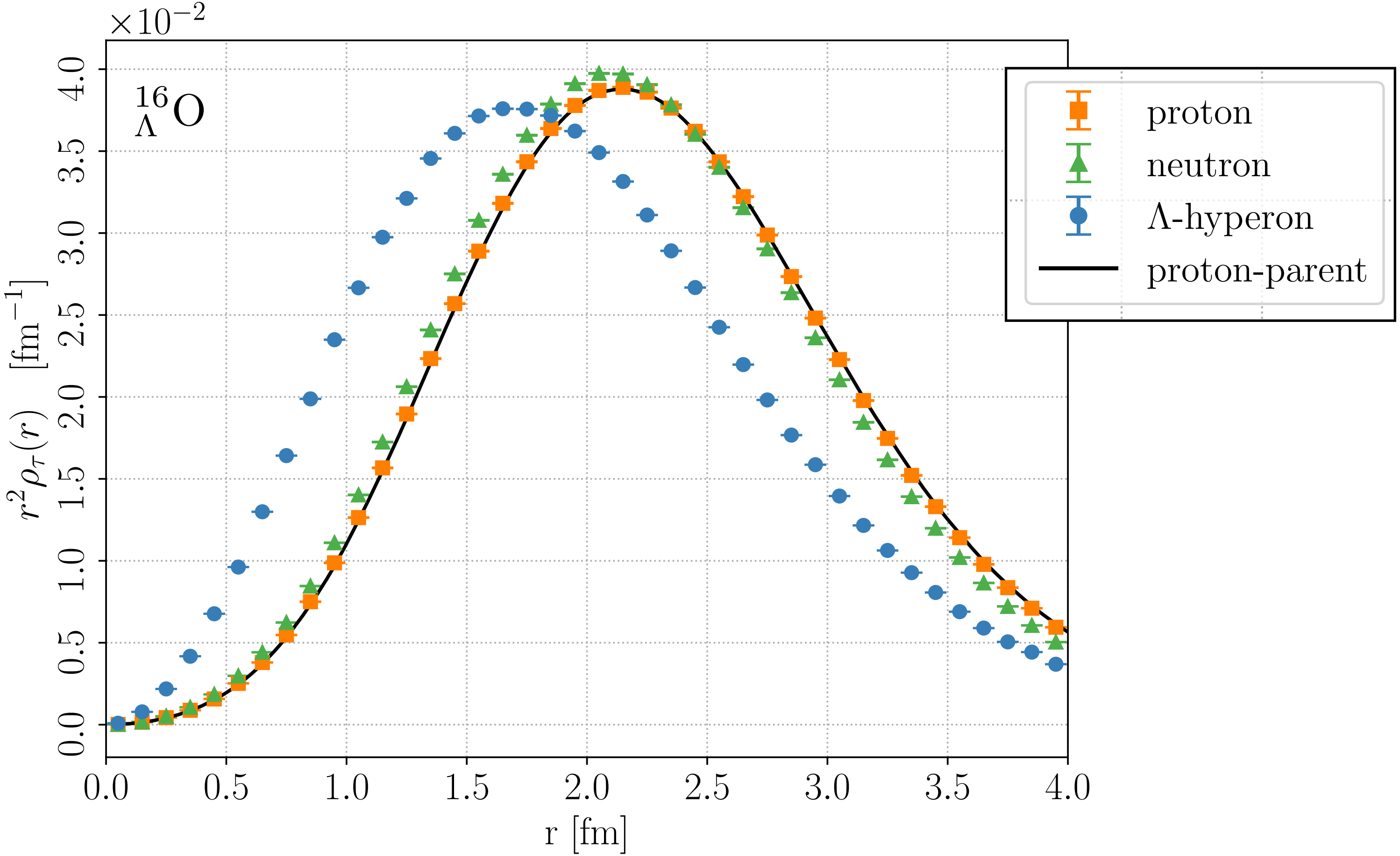
SINGLE-PARTICLE DENSITIES



SINGLE-PARTICLE DENSITIES



SINGLE-PARTICLE DENSITIES



CONCLUSIONS

- First application of NQS to “strange” systems, good agreement with experimental separation energy; we *distilled the essential elements of hypernuclear binding*
- “Easy” to reach A=40 on Argonne-Polaris. Exascale resources needed to compute heavy hypernuclei with more sophisticated interactions.
- Formalism directly applicable to study the onset of hyperons in neutron-star matter

PERSPECTIVES

- Use equivariant neural-network to target state with given quantum numbers (J in particular).
    - ➡ Efficient calculation of excited states
- Implement high-resolution phenomenological and chiral-EFT interactions.
    - ➡ Neutron-star matter equation of state
- Dynamical observables
    - ➡ Linear response with integral-transform techniques
    - ➡ Real-time quantum dynamics with the TdVMC

THANK YOU