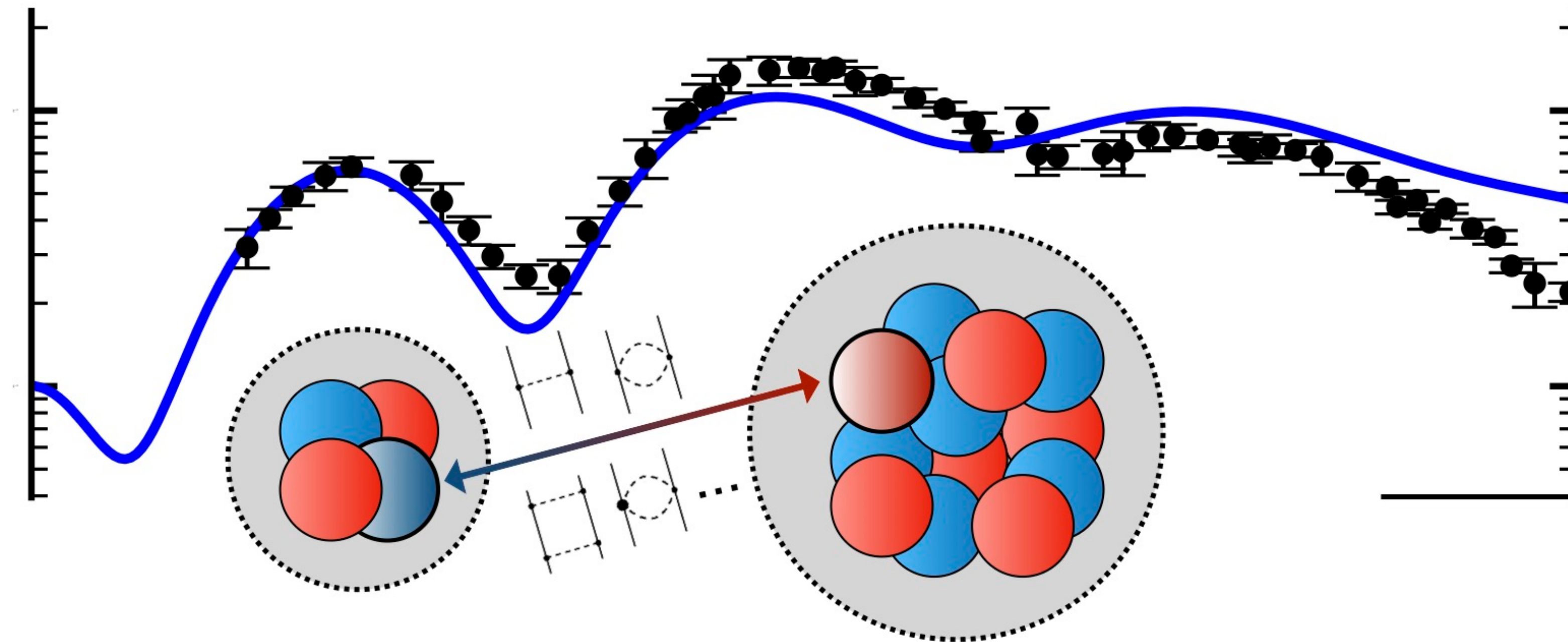


Microscopic Optical Potentials

New developments and achievements

Paolo Finelli, UniBO and INFN



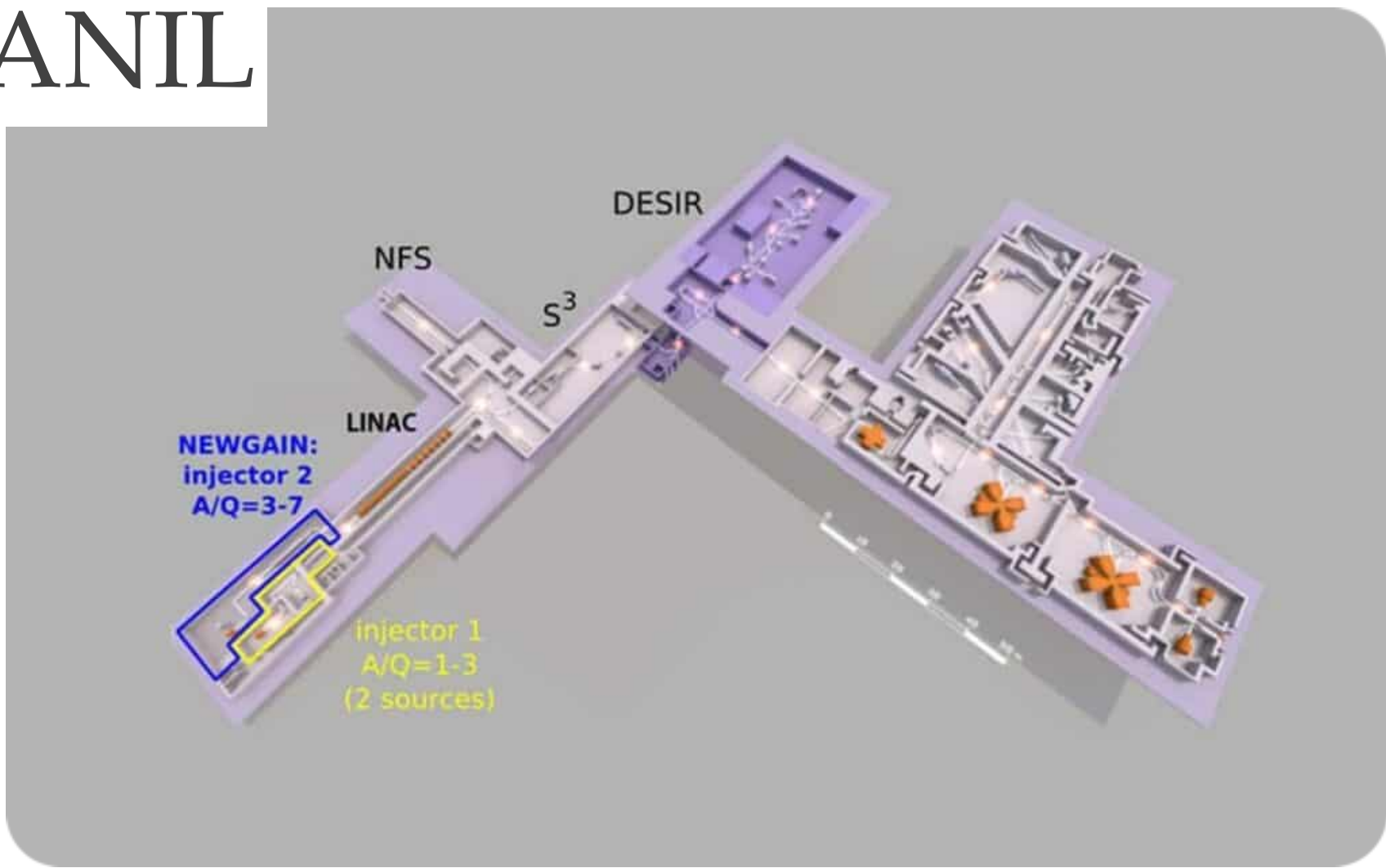
Collaborators

- Matteo Vorabbi
- Carlotta Giusti
- Carlo Barbieri
- Vittorio Somà
- Peter Navratil
- Michael Gennari

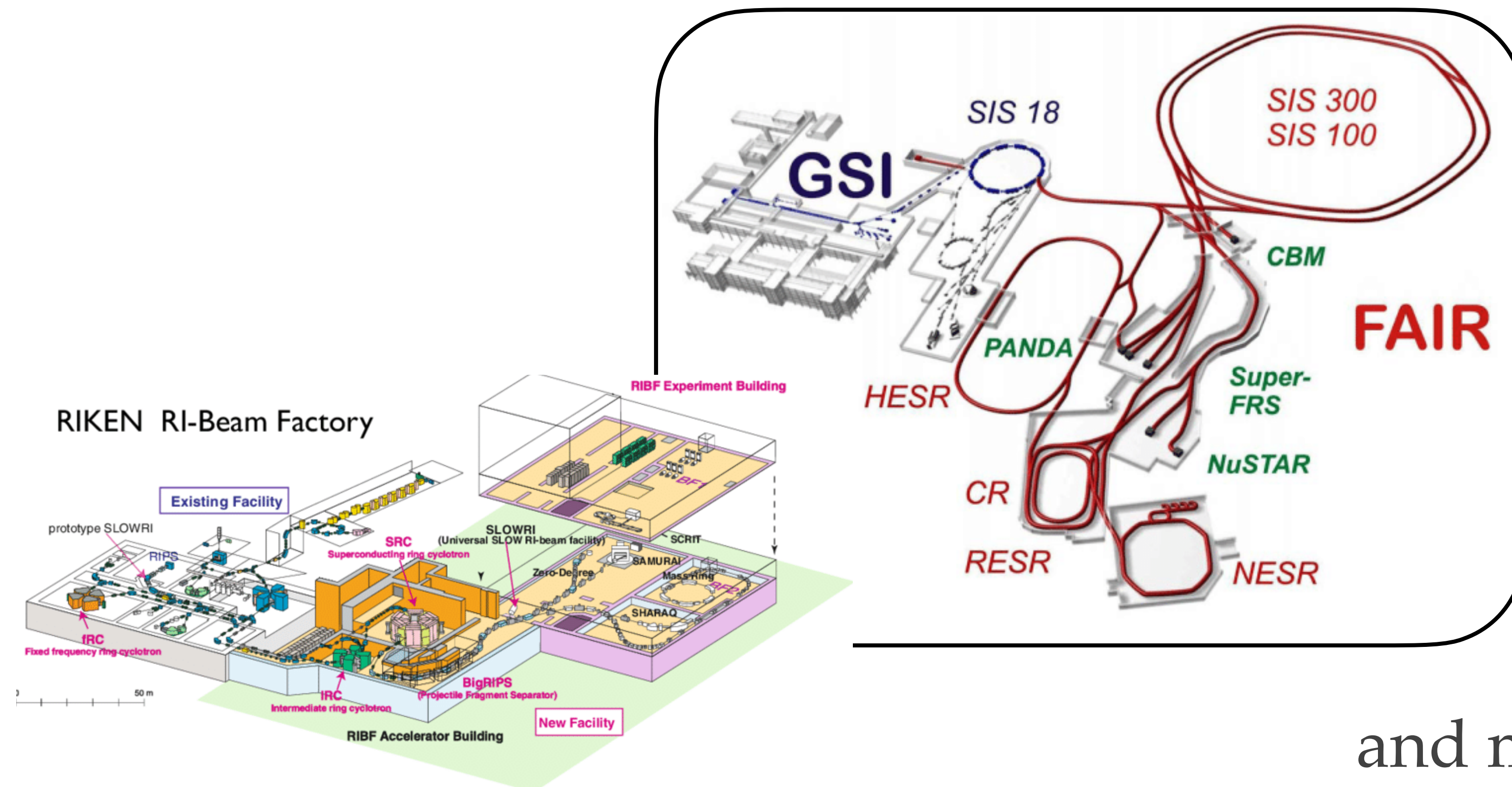
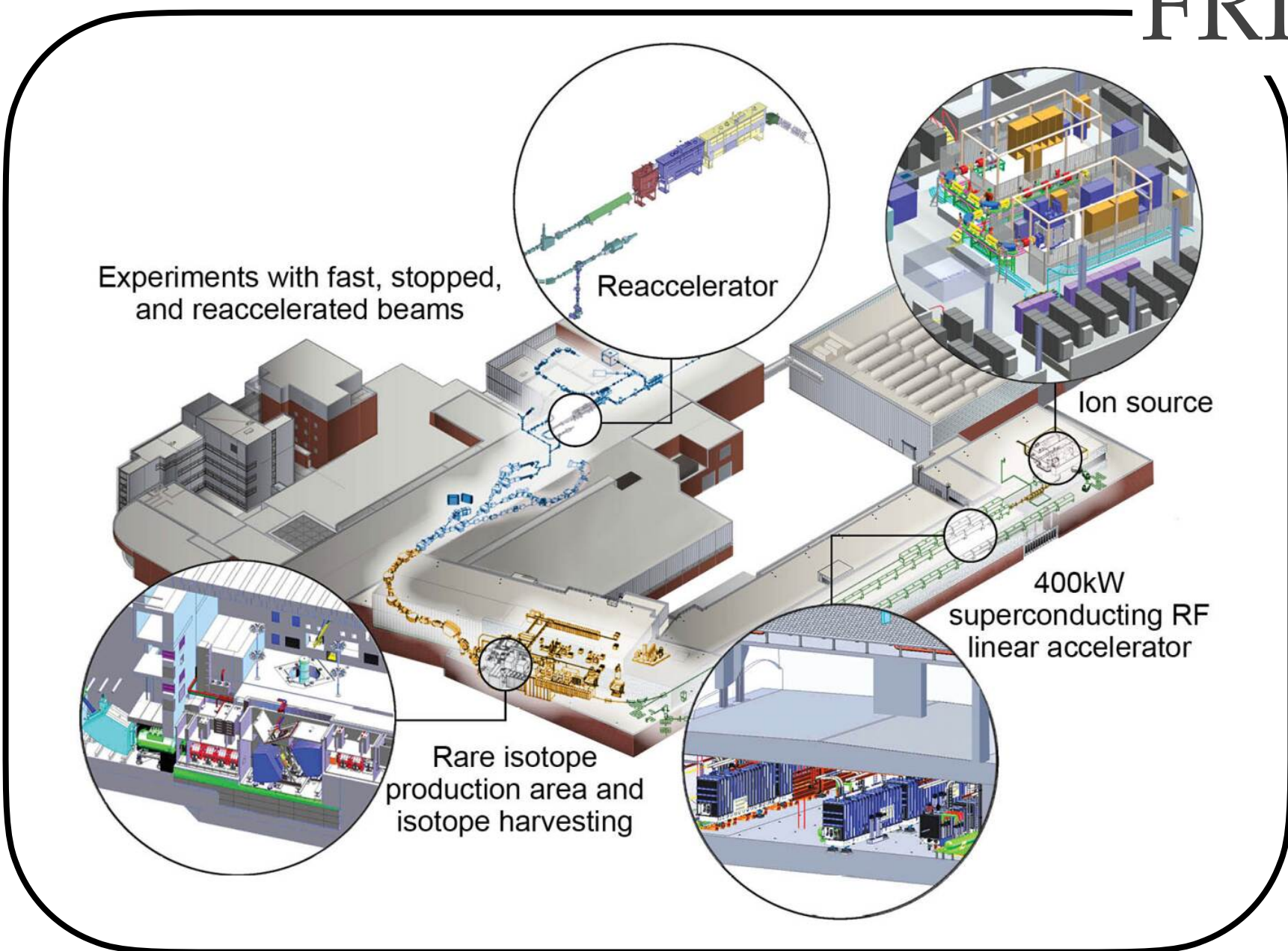
Motivation

Develop a microscopic framework to perform reliable predictions for nuclear reactions (elastic, inelastic, transfer,...) despite the nature of the projectiles (protons, neutrons, antiprotons, ions,...)

GANIL



FRIB



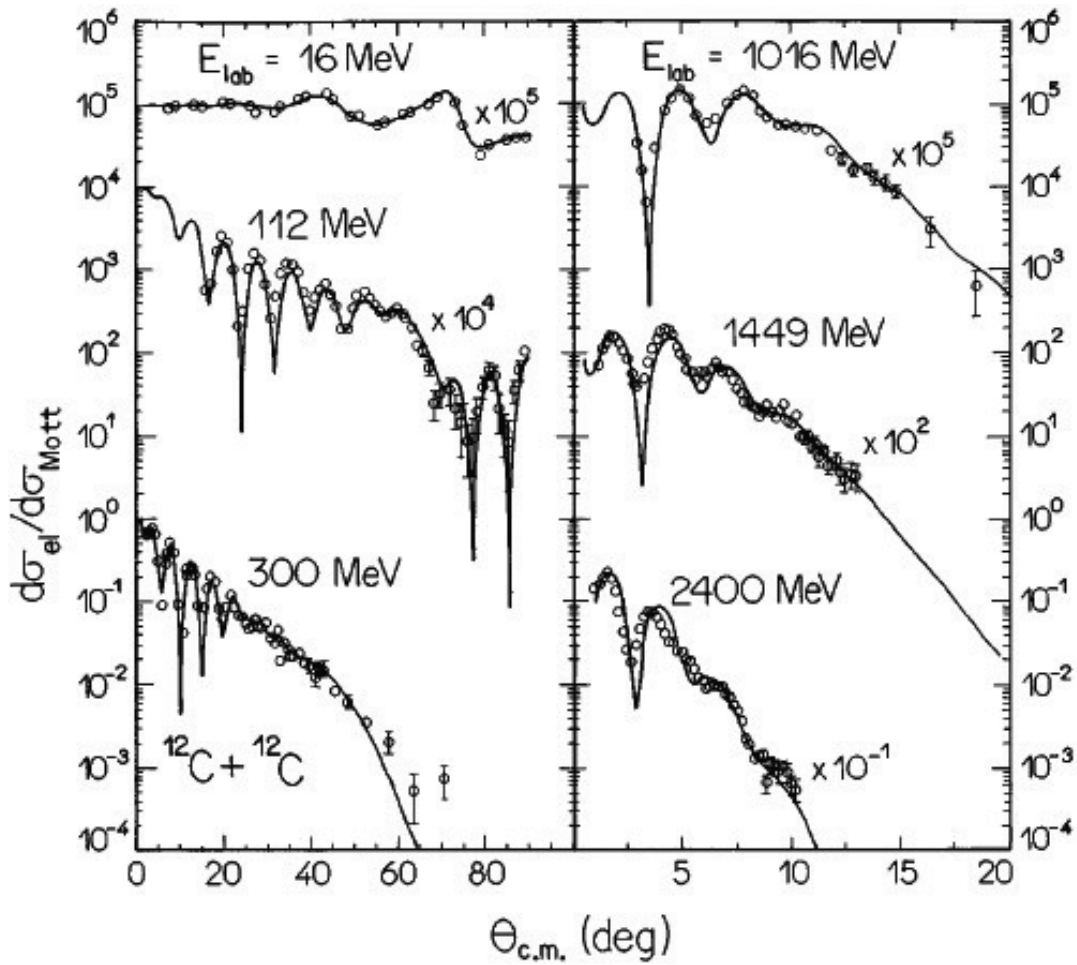
and many more experimental facilities...

Optical potentials

Phenomenological approach

- ✓ Very successful (large community)
- ✓ Easy to use (but not always)
- ✓ Open-access codes (TALYS, FRESCO, EMPIRE, FLUKA,...)

? How to extrapolate to exotic nuclei
? How (and where) to improve the description of data

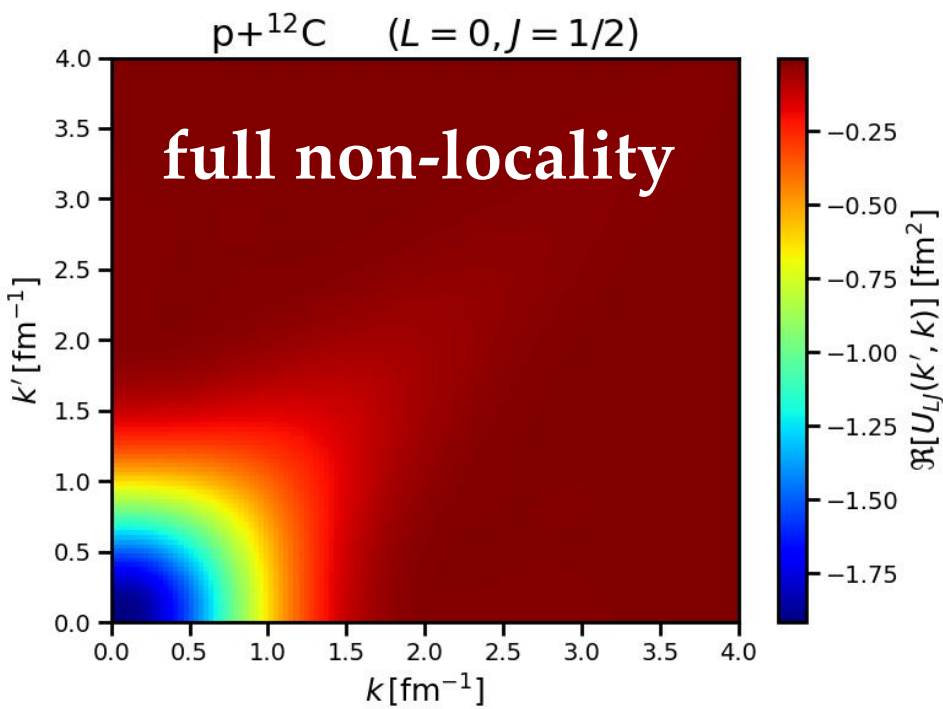
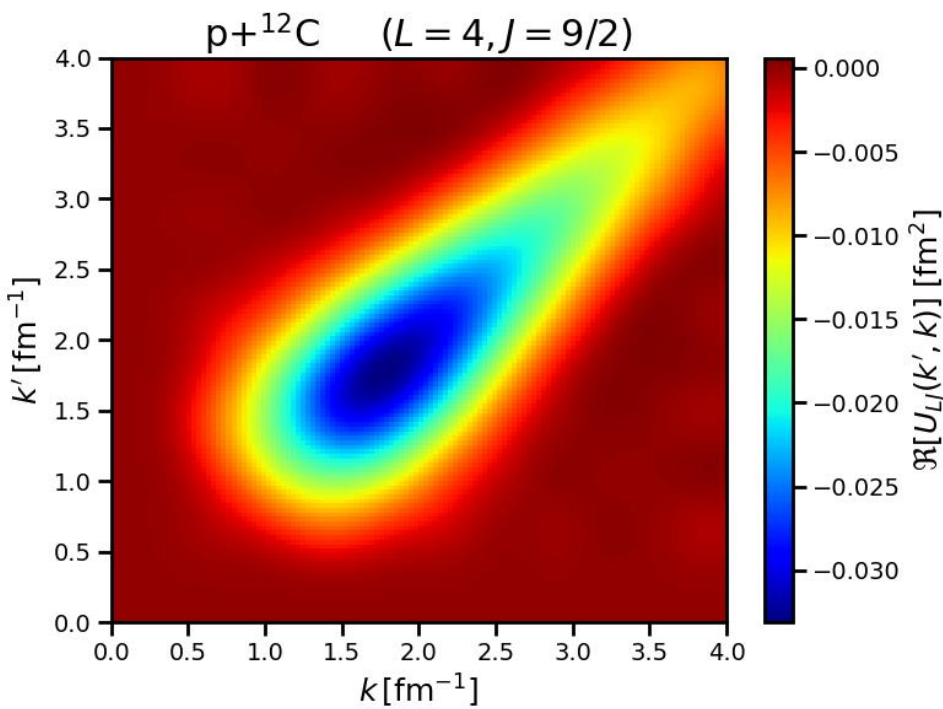


$^{12}\text{C} + ^{12}\text{C}$

Microscopic approach

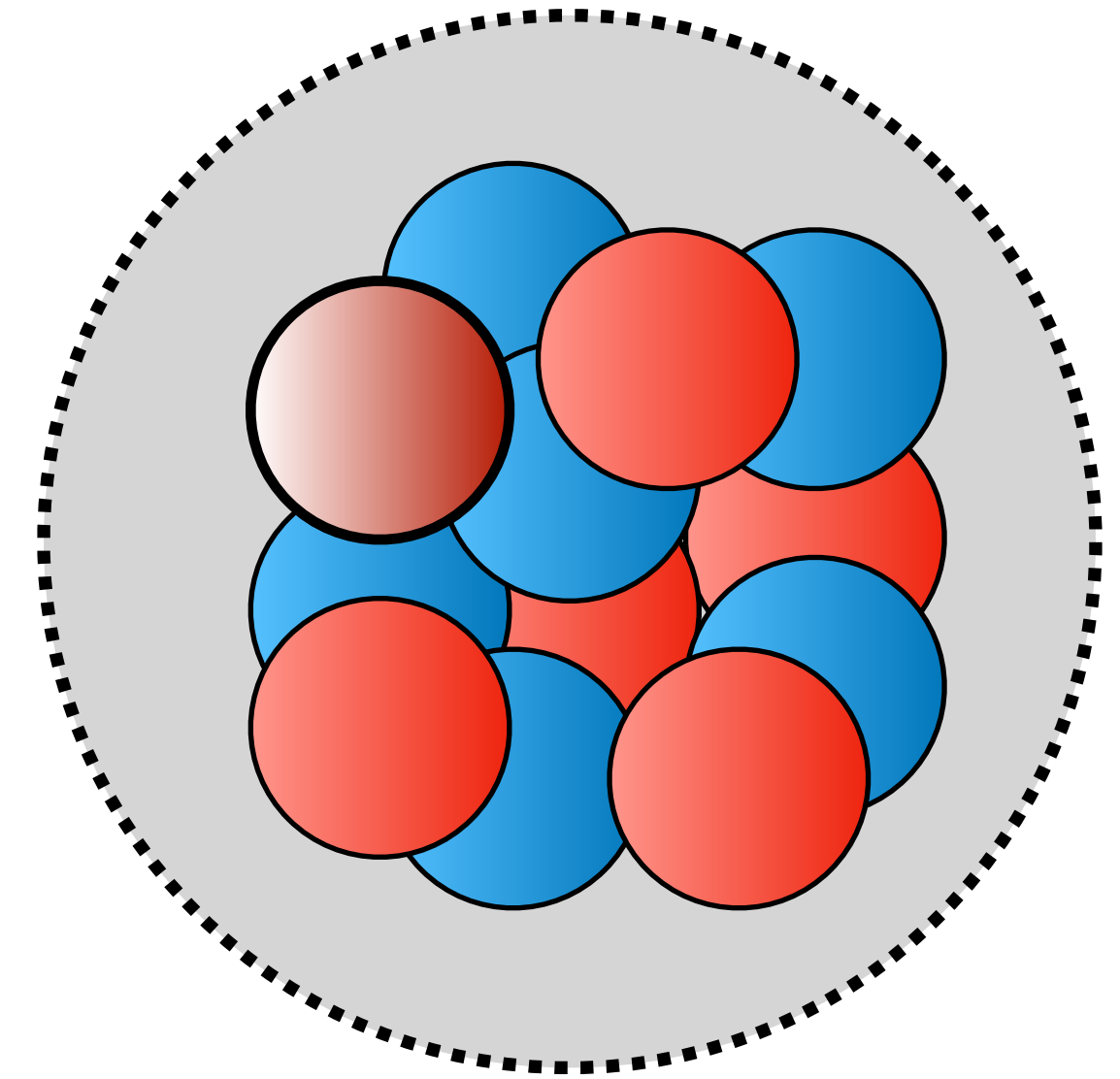
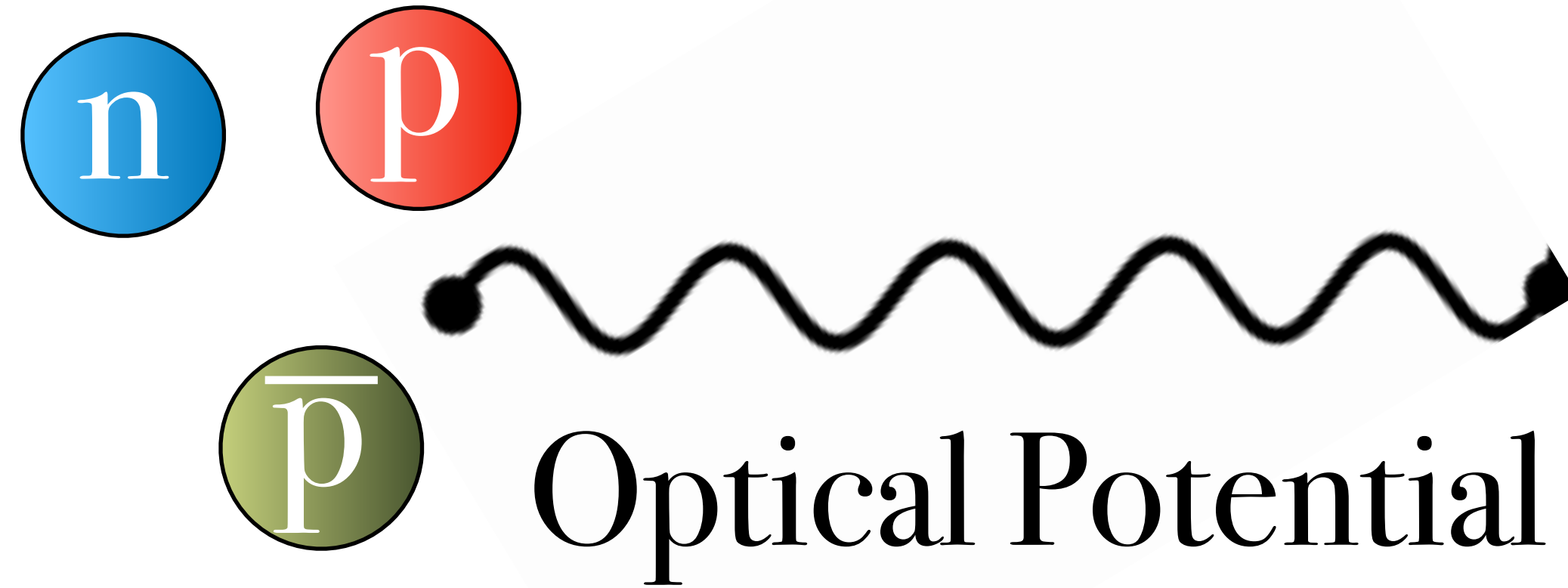
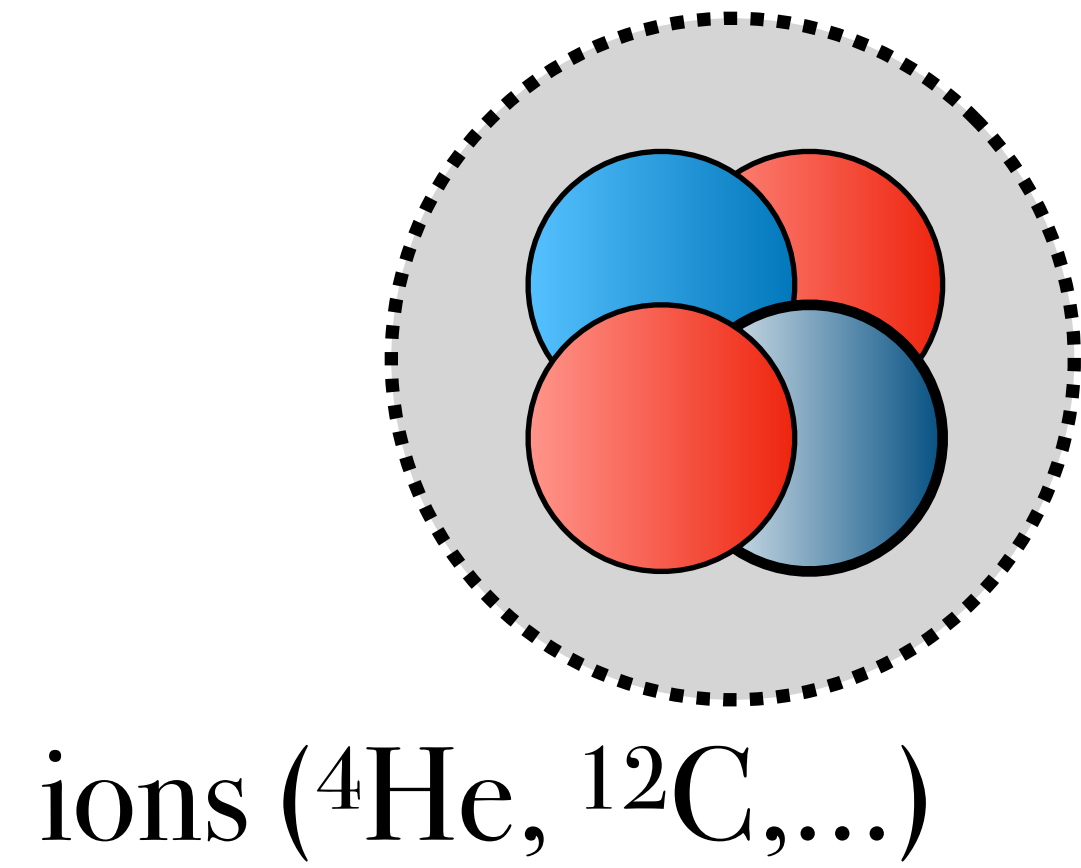
- ⚠ Difficult to manage
- ⚠ No open-access codes so far (only to generate NN potentials)
- ⚠ High-performance computers needed (depends on the reaction)

- ✓ Parameter free predictions (no fit)
- ✓ Reliable when extrapolate to exotic nuclei
- ✓ How (and where) to improve the description of data guided by theory (maybe difficult)

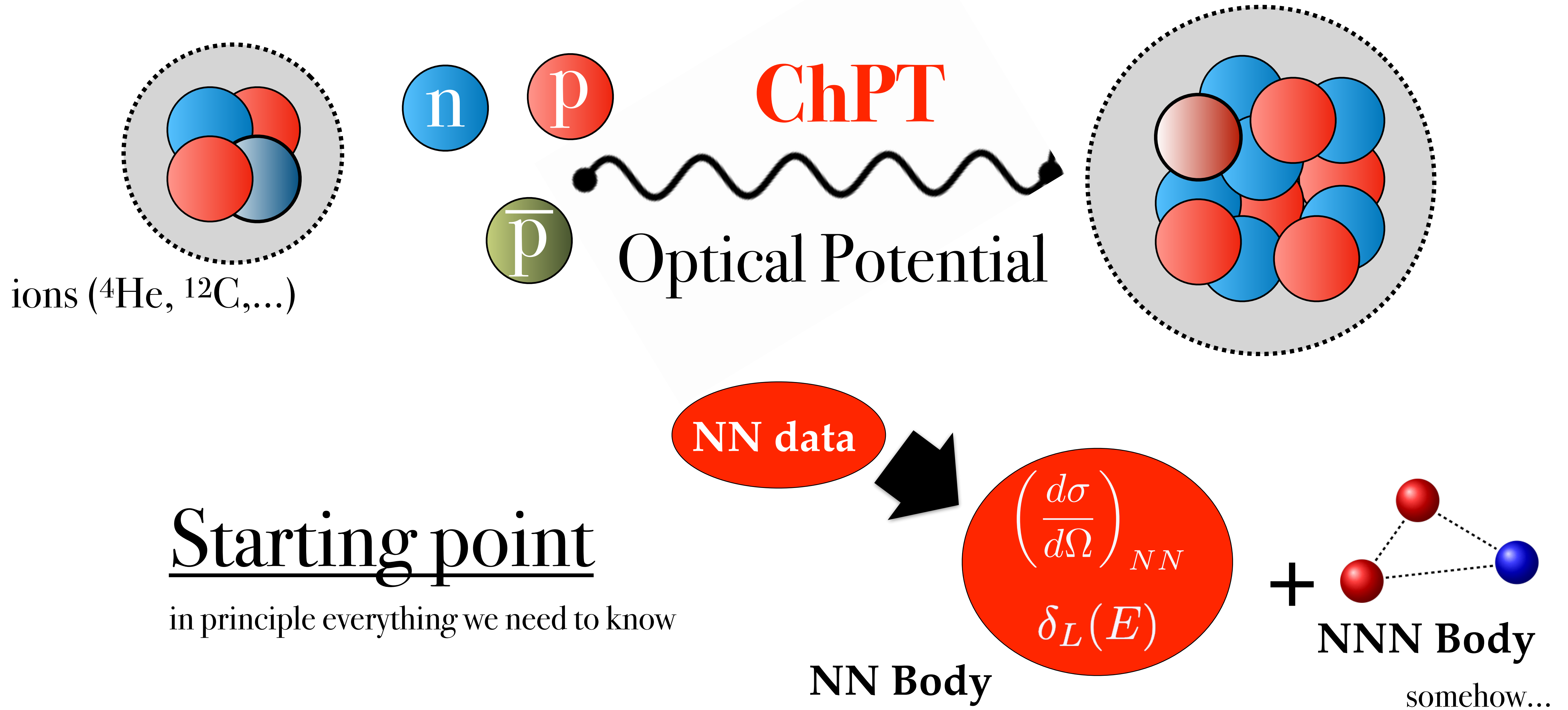


$p + ^{12}\text{C}$

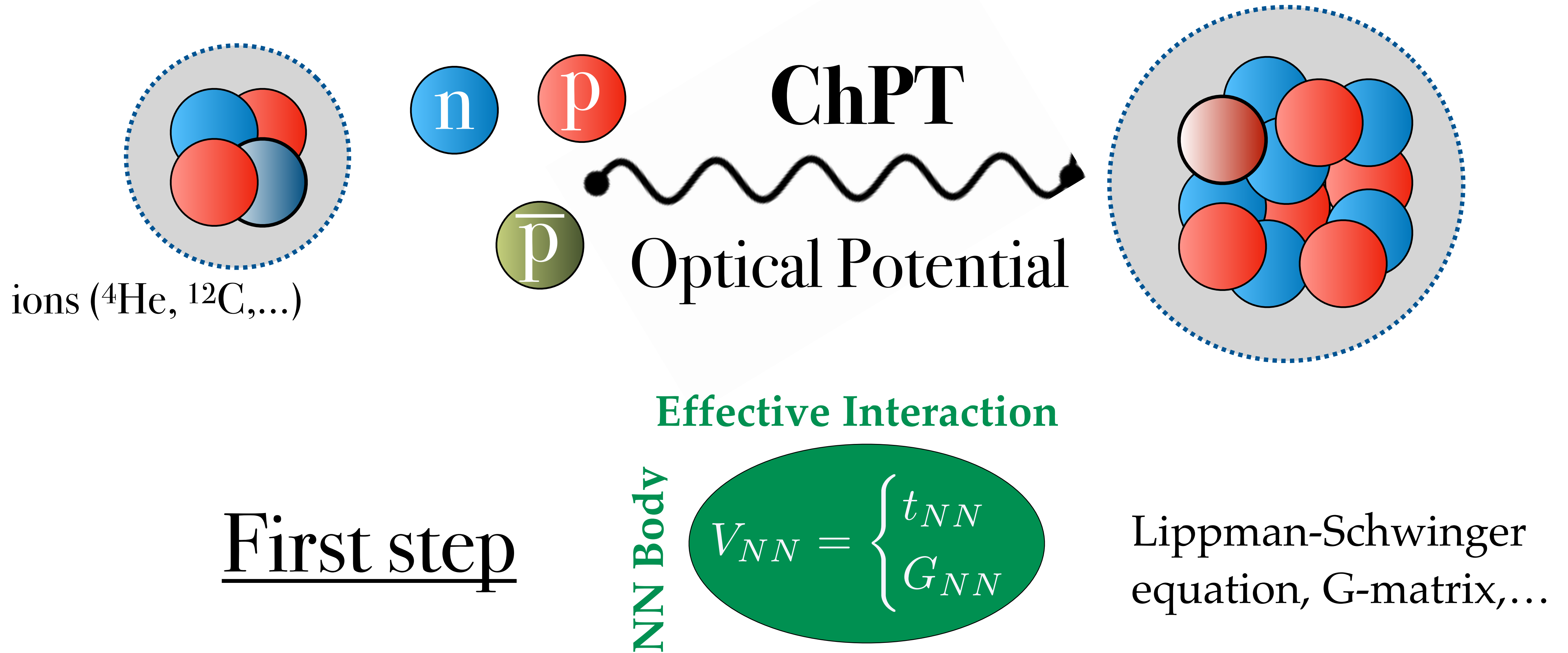
Roadmap towards microscopic optical potentials



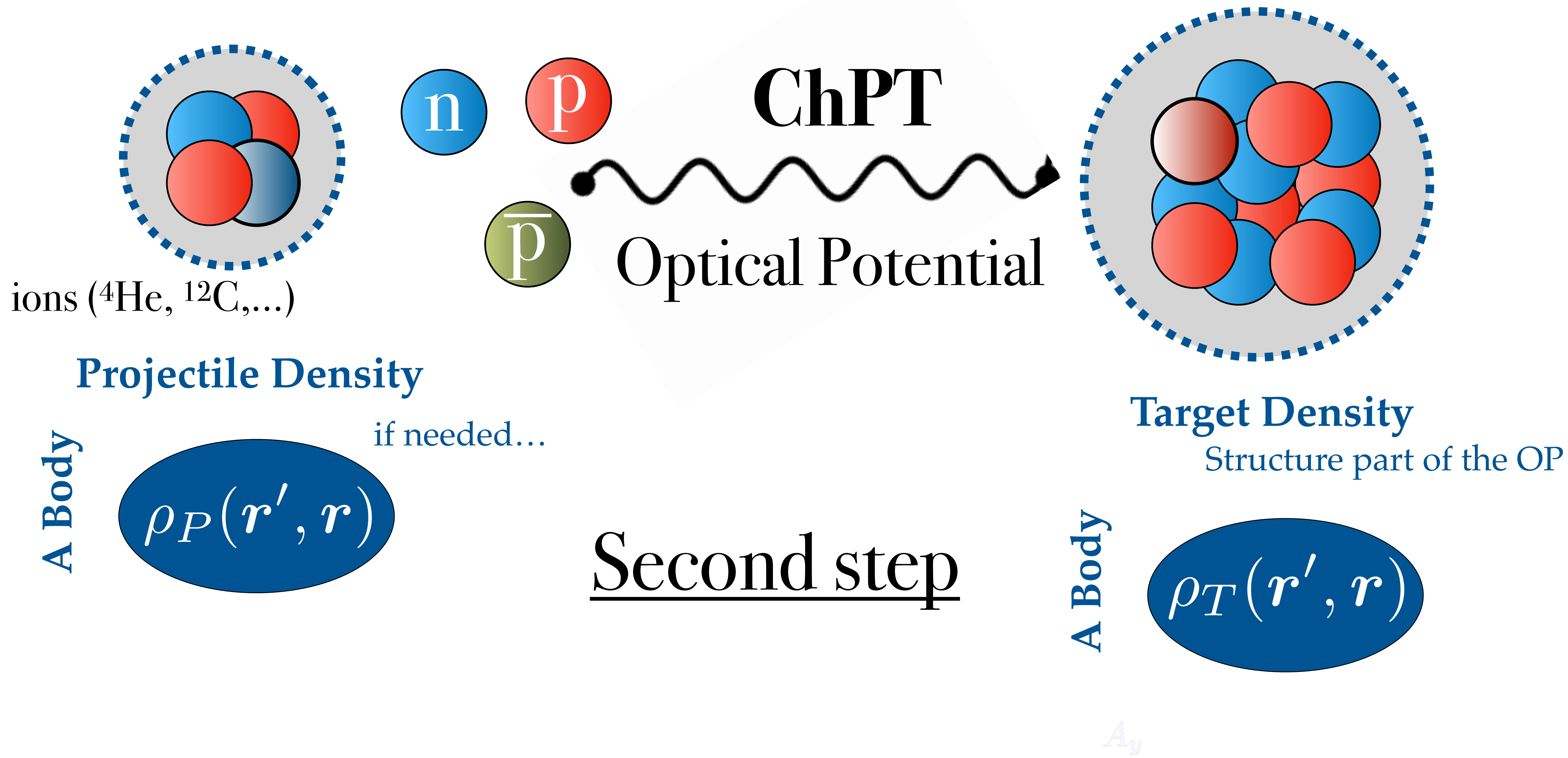
Roadmap towards microscopic optical potentials



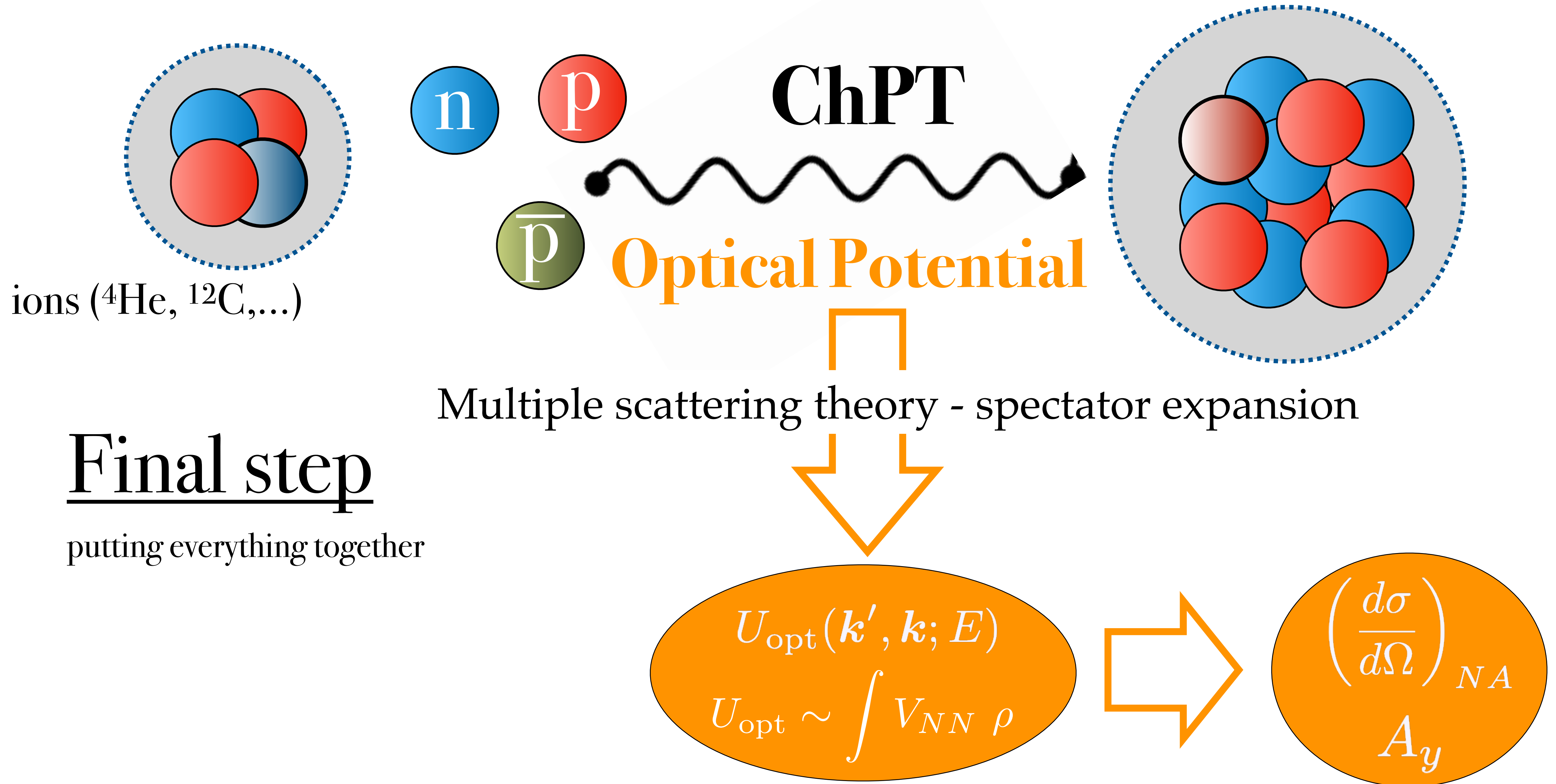
Roadmap towards microscopic optical potentials



Roadmap towards microscopic optical potentials



Roadmap towards microscopic optical potentials



Multiple scattering approximation for NA

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + V G_0(E) T \quad \text{elastic case}$$

Projectile-target interaction

$$V = \sum_{i=1}^A v_{0i} + \sum_{i < j}^A w_{0ij}$$

Full 3N
interaction is
still missing

Many-body propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

Multiple scattering approximation for NA

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + V G_0(E) T \quad \text{elastic case}$$

Let's introduce the optical potential U

Projection operators $P + Q = 1$

P space (elastic)

$$P = |\Psi_0\rangle\langle\Psi_0|$$

Q Space

$$Q = 1 - P$$

$$T = U + U G_0(E) P T$$

$$U = V + V G_0(E) Q U$$

The spectator expansion

Transition amplitude for elastic scattering

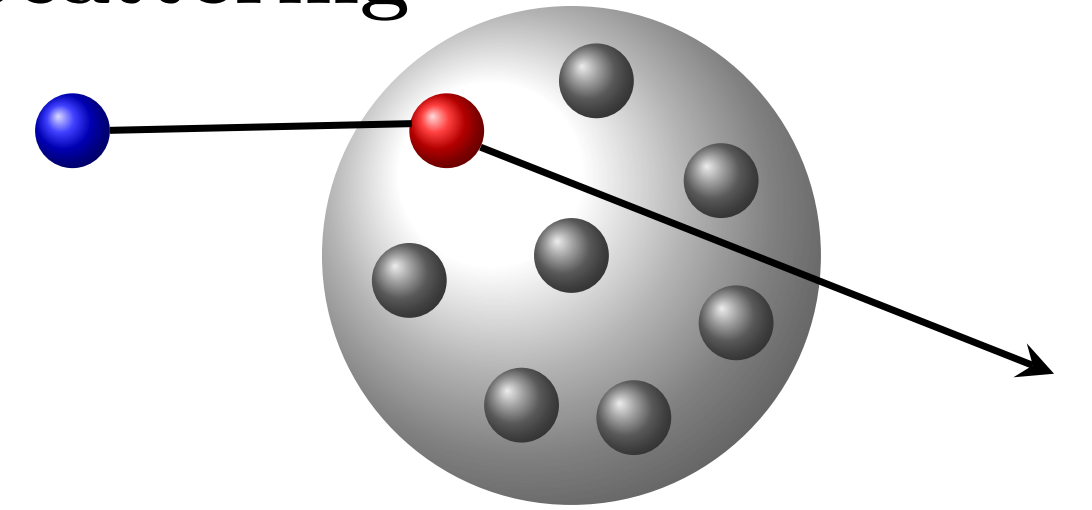
$$T_{\text{el}} \equiv PTP = PUP + PUPG_0(E)T_{\text{el}}$$

The spectator expansion

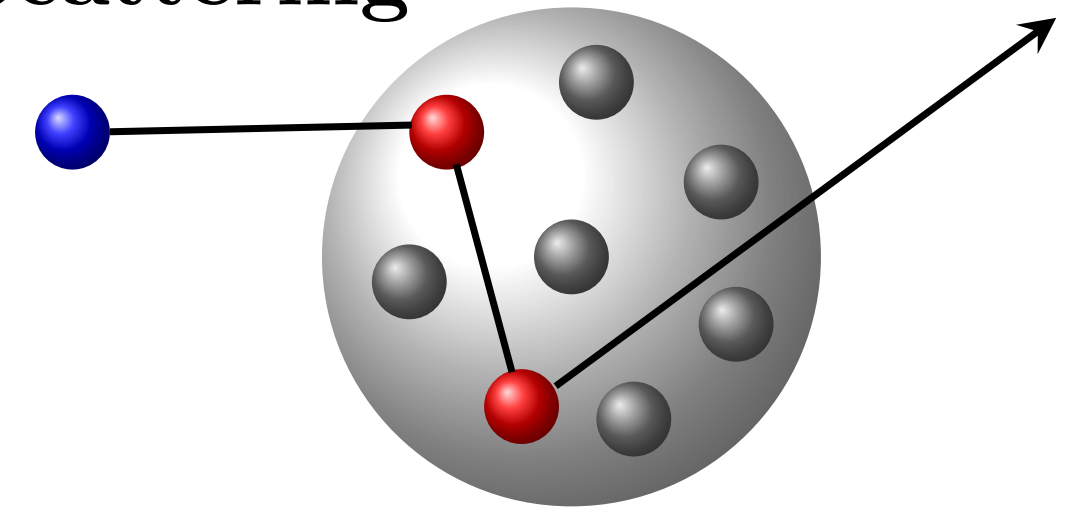
$$U = \sum_{i=1}^A \tau_{0i} + \sum_{i,j \neq i}^A \tau_{0ij} + \sum_{i,j \neq i, k \neq i,j}^A \tau_{0ijk} + \dots$$

A terms

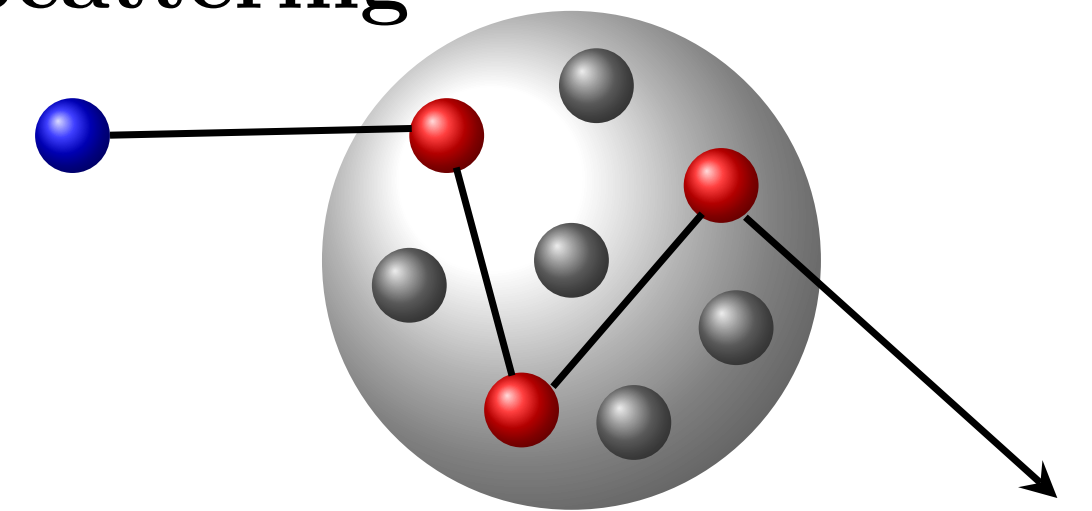
Single
Scattering



Double
Scattering



Triple
Scattering



+
.
.
.

The first-order spectator expansion

Transition amplitude for elastic scattering

$$T_{\text{el}} \equiv PTP = PUP + PUPG_0(E)T_{\text{el}}$$

The spectator expansion

$$U = \sum_{i=1}^A \tau_{0i} + \sum_{i,j \neq i}^A \tau_{0ij} + \sum_{i,j \neq i, k \neq i,j}^A \tau_{0ijk} + \dots$$

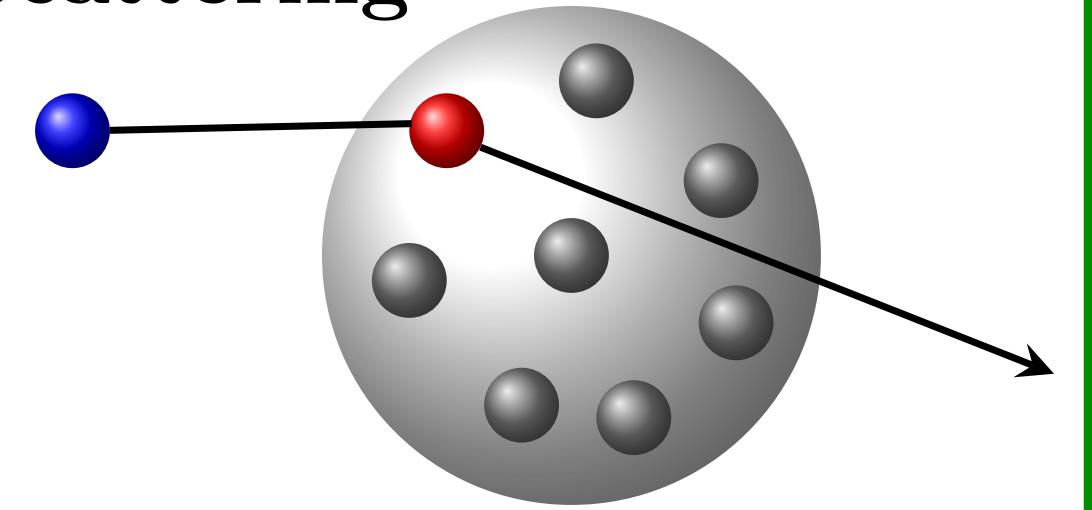
A terms

$$U \simeq \sum_{i=1}^A \tau_{0i} \longrightarrow \tau_{0i} \approx t_{0i} = v_{0i} + v_{0i} g_0(E) t_{0i}$$

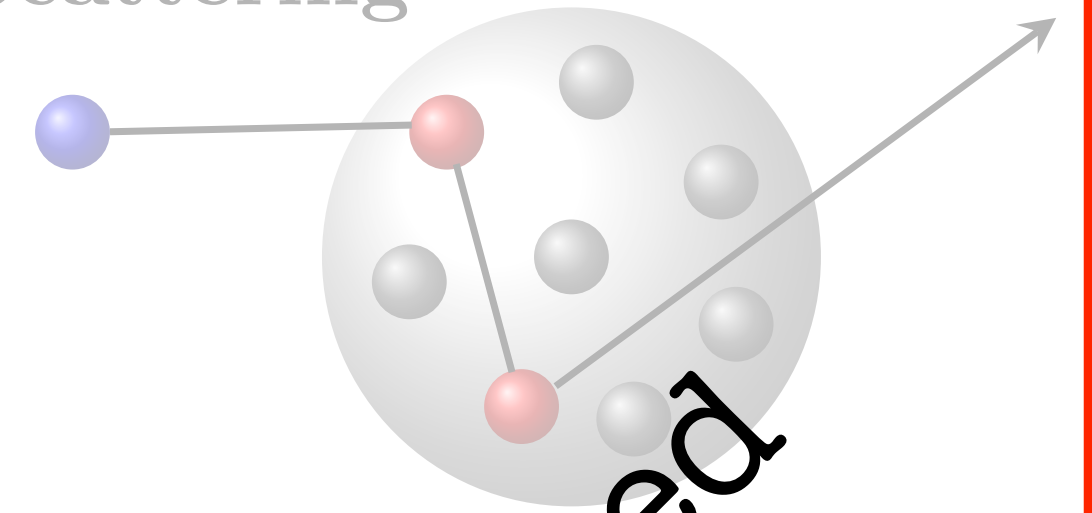
Impulse approximation

Two-body propagator

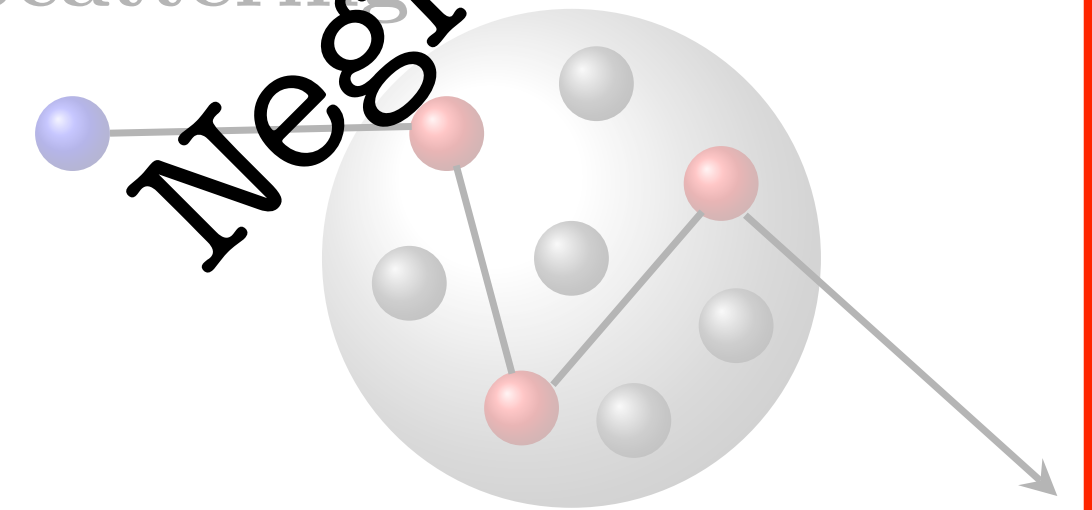
Single
Scattering



Double
Scattering



Triple
Scattering



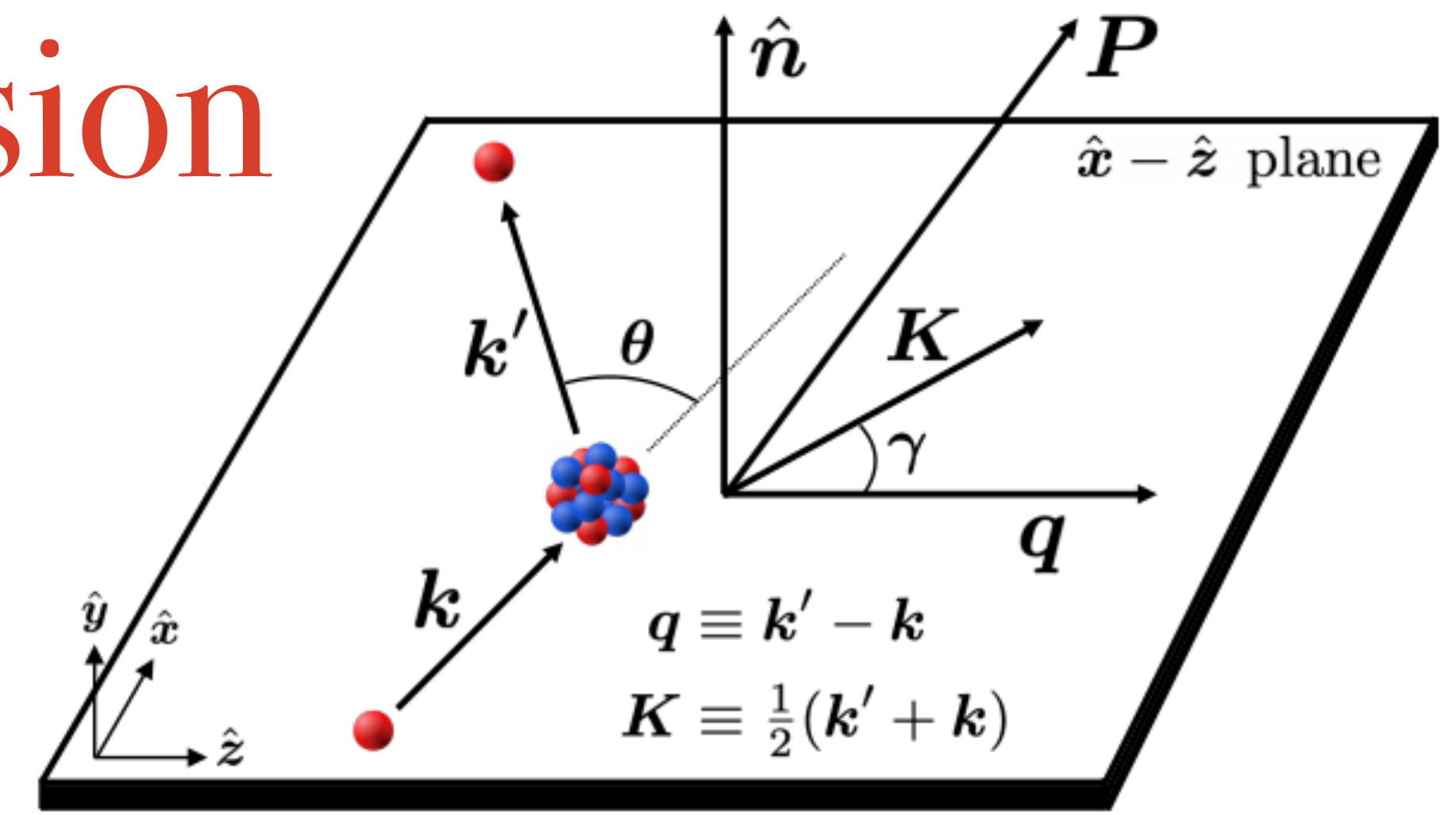
Neglected

+

⋮

+

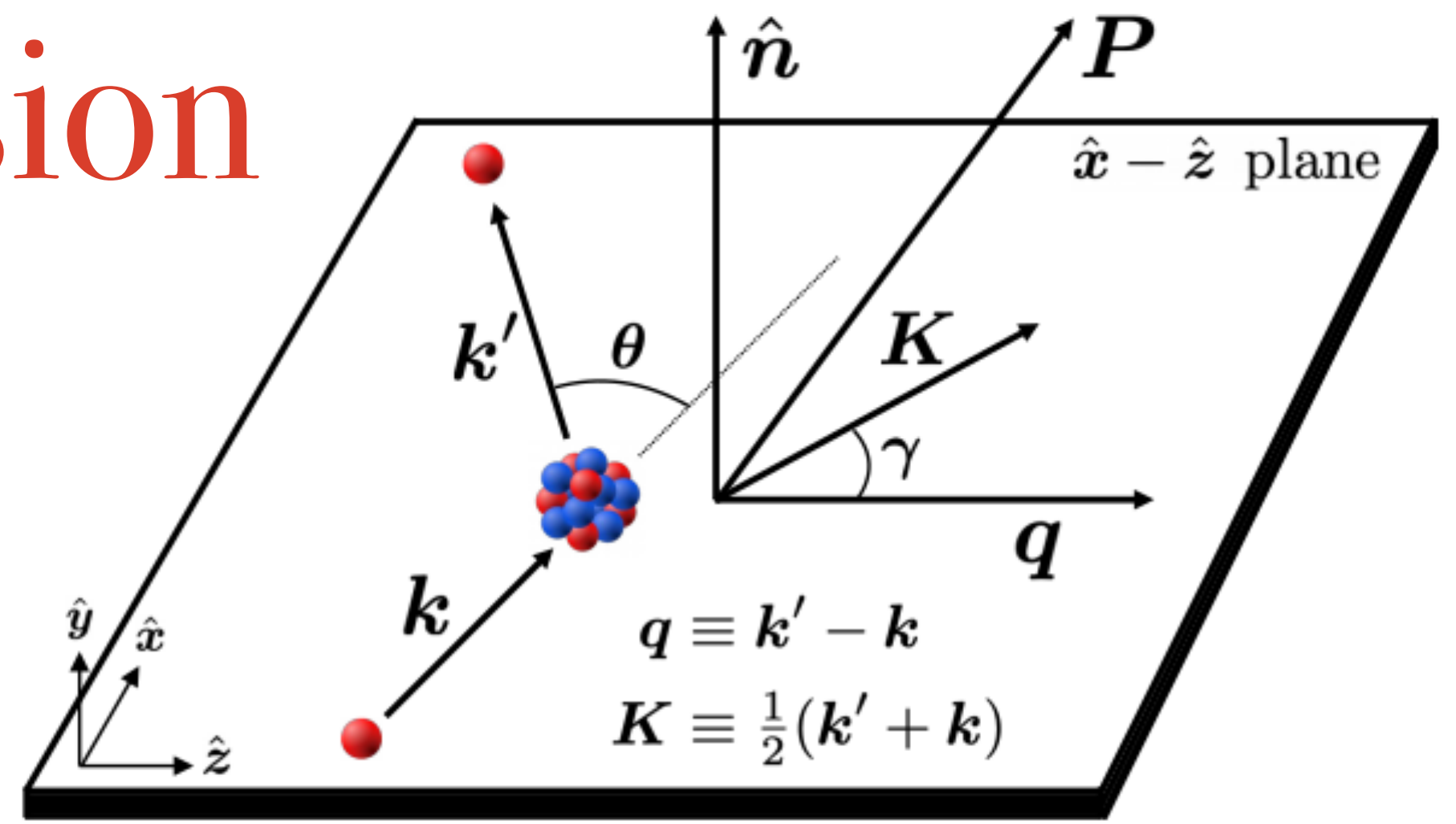
The first-order spectator expansion



$$U_{\mathbf{p}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{\mathbf{p}N}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N(\mathbf{q}, \mathbf{P})$$

1. Folding procedure with full non-locality
2. Many-body complexity has been strongly reduced
3. Well suited for intermediate energies (70 - 350 MeV), we need improvements (work is in progress) for lower and higher energies
4. It does not require any correction/fit, approximations should be considered

The first-order spectator expansion



$$U_{\mathbf{p}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{\mathbf{p}N}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N(\mathbf{q}, \mathbf{P})$$

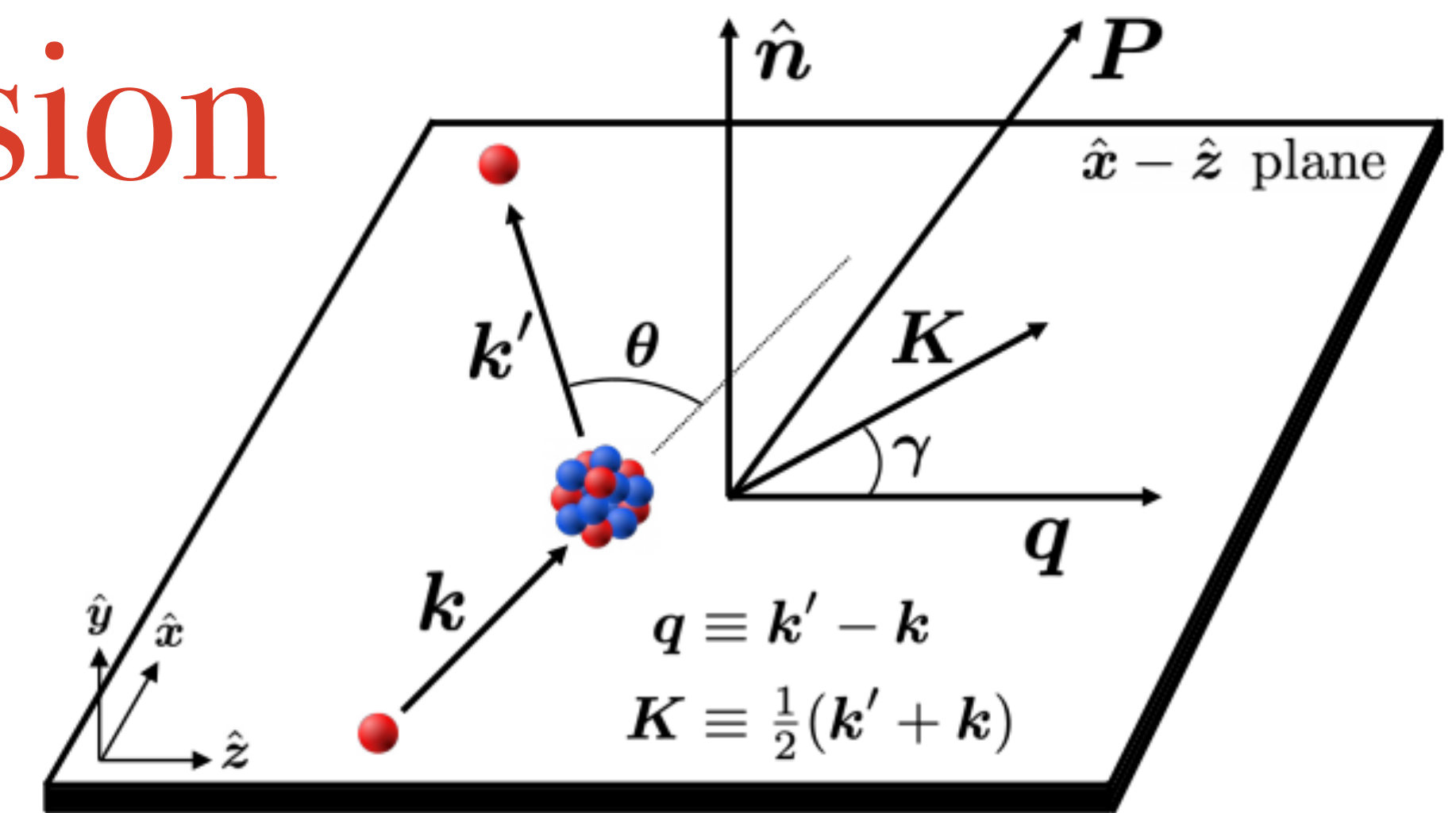
Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i} g_{0i} t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only NN interaction

The first-order spectator expansion



$$U_{\mathbf{p}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{\mathbf{p}N}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N(\mathbf{q}, \mathbf{P})$$

Free two-body scattering matrix

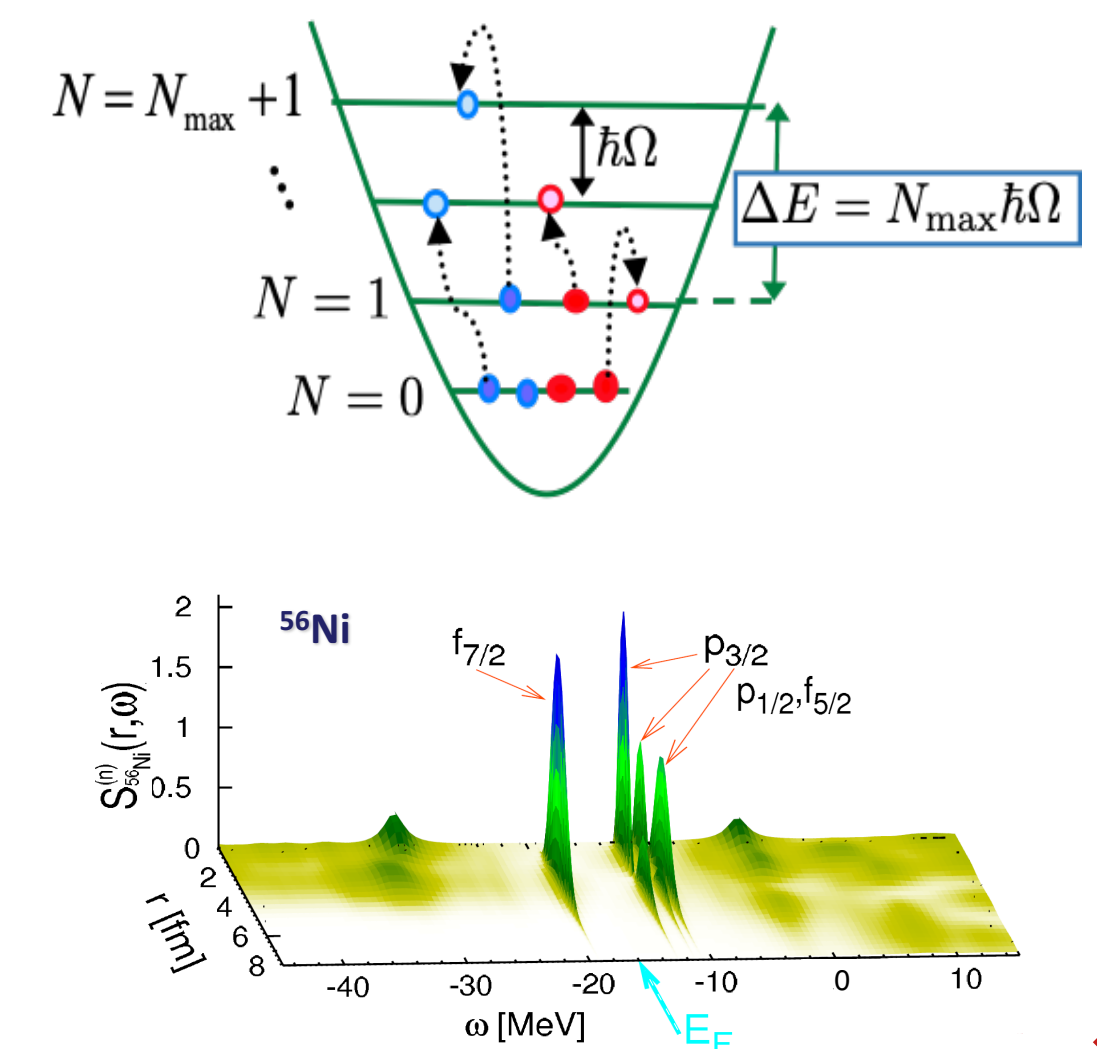
$$t_{0i} = v_{0i} + v_{0i} g_{0i} t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only NN interaction

Nonlocal one-body density

- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with NN and 3N interaction

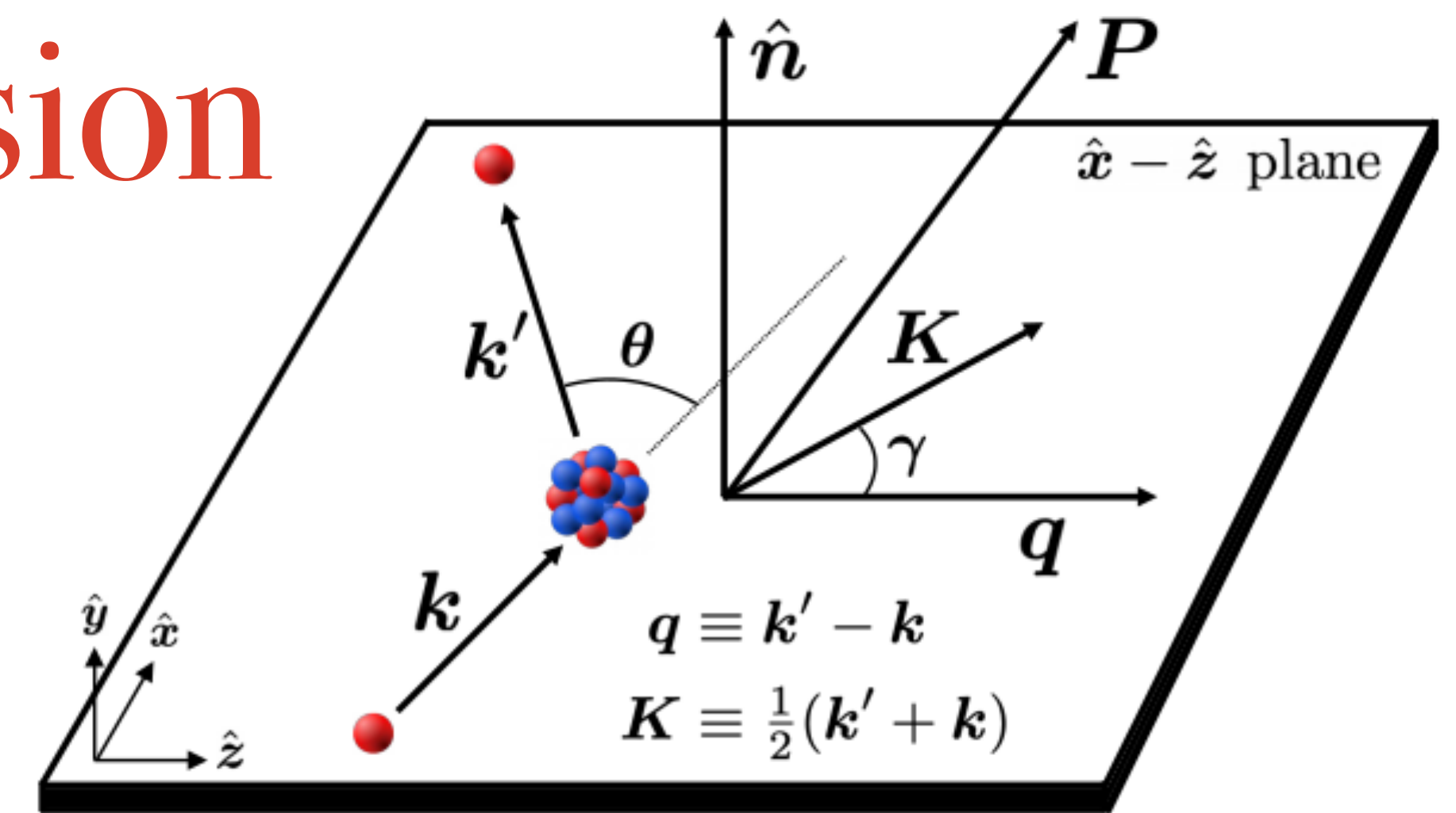


The first-order spectator expansion

Møller factor

$$t_{\mathbf{p}N}^{(NA)} = \eta t_{\mathbf{p}N}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated



$$U_{\mathbf{p}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{\mathbf{p}N}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N(\mathbf{q}, \mathbf{P})$$

Free two-body scattering matrix

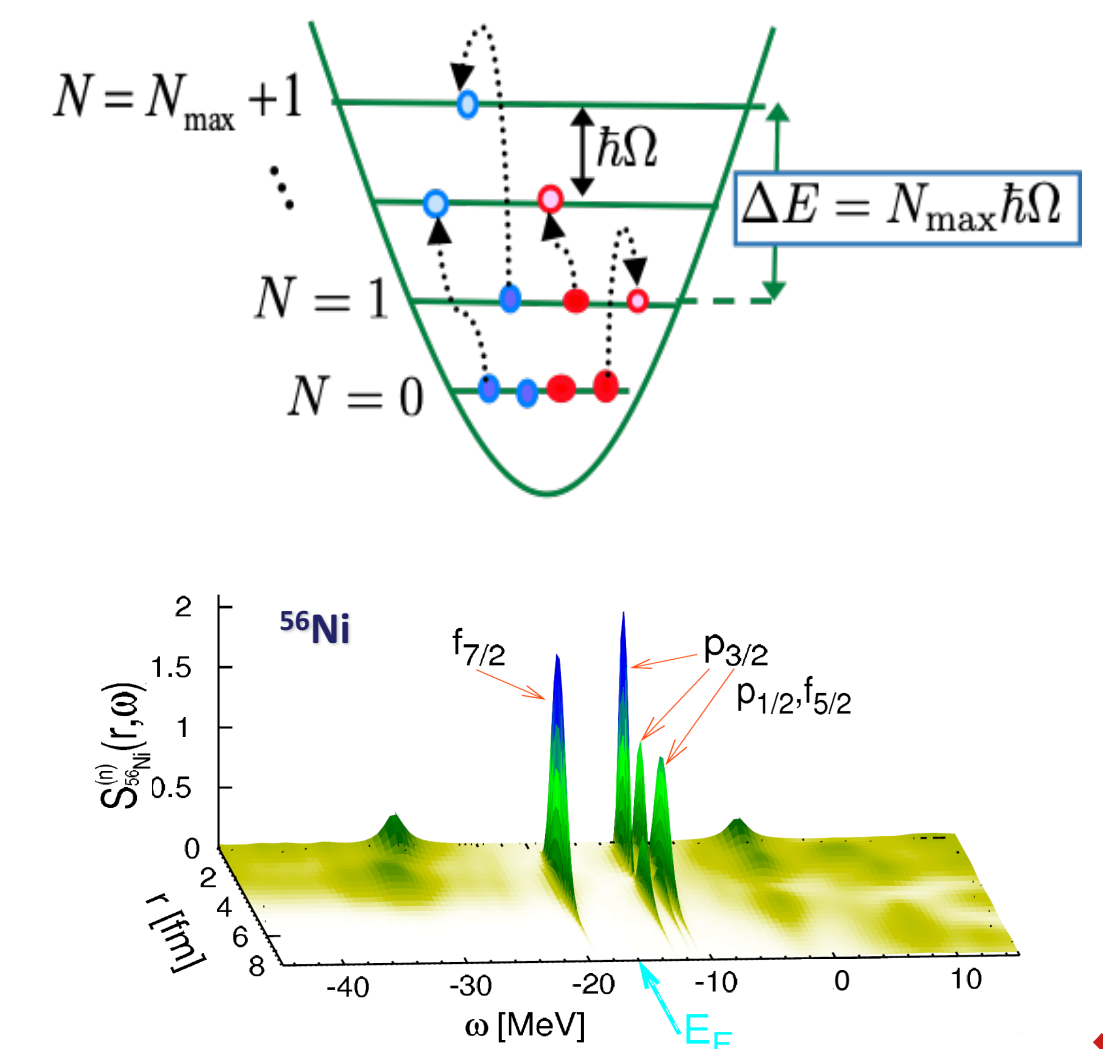
$$t_{0i} = v_{0i} + v_{0i} g_{0i} t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only NN interaction

Nonlocal one-body density

- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with NN and 3N interaction



Chiral interactions

Advantages

- ☑ QCD symmetries are consistently respected
- ☑ Systematic expansion (order by order we know exactly the terms to be included)
- ☑ Theoretical errors
- ☑ Two- and three-nucleon forces belong to the same framework

We use these interactions as the only input to calculate the effective interaction between projectile and the target and into the description of the target density

LO
 $(Q/\Lambda_\chi)^0$

NLO
 $(Q/\Lambda_\chi)^2$

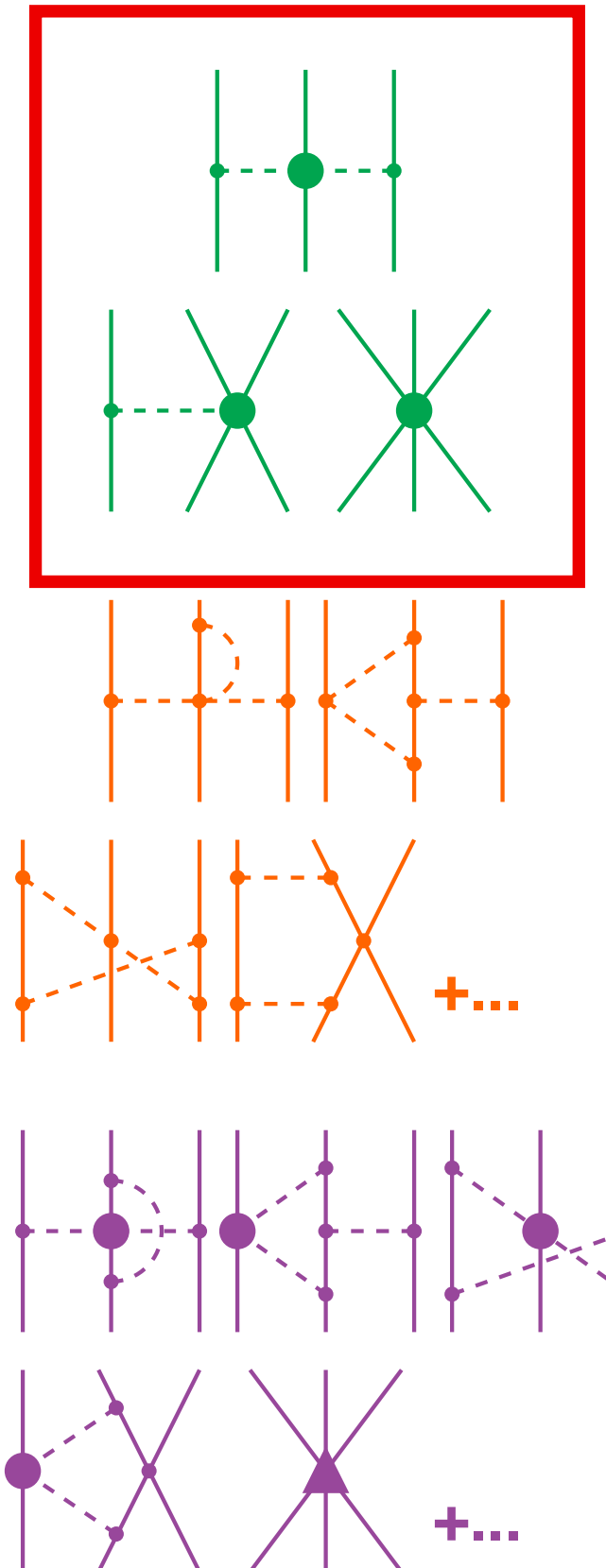
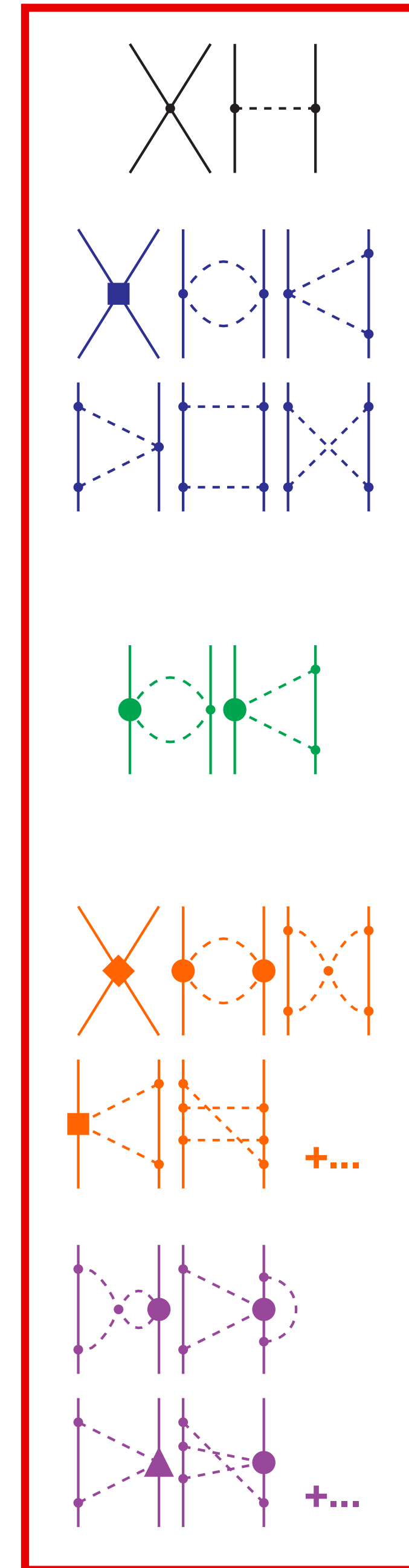
NNLO
 $(Q/\Lambda_\chi)^3$

N³LO
 $(Q/\Lambda_\chi)^4$

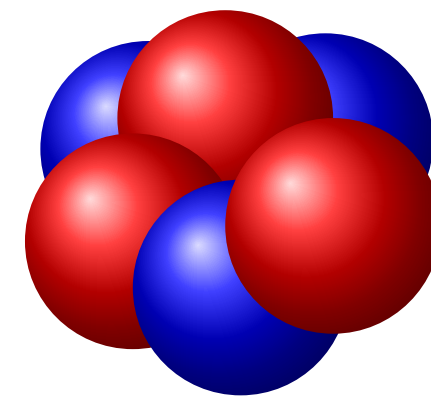
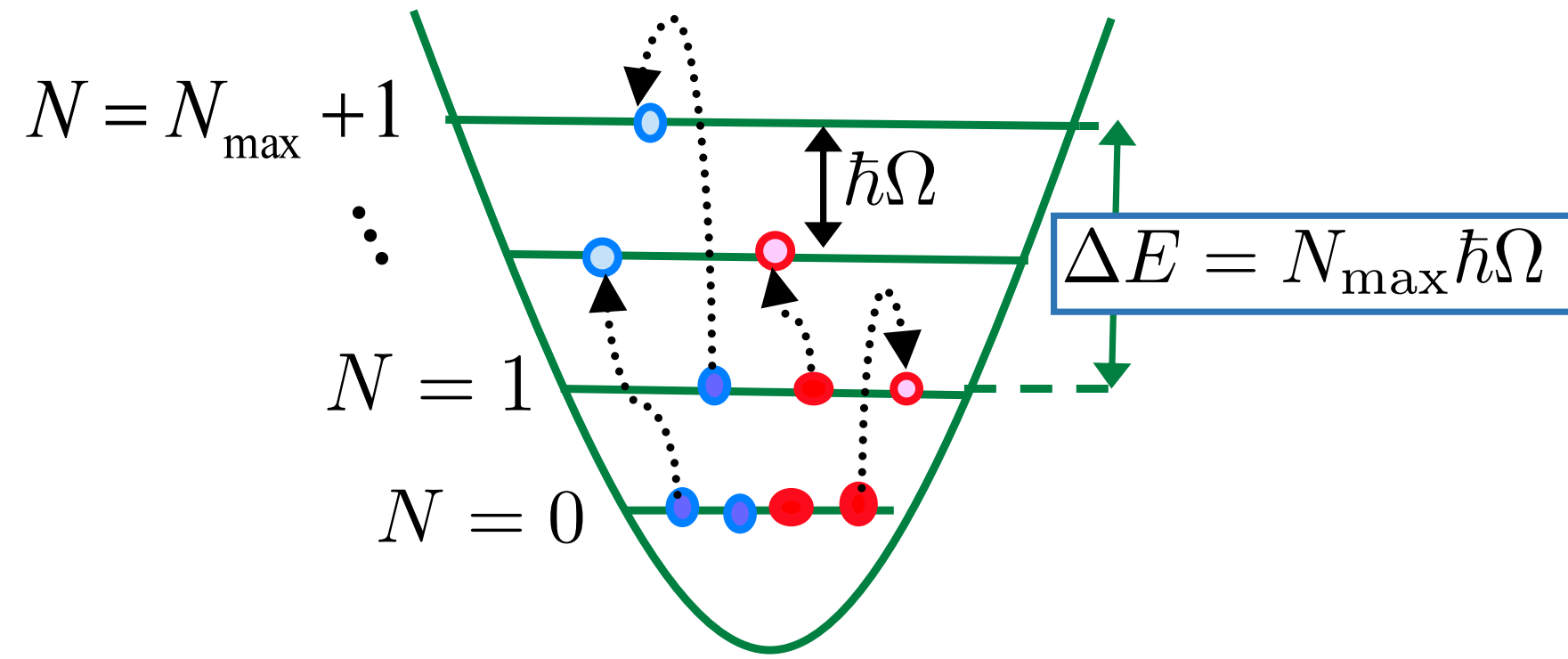
N⁴LO
 $(Q/\Lambda_\chi)^5$

2N Force

3N Force



Target description - light nuclei



NCSM

- NN-N⁴LO + 3Nlnl (¹²C, ¹⁶O)**

- N⁴LO: Entem et al., Phys. Rev. C **96**, 024004 (2017)
- 3Nlnl: Navrátil, Few-Body Syst. **41**, 117 (2007)
- c_D & c_E: Kravvaris et al., Phys. Rev. C **102**, 024616 (2020)

- NN-N³LO + 3Nlnl (^{9,13}C, ^{6,7}Li, ¹⁰B)**

- N³LO: E&M, Phys. Rev. C **68**, 041001(R) (2003)
- 3Nlnl: Navrátil, Few-Body Syst. **41**, 117 (2007)
- c_D & c_E: Somà et al., Phys. Rev. C **101**, 014318 (2020)

LO
 $(Q/\Lambda_\chi)^0$

NLO
 $(Q/\Lambda_\chi)^2$

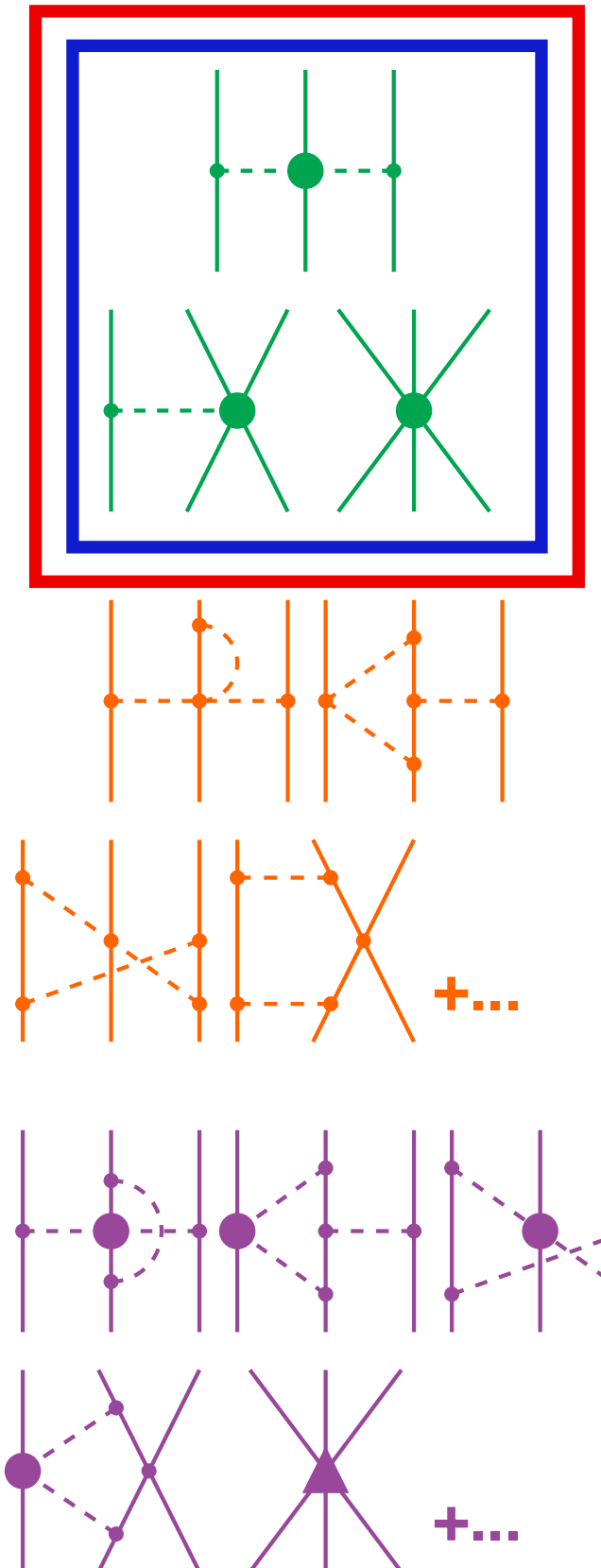
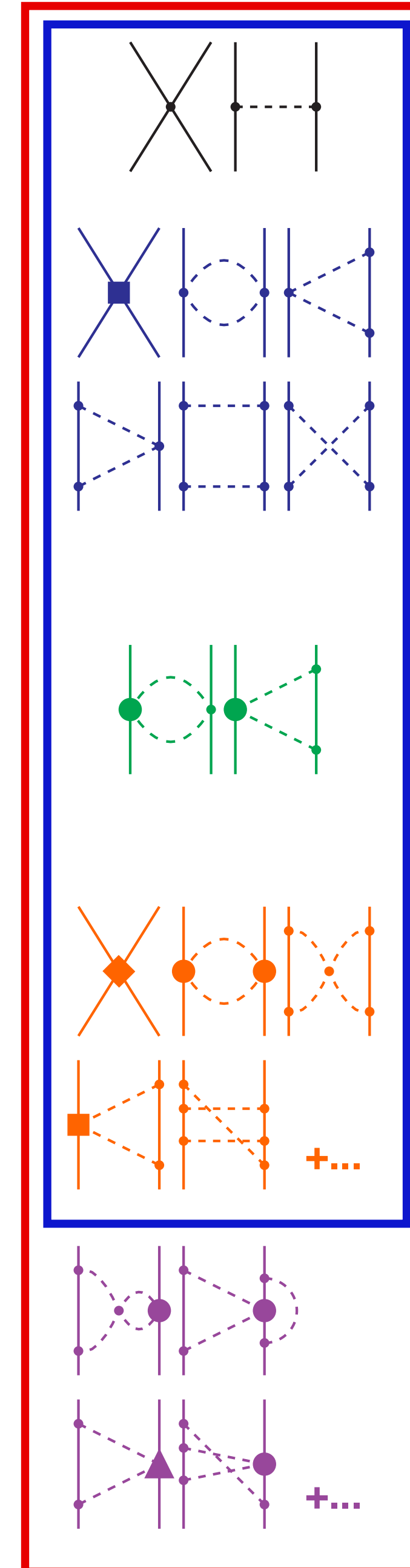
NNLO
 $(Q/\Lambda_\chi)^3$

N³LO
 $(Q/\Lambda_\chi)^4$

N⁴LO
 $(Q/\Lambda_\chi)^5$

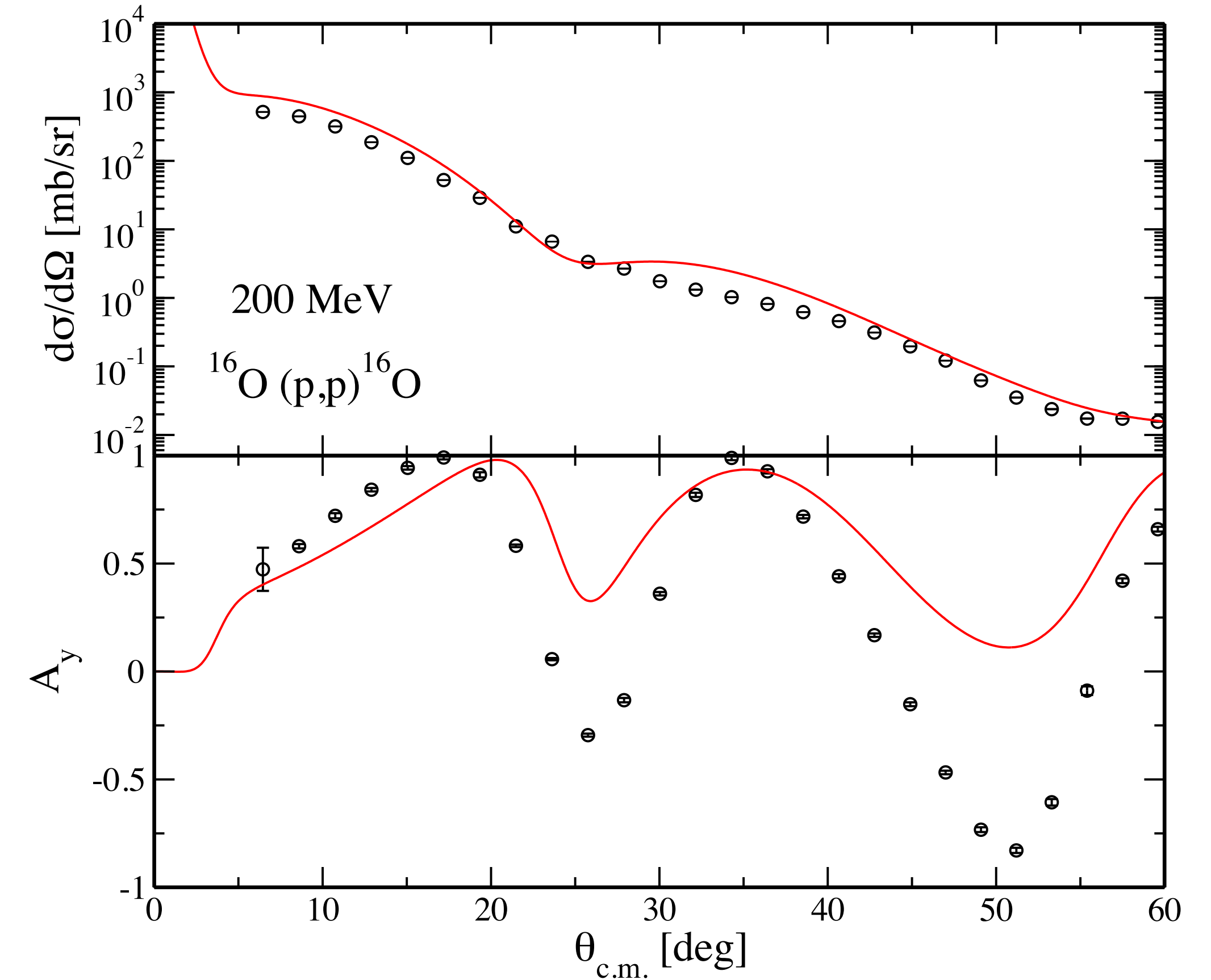
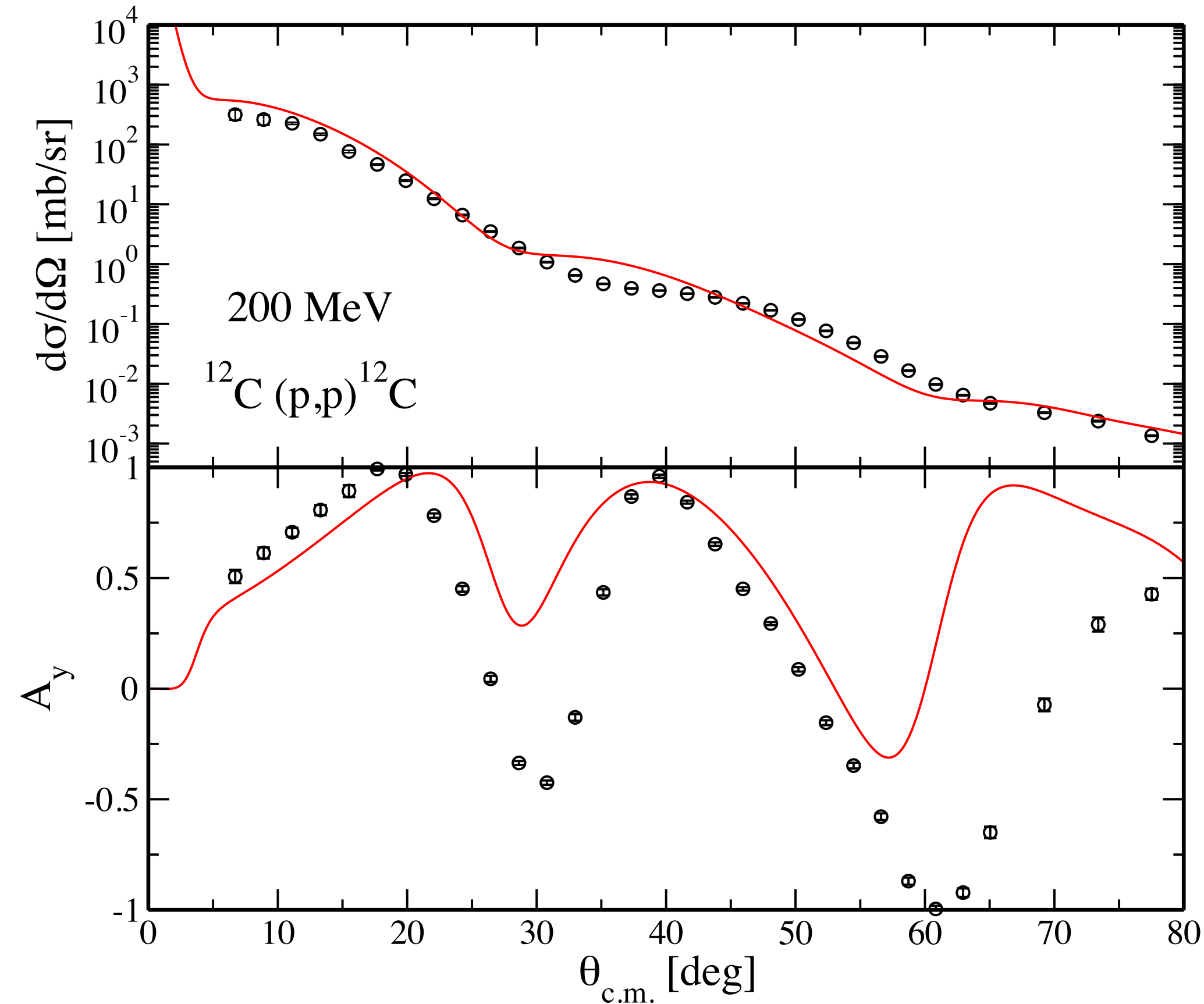
2N Force

3N Force



Elastic scattering: p+¹²C and p+¹⁶O

Vorabbi et al., PRC **103**, 024604 (2021)



$$U \sim \sum \int \eta \, t_{pN} \, \rho_N$$

- The t matrix is computed with the N⁴LO interaction
- The density is computed with the N⁴LO + 3Nlnl interaction

Assessing the impact of the 3N interaction

General equation for the optical potential

$$U = (V_{NN} + V_{3N}) + (V_{NN} + V_{3N})G_0(E)QU$$

Treatment of the 3N force

[Holt et al., Phys. Rev. C 81, 024002 (2010)]

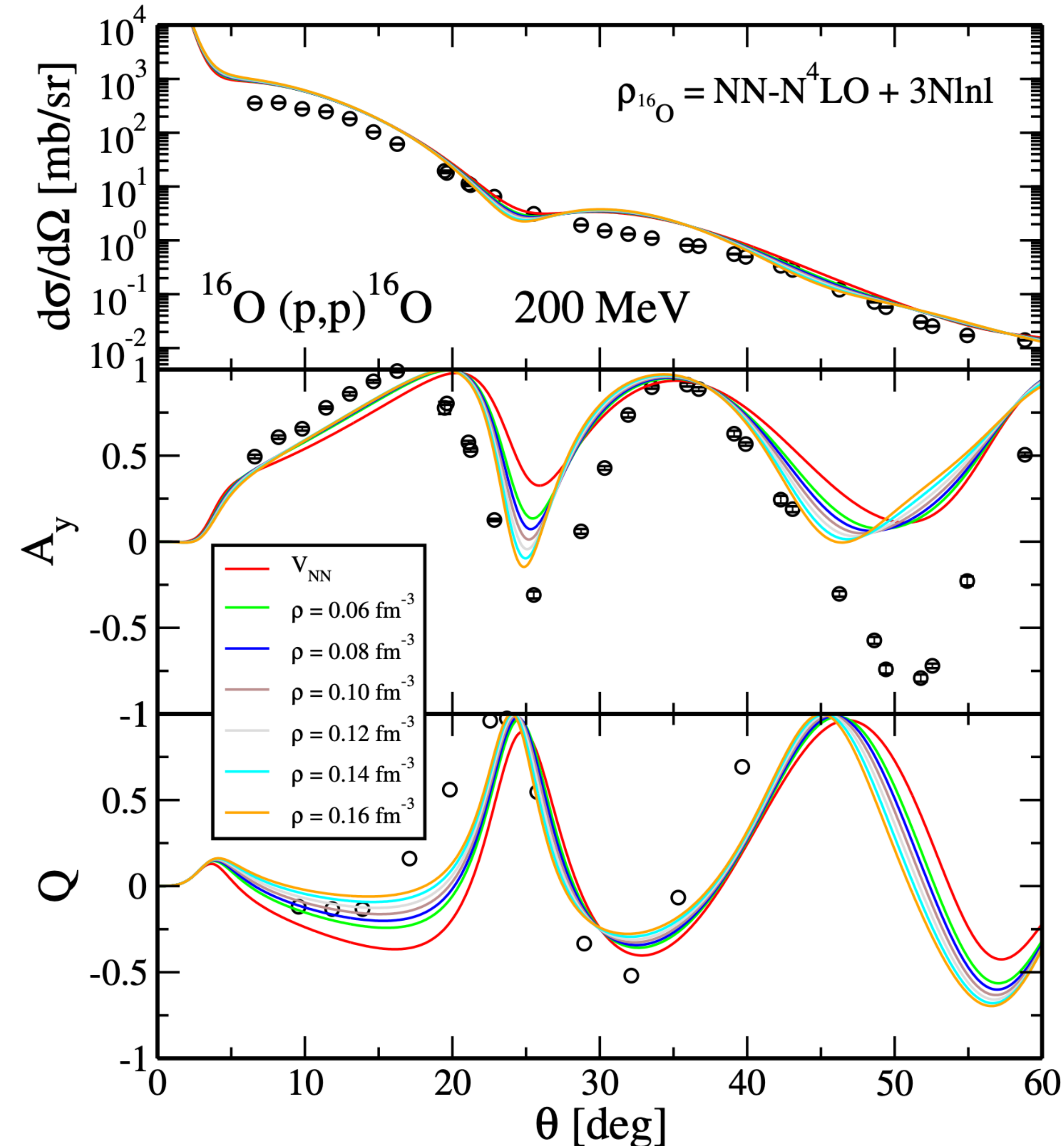
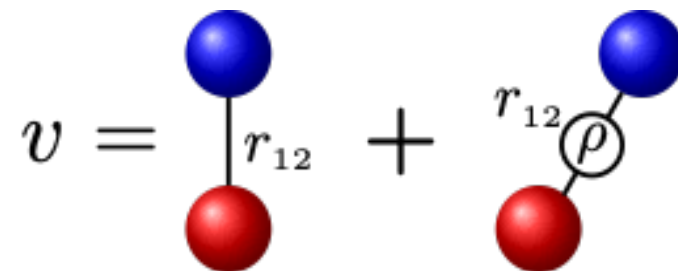
$$V_{3N} = \frac{1}{2} \sum_{i=1}^A \sum_{\substack{j=1 \\ j \neq i}}^A w_{0ij} \approx \sum_{i=1}^A \langle w_{0i} \rangle \quad \text{Density dependent}$$

Modification of the t matrix

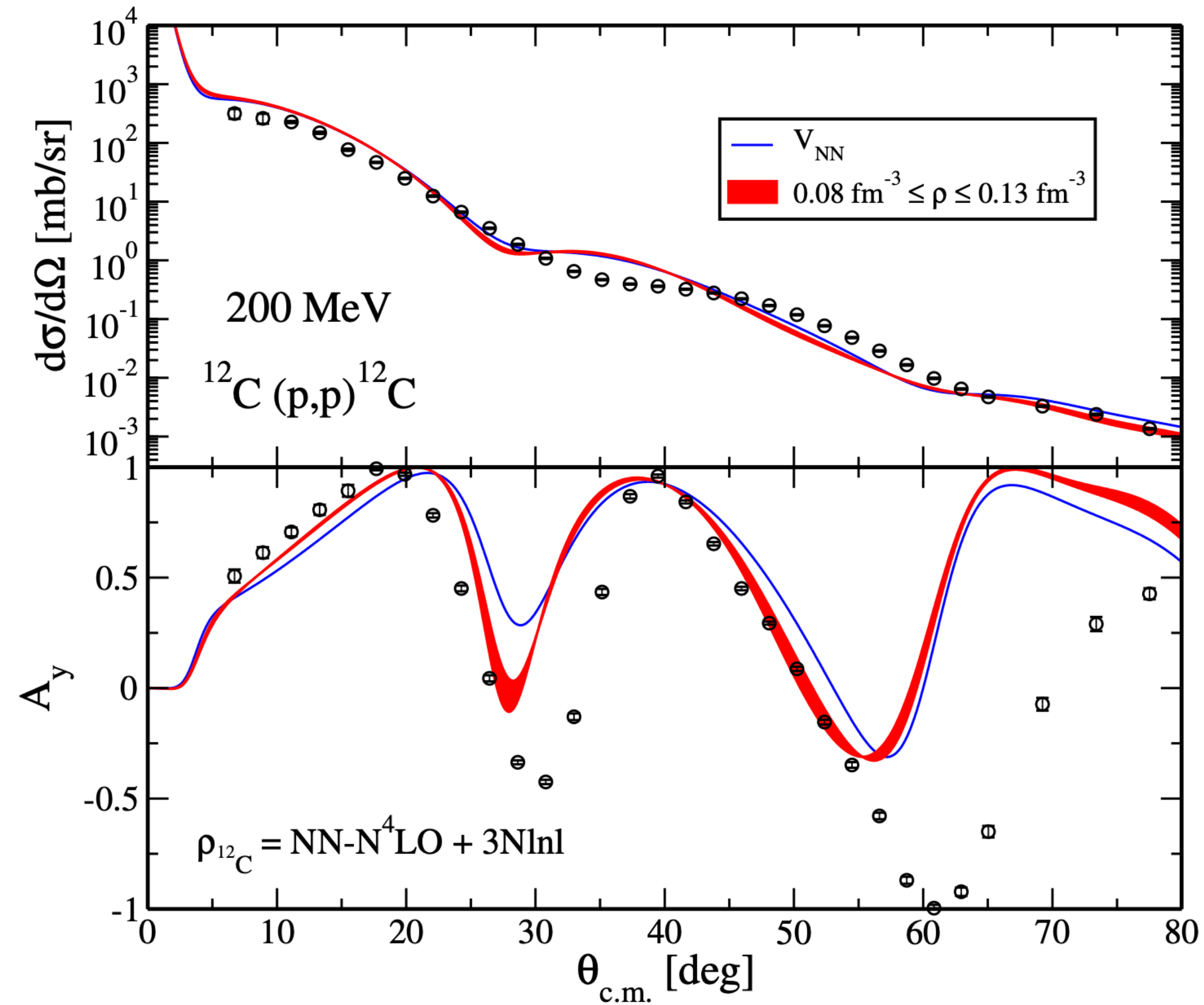
$$t_{0i} = v_{0i}^{(1)} + v_{0i}^{(2)} g_{0i} t_{0i}$$

$$v_{0i}^{(1)} = v_{0i} + \frac{1}{2} \langle w_{0i} \rangle$$

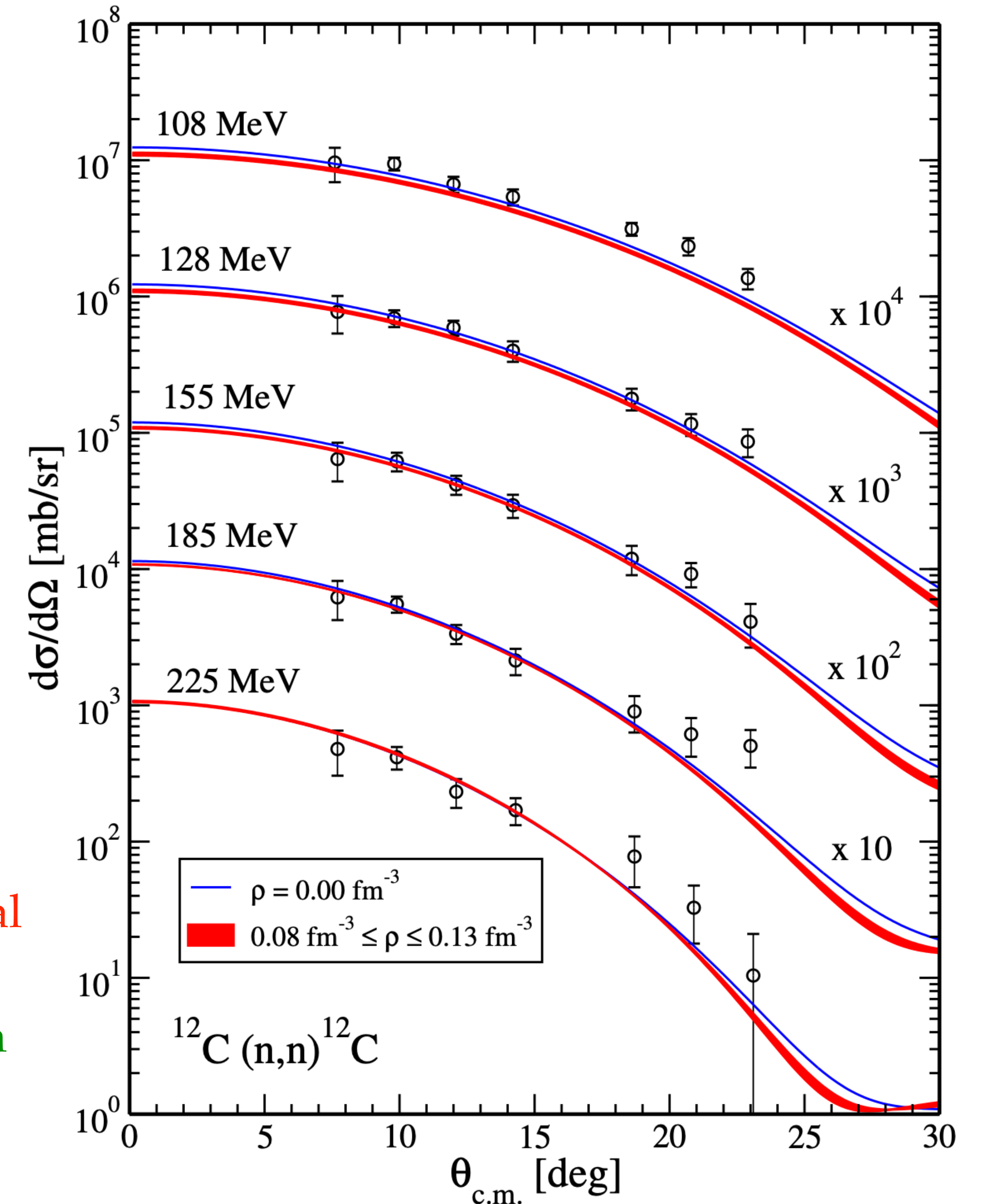
$$v_{0i}^{(2)} = v_{0i} + \langle w_{0i} \rangle$$



Assessing the impact of the 3N interaction

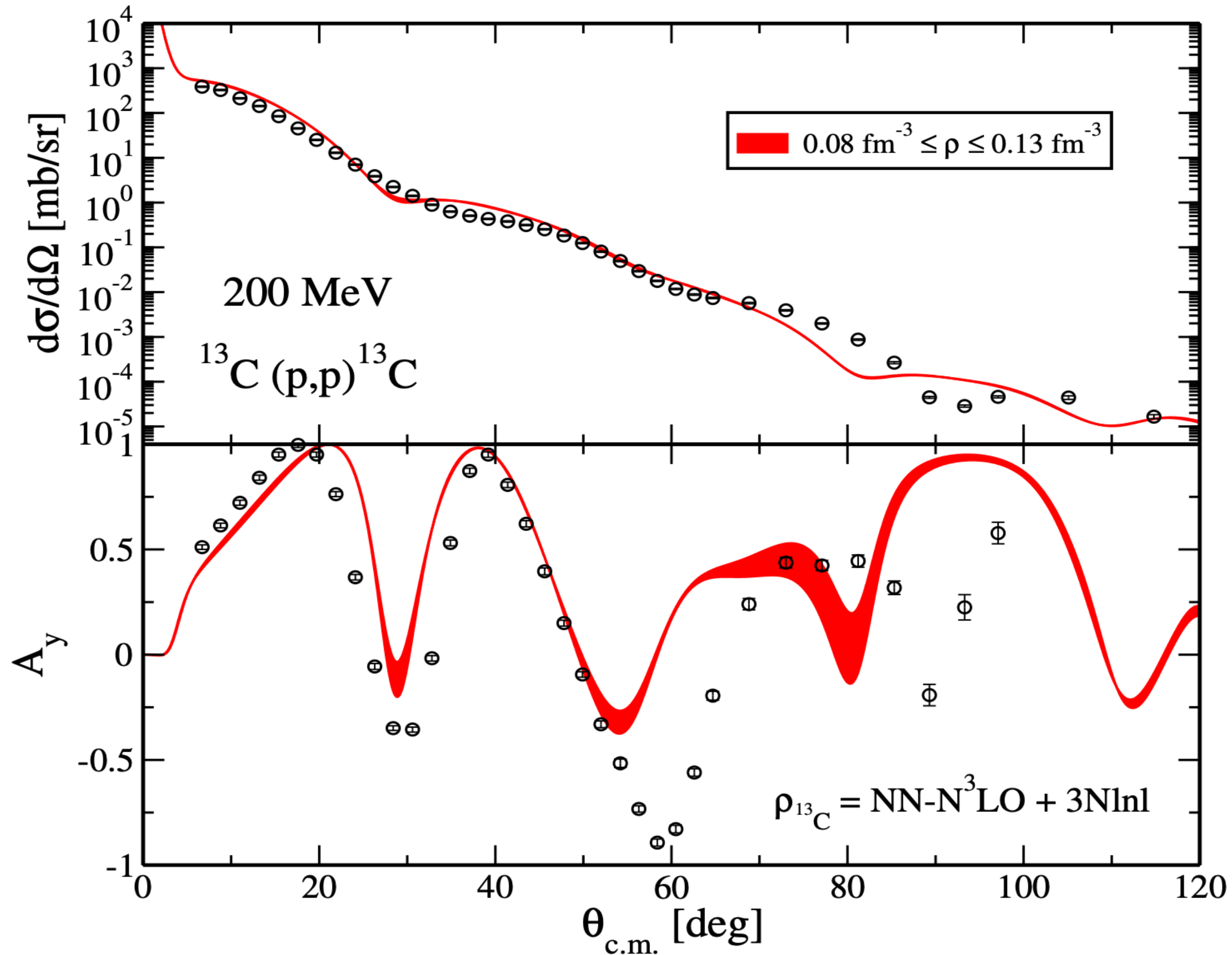


- For all nuclei we found very small contributions to the differential cross section
- The contributions to the spin observable are larger and they seem to improve the agreement with the data

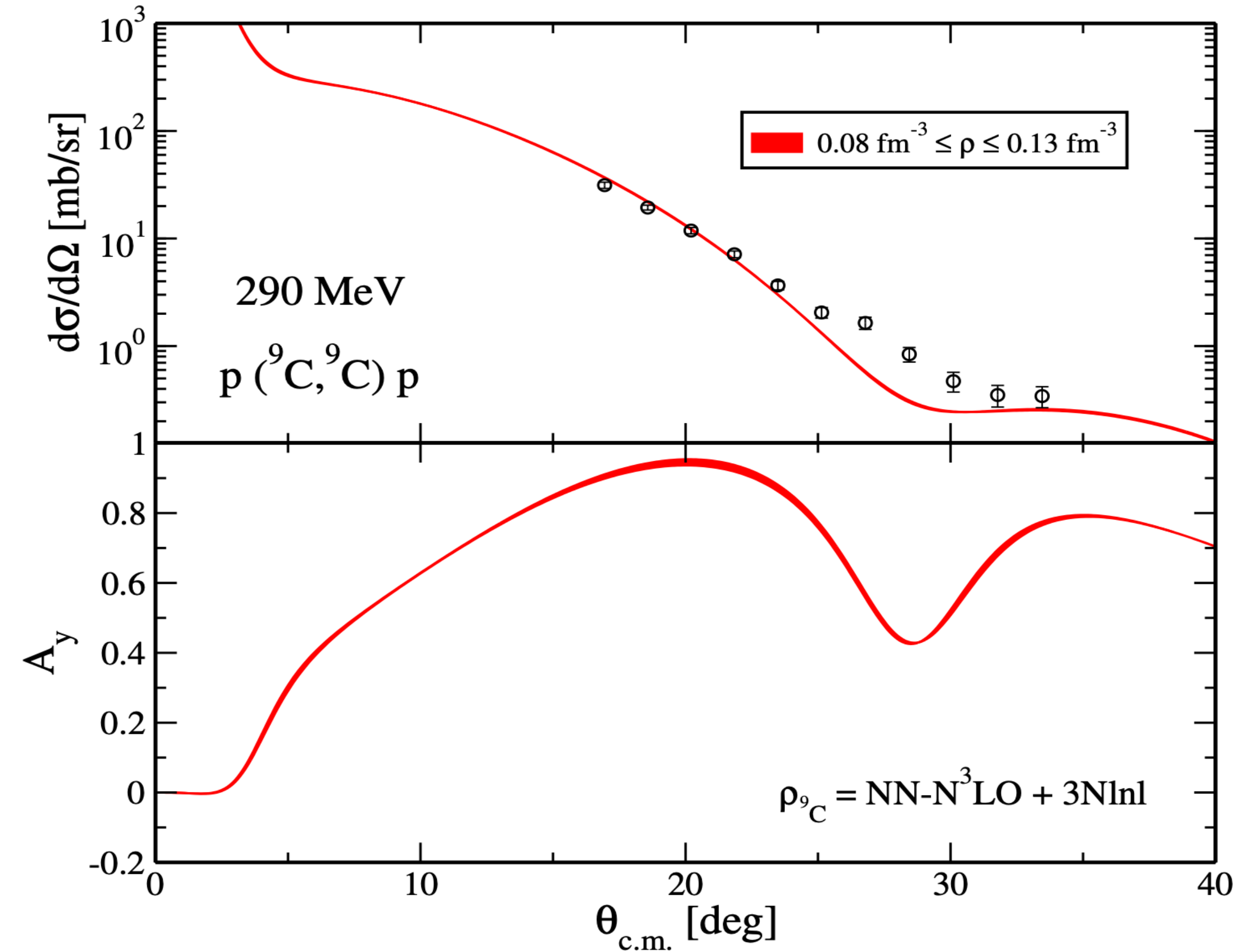


Extension to non-zero spin targets

$$J^\pi = 1/2^-$$



$$J^\pi = 3/2^-$$



Convergence (where to stop the chiral expansion)

- Density computed with NN+3N interaction

-NN interactions at all orders

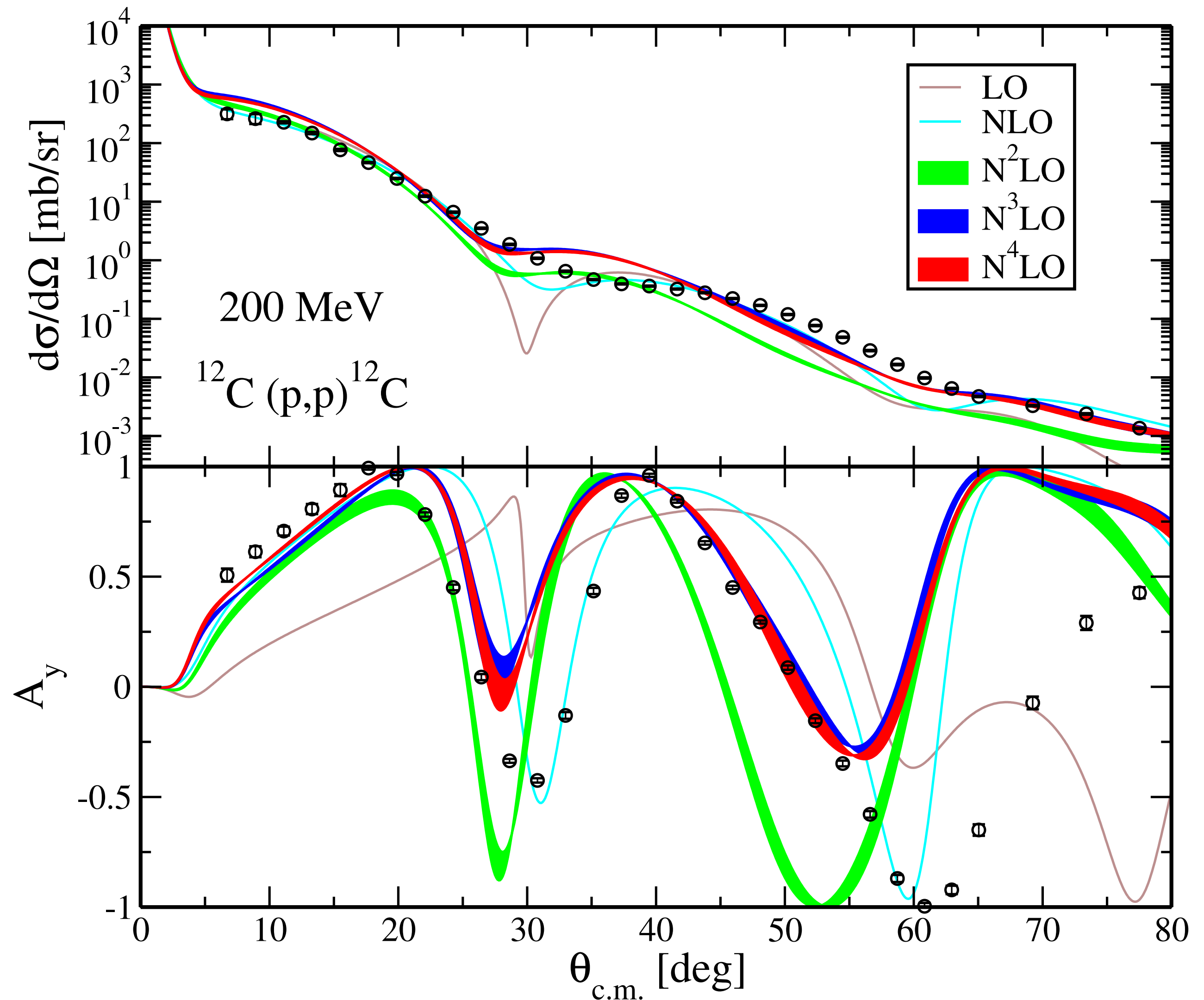
Entem et al., PRC **96**, 024004 (2017)

-3N only at N²LO with c_D and c_E refitted at each order

Kravvaris et al., PRC **102**, 024616 (2020)

- Bands are obtained starting from N²LO when the matter density ρ is allowed to vary between 0.08 and 0.13 fm⁻³

- At N³LO the results seem to achieve a good degree of convergence

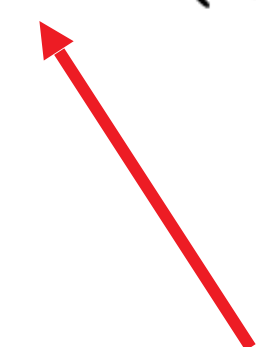


Vorabbi et al., PRC **103**, 024604 (2021)

Extension to antiproton-nucleus elastic scattering

$$U_p(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \, \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) \, t_{pN}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \, \rho_N(\mathbf{q}, \mathbf{P})$$

- The projectile information only enters the t_{pN} matrix. For antiprotons we make the following replacement

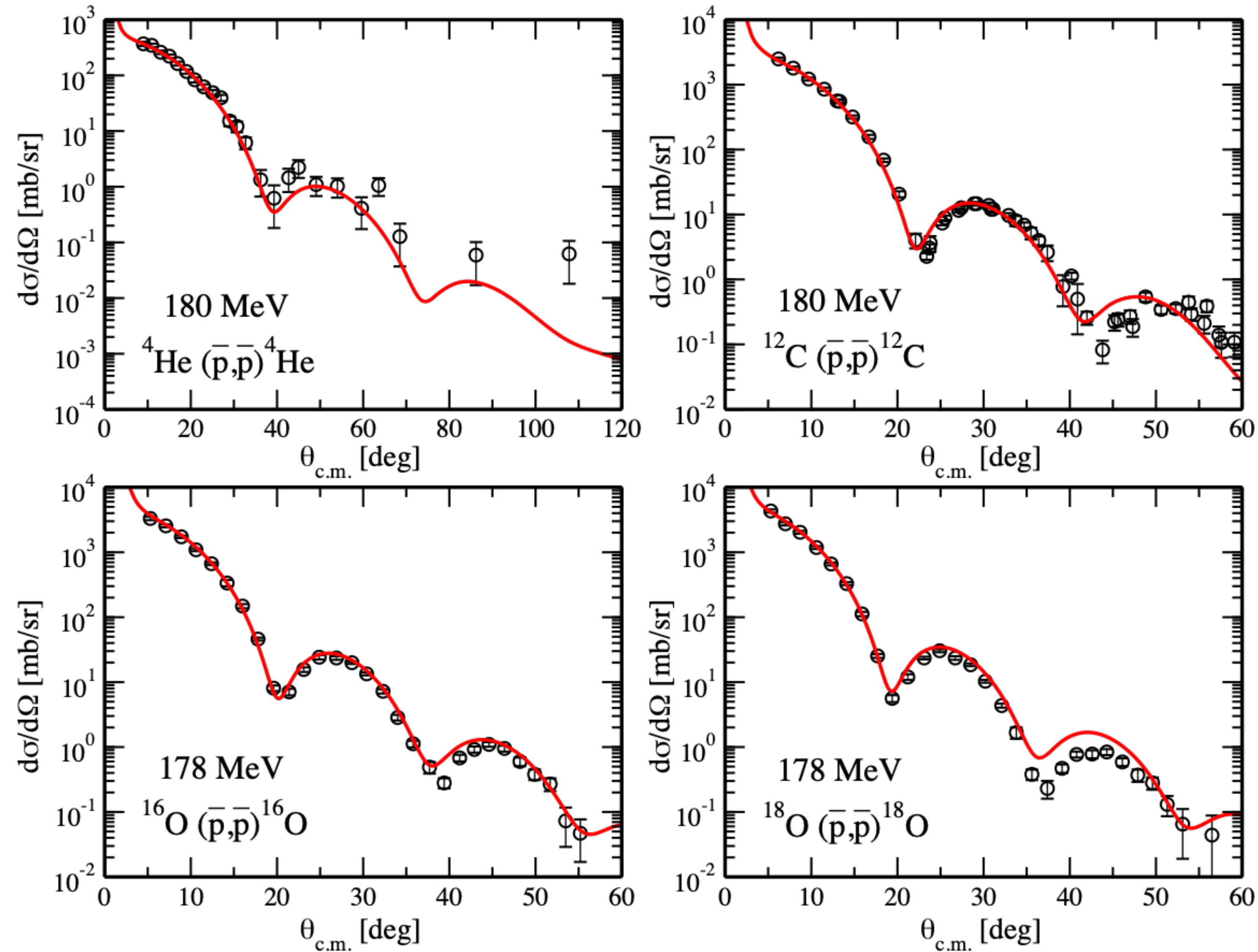

$$t_{pN} \Rightarrow t_{\bar{p}N}$$

- An antiproton-nucleon interaction is needed. $\bar{p}N$ chiral interaction derived up to N³LO

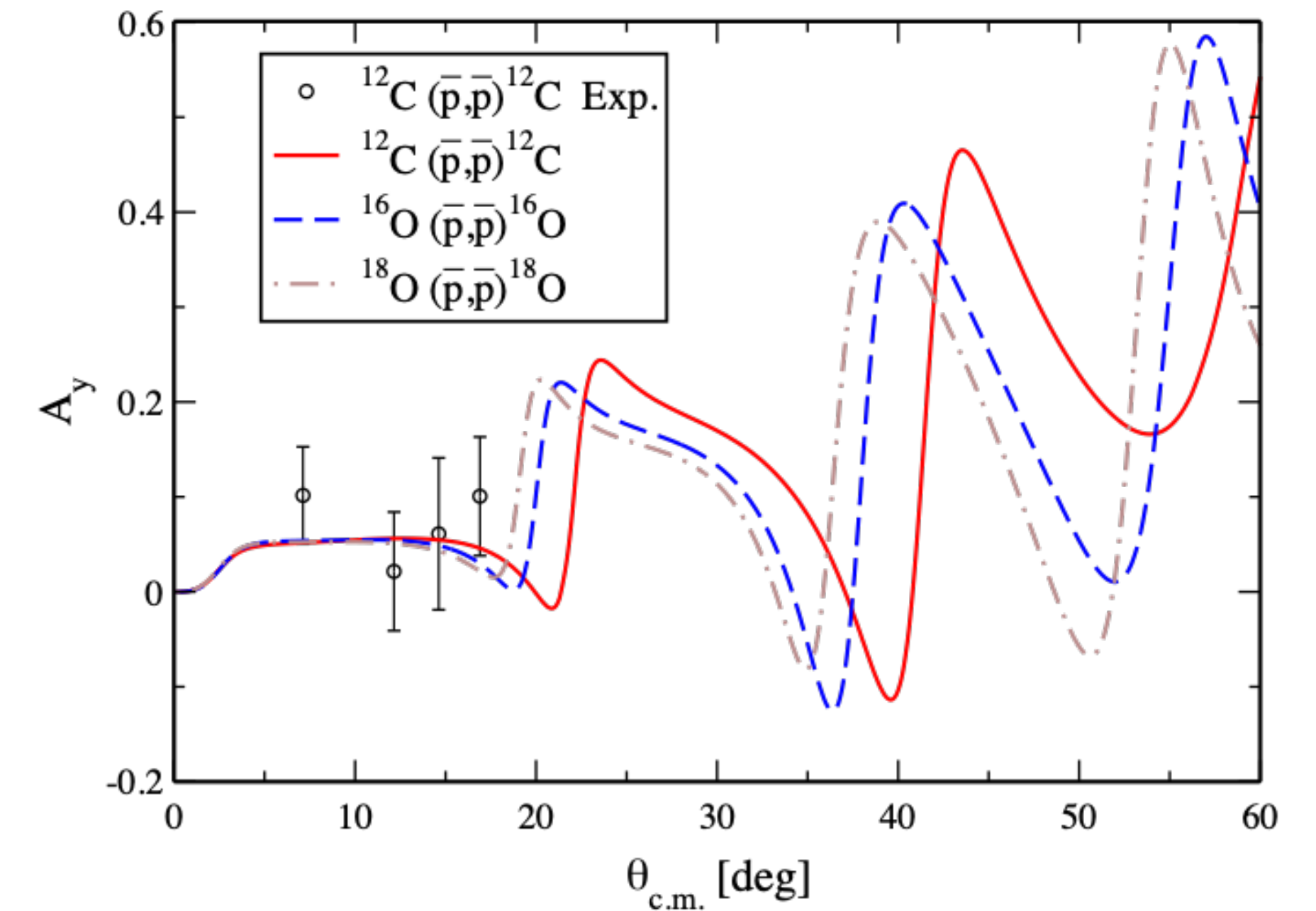
[Dai, Haidenbauer, Meißner, JHEP 07 (2017) 78]

- No projectile-target anti-symmetrisation
- Antiprotons are mostly absorbed at the surface of the nucleus so the first-order expansion should work better in this case!

Extension to antiproton-nucleus elastic scattering



Antiprotons mostly interact at the surface of the nucleus so the first-order expansion should work better in this case.

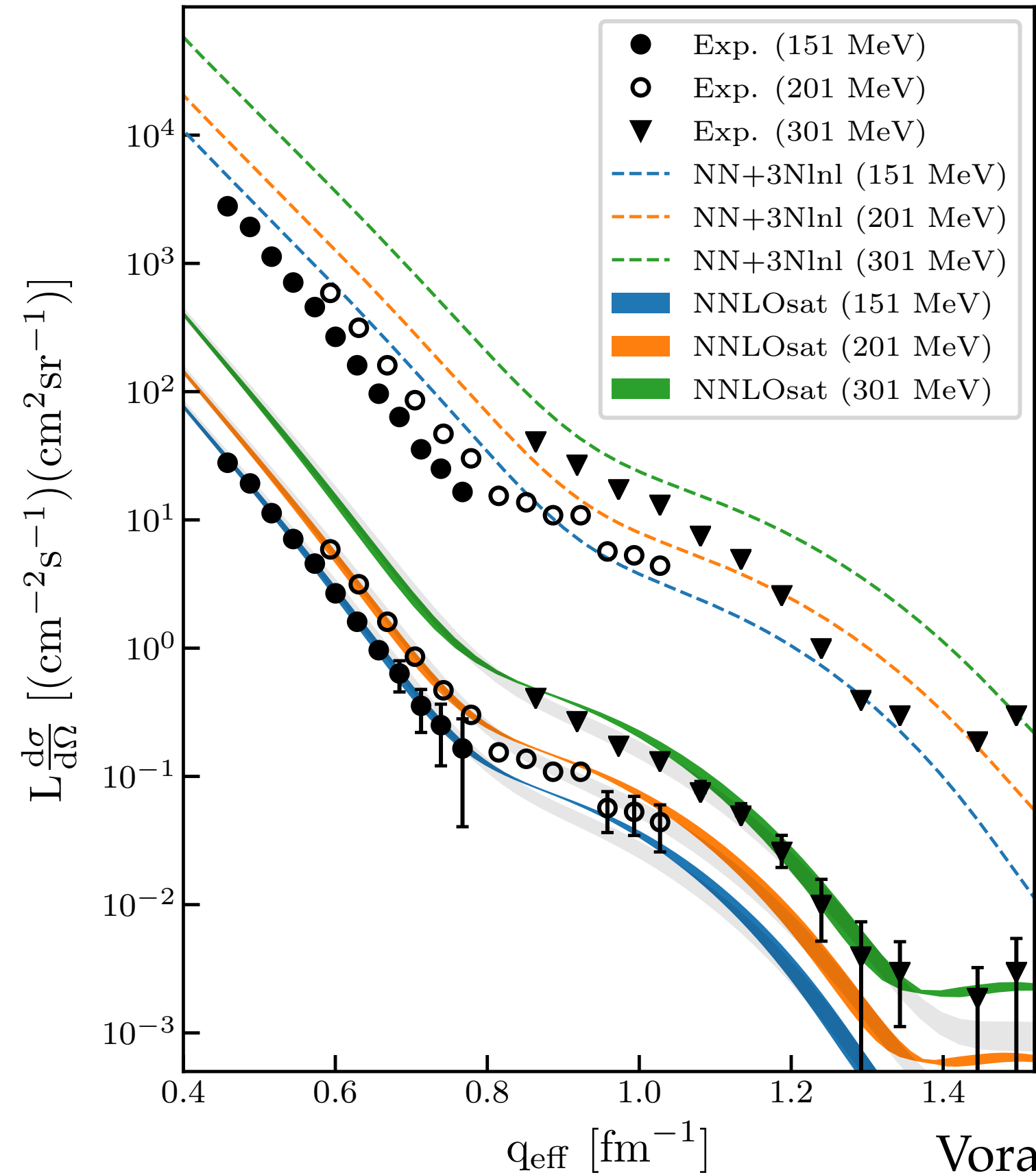


Extension to heavier nuclei - SCGF

Self Consistent Green's Function (**SCGF**)

$$U_{\mathbf{p}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \, \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) \, t_{\mathbf{p}N}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \, \rho_N(\mathbf{q}, \mathbf{P})$$

Somà, Frontiers 8 (2020) 340



Vorabbi et al., PRL **125**, 182501 (2020)

LO
 $(Q/\Lambda_\chi)^0$

NLO
 $(Q/\Lambda_\chi)^2$

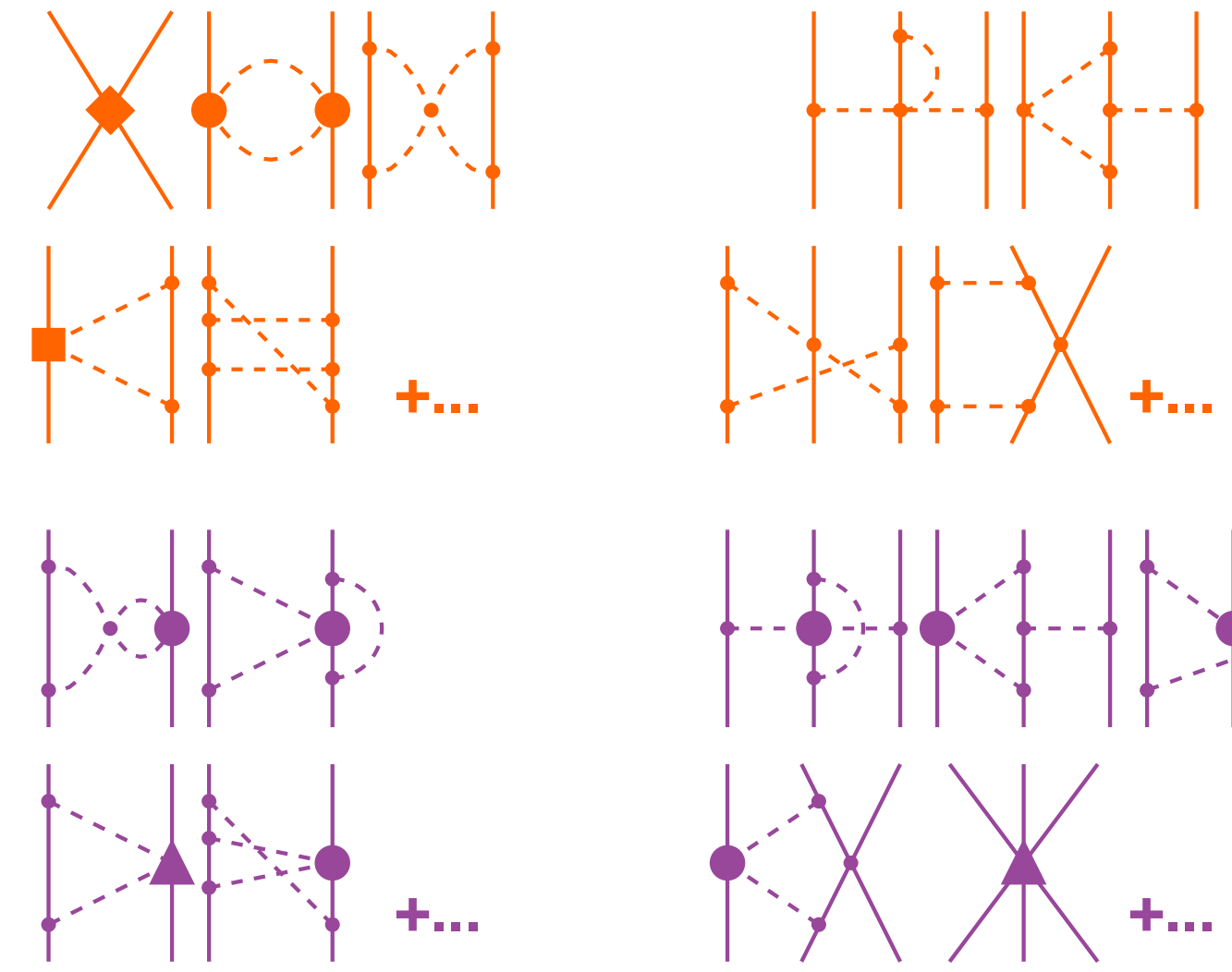
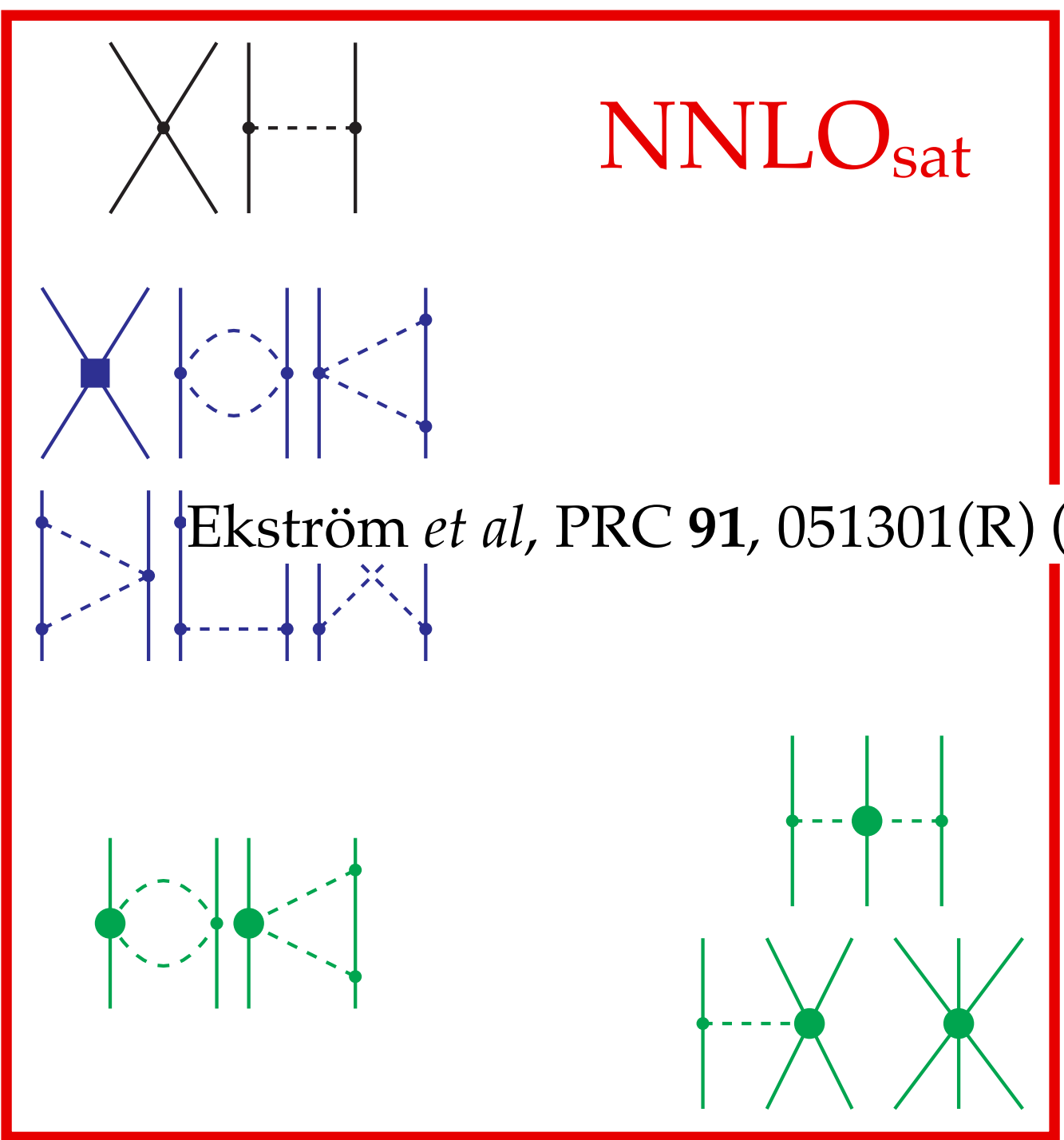
NNLO
 $(Q/\Lambda_\chi)^3$

N³LO
 $(Q/\Lambda_\chi)^4$

N⁴LO
 $(Q/\Lambda_\chi)^5$

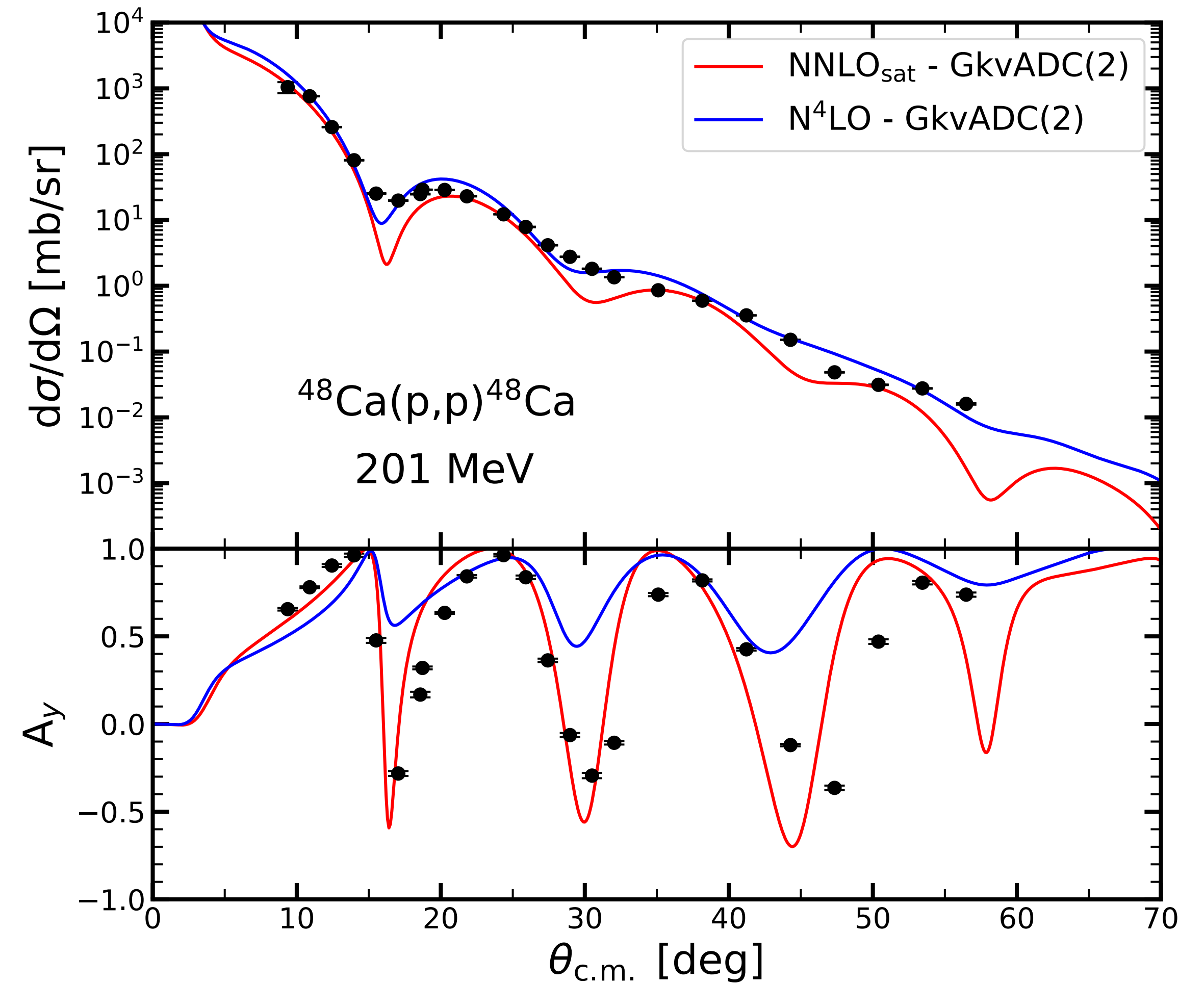
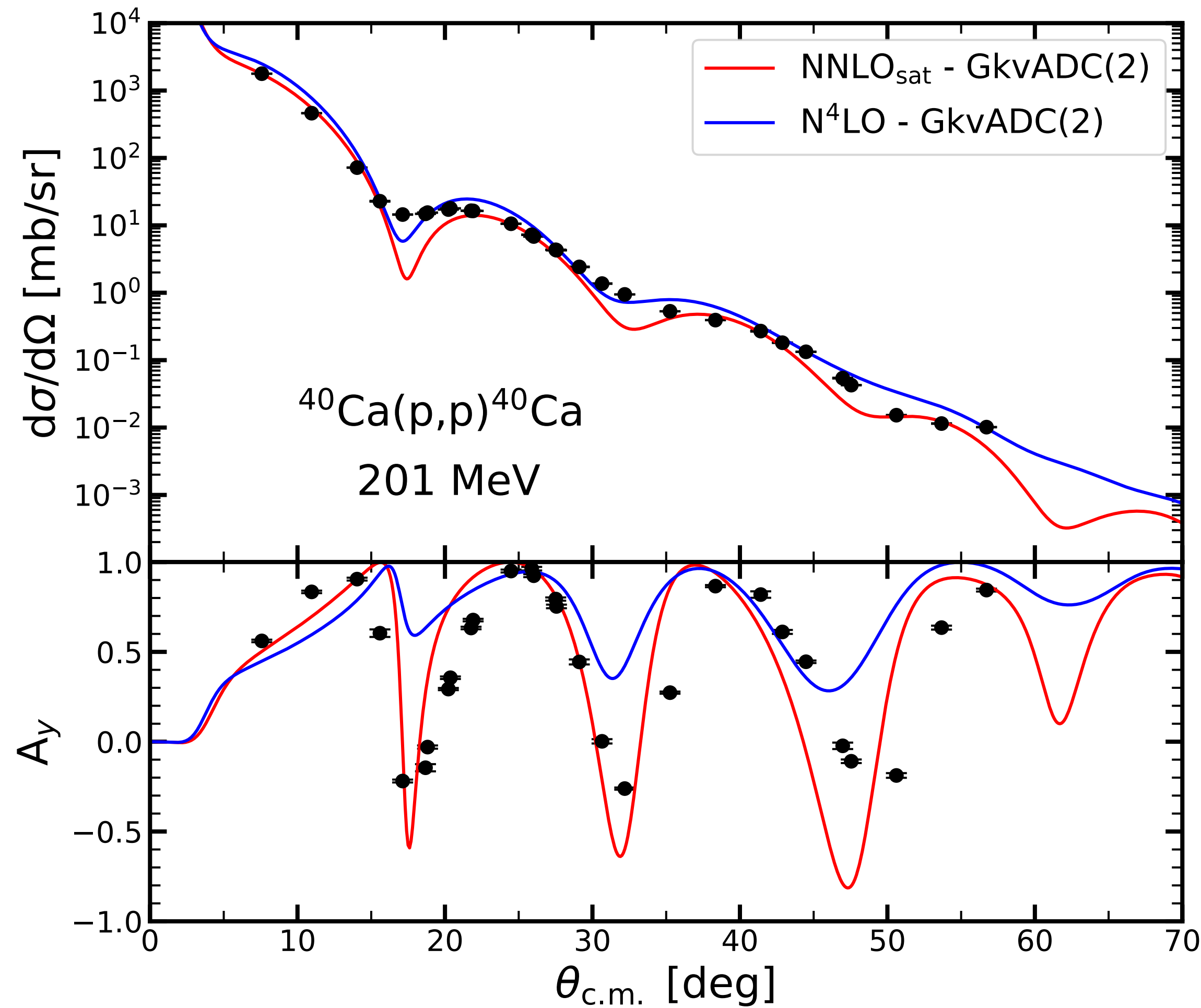
2N Force

3N Force



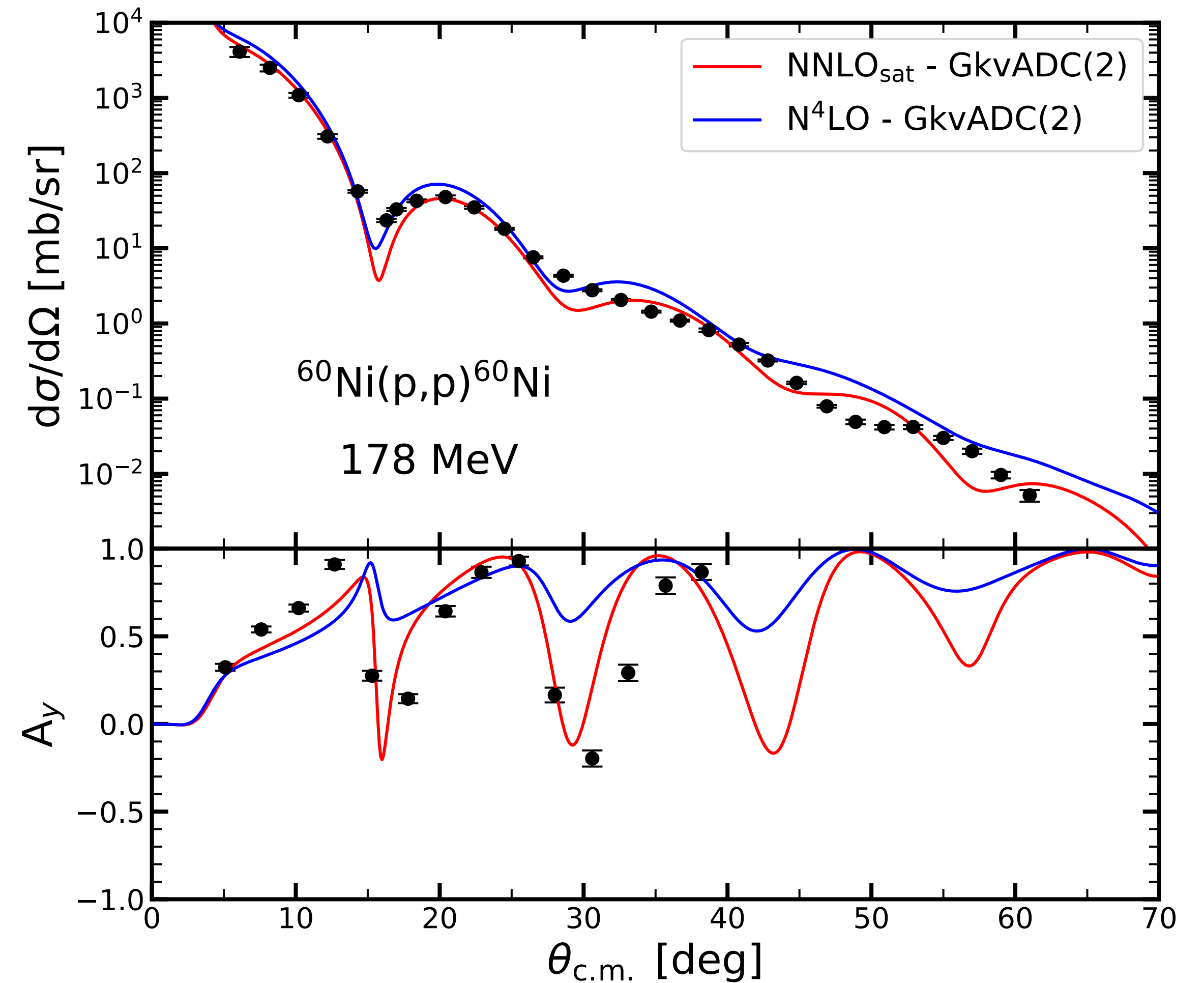
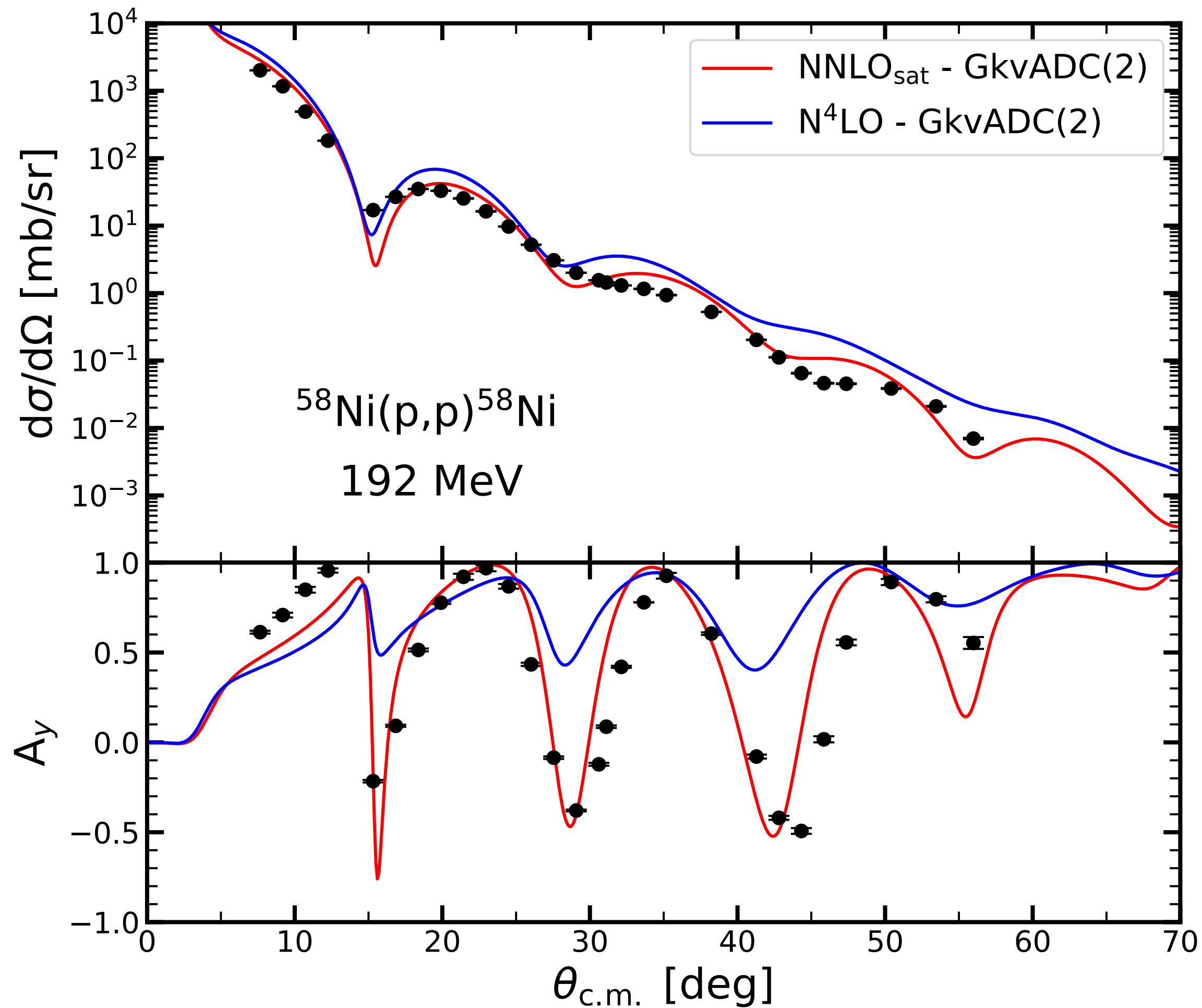
Results for proton scattering off $^{40,48}\text{Ca}$

Vorabbi et al., PRC **109**, 034613 (2024)



- First microscopic OP calculations for calcium and nickel from *ab initio* densities
- Densities are always computed with the NNLO_{sat}

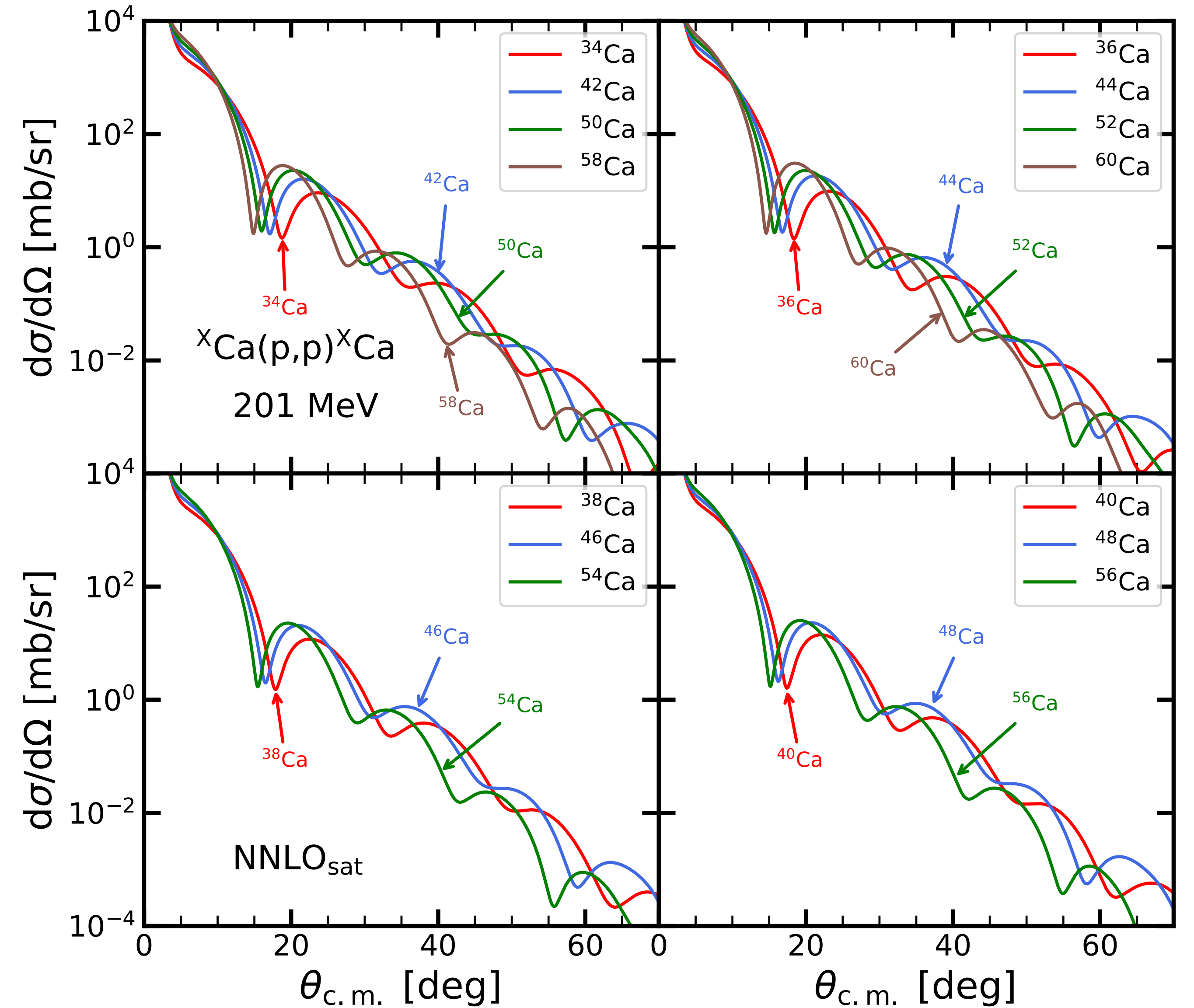
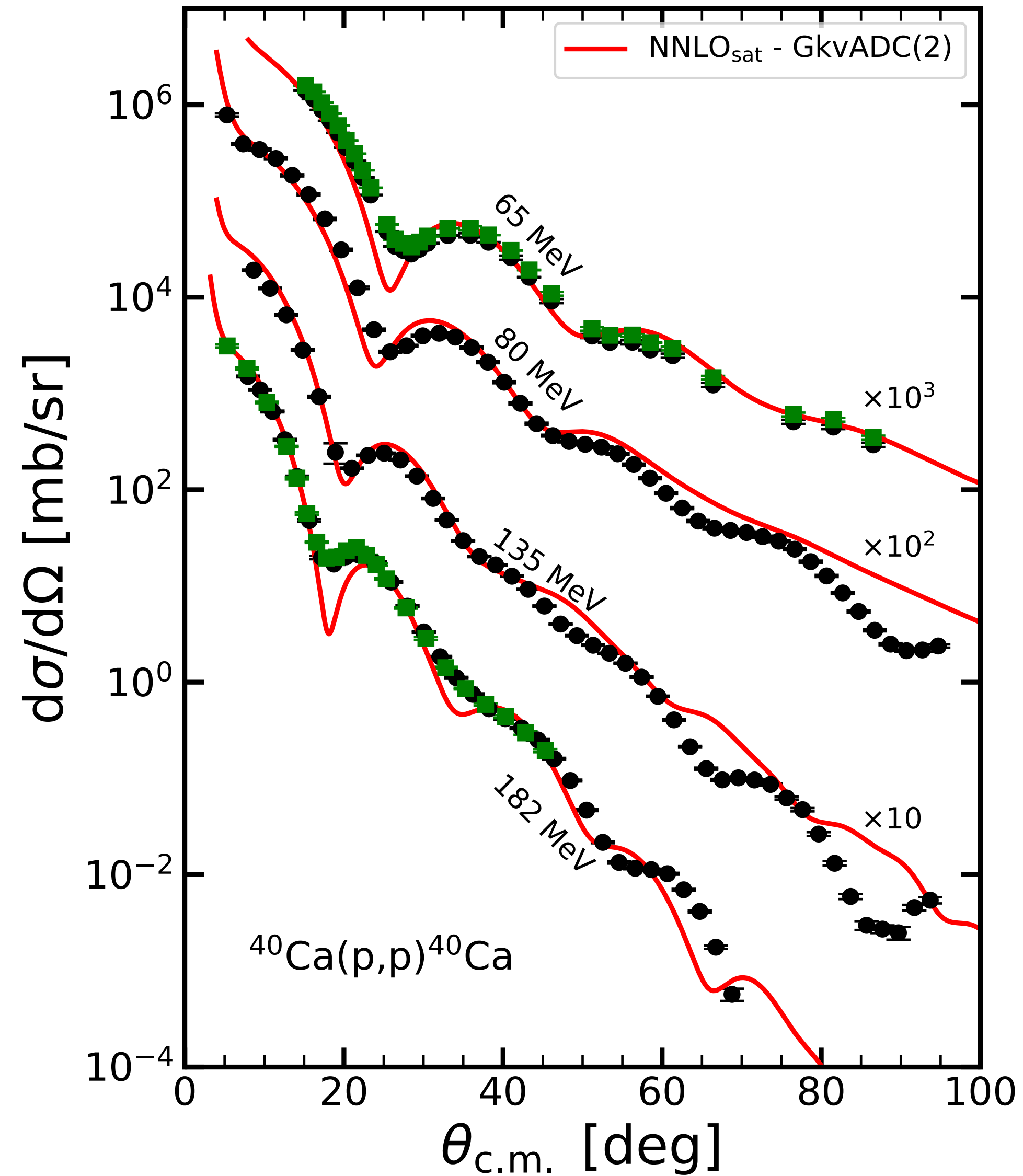
Results for proton scattering off $^{58,60}\text{Ni}$



The data for the analysing power are remarkably well described

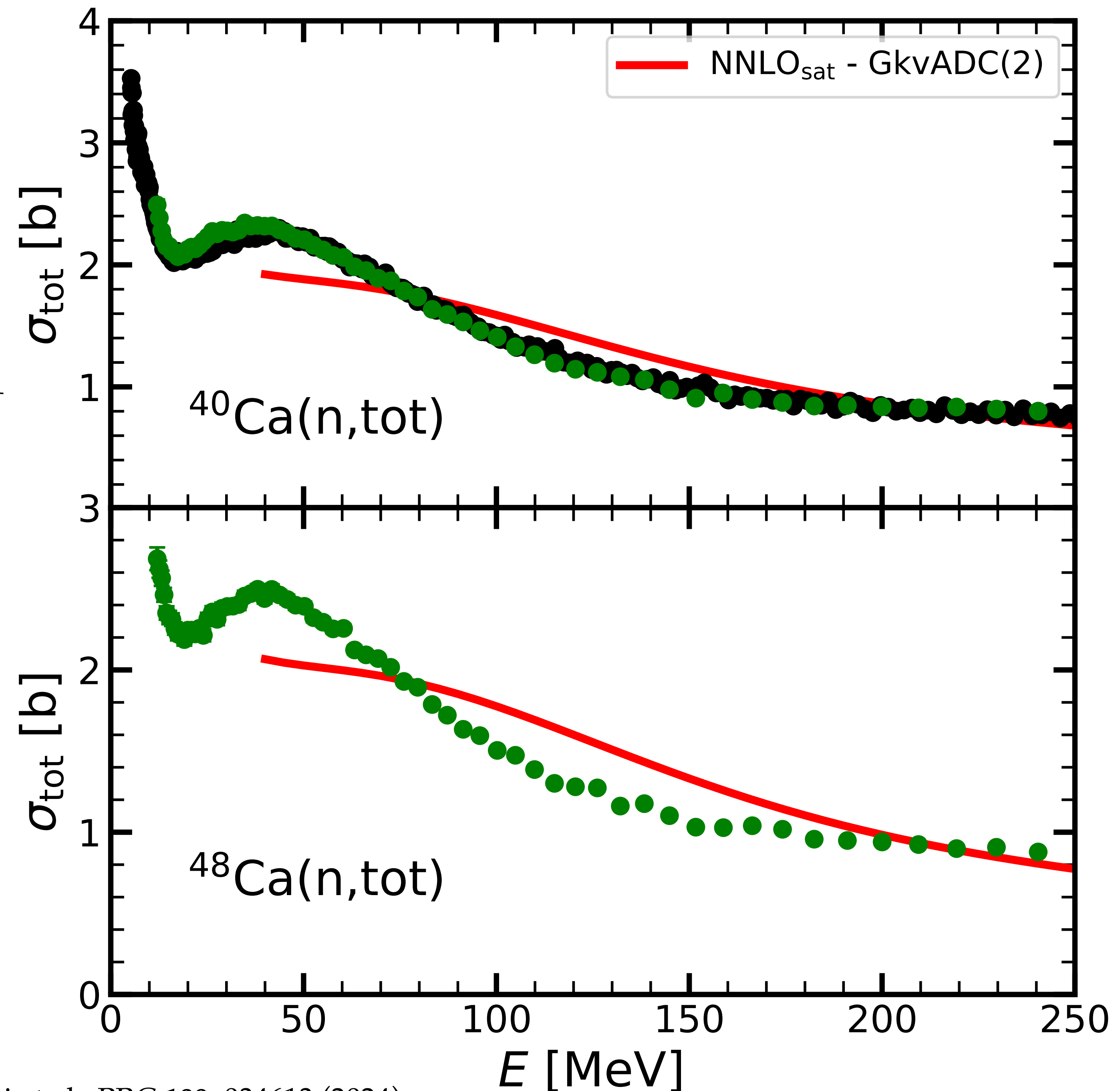
Results for Calcium isotopic chain

Vorabbi et al., PRC **109**, 034613 (2024)



Total cross sections

- At high energies, 180-250 MeV, our approach is able to describe the data
- The adopted impulse approximation gradually worsens as the energy decreases, without any abrupt divergence
- In the range 70-180 MeV, data are overestimated
- Below 70 MeV, the impulse approximation is questionable



Application of MST to inelastic scattering

The inelastic transition amplitude

$$T_{fi}^{\text{inel}} = \langle \psi_f^{(-)} | U_{\text{tr}} | \psi_i^{(+)} \rangle$$

Distorted waves

$$|\psi_i^{(+)}\rangle = |\Phi_i \mathbf{k}\rangle + G_0 U_{\text{el}} |\psi_i^{(+)}\rangle \quad \langle \psi_f^{(-)}| = \langle \mathbf{k}' \Phi_f| + \langle \psi_f^{(-)}| U_{\text{ex}} G_0$$

General expression of the potential

$$U = \sum_{k=1}^A t_{0k}$$

- U still defined within multiple scattering theory
- Impulse approximation
- New projection operators to obtain **OP** components

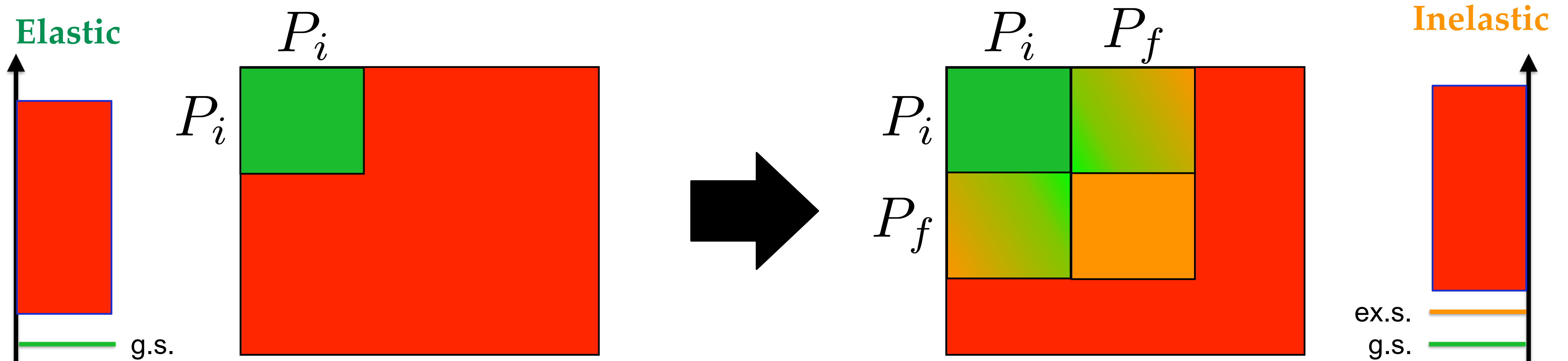
Application of MST to inelastic scattering

Projection operators

$$P_i = |\Phi_i\rangle\langle\Phi_i|$$

$$P_f = |\Phi_f\rangle\langle\Phi_f|$$

Eigenstates of
the target



Potentials

$$U_{\text{el}} = \sum_{k=1}^A P_i t_{0k} P_i$$

for the initial distorted wave

$$U_{\text{ex}} = \sum_{k=1}^A P_f t_{0k} P_f$$

for the final distorted wave

$$U_{\text{tr}} = \sum_{k=1}^A P_f t_{0k} P_i$$

Application of MST to inelastic scattering

The inelastic transition amplitude

$$T_{\text{inel}}(\mathbf{k}_*, \mathbf{k}_0) = \int d\mathbf{r}' \int d\mathbf{r} \psi^\dagger(\mathbf{k}_*, \mathbf{r}') U_{\text{tr}}(\mathbf{r}', \mathbf{r}) \psi(\mathbf{k}_0, \mathbf{r})$$

Required potentials

$$U_{\text{ex}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{pN}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N^{(\text{ex})}(\mathbf{q}, \mathbf{P})$$

$$U_{\text{tr}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{pN}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N^{(\text{tr})}(\mathbf{q}, \mathbf{P})$$

$$U_{\text{gs}}(\mathbf{q}, \mathbf{K}) = \sum_{N=p,n} \int d\mathbf{P} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}) t_{pN}(\mathbf{q}, \mathbf{K}, \mathbf{P}) \rho_N^{(\text{gs})}(\mathbf{q}, \mathbf{P})$$

Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)

Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)

Inelastic scattering: first results

- The general shape of the data is reproduced
- The NN t matrix adopted for the calculation of the 3 potentials contains only two terms so far

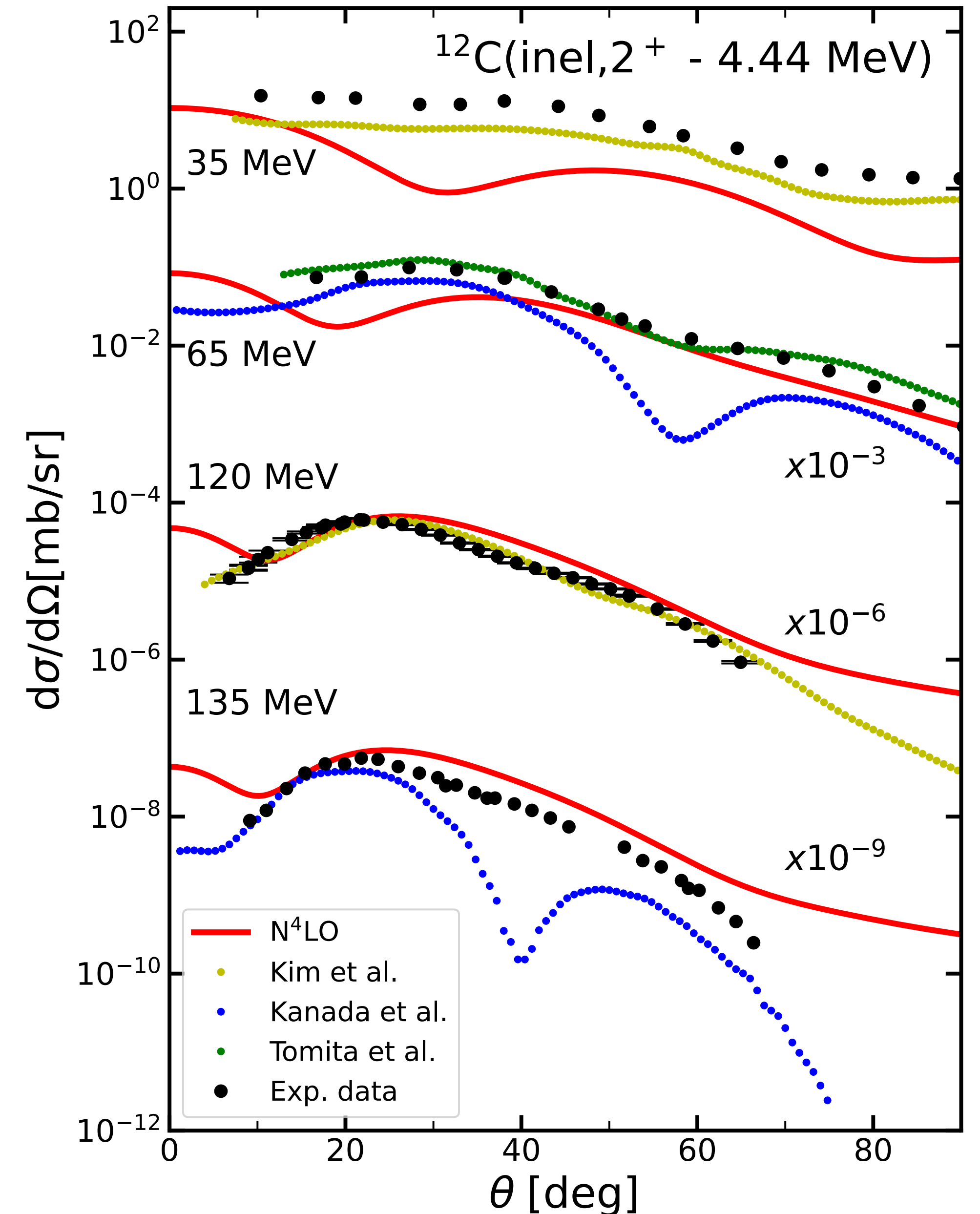
$$A + i(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}})C$$

- No adjustments, no free parameters
- Large angles behaviour needs improvement

Theoretical $B(E2; 0^+ \rightarrow 2^+) = 35.2987 \text{ e}^2 \text{ fm}^4$

Experimental $B(E2; 0^+ \rightarrow 2^+) = 39.7(33) \text{ e}^2 \text{ fm}^4$

Vorabbi et al., submission to PRC (2025)



Inelastic scattering: first results

Vorabbi et al., submission to PRC (2025)

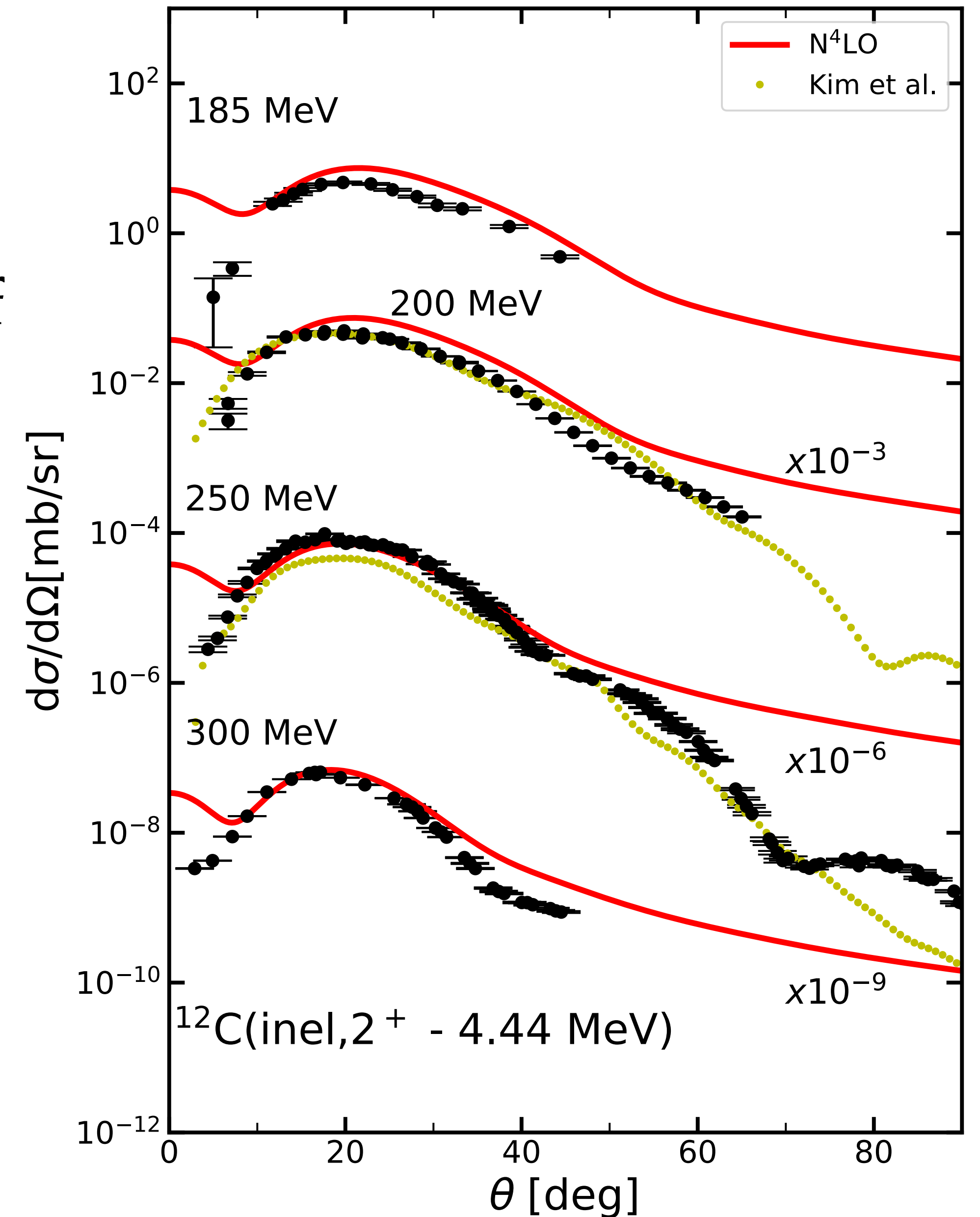
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OP for nucleus-nucleus elastic scattering

Transition amplitude for elastic scattering

$$T_{\text{el}} \equiv PTP = PUP + PUPG_0(E)T_{\text{el}}$$

The spectator expansion

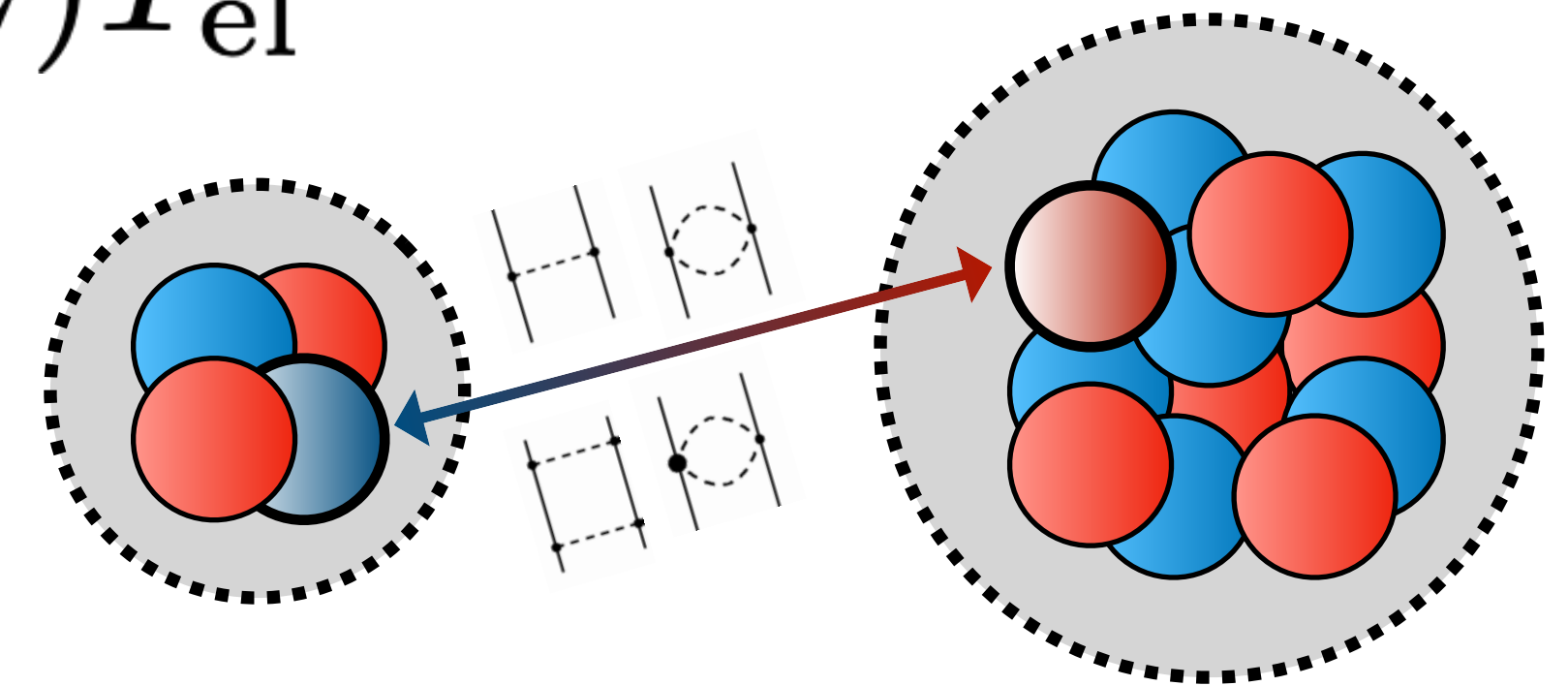
$$U \simeq \sum_{i=1}^A \sum_{j=A+1}^{A+B} \tau_{ij}$$

$$\tau_{ij} = v_{ij} + v_{ij}G_0(E)\tau_{ij}$$

$$\tau_{ij} \approx t_{ij} = v_{ij} + v_{ij}g_0(E)t_{ij}$$

Impulse approximation

Two-body propagator



(A+B)-body propagator
More complicated than
the NA case

First-order OP for nucleus-nucleus elastic scattering

Møller factor

$$t_{\mathbf{p}N}^{(NA)} = \eta t_{\mathbf{p}N}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

$$U(\mathbf{q}, \mathbf{K}) = \sum_{\alpha=n,p} \sum_{\beta=n,p} \int d\mathbf{P} \int d\mathbf{Q} \eta(\mathbf{q}, \mathbf{K}, \mathbf{P}, \mathbf{Q}) t_{\alpha\beta}(\mathbf{q}, \mathbf{K}, \mathbf{P}, \mathbf{Q}; \mathcal{E}) \rho_{\alpha}^{(\mathbb{P})}(\mathbf{q}, \mathbf{P}) \rho_{\beta}^{(\mathbb{T})}(\mathbf{q}, \mathbf{Q})$$

Projectile density

Target density

Free two-body scattering matrix

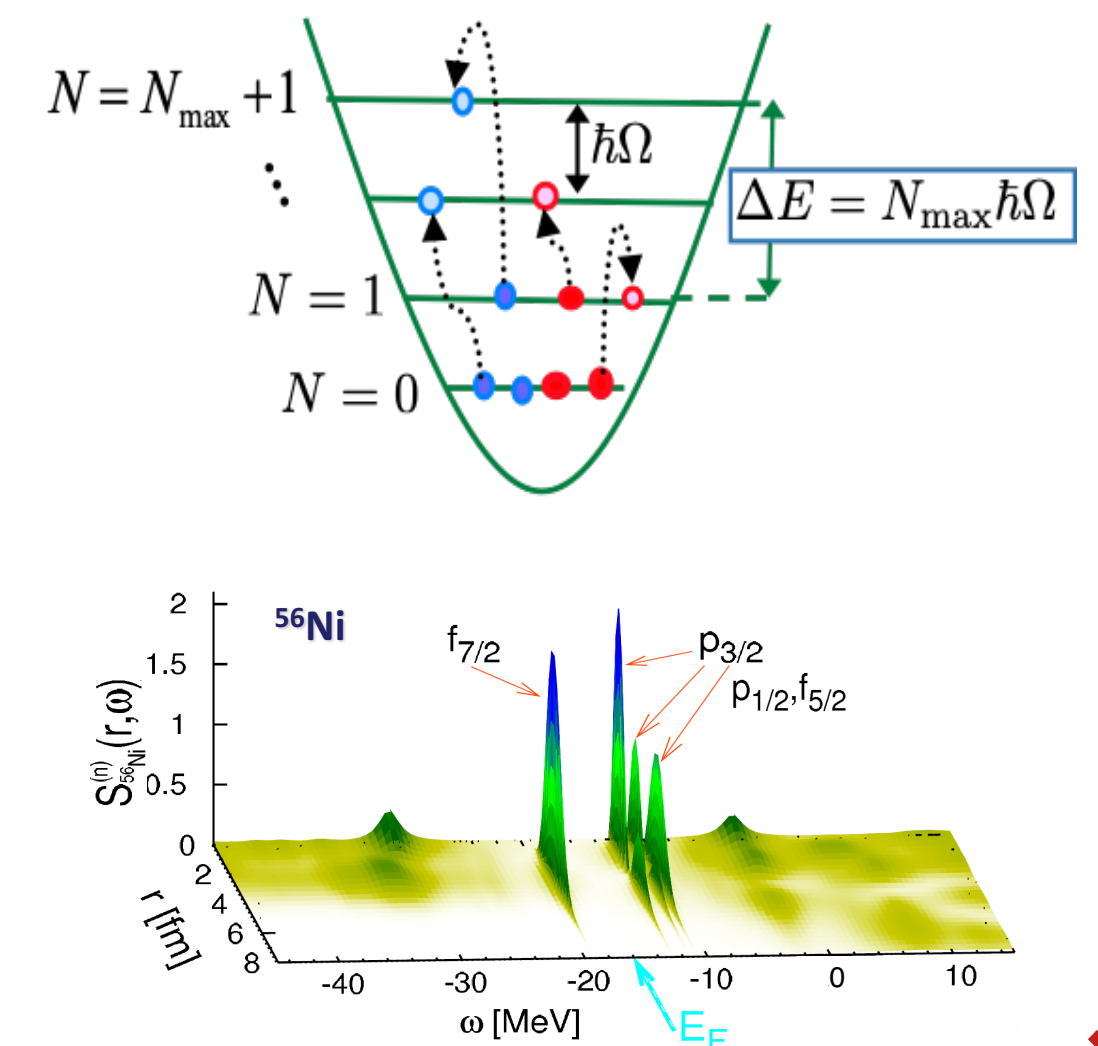
$$t_{0i} = v_{0i} + v_{0i} g_{0i} t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only NN interaction

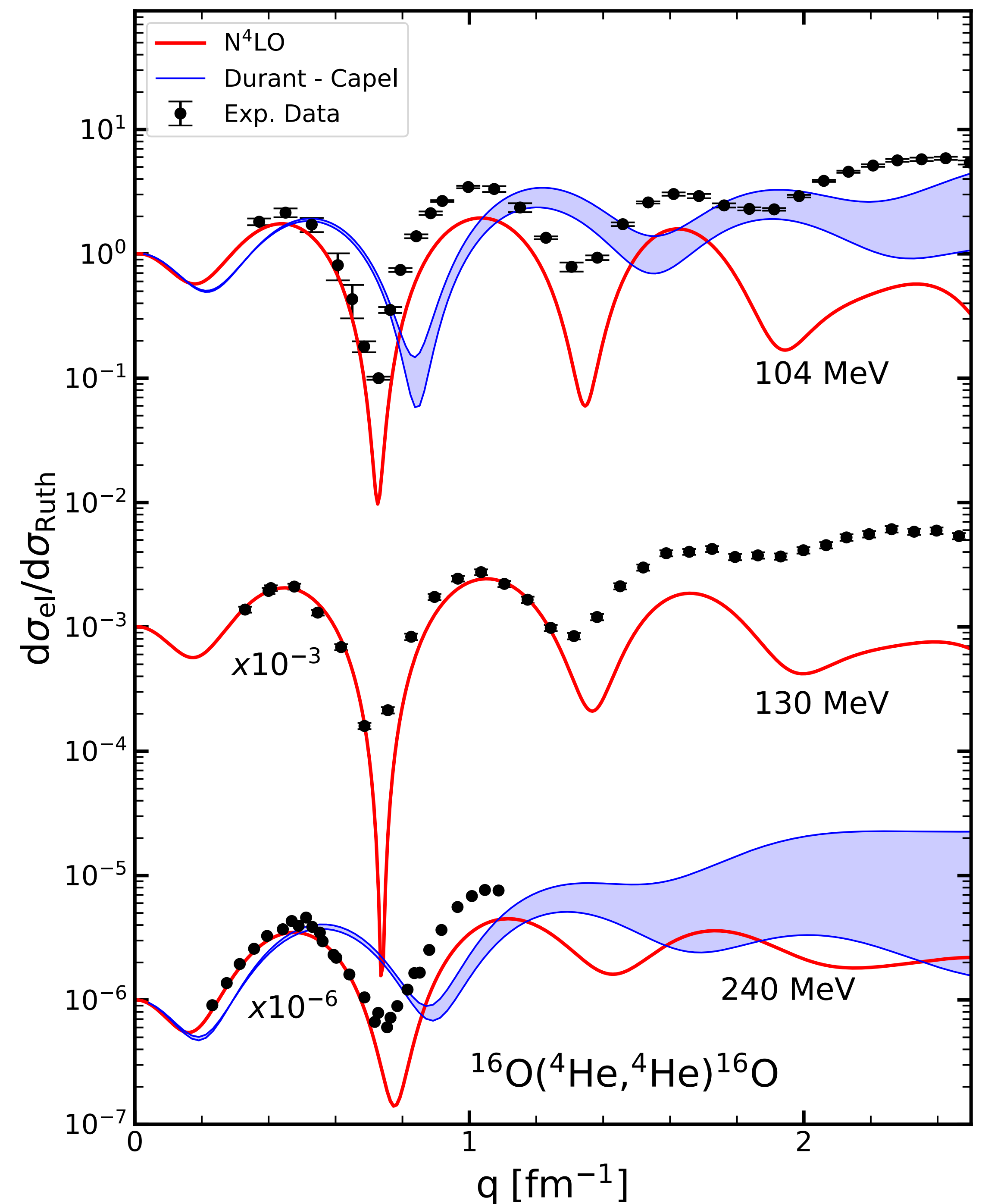
Nonlocal one-body density

- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with NN and 3N interaction



Elastic ^4He - ^{16}O scattering

- Parameter-free predictions
- Nice reproduction of experimental data at low-momentum transfer ($q \lesssim 1 \text{ fm}^{-1}$)
- Underestimates the high q behaviour, too much absorption?
- Competitive with a chiral microscopic *inspired* model by Durant and Capel

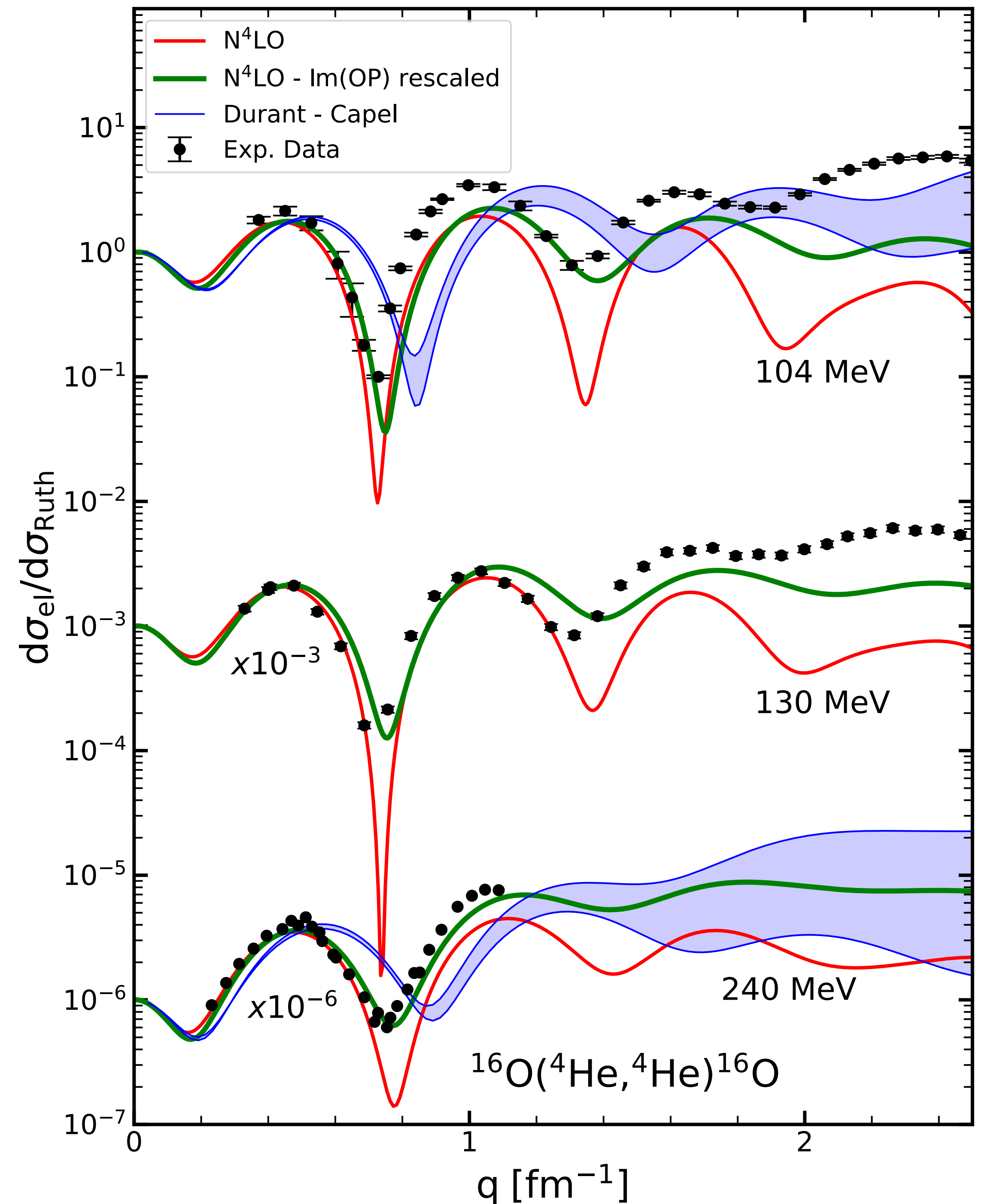


Elastic ^4He - ^{16}O scattering

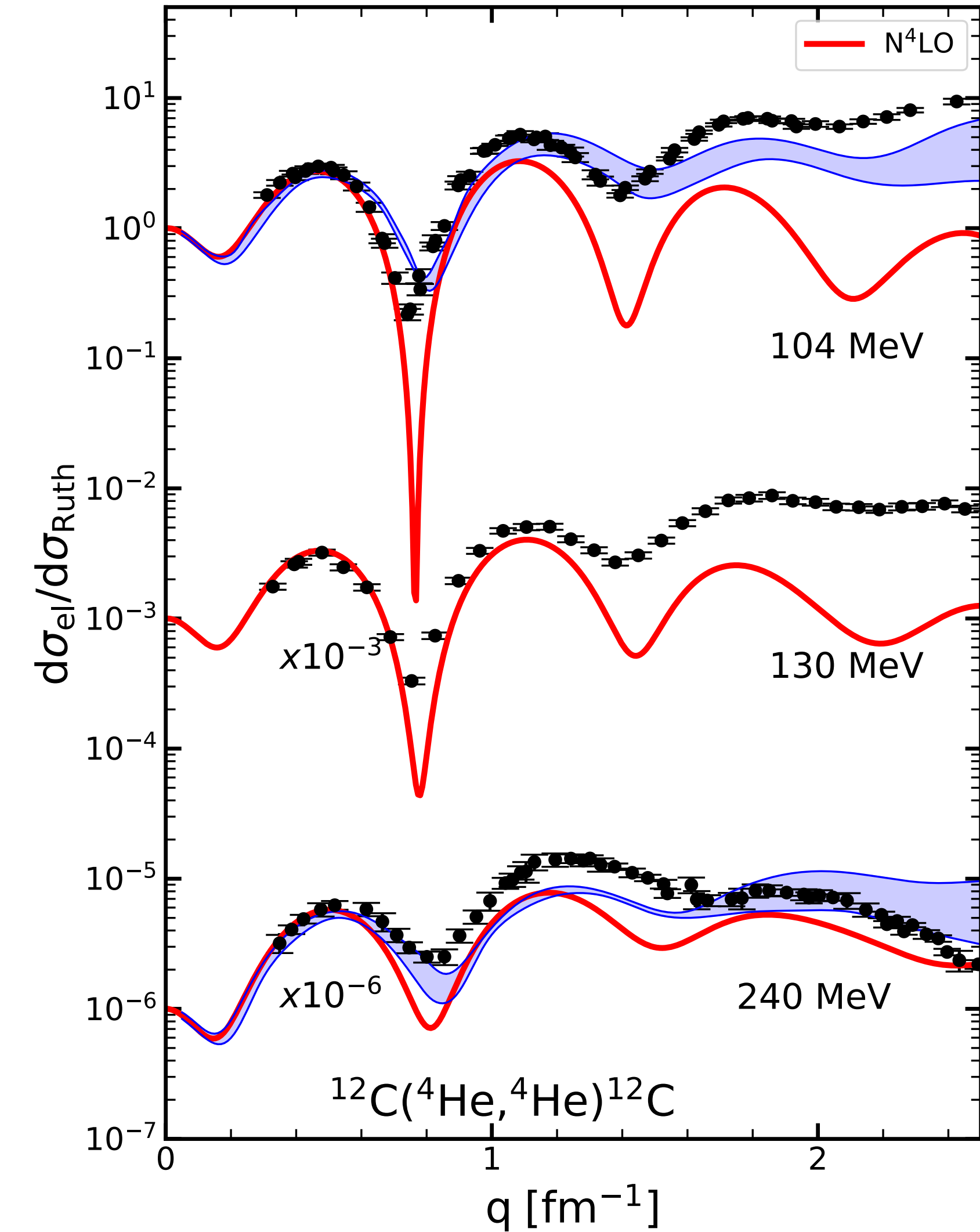
How to solve the high- q behaviour? Very likely the introduction of higher-order effects would help

In the meantime, inspired by phenomenological optical potentials, we can reduce by hand the imaginary part

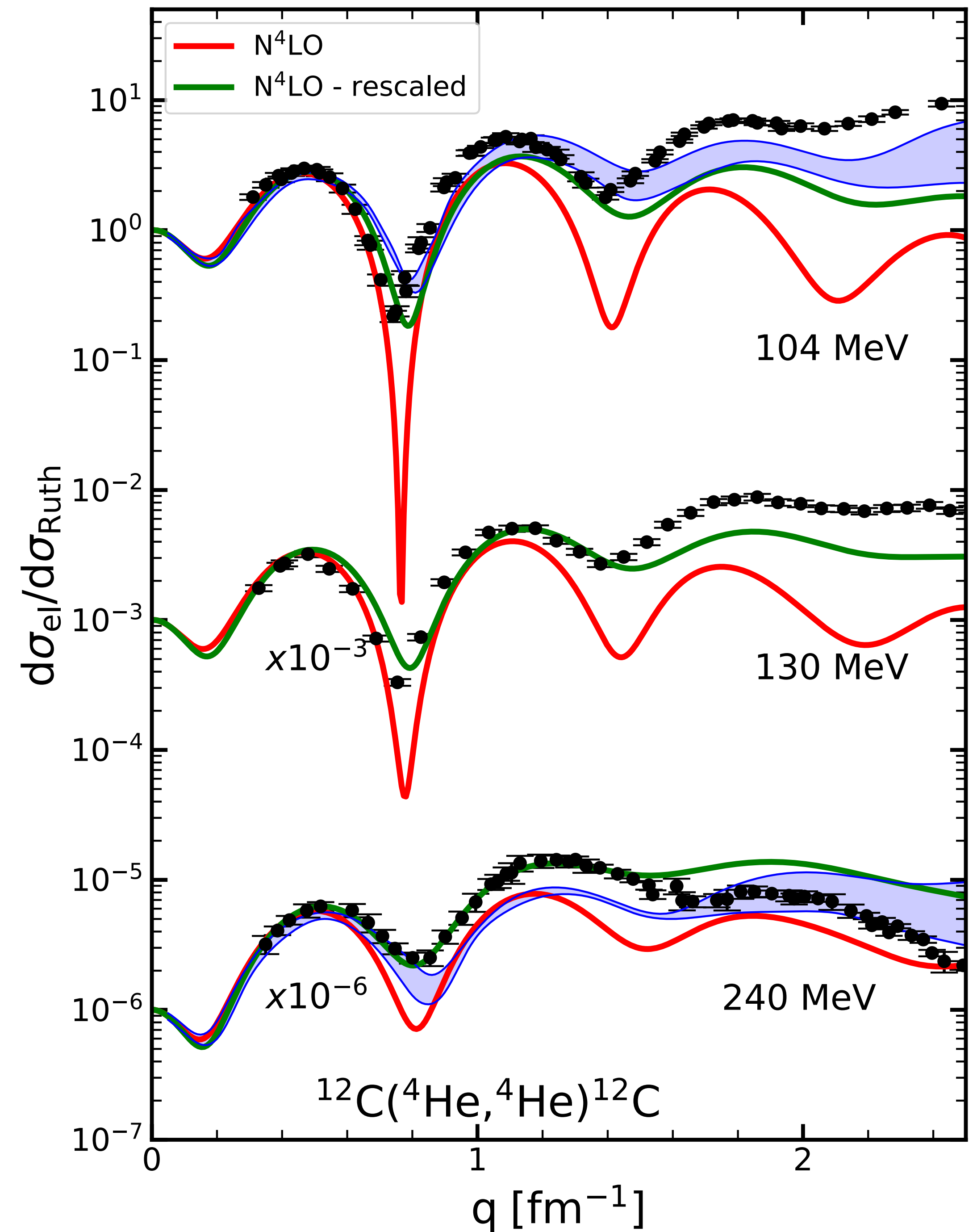
A guess in the range of 0.5/0.6 could be tested. We have chosen 0.5 (**green line**). The agreement is nicely improved.



Elastic ^4He - ^{12}C scattering



- Same behaviour as in the oxygen case.
- A highly non-trivial fact is the quality of agreement along different energies without any other corrections aside the absorption reduction.



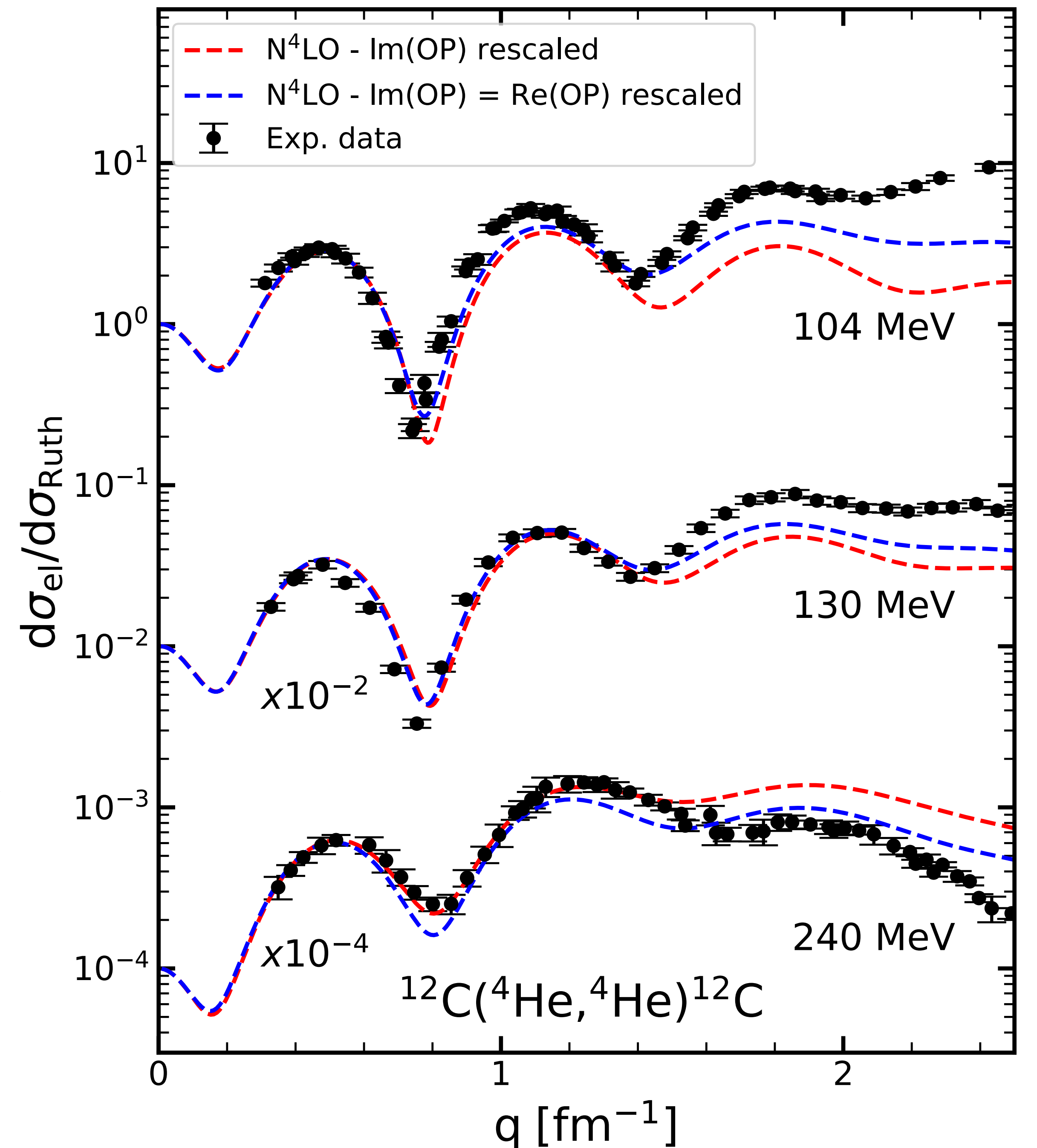
Elastic ^4He - ^{12}C scattering

It is interesting to study a connection between phenomenology and theoretically inspired approaches.

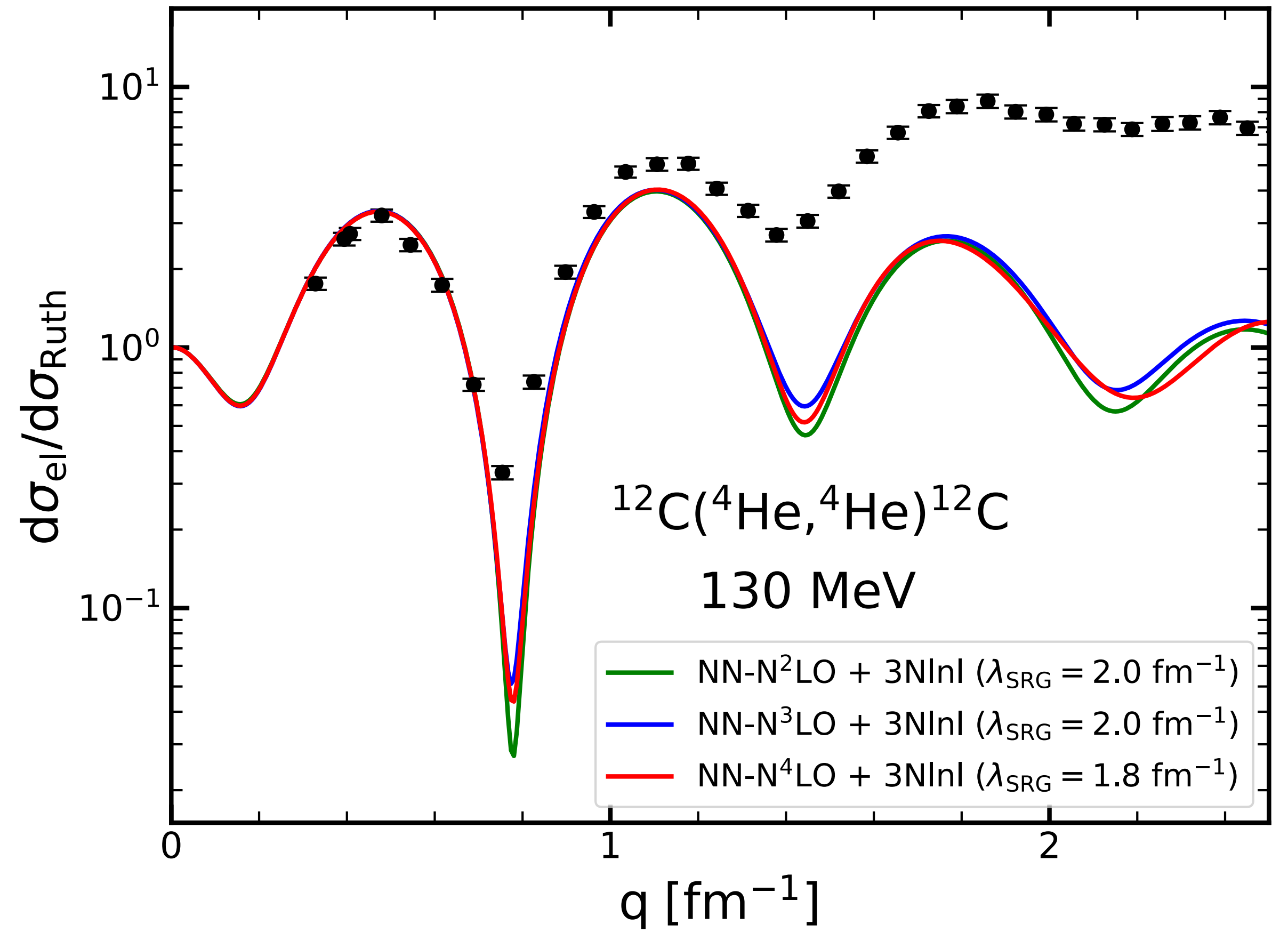
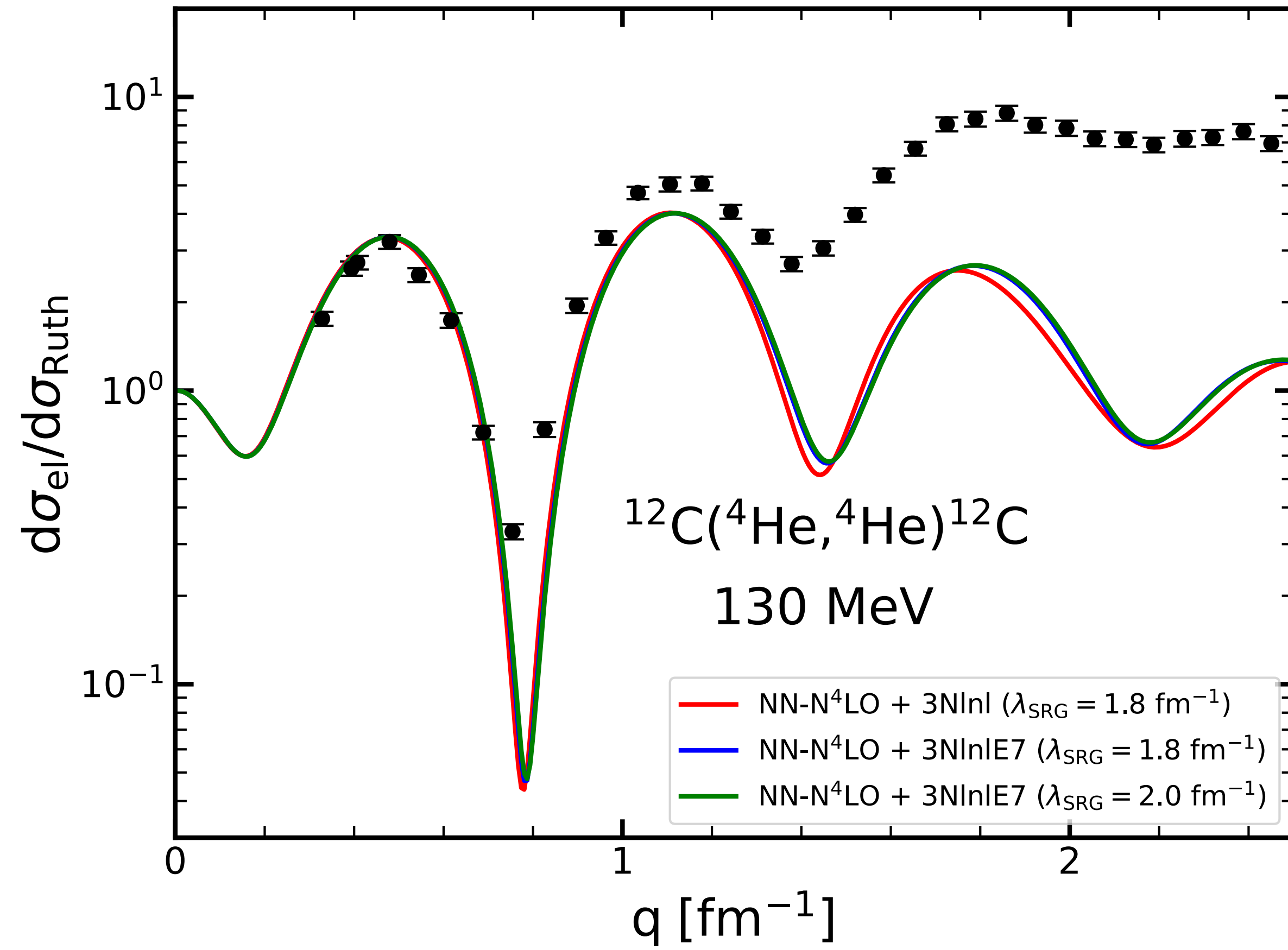
We tested the usual adopted convention that the imaginary component of the optical potential could be taken as follows

$$\text{Im } U_{\text{opt}} = 1/2 \text{ Re } U_{\text{opt}}$$

meaning that from a phenomenological point of view they have the same functional dependence. We confirm the prescription from a microscopic approach.



Nucleus-nucleus scattering: NN sensitivity



Conclusions

- ☑ Achieved a satisfactory description of nucleon **elastic** scattering of light and medium-mass nuclei at the first order of the spectator expansion
- ☑ Extension of the model to nucleon-nucleus **inelastic** scattering
- ☑ Achieved a first step in the derivation of a **nucleus-nucleus** optical potential
- ☐ Inclusion of medium effects / higher order effects (soon, end of 2025)
- ☐ Extend the high-energy limits of applicability of the optical potential
- ☐ Inclusion of the second-order term of the spectator expansion
- ☐ Consistent treatment of the full 3N interaction
- ☐ Extend the formalism to transfer/charge-exchange (**long term goal**)