# Charm and beauty quarks in the initial stages of proton-Ion Collisions

**Gabriele Parisi**

in collaboration with

Fabrizio Murgana, Lucia Oliva, Vincenzo Greco, Marco Ruggieri

XX Conference on Theoretical Nuclear Physics in Italy
October 2, 2025

# Outline

**General Topic**

**Heavy Ion Collisions**

**Specific Topic**

**Initial Stages**

of Heavy Ion Collisions

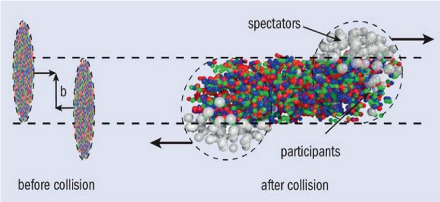
**More Specific Topic**

**Heavy Quarks**

in Initial Stages

of Heavy Ion Collisions

(my work)




D. Rashid, 2103.16899 [hep-ph]

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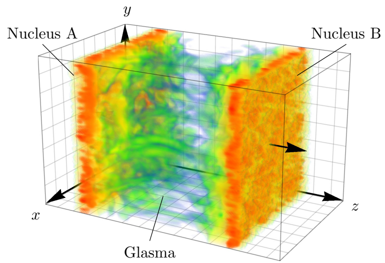


A. Ipp and D. Müller, Phys.Lett.B 771 (2017) 74-79

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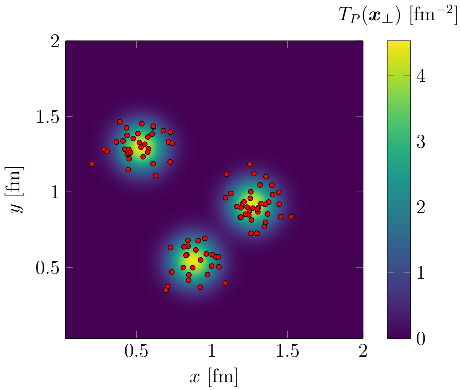
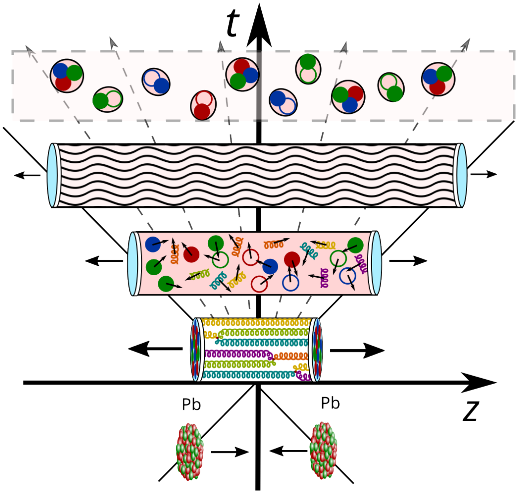



L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

# Heavy-ion collisions




Final stages $\tau \geq 10\,\mathrm{fm/c}$

$\uparrow$

Local equilibrium $\tau \sim 10\,\mathrm{fm/c}$

$\uparrow$

QGP phase $\tau \sim 1\,\mathrm{fm/c}$

$\uparrow$

Initial stage $\tau \sim 0.1\,\mathrm{fm/c}$

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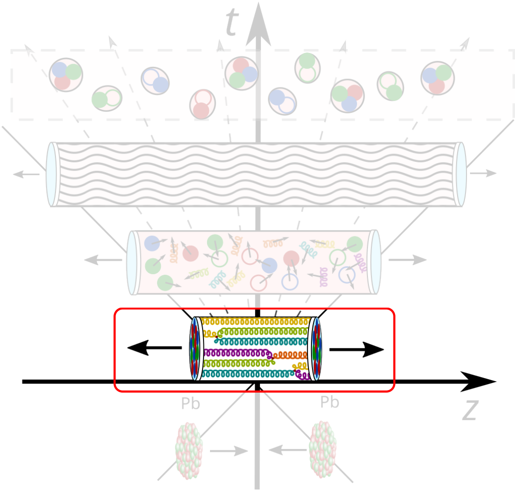
Before the collision $\tau \leq 0\,\mathrm{fm/c}$

A. Mazeliauskas: Stages of heavy ion collision

# Heavy-ion collisions

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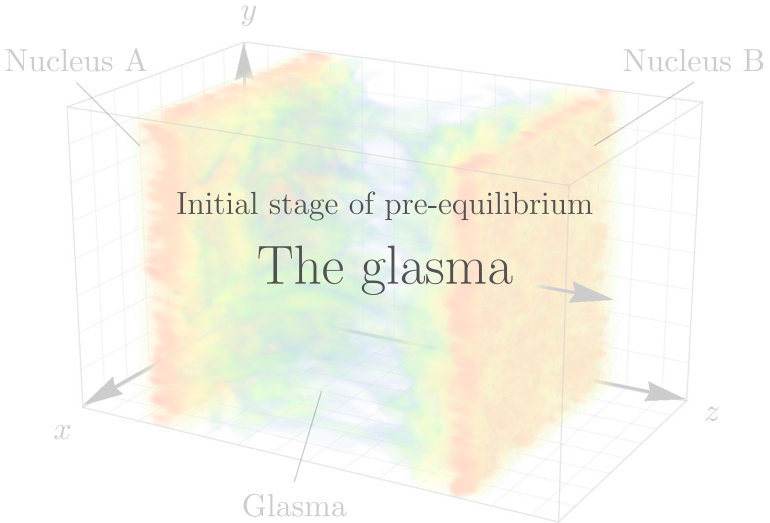
Before the collision $\tau \leq 0\,\mathrm{fm/c}$

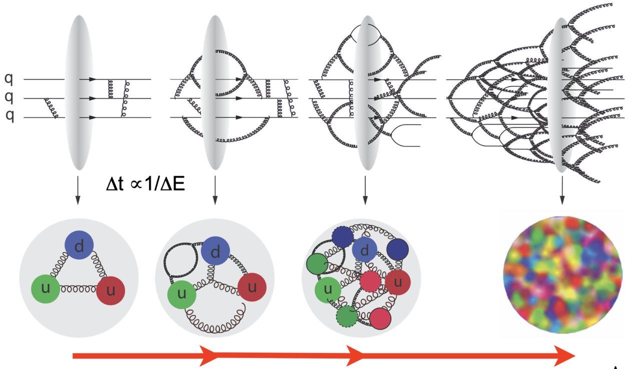
A. Mazeliauskas: Stages of heavy ion collision

Initial stage of pre-equilibrium

# The glasma



# BFKL evolution

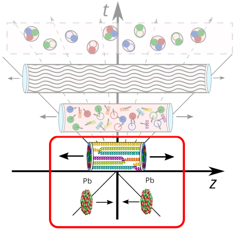



Smaller Bjorken $x$

At low energy,
valence quarks only

At high energy,
the nucleus contains
many soft gluons

# QCD at high energy




At LHC energies, collision among **dense gluon distributions**

$\Downarrow$

Emergence of a saturation scale $Q_s \sim 2$ GeV: **Glasma**

$\Downarrow$

described by the **Color Glass Condensate** formalism

L. McLerran and R. Venugopalan, Phys.Rev.D 49 (1994) 2233-2241
Phys.Rev.D 49 (1994) 3352-3355, Phys.Rev.D 50 (1994) 2225-2233

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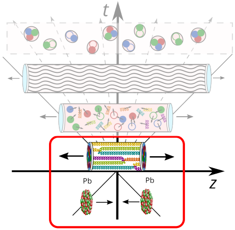


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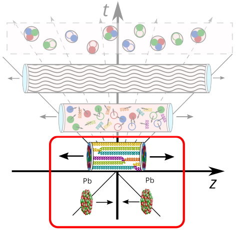


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# Color Glass Condensate

**Color**
QCD degrees of freedom

**Condensate**
High-gluon density,
classical dynamics

**Glass**
Quarks are static sources,
due to time dilation

L. McLerran and R. Venugopalan, Phys.Rev.D 49 (1994) 2233-2241
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# Color Glass Condensate in 2+1D

McLerran-Venugopalan (MV) initial conditions

$$\langle \rho^a(\mathbf{x}_T) \rangle = 0,$$

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T) \rangle = (g\mu)^2 \delta^{ab} \delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T),$$

$J^\mu$ by the hard partons $\Longrightarrow$ $A^\mu$ of the soft partons

Classical dynamics of glasma, Yang Mills equations

$$\mathcal{D}_\mu F^{\mu\nu} = 0$$

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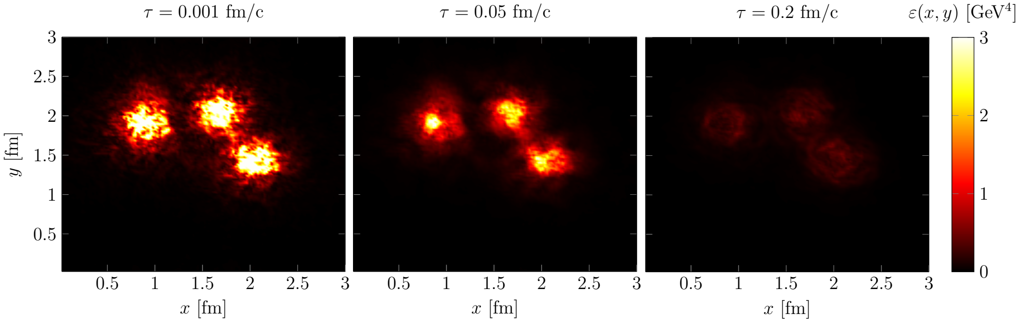
# Features of the glasma

## Energy density in pA vs $(x, y)$

$$\varepsilon = \mathrm{Tr}[E_L^2 + B_L^2 + E_T^2 + B_T^2]$$

We can have $\mu$ space-dependent: **proton structure**




Fields dilute after $\delta\tau \sim Q_s^{-1} \sim \mathcal{O}(0.1\ \mathrm{fm/c})$

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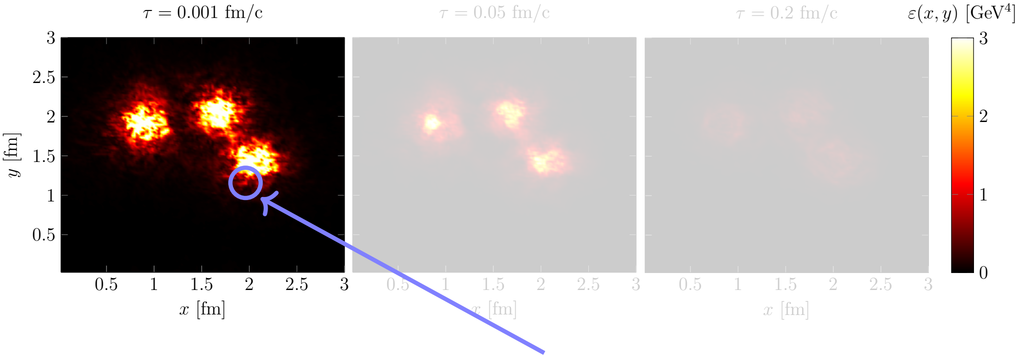
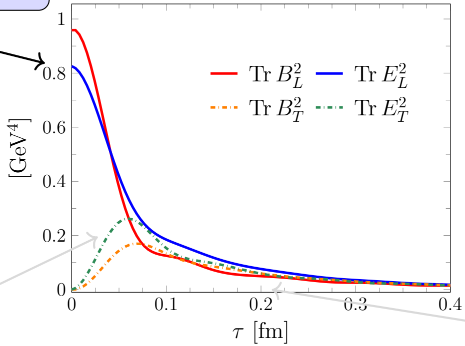



Hotspots of size $\delta x_T \sim Q_s^{-1} \sim \mathcal{O}(0.1 \text{ fm})$

# Features of the glasma

Fields in pA vs $\tau$

Longitudinal fields $\neq 0$




Transverse fields emerge later

Dilute fields after $\sim 0.1$ fm/c

32x32x32 lattice, $N_\tau = 400$

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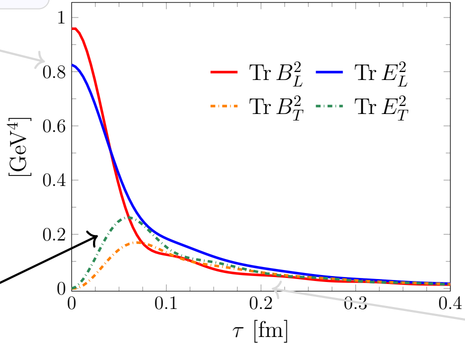


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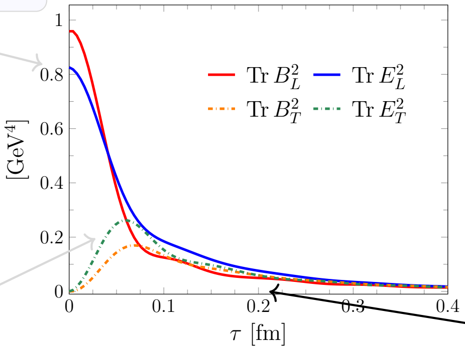


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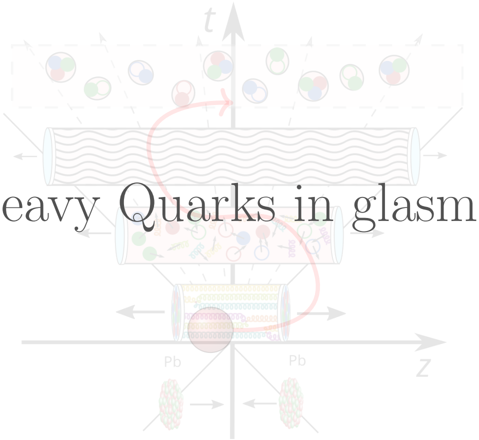
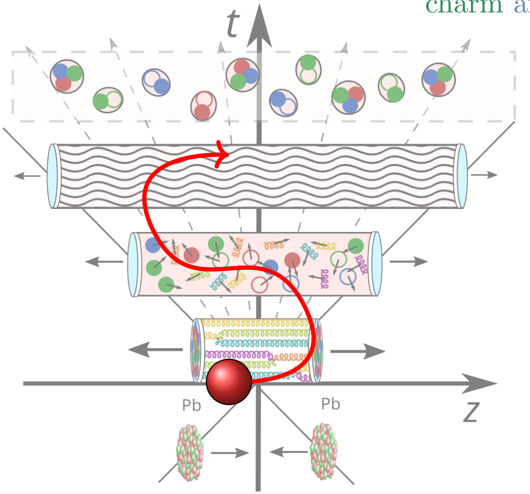
# Heavy Quarks in glasma



Check also this talk:
M. L. Sambataro Oct 2, 2025

# Heavy quarks

charm and beauty




**High mass**: probes of initial stages

- $M \gg \Lambda_{\mathrm{QCD}}$:
  early pQCD production
- $M \gg T_{\mathrm{QGP}}$:
  no thermal production
- $\tau_{\mathrm{creation}} \ll \tau_{\mathrm{QGP}}$:
  probes of the collision

$$M_{\mathrm{charm}} = 1.3 \text{ GeV}, \; M_{\mathrm{beauty}} = 4.2 \text{ GeV}$$

$$\tau_{\mathrm{creation}} \sim 1/2M$$

A. Mazeliauskas: Stages of heavy ion collision

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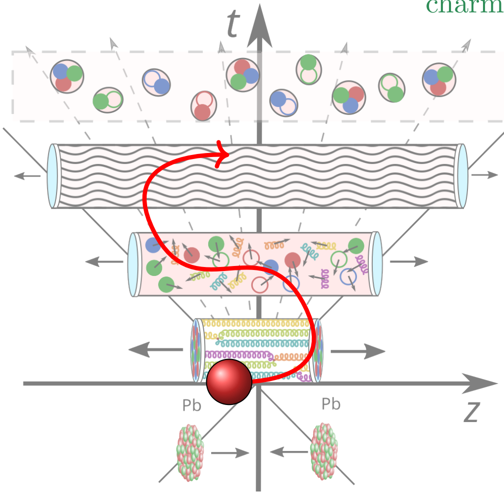


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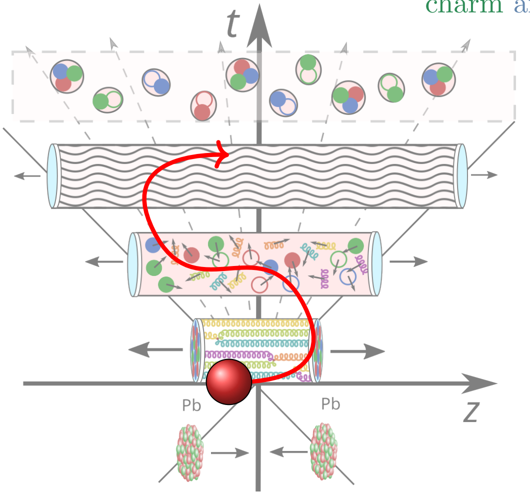


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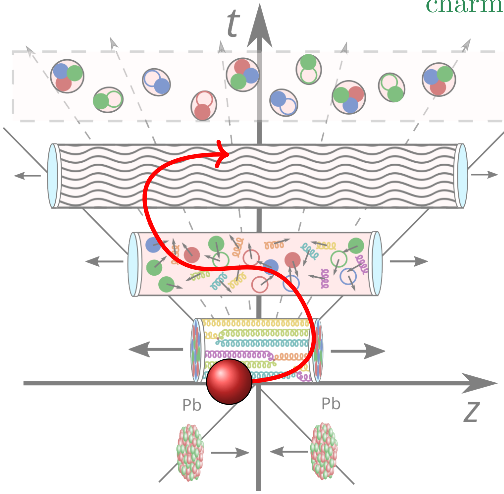


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# Heavy Quarks in Yang-Mills fields

## Wong equations

Classical transport in Yang-Mills background field $A^\mu$

**Coordinate** evolution

$$\frac{\mathrm{d}}{\mathrm{d}\tau} x^\mu = \frac{p^\mu}{m}$$

**Momentum** evolution

$$\frac{\mathrm{d}}{\mathrm{d}\tau} p^\mu = \frac{1}{T_R} g \,\mathrm{Tr}\left\{ Q F^{\mu\nu} \right\} \frac{p_\nu}{m}$$

**Color charge** evolution

$$\frac{\mathrm{d}}{\mathrm{d}\tau} Q = -\mathrm{i} g [A_\mu, Q] \frac{p^\mu}{m}$$

S. K. Wong, Nuovo Cim. A 65 (1970) 689-694
U. W. Heinz, Annals Phys. 161 (1985) 48

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*Basically* $\mathbf{p} = m\mathbf{v}$

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*Basically* $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

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*Basically* Color Rotation in SU(3)

Preserves Casimir invariants $q_2 = Q^a Q^a$ and $q_3 = d_{abc} Q^a Q^b Q^c$

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# Heavy Quarks in Yang-Mills fields: pair potential

$$\frac{\mathrm{d}p^i}{\mathrm{d}\tau} = \frac{1}{T_R} g \,\mathrm{Tr}\left\{QF^{i\nu}\right\} \frac{p_\nu}{m} - \frac{\partial V}{\partial x^i}$$

Classical projection of pQCD potential

$$\hat{V} = T^a \otimes \bar{T}^a \frac{\alpha_s}{r_{\mathrm{rel}}} \implies V = \frac{Q_a \bar{Q}_a}{N_c} \frac{\alpha_s}{r_{\mathrm{rel}}}$$

- For $\tau = \tau_{\mathrm{form}}$, $Q_a\bar{Q}_a = -4$: attractive potential
- For $\tau > \tau_{\mathrm{form}}$: dynamic potential

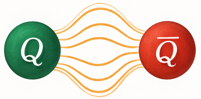


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Image courtesy of Chat-GPT

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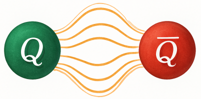


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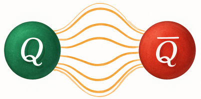


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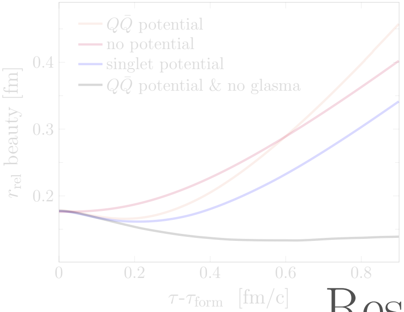
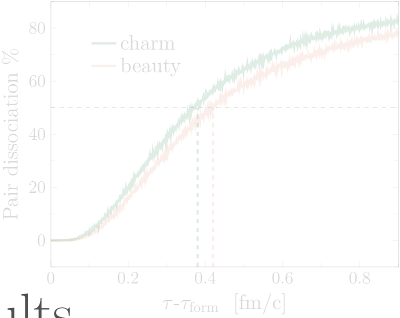
# Results



$Q\bar{Q}$ potential
no potential
singlet potential
$Q\bar{Q}$ potential & no glasma
$r_{\rm rel}$ beauty [fm]
$\tau$-$\tau_{\rm form}$ [fm/c]

charm
beauty
Pair dissociation %
$\tau$-$\tau_{\rm form}$ [fm/c]




charm
beauty
$r_{\rm rel}$ [fm]
$\tau$-$\tau_{\rm form}$ [fm/c]




$\langle P_S \rangle$
$\langle P_O \rangle/8$
$\langle P_S \rangle$ and $\langle P_O \rangle/8$, charm
$\tau$-$\tau_{\rm form}$ [fm]

# Relative distance

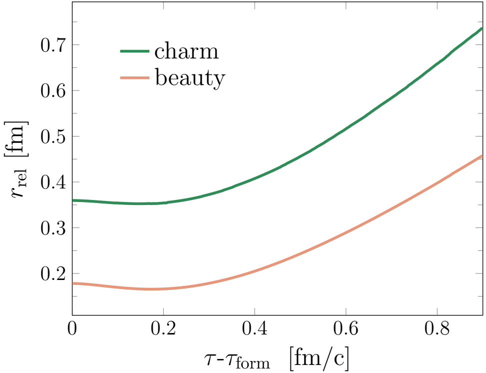
charm vs beauty

- Physical input: $r_{\rm rel}^{\rm beauty} < r_{\rm rel}^{\rm charm}$
- Higher mass, lower spread
- Initial balance between:

$Q\bar{Q}$ potential (attraction)

$\Updownarrow$

Glasma fields (diffusion)




**Distance not the main driver of pair decorrelation**

(within glasma timescales)

L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

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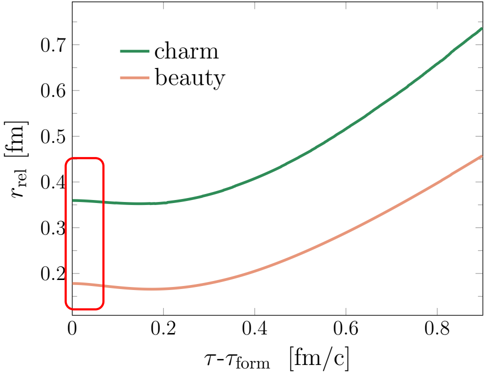


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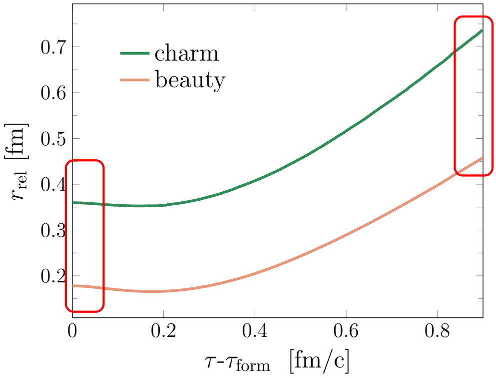


**Distance not the main driver of pair decorrelation**

(within glasma timescales)

L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

# Relative distance

charm vs beauty

- Physical input: $r_{\rm rel}^{\rm beauty} < r_{\rm rel}^{\rm charm}$
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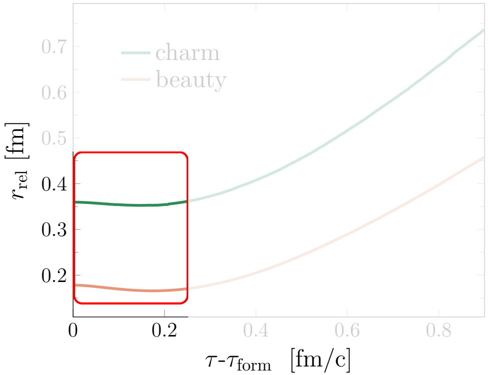


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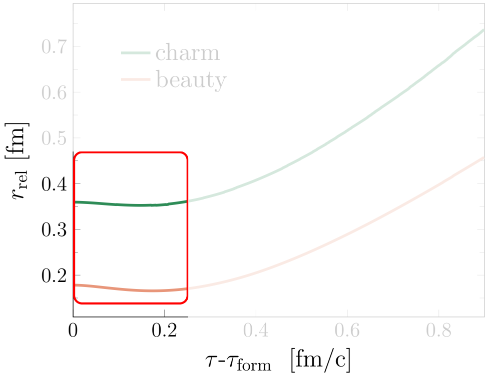


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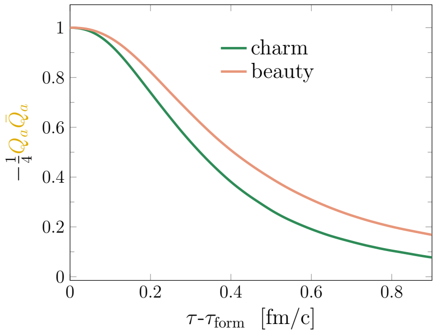
# Dissociation of pairs

Melting of quarkonia due to color decorrelation




For each pair, at each time, we define:

$$\mathcal{P}_{\rm melting} = 1 - \mathcal{P}_{\rm survival},$$

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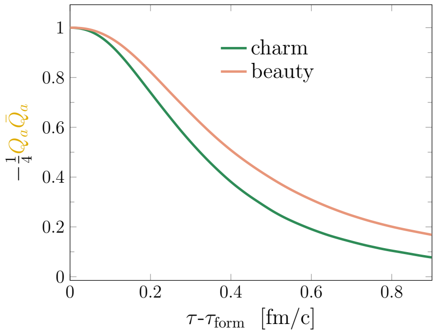


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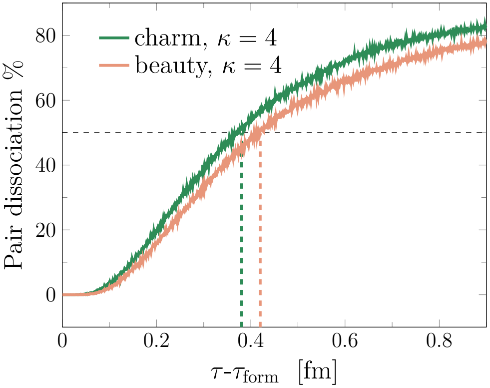


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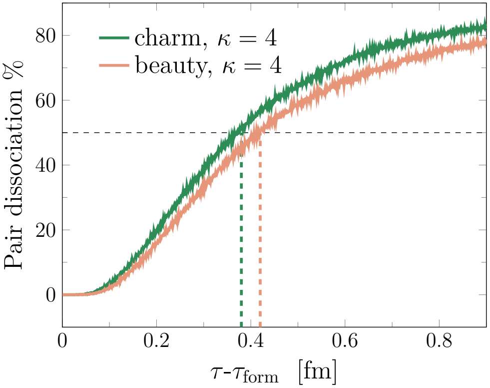


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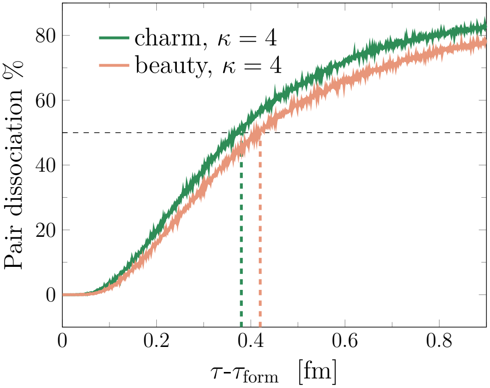


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# Elliptic flow $v_2$

Azimuthal **spatial** anisotropy $\implies$ Azimuthal **momentum** anisotropy

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\mathbf{p}_\mathrm{T}^2} \propto \{1 + v_1 \cos[\phi - \Psi] + v_2 \cos[2(\phi - \Psi)] + v_3 \cos[3(\phi - \Psi)] + \dots\}$$

But, which method to get $\Psi$?

Event Plane (**EP**):

strongest flow identifies $\Psi$.

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no need for $\Psi$, only $\langle \cos[2(\phi_1 - \phi_2)] \rangle$.

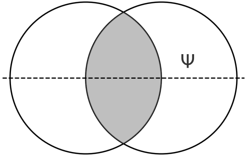


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Image courtesy of Chat-GPT

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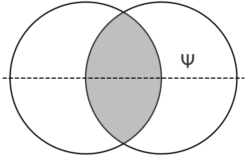


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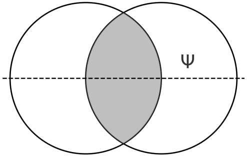


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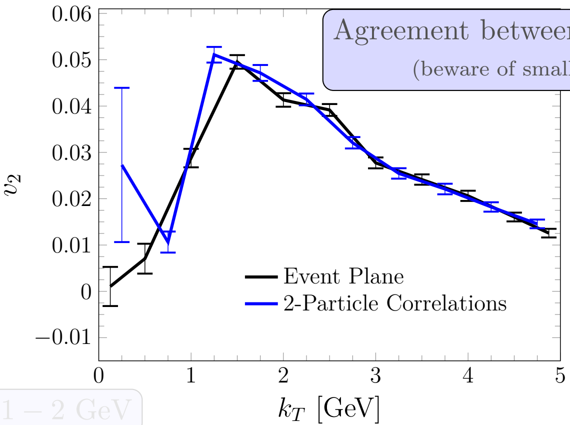


Agreement between **EP** and **2PC**
(beware of small momenta!)

$v_2 = 0.05$ for $k_T \sim 1 - 2$ GeV

F. Murgana, G.P., V. Greco and M. Ruggieri, in preparation

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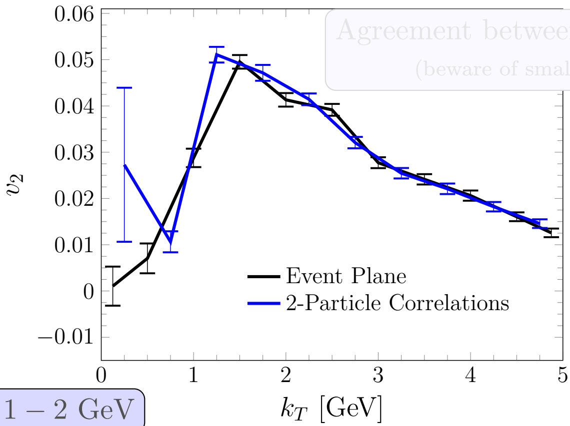


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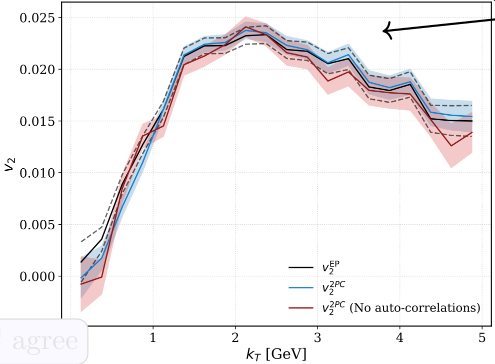
$v_2$ of glasma $\Longrightarrow$ $v_2$ of Heavy Quarks

Around 50% of glasma $v_2$

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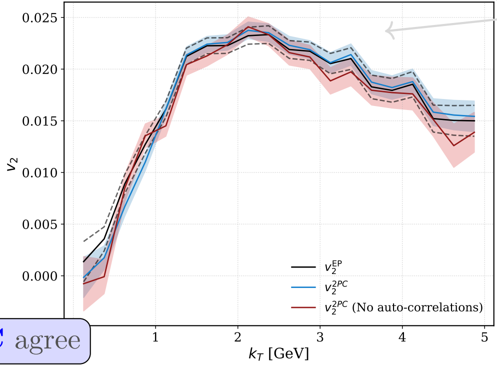


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# Outro

## Conclusions

- HQ pairs' color significantly decorrelated in glasma timescales
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## Outlook

- Coupling to later stages of Heavy Ion Collisions
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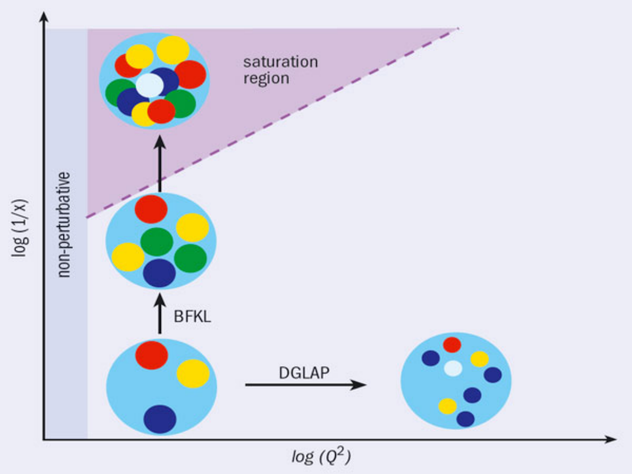
https://siciliamare.info/etna/

# Back-up

# BFKL and DGLAP evolutions

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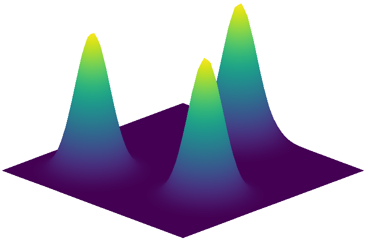
https://cerncourier.com/a/qcd-scattering-from-dglap-to-bfkl/

# Color charge generation in pA

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)\rangle = (g\mu)^2\delta^{ab}\delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T)$$

We can have $\mu$ space-dependent:
**proton structure**



- Three hotspots $\bar{\mathbf{x}}_T^i$ in a width $\sqrt{B_{qc}}$.
- Then:

$$\mu(\mathbf{x}_T) \propto \frac{1}{3}\sum_{i=1}^{3}\frac{1}{2\pi B_q}\exp\left[-\frac{(\mathbf{x}_T - \bar{\mathbf{x}}_T^i)^2}{2B_q}\right].$$

$$B_{qc} = (0.4\ \text{fm})^2, \quad B_q = (0.11\ \text{fm})^2$$

H. Mäntysaari and B. Schenke, Phys.Rev.Lett. 117 (2016) 5, 052301

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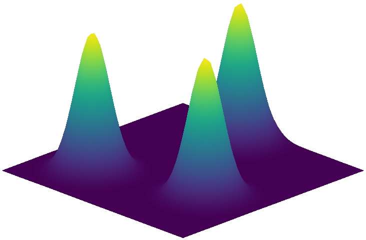

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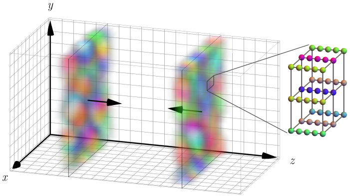


- Classical Yang-Mills discretized on lattice:

  color sheets

- Real-time lattice gauge theory techniques

Gauge links

$$V^{\dagger}(\mathbf{x}_T, x^-) = \mathcal{P}\exp\left[-ig\int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_T)\right]$$

Plaquettes

$$U_{x,\mu\nu} = U_{x,\mu}U_{x+\mu,\nu}U_{x+\mu+\nu,-\mu}U_{x+\nu,\nu}$$

Wilson lines

$$U_{\mathbf{x}_T,i} = V(\mathbf{x}_T)V^{\dagger}(\mathbf{x}_T + \Delta x_i)$$

D. Gelfand et al., Phys.Rev.D 94 (2016) 1, 014020

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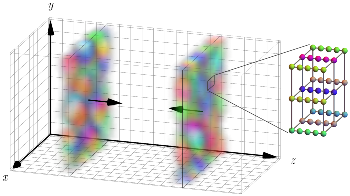


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D. Gelfand et al., Phys.Rev.D 94 (2016) 1, 014020

# CYM on lattice

After gauge choice, YM equations reduce to:

$$\Delta_\perp \alpha(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp) \implies \tilde{\alpha}^a_{n,k} = \frac{\tilde{\rho}^a_{n,k}}{\tilde{k}^2_\perp + m^2}$$

where

$$\tilde{k}^2_\perp = \sum_{i=x,y} \left(\frac{2}{a_\perp}\right)^2 \sin^2\left(\frac{k_i a_\perp}{2}\right).$$

# CYM on lattice

Solve previous eq. for $A_a^+$ in each nucleus (at $\tau = 0^-$). For $\tau = 0^+$?

$$A_{\rm tot}^+ = A_1^+ + A_2^+ \overset{?}{\Longrightarrow} U_{\rm tot} \propto \exp\left[iA_{\rm tot}^+\right] = U_1 \cdot U_2 \quad \text{NO!!}$$

In SU(3) no exact formula for $U_{\rm tot}$, rather we iteratively solve:

$$\mathrm{Tr}[t_a(U_{x,i}^A + U_{x,i}^B)(\mathbb{I} + U_{x,i}) - \text{h.c.}] = 0.$$

Those U are needed for the magnetic fields:

$$B_L^2 = \frac{2}{g^2 a_\perp^4}\mathrm{Tr}(\mathbb{I} - U_{xy}), \qquad B_T^2 = \frac{2}{(g a_\eta a_\perp \tau)^2}\sum_{i=x,y}\mathrm{Tr}(\mathbb{I} - U_{\eta i}).$$

M. C. Cautun, Master Thesis (2009)

# CYM on lattice

- **Initial conditions:**

$$
\begin{aligned}
E_x =& E_y = 0, \quad U_\eta = \mathbb{I}, \quad U_{x/y} = U_{\text{tot},x/y} \text{ as in previous slide} \\
E^\eta =& -\frac{i}{4ga_\perp^2} \sum_{i=x,y} \Big[ (U_i(\mathbf{x}_\perp) - \mathbb{I})(U_i^{B,\dagger}(\mathbf{x}_\perp) - U_i^{A,\dagger}(\mathbf{x}_\perp)) + \\
& (U_i^\dagger(\mathbf{x}_\perp - \hat{i}) - \mathbb{I})(U_i^B(\mathbf{x}_\perp - \hat{i}) - U_i^A(\mathbf{x}_\perp - \hat{i})) - h.c. \Big] .
\end{aligned}
$$

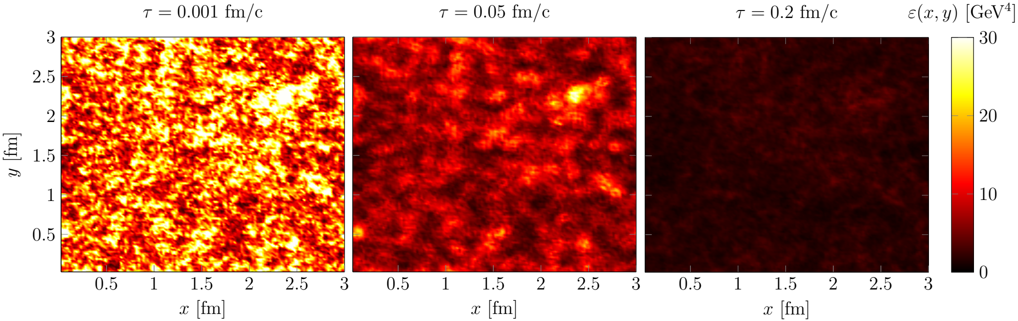
- **Eqs of motion:**

$$
\partial_\tau U_i(x) = \frac{-iga_\perp}{\tau} E^i(x) U_i(x), \qquad \partial_\tau U_\eta(x) = -iga_\eta \tau E^\eta(x) U_\eta(x).
$$

# Features of the glasma

## Energy density in nucleus-nucleus collisions

$$\varepsilon = \mathrm{Tr}[E_\mathrm{L}^2 + B_\mathrm{L}^2 + E_\mathrm{T}^2 + B_\mathrm{T}^2]$$




Fields dilute after $\delta\tau \sim Q_s^{-1} \sim \mathcal{O}(0.1 \text{ fm/c})$

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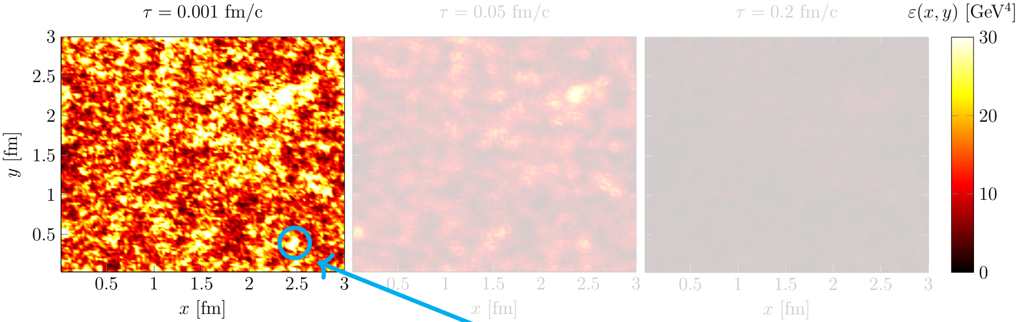


Hotspots of size $\delta x_T \sim Q_s^{-1} \sim \mathcal{O}(0.1\ \mathrm{fm})$

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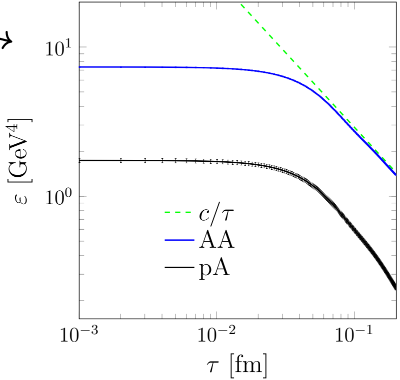
## Energy density in pA vs $\tau$

$\varepsilon_{pA}(0^+) < \varepsilon_{AA}(0^+)$




$\varepsilon \propto 1/\tau \implies$ dilute fields

32x32x32 lattice, $N_\tau = 400$

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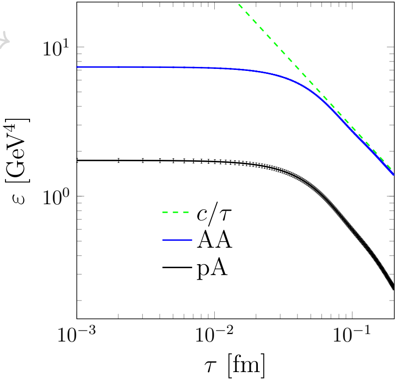


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# Heavy Quarks in Yang-Mills fields: initialization

- Center of Mass position:

$$P(\mathbf{x}_\perp) \propto \mu(\mathbf{x}_\perp)$$

- Relative position:

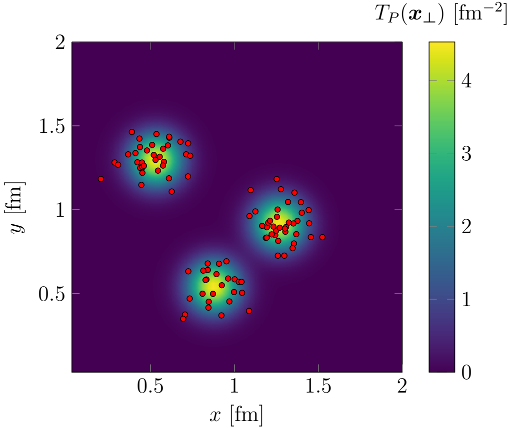
$$P(r_{\mathrm{rel}}) \propto r_{\mathrm{rel}} \exp\left(-r_{\mathrm{rel}}^2/\sigma^2\right)$$

- Relative momentum:

$$P(p_{\mathrm{rel}}) \propto p_{\mathrm{rel}} \exp\left(-p_{\mathrm{rel}}^2\sigma^2\right)$$

- Color charge:

$$\text{singlet}, \quad Q_a = -\bar{Q}_a$$




J. Zhao et al., Nuovo Cim. C 48, 5 (2025)

L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

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- Change in **coordinates** due to momentum kicks
- **Momentum** broadening due to color Lorentz force
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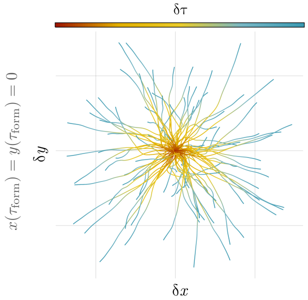


D. Avramescu et al., Phys.Rev.D 107 (2023) 11, 114021
Image courtesy of Dana Avramescu

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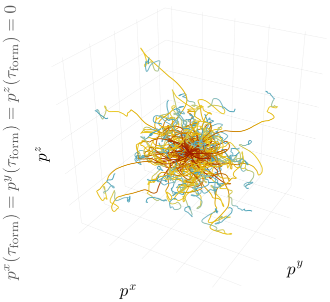


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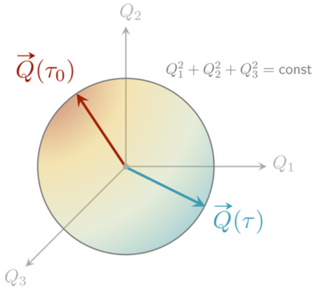


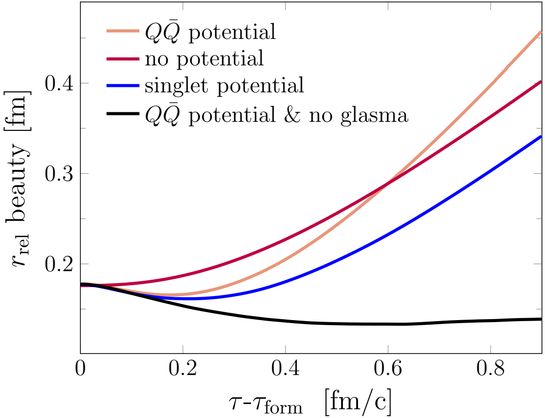

(Picture should actually be in 8 dims)

D. Avramescu et al., Phys.Rev.D 107 (2023) 11, 114021

Image courtesy of Dana Avramescu

# Relative distance

## Effect of potential on beauty quark

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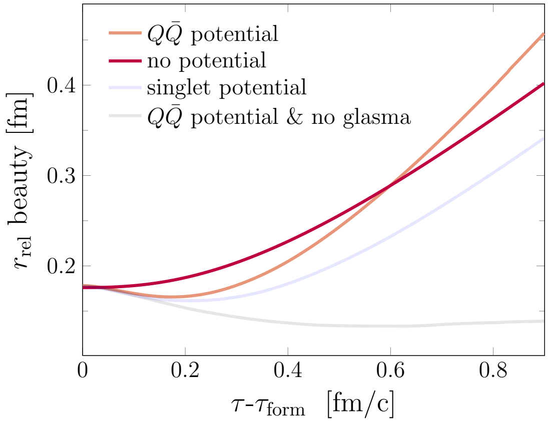


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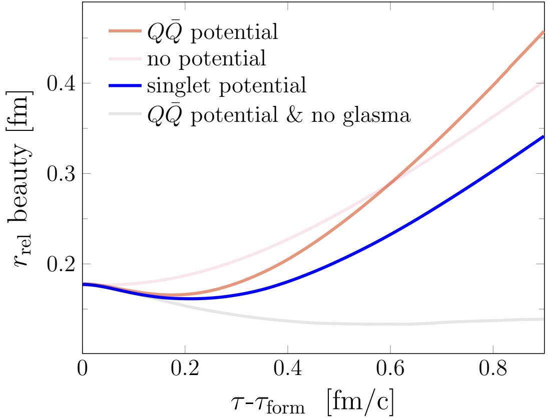


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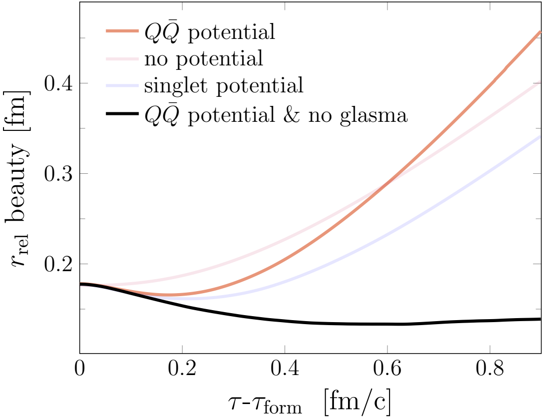
L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

# Relative distance

## Effect of potential on beauty quark




- Glasma fields only: diffusion
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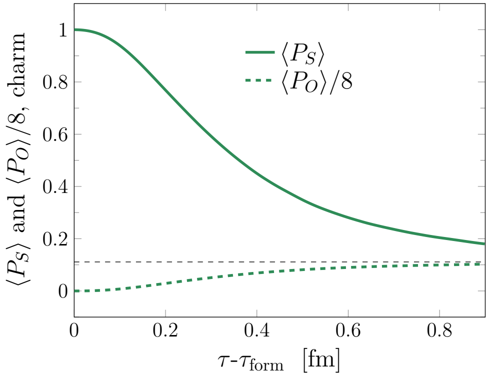
L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

# Octet and Singlet projectors

$$P_S = -\frac{2}{3}\frac{Q_a\bar{Q}_a}{N_c} + \frac{1}{9}, \quad P_O = \frac{2}{3}\frac{Q_a\bar{Q}_a}{N_c} + \frac{8}{9},$$




At late times

$$\langle P_S \rangle \approx \langle P_O \rangle/8 \approx 1/9.$$

Qualitative agreement with other approaches:

Brownian motion in QGP

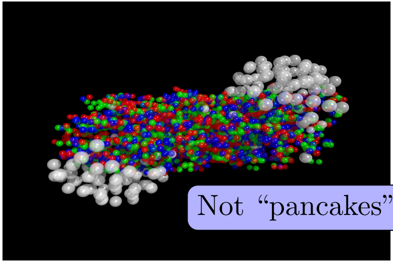

L. Oliva, G.P., V. Greco and M. Ruggieri, Phys. Rev. D **112** (2025), 014008

S. Delorme et al., JHEP 06 (2024) 060

# 3+1D simulations

Heavy Ion Collisions are **not** boost-invariant:

rapidity-dependence to be taken into account.




Same initial conditions + $\eta$-dependent terms such that:

$$D_i E_i(\eta) + D_\eta E_\eta(\eta) = 0 \quad \text{Gauss' Law}$$

Explicit expressions for $\delta E$ in backup

https://spacegid.com/kvark-glyuonnaya-plazma.html

G.P., V. Greco and M. Ruggieri, 2505.08441 [hep-ph]

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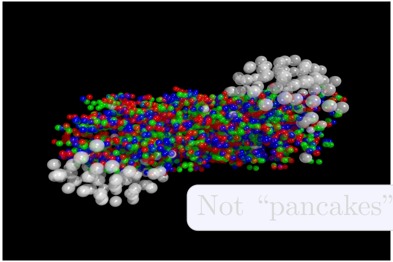

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# 3+1D simulations

## Momentum broadening for $M_{\text{HQ}} \to +\infty$

Focus on **momentum broadening**:

$$\delta p_i^2(\tau) \equiv p_i^2(\tau) - p_i^2(\tau_{\text{form}}), \qquad i = x, y, z.$$

One can prove that for $M_{\text{HQ}} \to +\infty$:

$$\langle \delta p_L^2(\tau) \rangle_\infty = g^2 \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' \langle \text{Tr}[E_z(\tau')E_z(\tau'')] \rangle$$

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D. Avramescu et al., Phys.Rev.D 107 (2023) 11, 114021

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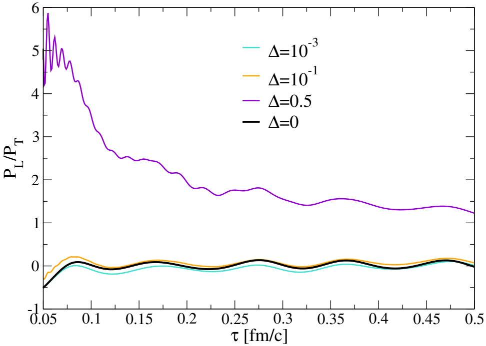
# Results: 3+1D simulations

## Pressures Ratio

Fluctuations scaling as $\delta E_i \sim \Delta$




Significant effect on pressure for high $\Delta$

So, maybe fluctuations favor isotropization...

G.P., V. Greco and M. Ruggieri, 2505.08441 [hep-ph]

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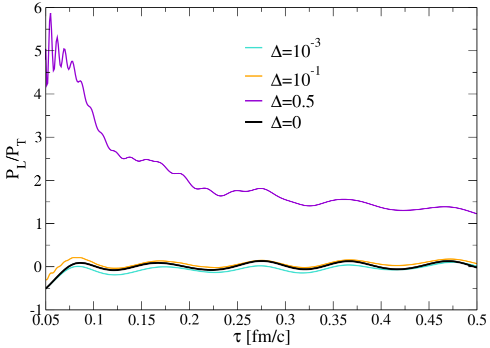


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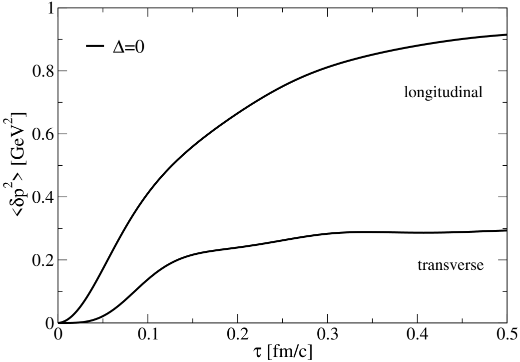
... or maybe not. Effect on momenta shifts irrelevant




$$\boxed{\begin{aligned}\langle \delta p_L^2(\tau)\rangle_\infty &= g^2 \int d\tau' \int d\tau'' \langle \mathrm{Tr}[E_z(\tau')E_z(\tau'')]\rangle \\ \langle \delta p_T^2(\tau)\rangle_\infty &= g^2 \int d\tau' \int d\tau'' \langle \mathrm{Tr}[E_i(\tau')E_i(\tau'')]\rangle/\tau'\tau''\end{aligned}}$$

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G.P., V. Greco and M. Ruggieri, 2505.08441 [hep-ph]

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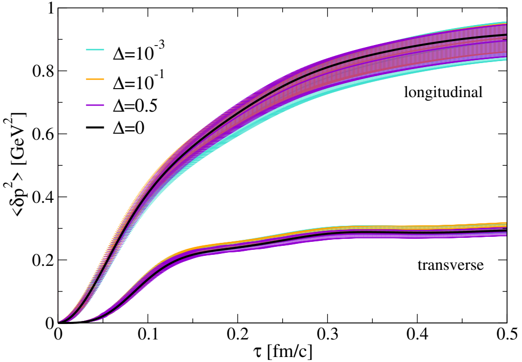


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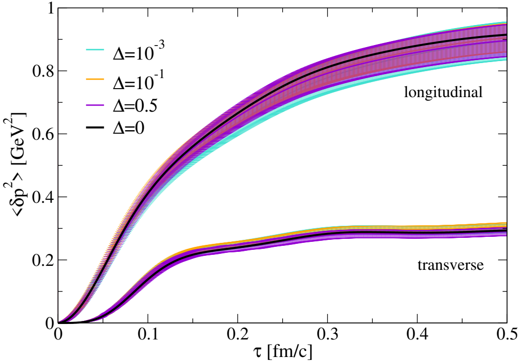


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