ALESSANDRO
ROGGERO

QUANTUM SIMULATION ALGORITHMS FOR MANY FERMION SYSTEMS IN FIRST QUANTIZATION

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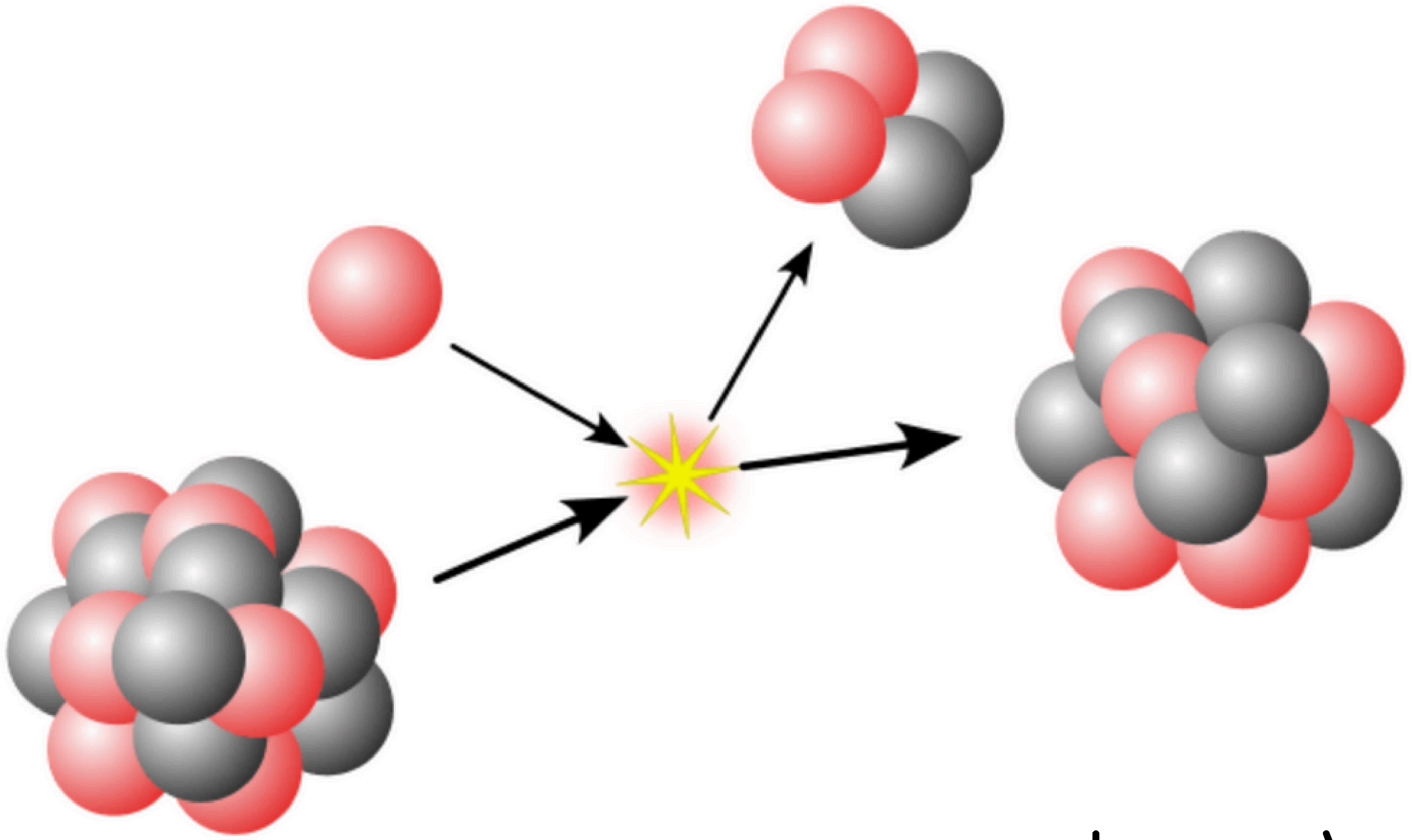
TIFPA

INFN

MOdeling
Nuclear
STructure and
REactions

NUCLEAR PHYSICS



We want to simulate scattering processes

$$|\psi_0\rangle \quad e^{-iHt} \quad |\psi_f\rangle$$

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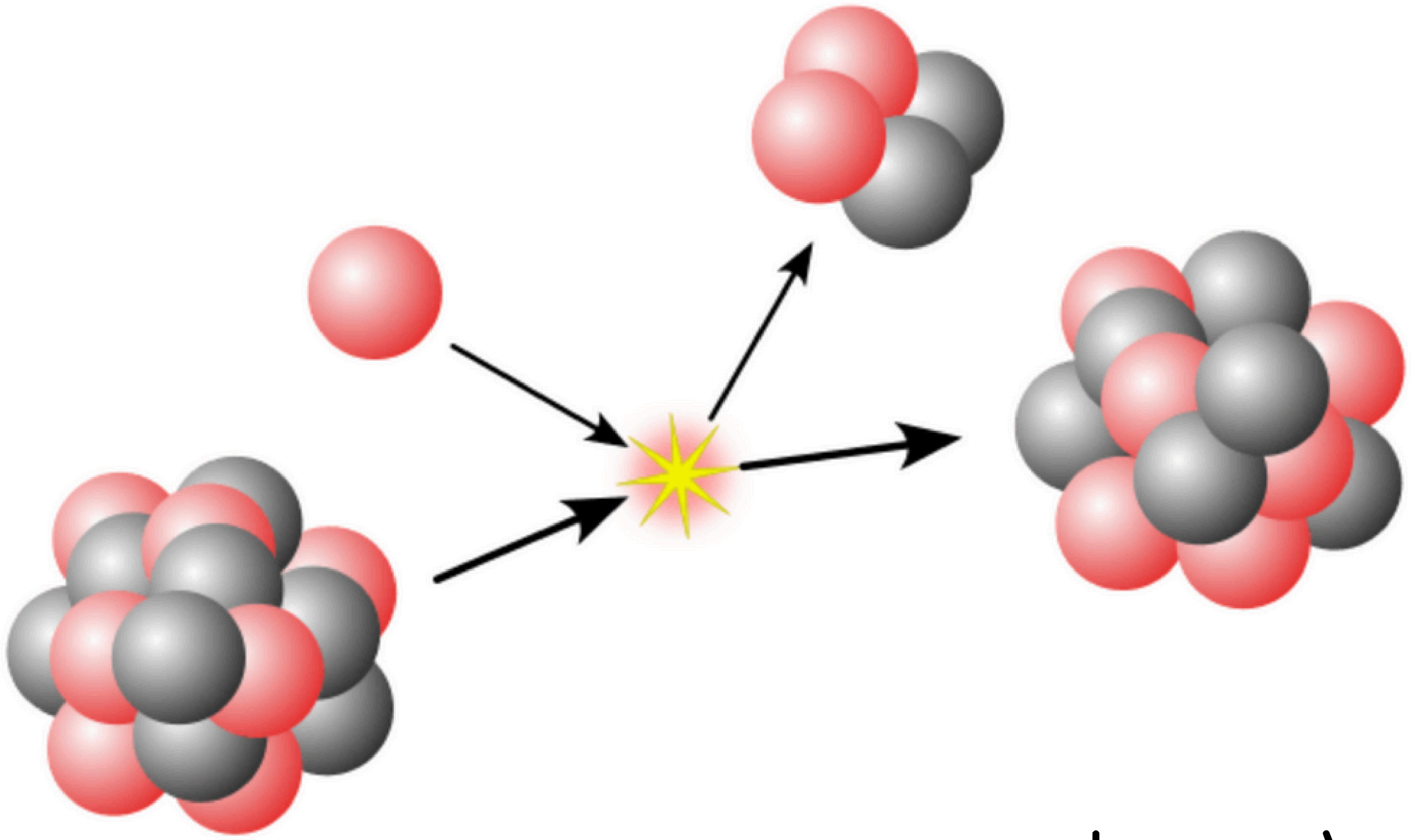

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Encoding

Evolution

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First quantization has an advantage over second quantization

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First quantisation

- Hilbert space: total number of particles conserved
- Operators: position, momentum

$$H = \underbrace{\sum_i \frac{p_i^2}{2m}}_{T} + \underbrace{\sum_{i<j} V(x_i - x_j)}_{V}$$

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Second quantization

- Fock space: total number of particles not conserved
- Operators: creation and anihilation operators

$$H = m \sum_i a_i^\dagger a_i + \sum_{i,j} \left(a_i^\dagger a_j + a_j^\dagger a_i \right)$$

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$$|\psi_3\rangle = \hat{A}|\phi_1\rangle \otimes |\phi_2\rangle \otimes |\phi_3\rangle, \quad |\phi_j\rangle = \sum_{k=0}^{\Omega-1} c_k |k\rangle$$

We need one wavefunction for every particle

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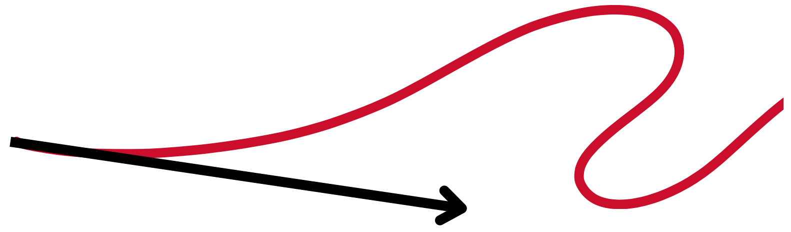
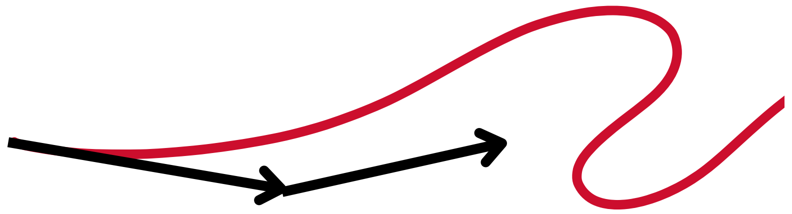


$$e^{-itH} \approx e^{-i\frac{t}{2}T} e^{-i\frac{t}{2}V} \cdot e^{-i\frac{t}{2}T} e^{-i\frac{t}{2}V} + O\left(\frac{t^2}{2}\right)$$



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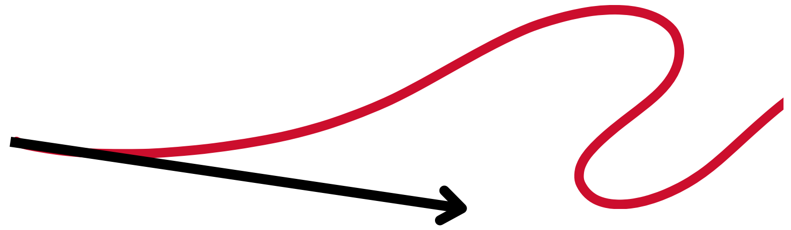

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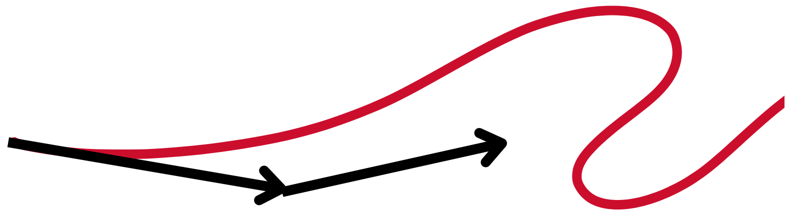
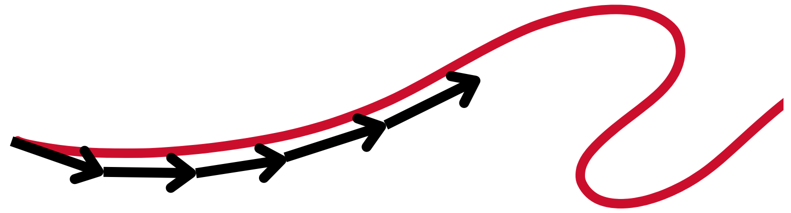


$$e^{-itH} \approx \underbrace{e^{-i\frac{t}{r}T} e^{-i\frac{t}{r}V} \cdots e^{-i\frac{t}{r}T} e^{-i\frac{t}{2}V}}_{r \text{ times}} + O\left(\frac{t^2}{r}\right)$$



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Kinetic term:

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$$\rightarrow O(\eta^3 \log(\Omega))$$

NUMBER OF GATES

Trotterization:

$$||e^{-itH} - \left(e^{-i\frac{t}{r}T}e^{-i\frac{t}{r}V}\right)^r|| \leq \epsilon \quad \Rightarrow \quad r = O\left(\frac{t^2\eta}{\epsilon}\right)$$

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The total cost will be: $r\left(C\left(e^{i\frac{t}{r}T}\right) + C\left(e^{i\frac{t}{r}V}\right)\right)$

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In second quantization: $$\widetilde{O}\left(\frac{t^2\eta}{\epsilon}\Omega\right)$$
arXiv:2312.05344

COMPARISON

	qubits	trotterization
First quantization: arXiv: 2507.22814	$O(\eta \log(\Omega))$	$\widetilde{O}\left(\frac{t^2\eta^4}{\epsilon}\log(\Omega)\right)$
Second quantization: arXiv: 2312.05344	$O(\Omega)$	$\widetilde{O}\left(\frac{t^2\eta}{\epsilon}\Omega\right)$

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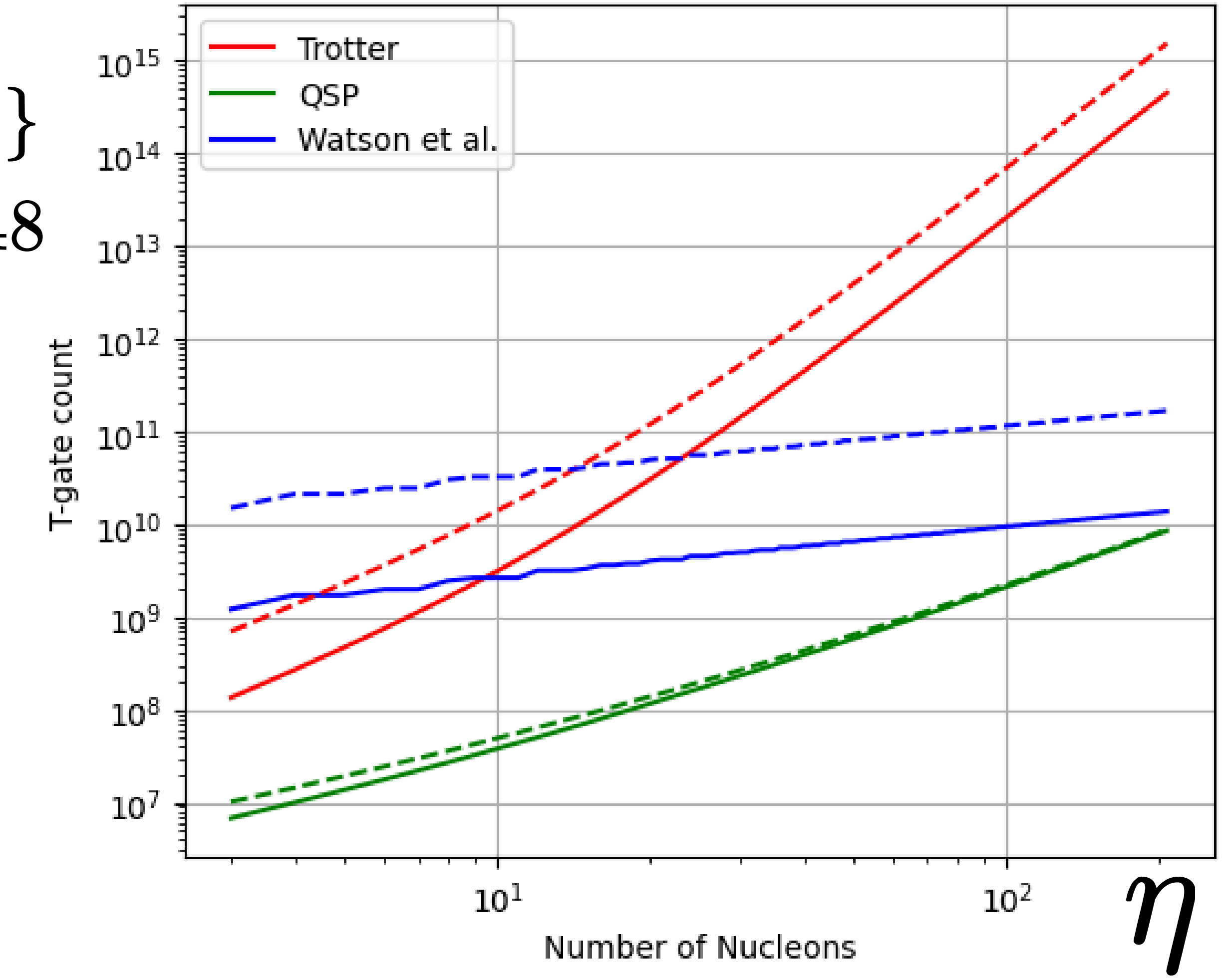
$$\epsilon = \{10^{-1}, 10^{-3}\}$$
$$\Omega = 4 \cdot 8^3 = 2048$$



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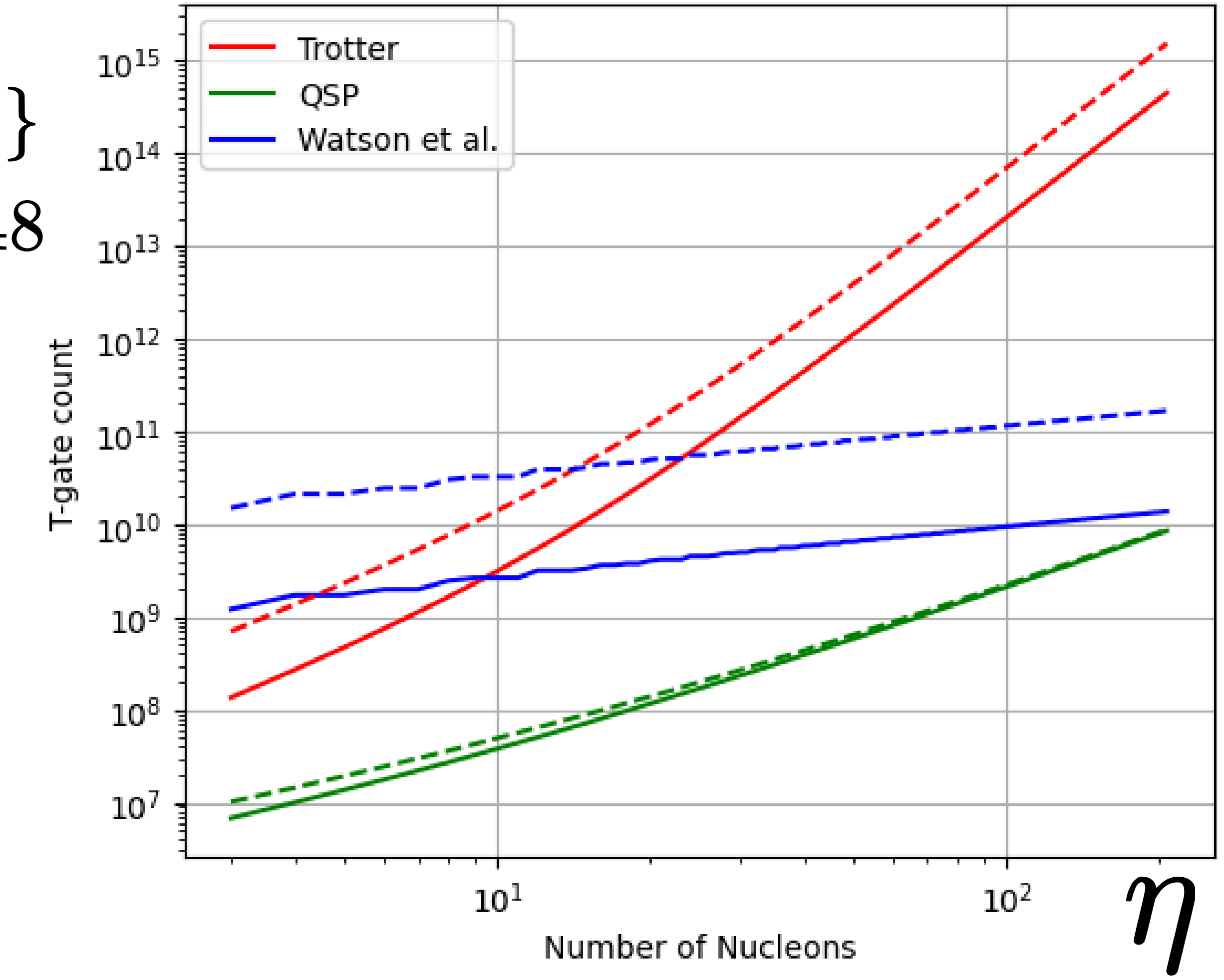
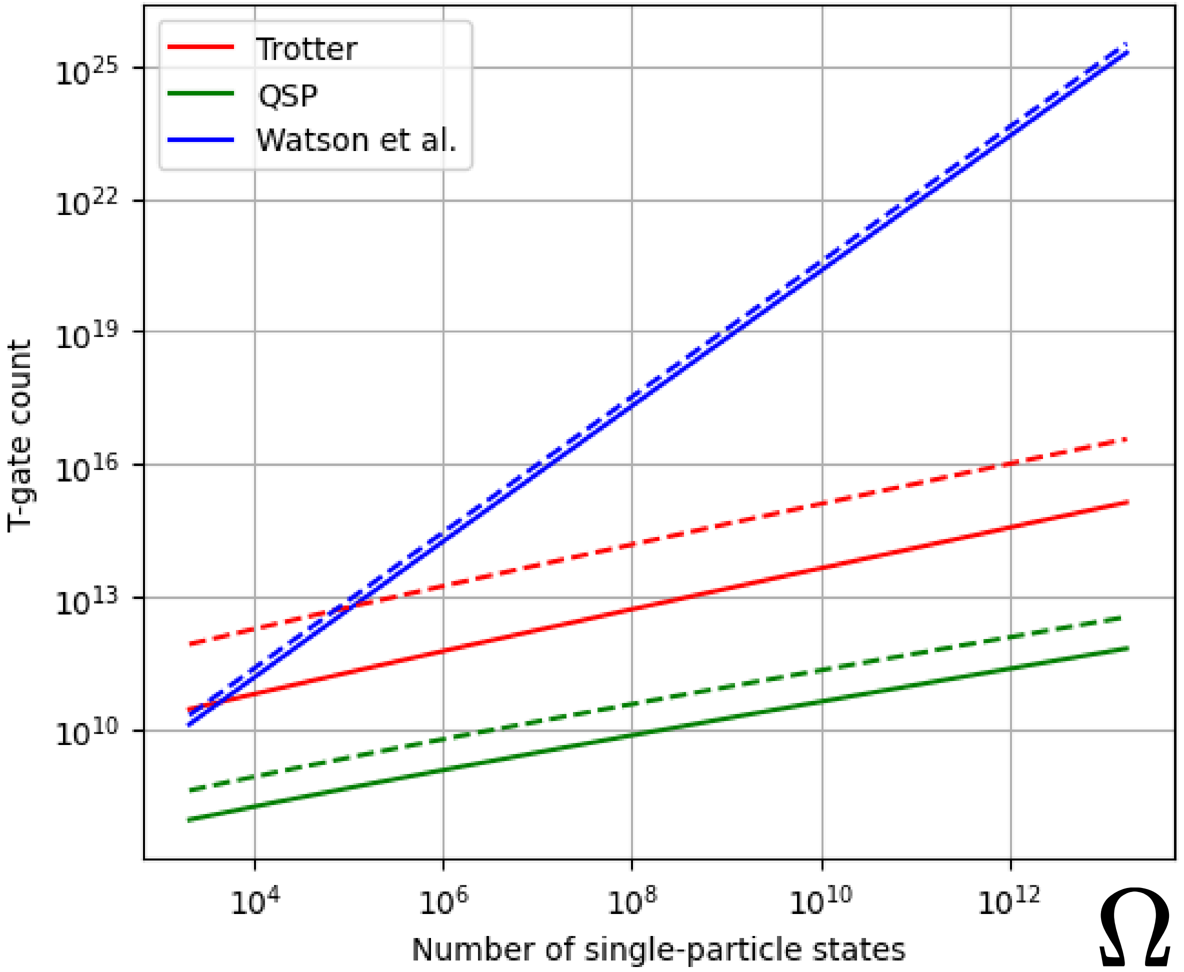


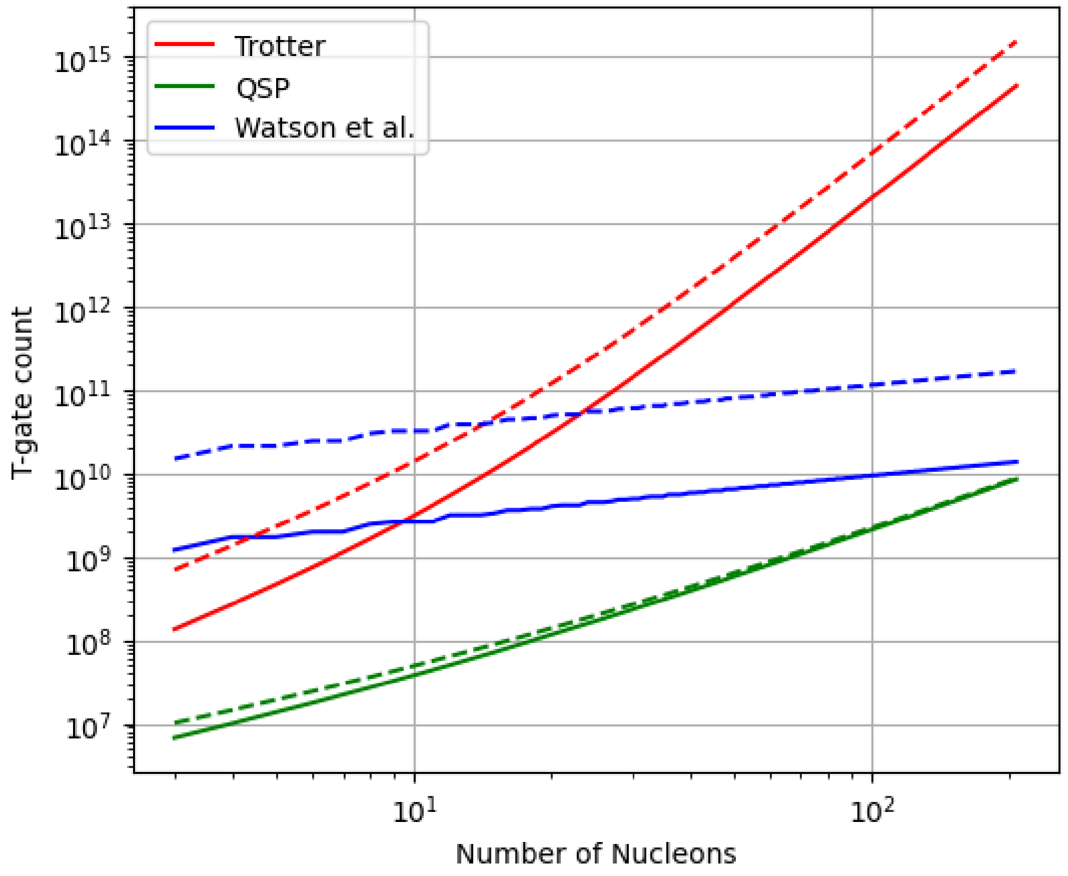
η



Ω

$$\epsilon = \{10^{-1}, 10^{-3}\}$$
$$\eta = 40$$

CONCLUSIONS



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First quantization is exponentially better in terms of the lattice size, while it is polynomially worst in terms of the number of particles

THANK YOU

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$$\rightarrow \quad ||e^{i\frac{t}{r}H} - e^{i\frac{t}{r}T}e^{i\frac{t}{r}V}|| \leq \frac{t^2}{2r^2}||[T,V]||$$

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The norm of the commutator scales as the number of particles:

$$||[T,V]|| = O(\eta)$$

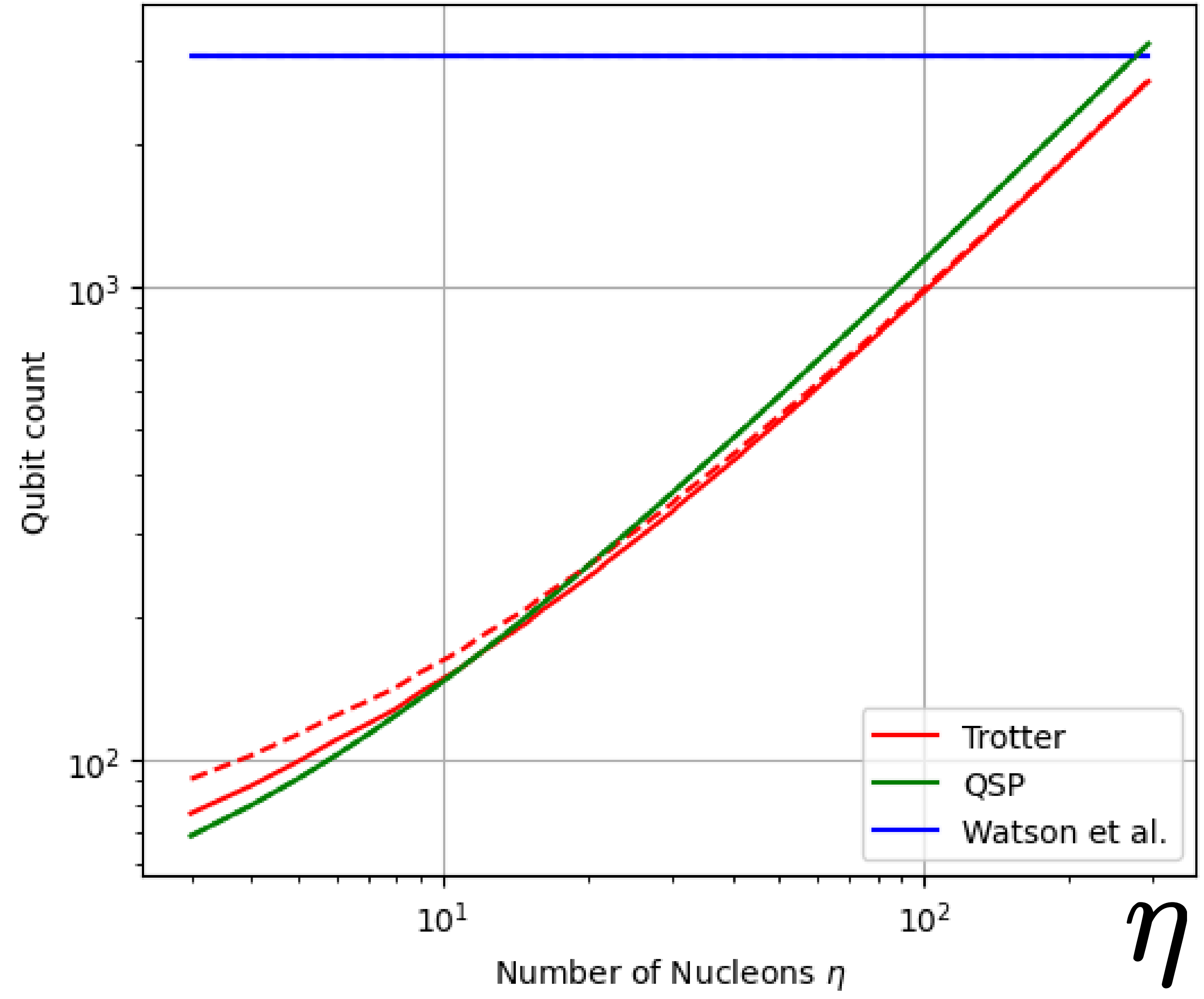
Which means that we can bound the error by choosing r as follows:

$$\frac{t^2}{r}\eta \leq \epsilon \;\Rightarrow\; r = O\left(\frac{t^2\eta}{\epsilon}\right)$$

The total cost will be:

$$r\left(C\left(e^{itT}\right) + C\left(e^{itV}\right)\right) = \widetilde{O}\left(\frac{t^2\eta^4}{\epsilon}\log(V)\right)$$

NUMBER OF QUBITS



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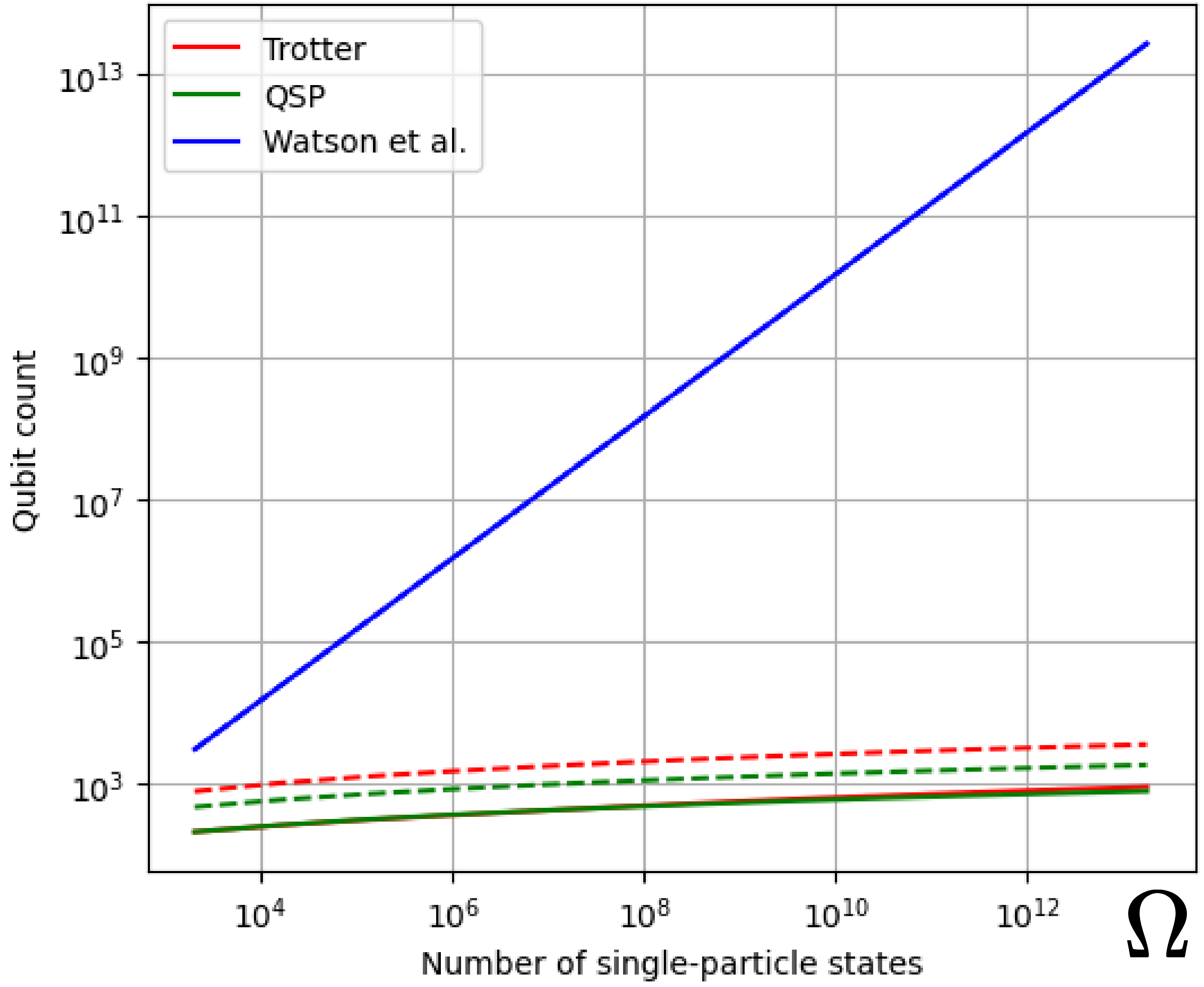

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