# Neutron star crust from Bayesian-constrained unified EoS with ab initio input

## XX Conference on Theoretical Nuclear Physics in Italy TNPI2025

Il Palazzone - Cortona (Arezzo), October 1st-3rd, 2025

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PHYSICAL REVIEW C **112**, 035802 (2025)

**Bayesian inference of neutron star crust properties using an *ab-initio*–benchmarked metamodel**

S. Burrello,[1,*] F. Gulminelli,[2] M. Antonelli,[2] M. Colonna,[1] and A. F. Fantina[3]
[1] *INFN, Laboratori Nazionali del Sud, I-95123 Catania, Italy*
[2] *Université de Caen Normandie, ENSICAEN, CNRS/IN2P3, LPC Caen UMR6534, 14000 Caen, France*
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# Outline of the presentation

1. **Inference of neutron star (NS) properties: nuclear & astrophysical constraints**
   - Unified modelization of the nuclear matter (NM) Equation of State (EoS)
   - Phenomenological models: energy density functionals (EDFs) & meta-model (MM)

2. **Upgraded version of MM: recent developments and results**
   - ✏ **Refined treatment at low-density: homogeneous & inhomogeneous matter**
     - Benchmark on ab-initio calculations of neutron matter: Y-MM
     - Thermodynamical properties of bulk matter in the inner crust
   - ✏ **Bayesian inference of NS crustal properties**
     - Crust-core (CC) transition and connection with symmetry enery and slope
     - Crustal fraction of the moment of inertia and NS crust EoS

3. **Further developments and outlooks**
   - Implementation in CUTER for interpreting gravitational waves (GW) signals
   - Joint analyses combining also nuclear structure and heavy-ion collision studies

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# Inferring EoS: nuclear & astrophysical constraints

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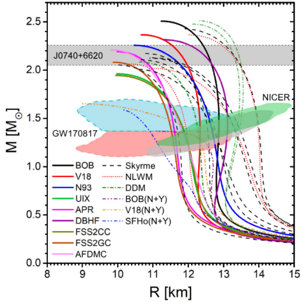
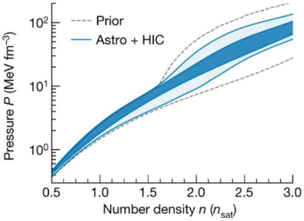
  - Insights on neutron stars (**NS**) from **observations**
  - Understanding nuclear **structure** and **reactions**
  - Joint **analyses** with heavy-ion collisions (**HICs**)
- **Bayesian** inference of most NS macroscopic **observables**
  - Leading role of **high-density** core EoS
  - Agnostic formulation ⇒ mismatch with **microscopics**
- Description of **low-density** EoS & NS **crust** composition
  - **Simulations** of proto-NS cooling processes
  - Precise **determination** of NS radii
  - **Understanding** the origin of **pulsar glitches**




[G. F. Burgio, PPNP 120, 103879 (2021)]




Huth et al. Nature 606, 276–280 (2022)

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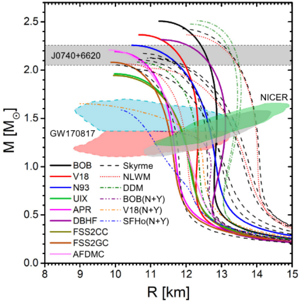


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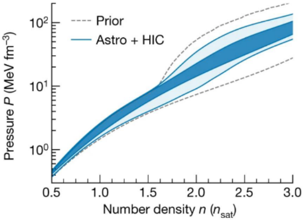


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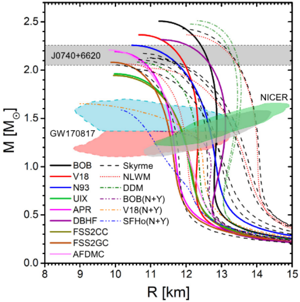


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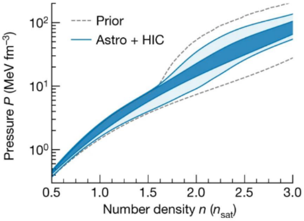


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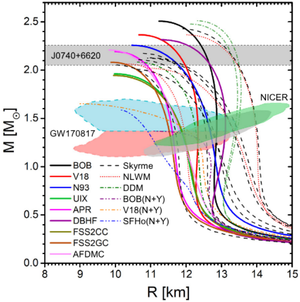


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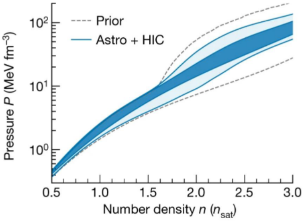


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# Unified EoS: phenomenological meta-model (MM)

- **Unified** models based on energy density functionals (**EDFs**) & **nucleonic** hypothesis

  ⇒ meta-modeling (**MM**) approach [J. Margueron et al., PRC 97, 025805 (2018)]

$$e_{\rm MM}(n_{\rm B},\delta) = t^*_{\rm FG}(n_{\rm B},\delta) + v_{\rm MM}(n_{\rm B},\delta) \qquad \delta = (n_{\rm n} - n_{\rm p})/n_{\rm B}$$

- Isoscalar (**IS**) & isovector (**IV**) expansion at symmetric NM (**SNM**) saturation $n_{\rm sat}$

  ✓ **Truncation** for $N = 4$ ($E_{\rm sat}$, $K_{\rm sat}$, $Q_{\rm sat}$, $Z_{\rm sat}$ & $E_{\rm sym}$, $L_{\rm sym}$, $K_{\rm sym}$, $Q_{\rm sym}$, $Z_{\rm sym}$)

$$v^N_{\rm MM} = \sum_{\alpha=0}^{N} \frac{1}{\alpha!}\left(v^{\rm IS}_\alpha + v^{\rm IV}_\alpha \delta^2\right) x^\alpha, \qquad x = \frac{n_{\rm B} - n_{\rm sat}}{3 n_{\rm sat}}$$

- **Span** over EDFs existing in literature
- **Probe** of **novel** $n_{\rm B}$-dependencies
- Pure neutron matter (**PNM**) at low-$n_{\rm B}$ ∼ unitary Fermi gas (**FG**) ⇒ **Lee-Yang**

$$\frac{e_{\rm B}}{t_{\rm FG}} = 1 + \frac{10}{9\pi}(ak_F) + \frac{4}{21\pi^2}(11 - 2\ln 2)(ak_F)^2 + \ldots$$

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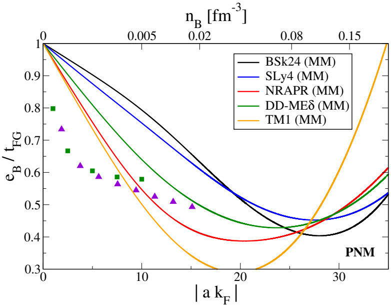


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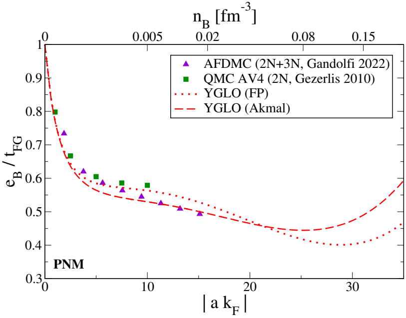
# Bridging EDFs with microscopic ab-initio methods

- New **class** of **EDFs** inspired by chiral effective field theory ($\chi$**EFT**): YGLO

$$e_{\rm Y}(n_{\rm B},\delta) = t_{\rm FG}(n_{\rm B},\delta) + v_{\rm Y}(n_{\rm B},\delta) \qquad v_{\rm Y}(n_{\rm B},\delta) = \frac{1}{n_{\rm B}}\left[\mathcal{V}^{\rm Y}_{\rm SNM} + \left(\mathcal{V}^{\rm Y}_{\rm PNM} - \mathcal{V}^{\rm Y}_{\rm SNM}\right)\delta^2\right]$$




$$\mathcal{V}^{\rm Y}_{\rm i} = Y_{\rm i}[n_{\rm B}]n_{\rm B}^2 + D_{\rm i}n_{\rm B}^{8/3} + F_{\rm i}n_{\rm B}^{\alpha+2}$$

$$Y_{\rm i}[n_{\rm B}] = \frac{B_{\rm i}}{1 - R_{\rm i}n_{\rm B}^{1/3} + C_{\rm i}n_{\rm B}^{2/3}}$$

$$B_{\rm i} = \frac{2\pi\hbar^2}{m}\frac{\nu_{\rm i}-1}{\nu_{\rm i}}a_{\rm i}$$

$$R_{\rm i} = \frac{6}{35\pi}\left(\frac{6\pi^2}{\nu_{\rm i}}\right)^{1/3}(11 - 2\ln 2)\,a_{\rm i}$$

$$(\nu_i = 2, 4 \text{ for PNM, SNM})$$

[C.J. Yang, M. Grasso, D. Lacroix, PRC 94, 031301 (2016)]

- Newly devised parameterization $\Rightarrow$ YGLO (MU)

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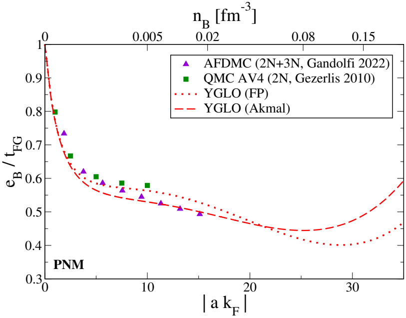


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PHYSICAL REVIEW C **103**, 064317 (2021)

**Application of an *ab-initio*-inspired energy density functional to nuclei: Impact of the effective mass and the slope of the symmetry energy on bulk and surface properties**

Stefano Burrello[1,*] Jérémy Bonnard[2,†] and Marcella Grasso[1,‡]

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Eur. Phys. J. A (2022) 58:22
https://doi.org/10.1140/epja/s10050-022-00665-2

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

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**Stefano Burrello[1,2,a], Marcella Grasso[2]**

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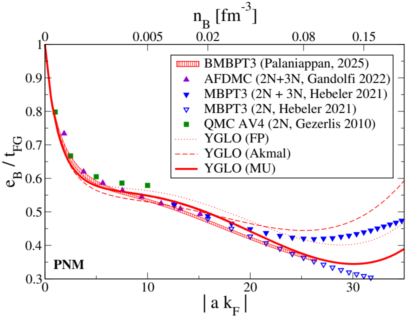


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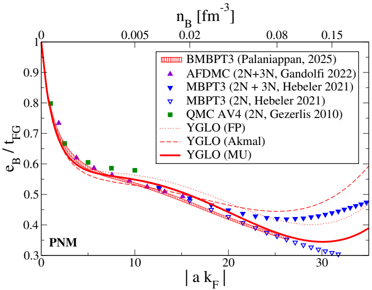
# Bulk matter and thermodynamical properties

- **Y-MM** $\Rightarrow$ smooth **interpolation** of MM with YGLO (MU) at low-density
  - Smooth-step transition **function** $\eta_\chi^{\rm MM} : \left[n_{\rm B}^\chi, n_{\rm B}^{\rm MM}\right] \to [0,1]$

$$e_{\rm B}(n_{\rm B},\delta) = e_{\rm Y}(n_{\rm B},\delta)\left(1-\eta_\chi^{\rm MM}\right) + e_{\rm MM}(n_{\rm B},\delta)\eta_\chi^{\rm MM}$$

[S. Burrello, F. Gulminelli, M. Antonelli, M. Colonna, A. Fantina, Phys. Rev. C 112, 035802 (2025)]




- $n_{\rm B} < n_{\rm B}^\chi = 0.02$ fm$^{-3}$ only **Monte-Carlo** (or Brueckner) calculations exist
- $n_{\rm B}^\chi \le n_{\rm B} < n_{\rm B}^{\rm MM}$ uncertainties on **3-body** forces of $\chi$EFT
- $n_{\rm B} \ge n_{\rm B}^{\rm MM}$ **empirical** MM, with $n_{\rm B}^{\rm MM} \le n_{\rm sat}$ variable **final endpoint**

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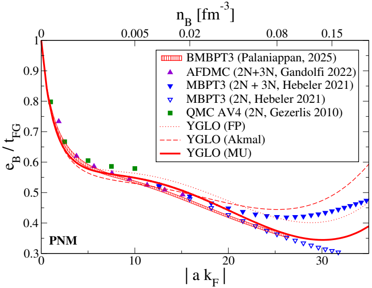


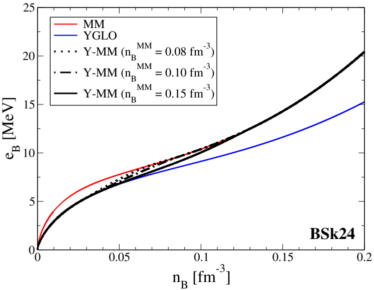


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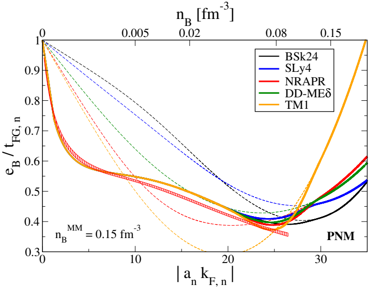


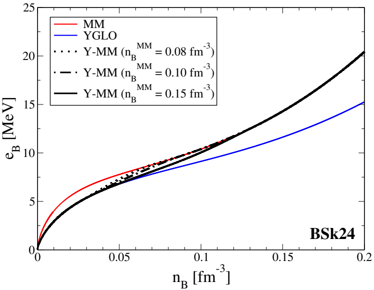


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# Bayesian analysis: (informed) prior & posterior

- **Bayes'** principle: by filtering **prior** $\Rightarrow$ **posterior** probability density functions (**PDF**)

$$p_{\text{post}}(\mathbf{X}) = \mathcal{C}\, w_{\mathrm{EFT}}(\mathbf{X})\, w_{\mathrm{IP}}(\mathbf{X})\, \mathrm{e}^{-\chi^2(\mathbf{X})/2}\, p_{\text{prior}}(\mathbf{X})$$

[S. Burrello, F. Gulminelli, M. Antonelli, M. Colonna, A. Fantina, Phys. Rev. C 112, 035802 (2025)]

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- Toggled $w_{\mathrm{EFT}}$ **strict band filter**[a] in $[n_{\mathrm{B}}^{\chi}, 0.20]$ fm$^{-3}$ $\Rightarrow$ Probe its **effectiveness** on (Y-)MM

| $X_k$ | $X_k^{\min}$ | $X_k^{\max}$ |
|---|---|---|
| $n_{\text{sat}}$ [fm$^{-3}$] | 0.15 | 0.17 |
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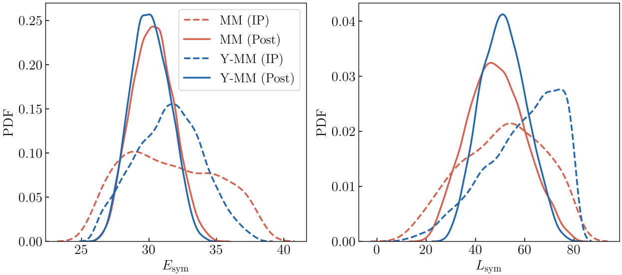
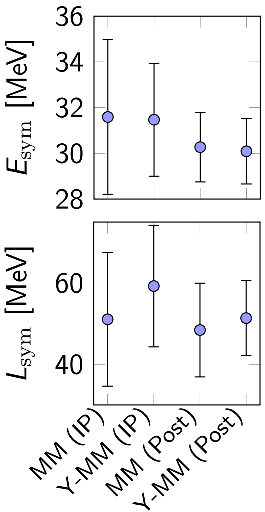
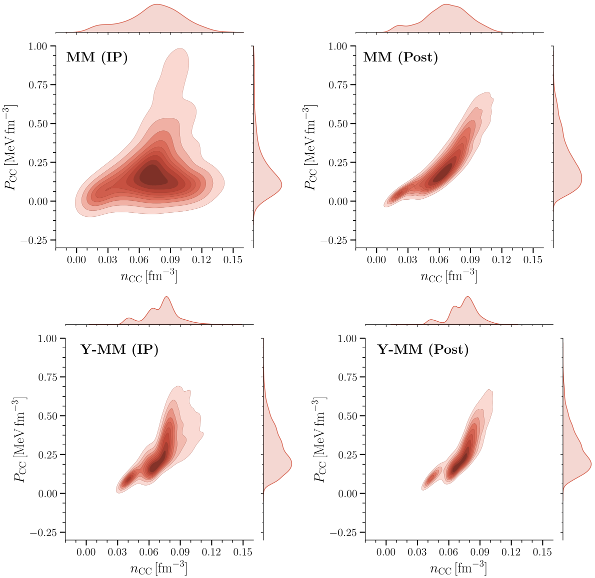
# Isovector empirical parameters: $E_{\rm sym}$ and $L_{\rm sym}$

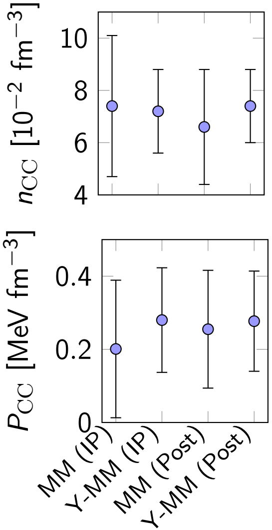



- IP filters **weakly** constraint $E_{\rm sym}$ & $L_{\rm sym}$
  - $\chi$EFT filter **crucial** to constraint $E_{\rm sym} \approx 30$ MeV
- Y-MM reduces **dispersion** & shifts PDF to **stiffer** EoS ($L_{\rm sym} \approx 51$ MeV)

# Crust-core transition density $n_{CC}$ and pressure $P_{CC}$



# Crustal fraction of the moment of inertia $\mathcal{I}_{\text{crust}}/\mathcal{I}$

- **Key** role for interpreting **glitch** activity
- **Slow-rotation** approximation $\left(\frac{\Omega^2 R^3}{GM} \ll 1\right)$
- $\frac{\mathcal{I}_{\text{crust}}}{\mathcal{I}}$ **decreases** while increasing $M$
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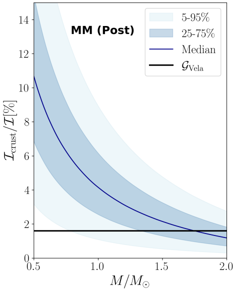


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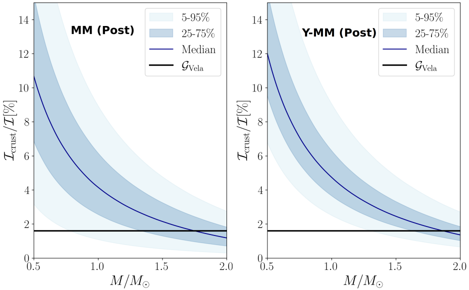


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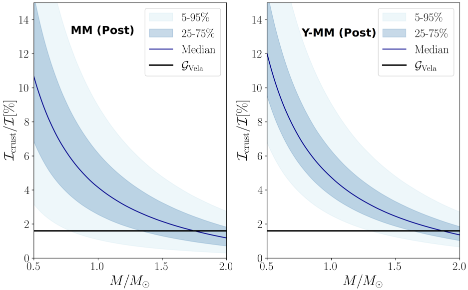


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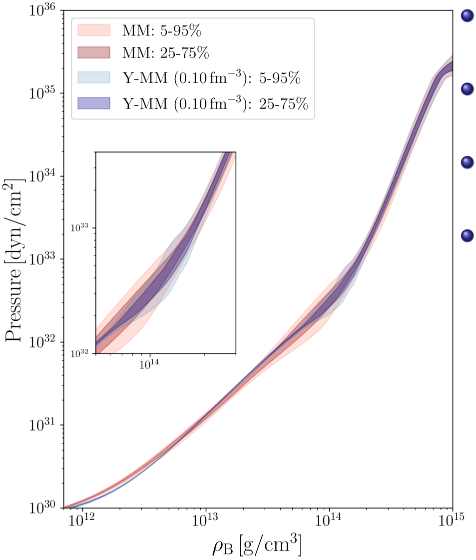
# Neutron star EoS: MM vs Y-MM

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| Model | Post [%] |
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| MM | 0.31 |
| Y-MM ($n_B^{MM} = 0.08$ fm$^{-3}$) | 0.16 |
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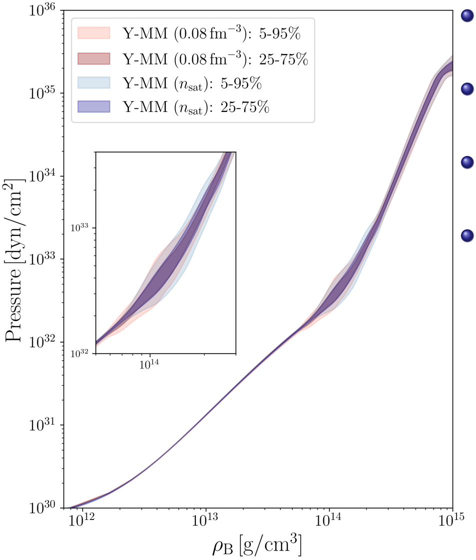


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# Outline of the presentation

1. Inference of neutron star (NS) properties: nuclear & astrophysical constraints
   - Unified modelization of the nuclear matter (NM) Equation of State (EoS)
   - Phenomenological models: energy density functionals (EDFs) & meta-model (MM)

2. Upgraded version of MM: recent developments and results
   - Refined treatment at low-density: homogeneous & inhomogeneous matter
   - Bayesian inference of NS crustal properties

3. **Further developments and outlooks**
   - Implementation in CUTER for interpreting gravitational waves (GW) signals
   - Joint analyses combining also nuclear structure and heavy-ion collision studies

4. **Summary**

# Final remarks and conclusions

## Main topic

- Unified **modeling** of **NS EoS** with a **phenomenological** MM based on EDFs
- Upgraded **Y-MM** through a **benchmark** on ab-initio calculations of **PNM**

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- Analytical procedure, adaptable to any EoS with **minimal computational** cost
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THANK YOU FOR YOUR ATTENTION!

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- **Inhomogeneous** matter in the **inner** crust
  $\Rightarrow$ Energy **minimization** in Wigner-Seitz (**WS**) cells
  + **C**ompressible **L**iquid **D**rop **M**odel ($\beta$-equilibrium)
  - Qualitative **agreement** with **microscopic** methods
  - **Unified** modeling of **core** & **crust** bulk-matter
- **Gas** density $n_g$ & **cluster** size increase with depth
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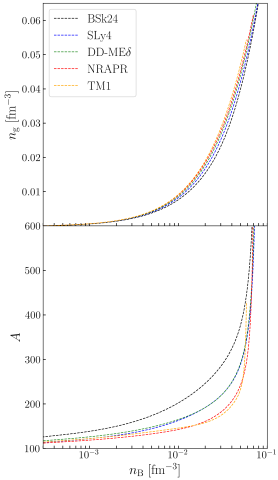

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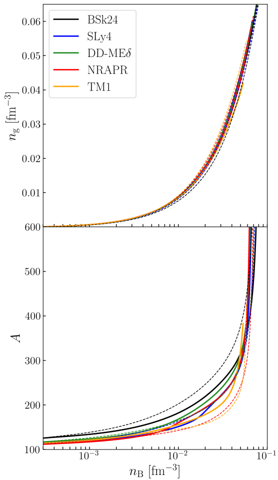

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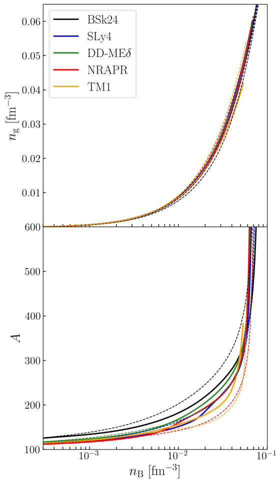

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