TPNI2025- XX conference on Theoretical Nuclear Physics

# Exploring neutron star's glitches with rotating supersolids

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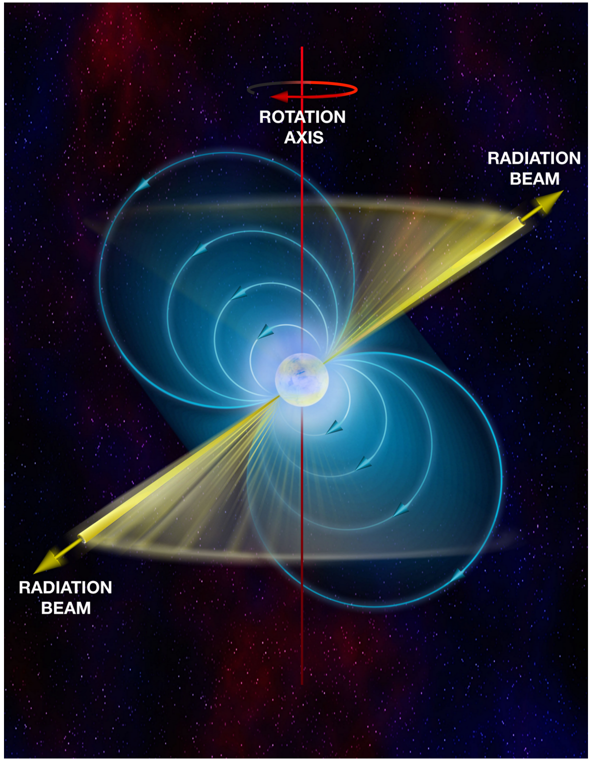
October 1, 2025

Bland, Ferlaino, Mannarelli, Poli, ST, Few-body Systems, 65, 81 (2024)
Poli, Bland, Mark, White, Ferlaino, ST, Mannarelli PRL 65, 81 (2023)

# Outline

- Quantum vortices
- The supersolid platform
- Angular momentum of vortices
- Vortices and Structure

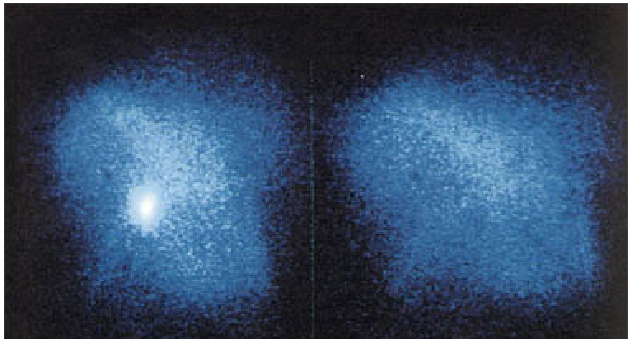
# Neutron stars




$$M \sim 1 \div 2 M_\odot$$
$$R \sim 10\text{km}$$
$$T \sim \text{keV}$$

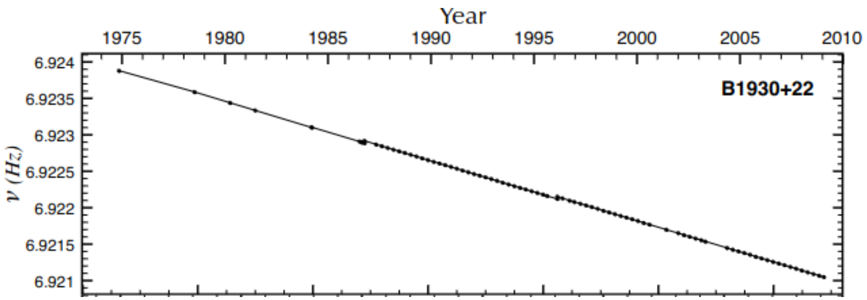
ON-PULSE OFF-PULSE

Crab



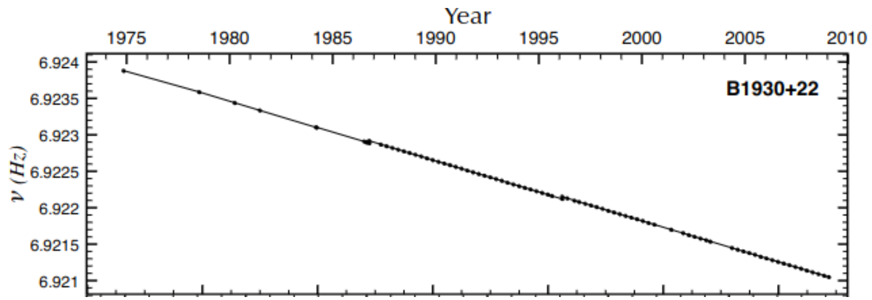
Credits:Wiki-Commons

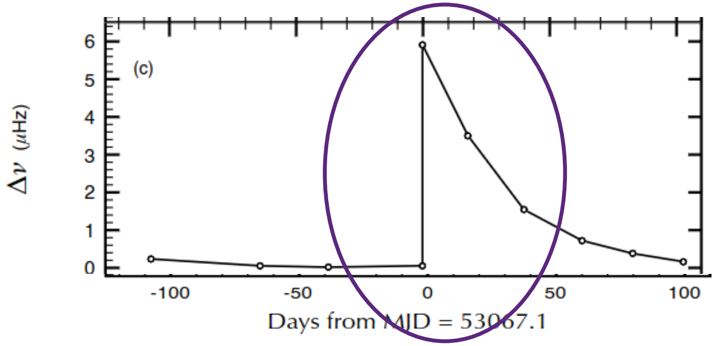
# The glitch signal




Espinoza et al, MNRAS, 414, 2, 1679-1704 (2011)

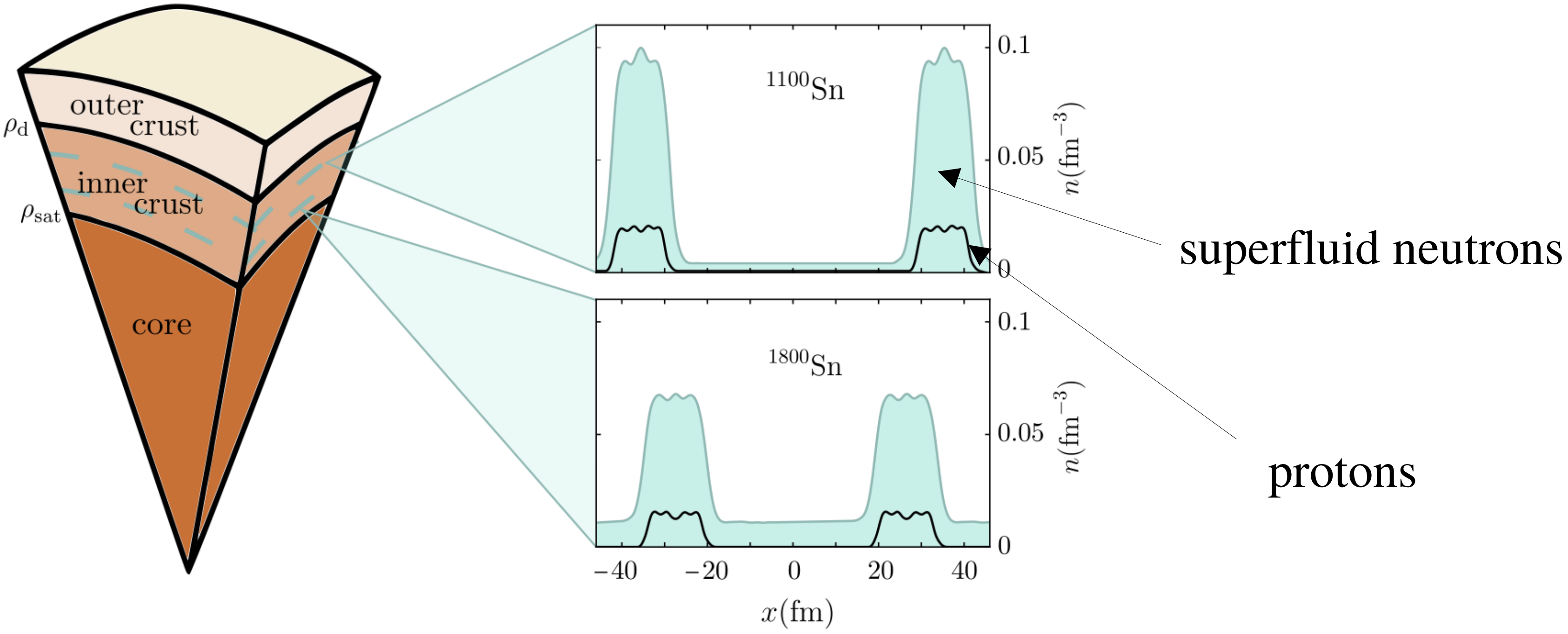
# The glitch signal




Espinoza et al, MNRAS, 414, 2, 1679-1704 (2011)

# Role of superfluidity



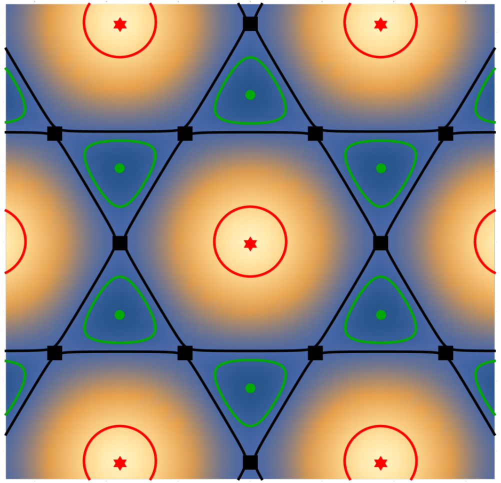

Negele, Vautherin, Nucl. Phys. A207, 298 (1973)
Warszawski, Melatos, MNRAS, 415,1611 (2011)

# Vortex trajectories (toy model)

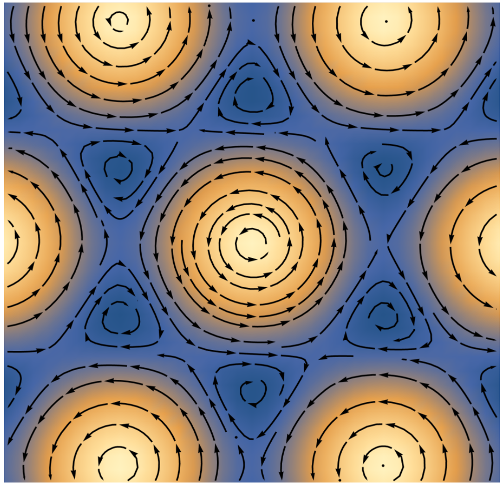
$$\rho(\boldsymbol{r}) = \rho_0 \left(1 + C \sum_{i=1}^{3} \cos(\boldsymbol{q}_i \cdot \boldsymbol{r})\right)$$

$$\boldsymbol{v}_v = \frac{\hbar}{m} \{\nabla(\Phi - \gamma \log \sqrt{\rho}) - \hat{\boldsymbol{z}} \times \nabla(\log \sqrt{\rho} + \gamma\Phi)\}$$

No Dissipation



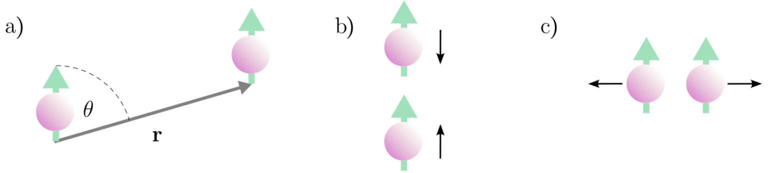
With dissipation

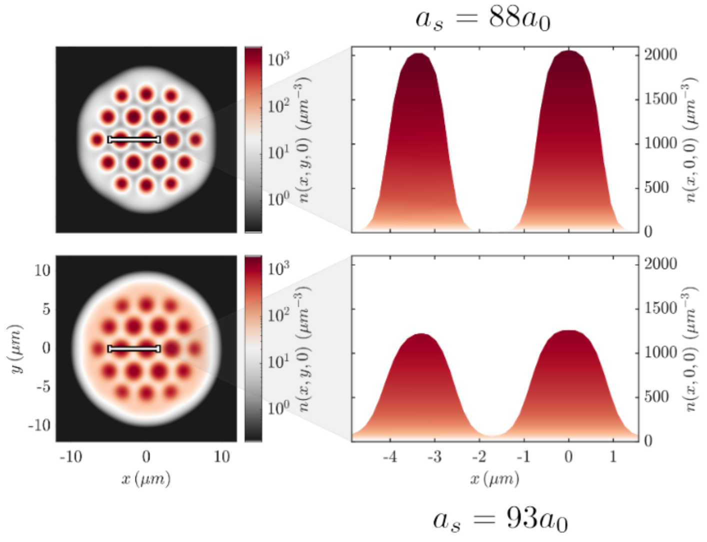


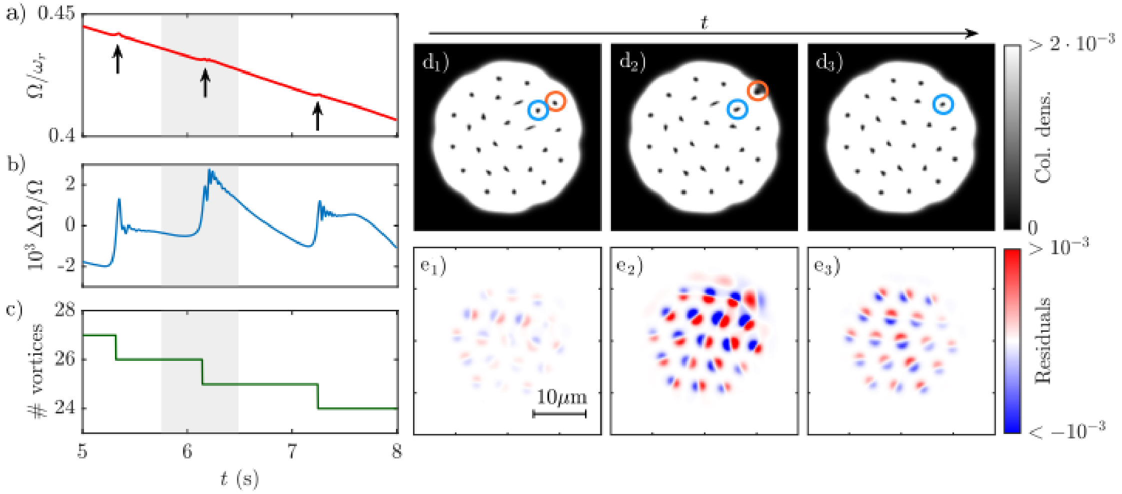

Pomeau, Rica, PRL 72, 2624 (1994)
Alana, Modugno, Capuzzi, Jezek, PRA 110, 023306 (2024)
Bland, Ferlaino, Mannarelli, Poli, ST, Few-body Systems, 65, 81 (2024)

# Glitches in rotating supersolids

Numerical simulations with ultracold highly magnetic atoms in the supersolid phase

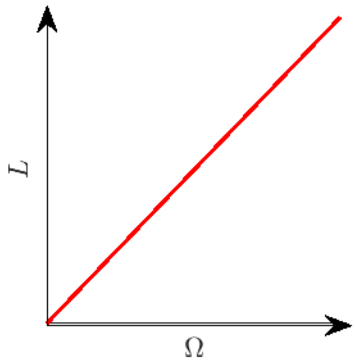


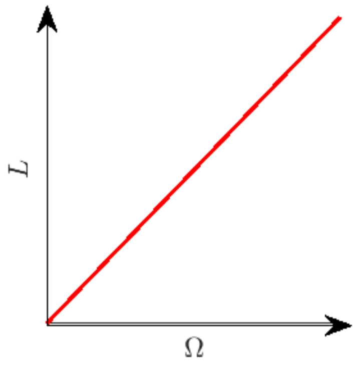
# Response to an external rotation

Solid-like

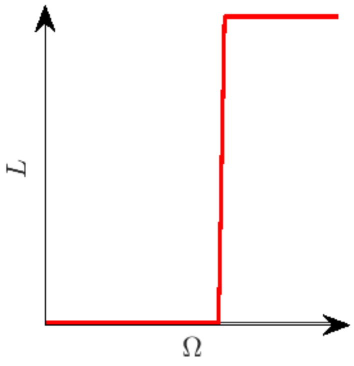


# Response to an external rotation

Solid-like



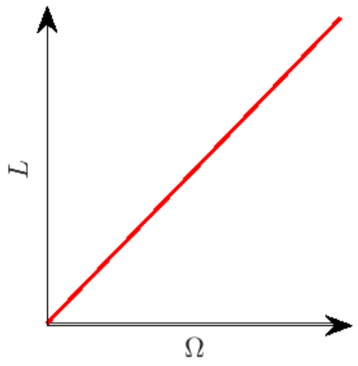

Superfluid-like



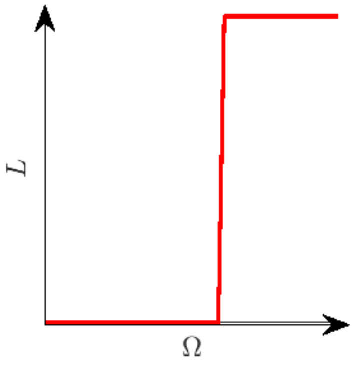

Supersolid-like

# Rotating a supersolid

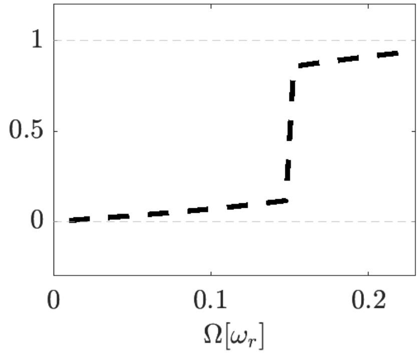
Solid-like




Superfluid-like




Supersolid




Bottcher, ..., Pfau, PRX 9, 011051 (2019)
Tanzi, ..., Modugno, PRL 122, 130405 (2019)
Chomaz, ..., Ferlaino, PRX 9, 021012 (2019)

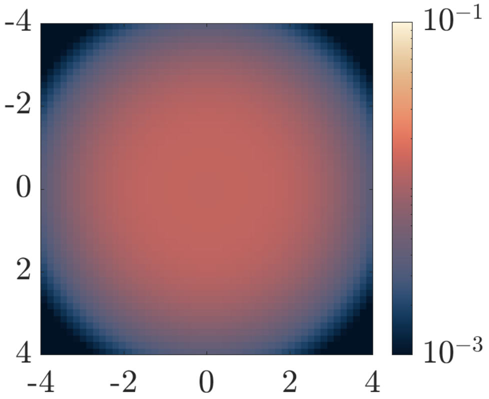
Casotti, Poli, ..., Ferlaino, Nature 635, 327–331 (2024)

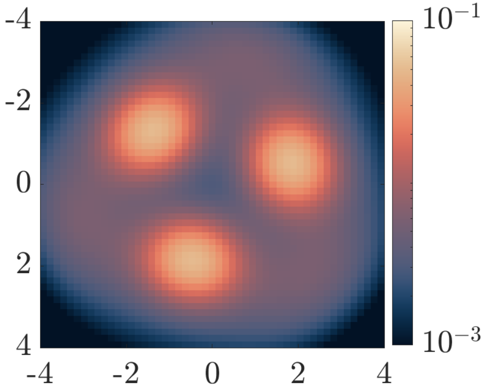
# Simulating the system

Solving the Gross- Pitaevskii equation we have the wavefunction

$$\psi(\mathbf{x}) = \sqrt{n(\mathbf{x})}e^{iS(\mathbf{x})} \Longrightarrow \boldsymbol{v} = \hbar \nabla S/m \Longrightarrow \langle \hat{L}_z \rangle$$

Uniform

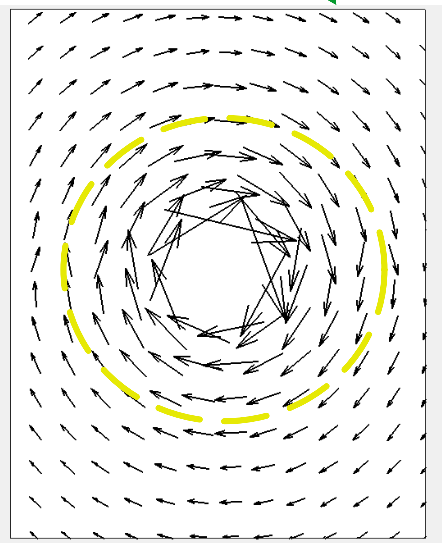



Supersolid

# Helmholtz decomposition

$$\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{v}_{\mathrm{c}}(\boldsymbol{x}) + \boldsymbol{v}_{\mathrm{i}}(\boldsymbol{x})$$



**Incompressible**

**Component**

Vortex contribution

Vortex
quantization

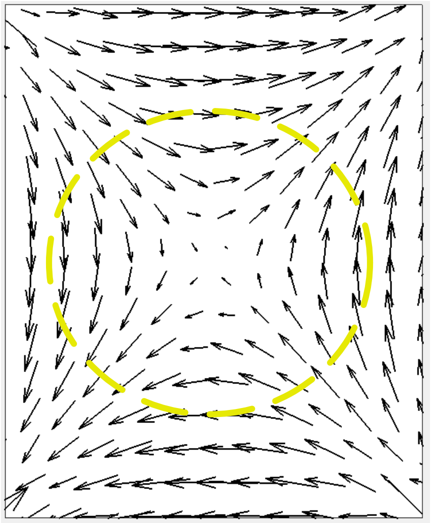
# Decomposing the velocity field

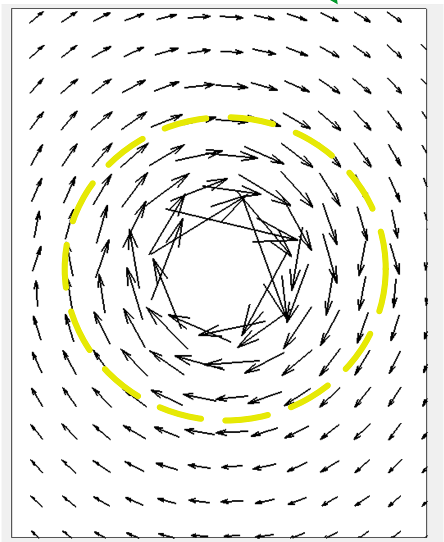
$$\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{v}_{\mathrm{c}}(\boldsymbol{x}) + \boldsymbol{v}_{\mathrm{i}}(\boldsymbol{x})$$

**Compressible Component**

Everything else

Vanishing circuitation





**Incompressible Component**

Vortex contribution

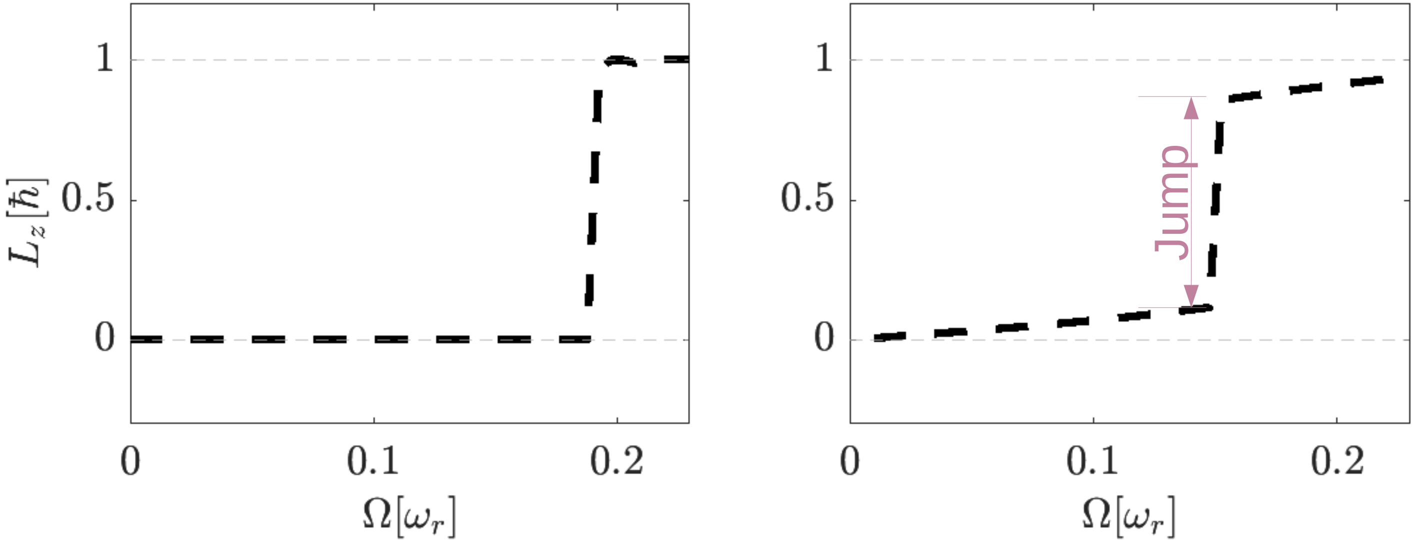
Vortex quantization

# Ground states

$$L_z = m \int d^3\boldsymbol{x}\, n(\boldsymbol{x}) [\boldsymbol{x} \times \boldsymbol{v}]_z$$

Uniform

Supersolid

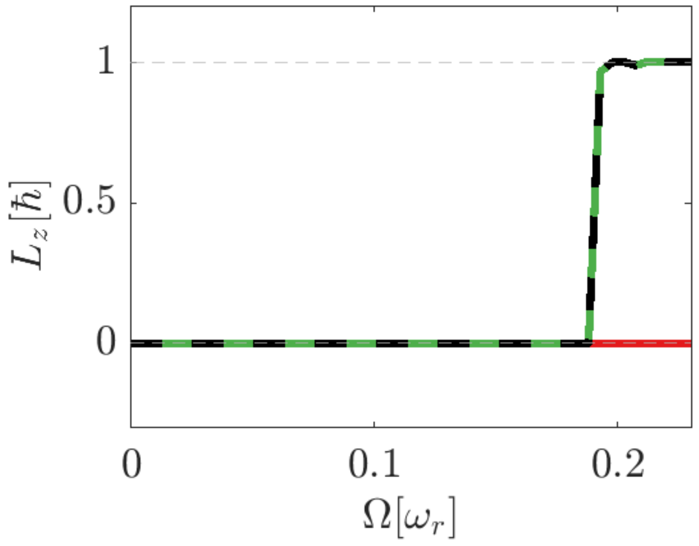


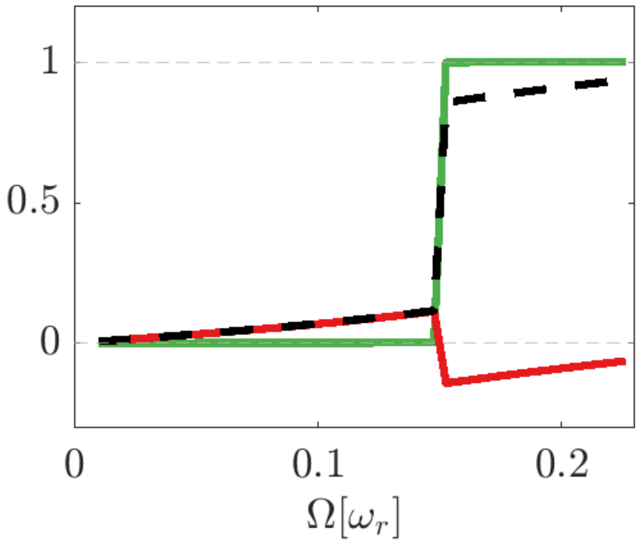

Roccuzzo, ..., Stringari, PRL 124, 045702 (2020)
Gallemi, ..., Recati PRA 102, 023322 (2020)

# Ground states

$$L_z = m \int d^3\boldsymbol{x}\, n(\boldsymbol{x})[\boldsymbol{x} \times \boldsymbol{v}]_z = L_c + L_i$$

Uniform

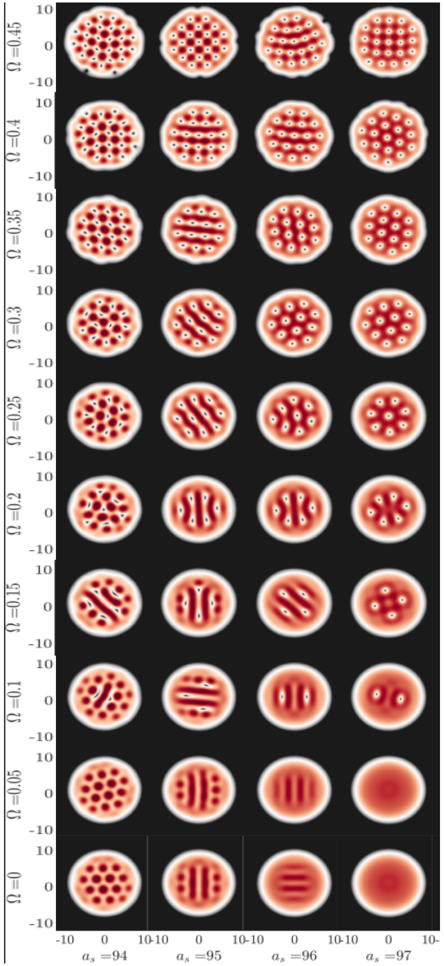


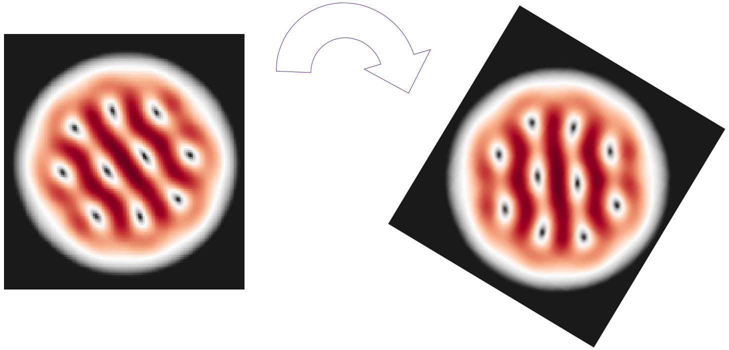




Supersolid

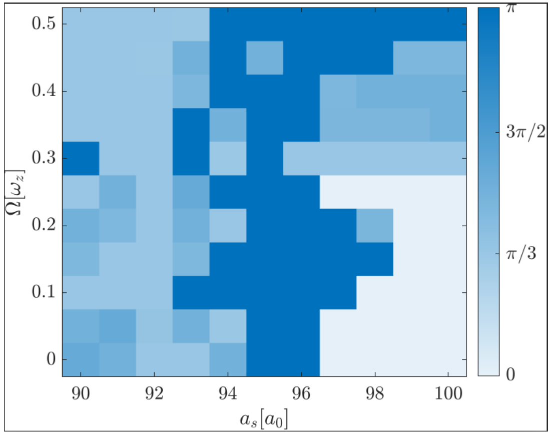
ST, Poli, Mannarelli, Ferlaino, *Manuscript in preparation*

# Vortex-induced modulation



Periodicity or angular correlation






ST, Poli, Mannarelli, Ferlaino, *Manuscript in preparation*

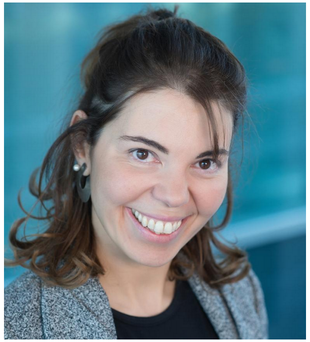
# Conclusions

Modulated superfluids have complex response to rotation

The presence of vortices can modify the structure

Massimo Mannarelli

Elena Poli



Francesca Ferlaino

# Thank you for listening

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# Backup slides

# The eGPE operator

$$\mathcal{L}[\psi; a_s, a_{\mathrm{dd}}, \boldsymbol{\omega}] = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\left[\omega_r^2(x^2+y^2) + \omega_z^2 z^2\right] + \int \mathrm{d}^3\mathbf{r}'\, U(\mathbf{r}-\mathbf{r}')\,|\psi(\mathbf{r}',t)|^2 - \mu + \gamma_{\mathrm{QF}}|\psi(\mathbf{r},t)|^3\,,$$

$$U(\mathbf{r}) = \left(4\pi\hbar^2 a_s/m\right)\delta(\mathbf{r}) + \left(3\hbar^2 a_{\mathrm{dd}}/m\right)\left[\left(1-3\cos^2\theta\right)/|\mathbf{r}|^3\right]$$

# Simulating the system

$$i\hbar\partial_t\psi = (\alpha - i\gamma)\left[\mathcal{L}[\psi; a_s, a_{\mathrm{dd}}, \boldsymbol{\omega}] - \Omega(t)\hat{L}_z\right]\psi$$

$$\psi(\mathbf{x}) = \sqrt{n(\mathbf{x})}e^{iS(\mathbf{x})} \implies \boldsymbol{v} = \hbar\nabla S/m \implies \langle\hat{L}_z\rangle$$

# Helmholtz decomposition

$$\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{v}_{\mathrm{c}}(\boldsymbol{x}) + \boldsymbol{v}_{\mathrm{i}}(\boldsymbol{x})$$

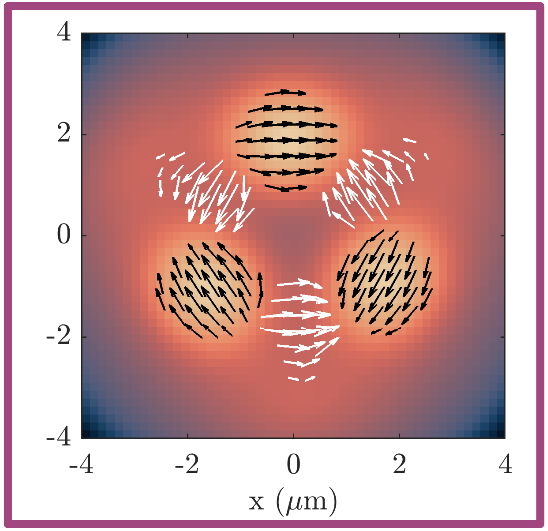
$$\hat{\boldsymbol{v}}_c = \frac{\boldsymbol{k}(\boldsymbol{k}\cdot\hat{\boldsymbol{v}})}{\boldsymbol{k}\cdot\boldsymbol{k}}$$

- Responsible of the linear response of the system
- Longitudinal component, describes a compression of the flow $\nabla \times \boldsymbol{v}_c = 0$
- Curl-free

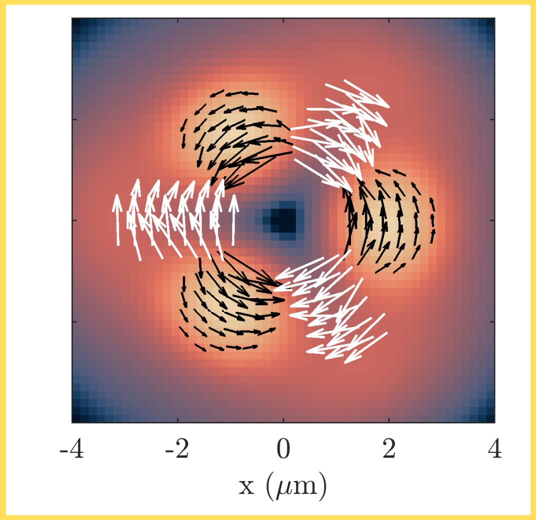
$$\hat{\boldsymbol{v}}_i = \hat{\boldsymbol{v}} - \frac{\boldsymbol{k}(\boldsymbol{k}\cdot\hat{\boldsymbol{v}})}{\boldsymbol{k}\cdot\boldsymbol{k}}$$

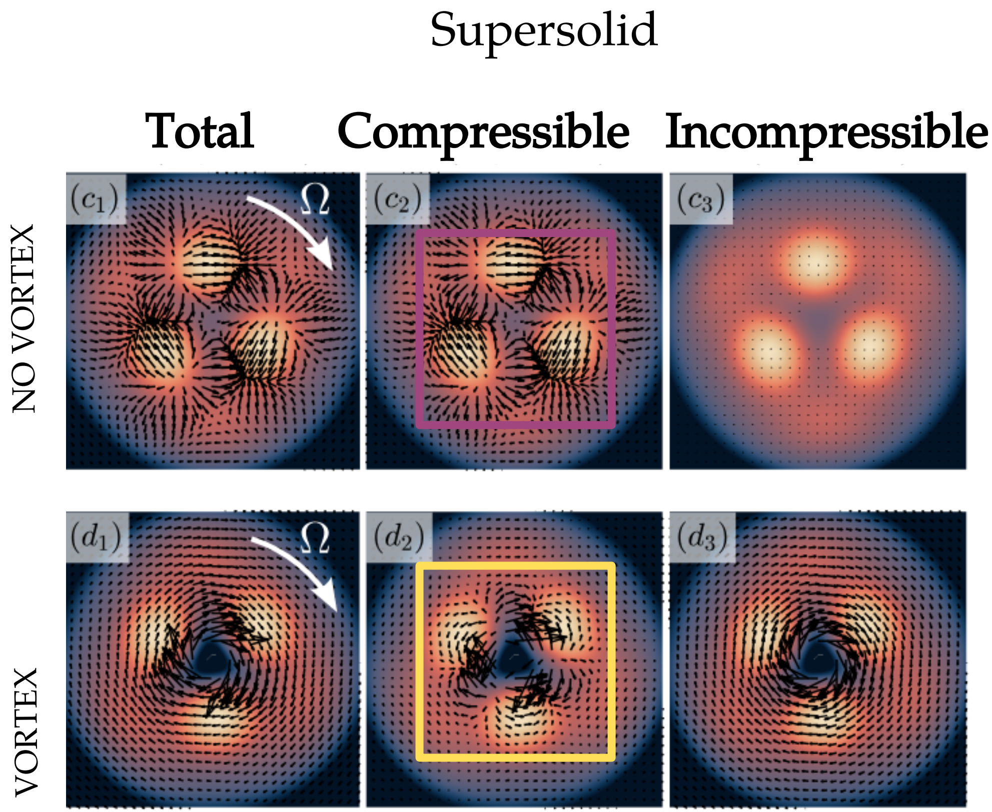
- Contains quantum vortices, and is related to vorticity $\nabla \times \boldsymbol{v} = \nabla \times \boldsymbol{v}_i = \kappa\hat{z}\delta(x - x_c)\delta(y - y_c)$
- Transverse component, describes incompressible flow $\nabla \cdot \boldsymbol{v}_i = 0$
- Divergence-free

Counterflow in the low density region



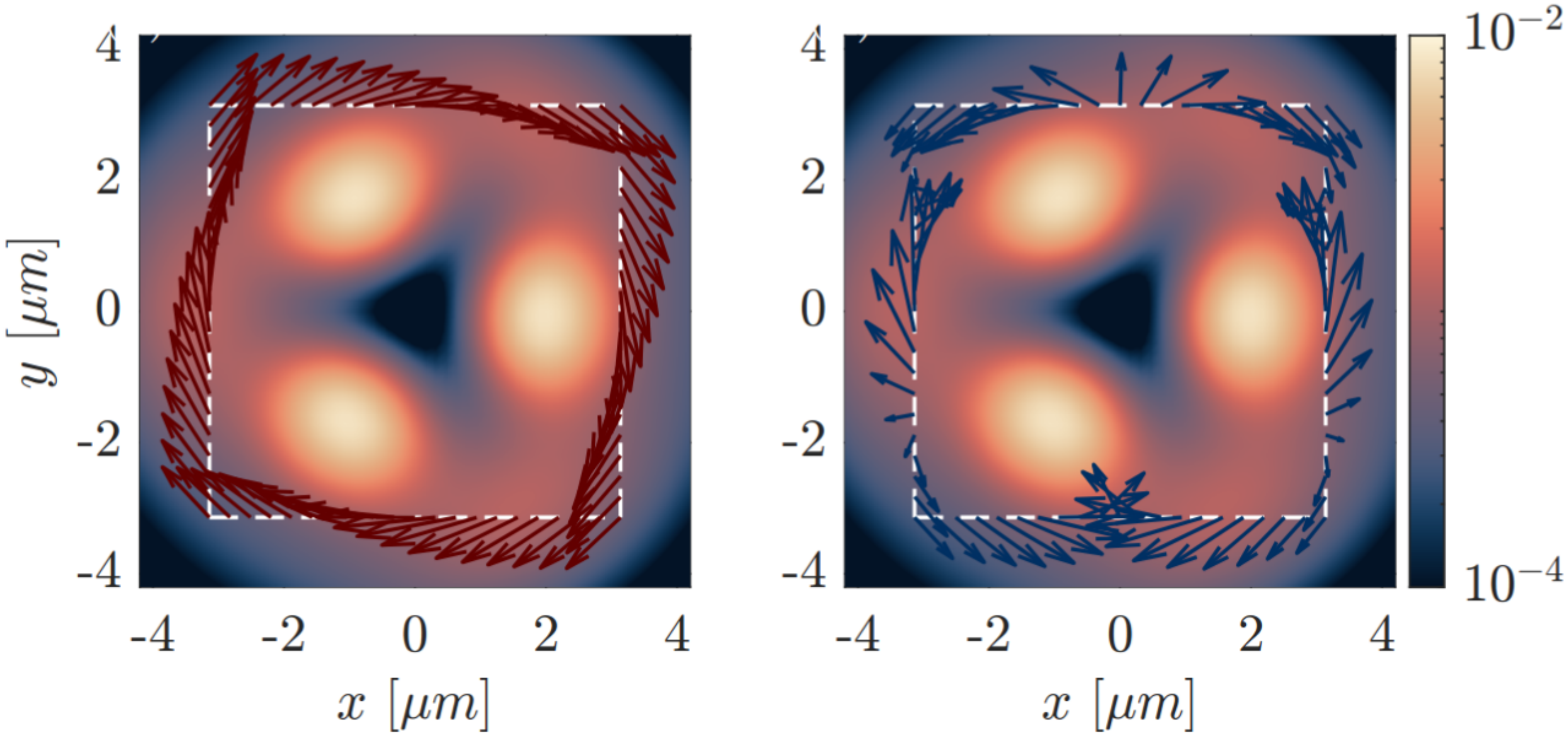

Counterflow in the density peaks



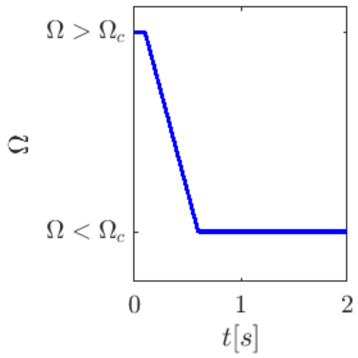



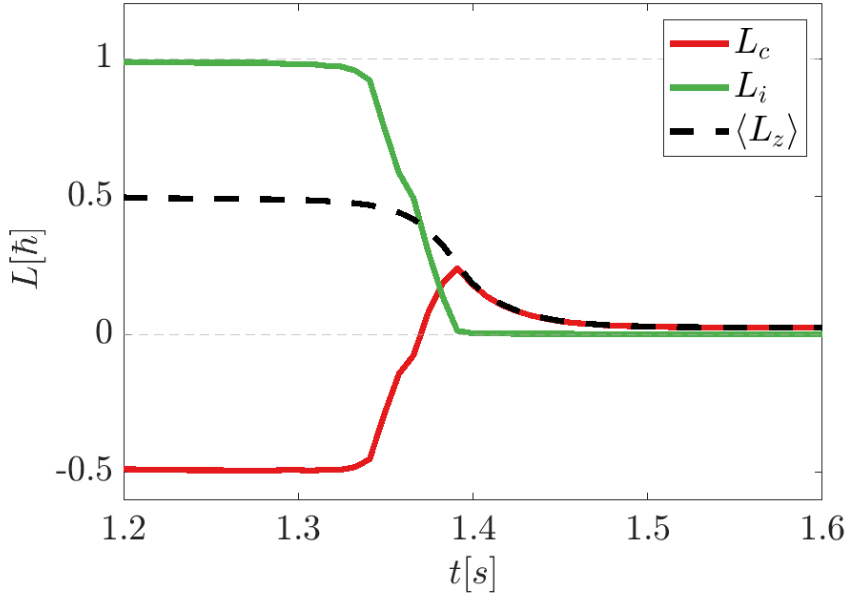

ST, Poli, Mannarelli, Ferlaino, *Manuscript in preparation*

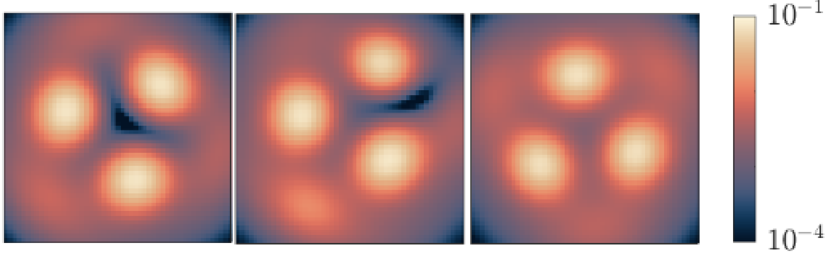
# A useful test



# Relaxation by means of vortex expulsion



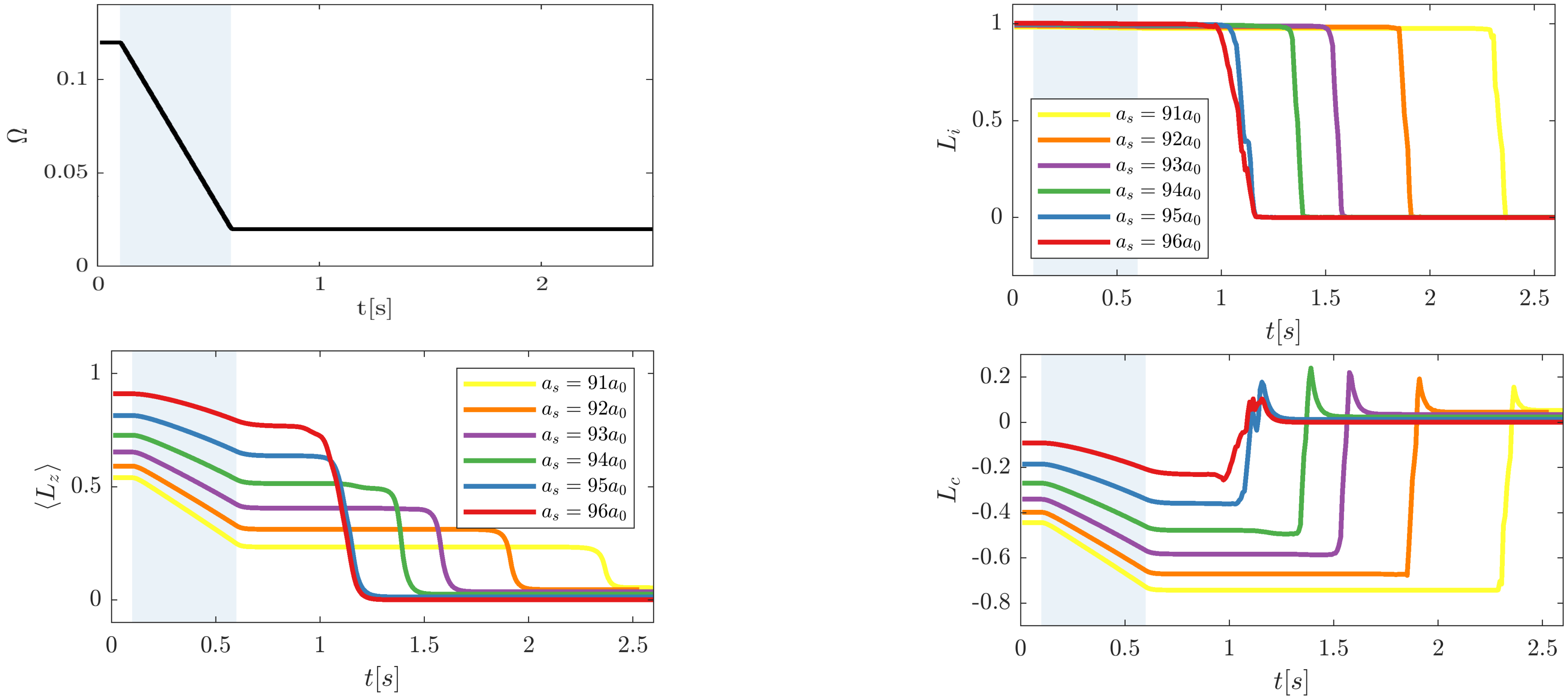



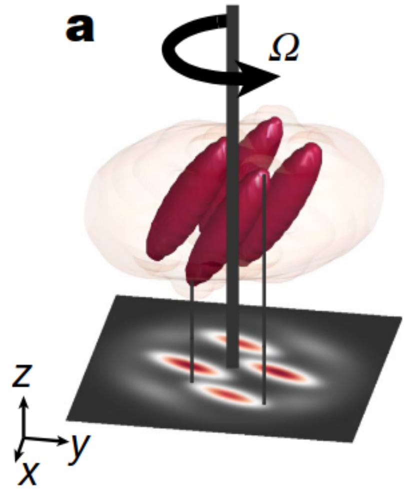

t = 1.24s     t = 1.37s     t = 1.56s

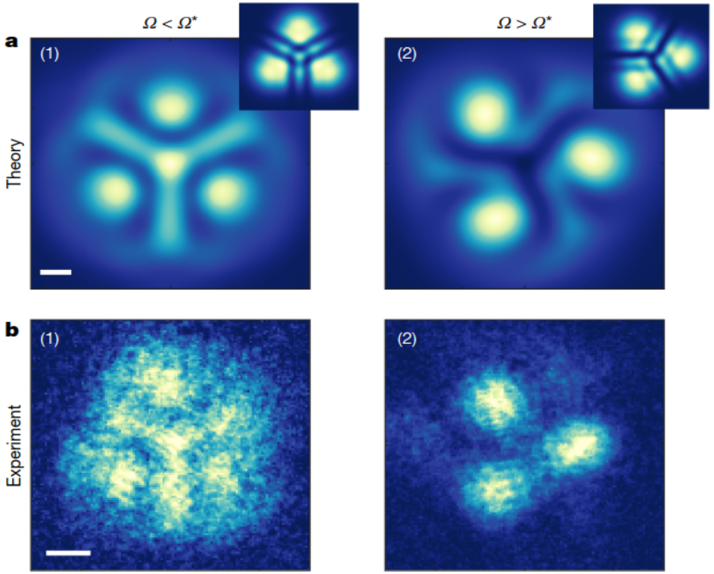
# Dynamics of vortex expulsion

Protocol for quasi-adiabatic vortex expulsion



# Observation of vortices in supersolids




Casotti, Poli, ..., Ferlaino, Nature 635, 327–331 (2024)