QCD EOS from effective Lagrangian and constraints from RMF models

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Istituto Nazionale di Fisica Nucleare

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Outline

- Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

  1. Extension of previous work in the pure gauge $SU(3)_c$ sector:
     - Inclusion of thermal fluctuations and excited glueballs
     - Comparison of results with lattice QCD data

  2. Contributions meson and baryons sector: σ, π, ω, ρ meson field and nucleons N
     - Investigation of the impact of thermal fluctuations for all fields
     - Exploration of phase diagram in the meson–baryon sector

- RMF predictions for the dense-matter EOS and application to NSs
  - Models generated using the MCMC approach and Bayesian statistics.
  - Models are filtered using astrophysical constraints

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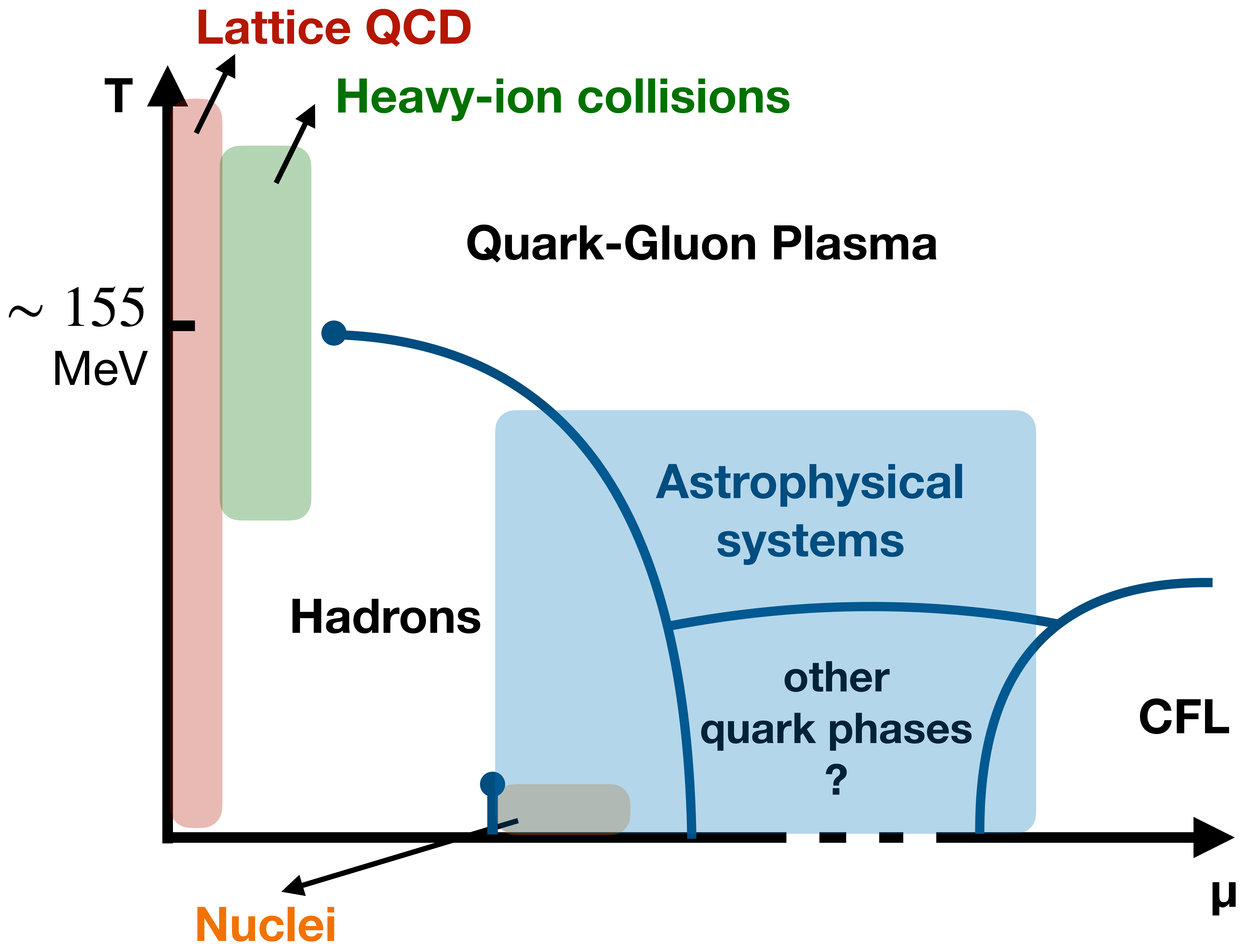
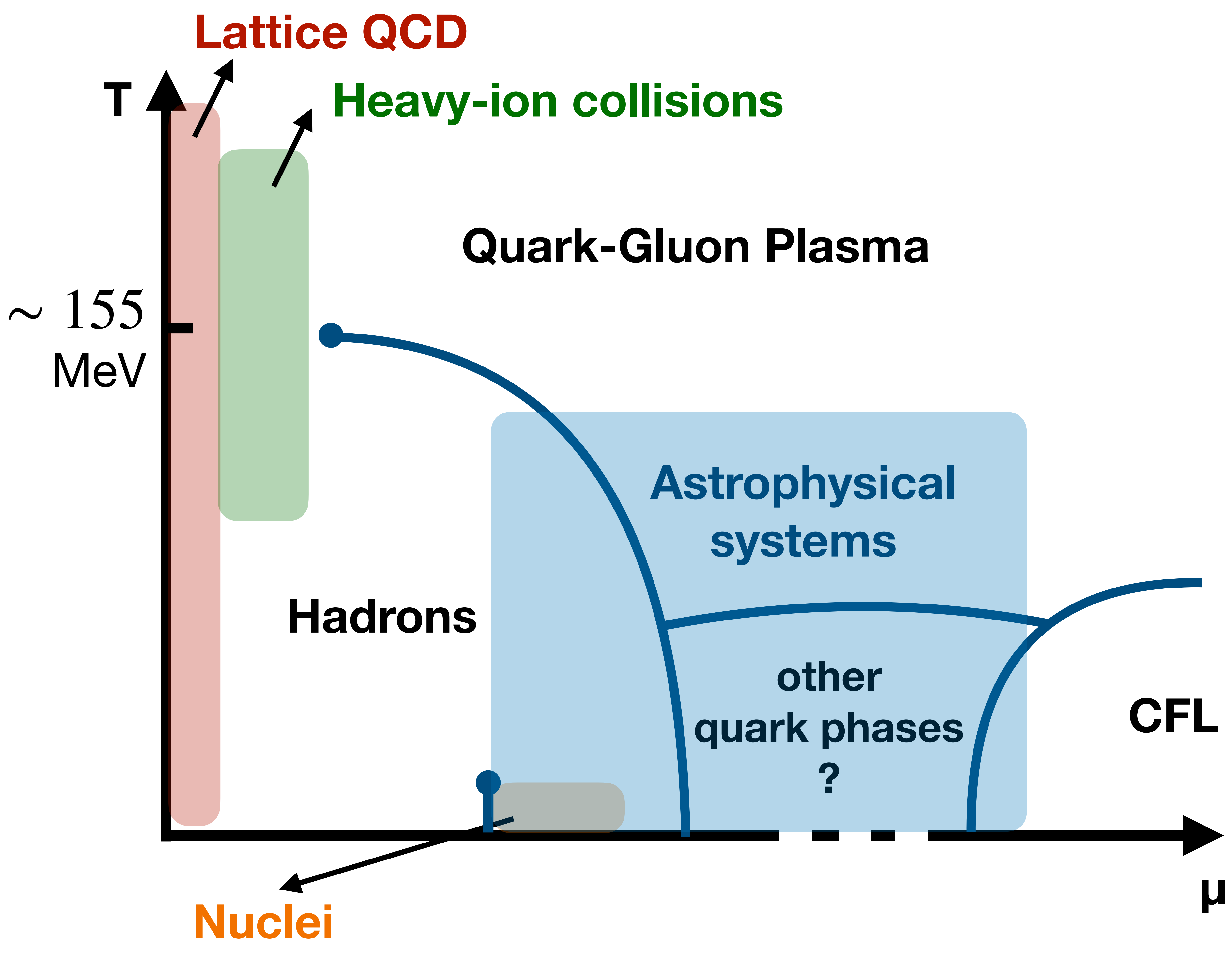
1 Introduction

QCD phase diagram



QCD phase diagram

- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions



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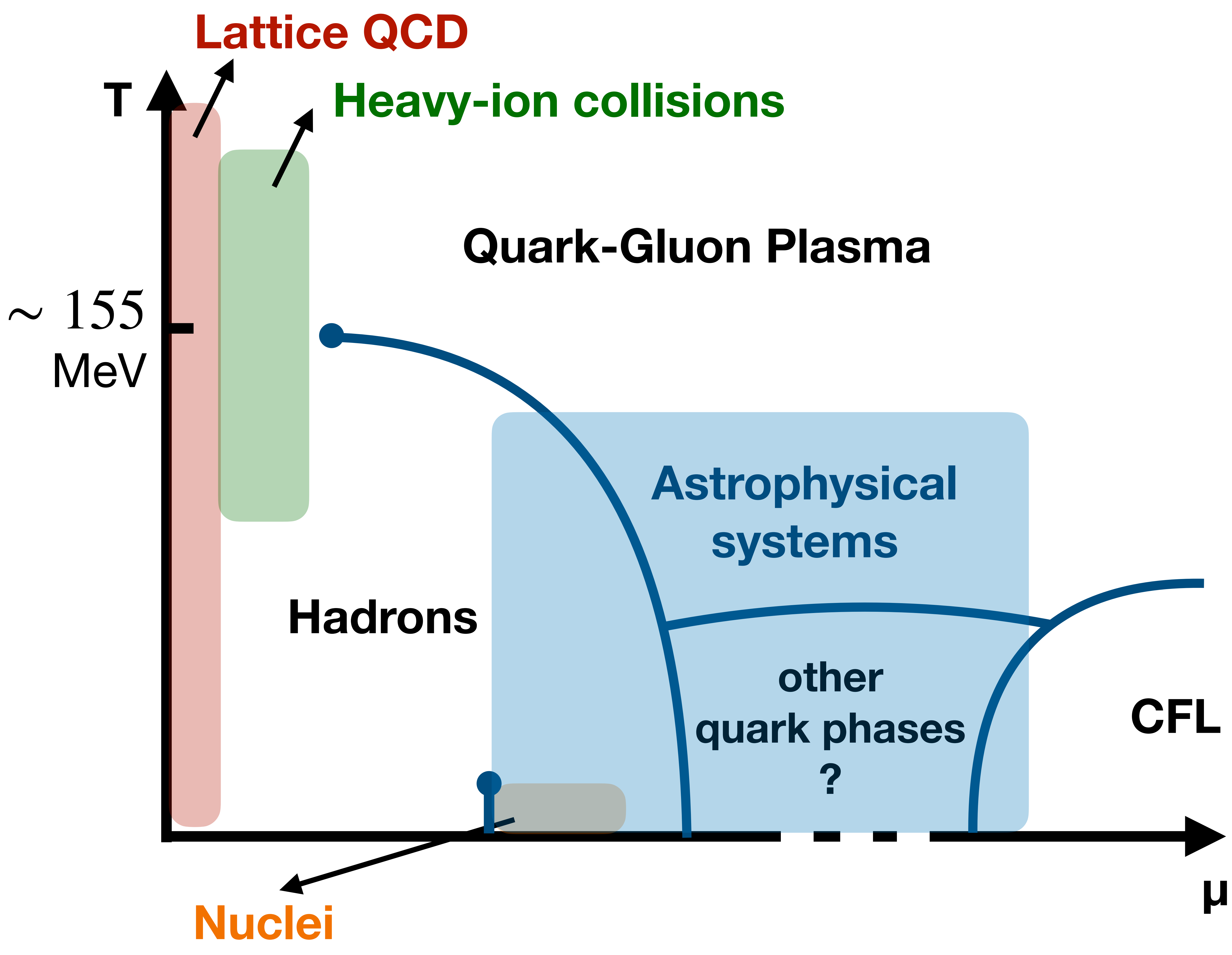
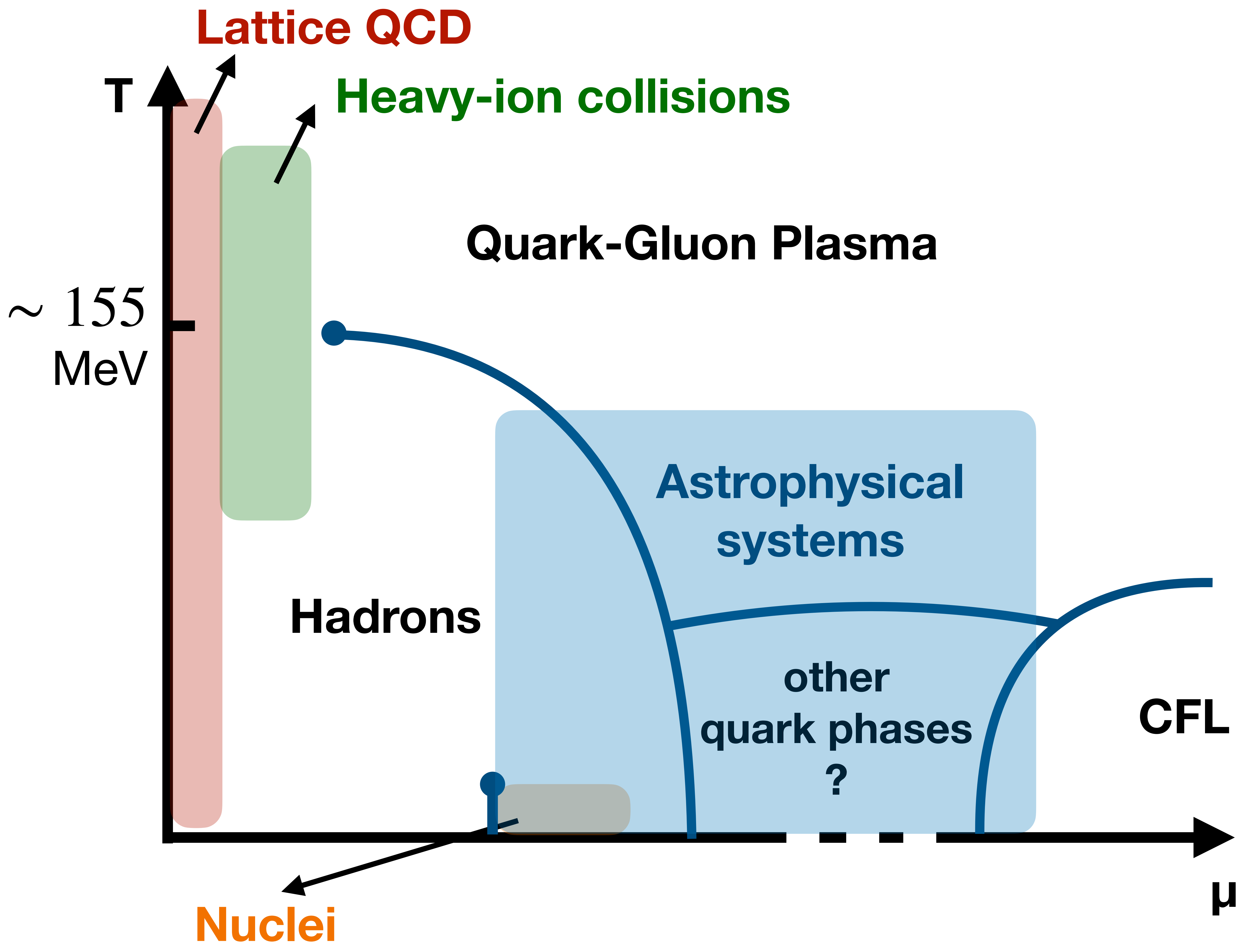


QCD phase diagram

- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions

- Regimes of low temperatures and high densities: neutron stars and compact stars

- Only a few EOS are studied in both regimes simultaneously and with several model-dependent parameters: a complete description is still challenging.



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2 Pure Gauge $\mathrm{SU(3)_c}$ sector

Pure gauge $\mathrm{SU(3)_c}$ sector

Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)

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Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln\frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln\chi\right).$$

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The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle\mathscr{V}(\chi)\rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2}m_\chi^{*2}\left\langle\Delta_\chi^2\right\rangle.$$

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Infrared cut-off $\quad K(\chi) = \dfrac{A}{1 - \chi}$

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It is possible to divide the glueball into two distinct components: the mean field $\bar{\chi}$ and the fluctuating part Δ_χ, with $\langle \Delta_\chi \rangle = 0$.

$$\chi = \bar{\chi} + \Delta_\chi$$

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$$\langle \Delta_\chi^2 \rangle = \frac{1}{2\pi^2 \phi_0^2} \int_0^\infty \frac{k^2}{\epsilon_\chi} \frac{1}{e^{\beta \epsilon_\chi} - 1} dk$$

$$P_\chi(z) = \sqrt{\frac{1}{2\pi \langle \Delta_\chi^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\chi^2 \rangle} \right)$$

$$\langle f(\chi) \rangle = \int_{-\infty}^{\infty} P_\chi(z) f(\bar{\chi} + z)\, dz$$

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Borsányi et al., JHEP 07 (2012) 056

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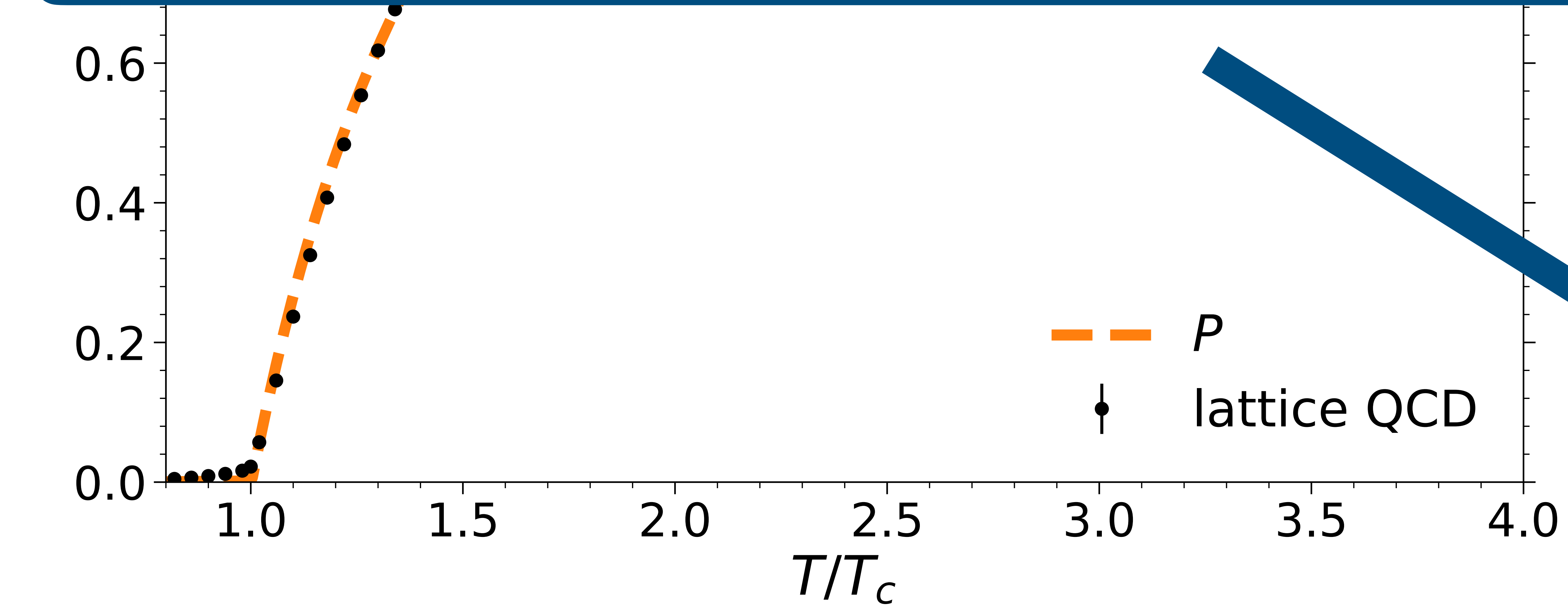
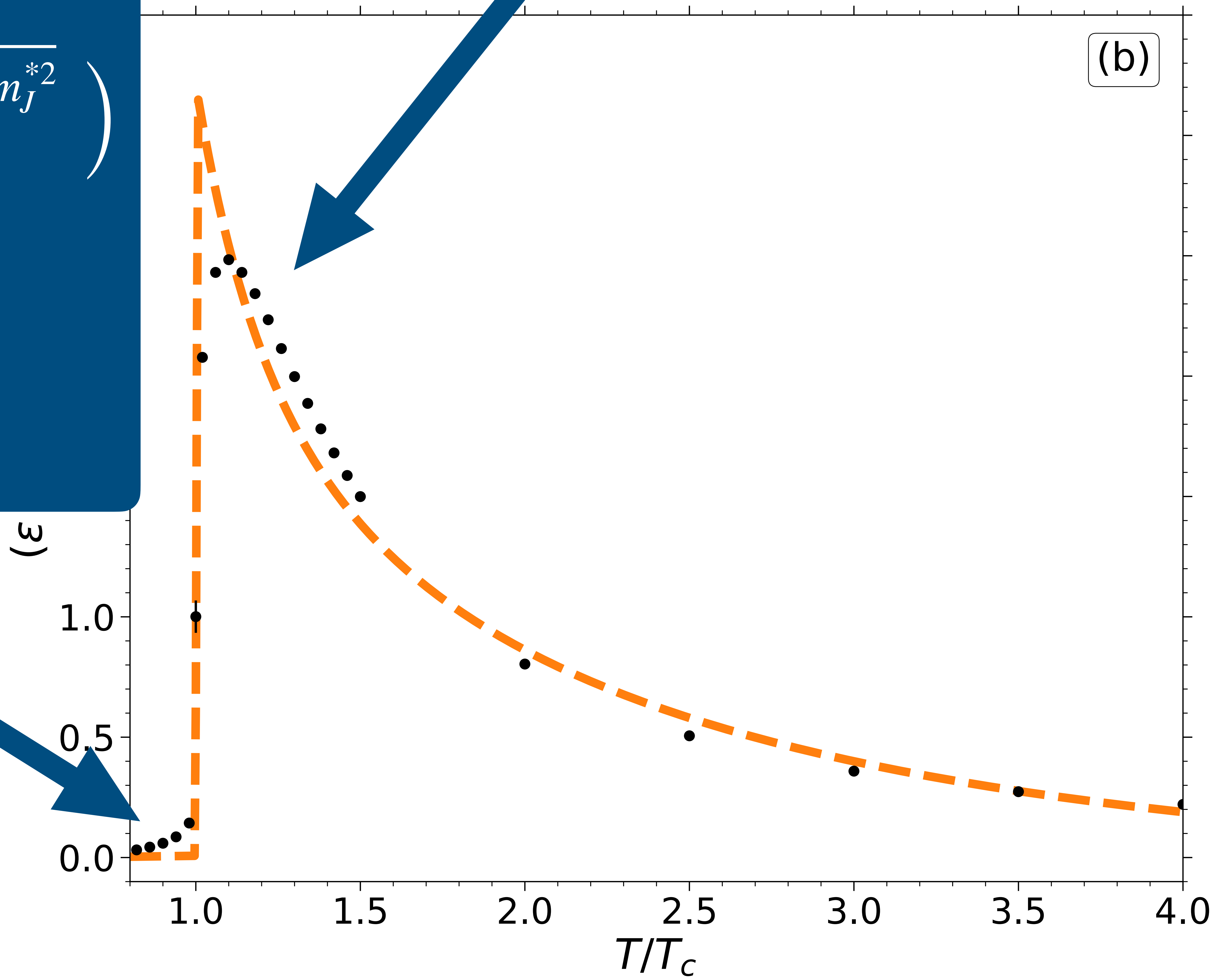
$$P_{\mathrm{dil}_J} = -\,(2J+1)T\int \frac{dk^3}{(2\pi)^3}\,\ln\!\left(1 - e^{-\beta\sqrt{k^2+m_J^{*2}}}\right)$$

$$\frac{\tilde{\Omega}(\chi,T)}{V} = \frac{\Omega(\chi,T)}{V} - \sum_J^N P_{\mathrm{dil}_J}$$





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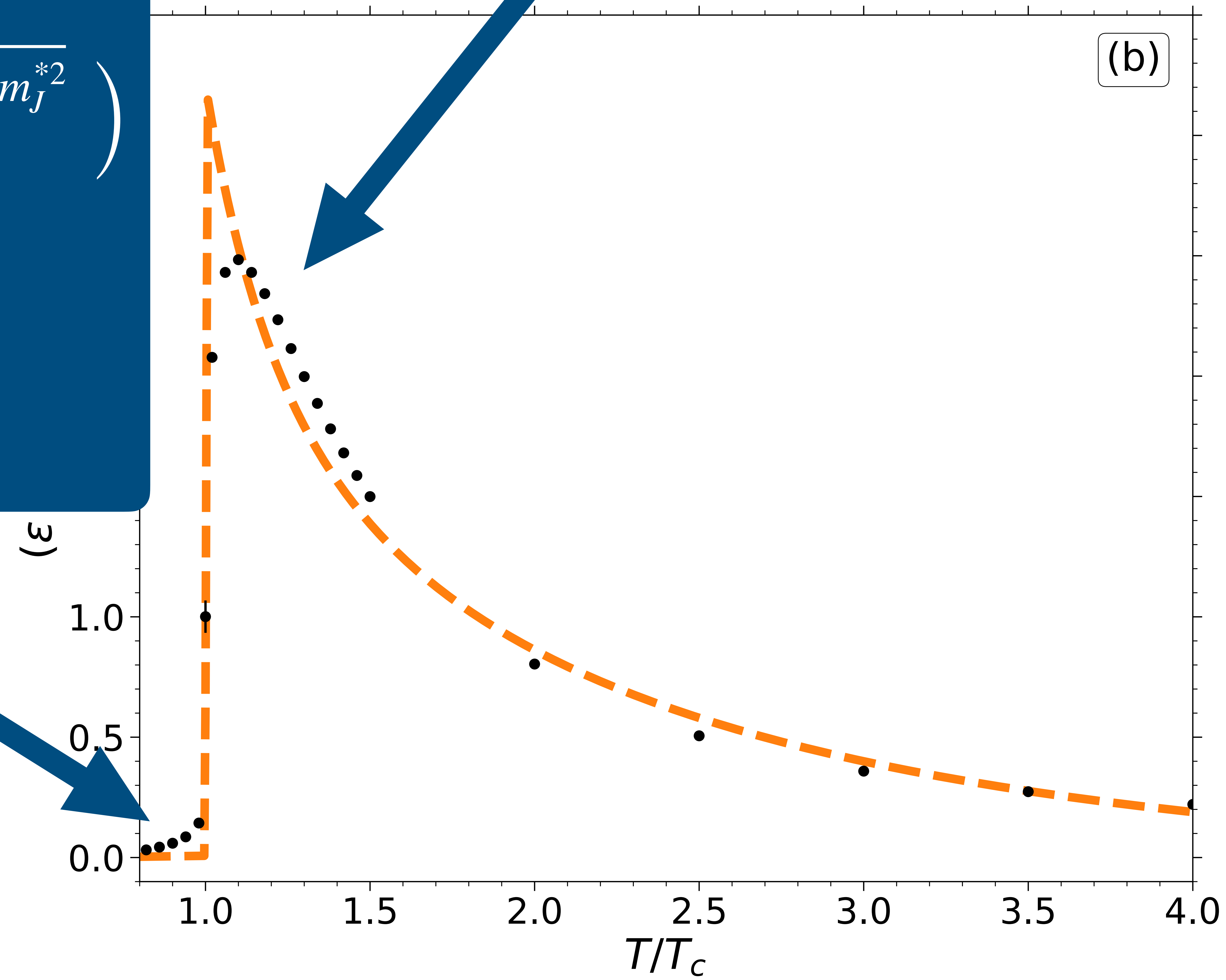
$$\frac{\tilde{\Omega}(\chi,T)}{V} = \frac{\Omega(\chi,T)}{V} - \sum_J^N P_{\mathrm{dil}_J}$$

Modified infrared cut-off

$$\tilde{K}(\chi,T) = \frac{A}{1-\chi^2} = \frac{A}{1-\bar{\chi}^2 - \langle\Delta_\chi^2\rangle\cdot F(T)}$$



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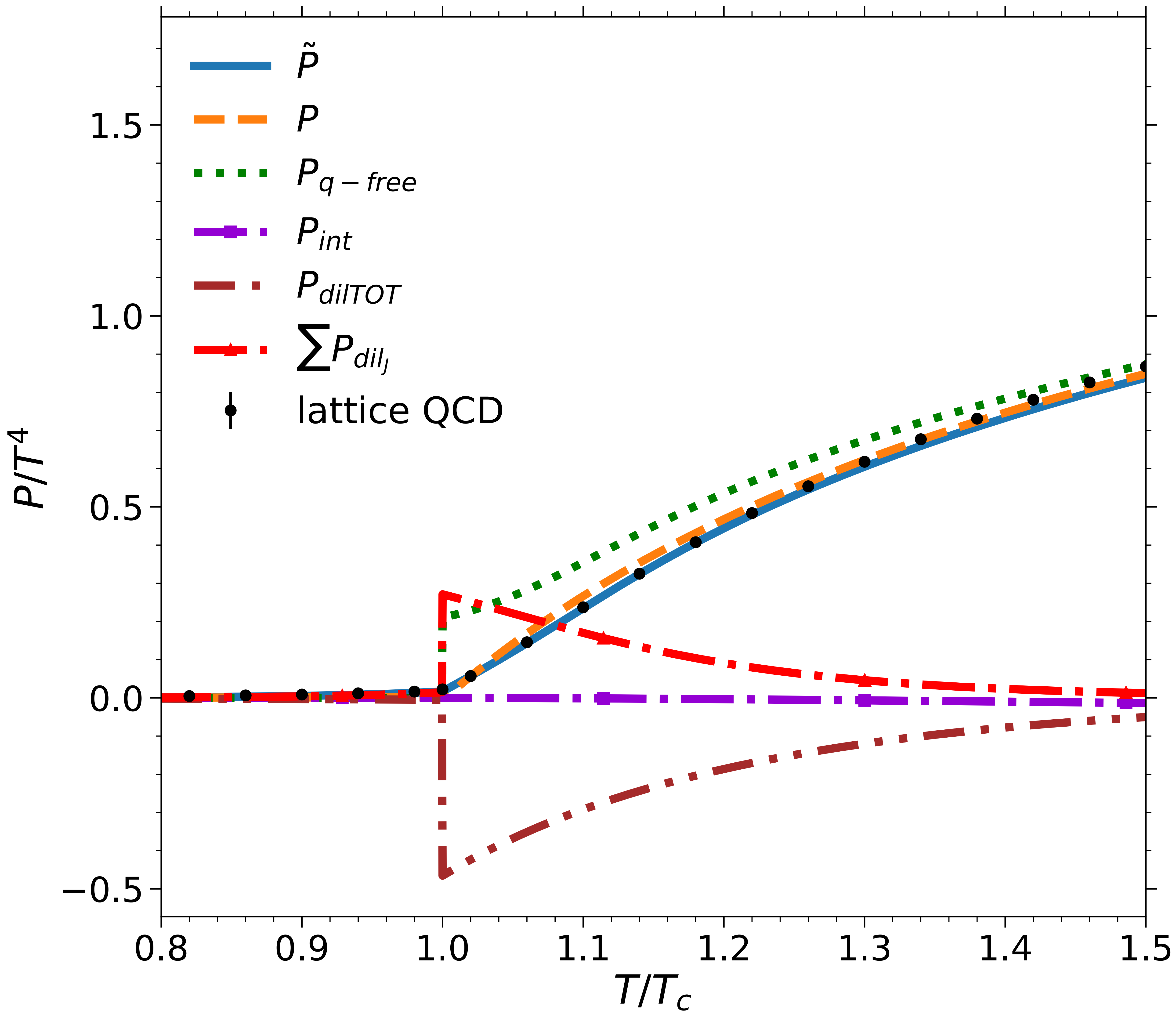

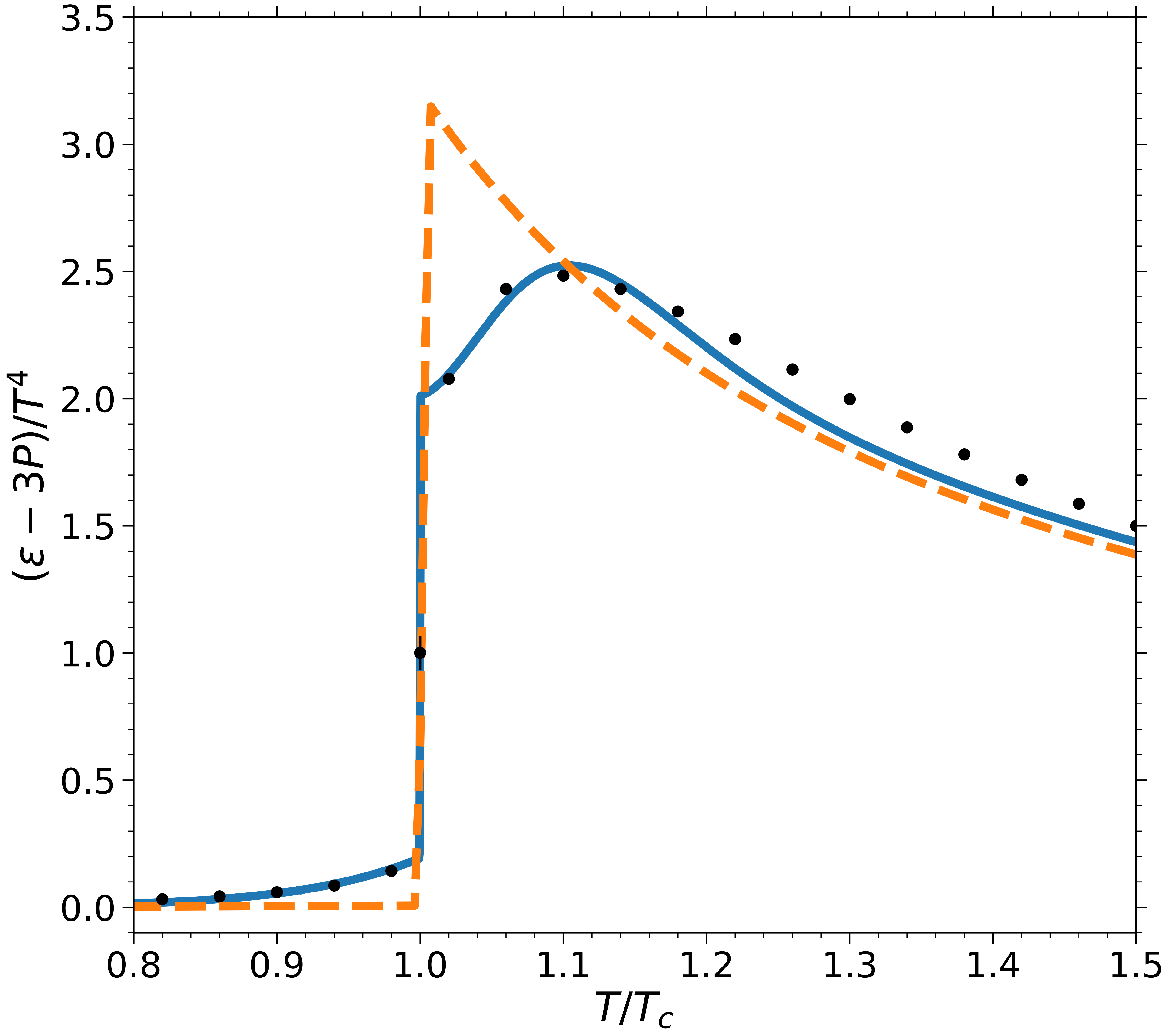

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3 Meson and baryon sector

Meson and baryon sector at finite chemical potential

We consider the **meson** and **baryon** sector at **finite chemical potential** through the introduction of an effective Lagrangian that incorporates broken scale in addition to explicit broken chiral symmetry

$$\tilde{\mathscr{L}} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{4}\boldsymbol{B}_{\mu\nu}\cdot\boldsymbol{B}^{\mu\nu} + \frac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu + \frac{1}{2}G_{b\phi}\phi^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu$$

$$+ \left[(G_4)^2\omega_\mu\omega^\mu\right]^2 - \tilde{\mathscr{V}} + \bar{N}\left[\gamma^\mu\left(i\partial_\mu - g_\omega\omega_\mu - \frac{1}{2}g_\rho\boldsymbol{b}_{\boldsymbol{\mu}}\cdot\boldsymbol{\tau}\right) - g\sqrt{\sigma^2+\pi^2}\right]N$$

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$$+\left[(G_4)^2\omega_\mu\omega^\mu\right]^2 - \tilde{\mathscr{V}} + \bar{N}\left[\gamma^\mu\left(i\partial_\mu - g_\omega\omega_\mu - \frac{1}{2}g_\rho\boldsymbol{b}_{\boldsymbol{\mu}}\cdot\boldsymbol{\tau}\right) - g\sqrt{\sigma^2+\pi^2}\right]N$$

$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2}B_0\delta\chi^4\ln\frac{\sigma^2+\pi^2}{\sigma_0^2} + \frac{1}{2}B_0\delta\chi^2\left[\frac{\sigma^2+\pi^2}{\sigma_0} - \frac{\chi^2}{2}\right] - \frac{1}{4}\epsilon_1'\chi^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) - \chi^2\right] - \frac{3}{4}\epsilon_1'\,.$$

Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\Big(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\Big)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\Big(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\Big)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \Big(B_0\,\delta + \epsilon_1'\Big)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$
$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

Meson and baryon sector at finite chemical potential

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\, P(z_i, \langle \Delta_i^2\rangle)\, A(\mathbf{\Phi}_i + z_i)$$

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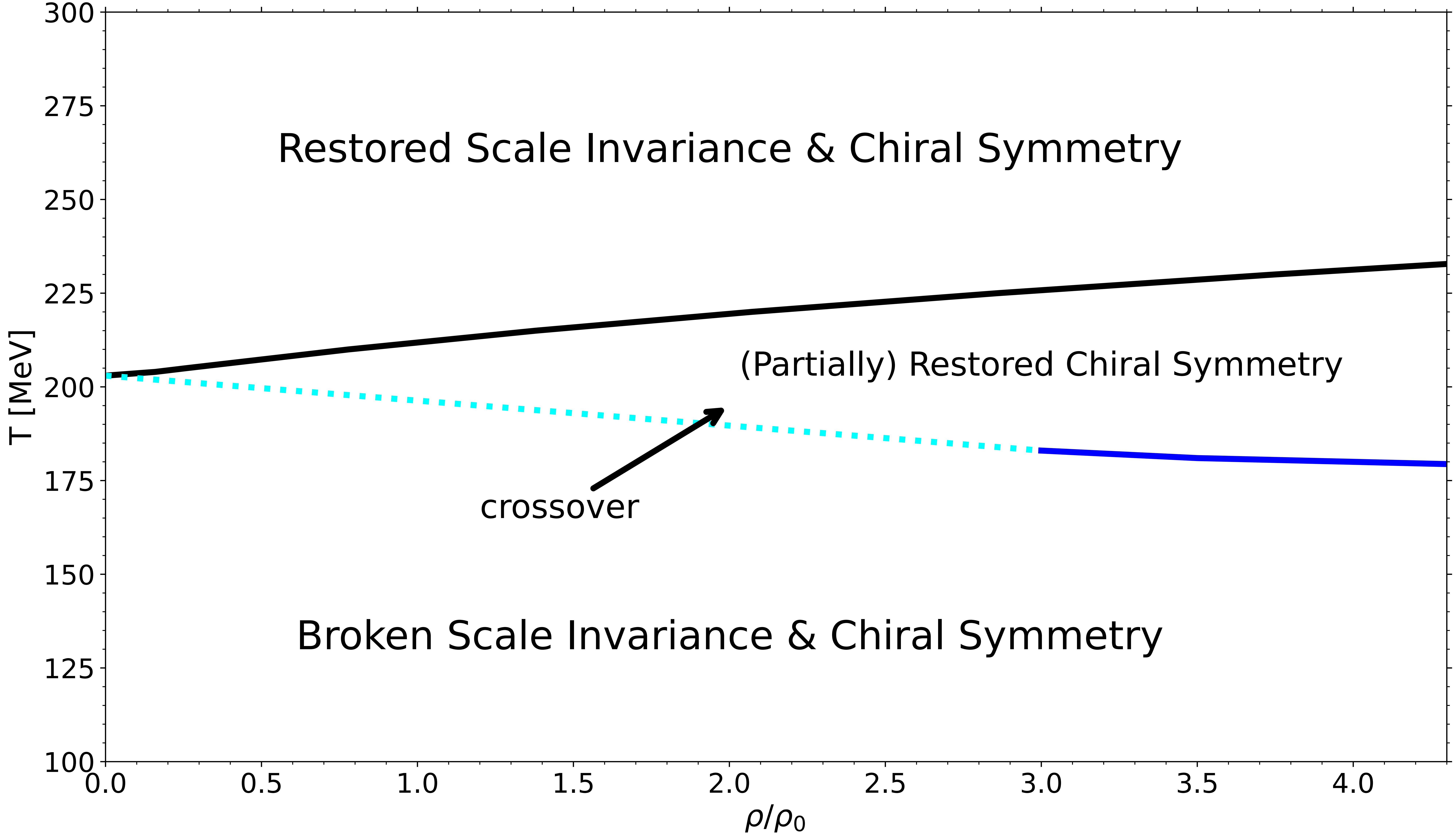
$$m_\pi = m_\pi(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\sigma = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\chi = m_\chi(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_\rho = m_\rho(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$
$$m_N = m_N(\chi, \sigma, \pi, \omega, \rho, \langle\Delta_i^2\rangle)$$

Meson and baryon sector at finite chemical potential

Meson and baryon sector: phase diagram



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4 RMF predictions through Bayesian analysis

RMF predictions for the dense-matter EOS and application to NSs

$$
\begin{aligned}
\mathscr{L} = \sum_N \bar{\psi}_N (i\gamma_\mu \partial^\mu - m_N + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu \\
-\frac{1}{2} g_\rho(n_b)\, \gamma_\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu) \psi_N + \frac{1}{2}(\partial_\mu \sigma \partial^\mu - m_\sigma^2 \sigma^2) \\
-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} \\
-\frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{3} b m_N (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4
\end{aligned}
$$

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- Five couplings constants:

$$g_\sigma, g_\omega, g_\rho, b, c$$

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- One more parameter a_ρ: describing the **density-dependent coupling** of $g_\rho(n_b)$.

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- Six Nuclear Empirical Parameters (NEPs):

$$E_{sat}, n_{sat}, K_{sat}, E_{sym,2}, L_{sym}, m^*_{D,sat}$$

NEPs	$E_{\rm sat}$	$n_{\rm sat}$	$K_{\rm sat}$	$E_{\rm sym,2}$	$L_{\rm sym}$	$m^*_{D,\rm sat}$
unif.dist.	[−16.4, −15.2]	[0.145, 0.165]	[180, 280]	[28, 36]	[30, 90]	[0.45, 1.05]

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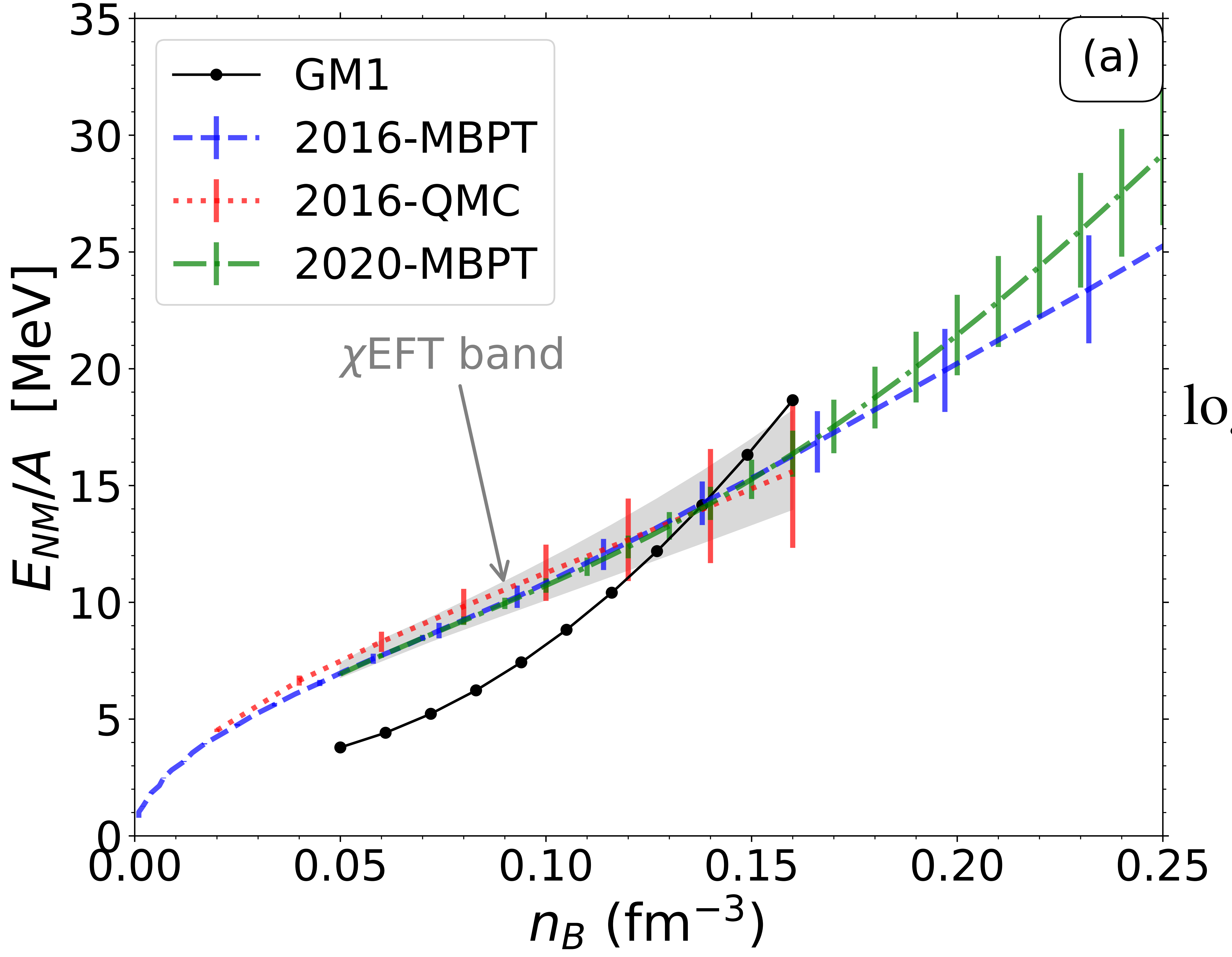

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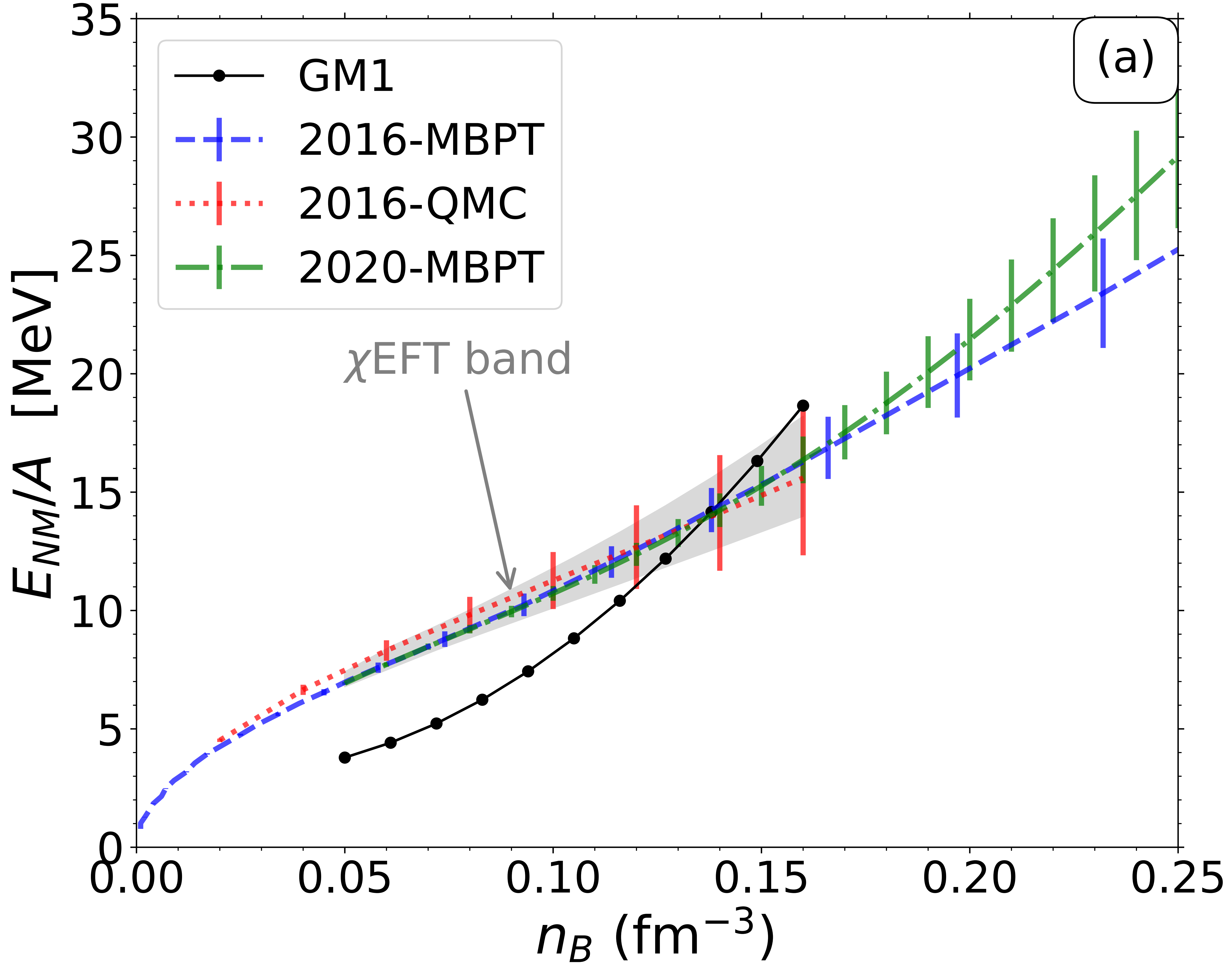

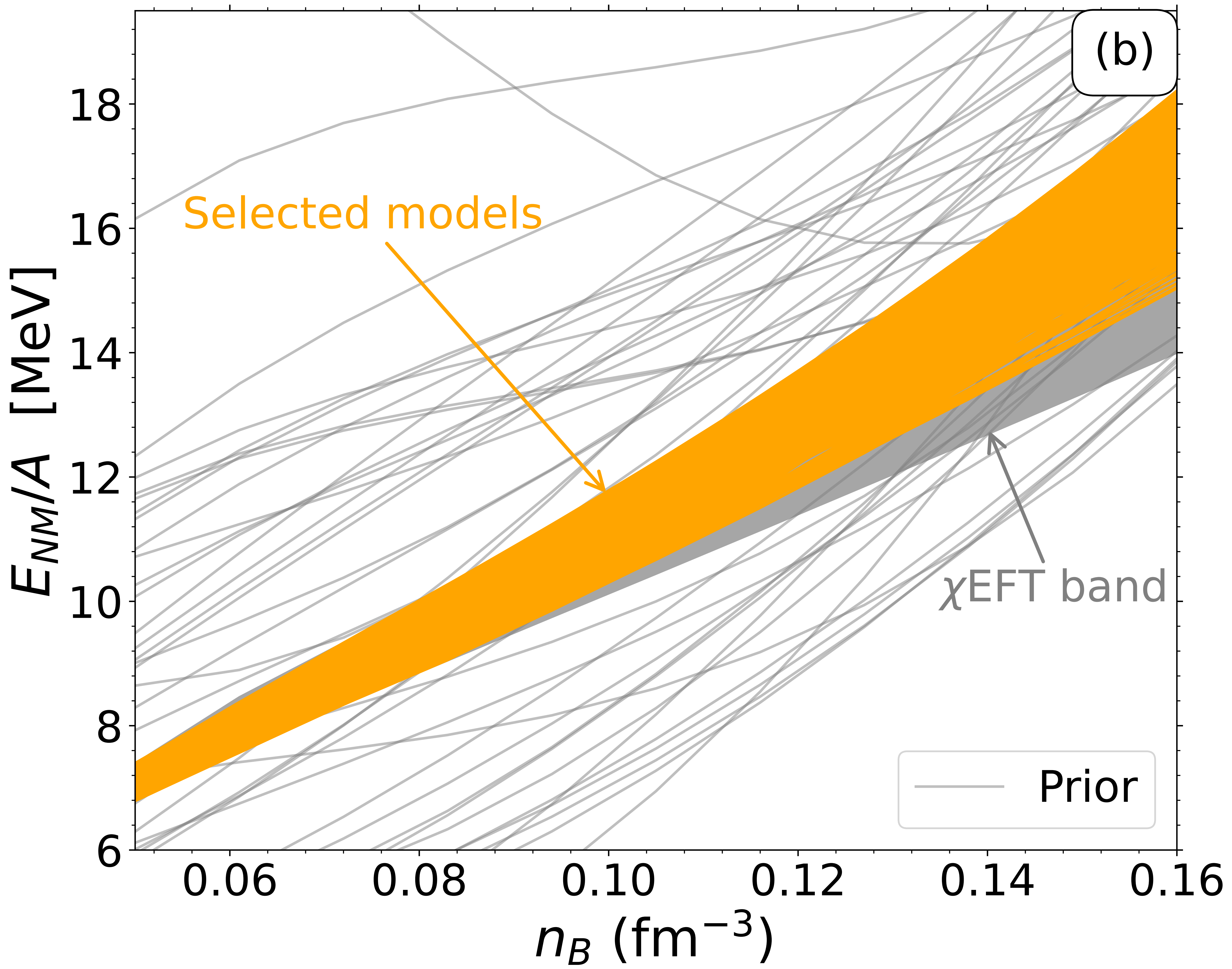

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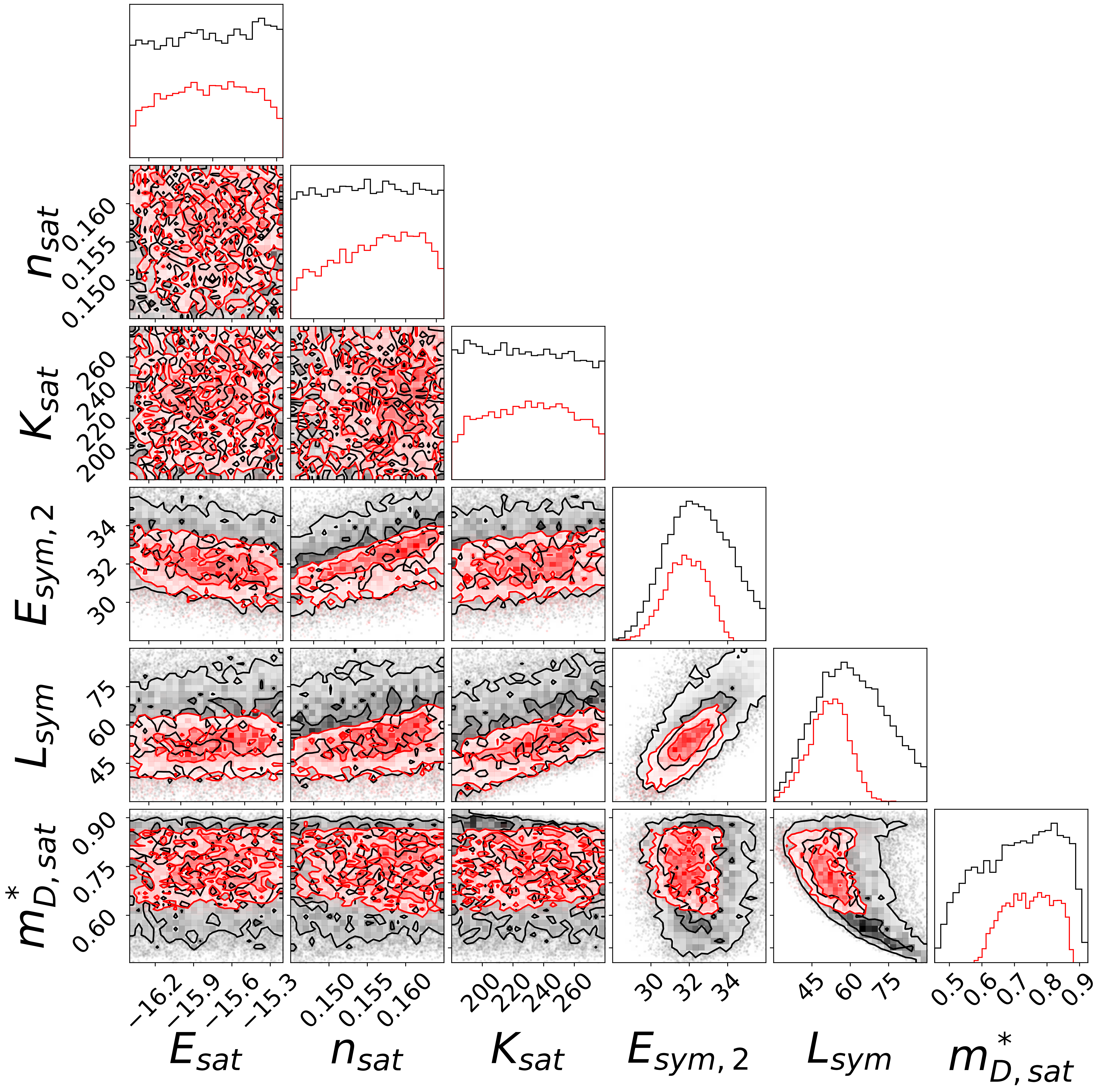
Astrophysical constraint:

- **Tidal deformability** from GW170817

$$\Lambda \in [70,770]$$



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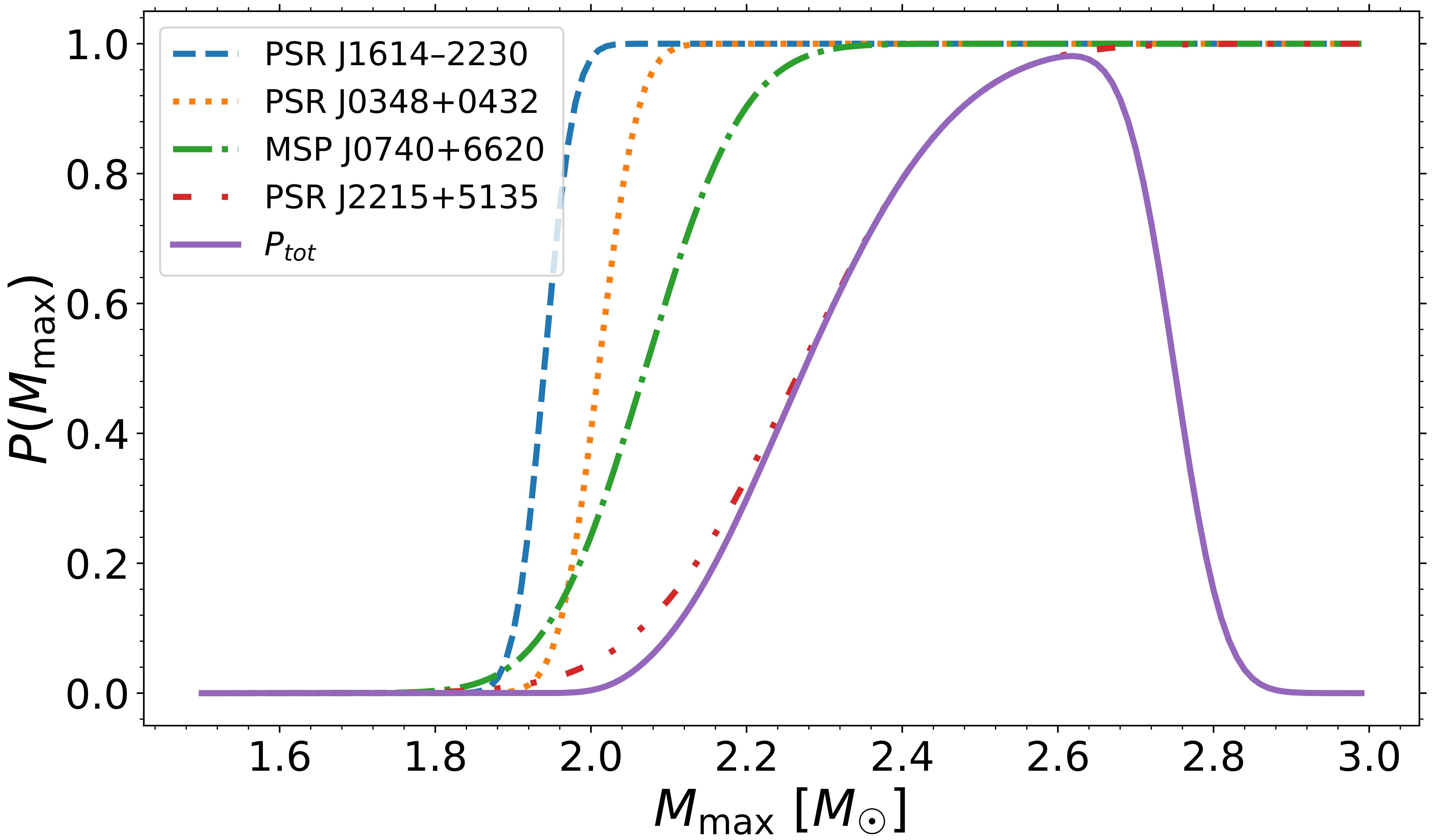
$$P_i(M_{\text{max}}) = \frac{\text{erf}[f_i(M_{\text{max}})] + 1}{2} \qquad f_i(M) = \frac{M - M_{\text{obs},i}}{\sqrt{2}\,\sigma_{M,i}}$$



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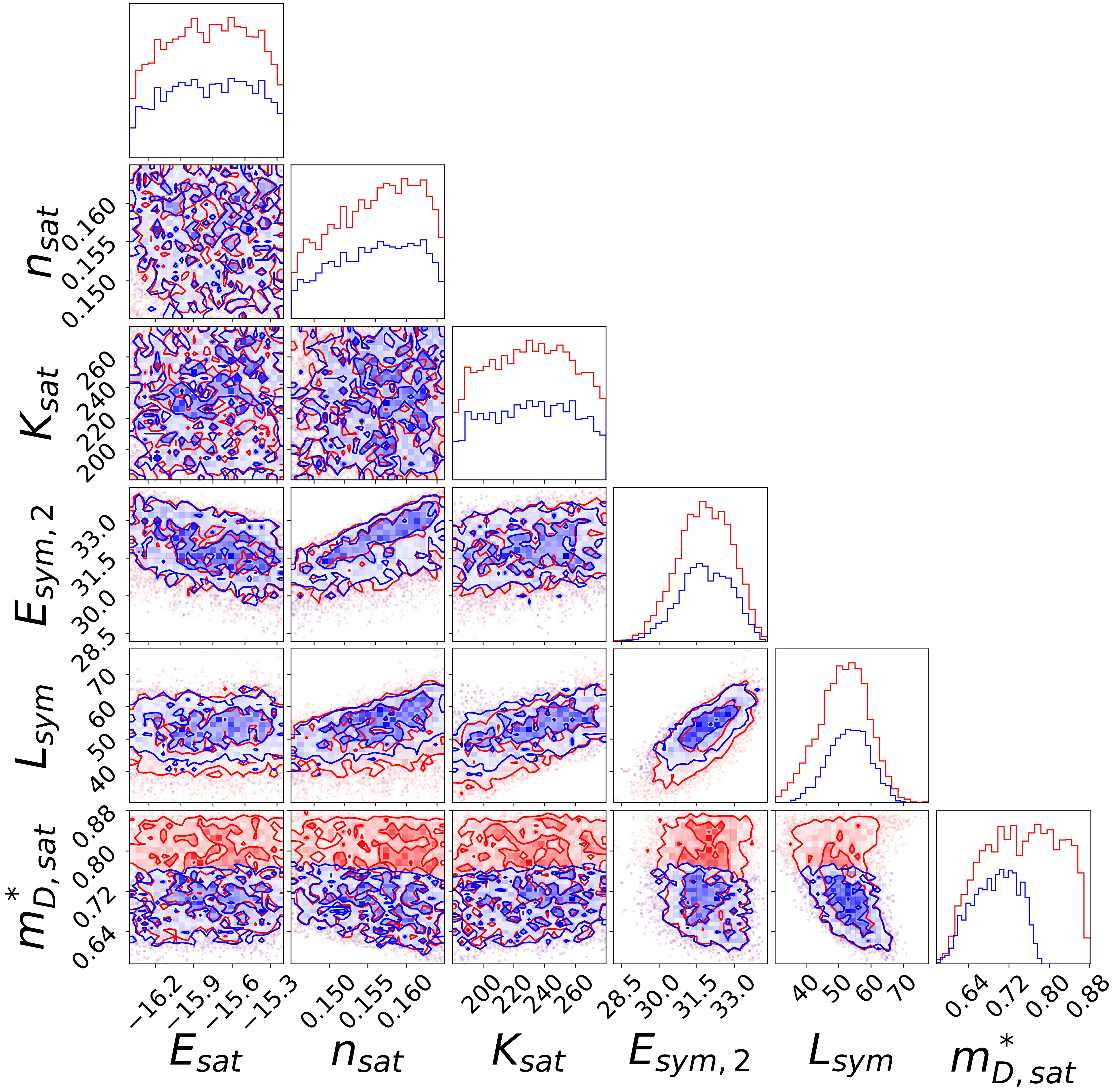

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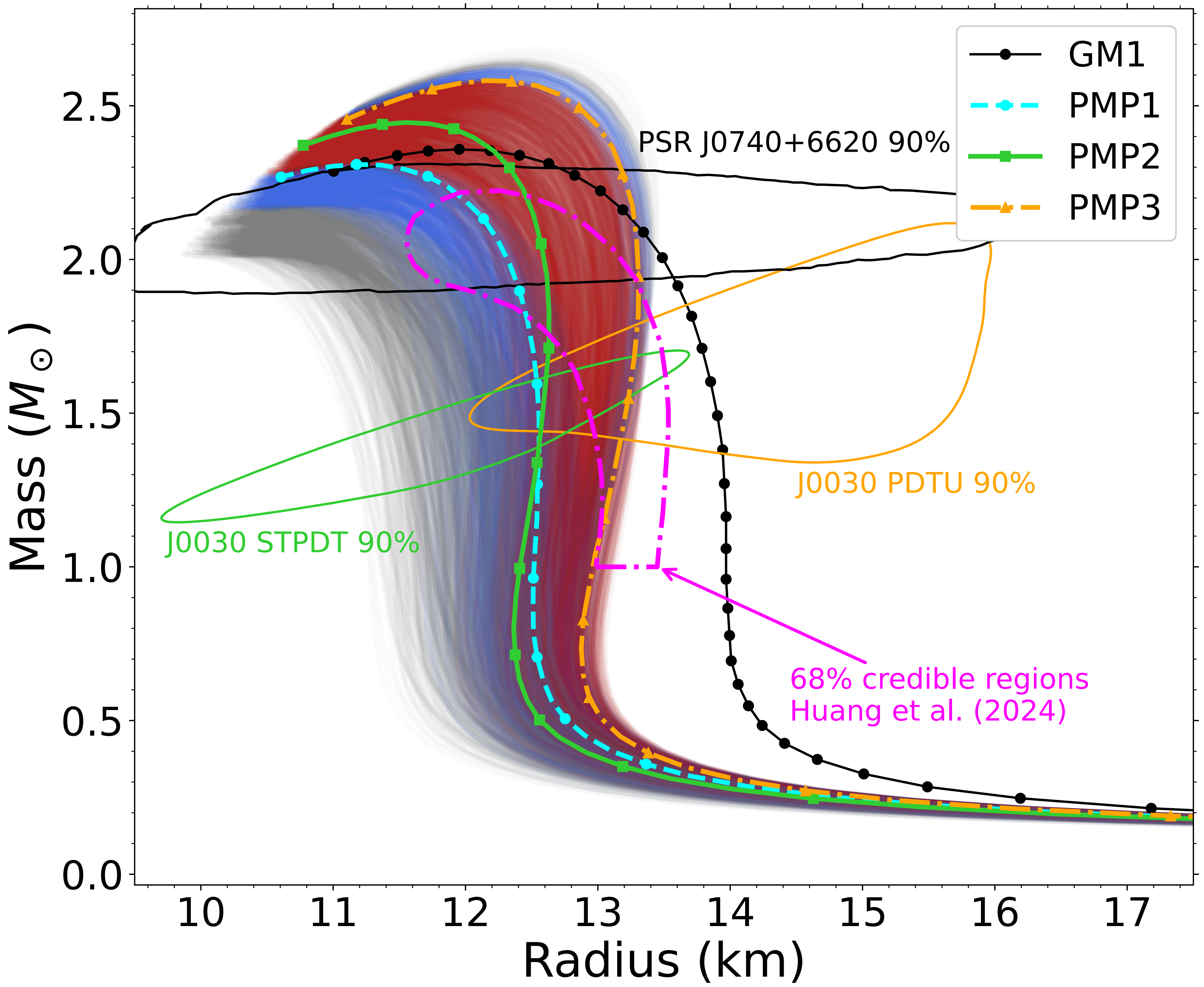

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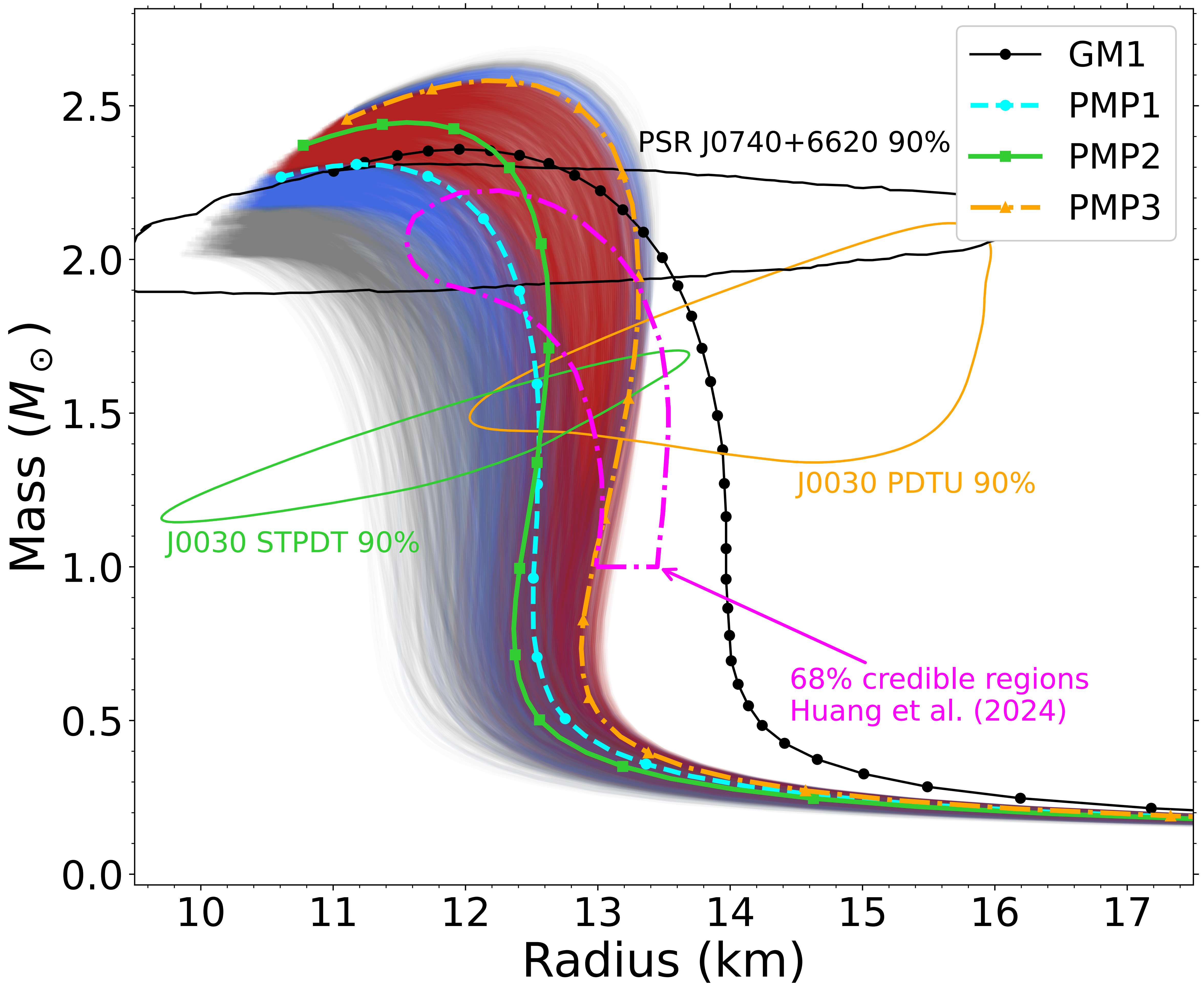

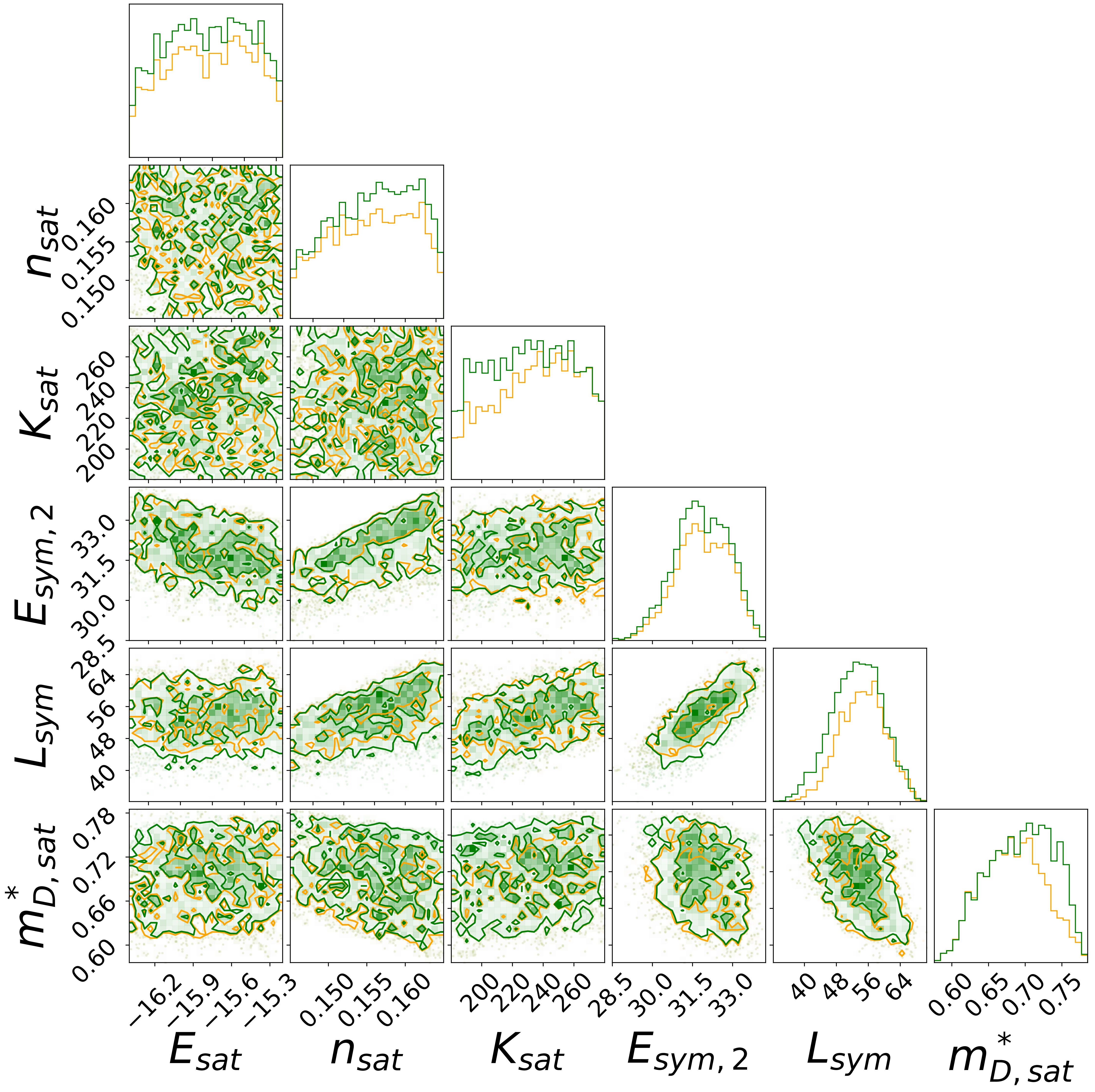

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3 Meson and baryon sector

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- Development of a numerical code for **RMF predictions** of the dense-matter EOS and **Bayesian analysis** applications to neutron stars

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3 Meson and baryon sector

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- Explore **double family scenario** through Bayesian analysis

Thank you

Backup slides

Numerical details

4 field equations:

$\chi, \sigma, \omega, \rho$

6 mass equations:

$$m_\chi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\sigma = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\pi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\omega = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\rho = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_N = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

2 conservation equation:

Total baryonic charge

Total electric charge

$$\begin{cases} \vdots \\ \langle \Delta_i^2 \rangle = \dfrac{n_i}{2\pi^2} \displaystyle\int_0^\infty dk \, \frac{k^2}{e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle)} \, \frac{1}{\exp\left[\beta\left(e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle) - \mu_i^*\right)\right] - 1} \\ \vdots \end{cases}$$

$$e_i^* = \sqrt{k^2 + m_i^{*2}}$$

Pure gauge $SU(3)_c$ sector

To improve accuracy with latticeQCD data:

- Inclusion **excited glueballs**:

$$m^2(n=1, J^{PC}) = a(n+J) + b_{J^{PC}}$$

J^{PC}	m [GeV]	$b_{J^{PC}}$ [GeV^2]
0^{++}	$1.647(25)$	-2.78 ± 0.21
2^{++}	$2.367(30)$	-10.87 ± 0.57
0^{-+}	$2.572(38)$	1.12 ± 0.27
2^{-+}	$3.11(5)$	-6.79 ± 0.66
1^{+-}	$2.955(37)$	-2.25 ± 0.45

$$P_{\mathrm{dil}_J} = -(2J+1)T\int \frac{dk^3}{(2\pi)^3} \ln\left(1 - e^{-\beta\sqrt{k^2+m_J^{*2}}}\right)$$

$$\frac{\tilde{\Omega}(\chi,T)}{V} = \frac{\Omega(\chi,T)}{V} - \sum_J^N P_{\mathrm{dil}_J}$$

String tension term

$$\sigma(T) = \left(1 - \frac{1}{1+e^{-(T-\alpha)/\delta}}\right)$$

- E. Trotti, S. Jafarzade, and F. Giacosa, *Eur.Phys.J.C* 83 (2023) 5, 390
- N. Cardoso and P. Bicudo, Phys. Rev. D 85, 077501 (2012).

Pure gauge $\mathrm{SU}(3)_c$ sector

To improve accuracy with latticeQCD data:

- Inclusion excited glueballs;
- Modification **infrared cut-off**:

$$I = \frac{s}{T^3} - \frac{4P}{T^4} \Rightarrow s/T^3 \gg 4P/T^4$$

Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1-\chi^2} = \frac{A}{1-\bar{\chi}^2 - \langle \Delta^2 \rangle \cdot F(T)}$$

Form factor

$$F(T) = \frac{\Theta(T-T_c)}{1+\left(\frac{T-T_c}{\gamma}\right)^2}$$

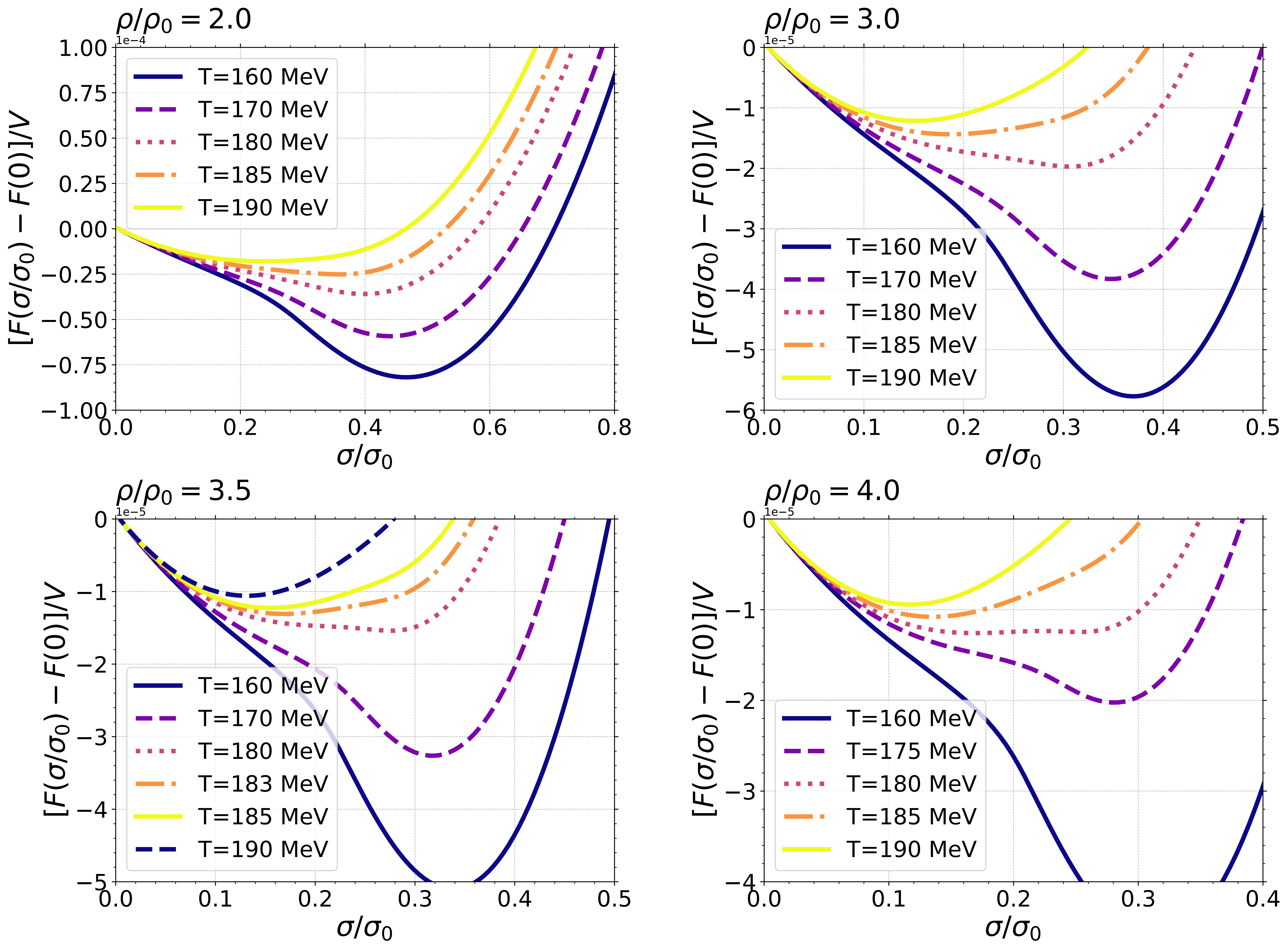
Meson and baryon sector at finite chemical potential

The **thermodynamical potential** is

$$\frac{\tilde{\Omega}}{V} = \langle \tilde{\mathscr{V}} \rangle - \frac{1}{2} m_\omega^2 \chi^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \chi^2 b_0^2 - \frac{G_4}{4} \omega_0^4 - \frac{1}{2} m_\sigma^{*2} \langle \Delta_\sigma^2 \rangle - \frac{1}{2} m_\pi^{*2} \langle \boldsymbol{\pi}^2 \rangle - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle$$
$$+ \frac{T}{2\pi^2} \int dk\, k^2 \left[\ln(1 - e^{-\beta \epsilon_\sigma}) + 3 \ln(1 - e^{-\beta \epsilon_\omega}) \right] + \frac{T}{2\pi^2} \int dk\, k^2 \left[\sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\pi^* - \mu_\pi^{*a})}) + 3 \sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\rho^* - \mu_\rho^{*a})}) \right]$$
$$- \frac{T}{\pi^2} \sum_{i=p,n} \int dk_i\, k_i^2 \left[\ln\left(1 + e^{-\beta(E_i^* - \mu_i^*)}\right) + \ln\left(1 + e^{-\beta(E_i^* + \mu_i^*)}\right) \right]$$

$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2} B_0 \delta \chi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} + \frac{1}{2} B_0 \delta \chi^2 \left[\frac{\sigma^2 + \pi^2}{\sigma_0} - \frac{\chi^2}{2} \right] - \frac{1}{4} \epsilon_1' \chi^2 \left[\frac{4\sigma}{\sigma_0} - 2 \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] - \frac{3}{4} \epsilon_1' .$$

Meson and baryon sector at finite chemical potential



Meson and baryon sector: thermal fluctuations

$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i^*} \frac{1}{e^{\beta(\epsilon_i^* - \mu_i^*)} - 1} dk$$

$$n_i = \begin{cases} 1 & \text{for } \phi, \sigma, \pi_0, \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu, b_{0\mu}, b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

$$P_\sigma(z) = \sqrt{\frac{1}{2\pi\langle \Delta_\sigma^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\sigma^2 \rangle} \right),$$

$$P_{\boldsymbol{\pi}}(y) = \sqrt{\frac{2}{\pi}} \left(\frac{3}{\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right)^{(3/2)} \exp\left(-\frac{3y^2}{2\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right),$$

$$\langle O(\bar{\sigma} + \Delta_\sigma, \boldsymbol{\pi}^2) \rangle = \int_{-\infty}^{\infty} dz P_\sigma(z) \int_0^\infty dy y^2 P_{\boldsymbol{\pi}}(y) O(\bar{\sigma} + z, y^2) \; .$$

The thermodynamical potential in the mean field approximation

$$\frac{\Omega(\chi,T)}{V} = \langle \mathscr{V}(\chi)\rangle - P_{\rm dil}(\chi,T) - P_{\rm q-free}(\chi,T) - P_{\rm int}(\chi,T) + \frac{B_0}{4} - \frac{1}{2}m_\chi^{*2}\left\langle \Delta_\chi^2\right\rangle$$

Where

$$P_{\rm dil}(\chi,T) = \frac{1}{6\pi}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_\chi^{*2}}}\frac{1}{e^{\beta\sqrt{k^2+m_\chi^{*2}}}-1}$$

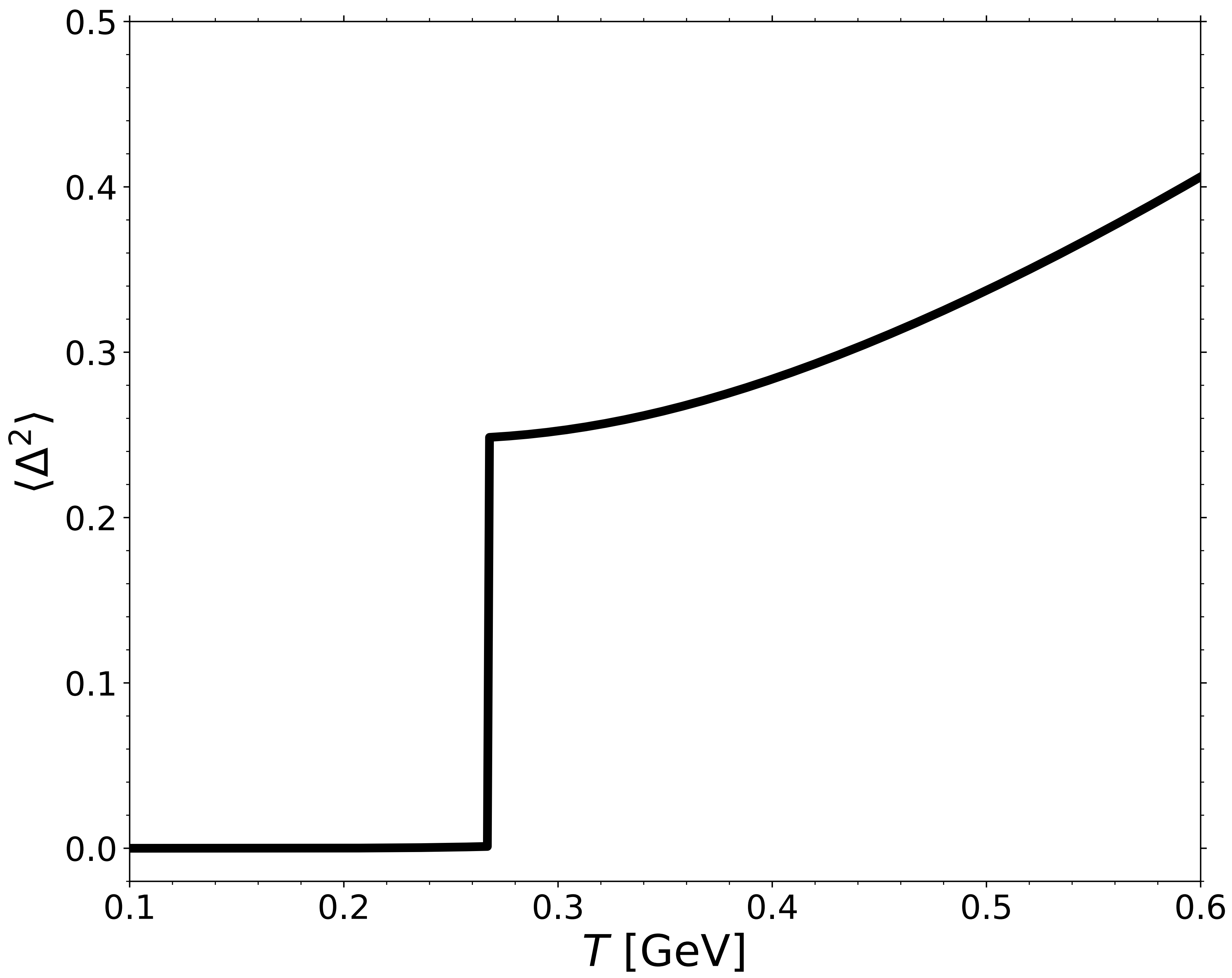
$$P_{\rm q-free}(\chi,T) = -2(N_c^2-1)T\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-k/T}\right)\Theta\left(k-K(\chi)\right)$$

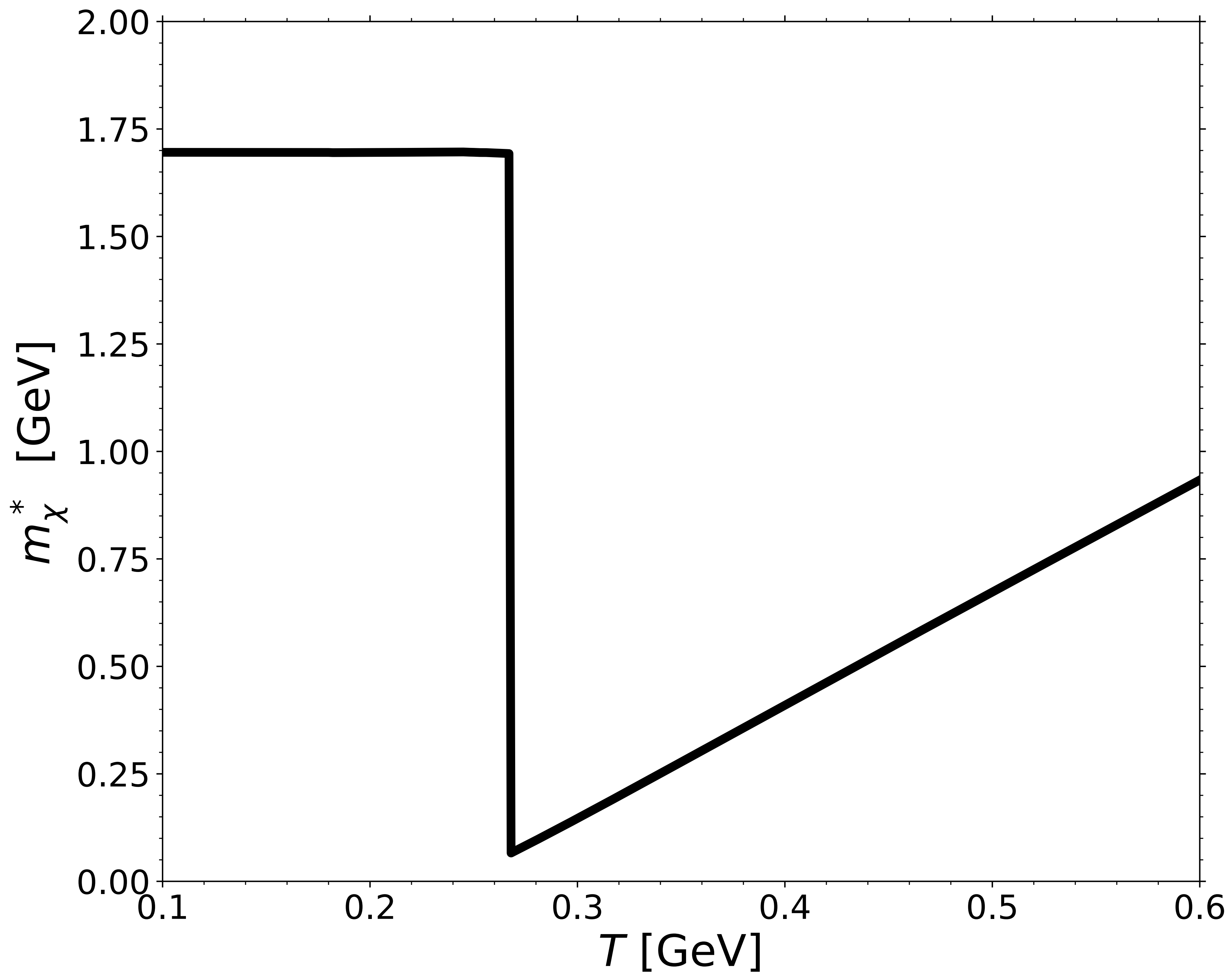
$$K(\chi) = \frac{A}{(1-\chi)}$$

$$g^2(T) = \frac{48\pi^2}{11N_c\ln\left(\frac{T^2+S^2}{\Lambda^2}\right)}$$

$$P_{\rm int}(\chi,T) = g^2N_c(N_c^2-1)\left\{(-3)\left[\int\frac{d^3k}{(2\pi)^3}\frac{1}{k}N_B\left(\frac{k}{T}\right)\Theta\left(k-K(\chi)\right)\right]^2 + \int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{1}{k_1k_2}N_B\left(\frac{k_1}{T}\right)N_B\left(\frac{k_2}{T}\right)\right.$$
$$\left.\times\Theta\left(k_1-K(\chi)\right)\Theta\left(k_2-K(\chi)\right)\times\left[\frac{9}{4}\Theta\left(|\boldsymbol{k}_1+\boldsymbol{k}_2|-K(\chi)\right)-\frac{1}{4}\Theta\left(|\boldsymbol{k}_1-\boldsymbol{k}_2|-K(\chi)\right)\right]\right\}$$

Pure gauge $\mathrm{SU(3)_c}$ sector





MCMC and Bayesian analysis

- If the set of parameters predicts at least one of the NEP to be out of the range given, then the prior probability is $p_{\rm prior} = 0$

$$\log p_{\rm MCMC} = -\frac{1}{2}\sum_{i=1}^{N}\left(\frac{E(\text{model}, n_i) - E(\chi\text{EFT}, n_i)}{\Delta E(\chi\text{EFT}, n_i)}\right)$$

i runs over the densities n_i, and where $E(\chi\text{EFT}, n_i)$ and $\Delta E(\chi\text{EFT}, n_i)$ represent the centroids and uncertainties of the χEFT.

- In case the RMF model is inside the χEFT band, the probability becomes constant: if $\log p_{\rm MCMC} \geq -N/2$, then $\log p_{\rm MCMC} = -N/2$.

NEPs	$E_{\rm sat}$	$n_{\rm sat}$	$K_{\rm sat}$	$E_{\rm sym,2}$	$L_{\rm sym}$	$m^*_{D,\rm sat}$
unif.dist.	[−16.4, −15.2]	[0.145, 0.165]	[180, 280]	[28, 36]	[30, 90]	[0.45, 1.05]

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$$-\frac{1}{2}g_\rho(n_b)\,\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu - m_\sigma^2\sigma^2)$$
$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu}$$
$$-\frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{3}bm_N(g_\sigma\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4$$

$$g_\rho(n_b) = g_\rho(n_{\rm sat})\exp\left[-a_\rho\left(\frac{n_b}{n_{\rm sat}} - 1\right)\right]$$

The density dependence of g_ρ introduces a rearrangement contribution Σ_r to the nucleon self-energies:

$$\Sigma_r = -\frac{1}{2}a_\rho g_\rho(n_b)\frac{n_p - n_n}{n_{\rm sat}}\rho_{03}.$$

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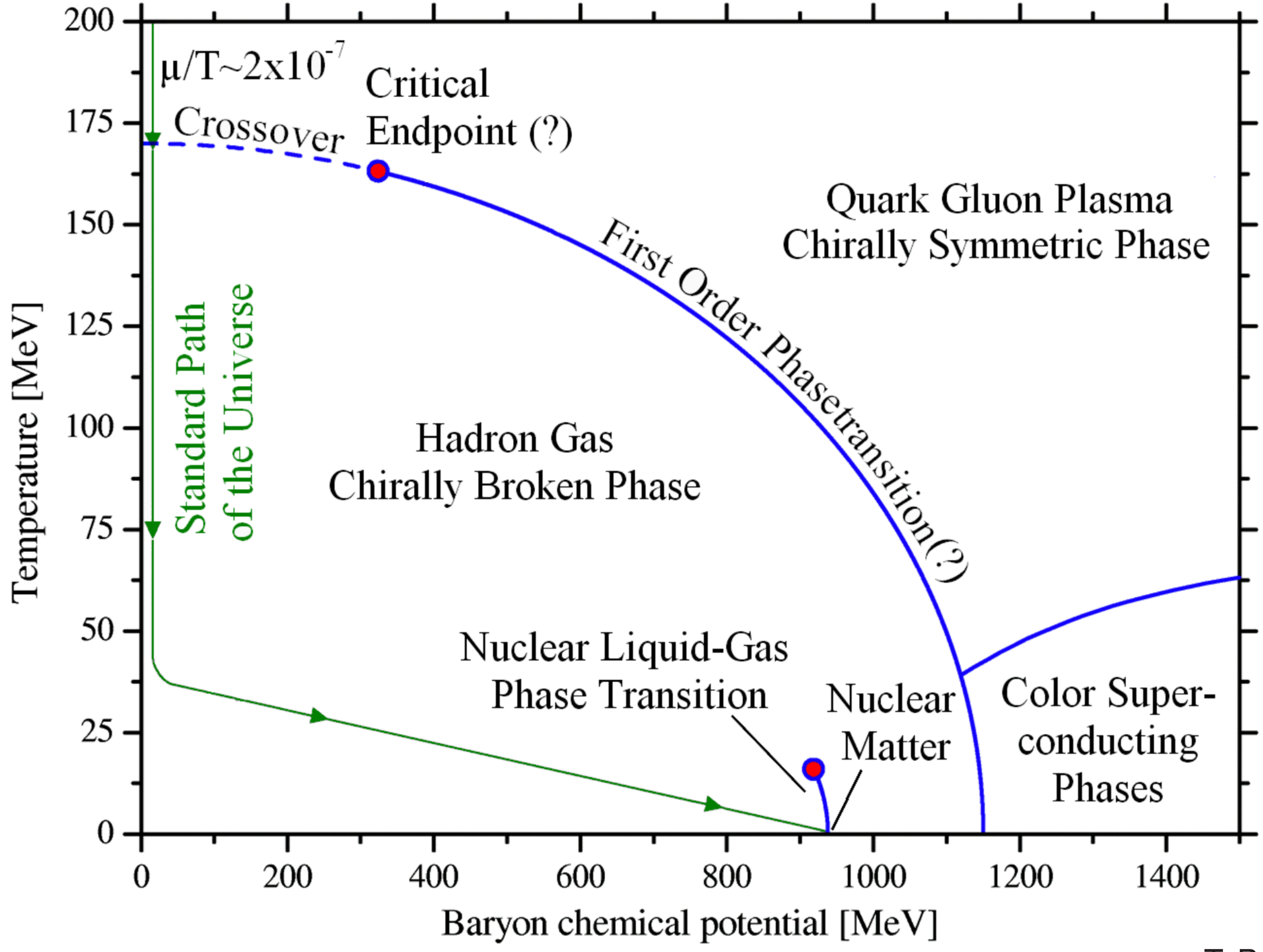
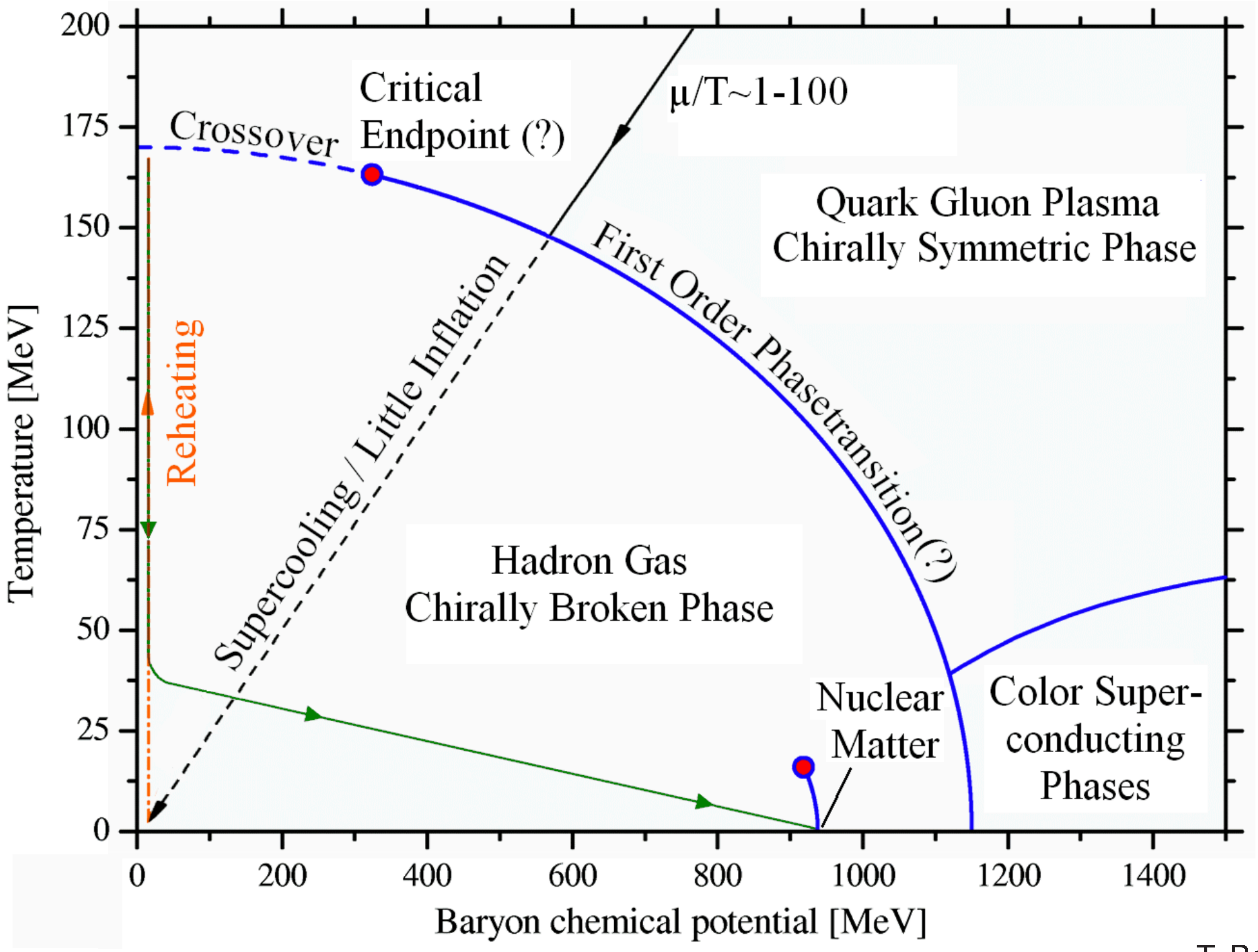
Little inflation scenario



- Negligible baryon density: $\mu_B \ll T$
- QCD phase transition: **crossover**

T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Little inflation scenario



- Non vanishing μ_B: $\mu_B / T \sim O(1)$
- $\Omega(T, V, \mu_B)$ has two minima
- System stucks in **false vacuum**

$\Rightarrow$ **LITTLE INFLATION**

- **First order phase transition**: release of false vacuum energy causes **reheating**

T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Little inflation scenario

T = 180 MeV, ρ = 4.00 ρ_0

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Conservation laws

l_α free input parameters

1. Lepton number: $l_\alpha s = n_\alpha + n_{\nu_\alpha}$, $\alpha = e, \mu, \tau$

$b = 8.6 \times 10^{-11}$

2. Baryon number: $bs = \sum_i B_i n_i$

$q = 0$ (charge neutrality)

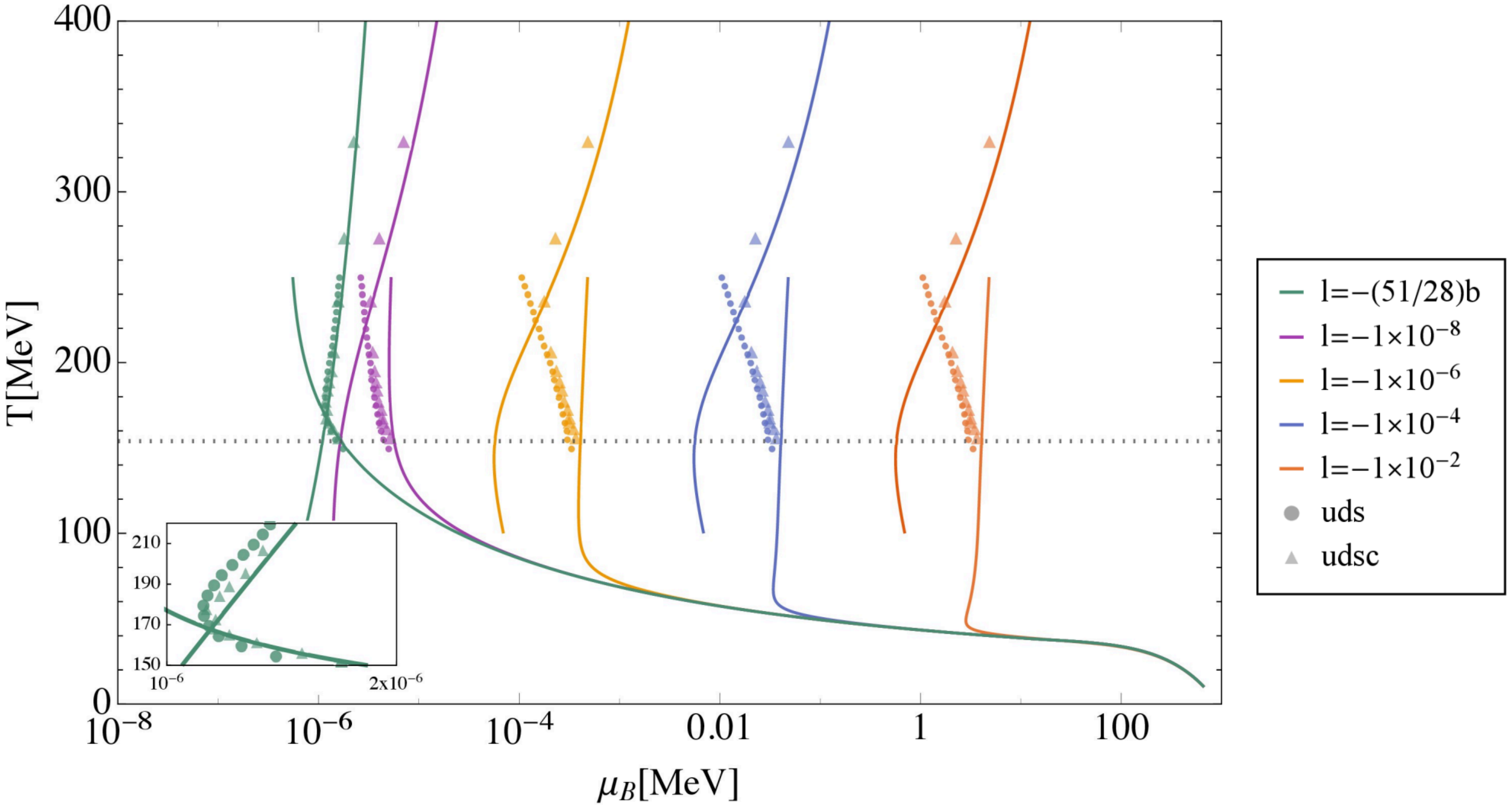
3. Electric charge: $qs = \sum_i Q_i n_i$

5 conservation laws

→5 equations

→5 chemical potentials

$\mu_{L_\alpha}, \mu_B, \mu_Q$



Mandy M. Wygas et al., PRL 121, 201302 (2018)

Color-dielectric model

$\mathscr{L}$ describes a system of interacting quarks, pions, sigmas and a scalar-isoscalar chiral singlet field χ, whose potential $U(\chi)$ has an absolute minimum for $\chi = 0$. Thus for the single nucleon problem, the effective quark mass $-g\sigma/\chi$ diverges outside the nucleon.

$$\mathscr{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{g}{\chi}\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi + \frac{1}{2}(\partial_\mu\chi)^2 - U(\chi) + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - W(\sigma,\vec{\pi})$$

$$U(\chi) = \frac{1}{2}M^2\chi^2 \qquad W(\sigma,\vec{\pi}) = k\cdot(\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2$$

QCD background

The theory that we currently use to describe strongly interacting matter is called Quantum Chromodynamics (**QCD**). It is a quantum field theory and the fields used to represent the fundamental degrees of freedom of the theory are quarks, which are spin-1/2 fermions, and gluons, which are bosons with spin 1.

Scale anomaly

A theory is invariant under scale transformations if its action remains constant when the fields are transformed as follows under the scale operator $U(\lambda)$:

$$\phi(x) \rightarrow U^{+}\phi(x)U(\lambda) = \lambda^{\Delta}\phi(\lambda x)$$

where Δ is the scale dimension of ϕ. The action remains unchanged *only if* no dimensional parameter is present: scale invariance is broken due to quantum corrections due to the appearance of a dimensional parameter Λ.

Chiral symmetry

Chiral symmetry is spontaneously broken in the QCD vacuum. This occurs since the vacuum of QCD is not trivial, and thus not invariant under the chiral $SU_A(2)$ transformation, even in the chiral limit, such that the order parameter $< \bar{q}_L^i q_R^j >$ acquires a nonvanishing expectation value.

Increasing temperature and/or density, one expects the quark condensate to melt, restoring chiral symmetry.