BLACK HOLES BEYOND GR

ENRICO TRINCHERINI
(SCUOLA NORMALE SUPERIORE)

LIGO-VIRGO 10 YEARS ANNIVERSARY

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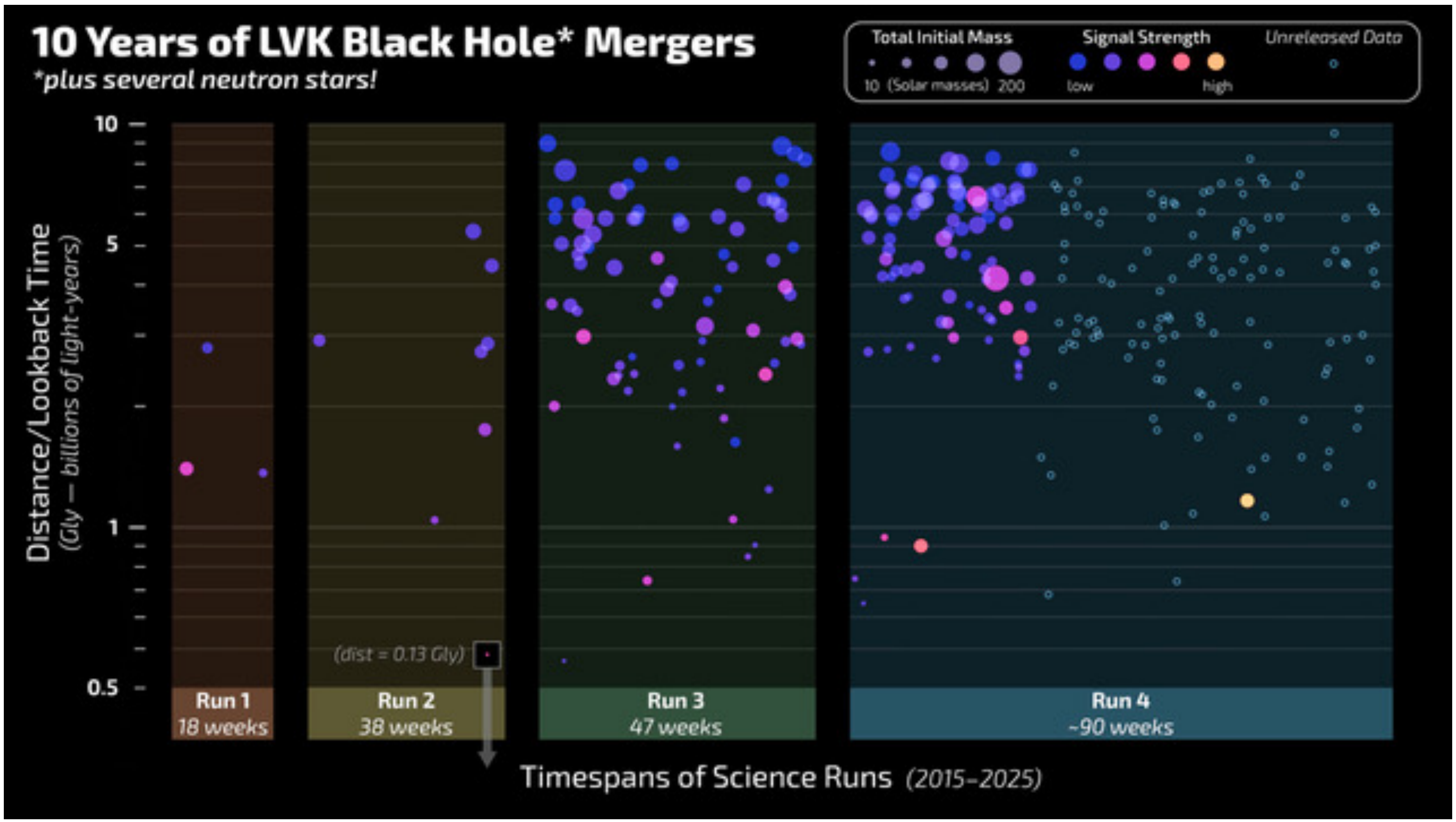

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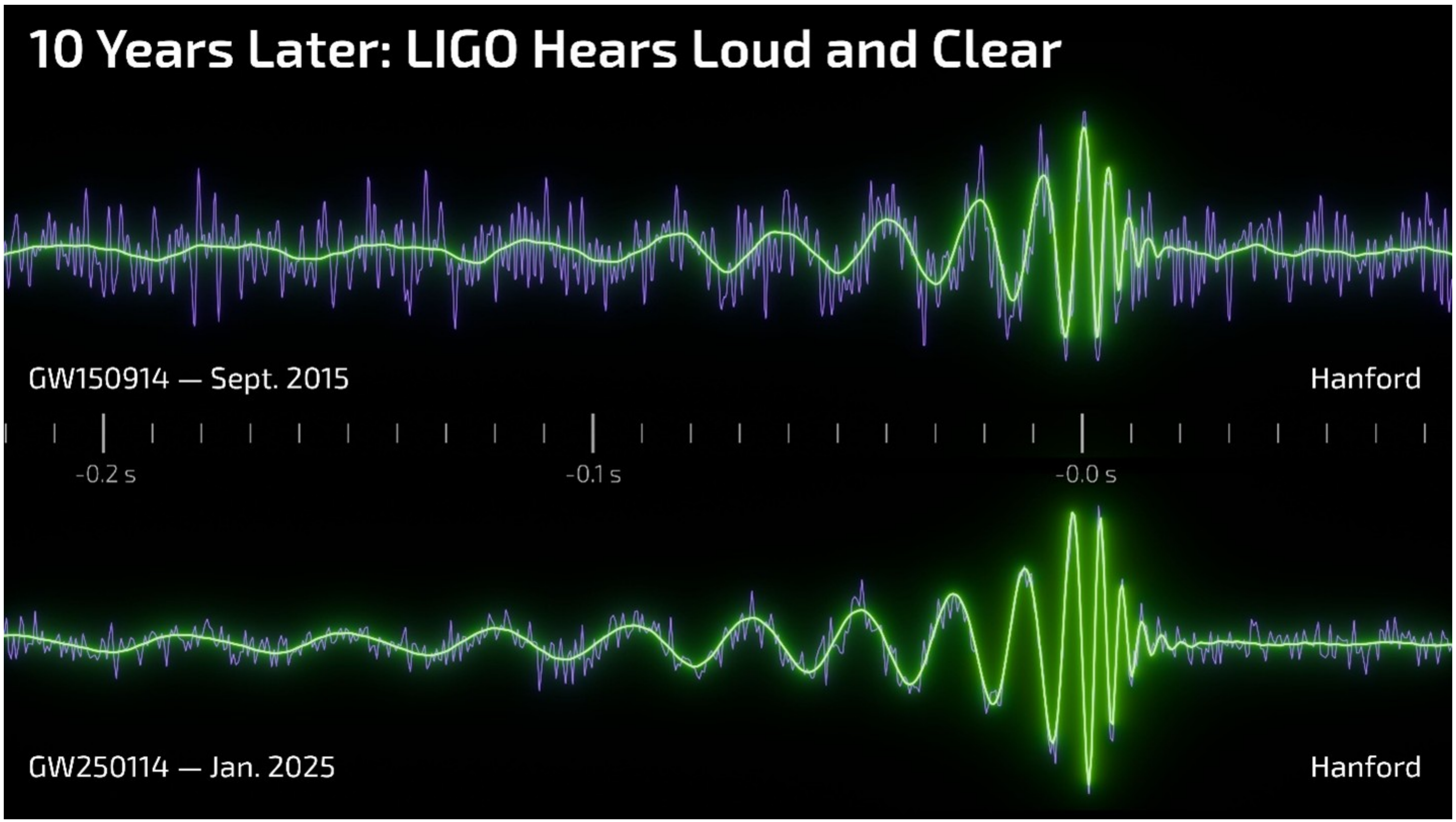
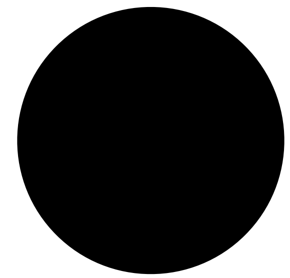


What is a black hole?



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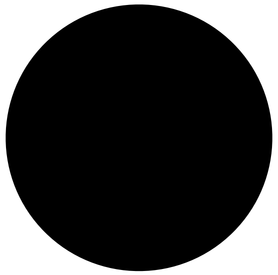

Geometry

Mass, Spin

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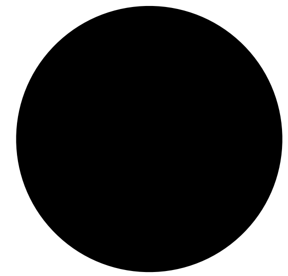

Geometry — Mass, Spin

BH response to an external field — Love numbers

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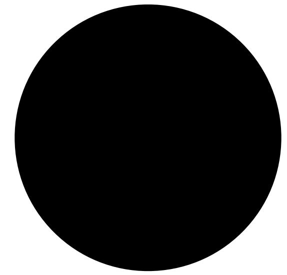

Geometry	Mass, Spin
BH response to an external field	Love numbers
Quasi Normal Modes (QNM)	Spectrum of characteristic (complex) frequencies

Quasi Normal Modes (QNM)

$$g_{\mu\nu} = g_{\mu\nu}^{\rm BH}(r) + \epsilon h_{\mu\nu}^{(1)}$$

$$G_{\mu\nu}^{(1)}\left[h^{(1)}\right] = 0$$

$$h(t,r,\theta,\phi) = \sum_{lm} h_{lm}(r) Y_{lm}(\theta,\phi) e^{i\omega t}$$

Classified accordingly to parity
- Even +
- Odd -

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Fix the gauge redundancy: 2 physical DOF described by 2 "master scalars" $\Psi^{(1)}_{lm,\pm}$

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Approriate boundary conditions $\Longrightarrow$ exponentially damped sinusoidal waves

1. Discrete complex frequencies $\omega = \omega_{lmn}$

n	$2M_\bullet\omega\ (L=2)$	$2M_\bullet\omega\ (L=3)$
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2. The amplitude is arbitrary
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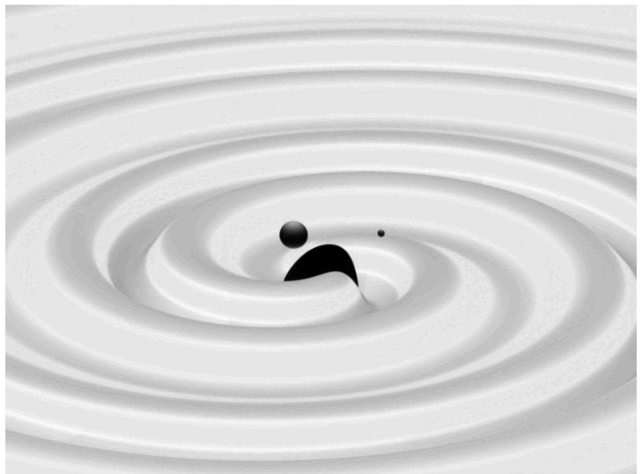
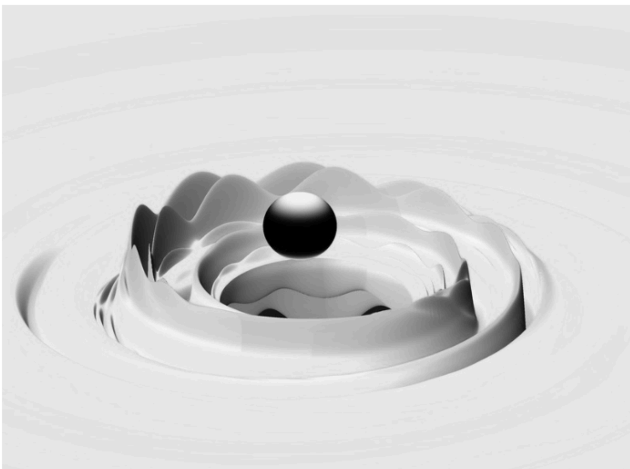
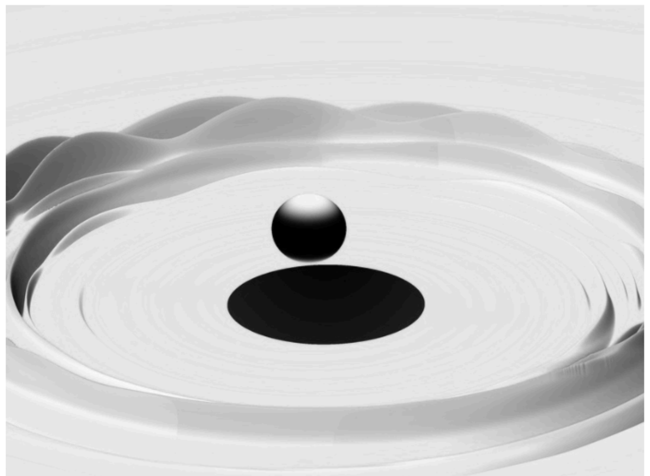


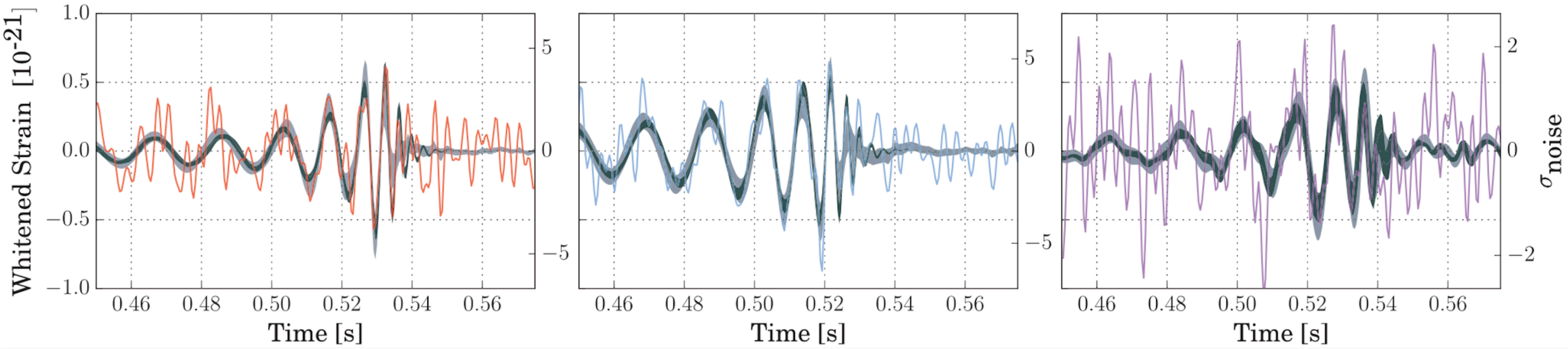
Inspiral

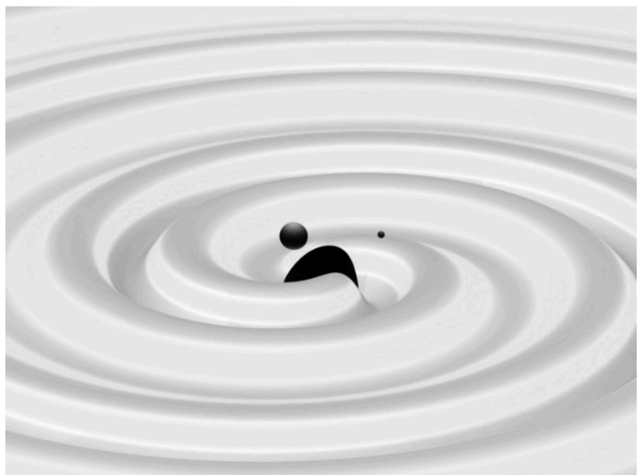


Merger



Ringdown





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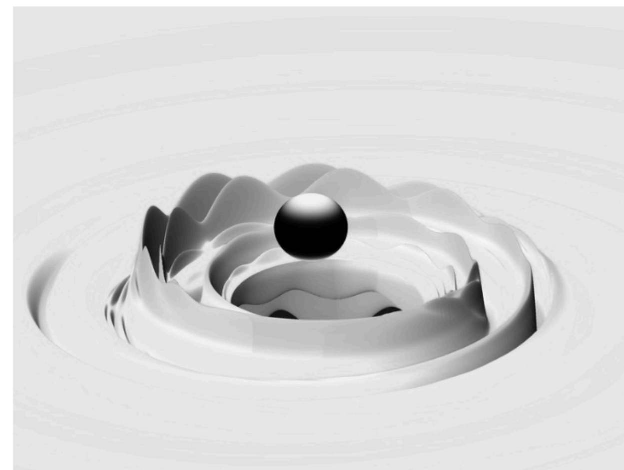

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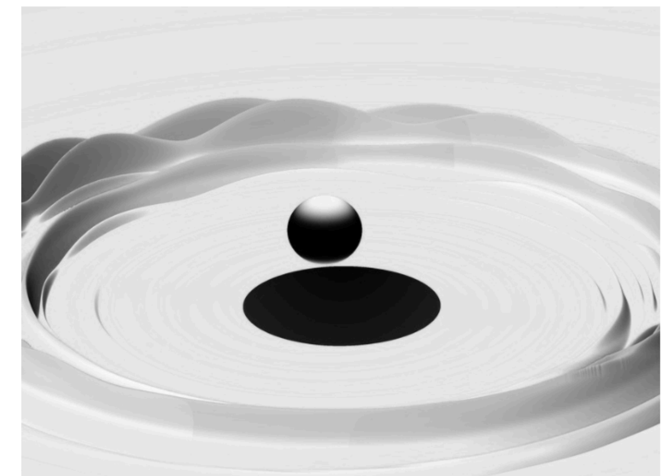

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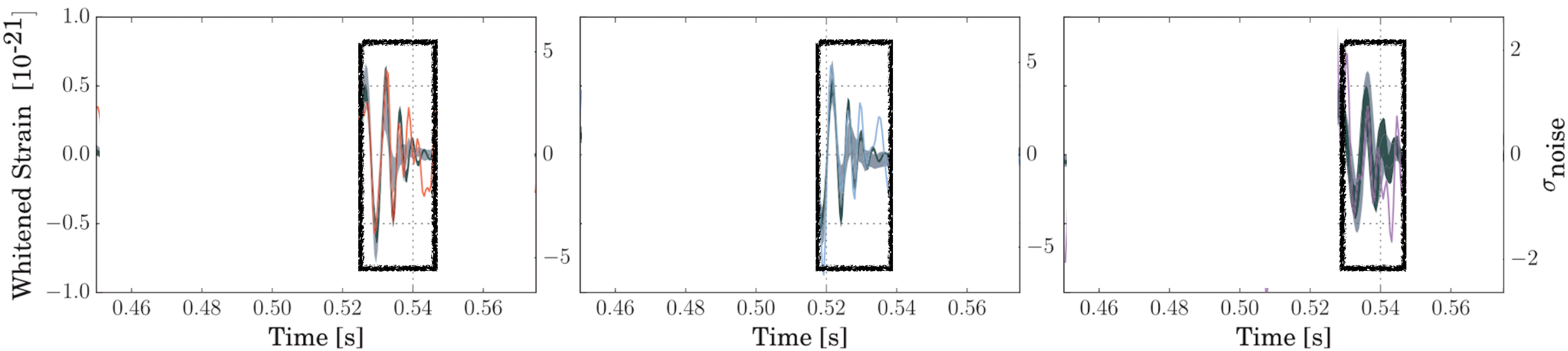

Quasi Normal Modes

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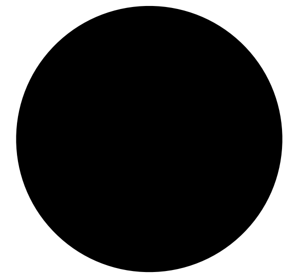

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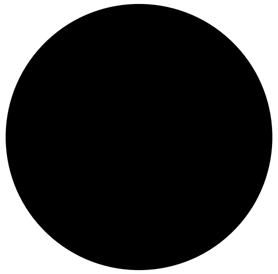

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Can there be deviations from GR ?

My perspective

Deviations

Hypothesis: observable by LIGO/Virgo, LISA, ET, …

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Need for precise modeling of the GR signal

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Are there theories:

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  - theoretically
  - with all the other observations

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Bucciotti, Kuntz, Serra, ET JHEP 2023

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Creminelli, Loayza, Serra, ET, Trombetta JHEP 2020

J. Serra, F. Serra, ET, Trombetta JHEP 2022

Juliano, ET *in progress*

Should we include non-linearities in QNM?

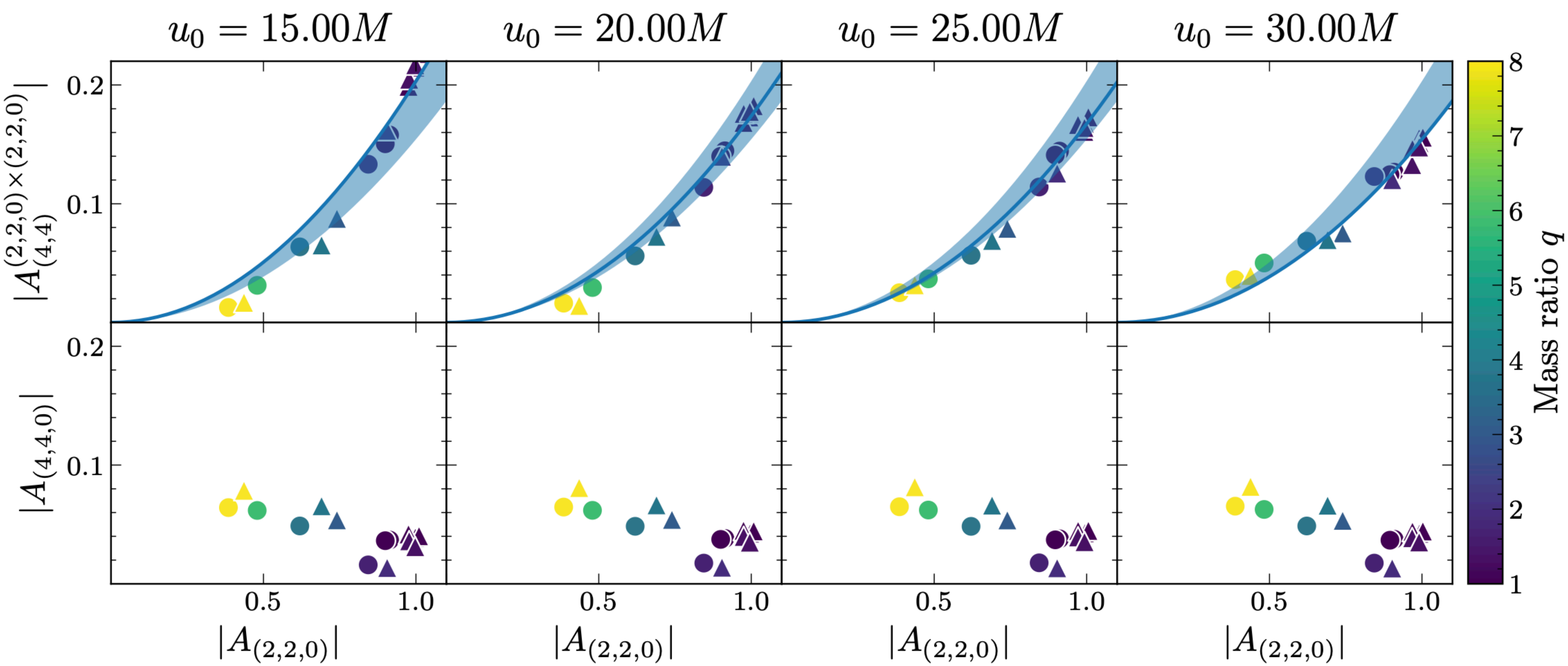

FIG. 1. Relationship between the peak amplitudes of the linear $(2,2,0)$ and the quadratic $(2,2,0)\times(2,2,0)$ QNMs (top) as well as the linear $(4,4,0)$ QNM (bottom), at different model start times u_0. Colors show different mass ratios q, and circles and triangles denote systems with remnant dimensionless spin $\chi_f \approx 0.5$ and $\chi_f \approx 0.7$, respectively. Each blue curve is a pure quadratic fit with start time u_0, and the shaded region brackets every one of the individual fits.

Mitman *et al.* 2022

Quadratic Quasi Normal Modes

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Source term

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Characteristics of the Quadratic modes

1. Frequencies are known: $\omega_1 + \omega_2 \qquad \omega_1 - \omega_2^*$
2. Amplitudes are calculable: a prediction of GR $\qquad \mathcal{R} = \mathcal{A}^{(2)}/\mathcal{A}^{(1)}_1\mathcal{A}^{(1)}_2$

A technical problem

The source goes as a constant for $r_* \to \infty$
The master scalars diverge as r_*^2

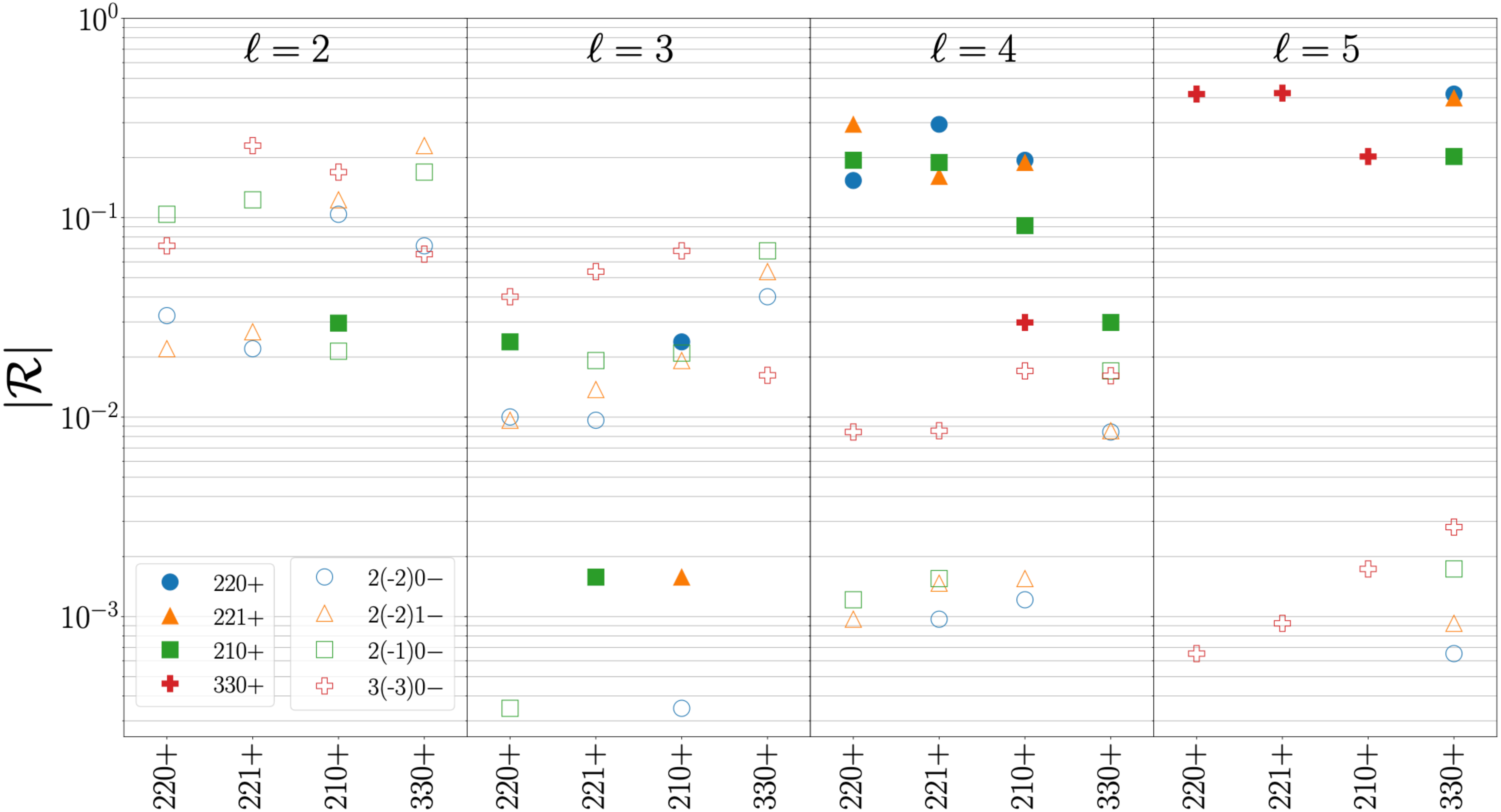
1. Hard to impose QNM boundary conditions

2. The physicsl amplitude is actually determined by the first regular term, which would be very hard to extract

Two different ways out:

Find new regular master scalars   Bucciotti, Juliano, Kuntz, ET 2024

Use the hyperboloidal frequency-domain framework   Bourg *et al.* 2025



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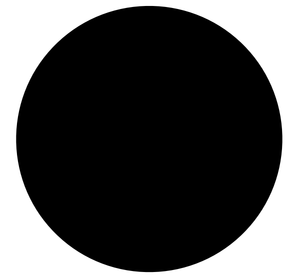

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BH response to an external field — Love numbers

Can we have observable deviations?

Two ways

(1) New states heavier than the BH curvature

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$$S = M_{\rm Pl}^2 \int d^4x \sqrt{-g}\Big(R + c_3 \frac{R^3_{\mu\nu\rho\sigma}}{\Lambda^4} + c_4 \frac{R^4_{\mu\nu\rho\sigma}}{\Lambda^6} + \ldots\Big)$$

Endlich, Gorbenko, Huang, Senatore 2017

Observable effects ⇒ $\Lambda \sim {\rm km}^{-1}$

Two ways

(2) Additional light DOF

Two ways

(2) Additional light DOF GR + photon

Two ways

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Geometry

Mass, Spin, Charge

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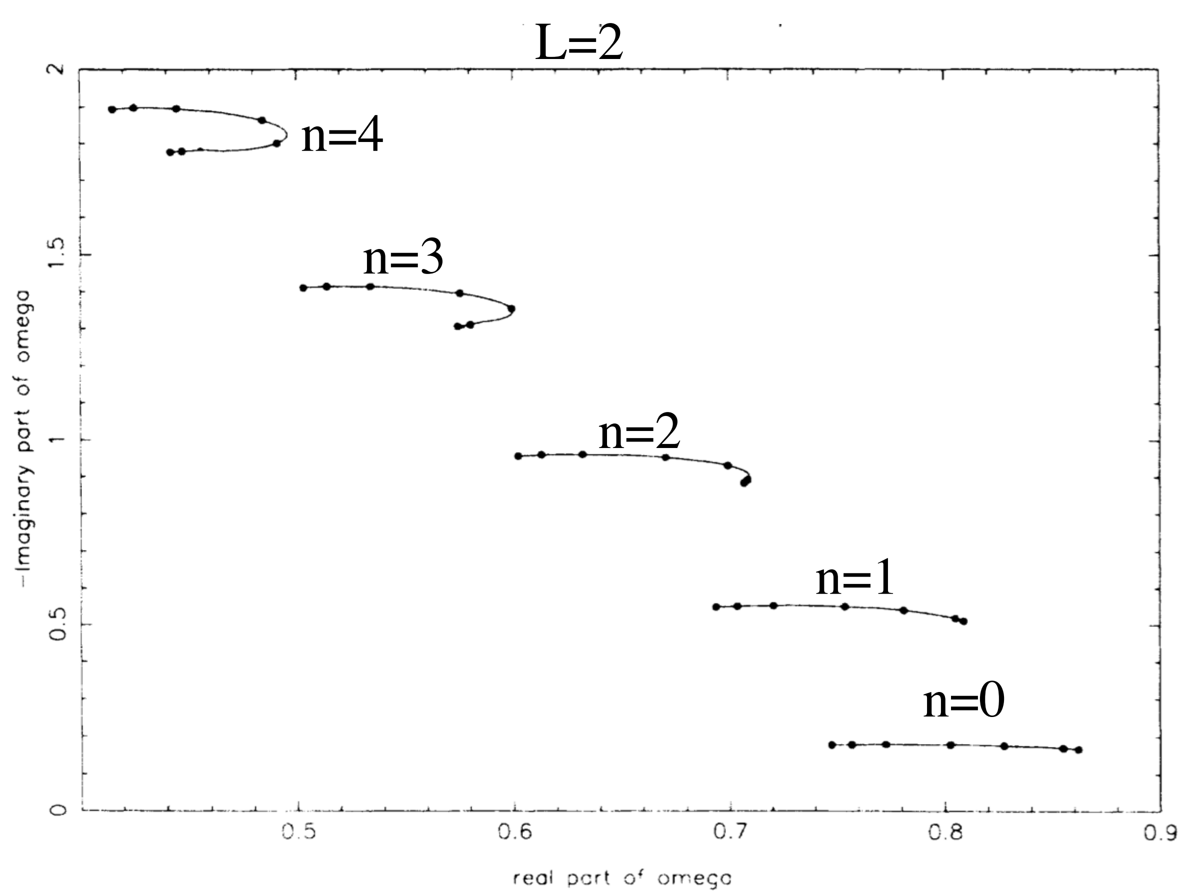

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GR + (shift-symmetric) scalar field
not coupled directly to matter

If the field has zero background ⇒ QNM are the same as GR + extra spectrum

If the field has a background ⇒ {
- GR frequencies are shifted
- Isospectrality can be broken
- even/odd mixing

A no-hair theorem

GR + shift-symmetric scalar field $$S = \int d^4x\sqrt{-g}\Big(\frac{M_{\rm Pl}}{2}R - \frac{1}{2}(\partial\phi)^2 + \mathcal{L}_{\rm int}(\phi, g)\Big)$$

Assuming spherically symmetric, time-independent solutions $$\phi'(r) = 0$$

Hui, Nicolis 2012

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EOM $\nabla_\mu J^\mu = 0$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \rho^2(r)d\Omega^2$$

$J^\mu J_\mu = (J^r)^2/f$ should be regular at the horizon $\Longrightarrow J^r = 0$

using the conservation of the current $\Longrightarrow J^r(r) = 0$

One last step to conclude that a vanishing current implies a constant scalar

If the dependence on the scalar in the Lagrangian starts quadratically then

$J^r = \phi' F[\phi', g, g']$ with a regular function

F asymptotes to a constant at infinity

Then $\phi'(r) = 0$

A no-hair theorem with a subtle exception

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$$S = \int d^4x \sqrt{-g}\Big(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \alpha M_{\rm Pl}\phi\mathcal{R}_{\rm GB}^2 + \dots\Big)$$

The Gauss-Bonnet invariant is a total derivative

The linear coupling gives a ϕ-independent contribution to the scalar EOM

$\phi'(r) = 0$ is no longer a solution

Sotiriou, Zhou 2013

"

Our mistake is not that we take our theories too seriously, but that we do not take them seriously enough

–Steven Weinberg

The EFT of scalar Gauss-Bonnet

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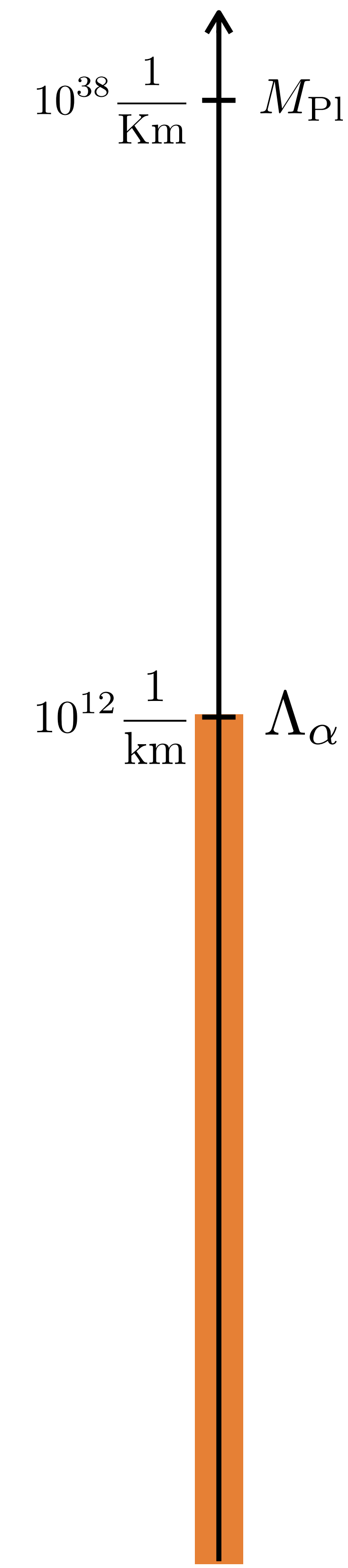
$\alpha \sim r_s^2$ observable effects

$$\frac{\phi\,\partial^2 h\,\partial^2 h}{\Lambda_\alpha^3} \qquad \Lambda_\alpha \equiv \left(\frac{M_{\rm Pl}}{\alpha}\right)^{1/3}$$



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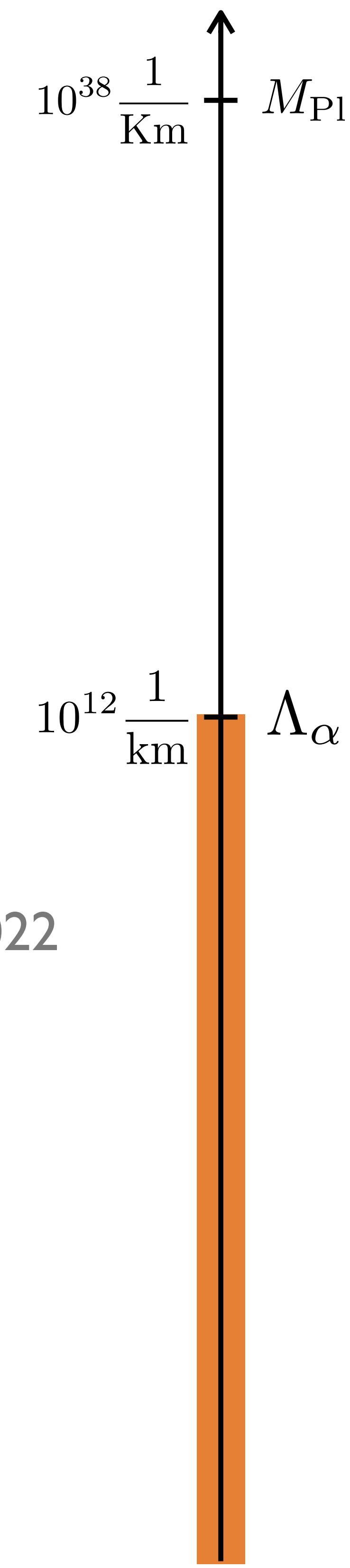

Such an EFT is constrained by causality in classical obs.

F. Serra, J. Serra, ET, L. Trombetta 2022

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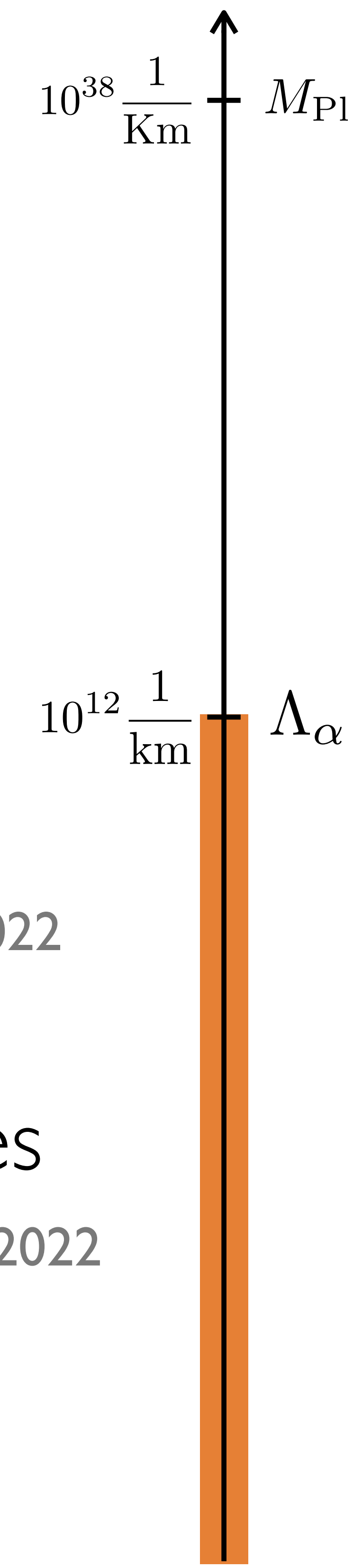
similar constraints from analiticity of scattering amplitudes

Caron-Huot, Li, Parra-Martinez, Simmons-Duffin, 2022



Causality Constraints from 3-point interactions

Consider higher derivative corrections to the graviton 3-point coupling

Camanho, Edelstein, Maldacena, Zhiboedov, 2014

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$$S = M_{\rm Pl}^2 \int d^4x \sqrt{-g}\Big(R + \alpha_4 R^3_{\mu\nu\rho\sigma}) \qquad \alpha_4 = [L]^4$$

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Small angle scattering of a graviton off a massive object

$m \gg \omega \gg |\vec{q}|$ Eikonal limit

It experiences a time delay

Both helicities contribute: eikonal phase $\longrightarrow$ eikonal phase matrix

$$\delta t \propto 4\frac{m}{M_{\rm Pl}^2}\left[\log\frac{b_0}{b} \pm \frac{\alpha_4}{b^4}\right]$$ observable while the computation is under control

Time delay becomes a time advance when $b \sim (\alpha_4)^{1/4}$

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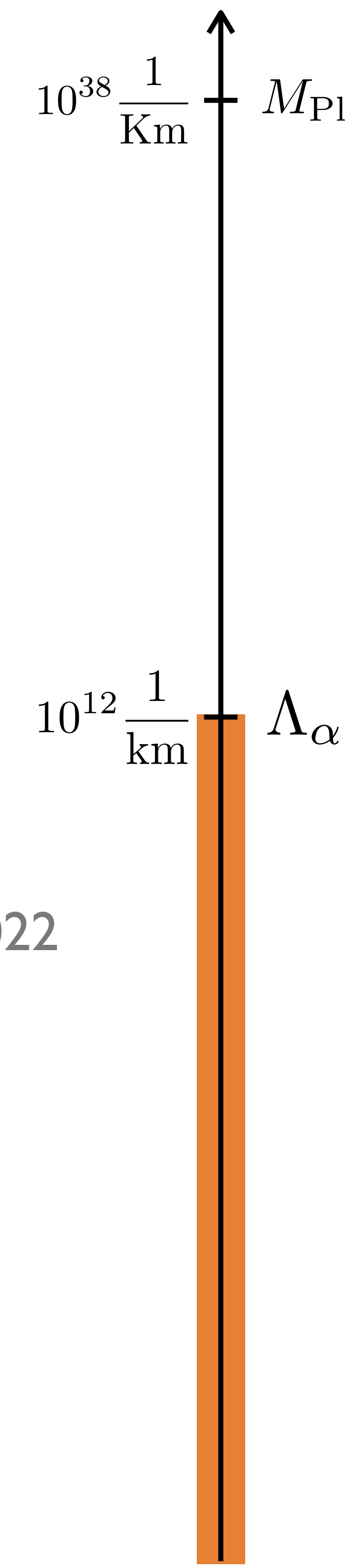

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Time advance when $b \sim \sqrt{\alpha}$

The EFT of scalar Gauss-Bonnet

$$S = \int d^4x \sqrt{-g} \Big(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 + \alpha M_{\rm Pl} \phi \mathcal{R}^2_{\rm GB} + \ldots \Big)$$

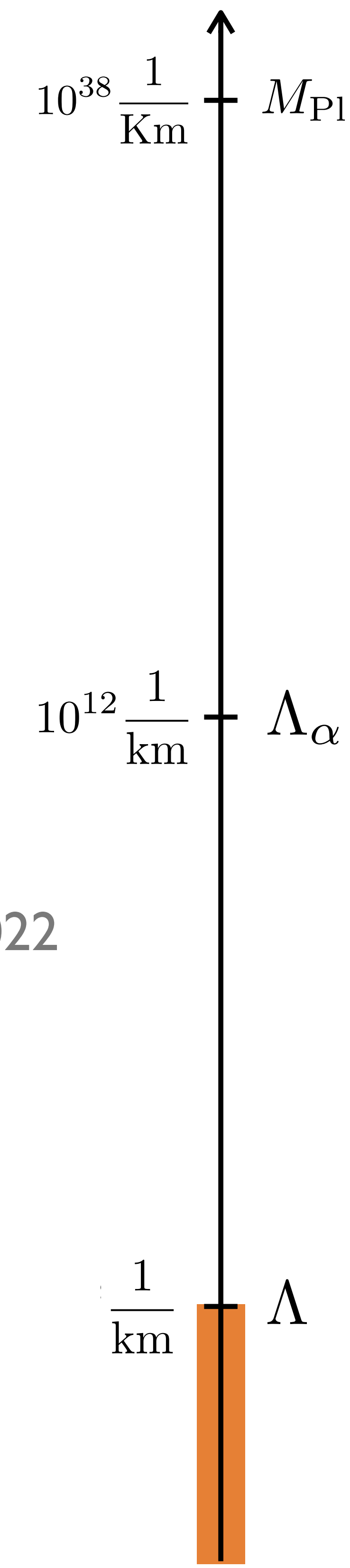
$\alpha \sim r_s^2$ observable effects

$$\frac{\phi\, \partial^2 h\, \partial^2 h}{\Lambda_\alpha^3} \qquad \Lambda_\alpha \equiv \Big(\frac{M_{\rm Pl}}{\alpha} \Big)^{1/3}$$

Such an EFT is constrained by causality in classical obs.

F. Serra, J. Serra, ET, L. Trombetta 2022

Time advance when $b \sim \sqrt{\alpha}$



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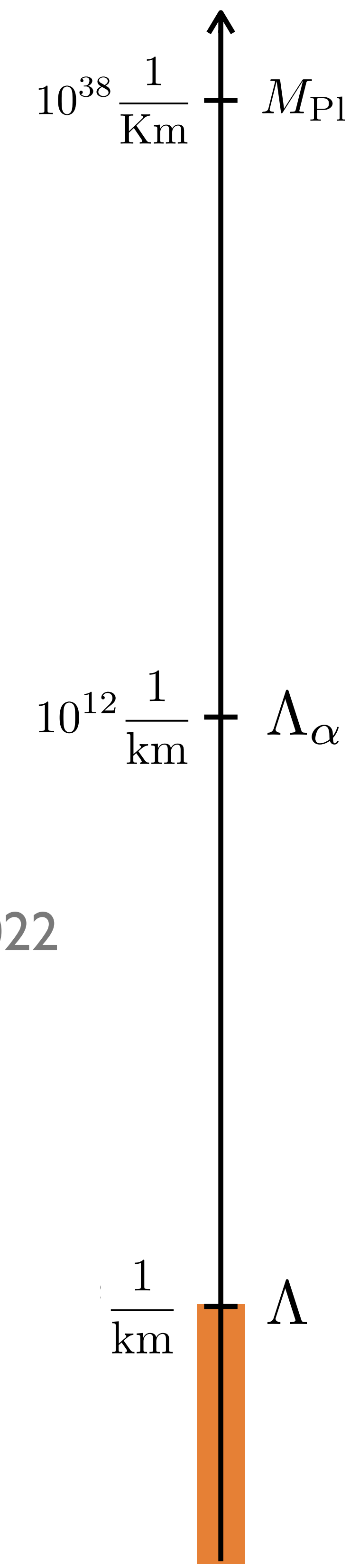
F. Serra, J. Serra, ET, L. Trombetta 2022

Time advance when $b \sim \sqrt{\alpha}$

$$M_{\rm Pl}^2 R + \frac{\Lambda^4}{g^2}\Big[\hat{\mathcal{L}}^{(0)}\Big(\frac{\partial}{\Lambda},\frac{R}{\Lambda^2},\frac{g\phi}{\Lambda}\Big) + \frac{g^2}{16\pi^2}\hat{\mathcal{L}}^{(1)}\Big(\frac{\partial}{\Lambda},\frac{R}{\Lambda^2},\frac{g\phi}{\Lambda}\Big) + \ldots\Big]$$

$$g \sim \frac{\Lambda}{M_{\rm Pl}}$$



My perspective

Deviations

Hypothesis: observable by LIGO/Virgo, LISA, ET, …

Need for precise modeling of the GR signal

Bucciotti, Kuntz, Serra, ET JHEP 2023

Bucciotti, Juliano, Kuntz, ET PRD 2024

Bucciotti, Juliano, Kuntz, ET JHEP 2024

Are there theories:

- (motivated)
- consistent → theoretically
- consistent → with all the other observations

Creminelli, Loayza, Serra, ET, Trombetta JHEP 2020

J. Serra, F. Serra, ET, Trombetta JHEP 2022

Juliano, ET *in progress*