The 3-dimensional map of the proton

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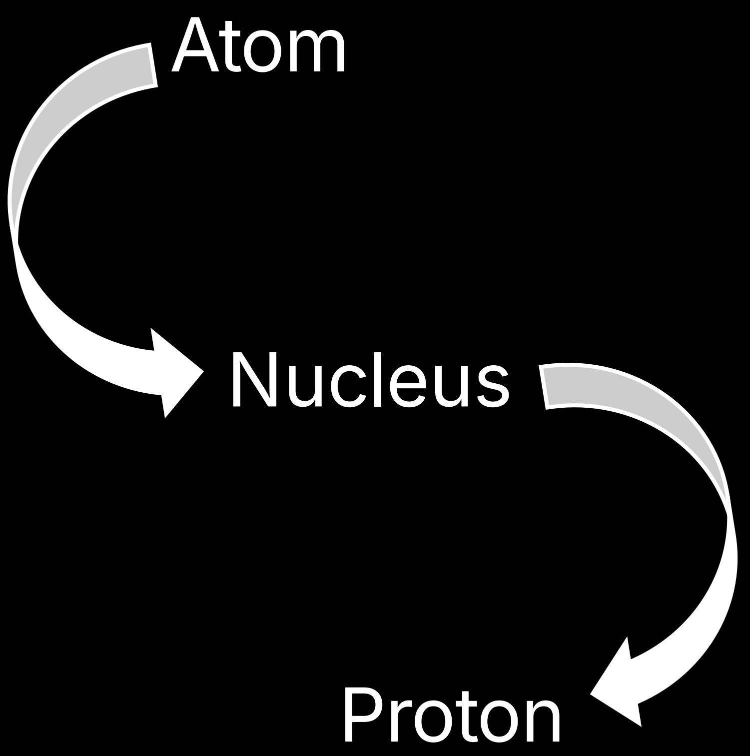
Quantum Chromodynamics



What is QCD, why it is relevant

$$\mathcal{L} = \bar{\psi}\left(i\not{\partial} - g\not{A} - m\right)\psi - \frac{F^2}{4}$$

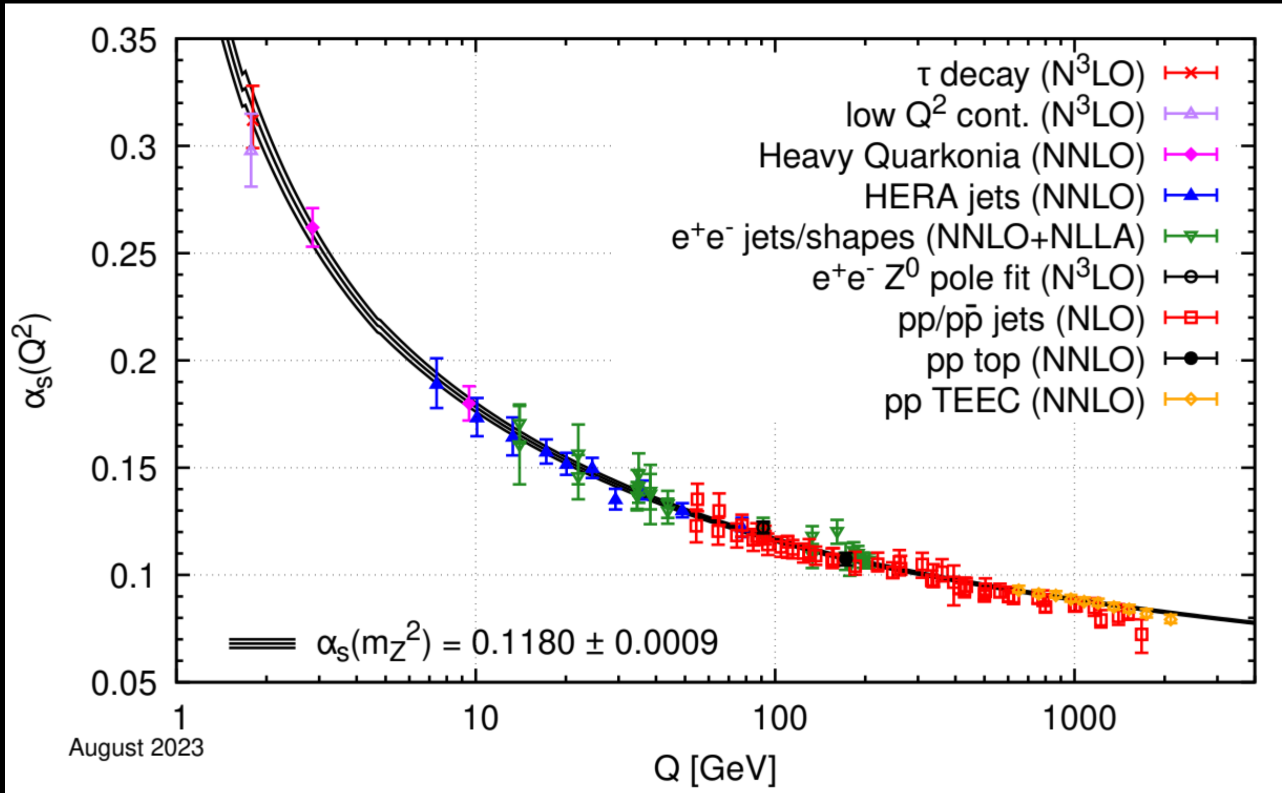
From PDG:
"Neither quarks nor gluons
are observed as free particles"

Quantum Chromodynamics

We have a problem

$$\frac{\partial \log a_s}{d \log \mu} = \beta(a_s(\mu))$$

$$a_s(\mu) = \frac{a_s(\mu_0)}{1 - a_s(\mu_0)\beta_0 \log(\mu/\mu_0)} + \dots$$



Quantum Chromodynamics

We cannot compute hadronic properties in perturbation theory

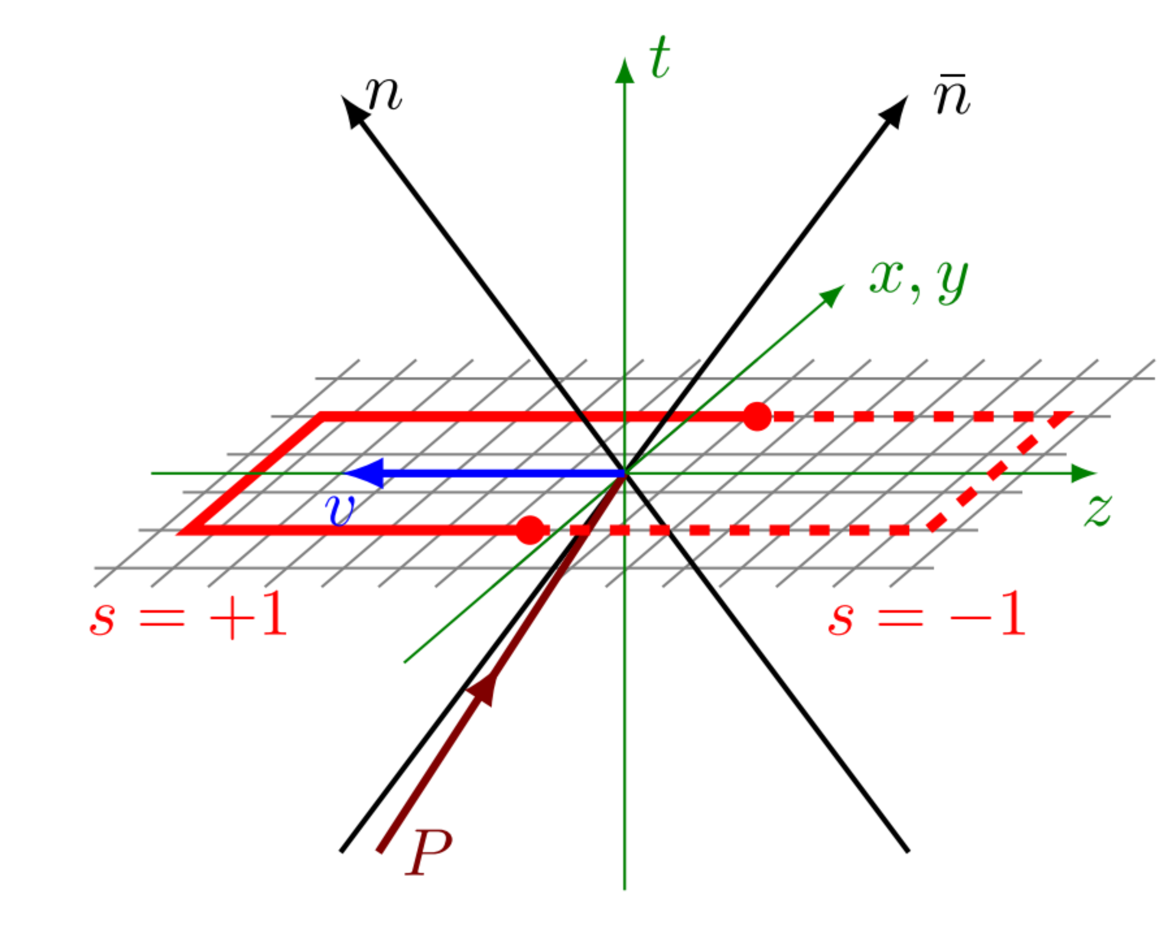
So, what CAN we do?

Simulate the full theory

Extract hadronic properties from experimental data

Lattice-QCD: non-perturbative methods

SR, A. Vladimirov JHEP 09 (2023) 117



Disclaimer: not only approach!

$$\Omega(\ell v + \boldsymbol{b}) = C_1^2 \Psi(\boldsymbol{b}) f(\ell, \boldsymbol{b})$$

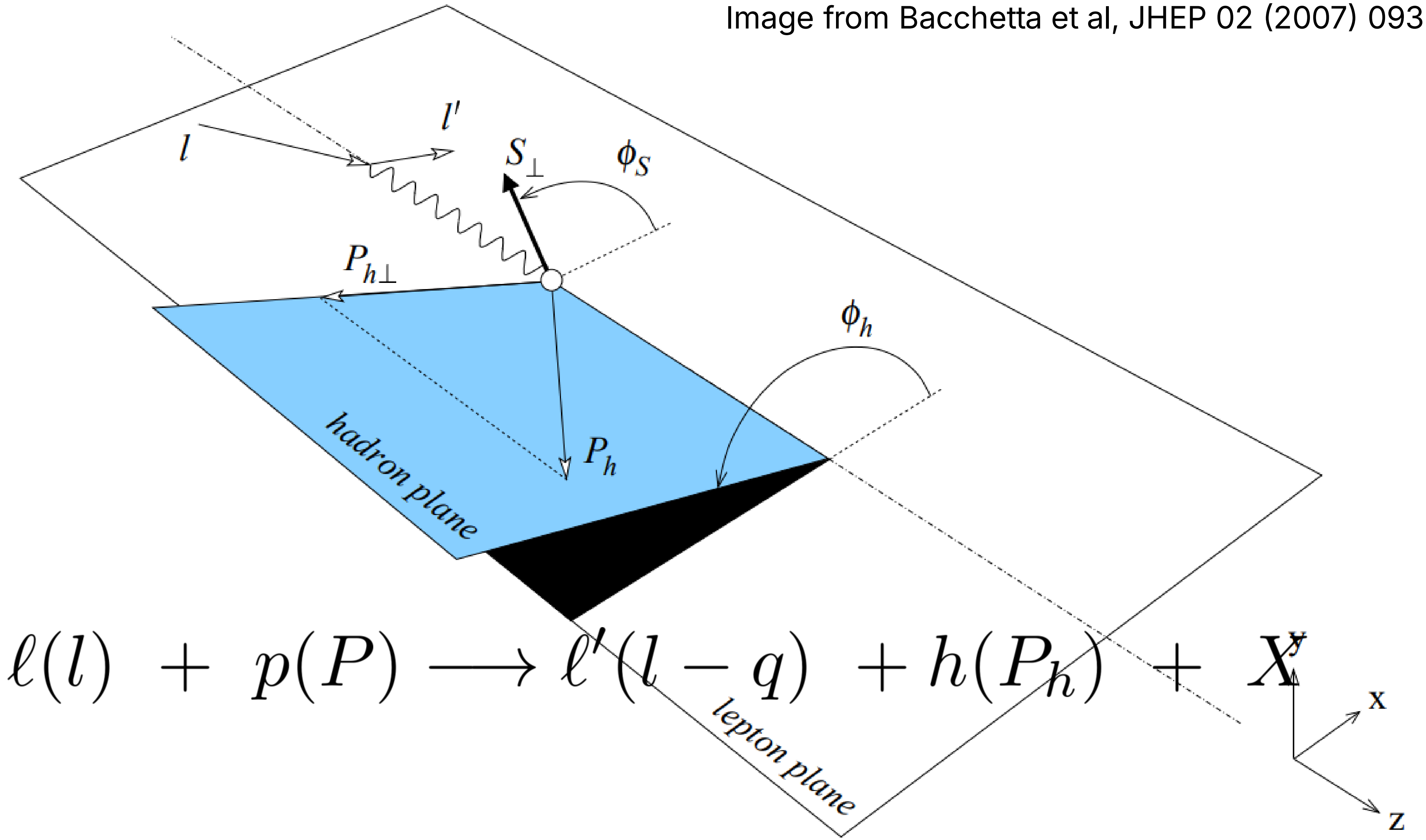
Perturbative calculable

Transverse Momentum distribution

Unknown non-perturbative function

Semi-Inclusive Deep Inelastic Scattering

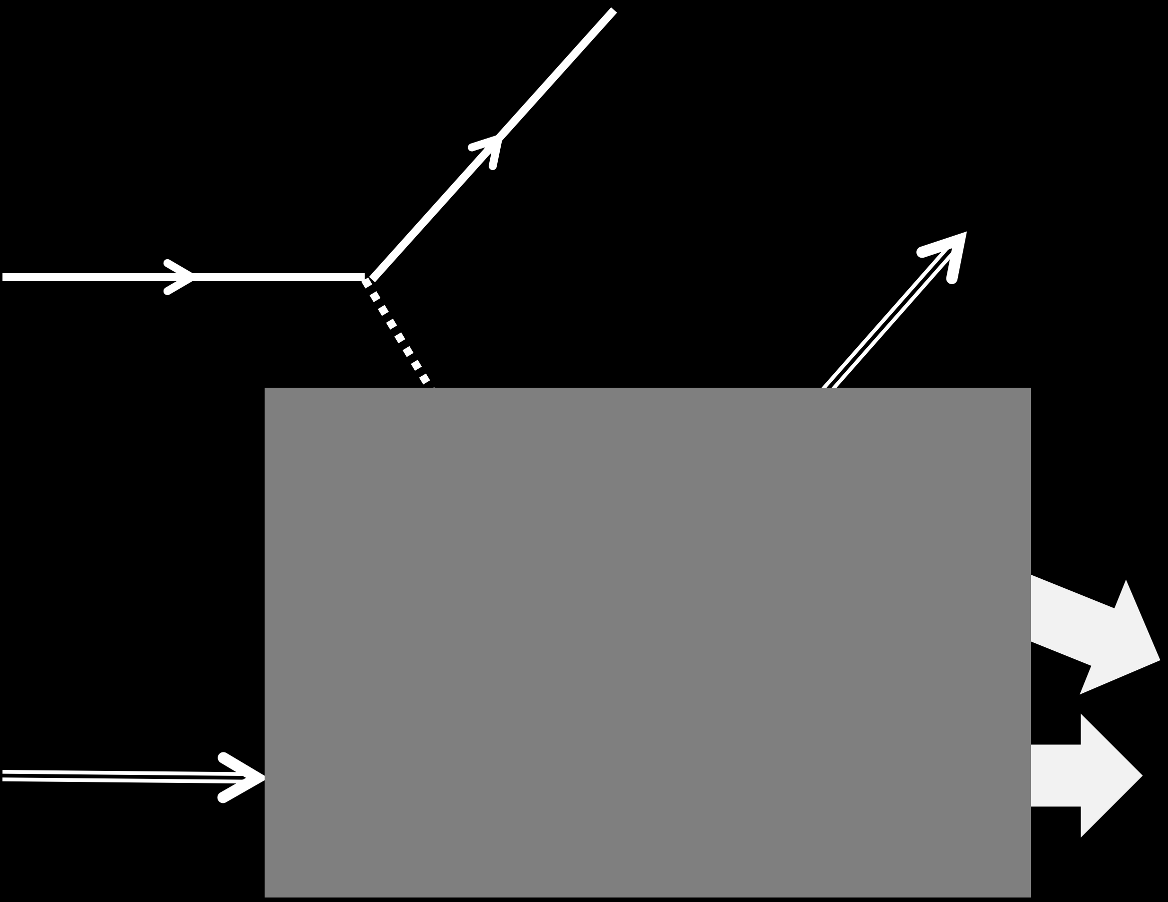
Image from Bacchetta et al, JHEP 02 (2007) 093



$$\ell(l) \; + \; p(P) \longrightarrow \ell'(l-q) \; + h(P_h) \; + \; X$$

Semi-Inclusive Deep Inelastic Scattering

$$\frac{d\sigma}{dx\ dy\ dz\ d\phi_S\ [d\phi_h\ dP_{h\perp}^2]}$$



$$x = \frac{Q^2}{2(Pq)} \quad y = \frac{(Pq)}{(Pl)} \quad z = \frac{(PP_h)}{(Pq)}$$

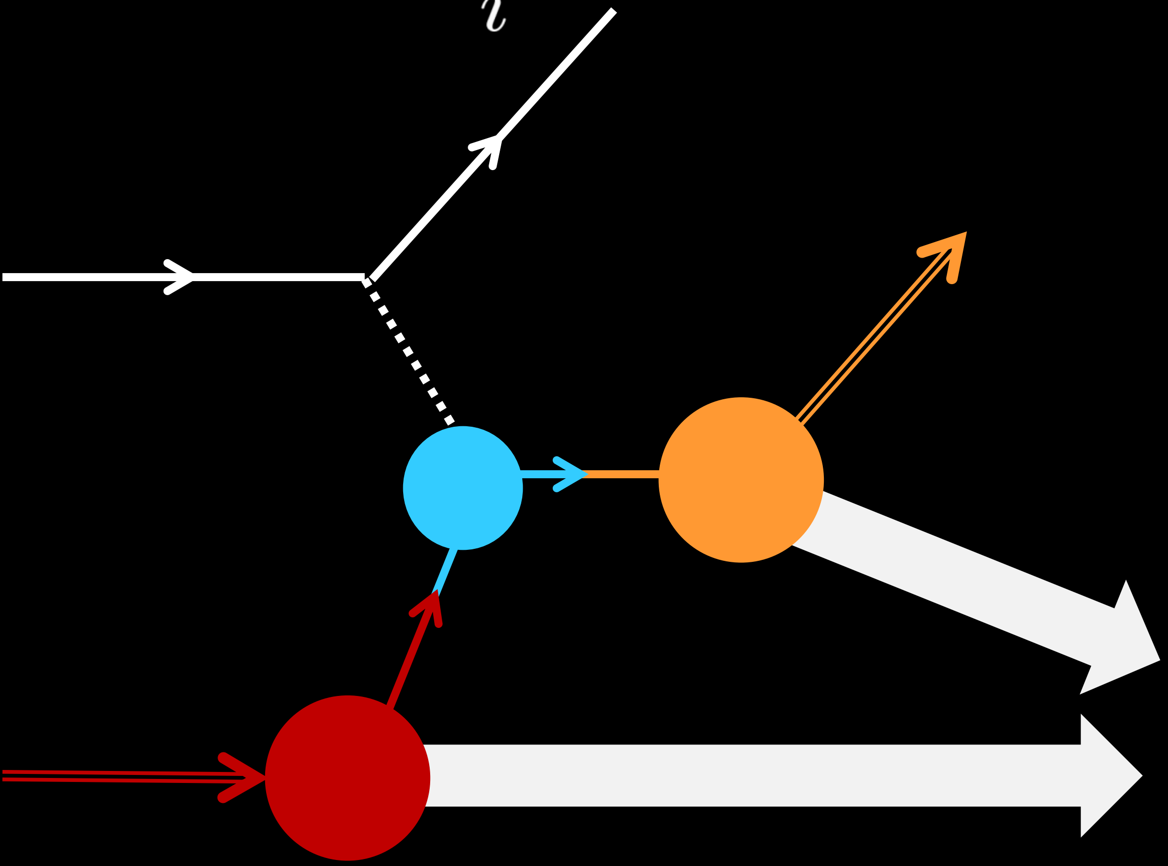
Factorization Theorem

$$d\sigma \sim \sum_i A_i F_i$$

Angular modulations

Structure Functions

$$Q \gg P_{h\perp}/z$$



Separate into

Parton Distribution Functions

Fragmentation Functions

Hard coefficient

Factorization Theorem

$$x|C_1(\mu^2, Q^2)|^2 \sum_i e_i^2 \int \frac{bdb}{2\pi} (bM)^n J_n\left(\frac{b|P_{h\perp}|}{z}\right) f_i(x, b; \mu, \zeta) D_i(z, b; \mu, \bar{\zeta})$$

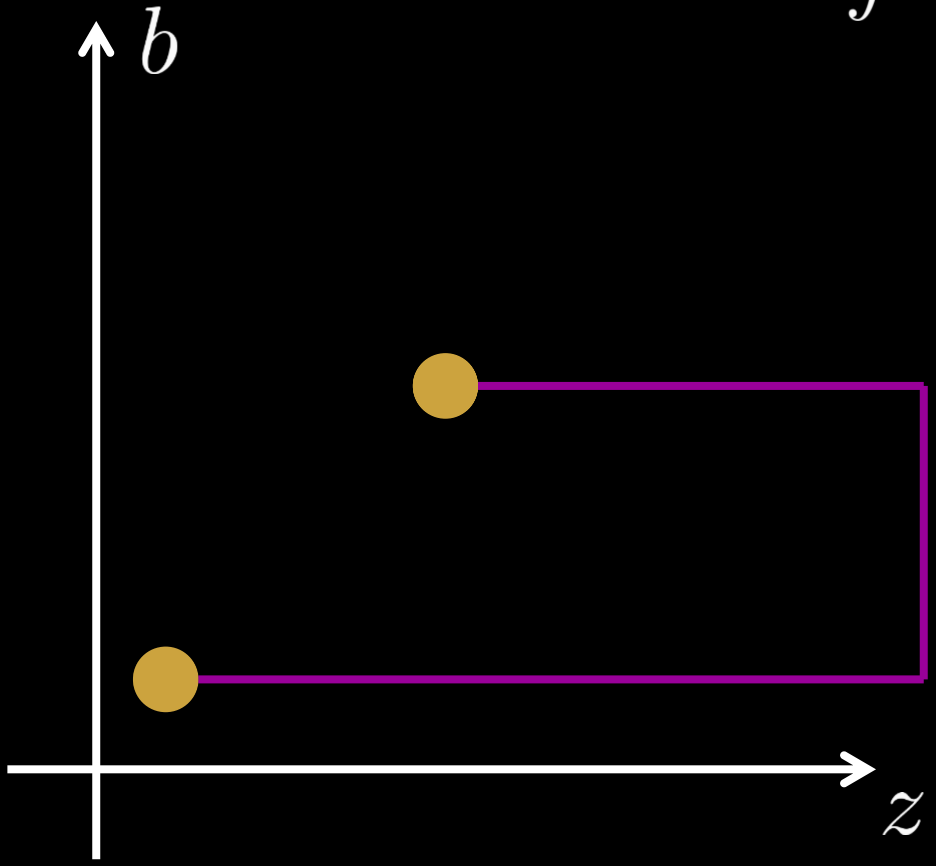
Quark Pol.

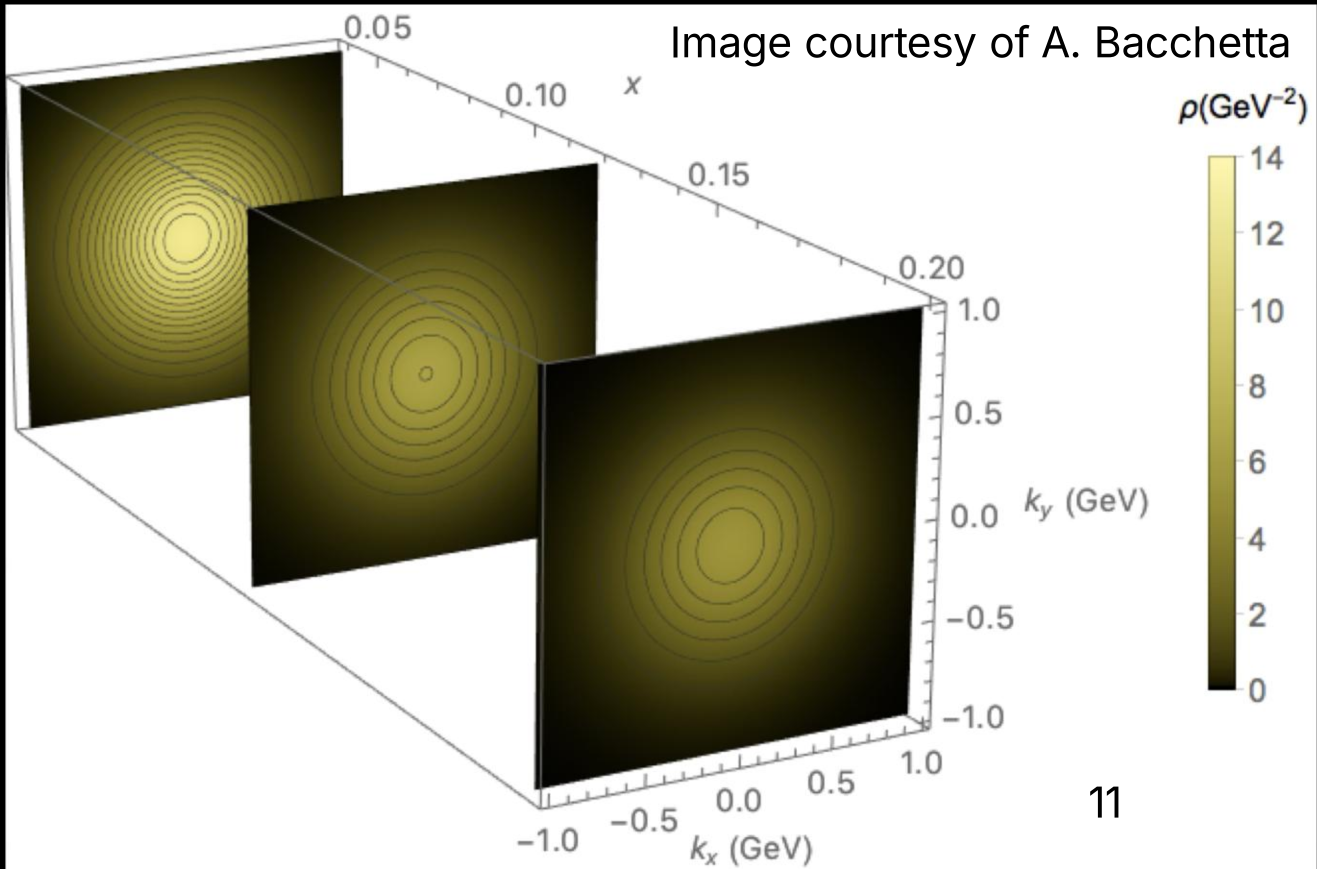
Proton Pol.

	U	H	T
U	f_1 (tw2)		$h_1^\perp$ (tw3)
L		g_1 (tw2)	$h_{1L}^\perp$ (tw2 & tw3)
T	$f_{1T}^\perp$ (tw3)	g_{1T} (tw2 & tw3)	h_1 (tw2) $h_{1T}^\perp$ (tw3 & tw4)

Properties: Evolution

$$f_i(x, b; \mu, \zeta) = \int \frac{dz}{4\pi} e^{-ixz} \langle p|\bar{\psi}((z/p_+), b)\Gamma W \psi(0)|p\rangle$$





Properties: Evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} f = \frac{\gamma_F(\mu^2, \zeta)}{2} f$$

$$\zeta \frac{\partial}{\partial \zeta} f = -\mathcal{D}(\mu^2, b) f$$

Perturbatively calculable

Collins-Soper kernel
Has non-perturbative part

Integrability condition

$$2\mu^2 \frac{\partial}{\partial \mu^2} \mathcal{D}(\mu^2, b) = \Gamma_{\text{cusp}}(\mu^2, \zeta) = -\zeta \frac{\partial}{\partial \zeta} \gamma_F$$

Properties: Evolution

Path-independence ⇒ simple solution

$$f(\mu^2,\zeta) = f(\mu_i^2,\zeta_i)\left(\frac{\zeta_f}{\zeta_i}\right)^{-\mathscr{D}(\mu_i^2,b)} \exp\left[\int_{\mu_i^2}^{\mu^2}\gamma_F(\mu^2,\zeta_f)\frac{d\mu}{\mu}\right]$$

So, what now?

Properties: Matching and f_{NP}

$$f(x,\boldsymbol{b}) = \int \frac{d\zeta}{4\pi} e^{-ix\zeta} \langle p|\bar{\psi}(\zeta,\boldsymbol{b})\Gamma W \psi(0)|p\rangle \xrightarrow[\boldsymbol{b}\to 0]{} ?$$

Operator Product Expansion

$$F(x,b) = C(x,\ln(\mu b)) \otimes f(x,\mu) + \mathcal{O}(b^2)$$

	U	H	T
U	f_1 (tw2)		$h_1^\perp$ (tw3)
L		g_1 (tw2)	$h_{1L}^\perp$ (tw2 & tw3)
T	$f_{1T}^\perp$ (tw3)	g_{1T} (tw2 & tw3)	h_1 (tw2) $h_{1T}^\perp$ (tw3 & tw4)

Properties: Matching and f_{NP}

$$F(x,b) = C(x, \ln(\mu b)) \otimes f(x,\mu) + \mathcal{O}(b^2)$$

Potentially large logs $\mu \,!\!\propto 1/b$

When b is large, scale is small ⟶ non-perturbative region

$$f(b,\mu_f^2,\zeta_f^2) = \left(\frac{\zeta_f}{\mu_*^2}\right)^{-\mathcal{D}_{\mathrm{OPE}}(\mu_*^2, b^*)} \exp\left(\int_{\mu_*}^{\mu_f} \frac{d\mu}{\mu}\gamma_F(\mu^2,\zeta_f)\right) f_{NP}(b,b^*,\mu_*^2) f_{\mathrm{OPE}}(b^*,\mu_*^2)$$

Two selected cases

The unpolarized distribution

Quark densities

Coefficient know @ N3LO

Usual unpolarized PDFs as input

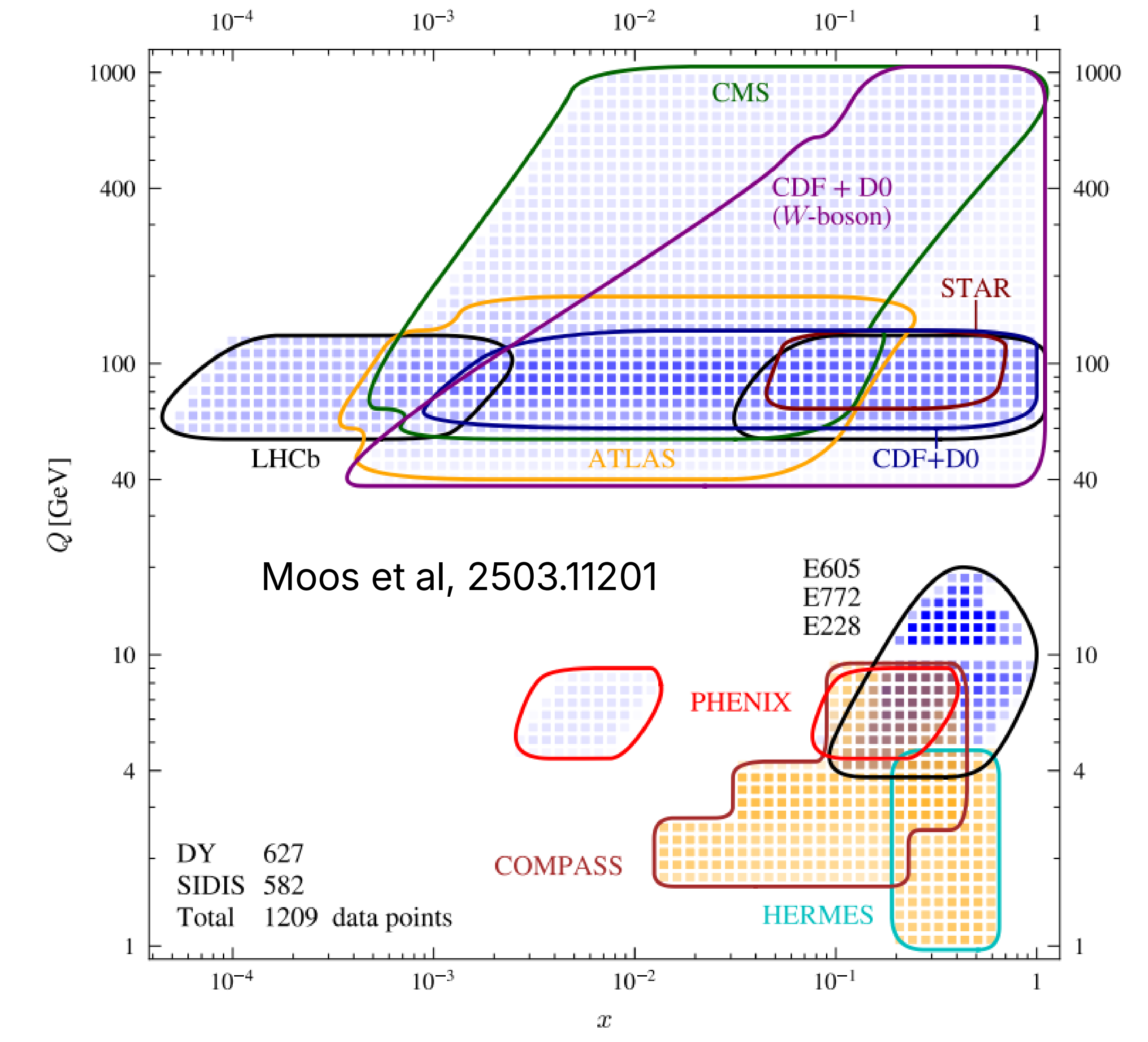
The Sivers distribution

Coefficient know @ NLO

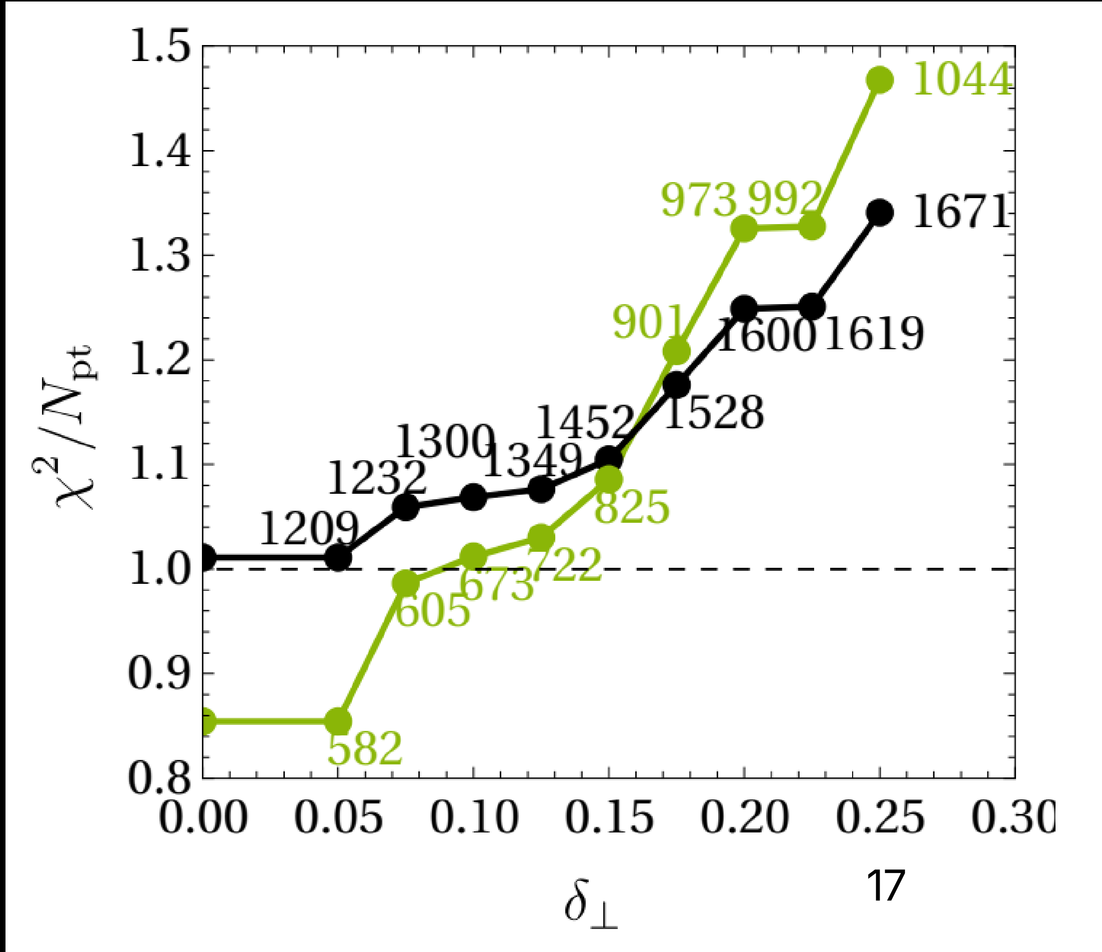
Quark Spin-Orbit correlations

Used to extract twist-3 PDFs

The case of Unpolarized Distribution

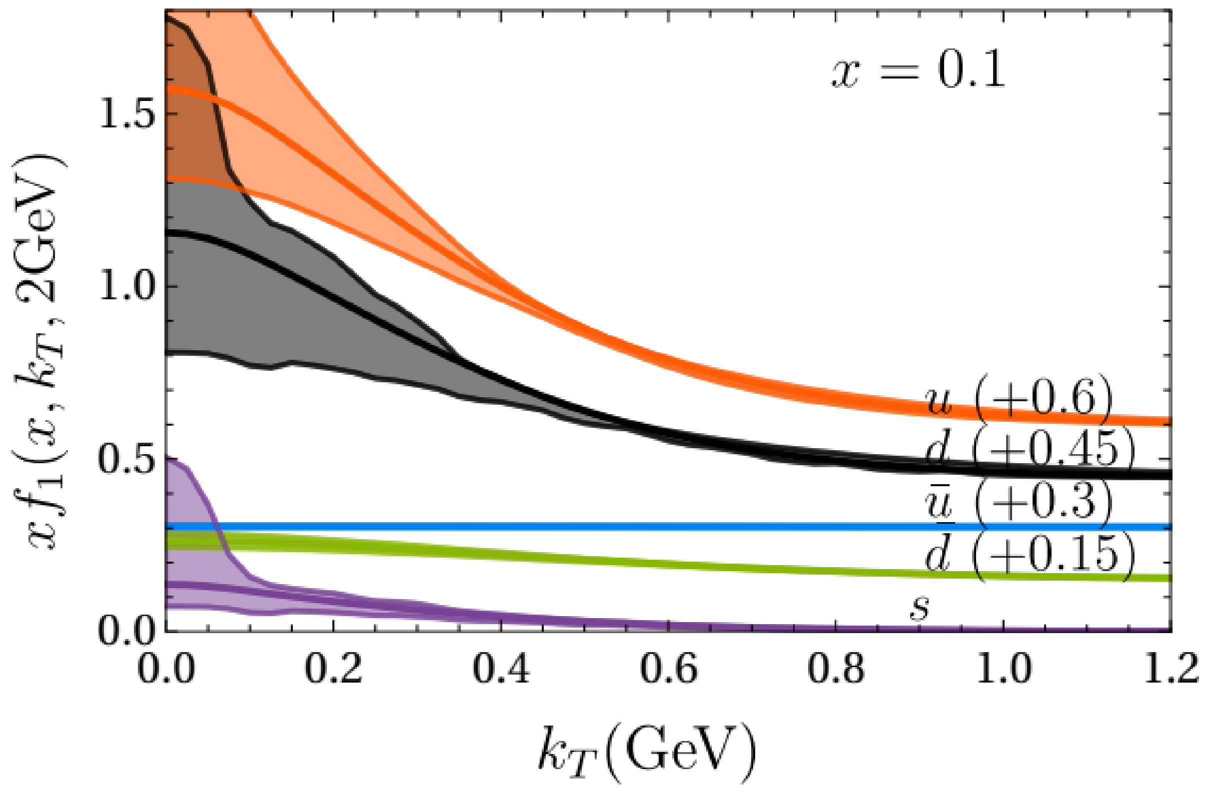


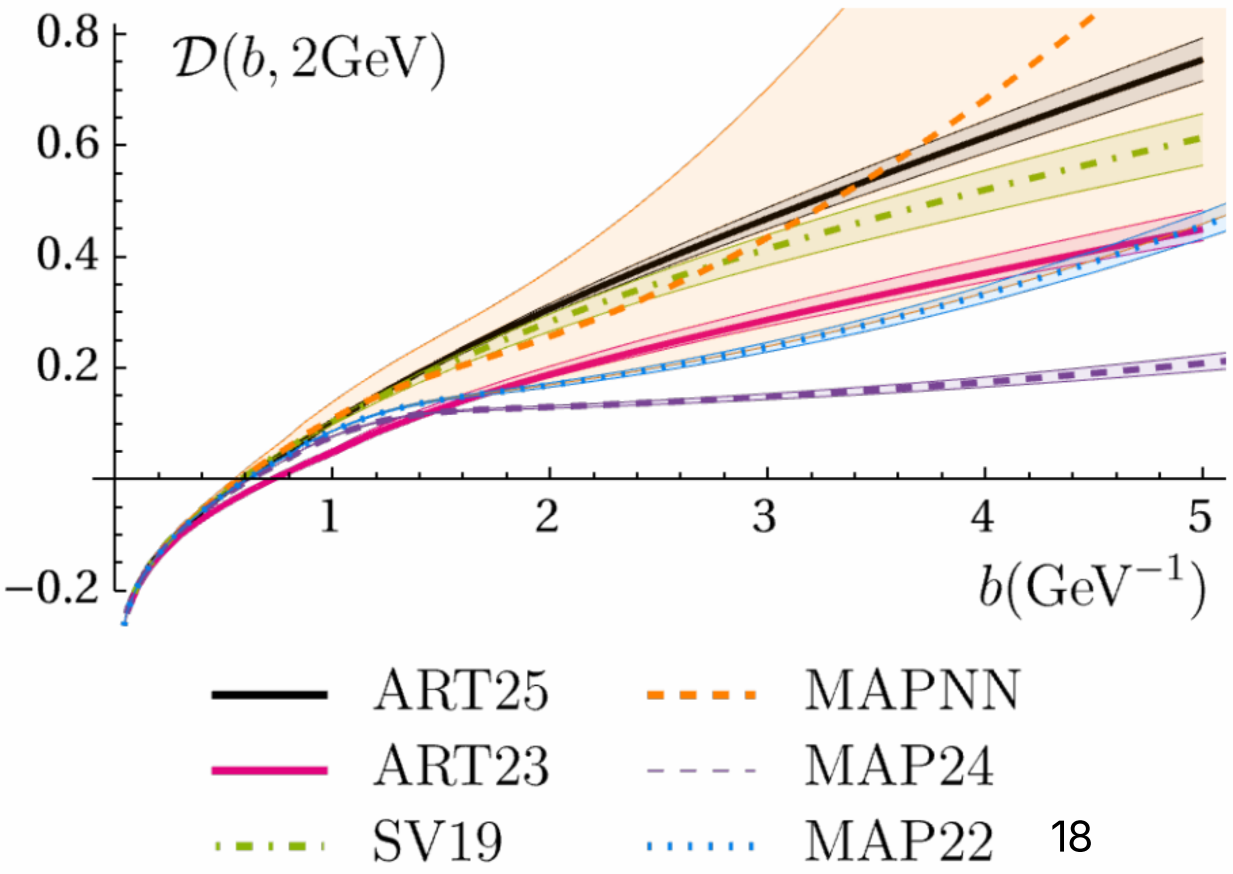
Extractions uses combination of SIDIS and Drell-Yan



The case of Unpolarized Distribution

Results from Moos et al, 2503.11201

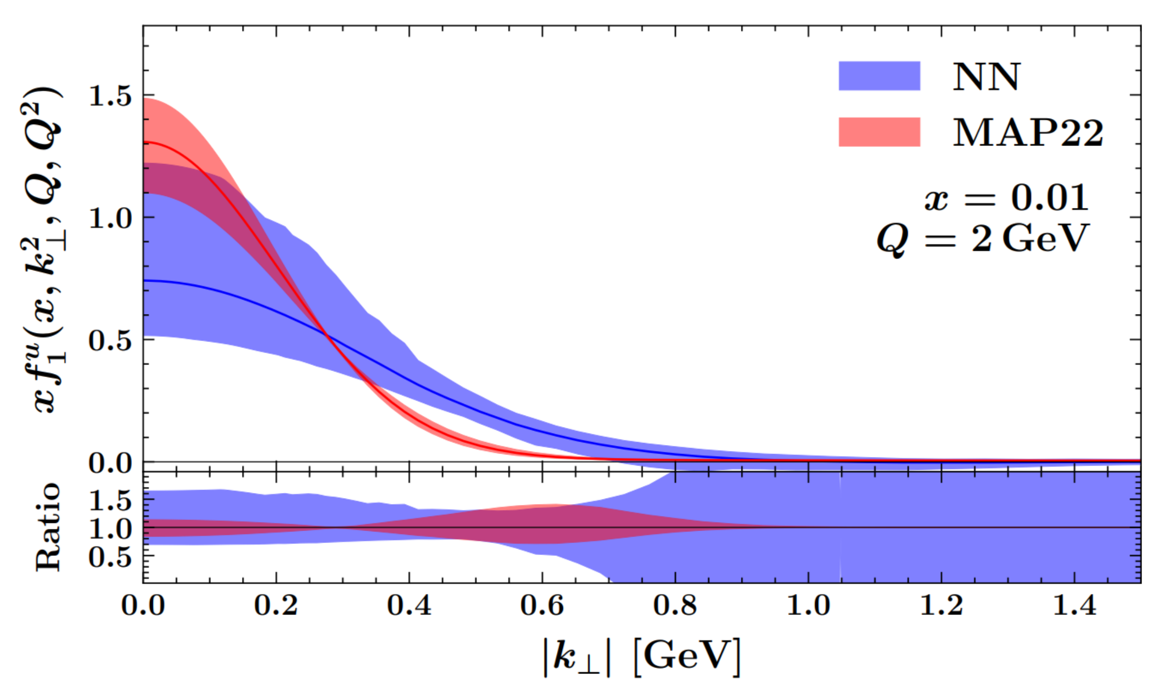


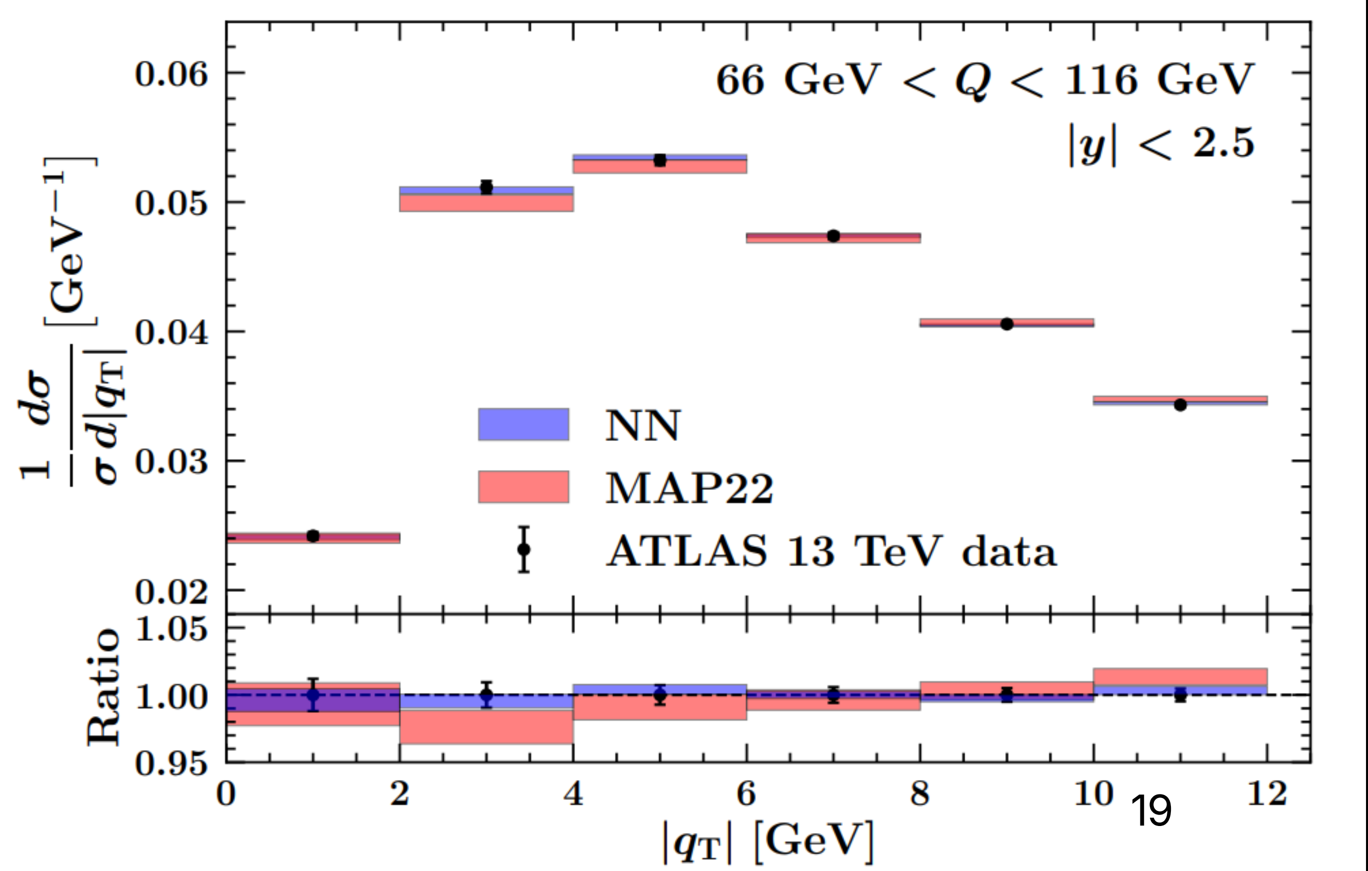


The case of Unpolarized Distribution

Drell-Yan only, Neural Network for more realistic uncertainties

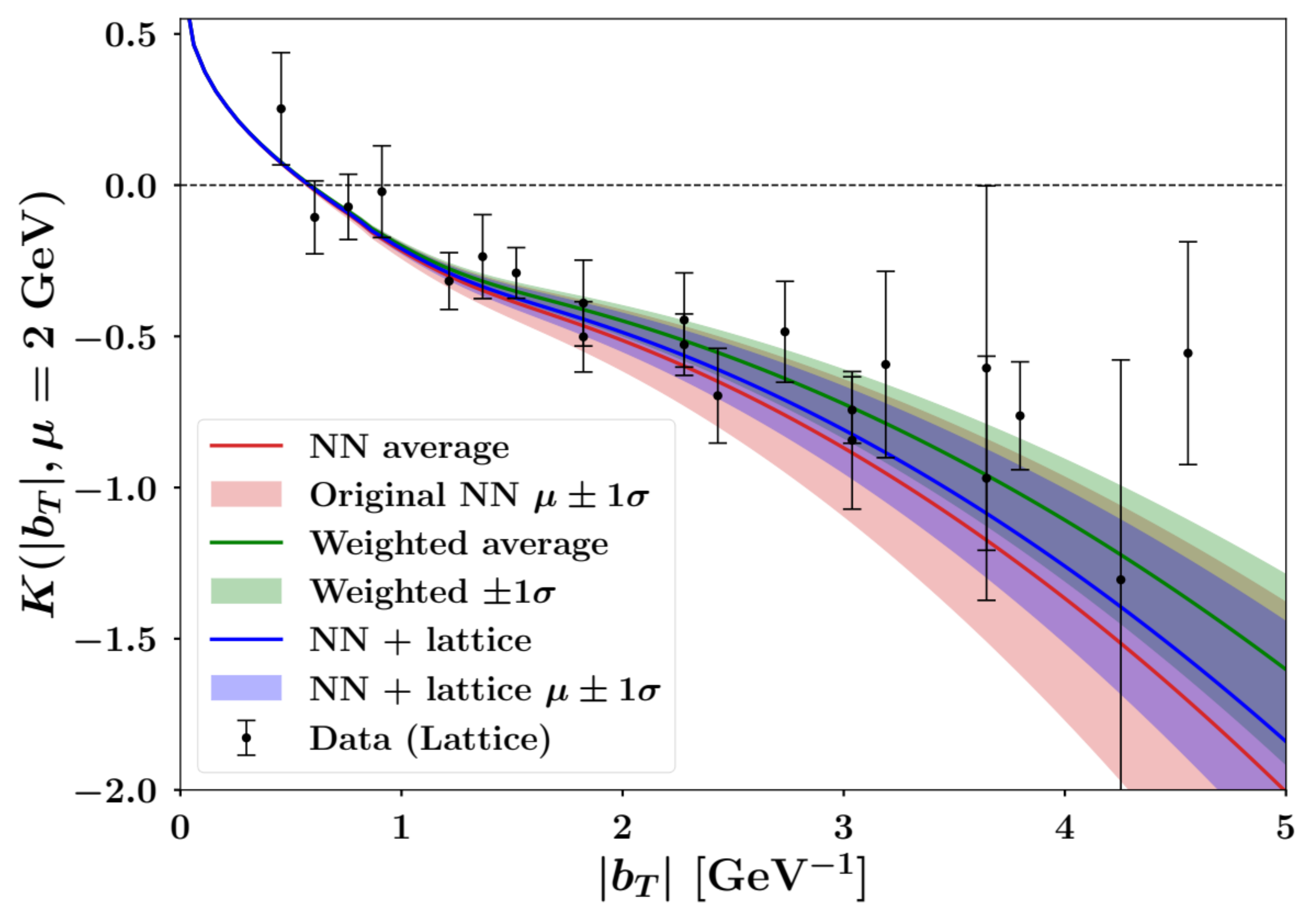
M.A.P. collaboration Phys.Rev.Lett. 135 (2025) 2, 021904





NN fit and Lattice Collins-Soper Kernel

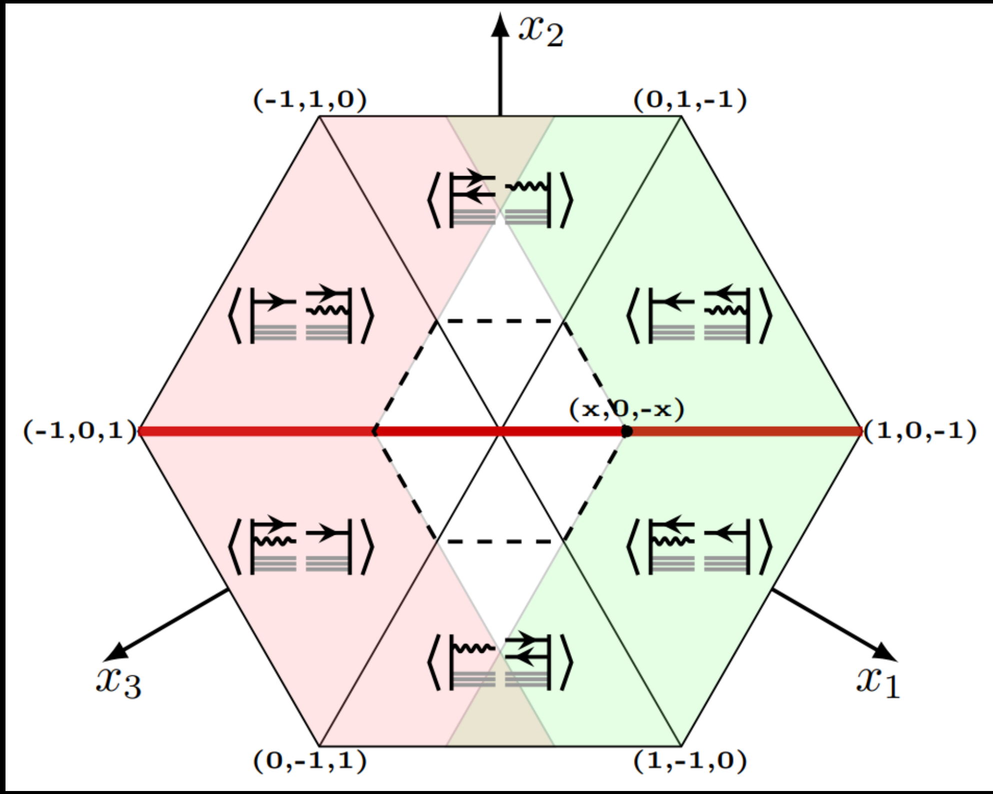
Experiment	N_{dat}	$\langle\overline{\chi}^2\rangle$		
		NN	NN rew	NN+lattice
Fixed-target	233	1.03	1.02	1.04
RHIC	7	1.01	1.03	1.02
Tevatron	71	0.86	0.86	0.88
LHCb	21	1.09	1.06	1.11
CMS	78	0.39	0.39	0.39
ATLAS	72	1.32	1.35	1.34
Lattice $a = 0.15$ fm	6	/		0.56
Lattice $a = 0.12$ fm	7	/		0.18
Lattice $a = 0.09$ fm	8	/		0.20
Total	503	0.95	0.95	0.95



The Sivers/twist-3 case

Much more challenging!

We need combined fit of TMD and collinear distributions



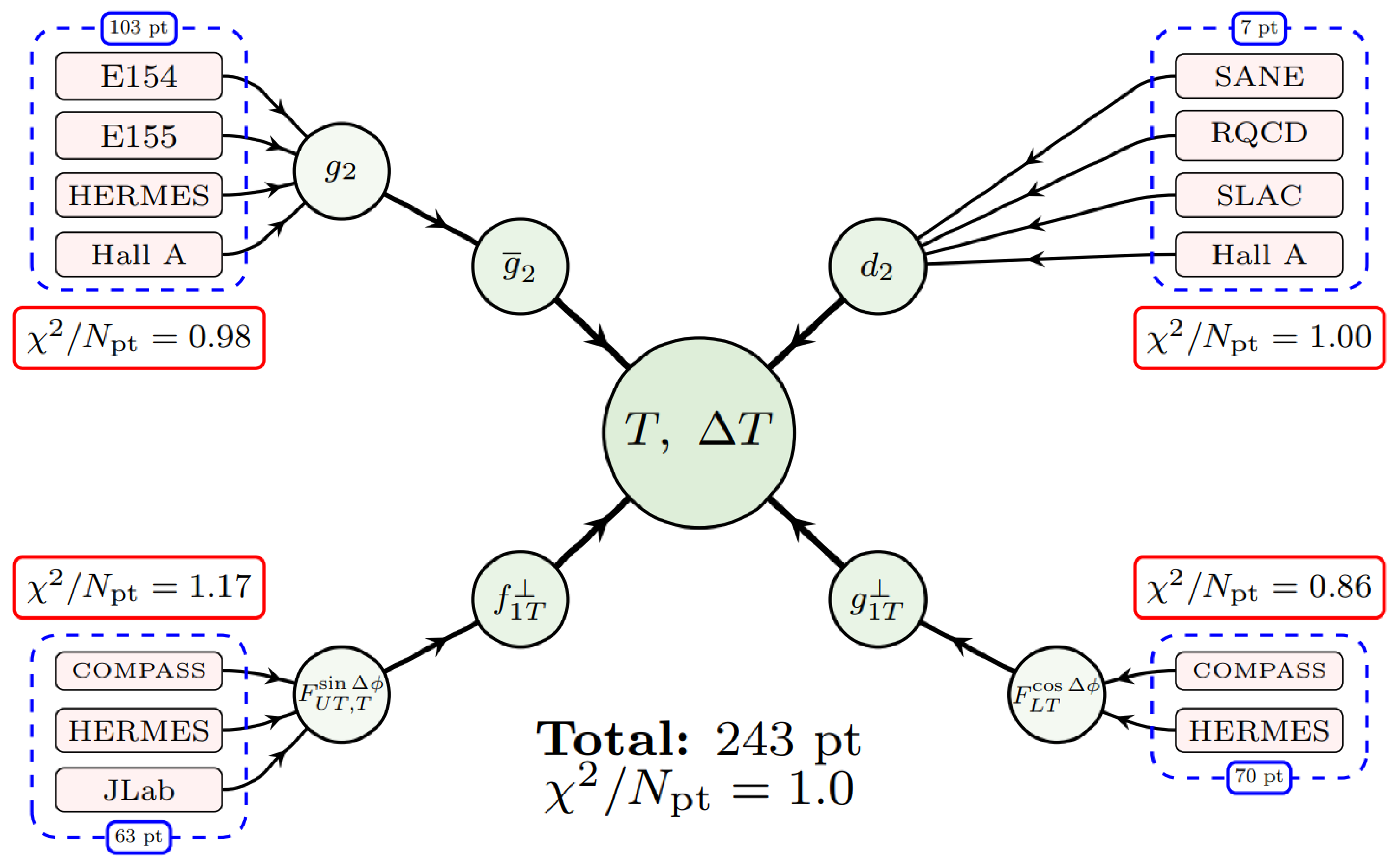
$$f_{1T,q}^{\perp}(x,b;\mu,\zeta) = \pm\pi T_q(-x,0,x) \pm \pi a_s\Big\{C_F\left(-\mathbf{L}_b^2 + 2\mathbf{L}_b\mathbf{l}_\zeta + 3\mathbf{L}_b - \frac{\pi^2}{6}\right)T_q(-x,0,x)$$

$$-2\mathbf{L}_b\mathbb{H}\otimes T_q(-x,0,x) + \delta\mathbf{f}_{1T}^{\perp}(x)\Big\} + \mathcal{O}(a_s^2,b^2)$$

$$\delta\mathbf{f}_{1T}^{\perp}(x) = \int_{-1}^{1}dy\int_0^1 d\alpha\delta(x-\alpha y)\Big[$$

$$\left(C_F - \frac{C_A}{2}\right)2\bar{\alpha}T_q(-y,0,y) + \frac{3\alpha\bar{\alpha}}{2}\frac{G_+(-y,0,y)+G_-(-y,0,y)}{y}\Big]$$

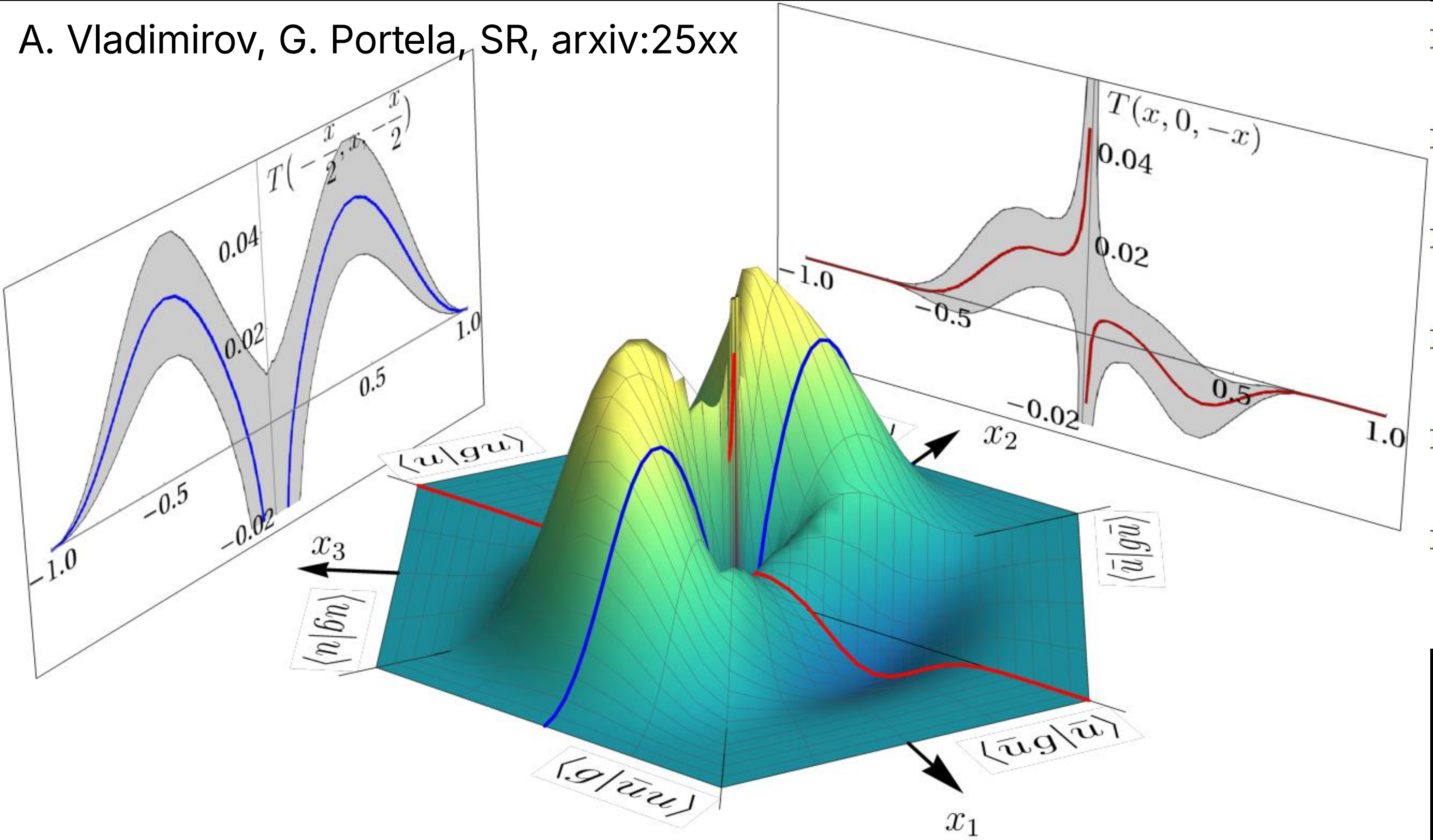
The Sivers/twist-3 case



The Sivers/twist-3 case

Twist-3 PDFs

A. Vladimirov, G. Portela, SR, arxiv:25xx



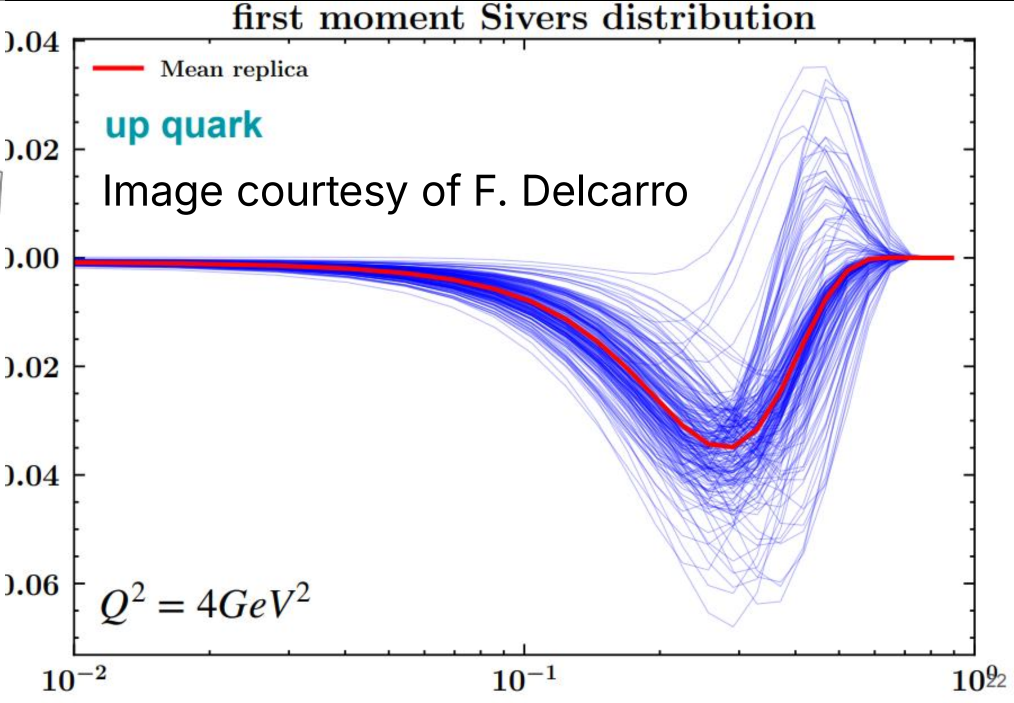


Image courtesy of F. Delcarro

Conclusions

Transverse-momentum distributions as the 3-dimensional momentum map of the proton

Lattice-QCD and experimental data synergies

Moving beyond the totally unpolarized sector