Semileptonic B decays at the junction of experiment and theory

Torino, June 12-13, 2025

Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

What we can learn from $B \to D^* \ell \nu_\ell$ decays without V_{cb}

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SAPIENZA UNIVERSITÀ DI ROMA

plan of the talk

- puzzles (anomalies ?) in $b \to c$ transitions
- the $B \to D^* \ell \nu_\ell$ channel: where are we with hadronic FFs from theory (LQCD)
- analysis of normalized angular distributions (independent of $|V_{cb}|$ and of theory)
- tensions (fluctuations ?) between experiments
- extraction of all SM FFs from light- and τ-lepton data ($\sim$ no theory)
- comparison with LQCD FFs
- extraction of $|V_{cb}|$ from the total decay rate (if there will be time …)
- conclusions

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What we can learn from the angular differential rates from semileptonic $B \to D^* \ell \nu_\ell$ decays

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THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

Semileptonic $B \to D^*$ decays from light to τ leptons: the extraction of the form factor F_2 from data

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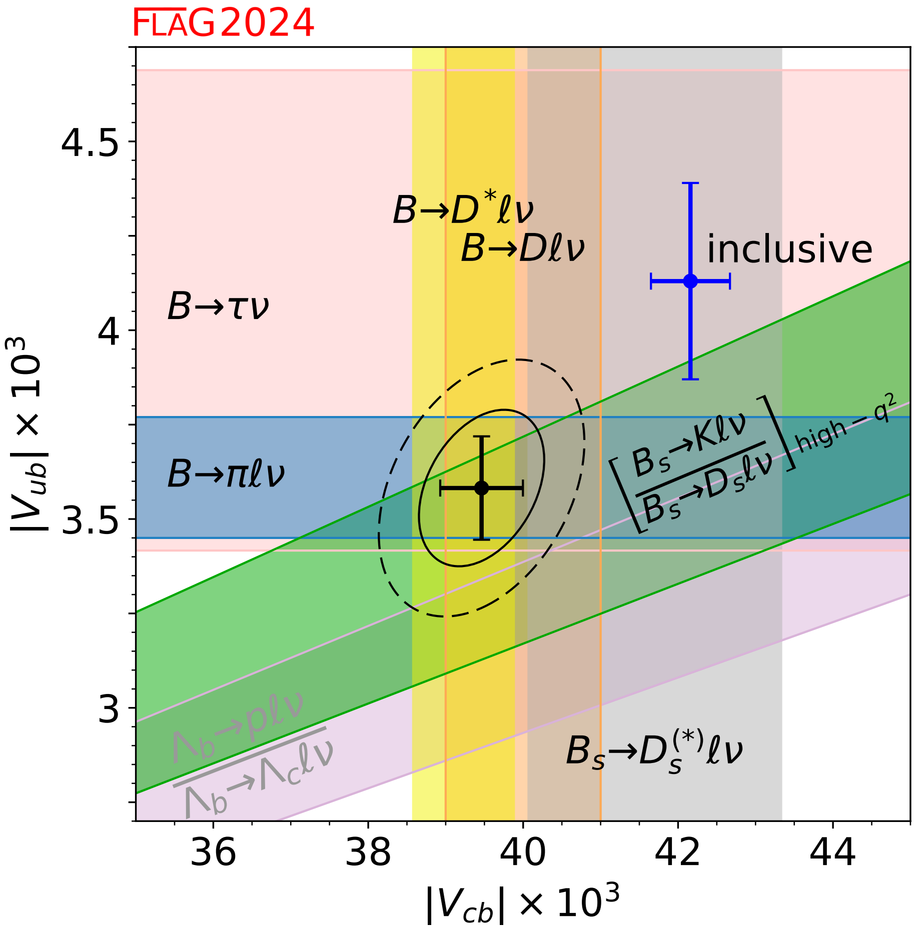
arXiv:2410.17974

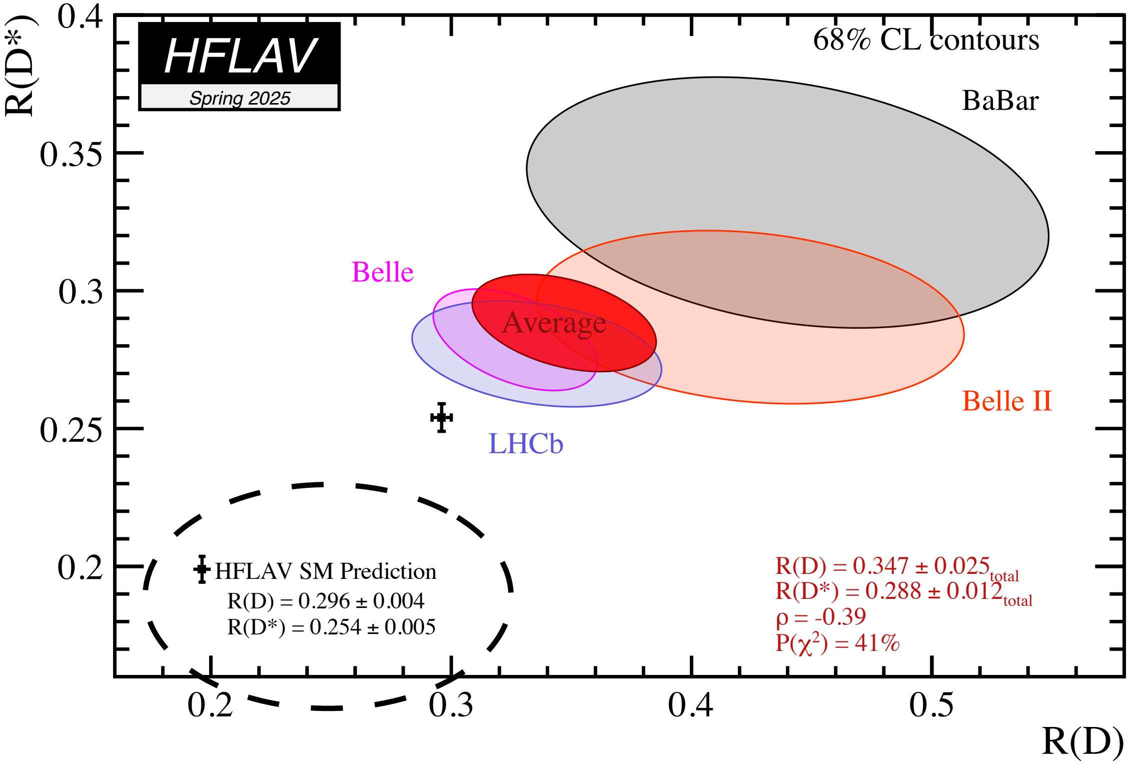
phenomenological challenges in $b \to c$ transitions

puzzles in CKM entries (inclusive/exclusive)



FLAG review 2024 (arXiv:2411.04268)

Lepton Flavour Universality (or violation ?)



HFLAV update as Spring 2025

https://hflav-eos.web.cern.ch/hflav-eos/semi/spring25/html/RDsDsstar/RDRDs.html

$$R(D^{(*)}) = \mathscr{B}(B \to D^{(*)}\tau\nu)\,/\,\mathscr{B}(B \to D^{(*)}\ell\nu)$$

differences $\approx 3-4\,\sigma$

let's look to $R(D^{(*)})$

$$R(D^{(*)}) = \mathcal{B}(B \to D^{(*)}\tau\nu)\,/\,\mathcal{B}(B \to D^{(*)}\ell\nu)$$

1. by definition $R(D^{(*)})$ is independent of $|V_{cb}|$ and of the absolute normalization of the hadronic FFs
2. it depends only on the q^2-***shape*** of the hadronic FFs

at the present level of accuracy, lattice QCD is the only quantitative approach for evaluating hadronic FFs, where systematic uncertainties can be controlled and reduced

SM predictions used by HFLAV (Spring 2025)

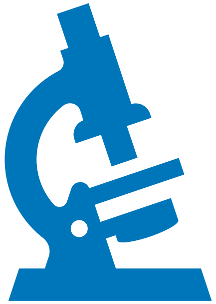
https://hflav-eos.web.cern.ch/hflav-eos/semi/spring25/html/RDsDsstar/RDRDs.html

	R(D)	**R(D*)**	**Compatibility in σ**
D.Bigi, P.Gambino, Phys.Rev. D94 (2016) no.9, 094008 [arXiv:1606.08030 [hep-ph]]	0.299 ± 0.003		1.89
M.Bordone, M.Jung, Danny van Dyk, Eur.Phy.J.C 80 (2020) 2, 74 [arXiv:1908.09398 [hep-ph]]	0.298 ± 0.003	0.247 ± 0.006	4.13
G. Martinelli, S. Simula, L. Vittorio, Phys.RevD 105 (2022) 3, 034503 [arXiv:2105.08674 [hep-ph]]	0.296 ± 0.008		1.93
F. U. Bernlochner, Z. Ligeti, M. Papucci, M. T. Prim, D. J. Robinson, C. Xiong, Phys. Rev. D 106 (2022) 096015 [arXiv:2206.11281 [hep-ph]]	0.288 ± 0.004	0.249 ± 0.003	4.65
I.Ray, S.Nandi, JHEP 01 (2024) 022 [arXiv:2305.11855 [hep-lat]]	0.304 ± 0.003	0.258 ± 0.012	2.44
FLAG Collaboration, [arXiv:2411.04268 [hep-lat]]	0.2938 ± 0.0054		2.06
BaBar Collaboration, Phys.Rev.Lett. 123 (2019) 9, 091801 [arXiv:1903.10002 [hep-ex]]		0.253 ± 0.005	2.79
P.Gambino, M.Jung, S.Schacht, Phys.Lett.B795 (2019) 386 [arXiv:1905.08209 [hep-ph]]		0.254 + 0.007 − 0.006	2.58
G. Martinelli, S. Simula, L. Vittorio, Eur. Phys. J. C 84 (2024) 400 [arXiv:2310.03680 [hep-ph]]		0.262 ± 0.009	1.79
Arithmetic average	**0.296 ± 0.004**	**0.254 ± 0.005**	**3.77**

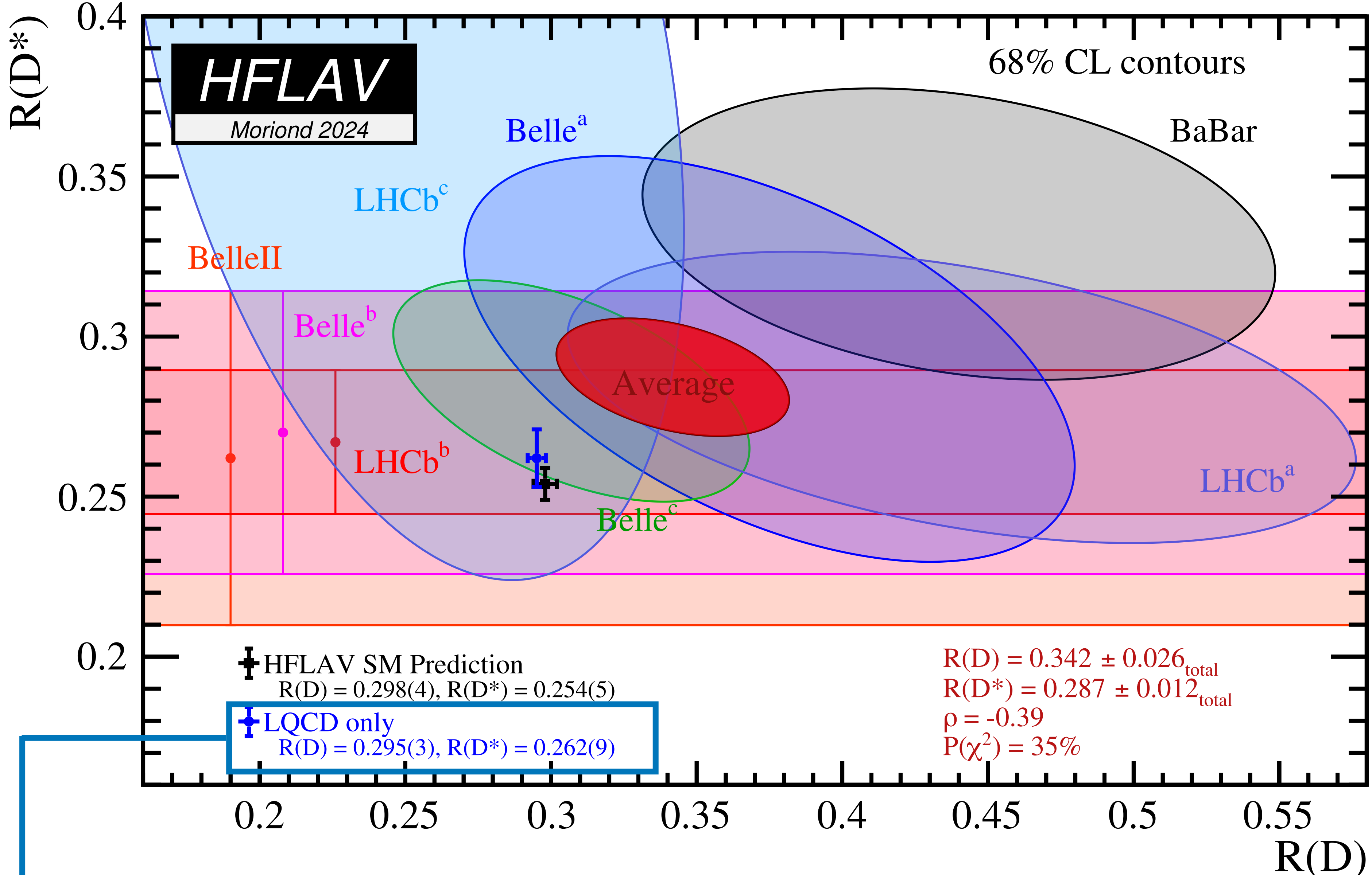
R(D) and R(D*) exceed the SM predictions (arithmetic average) given above, by 1.9σ and 2.7σ respectively. Considering the R(D)-R(D*)) correlation of -0.39, the resulting combined χ^2 is 16.92 for 2 degree of freedom, corresponding to a p-value of 2.12 x 10^{-4}. The difference with the SM predictions reported above, corresponds to about 3.8σ.

- arithmetic average ignores possible correlations among individual determinations (due to common inputs)

- which predictions for $R(D^*)$ are rigorously theoretical only (i.e., based on LQCD only) ?

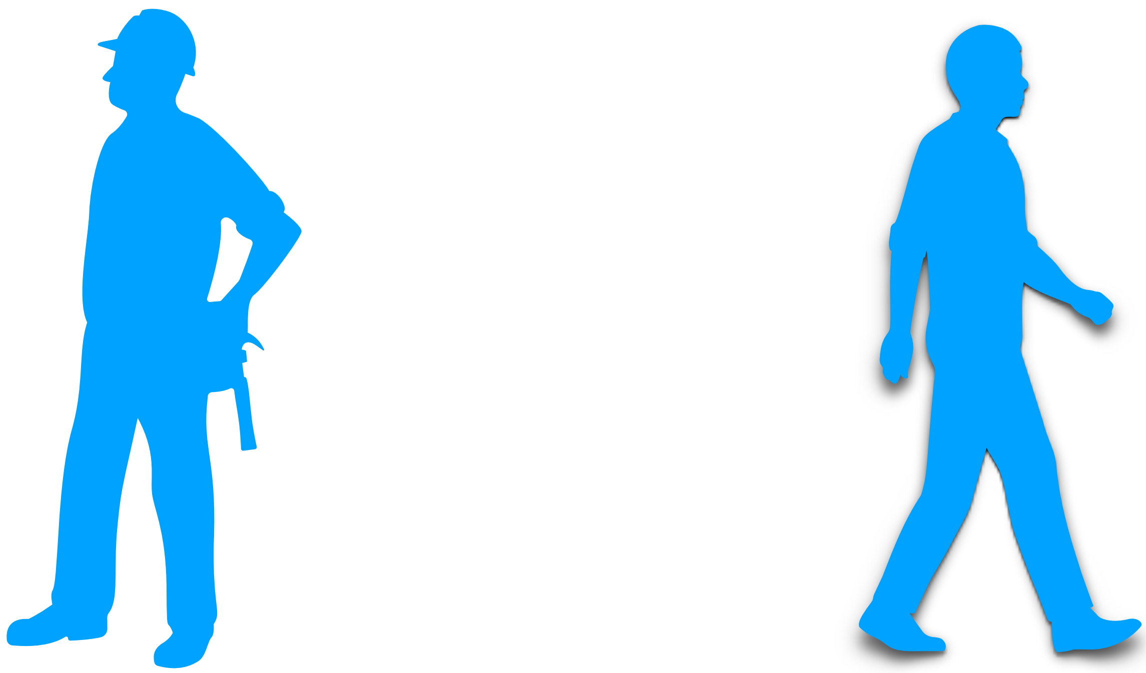


nice review from Klaver and Rotondo: Symmetry '24 (doi=10.3390/sym16080964)



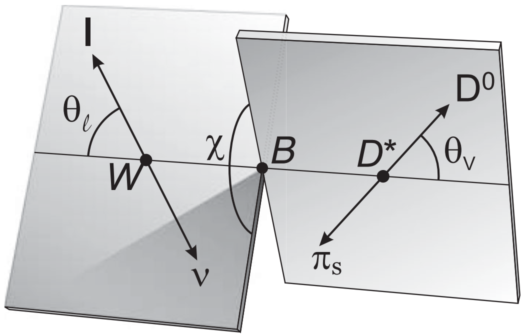
HFLAV SM prediction: fits of exp. data and LQCD results for the FFs

R(D) is stable, while R(D*) changes significantly (the error by a factor of ~ 2) are the experimental data and the theoretical shapes of the FFs consistent ?



the $B \to D^* \ell \nu_\ell$ channel

- four-fold differential decay rate in the SM and in massless lepton limit ($m_\ell = 0$)



$$\frac{d^4\Gamma}{dw\, dcos\theta_v\, dcos\theta_\ell\, d\chi} = \frac{3}{16\pi}\Gamma_0(w)|V_{cb}|^2 \Big\{ H_+^2(w)\ sin^2\theta_v\ (1 - cos\theta_\ell)^2$$
$$+ H_-^2(w)\ sin^2\theta_v\ (1 + cos\theta_\ell)^2 + 4\ H_0^2(w)\ cos^2\theta_v\ sin^2\theta_\ell$$
$$-2\ H_-(w)H_+(w)\ sin^2\theta_v\ sin^2\theta_\ell\ cos2\chi$$
$$-2\ H_+(w)H_0(w)\ sin2\theta_v\ sin\theta_\ell\ (1 - cos\theta_\ell)\ cos\chi$$
$$+2\ H_-(w)H_0(w)\ sin2\theta_v\ sin\theta_\ell\ (1 + cos\theta_\ell)\ cos\chi \Big\}$$

$$w \equiv \frac{1 + r^2 - q^2/m_B^2}{2r}$$

$$\Gamma_0(w) \equiv \frac{\eta_{EW}^2 m_B m_{D^*}^2}{(4\pi)^3} G_F^2 \sqrt{w^2 - 1}(1 - 2rw + r^2)$$

$$r \equiv \frac{m_{D^*}}{m_B}$$

- helicity FFs $H_{\pm,0,t}(w)$ in terms of spin-parity ones $g(w), f(w), F_1(w), F_2(w)$ $(1^-,1^+,1^+,0^+)$

$$H_\pm(w) = f(w) \mp m_B^2 r\sqrt{w^2 - 1}\, g(w)$$

$$H_0(w) = F_1(w)/(m_B\sqrt{1 - 2rw + r^2})$$

$$H_t(w) = F_2(w)\, m_B r\sqrt{w^2 - 1}/\sqrt{1 - 2rw + r^2}$$

this FF appears only for $m_\ell \neq 0$

- kinematical constraints

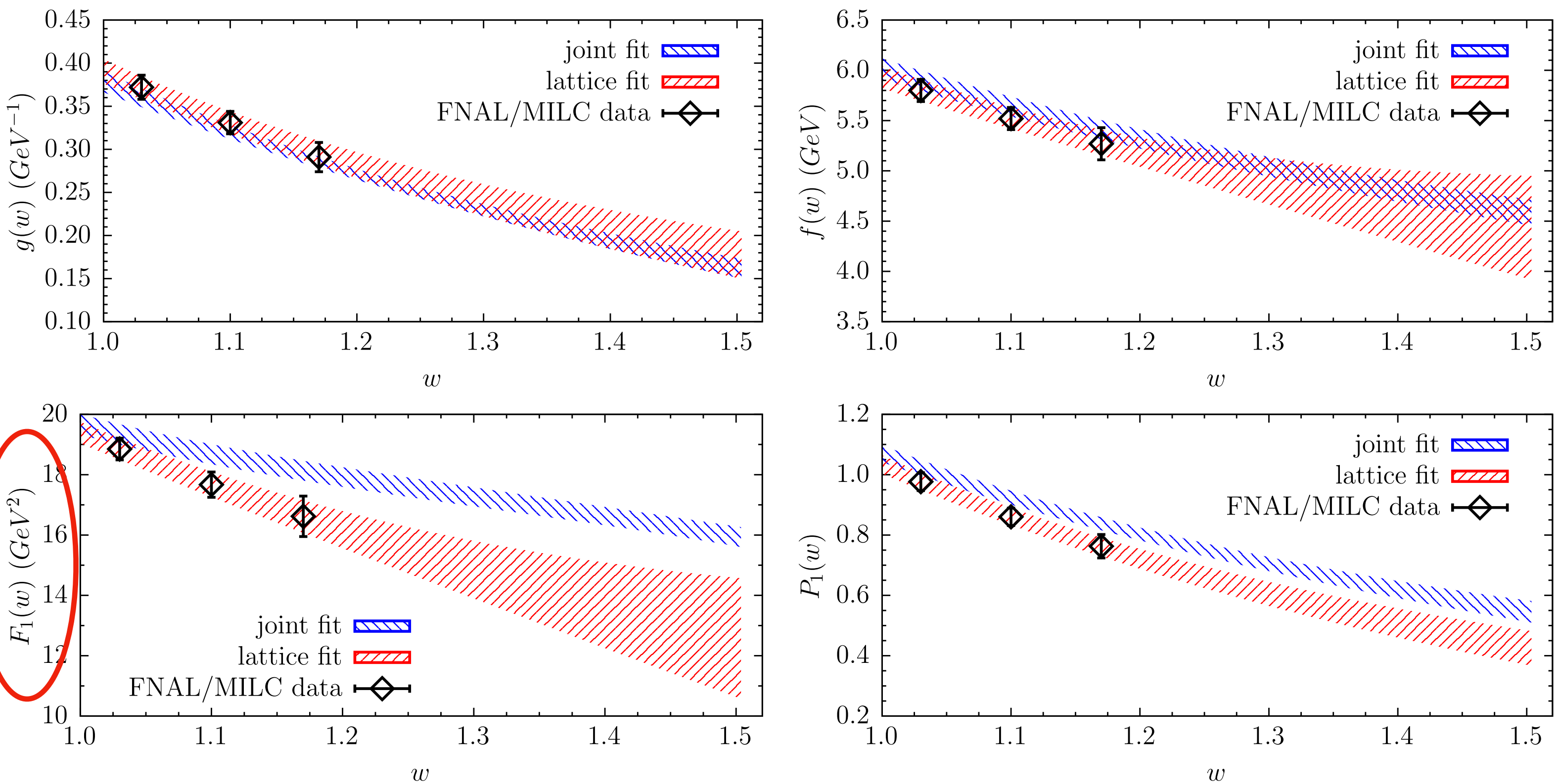
$$F_1(w = 1) = m_B(1 - r) f(w = 1)$$

$$F_2(w = w_{max}) = 2F_1(w = w_{max})/[m_B^2(1 - r^2)]$$

- first lattice determination of the spin-parity FFs at w > 1 by FNAL/MILC Coll. (arXiv:2105.14019)
- experimental measurements of four single-differential rates $\dfrac{d\Gamma}{dx}$ $\quad x = \{w, cos\theta_\ell, cos\theta_v, \chi\}$
  available at that time from Belle Coll. (arXiv:1809.03290)

why not doing a global fit of both lattice and exp. data ?

that was done by FNAL/MILC Coll. in arXiv:2105.14019 adopting BGL z-expansions for the FFs



joint fit
Belle (+ Babar) exp. data + FNAL/MILC data

$R(D^*) = 0.2483 \pm 0.0013$

lattice fit
FNAL/MILC data only

$R(D^*) = 0.265 \pm 0.013$

S.S. @ Barolo '22

different shapes of $F_1(w)$ between lattice results and experimental measurements

experimental data cover the full kinematical range in w, while LQCD results are (typically) restricted to low recoils (this allows a better control over discretization errors) different extrapolation methods of the LQCD FFs are consistent ?

BGL z-expansion

a widely used extrapolation method is based on the Boyd-Grinstein-Lebed (BGL) expansion of the FFs in terms of the conformal variable z

hep-ph/9412324, hep-ph/9504235, hep-ph/9508211, hep-ph/9705252

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \qquad t \equiv q^2 \qquad t_+ = (m_B + m_{D^*})^2$$

$$\boxed{f_\alpha(z) = \frac{\sqrt{\chi_\alpha}}{\phi_\alpha(z)\, B_\alpha(z)} \sum_{k=0}^{\infty} b_k^{(\alpha)} z^k} \qquad \alpha = \{g, f, F_1, F_2\}$$

outer functions

Blaschke products

the functions $\phi_\alpha B_\alpha f_\alpha$ are analytic inside the unit disk $|z| < 1$

$$\oint_{|z|=1} \frac{dz}{z} |\phi_\alpha(z) f_\alpha(z)|^2 \leq \chi_\alpha = \text{unitarity bounds} \quad \Longrightarrow \quad \sum_{k=0}^{\infty} [b_k^{(\alpha)}]^2 \leq 1$$

a suitable choice of ϕ_α allows to relate χ_α to derivatives of vacuum polarization functions, which can be estimated using pQCD

Meinan ’63, Okubo et al. ’71,
Bourrely et al. ’81,
see also Bharucha et al. arXiv:1004.3249

unitarity constraints for $B \to D^* \ell \nu_\ell$ decays involve the $1^-, 1^+, 0^+$ channels

$$\sum_{k=0}^{\infty} [b_k^{(g)}]^2 \leq 1 \qquad \sum_{k=0}^{\infty} [b_k^{(f)}]^2 + [b_k^{(F_1)}]^2 \leq 1 \qquad \sum_{k=0}^{\infty} [b_k^{(F_2)}]^2 \leq 1$$

recently the unitarity bounds $\chi_{1^-,1^+,0^+,0^-}$ have been calculated on the lattice

Martinelli, SS, Vittorio: arXiv:2105.07851 ($b \to c$ transitions)
arXiv:2105.02497 ($c \to s$ transitions)
arXiv:2202.10285 ($b \to u$ and $c \to d$ transitions)
SS, Vittorio: arXiv:2309.02135 (pion FF)

$b \to c$ transitions
Melis, Sanfilippo, SS: arXiv:2401.03920
Harrison: arXiv:2405.01390

two main variants of BGL z-expansions applied to a set of N_{data} known values of the FFs

- frequentist truncated fits: involve a finite number of parameters $N_{parms} < N_{data}$ (truncation error)
  for $B \to D^*$ Gambino et al. '19, Bordone et al. '19, Bernlocher et al. '22, Nandi et al. '23, …

- Bayesian inference fits: include higher-order terms constrained by unitarity (as a prior)
  RBC/UKQCD arXiv:2303.11280, Flynn, Jüttner, Tsang arXiv:2303.11285, Bordone, Jüttner arXiv:2406.19974

Dispersion Matrix (DM) approach

Di Carlo et al.
arXiv:2105.02497

reappraisal and improvement of the method originally proposed by Bourrely et al. '81 and Lellouch '96

$$\mathcal{M} = \begin{pmatrix} <\phi f | \phi f> & <\phi f | g_t> & <\phi f | g_{t_1}> & \cdots & <\phi f | g_{t_N}> \\ <g_t | \phi f> & <g_t | g_t> & <g_t | g_{t_1}> & \cdots & <g_t | g_{t_N}> \\ <g_{t_1} | \phi f> & <g_{t_1} | g_t> & <g_{t_1} | g_{t_1}> & \cdots & <g_{t_1} | g_{t_N}> \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ <g_{t_N} | \phi f> & <g_{t_N} | g_t> & <g_{t_N} | g_{t_1}> & \cdots & <g_{t_N} | g_{t_N}> \end{pmatrix}$$

inner product: $<g | h> \equiv \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \, g^*(z) \, h(z)$

$$g_t(z) \equiv \frac{1}{1 - z^*(t) \, z}$$

$$<g_t | \phi f> \equiv \phi(z) \, f(z) \qquad <g_t | g_{t_m}> = \frac{1}{1 - z^*(t_m) \, z(t)}$$

$t_1, t_2, \ldots, t_N$ are N values of the squared 4-momentum transfer where the form factor f(t) is known and t is its value where we want to compute f(t)

$$\text{unitarity bound:} \quad <\phi f | \phi f> \equiv \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \, |\phi(z) \, f(z)|^2 \leq \chi$$

in the case of interest $z_i \equiv z(t_i)$ are real numbers and the positivity of the inner product implies:

$$\det[\overline{\mathcal{M}}] = \begin{vmatrix} \chi & \phi f & \phi_1 f_1 & \cdots & \phi_N f_N \\ \phi f & \frac{1}{1 - z^2} & \frac{1}{1 - z z_1} & \cdots & \frac{1}{1 - z z_N} \\ \phi_1 f_1 & \frac{1}{1 - z_1 z} & \frac{1}{1 - z_1^2} & \cdots & \frac{1}{1 - z_1 z_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \phi_N f_N & \frac{1}{1 - z_N z} & \frac{1}{1 - z_N z_1} & \cdots & \frac{1}{1 - z_N^2} \end{vmatrix} \geq 0$$

* the **explicit solution** is a band of values: $\boxed{\beta(z) - \sqrt{\gamma(z)} \leq f(z) \leq \beta(z) + \sqrt{\gamma(z)}}$ arXiv:2105.02497

$$\beta(z) = \frac{1}{d(z)\,\phi(z)} \sum_{j=1}^{N} f_j\, \phi_j d_j \frac{1 - z_j^2}{z - z_j} \qquad \gamma(z) = \frac{1}{d^2(z)\,\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j\, \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

$d(z) = \Pi_{m=1}^{N}(1 - z z_m)/(z - z_m)$ $\qquad d_i = \Pi_{m \neq i}(1 - z_i z_m)/(z_i - z_m)$

* **no free parameters, no explicit z-expansion (exact and automatic truncation independence)**

* important feature: when $z \to z_j$ one has $\beta(z_j) \to f_j$ and $\gamma(z_j) \to 0$, i.e. the DM band collapses to f_j for $z = z_j$

* unitarity is satisfied if $\gamma(z) \geq 0$, which occurs when $\chi \geq \chi_{\{f\}}^{DM} \equiv \sum_{i,j=1}^{N} f_i f_j\, \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$

generate a sample of events $\{f_j\}$ → unitarity filter $\chi <> \chi_{\{f\}}^{DM}$ → subset of unitary events $\{f_j\}$ with $\chi \geq \chi_{\{f\}}^{DM}$ / subset of non-unitary events $\{f_j\}$ with $\chi < \chi_{\{f\}}^{DM}$

for any given set of input data the DM approach reproduces exactly the known data and it allows to extrapolate the form factor in the whole kinematical range in a parameterization-independent way providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points

* the DM band represents a uniform distribution which is combined with the multivariate distribution of the input data $\{f_j\}$ to generate the final band for the FF $f(z)$

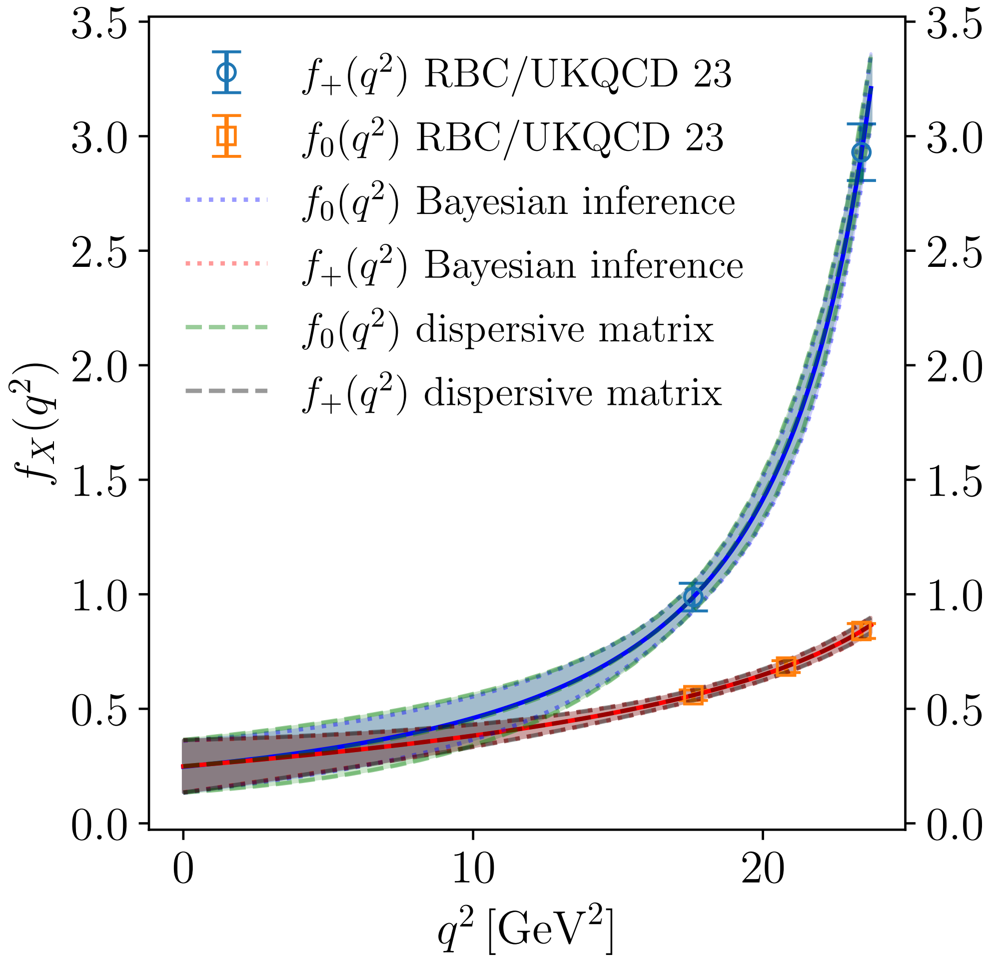
* kinematical constraint(s) can be easily and rigorously implemented in the DM approach [2105.02497, 2105.08674, 2109.15248, 2202.10285, 2204.05925, 2309.02135, 2310.03680]

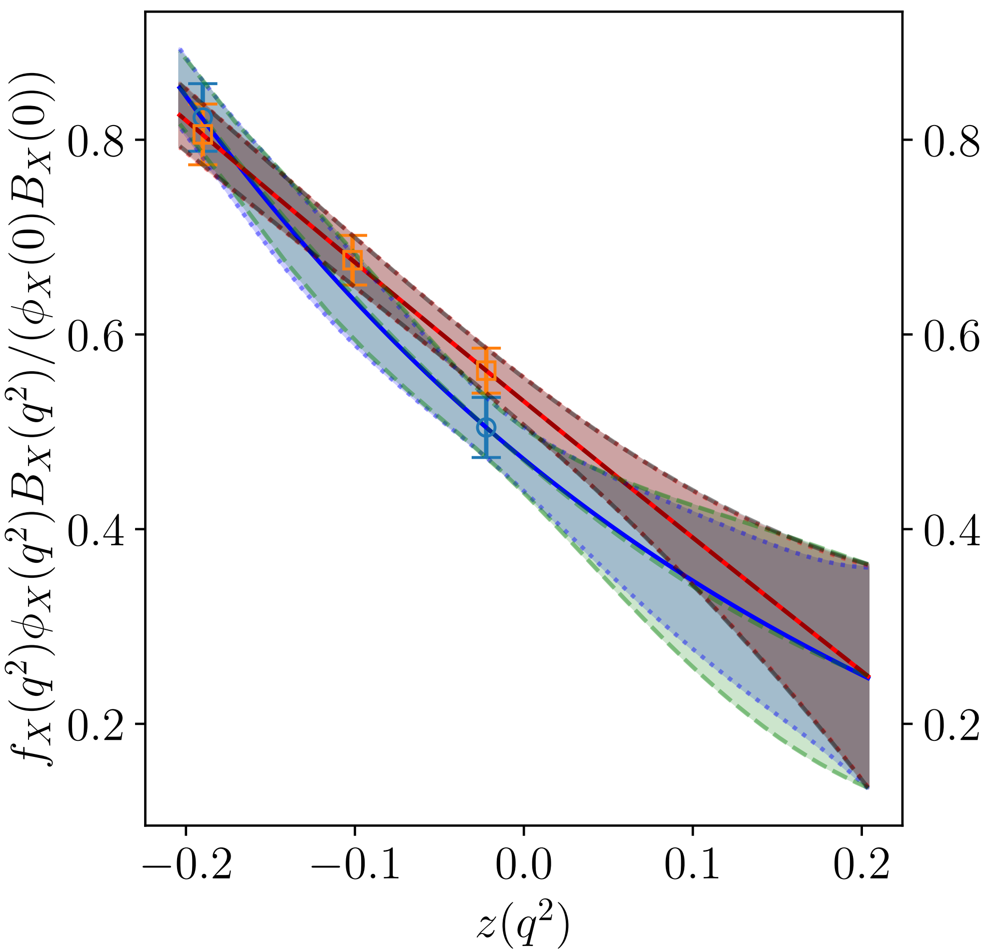
Bayesian inference BGL fits versus DM

the $B_s \to K$ channel

Flynn, Jüttner, Tsang arXiv:2303.11285

LQCD data: RBC/UKQCD arXiv:2303.11280





*** extrapolations to low q^2 consistent ***

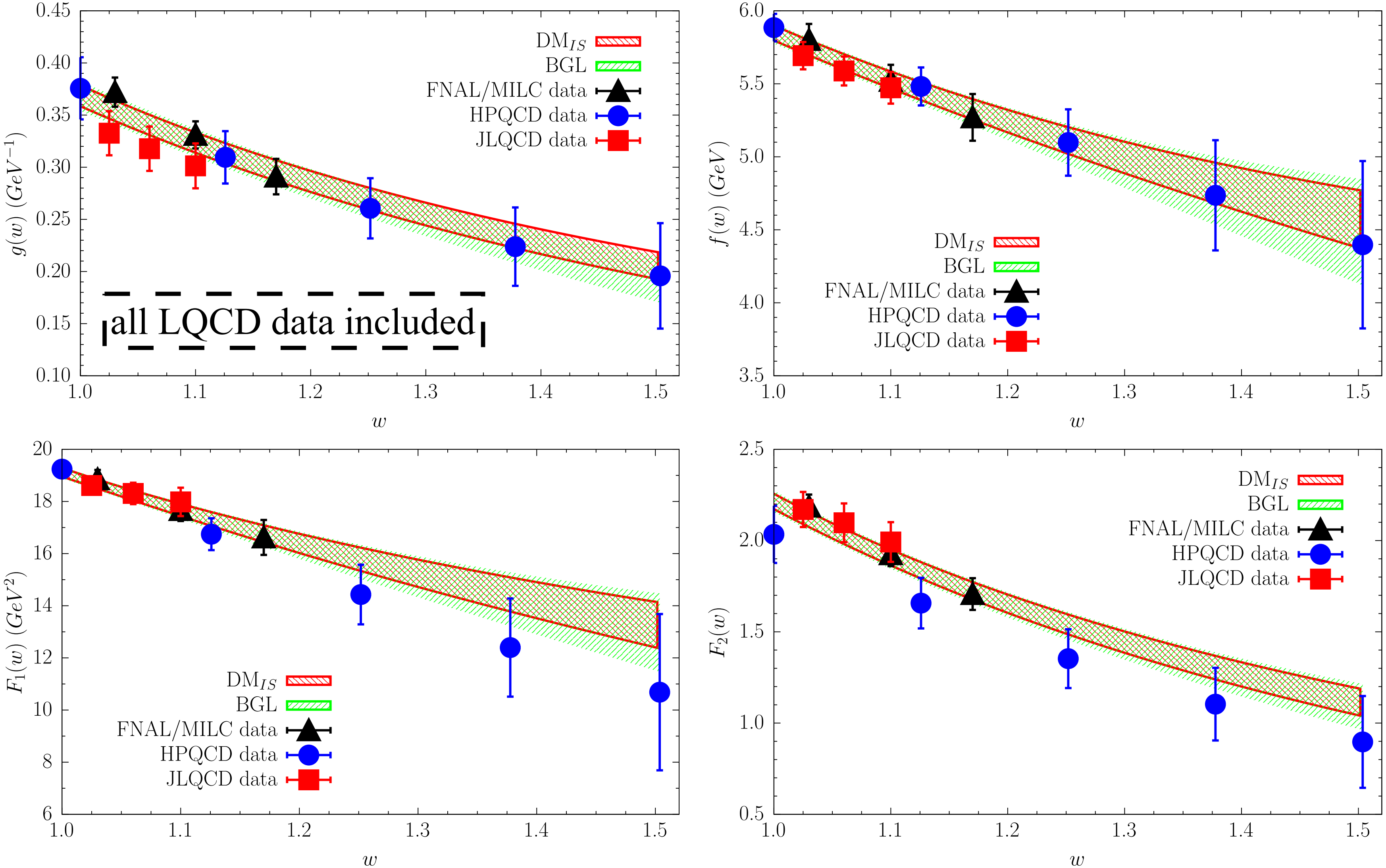
expected since
the RBC/UKQCD data
largely satisfy the DM filter

frequentist BGL fits versus DM

the $B \to D^*$ channel

Martinelli, SS, Vittorio arXiv:2310.03680

LQCD data: FNAL/MILC arXiv:2105.14019
HPQCD arXiv:2304.03137 (v2)
JLQCD arXiv:2306.05657



DM is more precise at large recoil (low q^2)

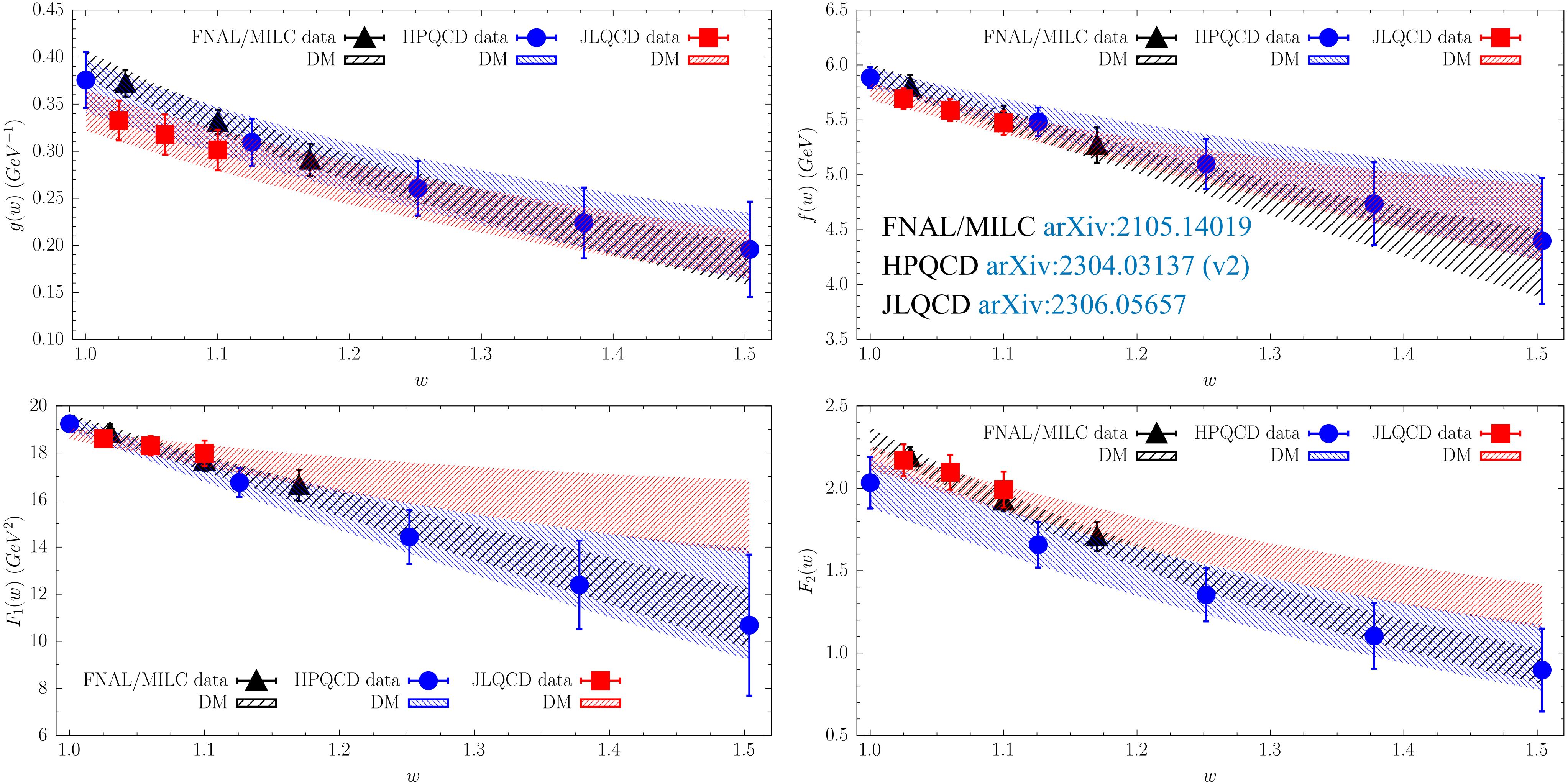
expected since
the LQCD data partially
satisfy the DM filter

theoretical questions in the $B \to D^*$ channel

- extrapolation methods in the full kinematical range: **not an issue**
- use of unitarity and kinematical constraints: **not an issue**
- perturbative renormalization of the weak current: **not an issue** at the current level of precision
- D^*-meson as a stable particle: **not an issue** at the current level of precision (see FLAG '24)
- differences in LQCD calculations by various Collaborations using different versions of the QCD action on the lattice: presently **the most important issue** $\Rightarrow$

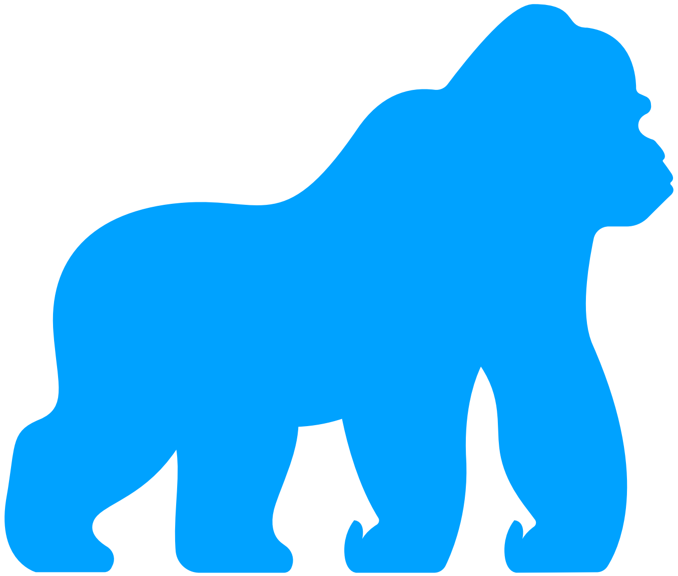
hadronic FFs in the $B \to D^*$ channel

Martinelli, SS, Vittorio arXiv:2310.03680



- the results for all the FFs but $F_2(w)$ are **consistent at low recoil**, where the computations have been done ($w \lesssim 1.2$)
- although $F_2(w)$ is consistent at low w between FNAL/MILC and JLQCD the extrapolated values are different
- at large w the allowed band for $F_1(w)$ from JLQCD is very different from the bands obtained using FNAL/MILC or HPQCD
- differences between $F_2(w)$ from HPQCD and the other two Collaborations at low recoil

what about experiments ?



experimental measurements of the $B \to D^* \ell \nu_\ell$ decays for $\ell = e, \mu$

* three datasets for the single differential decay rates $\dfrac{1}{\Gamma}\dfrac{d\Gamma}{dx}$ $\quad x = \{w, cos\theta_\ell, cos\theta_v, \chi\}$

Belle18: PRD 100 (2019) 052007 [1809.03290], erratum: PRD 103 (2021) 079901

Belle23: PRD 108 (2023) 012002 [2301.07529]

BelleII23: PRD 108 (2023) 092013 [2310.01170]

* a recent dataset for the (twelve) angular coefficients of the four-fold differential decay rate

$$\frac{1}{\Gamma}\frac{d^4\Gamma}{dw\, dcos\theta_v\, dcos\theta_\ell\, d\chi} = \frac{1}{\mathcal{N}}\Big\{ J_{1s}(w)\ sin^2\theta_v + J_{1c}(w)\ cos^2\theta_v + J_{2s}(w)\ sin^2\theta_v cos2\theta_\ell + J_{2c}(w)\ cos^2\theta_v cos2\theta_\ell$$
$$+ J_3(w)\ sin^2\theta_v sin^2\theta_\ell cos2\chi + J_4(w)\ sin2\theta_v sin2\theta_\ell cos\chi + J_5(w)\ sin2\theta_v sin\theta_\ell cos\chi$$
$$+ J_{6s}(w)\ sin^2\theta_v cos\theta_\ell + J_{6c}(w)\ cos^2\theta_v cos\theta_\ell + J_7(w)\ sin2\theta_v sin\theta_\ell sin\chi$$
$$+ J_8(w)\ sin2\theta_v sin2\theta_\ell sin\chi + J_9(w)\ sin^2\theta_v sin^2\theta_\ell sin2\chi \Big\}$$

valid both within the SM and Beyond the SM

$$\mathcal{N} \equiv \frac{8\pi}{9}\left[6\bar{J}_{1s} + 3\bar{J}_{1c} - 2\bar{J}_{2s} - \bar{J}_{2c}\right] \qquad \bar{J}_i \equiv \int_1^{w_{max}} dw\, J_i(w)$$

integrated angular coefficients

Belle23(Ji): PRL 133 (202) 131801 [2310.20286]

share the same dataset for $d^4\Gamma$ with Belle23

analysis of the normalized angular distributions

following the strategy proposed in Bobeth et al. ’21 (arXiv:2104.02094)

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_v} = \frac{3}{4(1+\eta)}\left\{\eta + (2-\eta)\cos^2\theta_v\right\} ,$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{8(1+\eta')}\left\{2+\eta' - 2\delta\cos\theta_\ell - (2-\eta')\cos^2\theta_\ell\right\} ,$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\chi} = \frac{1}{2\pi}\left\{1 - \frac{\epsilon}{1+\eta}\cos 2\chi - \frac{\epsilon'}{1+\eta}\sin 2\chi\right\} .$$

five basic parameters

$\{\eta, \eta', \delta, \epsilon, \epsilon'\}$

independent of $|V_{cb}|$ and of theory

within the SM

$\epsilon' = 0$ and $\eta' = \eta + \mathcal{O}(m_\ell^2)$

relation with helicity amplitudes with the SM

$$\eta = \frac{H_{++} + H_{--} + \frac{m_\ell^2}{2m_B^2}\left(\widetilde{H}_{++} + \widetilde{H}_{--}\right)}{H_{00} + \frac{m_\ell^2}{2m_B^2}\left(\widetilde{H}_{00} + 3\widetilde{H}_{tt}\right)} ,$$

$$\eta' = \frac{H_{++} + H_{--} + \frac{m_\ell^2}{m_B^2}\left(\widetilde{H}_{00} + \widetilde{H}_{tt}\right)}{H_{00} + \frac{m_\ell^2}{2m_B^2}\left(\widetilde{H}_{++} + \widetilde{H}_{--} - \widetilde{H}_{00} + \widetilde{H}_{tt}\right)} ,$$

$$\delta = \frac{H_{++} - H_{--} + \frac{2m_\ell^2}{m_B^2}\widetilde{H}_{0t}}{H_{00} + \frac{m_\ell^2}{2m_B^2}\left(\widetilde{H}_{++} + \widetilde{H}_{--} - \widetilde{H}_{00} + \widetilde{H}_{tt}\right)} ,$$

$$\epsilon = \frac{H_{+-} - \frac{m_\ell^2}{m_B^2}\widetilde{H}_{+-}}{H_{00} + \frac{m_\ell^2}{2m_B^2}\left(\widetilde{H}_{00} + 3\widetilde{H}_{tt}\right)} ,$$

$$H_{ij} \equiv \int_1^{w_{max}^\ell} dw\sqrt{w^2-1}(1-2rw+r^2)\left(1-\frac{m_\ell^2}{q^2}\right)^2 H_i(w)\, H_j(w) ,$$

$$\widetilde{H}_{ij} \equiv \int_1^{w_{max}^\ell} dw\sqrt{w^2-1}\left(1-\frac{m_\ell^2}{q^2}\right)^2 H_i(w)\, H_j(w) .$$

measurable observables

$$A_{FB} = -\frac{3}{4}\frac{\delta}{1+\eta'} , \qquad F_L = \frac{1}{1+\eta}$$

$$A_{1c} = -\frac{\epsilon}{1+\eta} , \qquad A_{9c} = -\frac{\epsilon'}{1+\eta}$$

model-independent analysis of the angular distributions

Martinelli, SS, Vittorio arXiv:2409.10492

	η	η'	δ	ϵ	ϵ'
Belle18	0.894 (29)	0.846 (47)	−0.534 (37)	0.346 (28)	0.004 (28)
Belle23	1.026 (59)	0.943 (81)	−0.595 (41)	0.333 (61)	0.046 (59)
BelleII23	0.912 (28)	0.908 (47)	−0.507 (28)	0.342 (22)	0.005 (19)
Belle18 + Belle23 + BelleII23	0.922 (18)	0.875 (29)	−0.540 (18)	0.337 (16)	0.005 (16)
LQCD	1.109 (66)	1.121 (66)	−0.705 (48)	0.415 (26)	0.0

	A_{FB}	F_L	A_{1c}	A_{9c}
Belle18	0.217 (13)	0.528 (8)	−0.183 (15)	−0.002 (15)
Belle23	0.230 (14)	0.494 (14)	−0.165 (30)	−0.023 (29)
BelleII23	0.200 (12)	0.523 (8)	−0.179 (13)	−0.003 (10)
Belle18 + Belle23 + BelleII23	0.216 (7)	0.520 (5)	−0.176 (9)	−0.003 (8)
LQCD	0.249 (10)	0.475 (15)	−0.196 (7)	0.0

LQCD predictions calculated with the hadronic FFs obtained using all the available LQCD determinations

- for all datasets the results for ϵ' (and A_{9c}) are consistent with zero as expected in the SM
- the **LQCD** predictions are consistent with the results from the **Belle23** dataset, while they show some tension with Belle18 and BelleII23, as well as with the results obtained using all the three datasets (~ dominated by BelleII23)

model-independent analysis of the angular distributions

Martinelli, SS, Vittorio arXiv:2409.10492

$$\eta \equiv 2\frac{3\bar{J}_{1s}-\bar{J}_{2s}}{3\bar{J}_{1c}-\bar{J}_{2c}}, \qquad \delta = -2\frac{2\bar{J}_{6s}+\bar{J}_{6c}}{2\bar{J}_{1s}+\bar{J}_{1c}-3(2\bar{J}_{2s}+\bar{J}_{2c})}, \qquad \epsilon' \equiv -4\frac{\bar{J}_9}{3\bar{J}_{1c}-\bar{J}_{2c}},$$

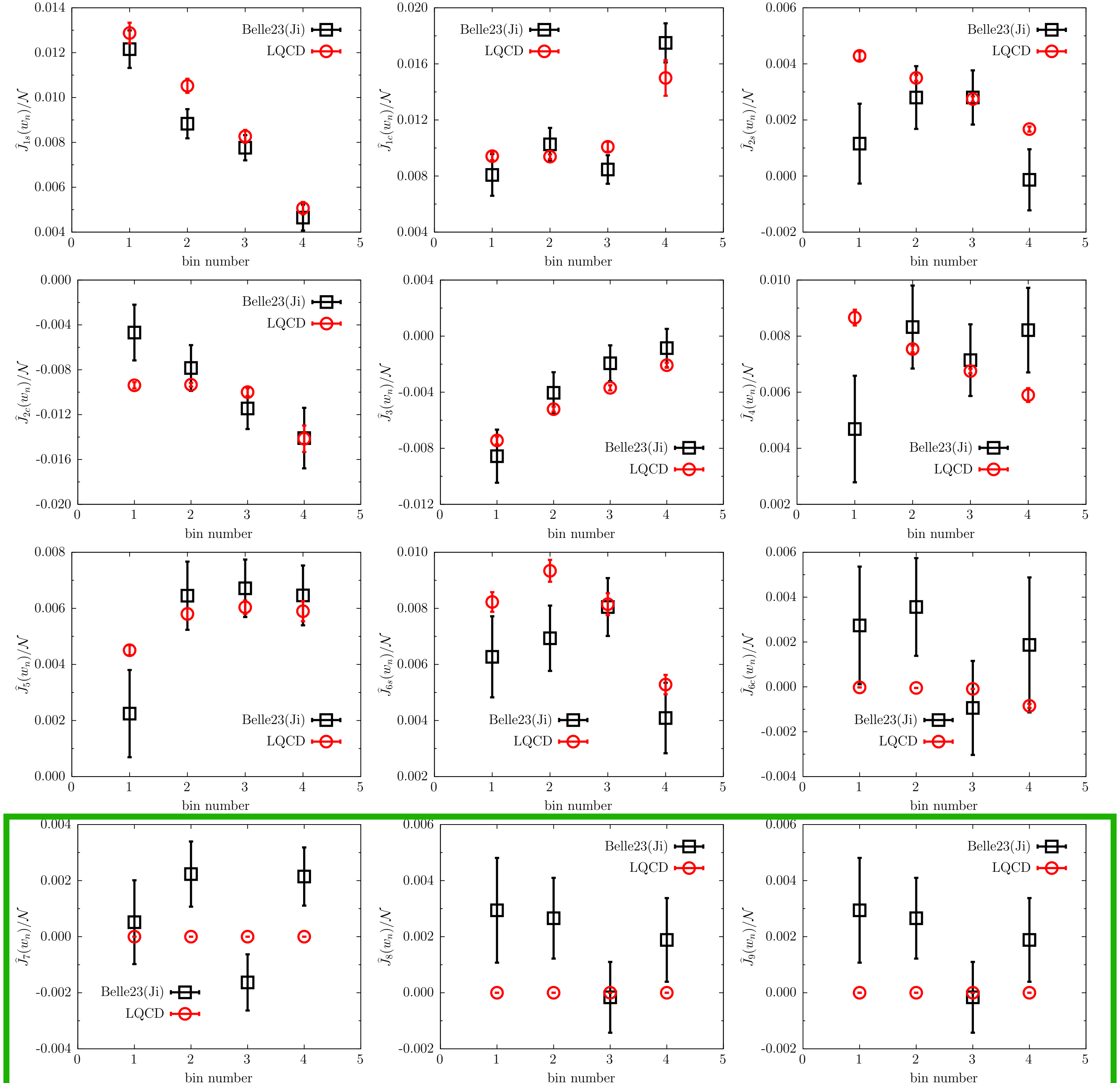
$$\eta' \equiv 2\frac{2\bar{J}_{1s}+\bar{J}_{1c}+2\bar{J}_{2s}+\bar{J}_{2c}}{2\bar{J}_{1s}+\bar{J}_{1c}-3(2\bar{J}_{2s}+\bar{J}_{2c})}, \qquad \epsilon \equiv -4\frac{\bar{J}_3}{3\bar{J}_{1c}-\bar{J}_{2c}},$$

	η	η'	δ	ϵ	ϵ'
Belle18	0.894 (29)	0.846 (47)	−0.534 (37)	0.346 (28)	0.004 (28)
Belle23	1.026 (59)	0.943 (81)	−0.595 (41)	0.333 (61)	0.046 (59)
BelleII23	0.912 (28)	0.908 (47)	−0.507 (28)	0.342 (22)	0.005 (19)
Belle18 + Belle23 + BelleII23	0.922 (18)	0.875 (29)	−0.540 (18)	0.337 (16)	0.005 (16)
Belle23(Ji)	1.097 (73)	0.934 (86)	−0.626 (49)	0.361 (69)	−0.054 (67)
LQCD	1.109 (66)	1.121 (66)	−0.705 (48)	0.415 (26)	0.0

	A_{FB}	F_L	A_{1c}	A_{9c}
Belle18	0.217 (13)	0.528 (8)	−0.183 (15)	−0.002 (15)
Belle23	0.230 (14)	0.494 (14)	−0.165 (30)	−0.023 (29)
BelleII23	0.200 (12)	0.523 (8)	−0.179 (13)	−0.003 (10)
Belle18 + Belle23 + BelleII23	0.216 (7)	0.520 (5)	−0.176 (9)	−0.003 (8)
Belle23(Ji)	0.243 (14)	0.477 (17)	−0.172 (32)	0.003 (32)
LQCD	0.249 (10)	0.475 (15)	−0.196 (7)	0.0

the LQCD predictions are consistent with the results from the **Belle23(Ji)** dataset

(normalized) angular coefficients J_i from **Belle23** and **LQCD**



$$\hat{J}_i(w_n) \equiv \int_{w_{n-1}}^{w_n} dw\, J_i(w)$$

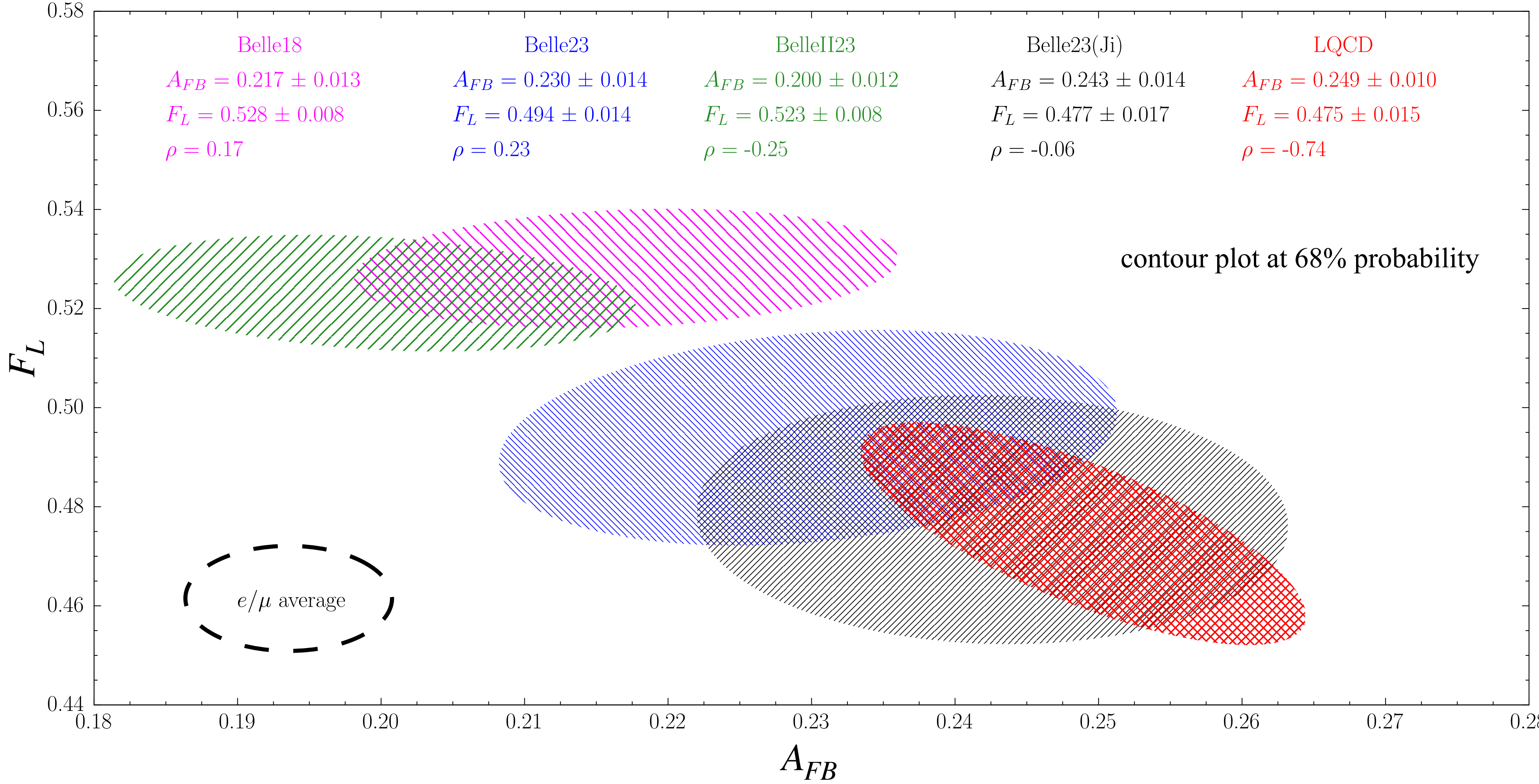
$$\{w_n\} = \{1.0, 1.15, 1.25, w_{max}\}$$

differences do not exceed the 2σ level

exactly zero in the SM

contour plots for light-lepton asymmetries

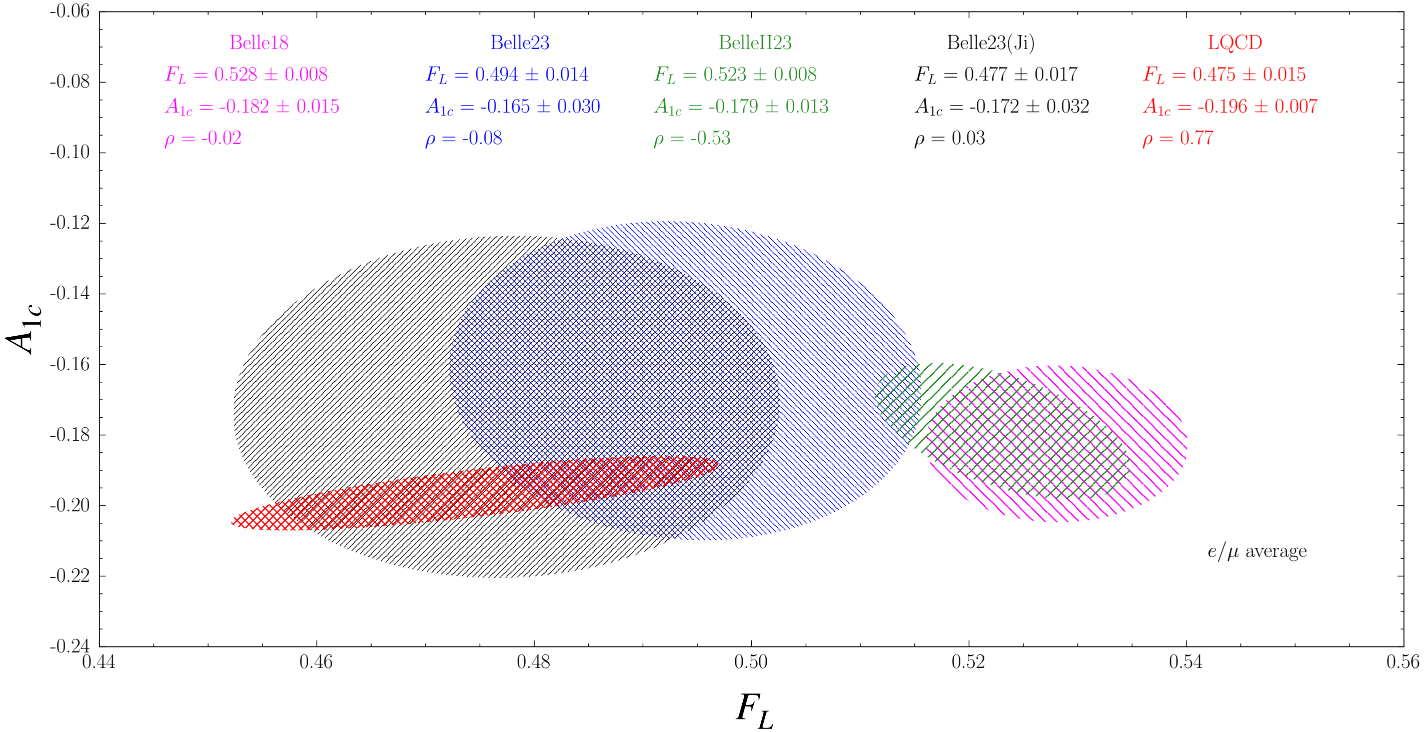
Martinelli, SS, Vittorio arXiv:2409.10492

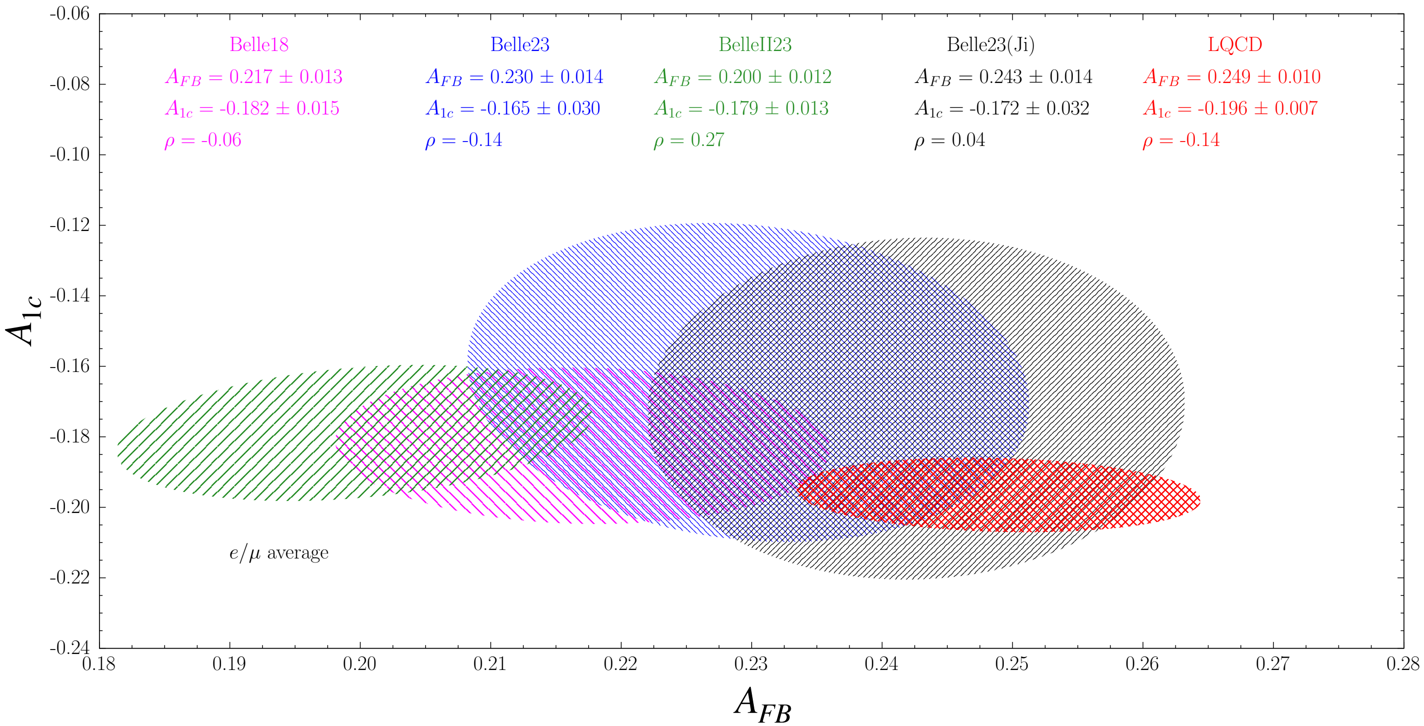


- relevant differences among the results corresponding to different datasets (cfr. Belle18, BelleII23 with Belle23)
- the SM LQCD predictions are consistent with the results from the Belle23 (or Belle23(Ji)) dataset, while the largest deviations occur with BelleII23

if Athens cries, Sparta does not laugh (G. Martinelli, talk @CERN ’24)

contour plots for light-lepton asymmetries (cont'd)





can we extract all the SM FFs from data with almost no theory inputs ?

YES

extraction of the SM FFs from experiments

Martinelli, SS, Vittorio arXiv:2410.17974

let's add to the analysis of the angular differential decay rates also the one in the recoil variable w

$$\frac{1}{\Gamma}\frac{d\Gamma}{dw} = \sqrt{w^2-1}(1-2rw+r^2)\,\frac{H_+^2(w)+H_-^2(w)+H_0^2(w)}{\overline{H}} \qquad (m_\ell = 0)$$

up to a common factor, e.g. $f(1)$, the ***reduced*** hadronic FFs $\{\widetilde{g}(w), \tilde{f}(w), \widetilde{F}_1(w), \widetilde{F}_2(w)\}$ can be determined through a (truncated) BGL fit applied to the **light-lepton datasets** (Belle18, Belle23, BelleII23) on $\frac{1}{\Gamma}\frac{d\Gamma}{dx}$ plus the experimental values of **few τ-observables**

all observables are independent of $|V_{cb}|$

$$R(D^*) = 0.286 \pm 0.012 \qquad (HFLAV\ 2411.18639)$$

$$F_{L,\tau} = 0.48 \pm 0.09 \qquad (LHCb\ 2311.05224 + Belle\ 1903.03102)$$

$$(F_{L,\tau})_{q^2<7\ GeV^2} = 0.52 \pm 0.08 \qquad (LHCb\ 2311.05224)$$

$$(F_{L,\tau})_{q^2>7\ GeV^2} = 0.34 \pm 0.08 \qquad (LHCb\ 2311.05224)$$

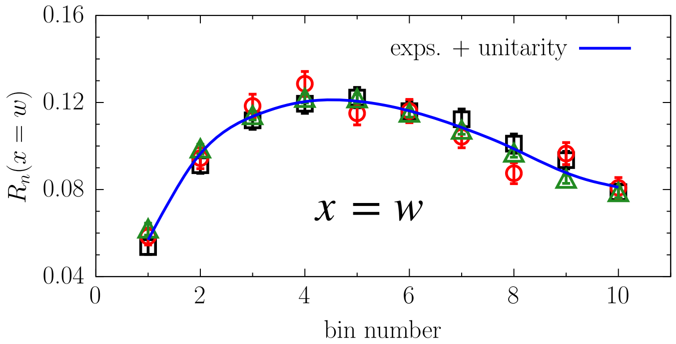
$\rho = -0.18$

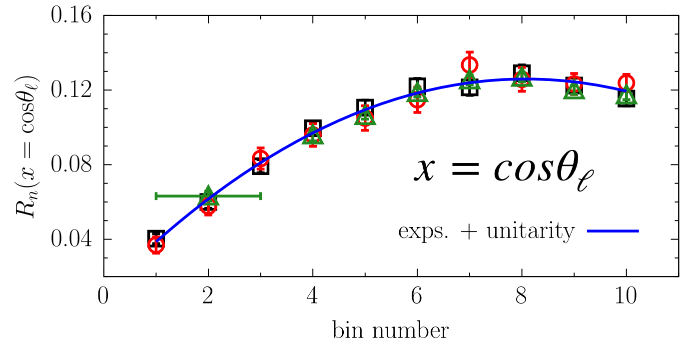
$$P_\tau(D^*) = -0.38 \pm 0.51^{+0.21}_{-0.16} \qquad (Belle\ 1612.00529)$$

$$\widetilde{g}(w) \equiv \frac{g(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^-}}}{\phi_g(z)B_g(z)}\sum_{i=0}^{N_g} a_i^g z^i \qquad \widetilde{F}_1(w) \equiv \frac{F_1(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^+}}}{\phi_{F_1}(z)B_{F_1}(z)}\sum_{i=0}^{N_{F_1}} a_i^{F_1} z^i$$

$$\tilde{f}(w) \equiv \frac{f(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^+}}}{\phi_f(z)B_f(z)}\sum_{i=0}^{N_f} a_i^f z^i \qquad \widetilde{F}_2(w) \equiv \frac{F_2(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{0^+}}}{\phi_{F_2}(z)B_{F_2}(z)}\sum_{i=0}^{N_{F_2}} a_i^{F_2} z^i$$

<u>to guarantee unitarity</u>: use of the three *reduced* susceptibilities $\widetilde{\chi}_{1^-,1^+,0^+} \equiv \frac{\chi_{1^-,1^+,0^+}}{f^2(1)}$ (**the only inputs from theory**)

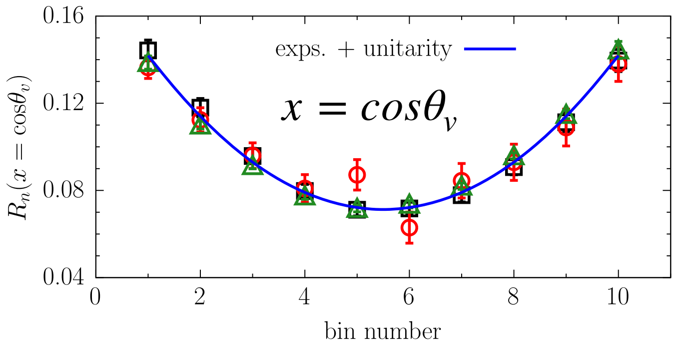


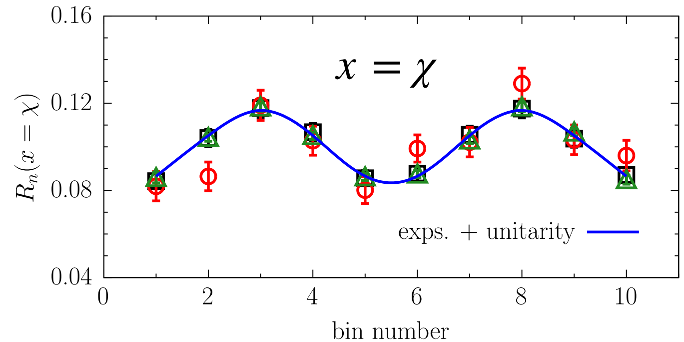


nice reproduction of

light-lepton data on

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx}$$

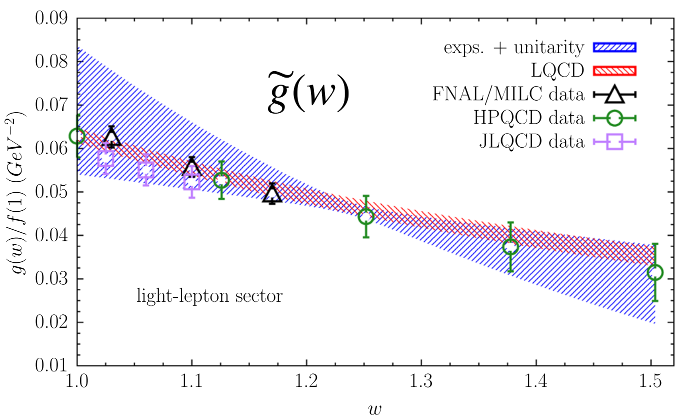


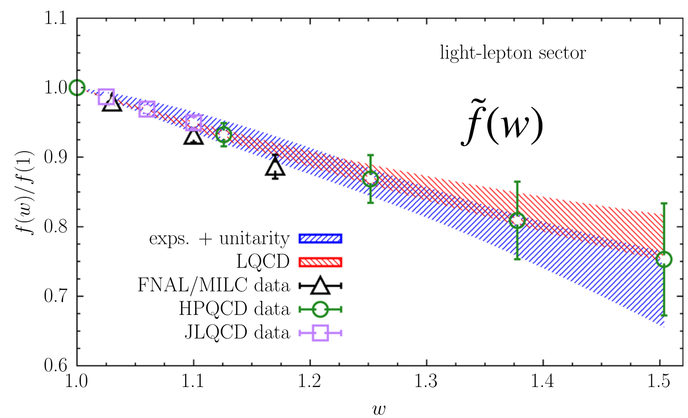


Belle18: black squares

Belle23 red circles

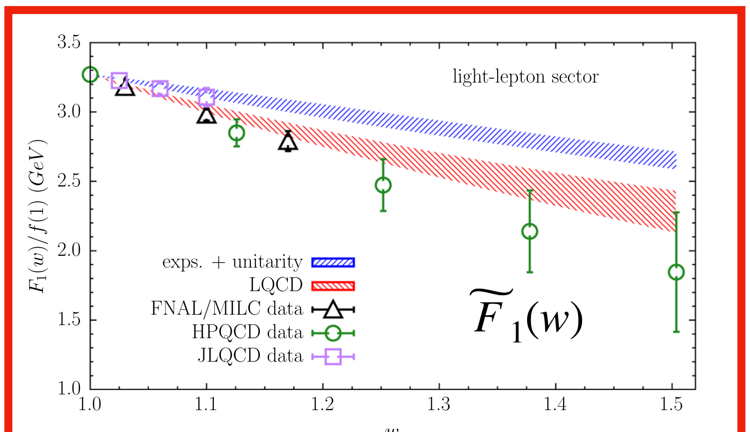
BelleII23: green triangles





exps. + unitarity reduced FFs

LQCD reduced FFs

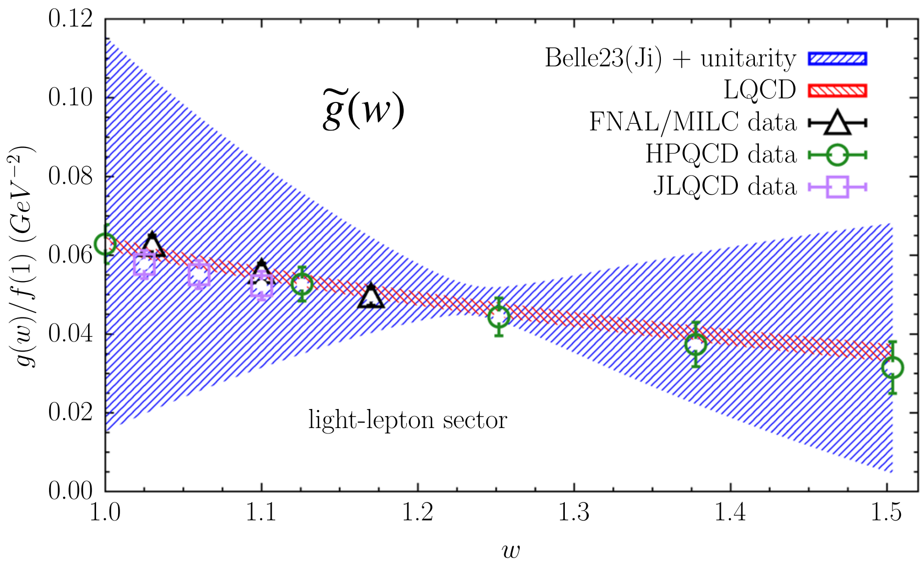


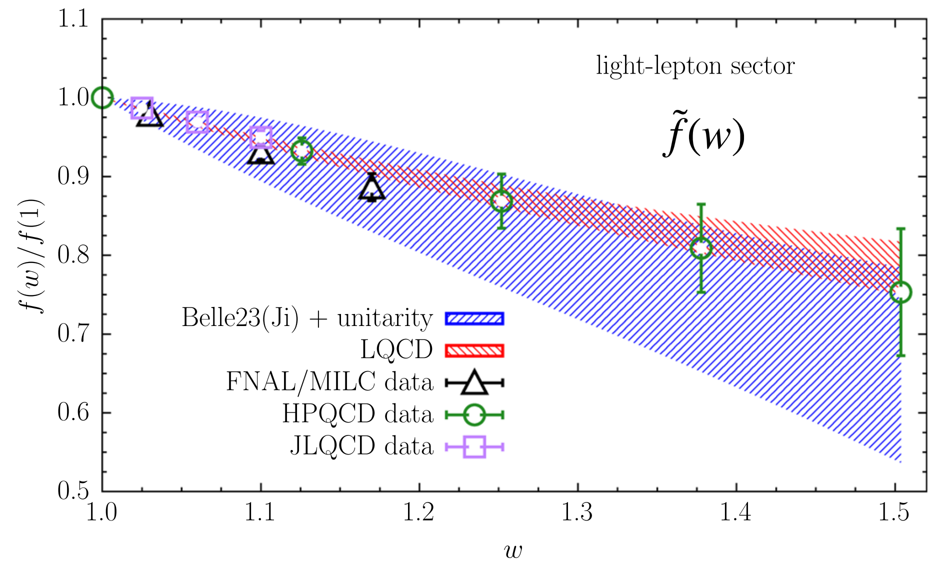
well-known *issue of the slope* of $\widetilde{F}_1(w)$
exp. slope consistent with the JLQCD one,
not consistent with FNAL/MILC and HPQCD

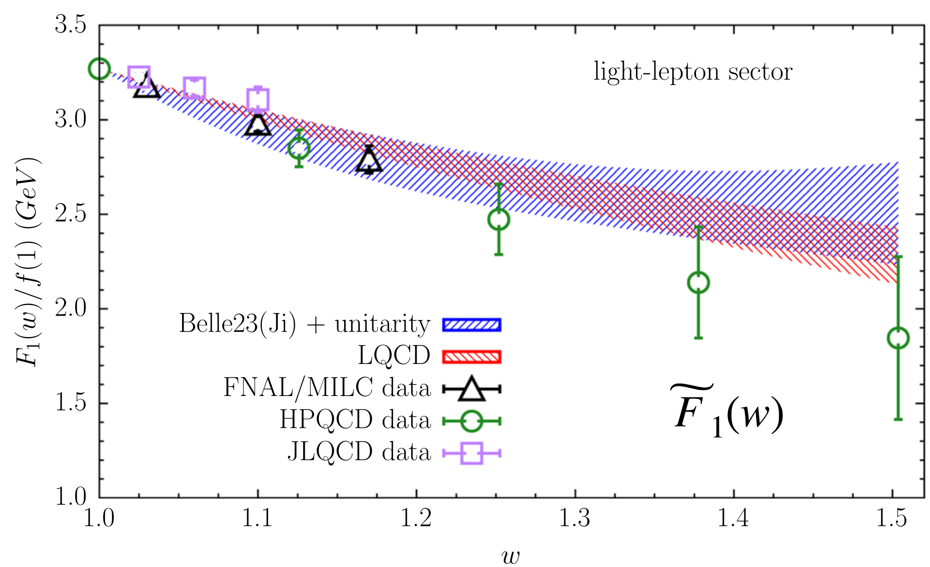
similar conclusion also from the LQCD + exp. fit
from Bordone and Jüttner arXiv: 2406.10074

however, it turns out that BelleII23 is the most precise dataset and dominates when combined with the Belle18 and Belle23 datasets

using as input only the angular coefficients from Belle23(Ji)







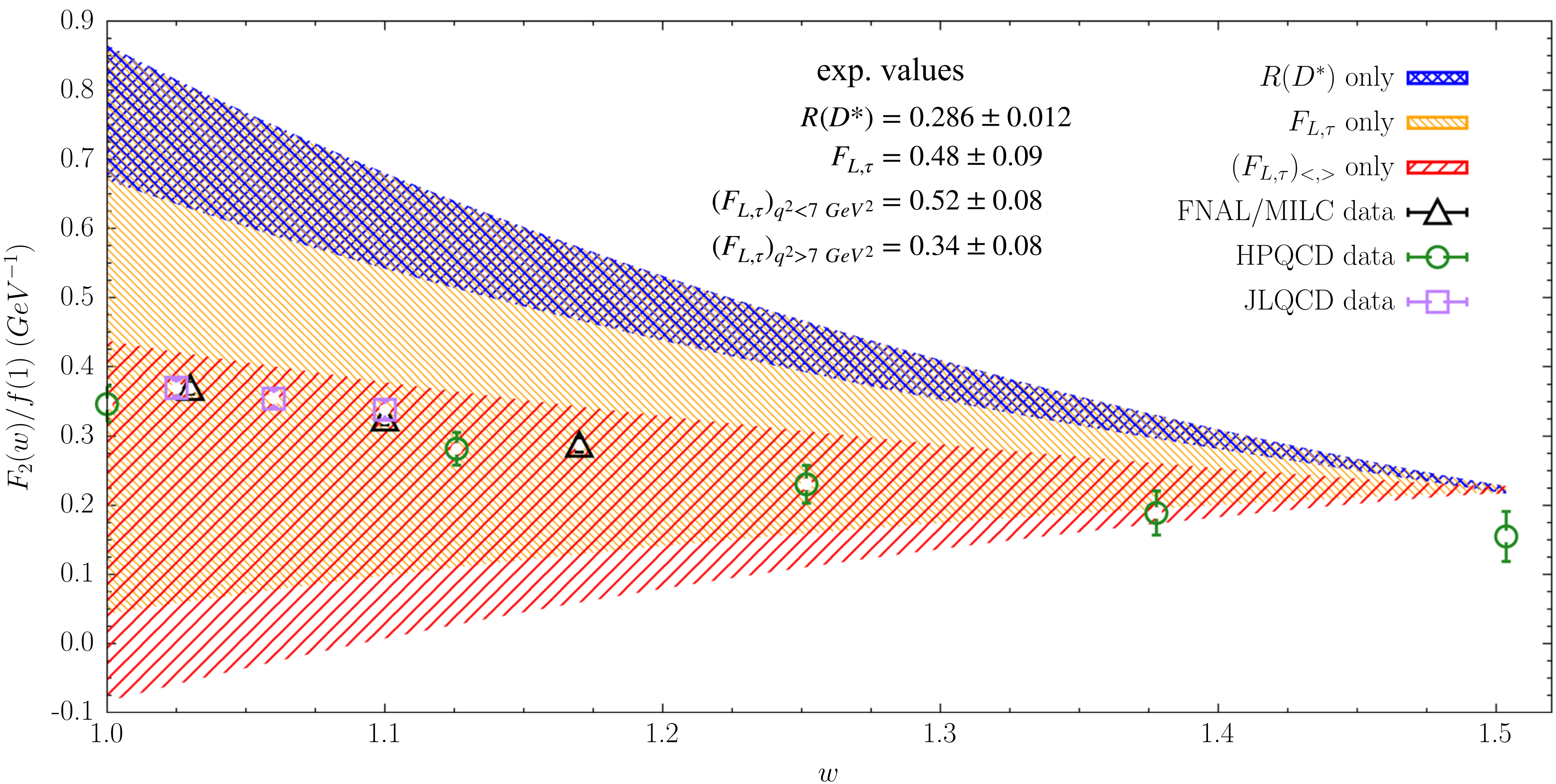
Belle23(Ji) slope consistent with FNAL/MILC and HPQCD, and marginally consistent with JLQCD for $\widetilde{F}_1(w)$

similar conclusion observed already for A_{FB}, F_L, A_{1c}, . . .

what about the pseudoscalar FF $F_2(w)$?

extraction of $\widetilde{F}_2(w)$ from data

Martinelli, SS, Vittorio arXiv:2410.17974

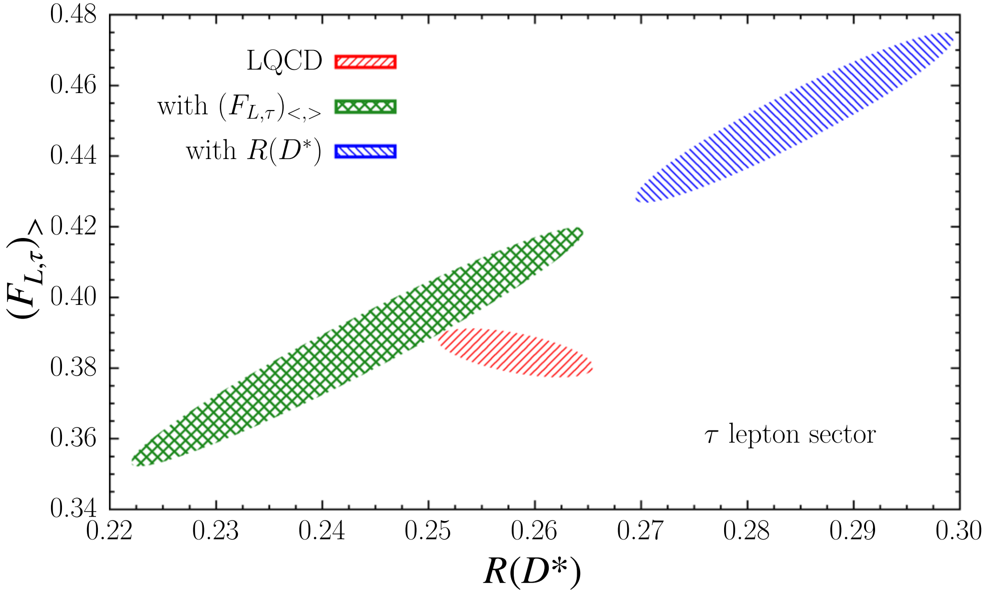


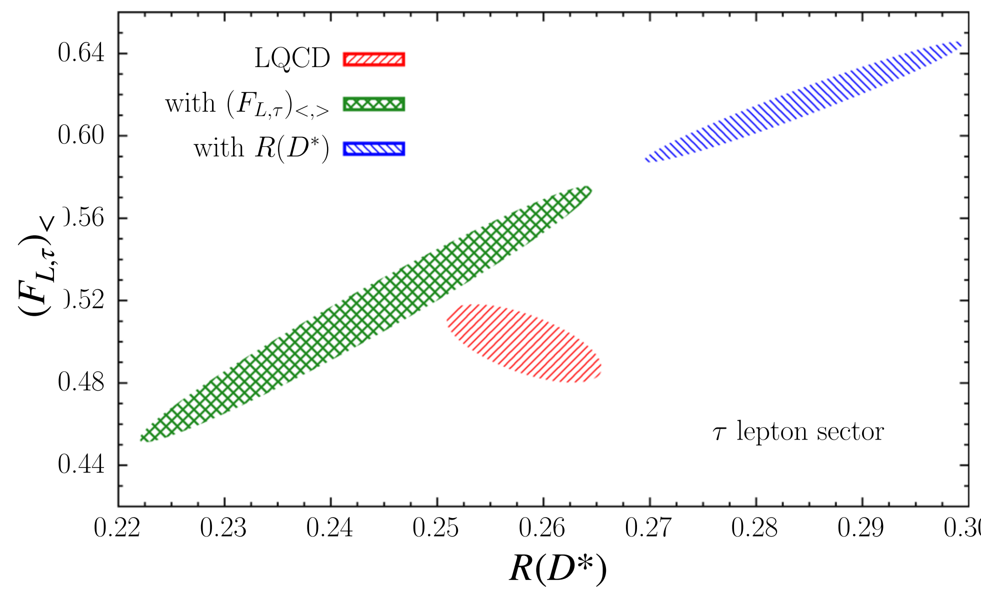
- $R(D^*)$ is the most precisely determined observable in the τ-sector
- when $R(D^*)$ is included, the ***exps.+unitarity*** reduced FF $\widetilde{F}_2(w)$ deviates from LQCD results by a factor of ~ 2

similar results using simultaneously both $R(D^*)$ and $F_{L,\tau}|_{<,>}$

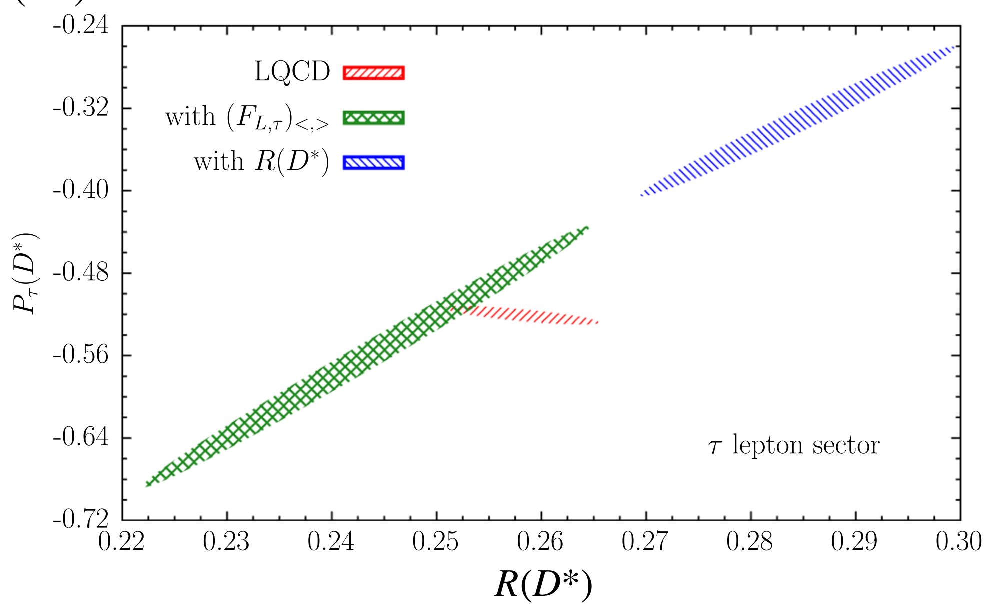
contour plots for τ-lepton observables

Martinelli, SS, Vittorio arXiv:2410.17974





contour plot at 68% probability



exp. values

$$R(D^*) = 0.286 \pm 0.012$$
$$(F_{L,\tau})_{q^2<7\ GeV^2} = 0.52 \pm 0.08$$
$$(F_{L,\tau})_{q^2>7\ GeV^2} = 0.34 \pm 0.08$$
$$P_\tau(D^*) = -0.38 \pm 0.51^{+0.21}_{-0.16}$$

- some tension between $R(D^*)$ and $(F_{L,\tau})_{<,>}$
- we found a strong correlation between $R(D^*)$ and $A_{FB,\tau}$ (not yet measured)

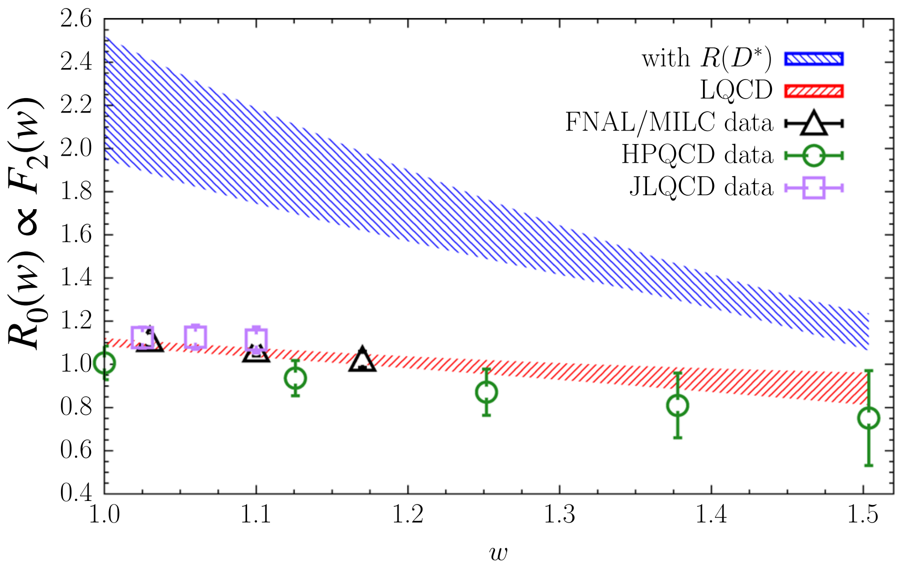
HQET-inspired FFs

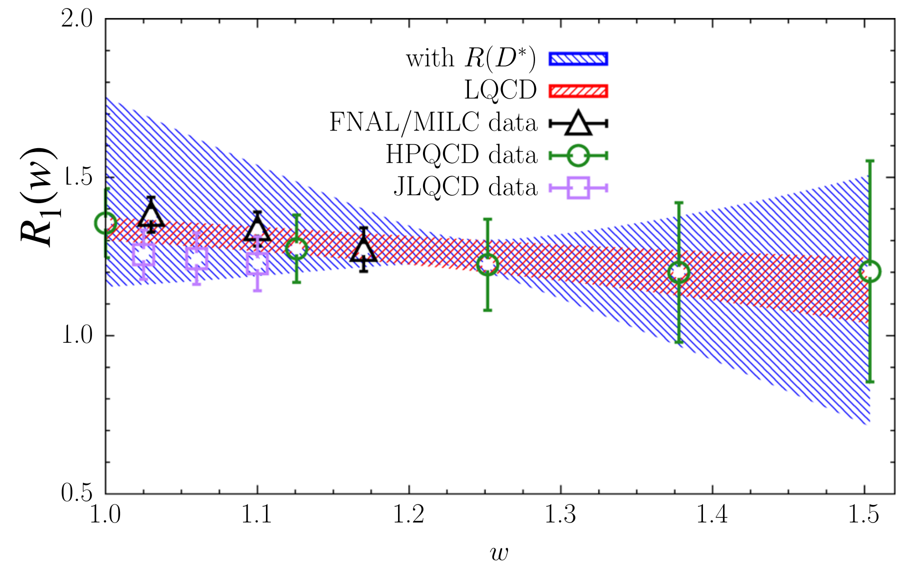
$$h_V(w) \propto g(w)$$
$$h_{A_1}(w) \propto f(w)$$
$h_{A_2}(w)$ and $h_{A_3}(w)$ depend on $f(w), F_1(w), F_2(w)$

ratio of HQET FFs

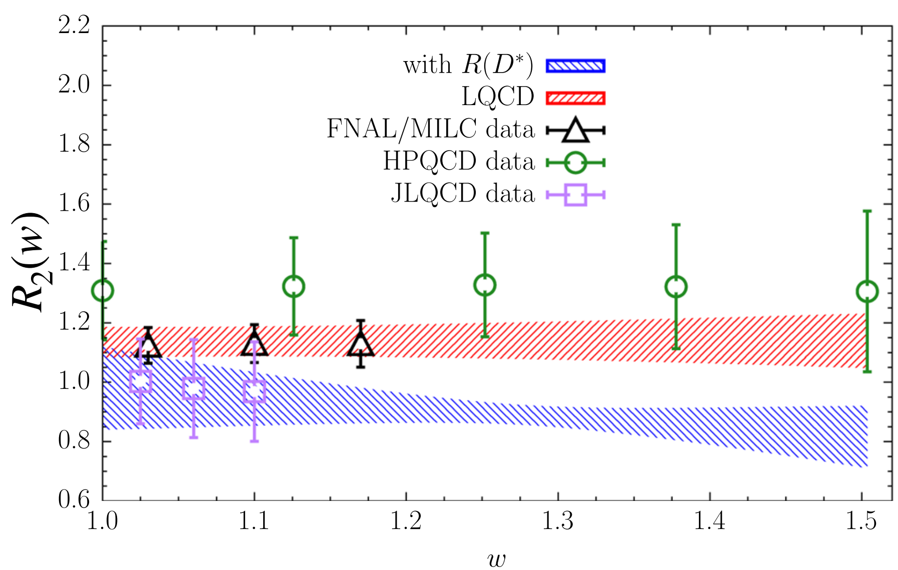
$$R_0(w) = \frac{1}{1+r}\left[1 + w + (rw-1)\frac{h_{A_2}(w)}{h_{A_1}(w)} + (r-w)\frac{h_{A_3}(w)}{h_{A_1}(w)}\right]$$

$$R_1(w) = \frac{h_v(w)}{h_{A_1}(w)}, \qquad R_2(w) = \frac{r h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$





$R_2(w=1)$ contributes to the slope of $F_1(w)$ at $w=1$



future LQCD calculations must carefully determine the value of $R_2(1)$ (not directly accessible)

in the HQ limit one has: $R_0(w) = R_1(w) = R_2(w) = 1$

determination of $|V_{cb}|$ from Γ

Martinelli, SS, Vittorio arXiv:2410.17974

determination of $|V_{cb}\, f(1)|$ using the total decay rate $\Gamma_{exp}(B \to D^* \ell \nu_\ell)$

$$\Gamma(B \to D^* \ell \nu_\ell) \to_{m_\ell = 0} \frac{4\eta_{EW}^2 m_B m_{D^*}^2 G_F^2}{3(4\pi)^3} |V_{cb}\, f(1)|^2 \left[\widetilde{H}_{++} + \widetilde{H}_{--} + \widetilde{H}_{00}\right]$$

where $\Gamma_{exp} = 21.74\,(51) \cdot 10^{-15}$ GeV from PDG '24, while $\widetilde{H}_{++,--,00}$ can be calculated using the reduced FFs $\tilde{g}, \tilde{f}, \widetilde{F}_1$ obtained either from the ***exps+unitarity*** fit or from LQCD

	$\lvert V_{cb}\, f(1)\rvert \cdot 10^3$ GeV^{-1}	$\lvert V_{cb}\rvert \cdot 10^3$
Belle18	222.9 (8.5)	38.1 (1.5)
Belle23	234.2 (9.3)	40.1 (1.6)
BelleII23	236.5 (5.6)	40.5 (1.0)
Belle18 + Belle23 + BelleII23	231.8 (4.6)	$\boxed{39.7\ (0.8)}$

using $f(1)|_{LQCD} = 5.845\,(50)$ GeV^{-1}

	$\lvert V_{cb}\, f(1)\rvert \cdot 10^3$ GeV^{-1}	$f(1)$ GeV^{-1}	$\lvert V_{cb}\rvert \cdot 10^3$
FNAL/MILC	253.2 (9.2)	5.951 (91)	42.6 (1.7)
HPQCD	253.4 (11.7)	5.885 (94)	43.1 (2.1)
JLQCD	231.3 (9.5)	5.776 (90)	40.0 (1.8)
LQCD	243.2 (5.9)	5.845 (50)	$\boxed{41.6\ (1.1)}$

larger values due to the LQCD versus exp. **slope of $F_1(w)$**

bin-per-bin analysis of the same datasets within DM

$$|V_{cb}| \cdot 10^3 = 39.92\,(64)$$

Martinelli, SS, Vittorio arXiv:2310.03680

Bayesian inference based analysis

$$|V_{cb}| \cdot 10^3 = 40.42\,(71)$$

Bordone, Jüttner arXiv:2406.10074

UTfit **prediction** arXiv:2212.03894

$$|V_{cb}| \cdot 10^3 = 42.19\,(48)$$

~ updated inclusive value (Fael et al. arXiv:2212.03894)

$$|V_{cb}| \cdot 10^3 = 42.00\,(47)$$

conclusions

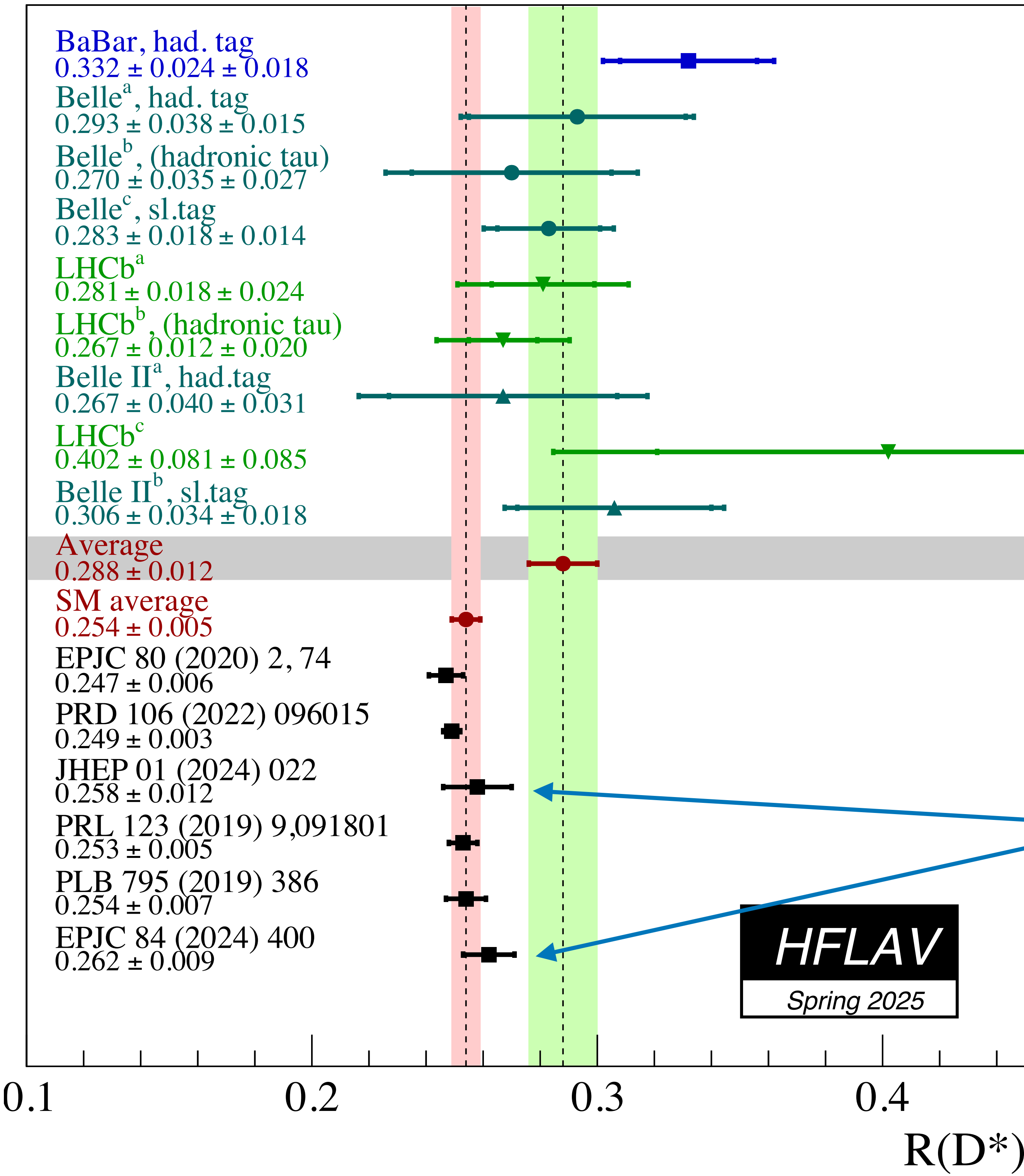
- systematic effects on the lattice results for the FFs obtained by different collaborations are not yet under full control ====> different ***slopes of*** F_1 between JLQCD and (FNAL/MILC, HPQCD)
- different methods for extrapolating the FFs in the whole kinematical range are consistent
- the (normalized) single differential decay rates $(1/\Gamma)\, d\Gamma/dx$ with $x = \{w, cos\theta_\ell, cos\theta_v, \chi\}$ are available from light-lepton experiments (Belle18, Belle23, BelleII23). They are independent of $|V_{cb}|$ and the angular ones depend in general on five hadronic parameters $\{\eta, \eta', \delta, \epsilon, \epsilon'\}$ both in the SM and beyond
- differences of $\{\eta, \eta', \delta, \epsilon, \epsilon'\}$ among the various experiments as well as with LQCD predictions
- the angular coefficients $J_i(w)$, determined by Belle, are consistent with LQCD predictions
- extraction of (reduced) hadronic FFs using unitary BGL fits applied to light-lepton and τ-lepton exp. data ($R(D^*)$ and $F_{L,\tau}$) with minimal input from theory: possible and independent of $|V_{cb}|$
- while the inclusion of $(F_{L,\tau})_{<,>}|_{exp}$ leads to a pseudoscalar FF $F_2(w)$ compatible with LQCD results, the inclusion of $R(D^*)|_{exp}$ requires instead a much larger $F_2(w)$ (by a factor of ~ 2)
- **thus**: $R(D^*)$ (or F_2) is problematic within the SM

outlooks

- a precise determination of the HQET ratio $R_2(1)$ is crucial for understanding the slope of F_1
- accurate determinations of $(F_{L,\tau})_{<,>}$ and $A_{FB,\tau}$ will be very valuable for assessing the tension with the SM

backup slides

updated HFLAV table of experimental and SM values of $R(D^*)$



experimental average

SM average

differences $\simeq 1.8\,\sigma$

unitarity constraints

BGL z-expansion of the *reduced* FFs

$$\widetilde{g}(w) \equiv \frac{g(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^-}}}{\phi_g(z)B_g(z)} \sum_{i=0}^{N_g} a_i^g z^i \qquad \widetilde{F}_1(w) \equiv \frac{F_1(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^+}}}{\phi_{F_1}(z)B_{F_1}(z)} \sum_{i=0}^{N_{F_1}} a_i^{F_1} z^i$$

$$\tilde{f}(w) \equiv \frac{f(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{1^+}}}{\phi_f(z)B_f(z)} \sum_{i=0}^{N_f} a_i^f z^i \qquad \widetilde{F}_2(w) \equiv \frac{F_2(w)}{f(1)} = \frac{\sqrt{\widetilde{\chi}_{0^+}}}{\phi_{F_2}(z)B_{F_2}(z)} \sum_{i=0}^{N_{F_2}} a_i^{F_2} z^i$$

unitarity constraints for $B \to D^* \ell \nu_\ell$ decays

$$\sum_{i=0}^{N_g} [a_i^{(g)}]^2 \leq 1 \qquad \sum_{i=0}^{N_f} [a_i^{(f)}]^2 + \sum_{i=0}^{N_{F_1}} [a_i^{(F_1)}]^2 \leq 1 \qquad \sum_{i=0}^{N_{F_2}} [a_i^{(F_2)}]^2 \leq 1$$

reduced susceptibilities $\widetilde{\chi}_{1^-,1^+,0^+} \equiv \dfrac{\chi_{1^-,1^+,0^+}}{f^2(1)}$

no. of parameters with two KCs + $\tilde{f}(1) = 1$

$$N_{parms} = N_g + N_f + N_{F_1} + N_{F_2} + 1$$

$$\chi_{1^-} = (5.84 \pm 0.44) \cdot 10^{-4}\,\mathrm{GeV}^{-2}$$

$$\chi_{1^+} = (4.69 \pm 0.30) \cdot 10^{-4}\,\mathrm{GeV}^{-2}$$

$$\chi_{0^+} = (21.9 \pm 1.9) \cdot 10^{-4}\,\mathrm{GeV}^{-2}$$

Martinelli, SS, Vittorio arXiv:2105.07851

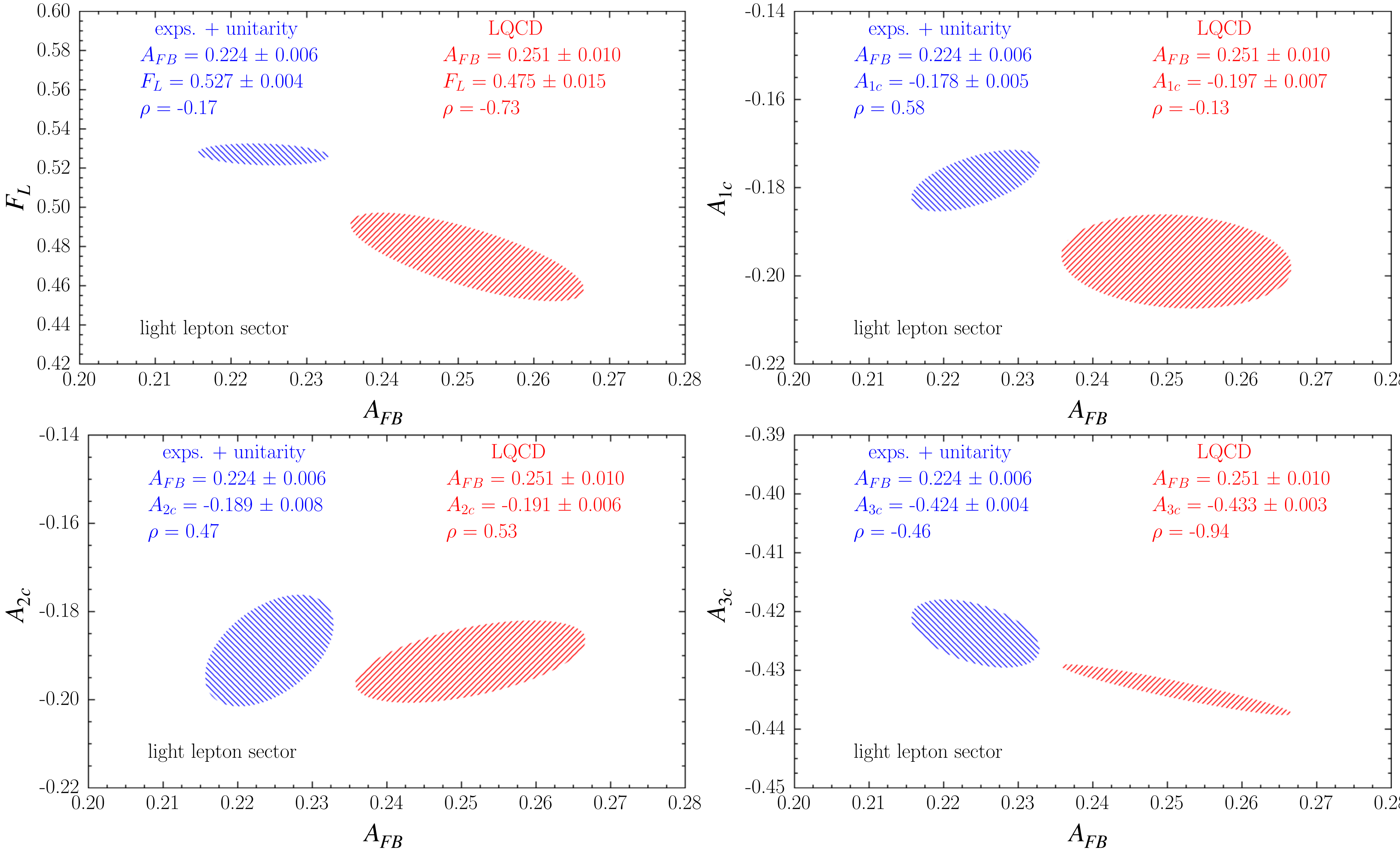
$$f(1) = 5.845\,(50)\,\mathrm{GeV}^{-1}$$

Martinelli, SS, Vittorio arXiv:2310.03680

[DM method applied to all LQCD results from FNAL/MILC, HPQCD, JLQCD]

contour plots for light-lepton asymmetries

Martinelli, SS, Vittorio arXiv: 2410.17974



differences at the $\approx 2\sigma$ level, except for F_L at 3.3σ level

HQET-inspired FFs

Martinelli, SS, Vittorio arXiv: 2410.17974

$$h_V(w) \propto g(w), \quad h_{A_1}(w) \propto f(w), \quad h_{A_2}(w) \text{ and } h_{A_3}(w) \text{ depend on } f(w), F_1(w), F_2(w)$$

in the HQ limit one has: $h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi_{IW}(w), \quad h_{A_2}(w) = 0$