Tensor-polarized twist-3 distribution function f_{LT} of spin-1 deuteron

S. Kumano and K. Kuroki, Phys. Lett. B 875 (2026) 140348
S. Kumano and K. Kuroki, arXiv:2605.00430

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https://kenshikuroki.github.io/

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DIS2026

Introduction

- Spin Tensor
- Structure Functions $b_{1,\ldots,4}$
- Deuteron b_1
- Tensor Structure Puzzle

Spin Tensor

Spin density matrix in the particle rest frame

■ Spin-1/2

$$\boldsymbol{\rho}_{1/2} = \frac{1}{2}\left(\mathbf{1} + s^i \boldsymbol{\sigma}^i\right)$$

$\boldsymbol{\sigma}$: Pauli matrices

$s^i = \mathrm{Tr}\left[\boldsymbol{\rho}_{1/2}\boldsymbol{\sigma}^i\right]$: **Spin vector**

■ Spin-1 (e.g., deuteron)

$$\boldsymbol{\rho}_1 = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2}S^i\boldsymbol{\Sigma}^i + 3T^{ij}\boldsymbol{\Sigma}^{ij}\right)$$

$\boldsymbol{\Sigma}$: 3×3 spin matrices, $S^i = \mathrm{Tr}\left[\boldsymbol{\rho}_1\boldsymbol{\Sigma}^i\right]$

$$\boldsymbol{\Sigma}^{ij} = \frac{1}{2}\left(\boldsymbol{\Sigma}^i\boldsymbol{\Sigma}^j + \boldsymbol{\Sigma}^j\boldsymbol{\Sigma}^i\right) - \frac{2}{3}\mathbf{1}\delta^{ij}$$

$T^{ij} = \mathrm{Tr}\left[\boldsymbol{\rho}_1\boldsymbol{\Sigma}^{ij}\right]$: **Spin tensor**

New tensor-polarization DOF in spin-1 hadrons

Covariant generalization: $\boldsymbol{S} \to S^\mu$, $\boldsymbol{T} \to T^{\mu\nu}$ with $P_\mu S^\mu = 0, P_\mu T^{\mu\nu} = 0$

Structure Functions $b_{1,\ldots,4}$

In charged-lepton DIS, P. Hoodbhoy, R. L. Jaffe, A. Manohar, NPB 312 (1989) 571

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} + g_1 \frac{i}{\nu} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)$$
$$- b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu})$$

Tensor-polarized Structure Functions

$$b_1, b_2, b_3, b_4$$

which vanish under spin average

- b_1, b_2: leading-twist
  $2x b_1(x) = b_2(x)$ in Bjorken limit
- b_3, b_4: higher-twist

Symmetric under $\mu \leftrightarrow \nu$
→ Unpolarized lepton beam ✓

$$r_{\mu\nu} = \frac{1}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) g_{\mu\nu}$$
$$s_{\mu\nu} = \frac{2}{\nu^2}\left(q \cdot E^* q \cdot E - \frac{1}{3}\nu^2 \kappa\right) \frac{p^\mu p^\nu}{\nu}$$
$$t_{\mu\nu} = \frac{1}{2\nu^2}\left(q \cdot E^* p_{\{\mu} E_{\nu\}} + q \cdot E p_{\{\mu} E^*_{\nu\}} - \frac{4}{3}\nu p_\mu p_\nu\right)$$
$$u_{\mu\nu} = \frac{1}{\nu}\left(E^*_{\{\mu} E_{\nu\}}\right) + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} p_\mu p_\nu$$

$\nu = p \cdot q$
$\kappa = 1 + M^2 Q^2 / \nu^2$
E^μ: Polarization vector
$p \cdot E = 0, \quad E^2 = -M^2$
$\sum_\lambda E^\mu(\lambda) E^{*\nu}(\lambda) = -g^{\mu\nu} M^2 + p^\mu p^\nu$
$s^\sigma = -\frac{i}{M^2} \varepsilon^{\sigma\alpha\beta\tau} E^*_\alpha E_\beta p_\tau$
$a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$

b_1 sum rule F.E. Close, S. Kumano, PRD 42 (1990) 2377

$$\int dx\, b_1(x) = -\frac{5}{24} \lim_{t \to 0} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x) \quad \text{in parton model}$$

$$\rightarrow \int dx\, b_1(x) \neq 0 \text{ suggests } \delta_T \bar{q}_i \neq 0$$

Deuteron b_1

HERMES b_1 (2005)

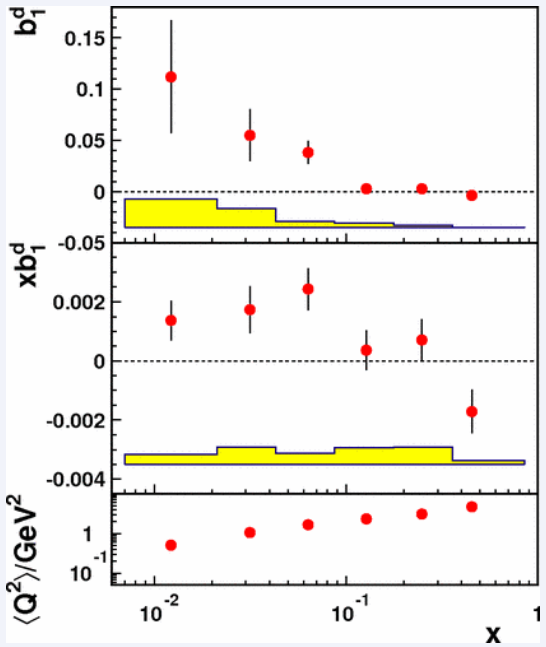
A. Airapetian et al. (HERMES), PRL 95 (2005) 242001

$$e^+ \xrightarrow{27.6\ \text{GeV}} D \rightleftarrows, 0$$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{d^2\sigma^U}{dxdQ^2}\left(1 + \frac{1}{2}P_{ZZ}A_{ZZ}^D\right), \quad P_{zz} \propto T_{ZZ}$$

$0.5 < Q^2 < 5\ \text{GeV}^2,\ 0.01 < x < 0.45$

Tensor asymmetry: $A_{ZZ}^D \xrightarrow[\text{Bjorken limit}]{} A_{ZZ}^D = -\frac{2}{3}\frac{b_1^D}{F_1^D}$



- Finite at 2σ level
  Positive at low-x
  Negative at high-x
- **Finite Sum**

$$\int_{0.02}^{0.85} dx\ b_1^D(x) = \begin{bmatrix} 0.0035 \\ \pm 0.0010\ (\text{stat}) \\ \pm 0.0018\ (\text{sys}) \end{bmatrix}$$

$$\delta_T \bar{q}_i \neq 0\ ?$$

JLab E12-13-011 (Approved)

E12-13-011 Jeopardy Update to Jefferson Lab PAC 51

The Deuteron Tensor Structure Function b_1

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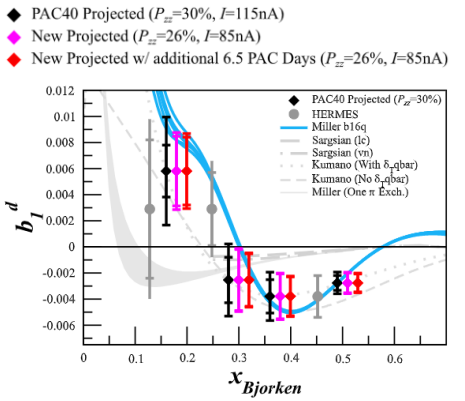
Executive Summary

We present an update to E12-13-011, which will precisely measure the leading twist deuteron tensor structure function b_1. This observable describes quark distributions within a spin polarized deuteron, providing unique insight into how nuclear properties arise from partonic degrees of freedom. The only existing data [1] revealed a surprisingly large negative value of b_1 at moderate x, in stark contrast to the predictions of all known conventional treatments of the deuteron. From the time of the original proposal, the motivation for measuring b_1 has only grown, as we discuss in this document. The collaboration is active and growing, as new students, post-docs and theorists join the effort. In August 2022, we were granted full approval status after satisfying a C1 Conditional Review, and we are now preparing for experiment readiness. As discussed in the Conditional Review panel summary, significant progress has been made in enhancing tensor polarization, understanding the tensor enhanced NMR lineshape for polarimetry, and in reducing the systematic uncertainties of our b_1 extraction via rapid changes in polarization state. We have made some changes in experimental setup from our initial request. A new polarizing magnet is being procured by the JLab target group. It now seems apparent that the existing 2 cm chicane used to bring the beam into the hall will be sufficient to deliver the beam on target before being collected in a local low power beam dump [2]. Our original request was for 30 days of 115nA beam in Hall C with an additional 10.8 days of overhead, assuming a tensor polarization of 30%. We update this request to 35.7 days of beam with 11.7 days of overhead (47.4 days total) assuming a more conservative tensor polarization of 26% at 85nA. This measurement represents a unique probe of hidden color effects [3] in the Deuteron and will provide valuable guidance to a series of planned experiments at JLab, Fermilab, NICA, EICC and the EIC.

[§] In memoriam

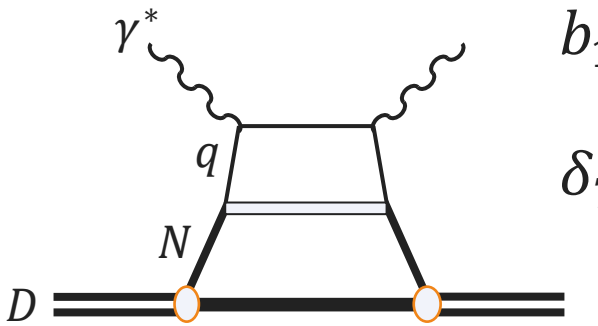
$$0.8 < Q^2 < 5\ \text{GeV}^2$$
$$0.16 < x < 0.49$$



Tensor Structure Puzzle

Convolution model

W. Cosyn, Y.-B. Dong, S. Kumano, M. Sargsian, PRD 95 (2017) 074036



$$b_1(x,Q^2) = \int \frac{dy}{y}\,\delta_T f(y)\,F_1^N\left(\frac{x}{y},Q^2\right)$$

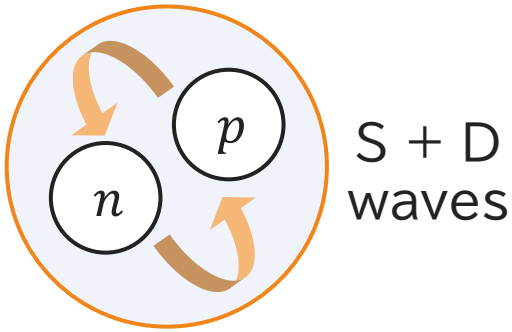
$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

$$= \int d^3p\; y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p)+\frac{3}{16\pi}|\phi_2(p)|^2\right]\left(3\cos^2\theta-1\right)\delta\left(y-\frac{p\cdot q}{M_N\nu}\right)$$

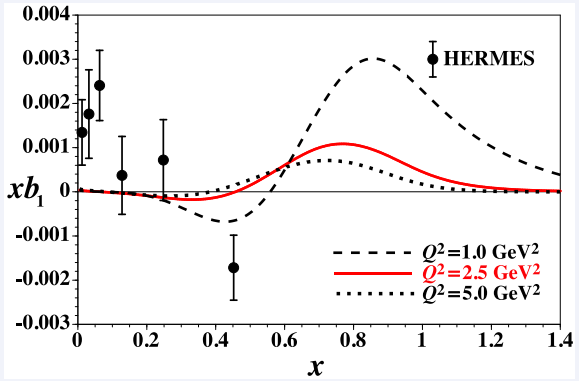
$$y = \frac{Mp\cdot q}{M_N P\cdot q}\cong\frac{2p^-}{P^-}$$

$$f^H(y) = \int d^3p\; y\left|\phi^H(p)\right|^2\delta\left(y-\frac{E-p_z}{M_N}\right)$$

$$\phi^H = \phi^H_{l=0} + \phi^H_{l=2}$$



S + D waves



indicate that these standard theoretical predications are much different from the HERMES measurements. Furthermore, significantly large distributions are predicted, at large x ($x > 0.8$) and even at extremely large x ($x > 1$). Since our results are very different from the HERMES measurement, new hadronic mechanisms could be needed for interpreting the data although there is still some room to improve the differences due to the higher-twist effects and the experimental extraction of b_1 from A_{zz}. The HERMES data have large uncertainties; however, upcoming JLab experimental measurements will improve on the size of the errors. In addition, there are

- New mechanism?
- Higher-twist?
- b_1 extraction from A_{zz}?

Purpose of this study

A quantitative estimate of twist-3 tensor-polarization

Theoretical Framework

- Collinear PDFs
- Twist-2 Relation

Collinear PDFs

S. Kumano, Q.-T. Song, PRD 103 (2021) 014025

Collinear correlation function

$$\Phi_{ij}(x,P,T)=\int\frac{d\xi^-}{2\pi}e^{ixP^+\xi^-}\left\langle P,T\left|\overline{\psi}_j(0)W(0,\xi|n)\psi_i(\xi)\right|P,T\right\rangle_{\xi^+=0,\vec{\xi}_T=0}$$

$0<x<1,\ \ P^\mu=P^+\bar{n}^\mu+\frac{M^2}{2P^+}n^\mu,\ \ W(0,\xi|n)$: Gauge link

$$n^\mu=\frac{1}{\sqrt{2}}(1,0,0,-1)$$
$$\bar{n}^\mu=\frac{1}{\sqrt{2}}(1,0,0,1)$$
$$g_T^{\mu\nu}=g^{\mu\nu}-\bar{n}^{\{\mu}n^{\nu\}}$$
$$a^{\{\mu}b^{\nu\}}=a^\mu b^\nu+a^\nu b^\mu$$

Spin tensor

A. Bacchetta, P.J. Mulders, PRD 62 (2000) 114004

$$T^{\mu\nu}=\frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M_D^2}\bar{n}^\mu\bar{n}^\nu-\frac{2}{3}S_{LL}\left(\bar{n}^{\{\mu}n^{\nu\}}-g_T^{\mu\nu}\right)+\frac{1}{3}S_{LL}\frac{M_D^2}{(P^+)^2}n^\mu n^\nu+\frac{P^+}{M_D}\bar{n}^{\{\mu}S_{LT}^{\nu\}}-\frac{M_D}{2P^+}n^{\{\mu}S_{LT}^{\nu\}}+S_{TT}^{\mu\nu}\right]$$

Tensor-polarization parameters: $S_{LL}, S_{LT}^\mu, S_{TT}^{\mu\nu}$

$$n\cdot S_{LT}=\bar{n}\cdot S_{LT}=0,\ \ n_\mu S_{TT}^{\mu\nu}=\bar{n}_\mu S_{TT}^{\mu\nu}=0,\ \ S_{TT}^{\mu\nu}=S_{TT}^{\nu\mu},\ \ g_{T\,\mu\nu}S_{TT}^{\mu\nu}=0$$

Collinear PDFs

$$\Phi(x,P,T)=\frac{1}{2}\left[S_{LL}(\bar{n}\cdot\gamma)f_{1LL}(x)+\frac{M_D}{P^+}S_{LL}1e_{LL}(x)+\frac{M_D}{P^+}(S_{LT}\cdot\gamma)f_{LT}(x)+\frac{M_D^2}{(P^+)^2}S_{LL}(n\cdot\gamma)f_{3LL}(x)\right]$$

Twist-2: f_{1LL} **Twist-3**: e_{LL}, f_{LT} **Twist-4**: f_{3LL}

e_{LL} is chiral-odd

Operator Product Expansion

S. Kumano, K. Kuroki, arXiv:2605.00430

The n-th moment of a non-local operator

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P,T|\bar{\psi}(0)\gamma^{\sigma}\psi(\xi)|P,T\rangle_{\substack{\xi^+=0 \\ \vec{\xi}_T=0}} \cong S_{LL}\bar{n}^{\sigma} \int_{-1}^{1} dx x^{n-1} f_{1LL}(x) + \frac{M_D}{P\cdot n} S_{LT}^{\sigma} \int_{-1}^{1} dx x^{n-1} f_{LT}(x)$$

LHS can also be written as

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P,T|\bar{\psi}(0)\gamma^{\sigma}\psi(\xi)|P,T\rangle_{\substack{\xi^+=0 \\ \vec{\xi}_T=0}} = \cdots = \frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n} \langle P,T|R^{\sigma\{\mu_1\cdots\mu_{n-1}\}}|P,T\rangle$$

Local operator

$$R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} = i^{n-1}\bar{\psi}(0)\gamma^{\sigma}D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi(0) - \text{traces}$$
$$= R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} + R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$$

$D^{\{\mu_1}\cdots D^{\mu_{n-1}\}} = \frac{1}{n!}(D^{\mu_1}\cdots D^{\mu_{n-1}} + \text{permutations})$

$R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}$	: twist-2
$R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$	: twist-3
traces	: twist ≥ 4

Expand matrix elements by momentum P^{μ} and spin tensor $T^{\mu\nu}$

$$\langle P,T|R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}a_n M^2 \left[\sum_{i=1}^{n-1}(T^{\sigma\mu_i}+T^{\mu_i\sigma})\prod_{j(\neq i)=1}^{n-1}P^{\mu_j} + \sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}T^{\mu_i\mu_j}P^{\sigma}\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

$$\langle P,T|R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}d_n M^2 \left[\sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}(P^{\sigma}T^{\mu_i\mu_j} - P^{\mu_i}T^{\sigma\mu_j})\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

a_n and d_n are overall constants

Twist-2 Relation

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141
S. Kumano, K. Kuroki, arXiv:2605.00430

Finally, LHS becomes

$$\frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n}\langle P,T|R^{\sigma\{\mu_1\cdots\mu_{n-1}\}}|P,T\rangle \cong S_{LL}\bar{n}^{\sigma}\left[\frac{2(n-1)}{3}a_n\right]+\frac{M_D}{P\cdot n}S_{LT}^{\sigma}\left[\frac{n-1}{n}a_n-\frac{(n-1)(n-2)}{2n}d_n\right]$$

Compare with RHS, **Moments of PDFs** are

$$\int_{-1}^{1}dx\,x^{n-1}\,f_{1LL}(x)=\int_{0}^{1}dx\,x^{n-1}\,f_{1LL}^{\pm}(x)=\frac{2(n-1)}{3}a_n$$

$$\int_{-1}^{1}dx\,x^{n-1}\,f_{LT}(x)=\int_{0}^{1}dx\,x^{n-1}\,f_{LT}^{\pm}(x)=\frac{n-1}{n}a_n-\frac{(n-1)(n-2)}{2n}d_n$$

$f^{\pm}(x)=f(x)\pm\bar{f}(x)$
$+\,(-)$ for $n=$ even (odd) integer

$$\int_0^1 dx x^{n-1}\int_x^1\frac{dy}{y}f^{\pm}(y)=\frac{1}{n}\int_0^1 dx x^{n-1}f^{\pm}(x)$$

Twist-2 Relation

$$f_{LT}^{\pm}(x)=f_{LT}^{\pm\,\text{twist-2}}(x)+f_{LT}^{\pm\,(\text{HT})}(x)$$

Contribution from twist-2 — Dynamical twist-3

$$f_{LT}^{\pm\,\text{twist-2}}(x)=\frac{3}{2}\int_x^1\frac{dy}{y}f_{1LL}^{\pm}(y)$$

$$\int_0^1 dx\,x^{n-1}f_{LT}^{\pm\,(\text{HT})}(x)=-\frac{(n-1)(n-2)}{2n}d_n$$

Sum rule

$$f_{2LT}^{\pm\,\text{twist-2}}(x)=\frac{2}{3}f_{LT}^{\pm\,\text{twist-2}}(x)-f_{1LL}^{\pm}(x)$$

$$\int_0^1 dx\,f_{2LT}^{\pm\,\text{twist-2}}(x)=0$$

If $\int_0^1 dx\,f_{LT}^{\pm}(x)\neq 0$

↓

$f_{LT}^{\pm\,(\text{HT})}(x)\neq 0$?

Cf. vector-polarized structure functions g_1 and g_2

Wandzura-Wilczek relation
S. Wandzura, F. Wilczek, PLB 72 (1977) 195

Burkhardt-Cottingham sum rule
H. Burkhardt, W.N. Cottingham, Annals. Phys. 56 (1970) 453

Numerical Results

- Phenomenological Distribution
- Twist-2 PDF $f_{1LL}(x)$
- Twist-3 PDF $f_{LT}(x)$

Phenomenological Distribution

Parton model

$$b_1^D = \frac{1}{2}\sum_q e_q^2[\delta_T q^D + \delta_T \bar{q}^D]$$

$$\delta_T q^D = q^{D0} - \frac{q^{D+}+q^{D-}}{2}$$

$q^{D\lambda}$: Unpolarized quark distribution inside the spin-λ deuteron

Parametrization S. Kumano, PRD 82 (2010) 017501

Assumption: $q = u, d, s$, flavor independent polarization

$$\delta_T q_v^D(x) = \delta_T w(x)\, q_v^D(x)$$
$$\delta_T \bar{q}^D(x) = \alpha_{\bar{q}}\, \delta_T w(x)\, \bar{q}^D(x)$$
$$\delta_T w(x) = a x^b (1-x)^c (x_0 - x)$$

$q_v^D, \bar{q}^D$: Unpolarized distribution in nucleon

A.D. Martin et al., EPJ C 63 (2009) 189

Parameters

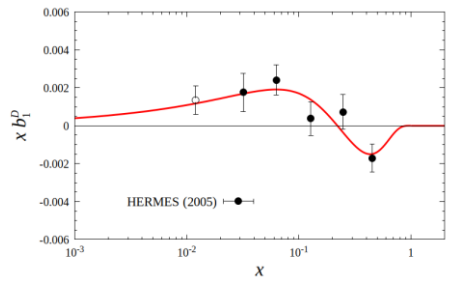
a, b

$c = 1$ (fixed)

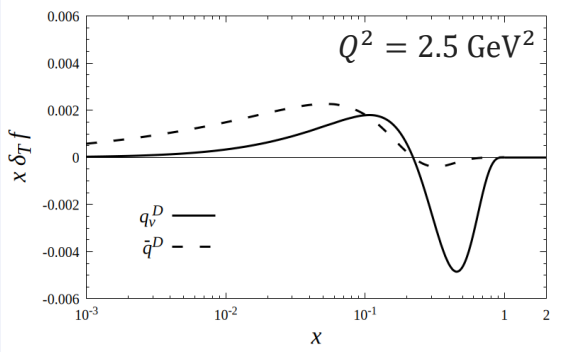
$x_0, \alpha_{\bar{q}}$

HERMES b_1

$\chi^2/\text{d.o.f.} = 1.57$



$a = 0.221$
$b = 0.648$
$x_0 = 0.221$
$\alpha_{\bar{q}} = 3.20$



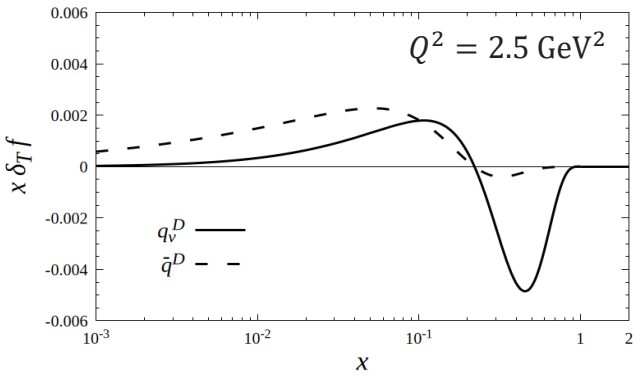
- Positive at $x \lesssim 0.22$
  Negative at $x \gtrsim 0.22$
- Finite $\delta_T \bar{q}^D$

Twist-2 PDF $f_{1LL}(x)$

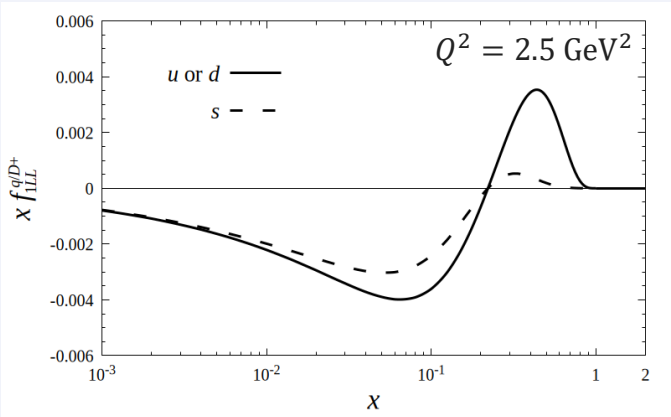
$$f_{1LL}^{q/D+}(x) = f_{1LL}^{q/D}(x) + f_{1LL}^{\bar{q}/D+}(x)$$

$$= -\frac{2}{3}[\delta_T q^D(x) + \delta_T \bar{q}^D(x)]$$

$$= -\frac{2}{3}[\delta_T q_v^D(x) + 2\delta_T \bar{q}^D(x)]$$



Twist-2 f_{1LL} of deuteron



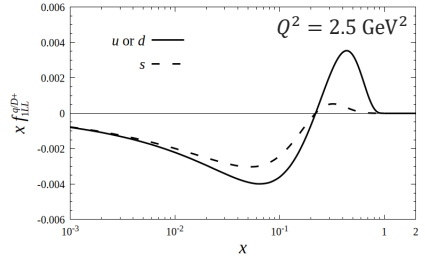
- Negative at $x \lesssim 0.22$
  Positive at $x \gtrsim 0.22$
- Magnitude $\sim \mathcal{O}(10^{-3})$

Use **twist-2 relation** to calculate twist-3 PDF f_{LT}

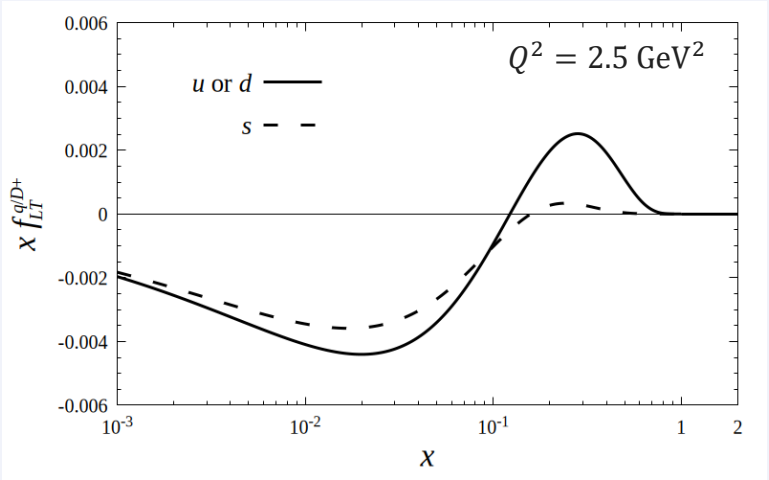
Twist-3 PDF $f_{LT}(x)$

Twist-2 relation (neglecting dynamical twist-3)

$$f_{LT}^{q/D+}(x) \cong f_{LT}^{q/D+\ \mathrm{twist-2}}(x) = \frac{3}{2}\int_x^2 \frac{dy}{y} f_{1LL}^{q/D+}(y)$$



Twist-3 f_{LT} of deuteron



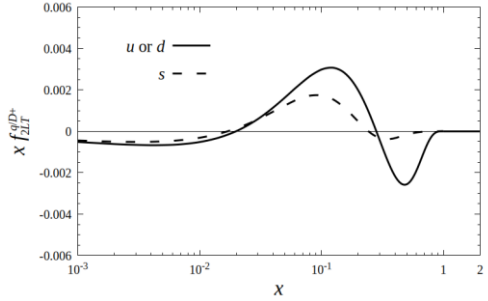
- Negative at $x \lesssim 0.1$
  Positive at $x \gtrsim 0.1$
- Magnitude $\sim \mathcal{O}(10^{-3})$

  About the same as twist-2 f_{1LL}

Sum rule

$$f_{2LT}^{q/D+}(x) = \frac{2}{3} f_{LT}^{q/D+}(x) - f_{1LL}^{q/D+}(x)$$

$$\int dx\, f_{2LT}^{q/D+}(x) = 0$$



Summary

First numerical estimate of twist-3 tensor-polarized PDF

Phenomenological twist-2 f_{1LL} ← HERMES b_1 data

Twist-2 relation

Twist-3 f_{LT}

Magnitude of twist-3 f_{LT} is the same as twist-2 f_{1LL}
→ Significant in the few-GeV^2 region (e.g., JLab)?

Accessible through

- DIS, SIDIS J. Zhao, A. Bacchetta, S. Kumano, T. Liu, Y.-J. Zhou, JHEP 12 (2025) 067
- Drell-Yan S.-Y. Qiao, Q.-T. Song, PRD 111 (2025) 054026