Theory updates for parton distributions in Hessian formalism

Petja Paakkinen (CERN)
in collab. w/ H. Paukkunen & S. Yrjänheikki (U. Jyväskylä)

DIS2026
5 May 2026

Hessian formalism for PDF errors

Expand global-analysis figure of merit $\chi^2_{\mathtt{GA}}$ around best-fit parameters $\mathbf{a}_{\mathtt{min,GA}}$[†]

$$\chi^2_{\mathtt{GA}}(\overbrace{\mathbf{a}}^{\text{PDF parameters}}) = (\mathbf{D} - \mathbf{T}(\mathbf{a}))^T \, C^{-1} \, (\overbrace{\mathbf{D}}^{\text{data}} - \overbrace{\mathbf{T}(\mathbf{a})}^{\text{theory}})$$
$$\approx \chi^2_{\mathtt{min,GA}} + (\mathbf{a} - \mathbf{a}_{\mathtt{min,GA}})^T \, H_{\mathtt{GA}} \, (\mathbf{a} - \mathbf{a}_{\mathtt{min,GA}})$$
$$= \chi^2_{\mathtt{min,GA}} + \underbrace{\mathbf{z}^2}_{\text{whitened parameters, } \mathbf{z} = W_{\mathtt{GA}}(\mathbf{a} - \mathbf{a}_{\mathtt{min,GA}}) \text{ such that } H_{\mathtt{GA}} = W_{\mathtt{GA}}^T W_{\mathtt{GA}}\text{, e.g. through EVD}}$$

Hessian PDF central and error sets given in terms of

$$f_i[S_{\mathtt{GA},0}] = f_i(\mathbf{z}[S_{\mathtt{GA},0}]), \qquad z_j[S_{\mathtt{GA},0}] = 0 \text{ for all } j$$
$$f_i[S^{\pm}_{\mathtt{GA},k}] = f_i(\mathbf{z}[S^{\pm}_{\mathtt{GA},k}]), \qquad z_j[S^{\pm}_{\mathtt{GA},k}] = \pm\, \delta_{jk} \underbrace{\sqrt{\Delta\chi^2}}_{\text{tolerance!}}$$

Symmetric error prescription:

$$\Delta f_i = \frac{1}{2}\sqrt{\sum_k (f_i[S^+_k] - f_i[S^-_k])^2}$$

Pumplin *et al.*, PRD 65 (2001) 014013

[†]We drop any terms that would contribute to χ^2 at $\mathcal{O}(a_j^3)$ in all formulas

Hessian PDF reweighting (a.k.a. profiling)

Want to study the impact of adding a new dataset $\approx \mathbf{T}_{\mathtt{new},0}+J_{\mathtt{new}}\mathbf{z},\ (J_{\mathtt{new}})_{ik}=\frac{(\mathbf{T}_{\mathtt{new}})_i[S^+_{\mathtt{GA},k}]-(\mathbf{T}_{\mathtt{new}})_i[S^-_{\mathtt{GA},k}]}{2\sqrt{\Delta\chi^2}}$

$$\begin{aligned}\chi^2_{\mathtt{rw}}(\mathbf{z}) &= \chi^2_{\mathtt{GA}}(\mathbf{z}) + (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new}}(\mathbf{z}))^T\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \overbrace{\mathbf{T}_{\mathtt{new}}(\mathbf{z})}) \\ &\approx \chi^2_{\mathtt{min,GA}} + \mathbf{z}^2 + (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0} - J_{\mathtt{new}}\mathbf{z})^T\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0} - J_{\mathtt{new}}\mathbf{z}) \\ &= \chi^2_{\mathtt{min,rw}} + \underbrace{\tilde{\mathbf{z}}^2}\end{aligned}$$

whitened parameters, $\tilde{\mathbf{z}} = W_{\mathtt{rw}}(\mathbf{z} - \mathbf{z}_{\mathtt{min,rw}})$ such that $H_{\mathtt{rw}} = W^T_{\mathtt{rw}} W_{\mathtt{rw}}$, e.g. through EVD

New minimum is at

$$\mathbf{z}_{\mathtt{min,rw}} = H^{-1}_{\mathtt{rw}}\, J^T_{\mathtt{new}}\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0})$$

and $\mathbf{z}$-space Hessian is

$$H_{\mathtt{rw}} = 1 + J^T_{\mathtt{new}}\, C^{-1}_{\mathtt{new}}\, J_{\mathtt{new}}$$

Reweighted central and error sets are given as

$$f_i[S_{\mathtt{rw},0}] = f_i[S_{\mathtt{GA},0}] + \textstyle\sum_j w_j[S_{\mathtt{rw},0}]\, \frac{1}{2}(f_i[S^+_{\mathtt{GA},j}] - f_i[S^-_{\mathtt{GA},j}]), \quad w_j[S_{\mathtt{rw},0}] = \frac{1}{\sqrt{\Delta\chi^2}}(\mathbf{z}_{\mathtt{min,rw}})_j$$

$$f_i[S^\pm_{\mathtt{rw},k}] = f_i[S_{\mathtt{GA},0}] + \textstyle\sum_j w_j[S^\pm_{\mathtt{rw},k}]\, \frac{1}{2}(f_i[S^+_{\mathtt{GA},j}] - f_i[S^-_{\mathtt{GA},j}]), \quad w_j[S^\pm_{\mathtt{rw},k}] = \frac{1}{\sqrt{\Delta\chi^2}}(\mathbf{z}_{\mathtt{min,rw}})_j \pm (W^{-1}_{\mathtt{rw}})_{jk}$$

Paukkunen & Zurita, JHEP 12 (2014) 100

Hessian PDF deweighting (≠ unweighting!)

Want to study the impact of *removing* an old dataset $\approx \mathbf{T}_{\mathtt{old},0}+J_{\mathtt{old}}\mathbf{z},\ (J_{\mathtt{old}})_{ik}=\frac{(\mathbf{T}_{\mathtt{old}})_i[S^+_{\mathtt{GA},k}]-(\mathbf{T}_{\mathtt{old}})_i[S^-_{\mathtt{GA},k}]}{2\sqrt{\Delta\chi^2}}$

$$\begin{aligned}\chi^2_{\mathtt{dw}}(\mathbf{z}) &= \chi^2_{\mathtt{GA}}(\mathbf{z}) - (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old}}(\mathbf{z}))^T\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \overbrace{\mathbf{T}_{\mathtt{old}}(\mathbf{z})}) \\ &\approx \chi^2_{\mathtt{min,GA}} + \mathbf{z}^2 - (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0} - J_{\mathtt{old}}\mathbf{z})^T\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0} - J_{\mathtt{old}}\mathbf{z}) \\ &= \chi^2_{\mathtt{min,dw}} + \underbrace{\tilde{\mathbf{z}}^2}\end{aligned}$$

whitened parameters, $\tilde{\mathbf{z}} = W_{\mathtt{dw}}(\mathbf{z} - \mathbf{z}_{\mathtt{min,dw}})$ such that $H_{\mathtt{dw}} = W^T_{\mathtt{dw}} W_{\mathtt{dw}}$, e.g. through EVD

New minimum is at

$$\mathbf{z}_{\mathtt{min,dw}} = H^{-1}_{\mathtt{dw}} \left[-J^T_{\mathtt{old}}\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0})\right]$$

and $\mathbf{z}$-space Hessian is

$$H_{\mathtt{dw}} = 1 - J^T_{\mathtt{old}}\, C^{-1}_{\mathtt{old}}\, J_{\mathtt{old}}$$

Deweighted central and error sets are given as

$$f_i[S_{\mathtt{dw},0}] = f_i[S_{\mathtt{GA},0}] + \textstyle\sum_j w_j[S_{\mathtt{dw},0}]\, \frac{1}{2}(f_i[S^+_{\mathtt{GA},j}] - f_i[S^-_{\mathtt{GA},j}]), \quad w_j[S_{\mathtt{dw},0}] = \frac{1}{\sqrt{\Delta\chi^2}}(\mathbf{z}_{\mathtt{min,dw}})_j$$

$$f_i[S^\pm_{\mathtt{dw},k}] = f_i[S_{\mathtt{GA},0}] + \textstyle\sum_j w_j[S^\pm_{\mathtt{dw},k}]\, \frac{1}{2}(f_i[S^+_{\mathtt{GA},j}] - f_i[S^-_{\mathtt{GA},j}]), \quad w_j[S^\pm_{\mathtt{dw},k}] = \frac{1}{\sqrt{\Delta\chi^2}}(\mathbf{z}_{\mathtt{min,dw}})_j \pm (W^{-1}_{\mathtt{dw}})_{jk}$$

Hessian PDF updating = deweighting + reweighting

Want to study the impact of removing *and* adding a dataset

$$\begin{aligned}
\chi^2_{\mathtt{ud}}(\mathbf{z}) &= \chi^2_{\mathtt{GA}}(\mathbf{z}) - (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old}}(\mathbf{z}))^T\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old}}(\mathbf{z})) \\
&\quad + (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new}}(\mathbf{z}))^T\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new}}(\mathbf{z})) \\
&\approx \chi^2_{\mathtt{min,GA}} + \mathbf{z}^2 - (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0} - J_{\mathtt{old}}\mathbf{z})^T\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0} - J_{\mathtt{old}}\mathbf{z}) \\
&\quad + (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0} - J_{\mathtt{new}}\mathbf{z})^T\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0} - J_{\mathtt{new}}\mathbf{z}) \\
&= \chi^2_{\mathtt{min,ud}} + \underbrace{\tilde{\mathbf{z}}^2}_{\text{whitened parameters, } \tilde{\mathbf{z}} = W_{\mathtt{ud}}(\mathbf{z} - \mathbf{z}_{\mathtt{min,ud}}) \text{ such that } H_{\mathtt{ud}} = W^T_{\mathtt{ud}} W_{\mathtt{ud}}\text{, e.g. through EVD}}
\end{aligned}$$

New minimum is at

$$\mathbf{z}_{\mathtt{min,ud}} = H^{-1}_{\mathtt{ud}} \left[- J^T_{\mathtt{old}}\, C^{-1}_{\mathtt{old}}\, (\mathbf{D}_{\mathtt{old}} - \mathbf{T}_{\mathtt{old},0}) + J^T_{\mathtt{new}}\, C^{-1}_{\mathtt{new}}\, (\mathbf{D}_{\mathtt{new}} - \mathbf{T}_{\mathtt{new},0}) \right]$$

and $\mathbf{z}$-space Hessian is

$$H_{\mathtt{ud}} = 1 - J^T_{\mathtt{old}}\, C^{-1}_{\mathtt{old}}\, J_{\mathtt{old}} + J^T_{\mathtt{new}}\, C^{-1}_{\mathtt{new}}\, J_{\mathtt{new}}$$

- If $\mathbf{D}_{\mathtt{new}} = \mathbf{D}_{\mathtt{old}}$, $C_{\mathtt{new}} = C_{\mathtt{old}}$, we are making a Hessian PDF *theory update*
- If instead $\mathbf{T}_{\mathtt{new}} = \mathbf{T}_{\mathtt{old}}$, it is a *data update*

Possible applications

Applications for theory updates:

- Test the impact of updating an observable from N^{n}LO to N^{n+1}LO precision
- Test the impact of electroweak, mass, resummation, or higher-twist effects
- Test the impact of a certain BSM scenario
- ⟨ insert your idea here ⟩

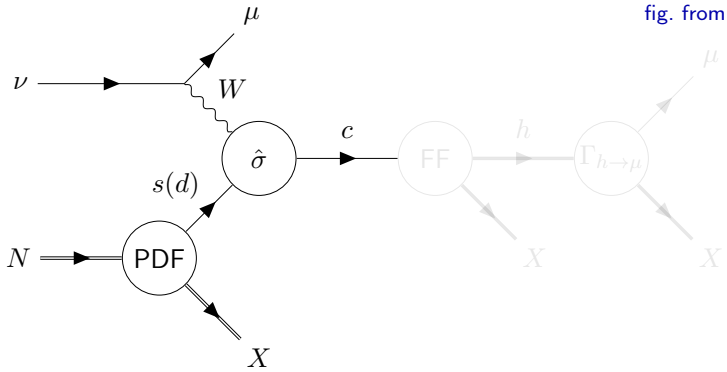
With a data update, one can test the impact of:

- Applying some correction to data
- Changing data correlation models
- Trading one dataset with another
- ⟨ insert your idea here ⟩

Next, we show a case study with dimuon production in $\nu + A$ DIS...

Application: Dimuon production in $\nu+A$ DIS

fig. from S. Yrjänheikki, DIS2025



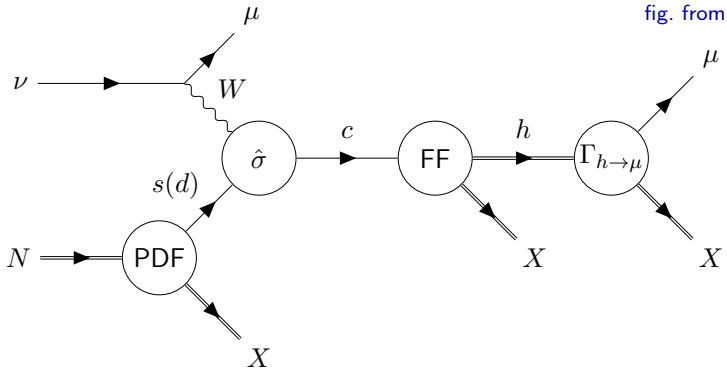
Traditional approach:

- Calculate charm-production cross section and apply semileptonic branching fraction $\mathcal{B}_\mu$ and acceptance correction $\mathcal{A}$: $\mathrm{d}\sigma(\nu N \to \mu\mu X) = \mathcal{A} \times \mathcal{B}_\mu \times \mathrm{d}\sigma(\nu N \to \mu c X)$

➔ Strictly valid only for LO @ fixed-flavour-number, $\mathcal{A}$ depends on event-generator evaluation

Application: Dimuon production in $\nu+A$ DIS

fig. from S. Yrjänheikki, DIS2025



Traditional approach:

- Calculate charm-production cross section and apply semileptonic branching fraction $\mathcal{B}_\mu$ and acceptance correction $\mathcal{A}$: $\mathrm{d}\sigma(\nu N \to \mu\mu X) = \mathcal{A} \times \mathcal{B}_\mu \times \mathrm{d}\sigma(\nu N \to \mu c X)$

➔ Strictly valid only for LO @ fixed-flavour-number, $\mathcal{A}$ depends on event-generator evaluation

New SIDIS approach:

- Use data-fitted charmed-hadron fragmentation functions (FF) and muonic decay width ($\Gamma_{h\to\mu}$)

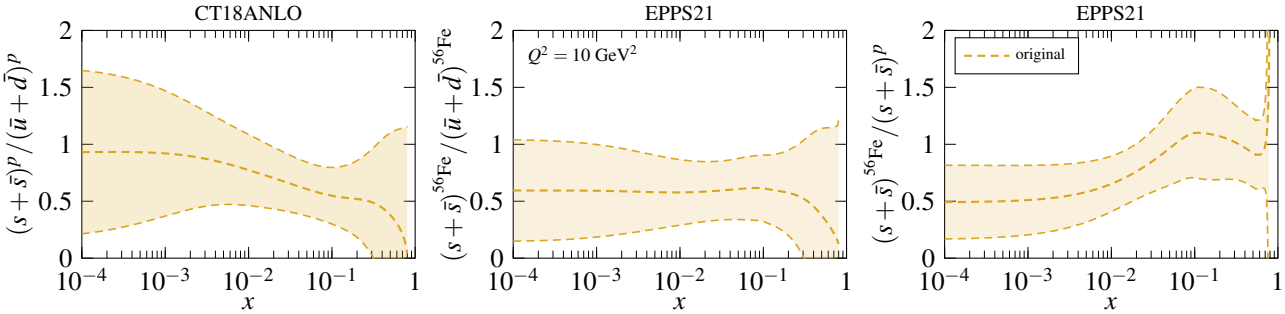
➔ Consistent and systematically improvable to arbitrary order in α_s[†]

Helenius, Paukkunen & Yrjänheikki, JHEP 09 (2024) 043; JHEP 01 (2026) 112; arXiv:2603.09646

[†]See S. Yrjänheikki Wed 15:10 for NNLO results

Updating (n)PDFs with dimuons in SIDIS formalism

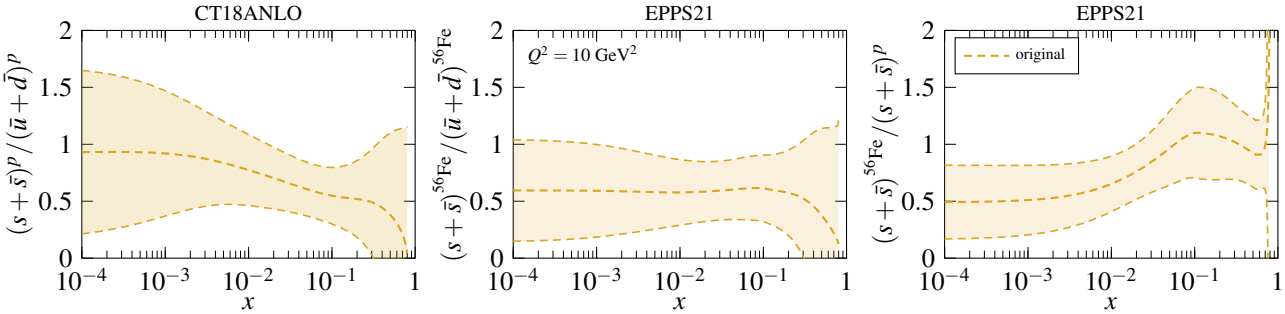
– Preliminary –



Want to study the impact on nPDFs, but proton-PDF fits traditionally include $\nu+A$ DIS constraints

Updating (n)PDFs with dimuons in SIDIS formalism

– Preliminary –

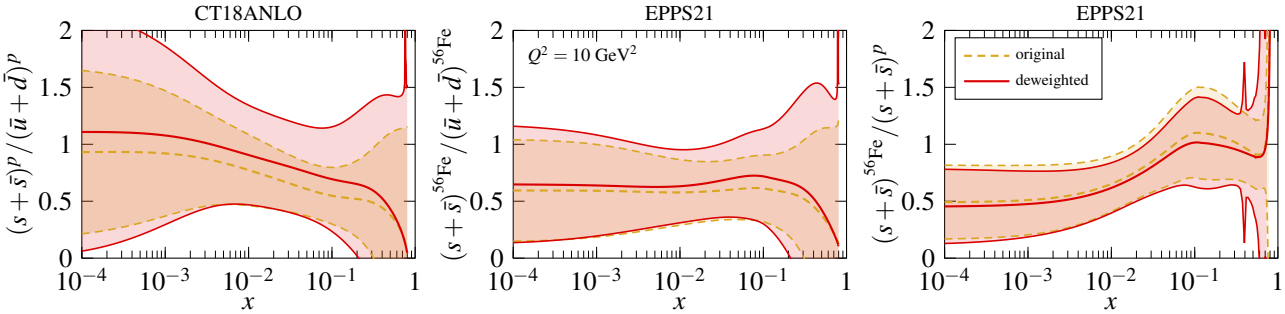


Want to study the impact on nPDFs, but proton-PDF fits traditionally include $\nu+A$ DIS constraints

We have now the tools we need!

Updating (n)PDFs with dimuons in SIDIS formalism

– Preliminary –



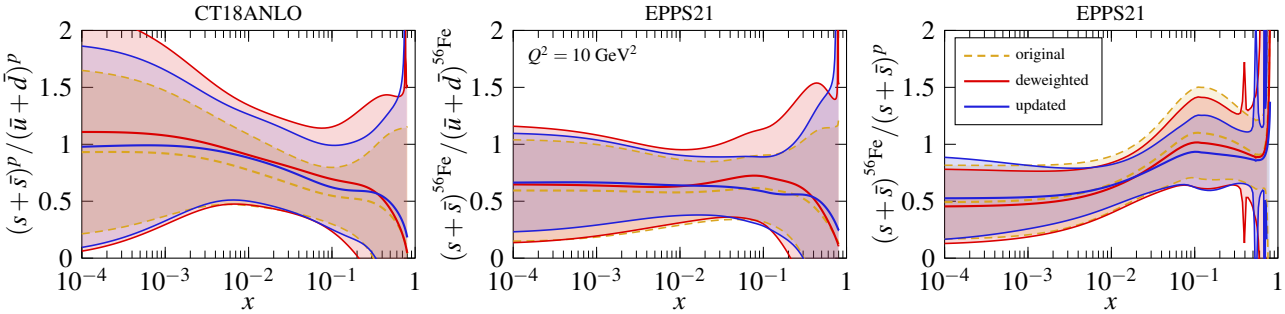
Want to study the impact on nPDFs, but proton-PDF fits traditionally include $\nu+A$ DIS constraints

We have now the tools we need!

Deweighted: Remove dimuon-DIS data w/ trad. approach from CT18ANLO with deweighting and propagate changes to EPPS21 nuclear PDFs

Updating (n)PDFs with dimuons in SIDIS formalism

– Preliminary –



Want to study the impact on nPDFs, but proton-PDF fits traditionally include $\nu+A$ DIS constraints

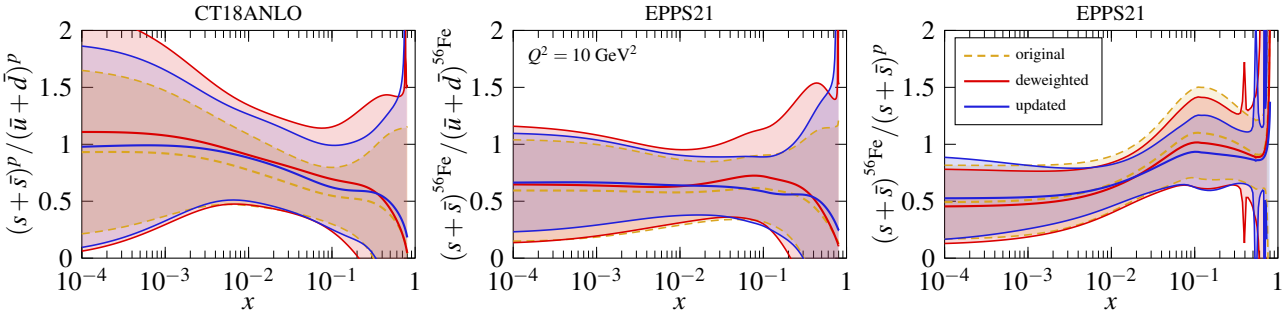
We have now the tools we need!

Deweighted: Remove dimuon-DIS data w/ trad. approach from CT18ANLO with deweighting and propagate changes to EPPS21 nuclear PDFs

Updated: Simultaneous reweighting of the deweighted CT18ANLO and EPPS21 PDFs with dimuon-DIS data using the SIDIS approach

Updating (n)PDFs with dimuons in SIDIS formalism

– Preliminary –



Want to study the impact on nPDFs, but proton-PDF fits traditionally include $\nu+A$ DIS constraints

We have now the tools we need!

Deweighted: Remove dimuon-DIS data w/ trad. approach from CT18ANLO with deweighting and propagate changes to EPPS21 nuclear PDFs

Updated: Simultaneous reweighting of the deweighted CT18ANLO and EPPS21 PDFs with dimuon-DIS data using the SIDIS approach

Result: More faithful representation of $\nu+A$ dimuon-DIS constraints!

. . . paper in writing w/ H. Paukkunen & S. Yrjänheikki

Thank you!