Correlations in theory uncertainties from missing higher orders

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DIS 2026

5 May 2026, Bologna, Italy

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Correlations

It is crucial to keep correlations into account when analysing data (think of luminosity uncertainty)

In experiments, the origin of correlation is usually "clear"
$\rightarrow$ often governed by parameters (nuisance parameters)

Theoretical predictions are uncertain too:

- due to uncertain parameters (couplings, masses, ...)
- due to **missing higher orders** (MHO)

Correlations are expected from both origins.
The first one is clear (parametric), but what about the second one?

The origin of correlations in MHOU

How do correlations arise in MHOU?

When the perturbative corrections are **related**, e.g. if the underlying physics origin is the same

How can I find out if perturbative corrections are related?

- either I *know* the **underlying physics** origin of the correction, and I'm able to parametrize it
- or I look for **similarities** in the perturbative expansions (this is more general, but also more arbitrary)

Moreover, there can be additional constraints, like:

- the obvious **continuity** requirement for a differential distribution
- the fact that different distributions for the same process must **integrate** to the same value (the total cross section)

$$\int dy \frac{d\Sigma}{dy} = \int dp_t \frac{d\Sigma}{dp_t} = \Sigma_{\text{tot}}$$

- ....

Theory Nuisance Parameters - Tackmann's approach [Tackmann 2411.18606]

Frank Tackmann suggested to use information from **resummation** to parametrize the MHOs
Resummation fixes the functional form of higher orders, in terms of unknown parameters

Example: threshold resummation for Higgs or Drell-Yan ($x = M^2/s$)

$$\int_0^1 dx\, x^N \Sigma(x) = \mathbf{PDFs}(N) \times g_0(\alpha_s)\, \exp S(\alpha_s, N)$$

$$S(\alpha_s, N) = \int_0^1 dz\, \frac{z^N - 1}{1-z} \left[\int_{\mu_F^2}^{M^2(1-z)^2} \frac{d\mu^2}{\mu^2} 2A\big(\alpha_s(\mu^2)\big) + D\big(\alpha_s([1-z]^2 M^2)\big) \right]$$

where

$$A(\alpha_s) = \alpha_s A_1 + \alpha_s^2 A_2 + \alpha_s^3 A_3 + \ldots$$
$$D(\alpha_s) = \alpha_s D_1 + \alpha_s^2 D_2 + \alpha_s^3 D_3 + \ldots$$
$$g_0(\alpha_s) = 1 + \alpha_s g_{01} + \alpha_s^2 g_{02} + \alpha_s^3 g_{03} + \ldots$$

The structure (functional dependence on N, i.e. on x after Mellin inversion) is fixed, the higher order coefficients of A, D, g_0 are unknown $\rightarrow$ nuisance parameters! ☺

If we have a probability distribution $P(A_3, D_3, g_{03}|\text{known orders})$ we can compute:

- $P(\Sigma(x)) \rightarrow$ uncertainty
- $P(\Sigma(x_1), \Sigma(x_2)) \rightarrow$ correlations! (even among different processes)

Determination of distributions of nuisance parameters

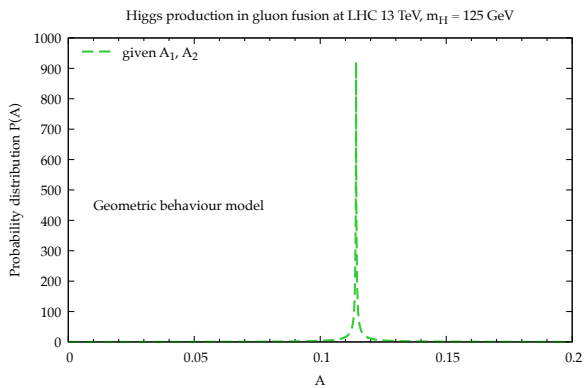
In the approach of [Tackmann 2411.18606], a gaussian probability distribution to the parameters is assigned 😕

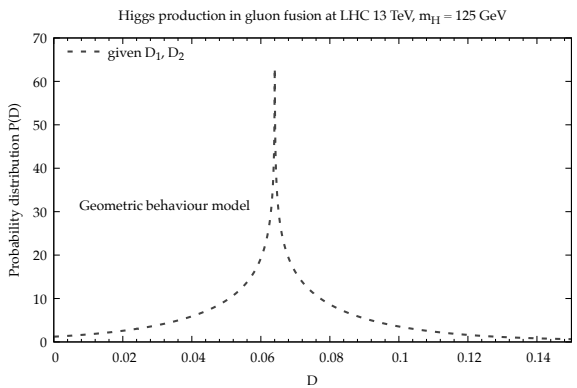
Good news: it is possible to promote the approach to a full-fledged Bayesian one, simply making inference on the parameters 🙂 [Bagnaschi,MB,Paparella (in prepraration)]

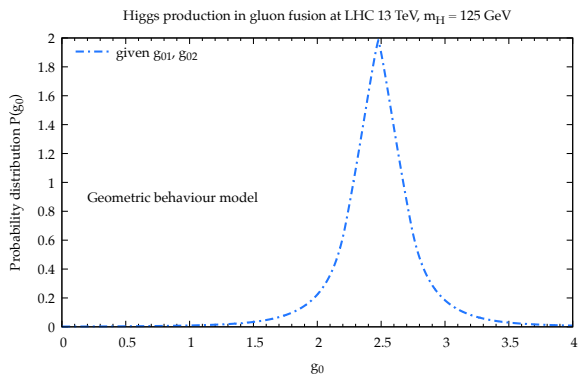
We use the Bayesian *geometric model* introduced in [MB 2006.16293]
With it, we can compute the probabilities

$$P(A_3|A_2, A_1) \qquad P(D_3|D_2, D_1) \qquad P(g_{03}|g_{02}, g_{01})$$

and use them to compute the uncertainty and the correlations on the cross section in a well-defined probabilistic way 🙂





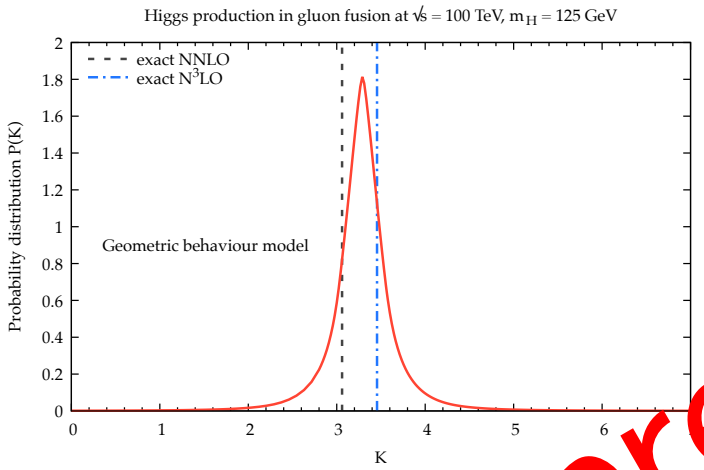


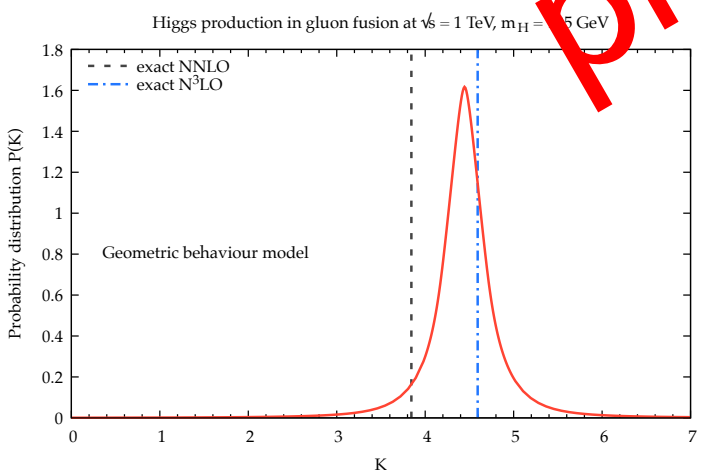
Preliminary results for ggH at NNLO

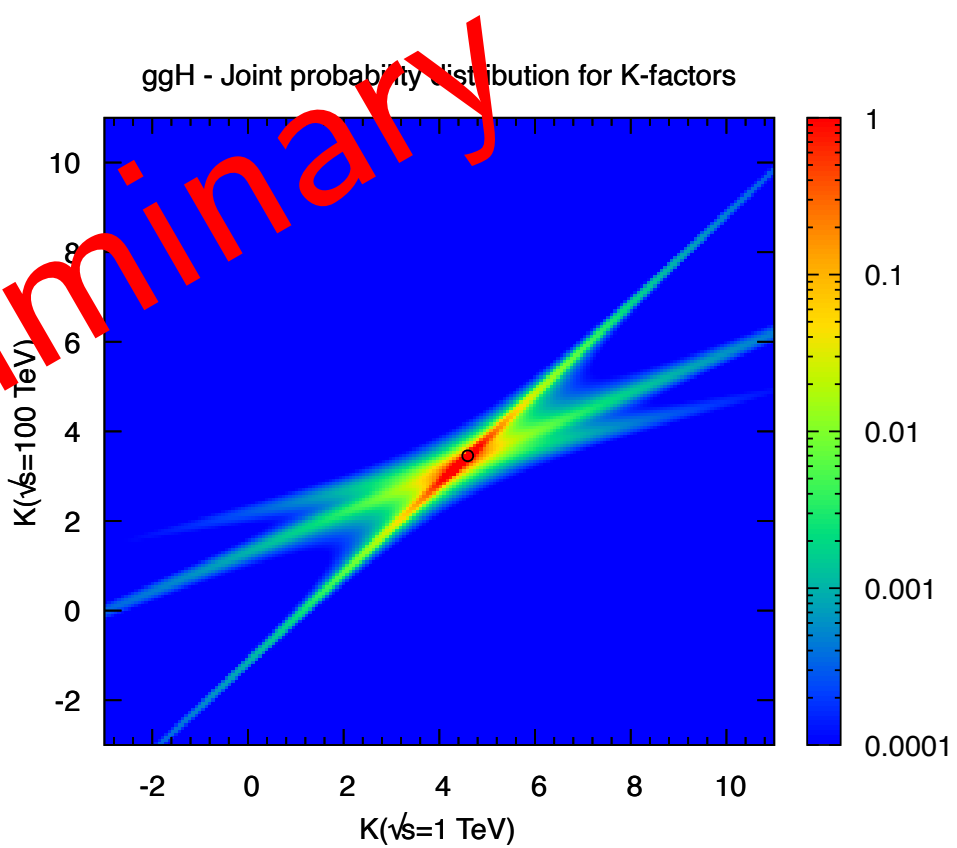
We considered the expansion of the resummed result at threshold, to predict the uncertainty on the NNLO cross section due to the missing N^{3}LO correction

$$\Sigma \simeq \Sigma_{\rm NNLO} + \Sigma_3, \qquad \Sigma_3 = \Sigma_3^{A_3=D_3=g_{03}=0} + \Sigma_3^A A_3 + \Sigma_3^D D_3 + \Sigma_3^g g_{03}$$

[For this exercise, we assumed that the resummed result is the exact one → validation]







preliminary

But resummation is not all!

Tackmann's approach (even in our Bayesian version) is valid only where resummation is dominant ☹

How to treat the other regions?

One could try to mix this approach with a more agnostic one which can be used far from the resummation region.

We can decompose an observable into two terms

$$\Sigma = \Sigma_{\text{from resummation}} + \delta\Sigma$$

We can apply a Tackmann-like approach to $\Sigma_{\text{from resummation}}$, and do something else for $\delta\Sigma$.

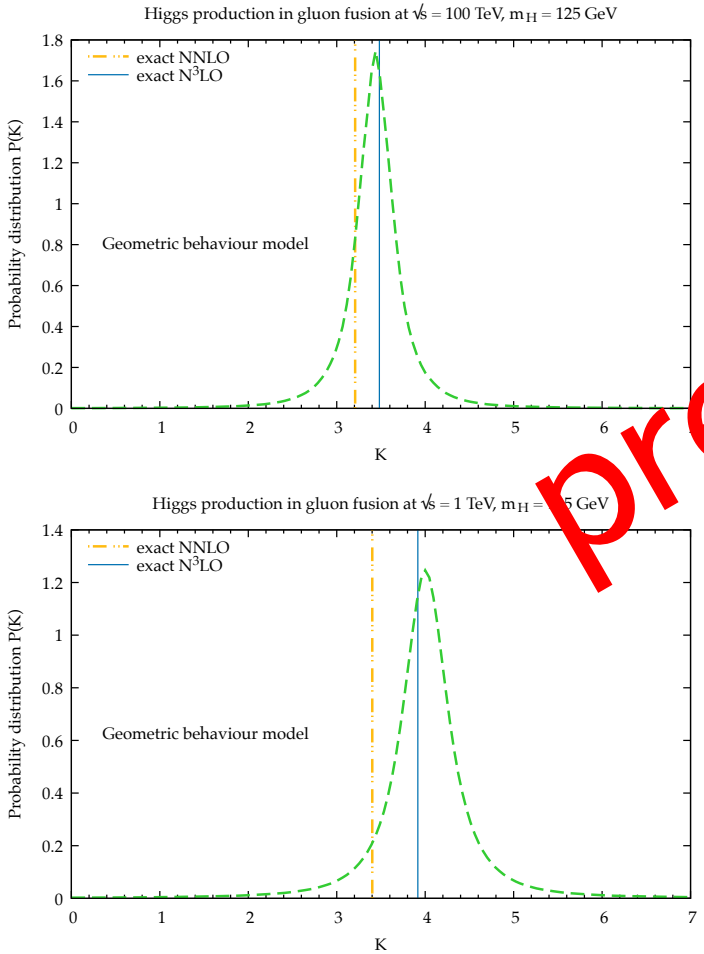
The easiest option is to treat $\delta\Sigma$ as uncorrelated:

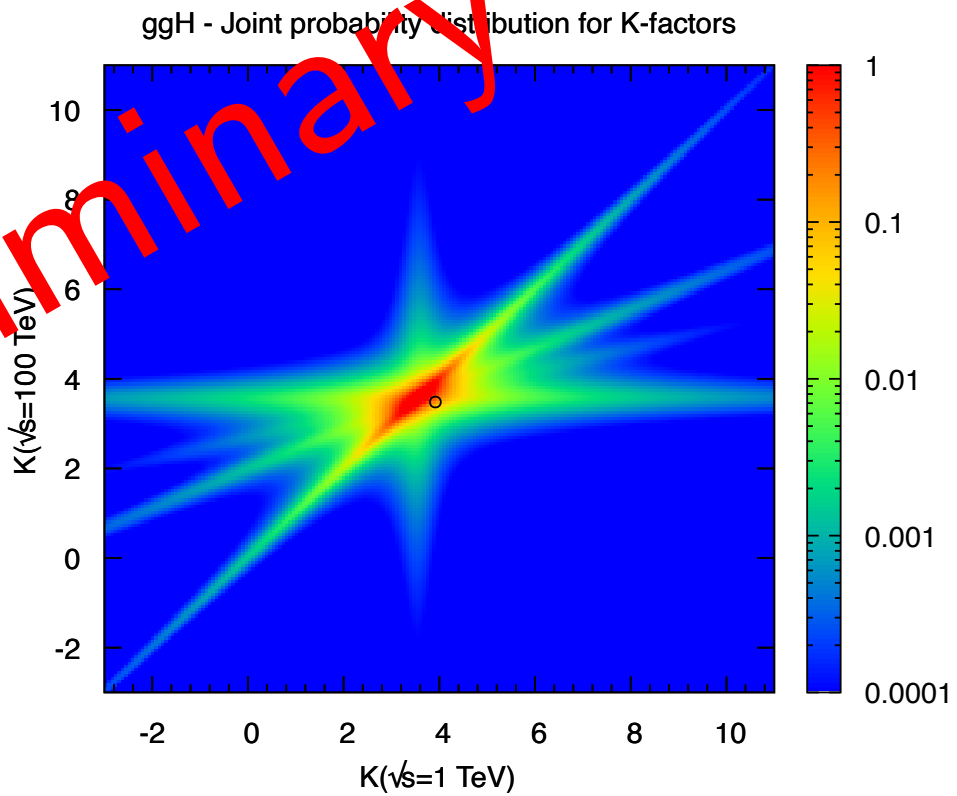
$$P(\delta\Sigma_1, \delta\Sigma_2) = P(\delta\Sigma_1)P(\delta\Sigma_2)$$

For each of them, we can apply the geometric model to construct their distributions

Preliminary results for ggH at NNLO, including uncorrelated piece

We considered the expansion of the resummed result at threshold, to predict the uncertainty on the NNLO cross section due to the missing N^{3}LO correction, plus the remaining part of the fixed-order result treated as uncorrelated





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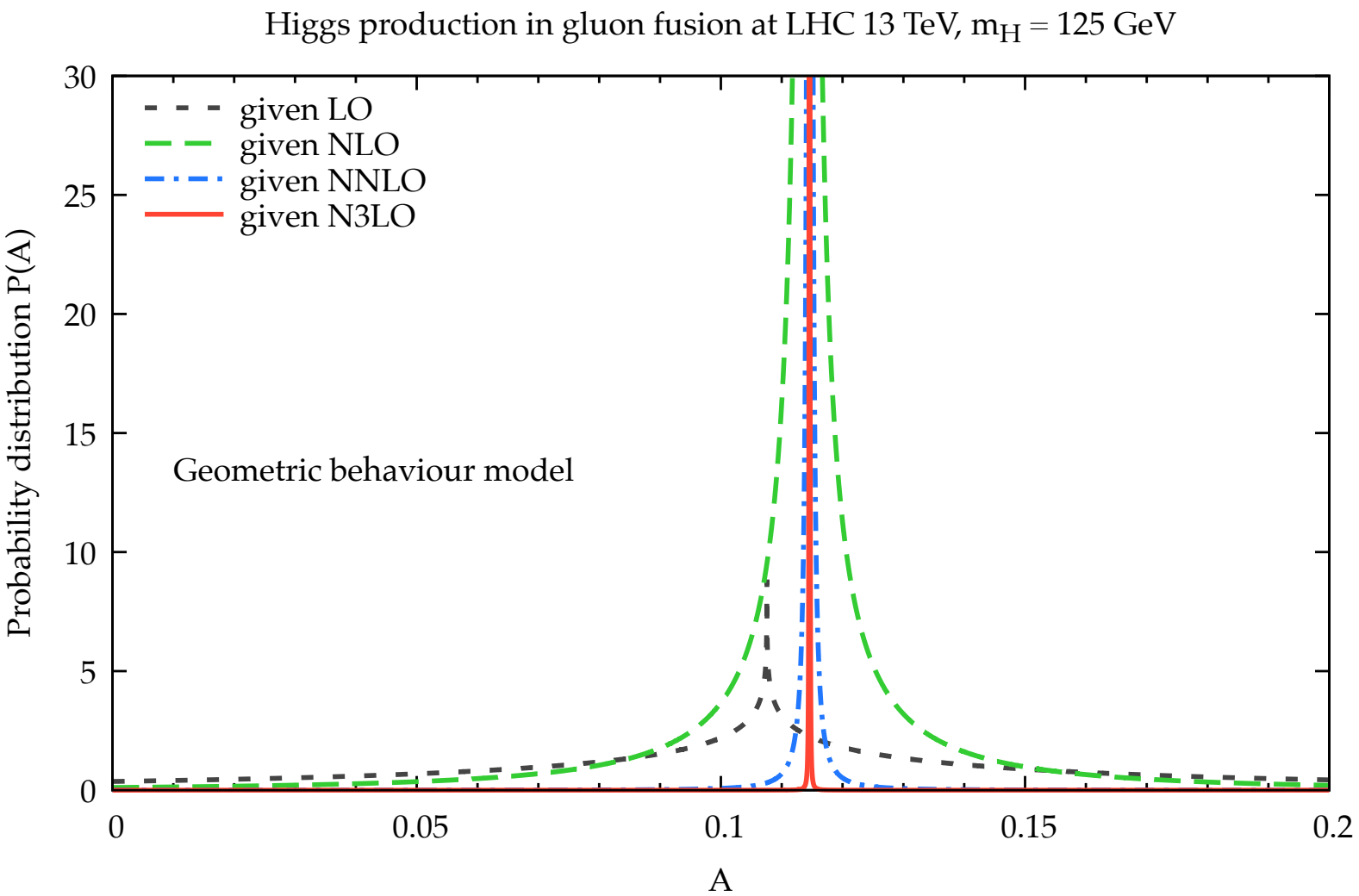
Summary

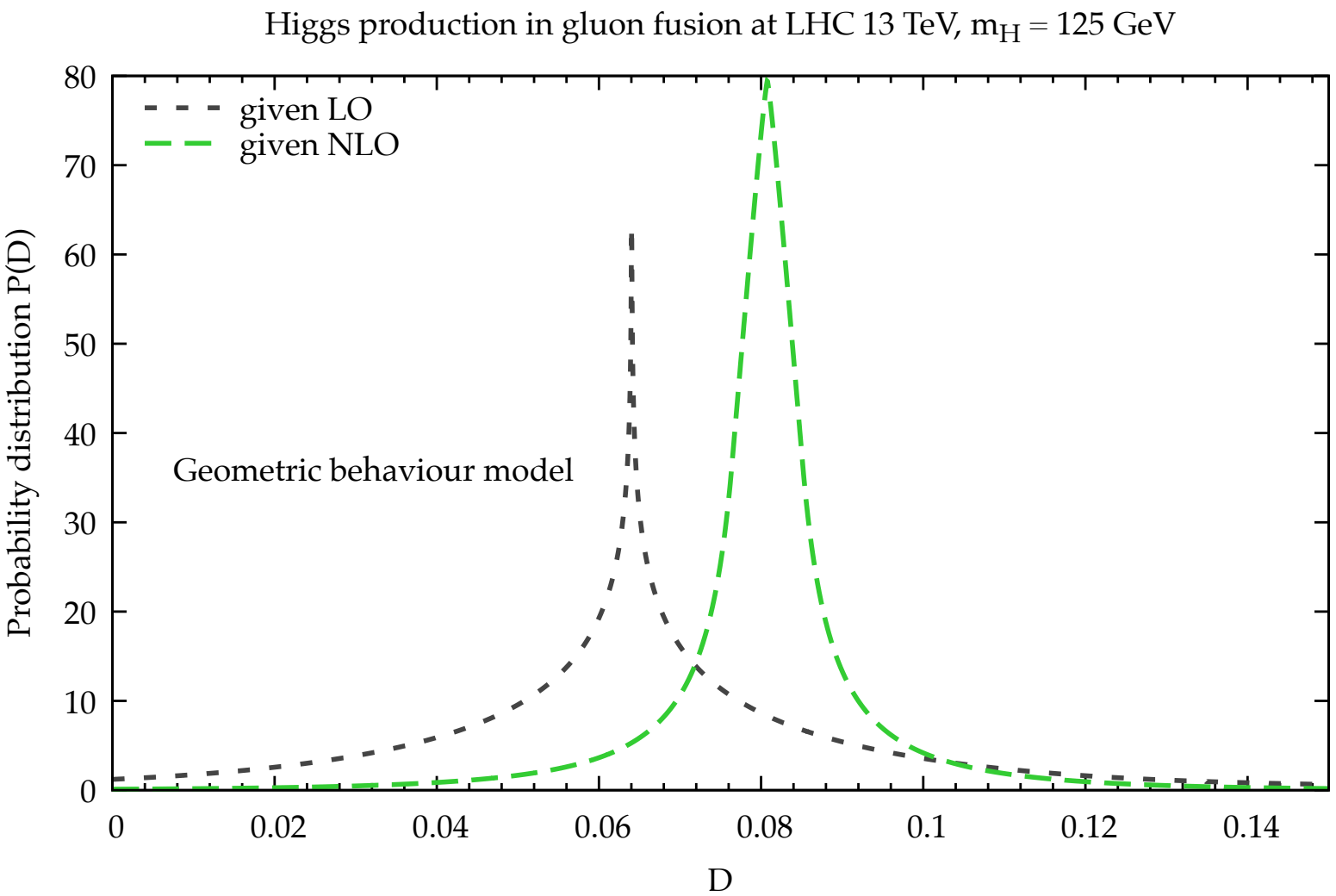
- Correlations in theoretical predictions can be induced by
  - the underlying physics, or ← this is more reliable
  - by similarities in perturbative expansion
- There is no physical reason why higher orders should be related to scale dependence
  → *it's a bad idea to use scale dependence to correlate*
- Tackmann's approach is powerful, and
  - we have "Bayesianised" it
  - we have supplemented it with an uncorrelated treatment of the contributions not coming from resummation
- More agnostic approaches, like the one proposed by Alex Huss, are more versatile
  → *we plan to explore these possibilities!*
- There is no "correct" way of treating MHOU. It is important to explore and test various options! Never use a single recipe!

Backup slides

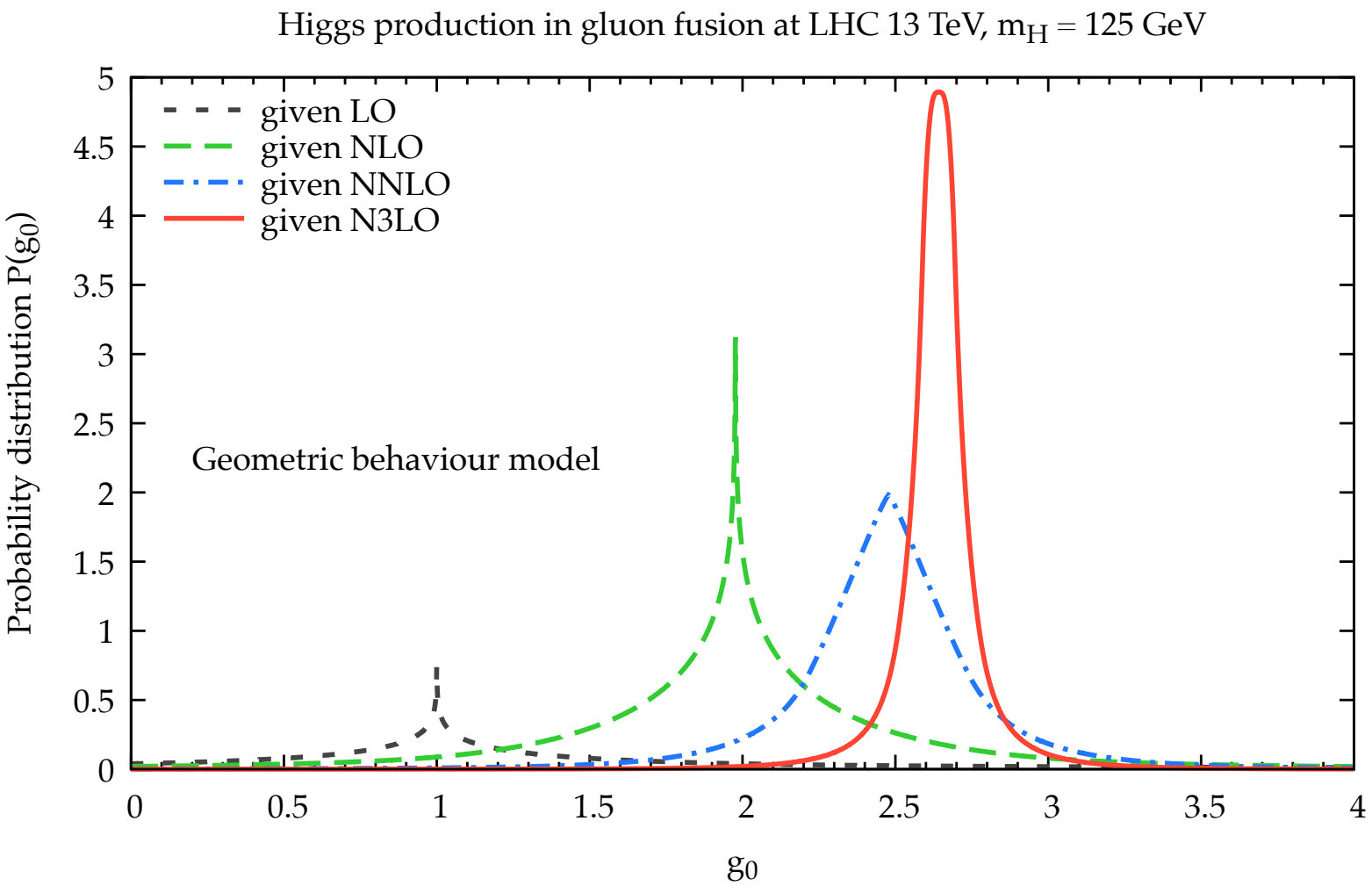
Distributions of threshold resummation parameters for ggH



Distributions of threshold resummation parameters for ggH

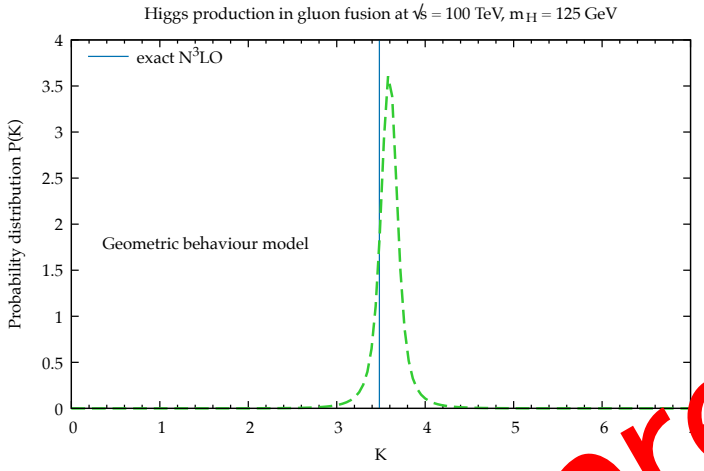


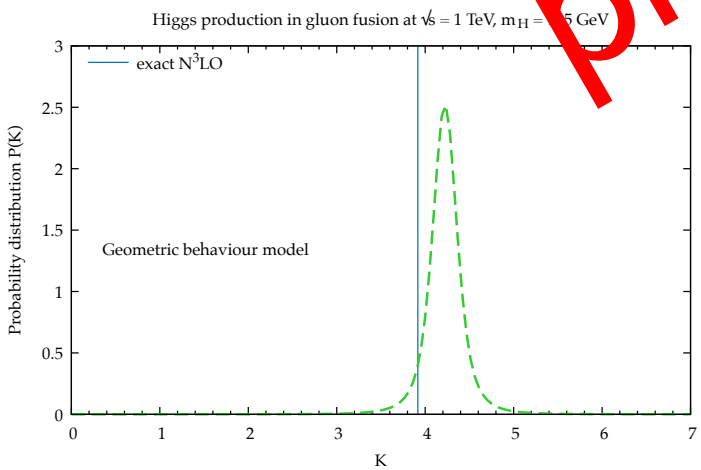
Distributions of threshold resummation parameters for ggH

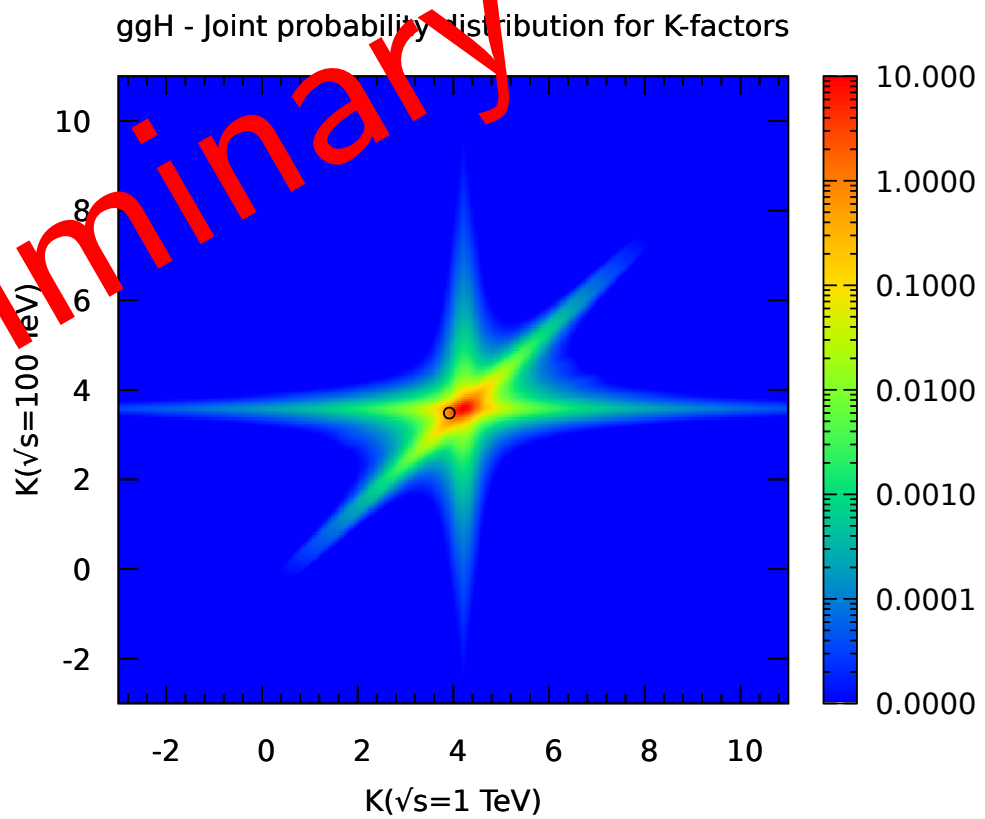


Preliminary results for ggH at N^{3}LO

We considered the expansion of the resummed result at threshold, to predict the uncertainty on the N^{3}LO cross section due to the missing N^{4}LO correction, plus the remaining part of the fixed-order result treated as uncorrelated







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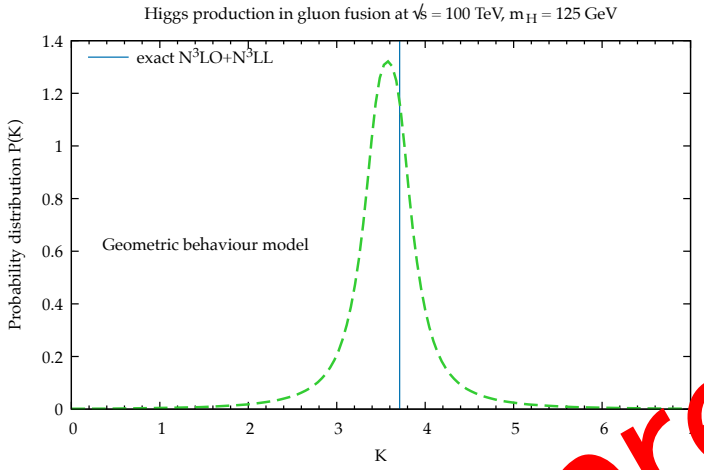
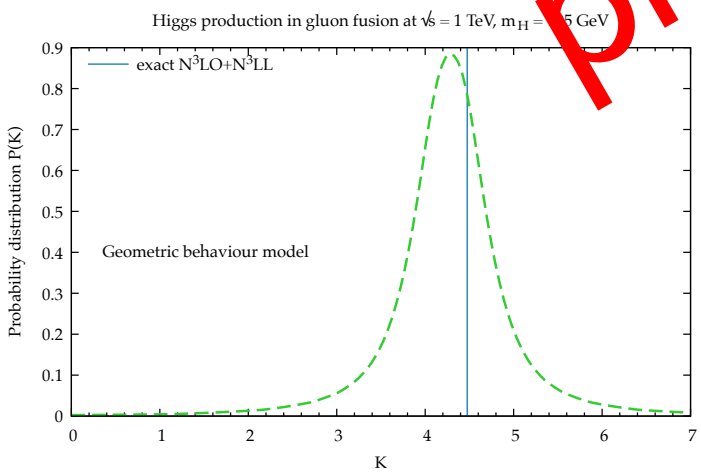
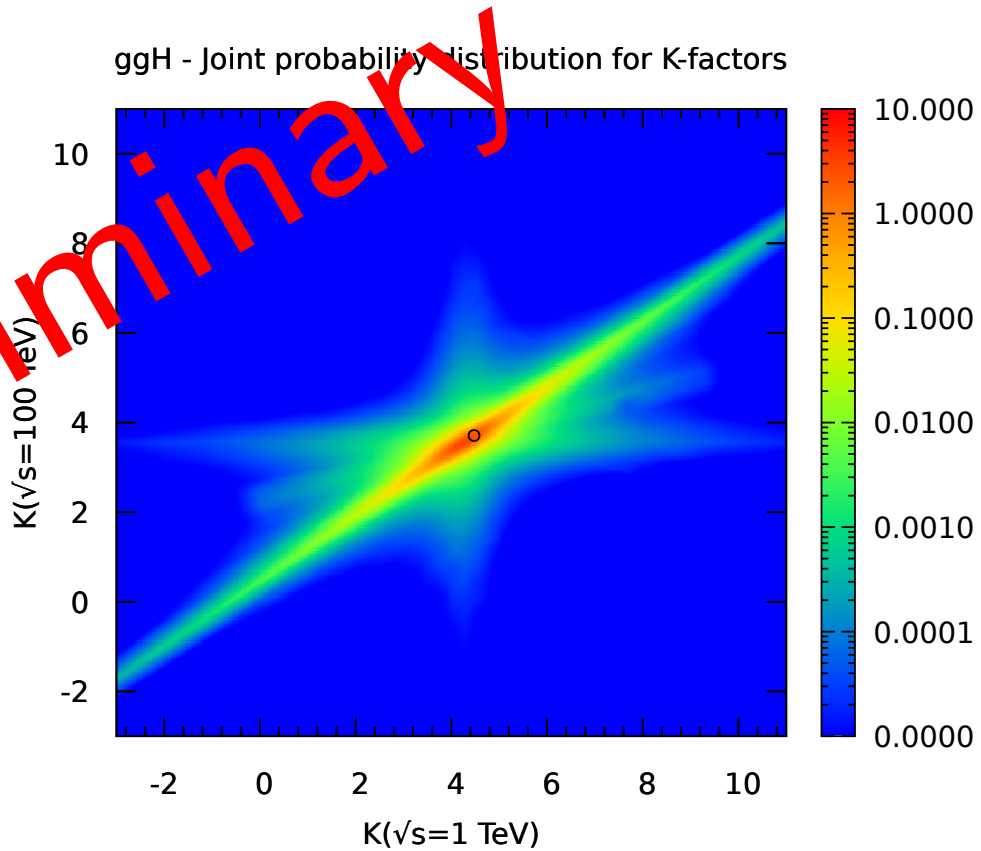
Preliminary results for ggH at N^{3}LL

We can also consider the complete resummed result at threshold, to predict the uncertainty on the N^{3}LL cross section due to the missing N^{4}LL correction, plus the remaining part of the fixed-order (N^{3}LO) result treated as uncorrelated







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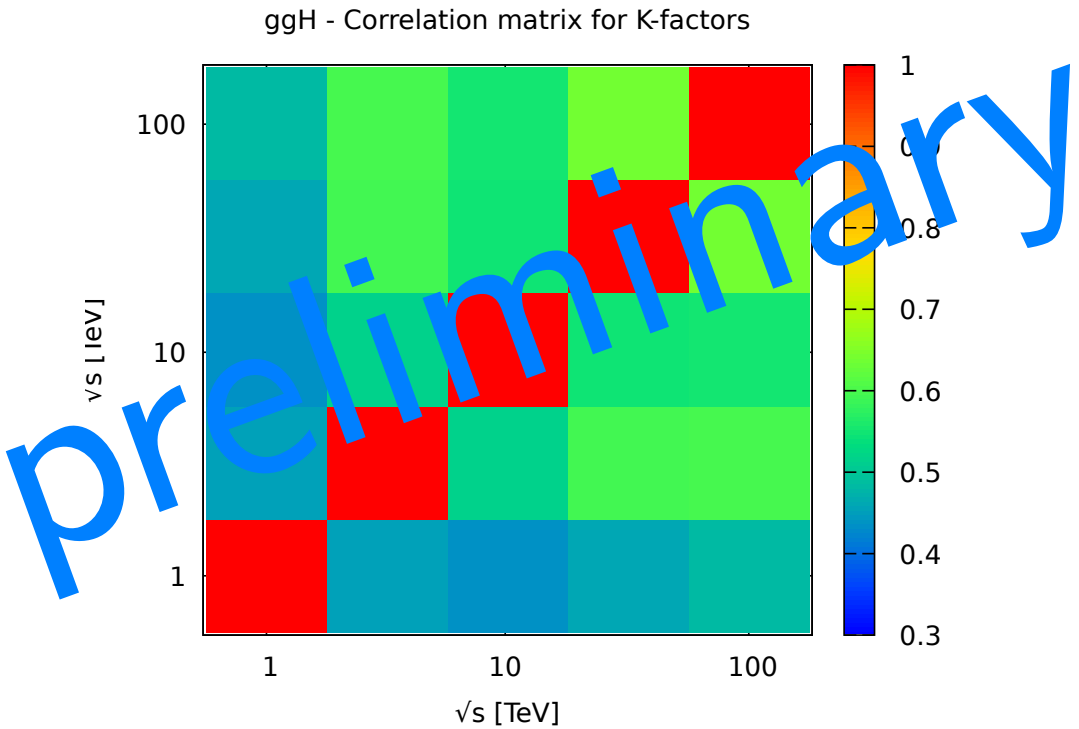
Preliminary results for ggH at N^{3}LO

Out of the joint distribution we can construct the correlation matrix for the $\boldsymbol{K}$-factor at different values of collider energy $\sqrt{s}$

For example, for the N^{3}LO $\boldsymbol{K}$-factor the correlation matrix is



Canonical scale variation

Scale dependence in perturbative computation is higher order

$$\Sigma_{\text{N}^n\text{LO}}(\mu) = \sum_{k=0}^{n} c_k(\mu)\alpha_s^k(\mu)$$

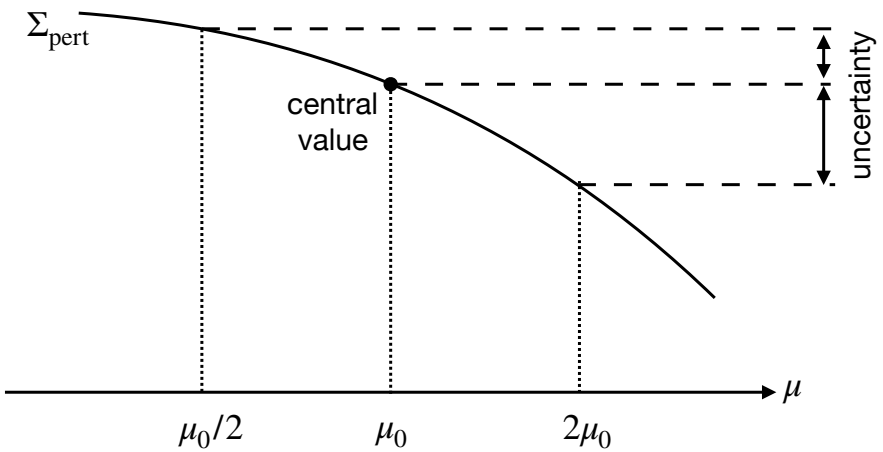
$$\mu\frac{d}{d\mu}\Sigma_{\text{N}^n\text{LO}}(\mu) = \mathcal{O}(\alpha_s^{n+1}) = \mathcal{O}(\Delta_{\text{MHO}})$$

← it's a fact ☺

Canonical Scale Variation:

Variation by a factor of 2 about a "central" scale μ_0

← it's a recipe 😕



Interpret it as a gaussian uncertainty

← it's an arbitrary assumption ☹

The easiest way: scale dependence (NB: easy≠good)

Every theoretical prediction depends on the unphysical (renormalization) scale $\boldsymbol{\mu}$

We can assume that $\boldsymbol{\mu}$ (or better, its ratio over the hard scale) is a parameter that provides the sought correlations. This *would* mean that

if $\boldsymbol{\Sigma_1^{\text{true}} = \Sigma_1^{\text{N}^n\text{LO}}(\bar{\mu})}$ for some $\boldsymbol{\bar{\mu}}$, then $\boldsymbol{\Sigma_2^{\text{true}} = \Sigma_2^{\text{N}^n\text{LO}}(\bar{\mu})}$ for the same $\boldsymbol{\bar{\mu}}$

(where $\mathbf{1}$ and $\mathbf{2}$ correspond to different kinematic points or different observables)

This would imply that at N^{n}LO it exists a single value of $\boldsymbol{\mu}$ such that the perturbative prediction equals the exact result for *any* kinematic point or observable! This is very unlikely... 😑

This procedure embeds the continuity condition of a differential distribution 🙂, but when the points are far away (or belonging to different observables) the correlation generated is **totally arbitrary** ☹