Taking dimuon production in DIS to NNLO precision

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Introduction

Strange-quark distribution

- Relevant for
  - W and Z production at the LHC
  - electroweak parameters
- Still quite poorly known, even for free protons
- Mostly constrained by neutrino-nucleus data
  - nuclear target for sufficient statistics
  - correlation between proton and nuclear PDFs
- Tensions between neutrino data and collider $W(+c)$ production data
  - neutrino data prefer suppressed strange sea
  - ATLAS data prefer unsuppressed sea

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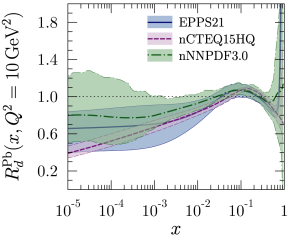
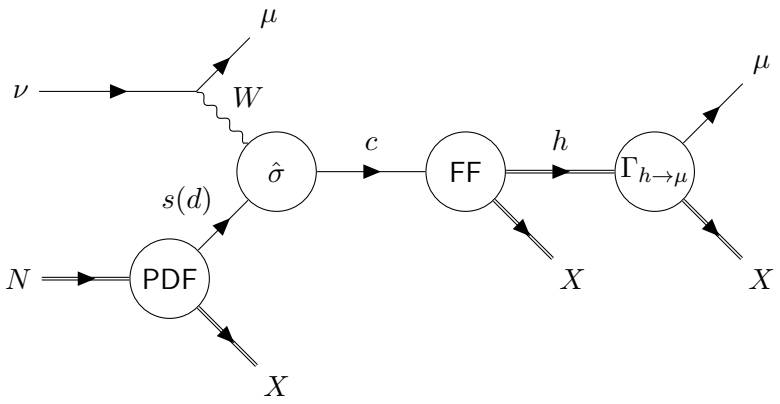


M. Klasen, H. Paukkunen; Ann. Rev. Nucl. Part. Sci. 2023

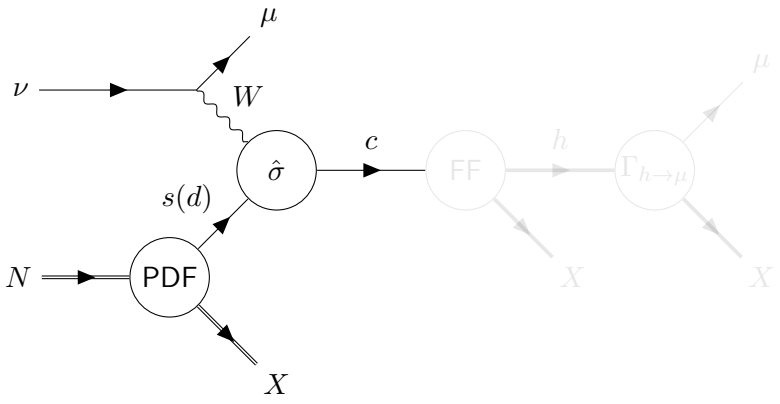
Dimuon production in νN collisions — the SIDIS approach

- Probe s with charm production $\rightarrow$ dimuon production
- SIDIS: charm fragmentation with DGLAP-evolved FFs
- Decay function: charmed-hadron decay to a muon



Dimuon production in νN collisions — the traditional approach

- Usually computed with $\mathrm{d}\sigma(\nu N \to \mu\mu X) \simeq \mathcal{A} \times \mathcal{B}_\mu \times \mathrm{d}\sigma(\nu N \to \mu c X)$
- Needs additional input: external acceptance $\mathcal{A}$, semileptonic branching fraction $\mathcal{B}_\mu$
- **Breaks down for NLO or variable-flavor number schemes**



Decay of charmed hadrons

General formalism

- Assume production of hadron h and its decay factorizes:

$$\sigma(\nu N \to \mu\mu X) = \int \mathrm{d}\sigma(\nu N \to \mu h X) \frac{\Gamma_{h\to\mu}}{\Gamma^h_{\mathrm{tot}}}$$

- Write the decay width in terms of a decay function

$$\Gamma_{h\to\mu} = \frac{1}{2m_h} \int \frac{\mathrm{d}^3\mathbf{P}_\mu}{E_\mu} d_{h\to\mu}(w); \quad w = \frac{P_\mu \cdot P_h}{m_h^2}$$

- Introduce cut on muon energy

$$\Gamma_{h\to\mu}(E_h, E_\mu^{\mathrm{min}}) = \frac{\pi}{m_h} \int \mathrm{d}\rho\, \rho E_h^2 \int \mathrm{d}(\cos\theta) d_{h\to\mu}(w) \Big|_{E_\mu = \rho E_h \geq E_\mu^{\mathrm{min}}}$$

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Fitting the decay function

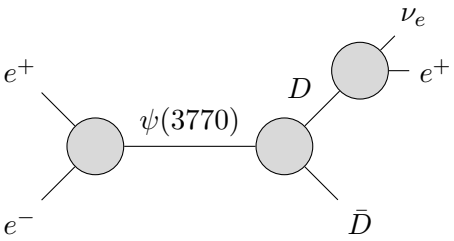
- Decay function parametrization

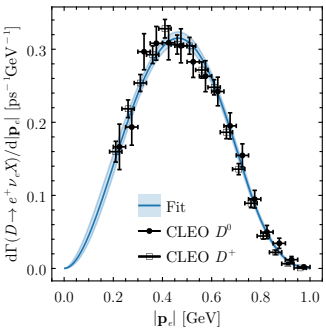
$$d_{D\to\mu}(w) = N w^{\alpha}(1-\gamma w)^{\beta}\theta(0 \le w \le 1/\gamma)$$

- Fit the function

$$\frac{\mathrm{d}\Gamma(D)}{\mathrm{d}\,|\mathbf{P}_e|} = \frac{\pi}{m_D}\,|\mathbf{P}_e| \int \mathrm{d}(\cos\theta) d_{D\to\mu}(w)$$

to CLEO data PRL 97 (2006) 251801





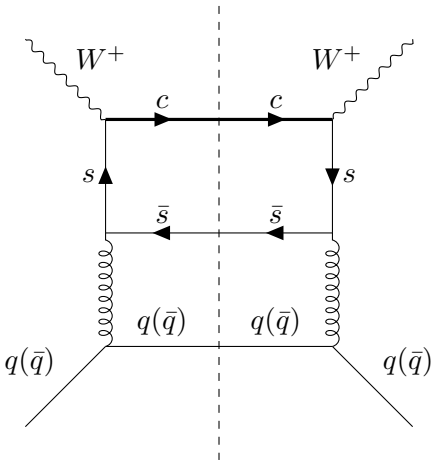
SIDIS at NNLO

NNLO corrections to SIDIS

- Charged-current NNLO SIDIS corrections have recently become available

  Bonino et. al., JHEP 10 (2025) 016
  Goyal et. al, 2603.30012 [hep-ph]

- Unlike at lower orders, charged-current not related to neutral-current via change of coupling
- New partonic channels
  - for example $u \to c$ could be significant due to large valence contribution



Approximate SACOT-χ scheme

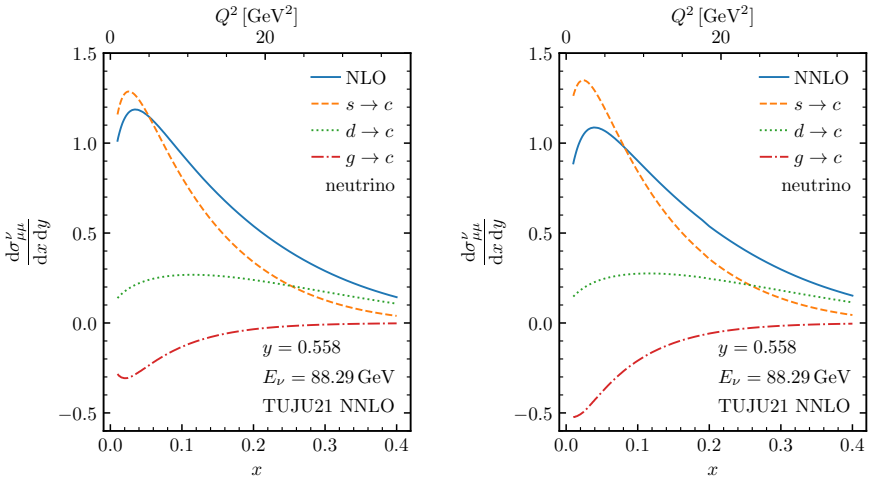
- Mass of charm quark $\sim 1.3 - 1.5\,\text{GeV}$ is non-negligible
- Rescaling variable $\chi = x(1 + m_c^2/Q^2)$ appears in convolution integrals instead of x
- At LO: just the rescaling variable (exact, no dynamical mass corrections)
- At NLO: SACOT-χ scheme Helenius et. al., JHEP 01 (2026) 112
  - charm mass appears explicitly in hard coefficient functions
  - mass logarithms are resummed to evolution of FFs
    $\rightarrow$ subtraction term to avoid double-counting
- At NNLO: approximate with the rescaling variable (misses terms $\mathcal{O}(\alpha_s^2 m_c^2/Q^2)$)
  - explicit dependence on number of active quarks N_f through quark-loop corrections

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0+1)}(\text{SACOT-}\chi) + \mathrm{d}\sigma^{(2)}(\text{ZM-VFNS + }\chi)$$

Results

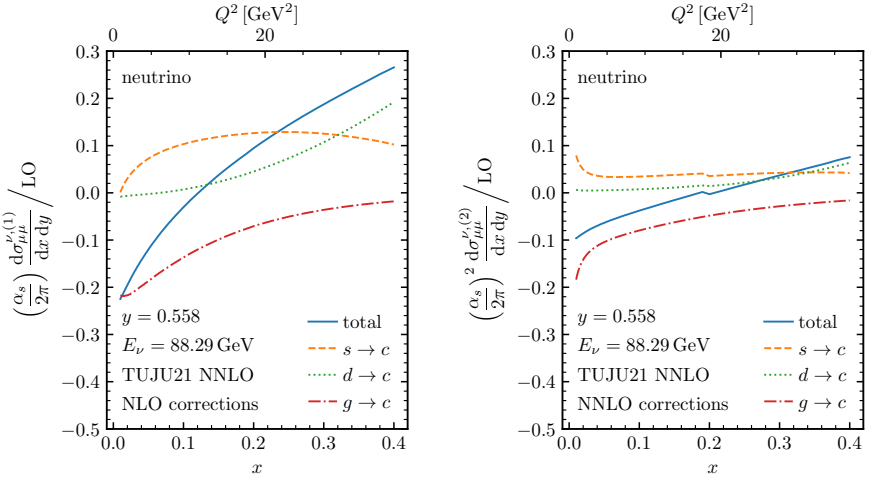
Leading channels — cross section

Cancellation between $s \to c$ and $g \to c$ becomes larger at NNLO



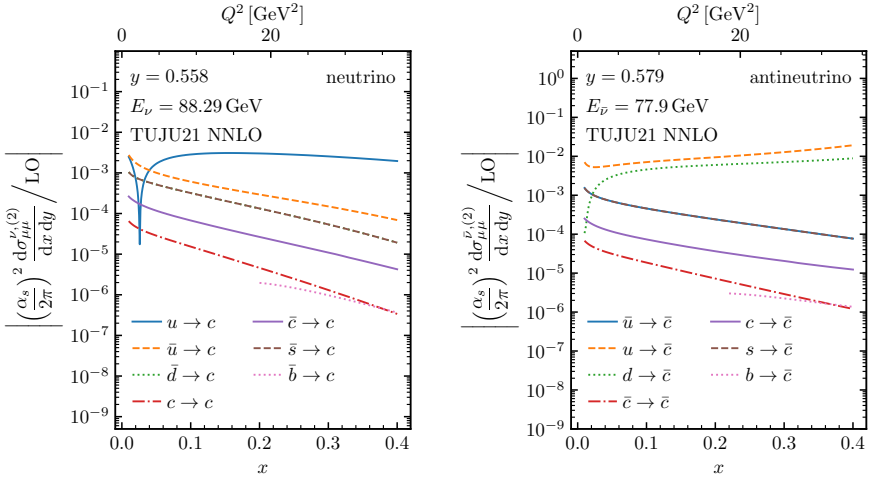
Leading channels — corrections

Corrections to quark channels shrink → convergence; gluon corrections remain similar



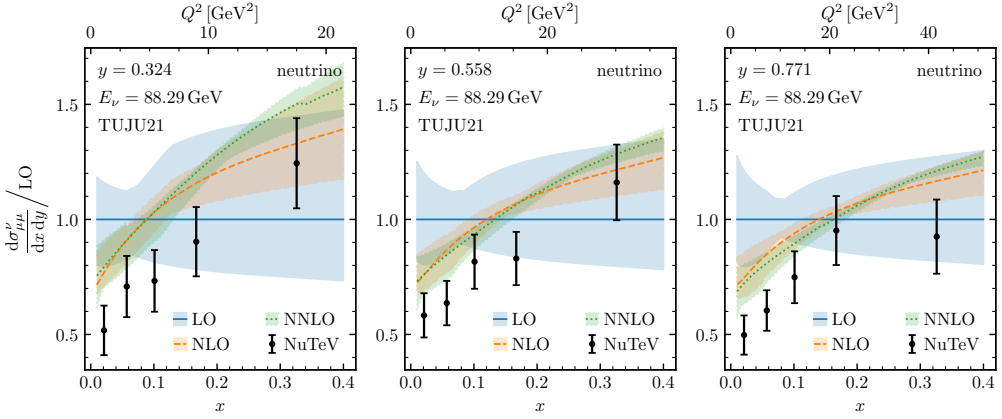
Subleading channels

New channels are orders of magnitude smaller than corrections to leading channels



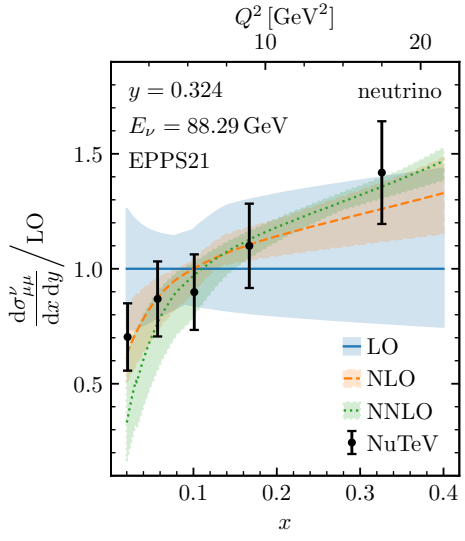
Perturbative convergence and scale uncertainties

- Scale uncertainties shrink at large x, but not at small x
  $\rightarrow$ due to larger cancellation between $s \rightarrow c$ and $g \rightarrow c$?
- NLO $\approx$ NNLO at small x



Suppression of the strange sea

- TUJU21 has unsuppressed strange sea, while EPPS21 allows for suppression
- For TUJU21, NLO $\approx$ NNLO at small x
- For EPPS21, NNLO corrections negative at small x
  $\rightarrow$ alleviates tension between neutrino and collider data



Conclusions & outlook

- We presented NNLO corrections the SIDIS-based approach to dimuon production
- This approach provides a more consistent and systematically improvable way to include NuTeV and CCFR dimuon data (+ future data) to PDF fits
- Dimuon production remains sensitive to strange quark at NNLO
- Potential to alleviate tensions between neutrino and collider data
- Future:
  - study the impact of including this calculation in a PDF fit
    See P. Paakkinen's talk Tue 11:20
  - study the impact of FF uncertainties
    See F. Hekhorn's talk Tue 10:00
  - release code + interpolation grids (later this year)

Backup

SIDIS in the SACOT-χ scheme

- The fixed-order structure functions below the charm-mass threshold $Q^2 < m_c^2$

  [Glück et. al., PLB 398 (1997) 381-386]

$$\hat{F}_i \sim s(\mu^2) \,\hat{\otimes}\, \left[1 + \frac{\alpha_s}{2\pi} H_i^{qq}\right] \tilde{\otimes}\, D_{c\to h} + g(\mu^2) \,\hat{\otimes}\, \left[\frac{\alpha_s}{2\pi} H_i^{qg}\right] \tilde{\otimes}\, D_{c\to h}$$

- Real expression includes all channels, here just $s \to c$ and $g \to c\bar{s}$ for simplicity
- Fragmentation function is scale-independent below charm threshold
- Logarithmic mass divergences in H_i^{qq} need to be resummed

Scale dependence in the charm fragmentation functions

- Based on DGLAP, the scale dependence enters as

$$D_{c\to h}(z,\mu^2) = D_{c\to h}(z) + \frac{\alpha_s}{2\pi}\log\left(\frac{\mu^2}{m_c^2}\right) D_{c\to h} \otimes (P_{qg} + \text{scheme}) + \mathcal{O}(\alpha_s^2)$$

- Easy to invert (up to terms $\mathcal{O}(\alpha_s^2)$):

$$D_{c\to h}(z) = D_{c\to h}(z,\mu^2) - \frac{\alpha_s}{2\pi}\log\left(\frac{\mu^2}{m_c^2}\right) D_{c\to h}(\mu^2) \otimes (P_{qg} + \text{scheme}) + \mathcal{O}(\alpha_s^2)$$

- Insert back to $\hat{F}_i$ to introduce scale dependence and avoid double-counting

$$F_i \sim s(\mu^2) \,\hat{\otimes}\left[1 + \frac{\alpha_s}{2\pi} H_i^{qq}\right] \tilde{\otimes}\, D_{c\to h}(\mu^2) + g(\mu^2) \,\hat{\otimes}\left[\frac{\alpha_s}{2\pi} H_i^{qg}\right] \tilde{\otimes}\, D_{c\to h}(\mu^2)$$
$$- \frac{\alpha_s}{2\pi}\log\left(\frac{\mu^2}{m_c^2}\right) s(\mu^2) \,\hat{\otimes}\left[1 + \frac{\alpha_s}{2\pi} H_i^{qq}\right] \tilde{\otimes}\, D_{c\to h}(\mu^2) \otimes (P_{qg} + \text{scheme})$$

Decay formalism (1/2)

- Assume that the production of a hadron h and its decay factorizes in $AB \to h \to CX$

$$\sigma(AB \to CX) = \frac{1}{2s} \int \mathrm{d(PS)} (2\pi)^4 \delta^{(4)}(P_A + P_B - P_C - P_{X_1} - P_{X_2}) \times (\text{production}) \frac{1}{(P_h^2 - m_h^2)^2 + m_h^2 \Gamma_{\text{tot}}^2} (\text{decay})$$

- Narrow-width approximation

$$\frac{1}{(P_h^2 - m_h^2)^2 + m_h^2 \Gamma_{\text{tot}}^2} \simeq \frac{\pi}{m_h \Gamma_{\text{tot}}} \delta(P_h^2 - m_h^2)$$

Decay formalism (2/2)

- Insert unity as

$$1 = \int \frac{\mathrm{d}P_h^2}{2\pi} \int \frac{\mathrm{d}^3\mathbf{P}_h}{(2\pi)^3 2E_h} (2\pi)^4 \delta^{(4)}(P_h - P_C - P_{X_2})$$

- Identify production cross section and decay width

$$\mathrm{d}\sigma(AB \to hX) = \frac{1}{2s} \int \mathrm{d}(\mathrm{PS})(\textbf{production}) \delta^{(4)}(P_A + P_B - P_{X_1} - P_h - P_C)$$

$$\Gamma_{h\to C} = \frac{1}{2m_h} \int \mathrm{d}(\mathrm{PS})(\textbf{decay}) \delta^{(4)}(P_h - P_{X_2} - P_C)$$

- With these, the cross section becomes

$$\sigma(AB \to CX) = \int \mathrm{d}(\mathrm{PS}) \mathrm{d}\sigma(AB \to hX) \frac{\Gamma_{h\to C}}{\Gamma_{\mathrm{tot}}}$$

Some convolutions written out

$$f(\mu^2) \otimes C(\mu^2) = \int_x^1 \frac{\mathrm{d}\xi}{\xi} f(x/\xi, \mu^2) C(\xi, \mu^2)$$

$$f(\mu^2) \hat{\otimes} C(\mu^2) = \int_\chi^1 \frac{\mathrm{d}\xi}{\xi} f(\chi/\xi, \mu^2) C(\xi, \mu^2)$$

$$f(\mu^2) \hat{\otimes} H \tilde{\otimes} D_c(\mu^2) = \int_\chi^1 \frac{\mathrm{d}\xi}{\xi} \int_{\max(z,\zeta_{\min})}^1 \frac{\mathrm{d}\zeta}{\zeta} f(\chi/\xi, \mu^2) H(\xi, \zeta, \mu^2) D_c(z/\zeta, \mu^2)$$