Constraining Proton Spin at Small x with Valence Quark Model

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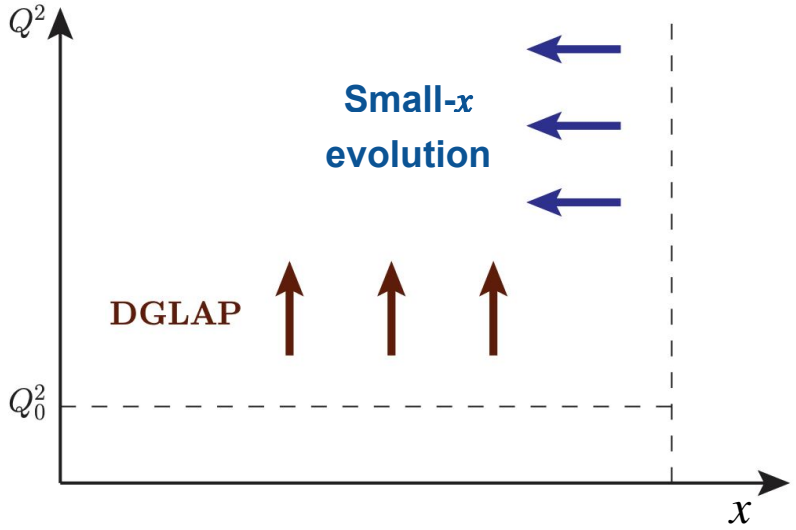
Based on [2407.08893, 2502.16604]

Outline

- (Quick) motivation
- Developments of polarized small-x physics
- The case of helicity
- Calculation within the valence quark model
- Impacts to the global fits
- Generalizability of the valence quark framework
- Conclusions and discussions

Motivations

- Physics at small Bjorken x is difficult to study experimentally, as it requires large center-of-mass energy.
- Small-x evolution could bridge the gap: parton (quark/gluon) densities at small x can be deduced from their values at moderate x.

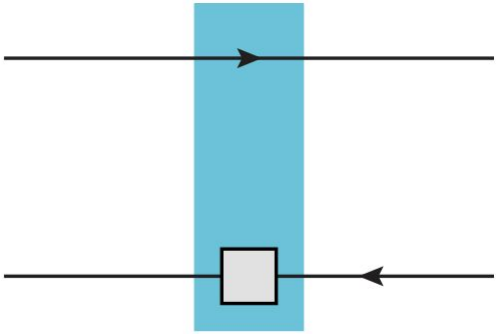


Developments in Polarized Small-x Physics

Plenty of developments in the past decade on small-x evolution of polarized PDFs & TMDs, together with other polarized observables.

- Helicity: [Kovchegov et al, 1511.06737, 1610.06197, 1808.09010, 2104.11765, 2204.11898, 2306.01651, 2406.11647, 2508.00195]
- Longitudinal spin asymmetry: [Kovchegov, Li, 2403.06959]
- Other polarized TMDs: [Kovchegov, Sievert, 1808.10354; Kovchegov, Santiago, 2003.12650, 2108.03667, 2209.03538; Adamiak, Santiago, Tawabutr, 2412.14154; Kovchegov, Li, 2504.12979]
- OAM: [Kovchegov, 1901.07453; Kovchegov, Manley, 2310.18404, 2401.05508, 2410.21260]

Polarized Small-x Evolutions



Things in common

- Involve sub-eikonal or sub-sub-eikonal dipole amplitudes
  - For each TMD or PDF, we keep the first surviving order in eikonality.
- They all resum $\alpha_s \ln^2(1/x)$, already at the leading order.
- Large-N_c or large-N_c&N_f limit: linear system of *polarized dipole amplitudes*
  - A (polarized) CGC-average of (beyond-eikonal) Wilson lines trace
- To specify moderate-x initial conditions (IC), need experimental inputs, which are limited in the polarized sector.

[Adamiak et al, 2308.07461]

The Case of Helicity

- Global analysis performed on polarized DIS and SIDIS data (**226 data pts**).
- Evolution begins at x_0 = 0.1, which is sensible as it resums $\alpha_s \ln^2(1/x)$.
- Include running couple correction with daughter-dipole prescription.
- Initial condition inspired by the Born-level calculation:

$$F(x_{10}^2, zs) = a \ln \frac{zs}{\Lambda^2} + b \ln \frac{1}{x_{10}^2 \Lambda^2} + c$$

for amplitudes $F \in \{Q_u, Q_d, Q_s, \overset{\text{NS}}{Q}_u, \overset{\text{NS}}{Q}_d, \overset{\text{NS}}{Q}_s \widetilde{G}, G_2\}$, which sums to **24 parameters**.

➢ Type-1 amplitude, Q, contains quark exchanges that can be flavor-dependent. Include flavor singlet ($f + \bar{f}$) and non-singlet ($f - \bar{f}$).

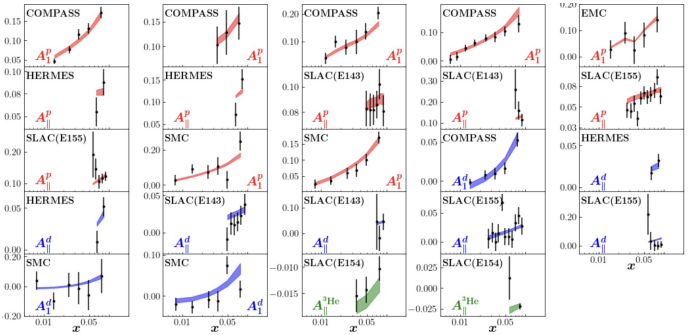
[Adamiak et al, 2308.07461]

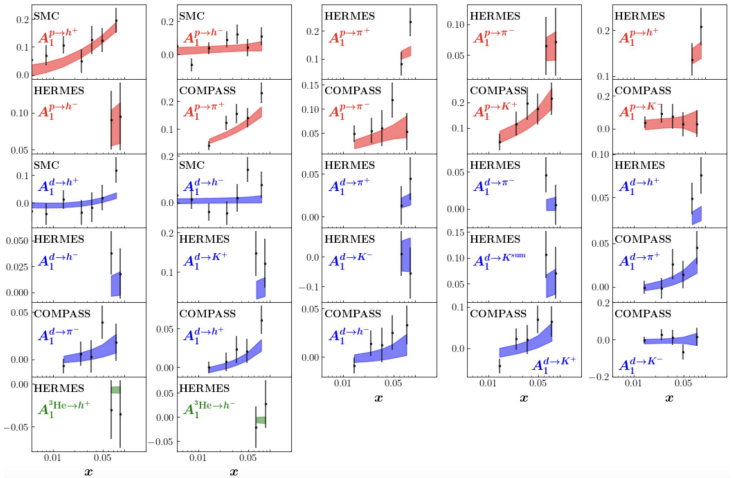
The Case of Helicity

- Great fit with $\chi^2 / N_{\text{pts}} = 1.03$
- Uncertainties of parameters remain large, as evidenced by e.g. total parton spin:

$$S_q + S_G \approx \int_{10^{-7}}^{1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta g\right)(x, Q^2) = -0.52 \pm 0.90$$

- The initial condition can benefit from additional physical constraints.





[Adamiak et al, 2503.21006]

The Case of Helicity

- Recently, include single-inclusive jet production in polarized pp collisions
  - 14 additional data points, now totaling 240 data points
  - $\chi^2 / N_{\text{pts}} = 1.02$
- Improved uncertainty in physical predictions, incl total parton spin

$$S_q + S_G \approx \int_{10^{-7}}^{1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta g\right)(x, Q^2) \;=\; -0.04 \pm 0.23$$

  but can we still do better?
- Other TMDs? Much less small-x data.

[Dumitru, Miller, Venugopalan, 1808.02501]
[Dumitru, Skokov, Stebel, 2001.04516]
[Dumitru, Paatelainen, 2010.11245]

Valence Quark Model as IC

- The initial condition can benefit from additional physical constraints.
- x_0 = 0.1 at the IC for polarized DLA evolutions, closer to the valence quark regime (x ~ 0.33)
  - BK/JIMWLK evolution is at LLA, and fits typically start at x_0 = 0.01.
- Model the **proton target as 3 valence quarks (uud),** together with a perturbative gluon (real or virtual emission)
  - Interactions with the target can be written as field operators sandwiched between these $|uud\rangle$ or $|uudg\rangle$ states (more later).

Valence Quark Model as IC

- We assume that the valence quark wave function is separable: $\psi = \Phi \cdot S$, per the non-relativistic limit of proton wave function [Schlumpf, hep-ph/9212250] (as used in [Dumitru, Mäntysaari, Paatelainen, 2103.11682] as IC for BK equation).
- The momentum-space wave function is Gaussian in transverse momenta with the width fixed by a scale related to longitudinal momentum fraction

$$\Phi(x_i, q_{i\perp}) = N \exp\left[-\frac{1}{2\beta^2}\sum_{i=1}^{3}\frac{q_{i\perp}^2 + M^2}{x_i}\right]$$

where the i-th valence quark has momentum ($x_i P^+$, $x_i P_\perp + q_{i\perp}$).

➢ β and M were fixed in [Schlumpf, hep-ph/9212250] by proton form factors.

[Dumitru, Mäntysaari, Tawabutr, 2407.08893]

Interaction with the Target: Gluon Exchange

- Two types of sub-eikonal Wilson line contribute to helicity:

$$V_{\underline{x}}^{\mathrm{G}[1]} = \frac{i\,g\,P^+}{s} \int_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-]\, F^{12}(x^-, \underline{x})\; V_{\underline{x}}[x^-, -\infty]$$

$$V_{\underline{z}}^{i\,\mathrm{G}[2]} \equiv \frac{P^+}{2s} \int_{-\infty}^{\infty} dz^-\, V_{\underline{z}}[\infty, z^-] \left[D^i(z^-, \underline{z}) - \overleftarrow{D}^i(z^-, \underline{z}) \right] V_{\underline{z}}[z^-, -\infty]$$

with contributions in the form of Wilson line trace, e.g.

$$Q_f^{\mathrm{G}}(x_{10}^2, zs) = \frac{zs}{2N_c} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) \mathrm{Re} \left\langle \mathrm{T\,tr} \left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{G}[1]\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{\mathrm{G}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle$$

where $\langle \cdots \rangle = \frac{1}{2}\,\mathcal{S}_L \sum_{\mathcal{S}_L} \frac{\langle P^+, \underline{P}, \mathcal{S}_L | \cdots | P^+, \underline{P}, \mathcal{S}_L \rangle}{\langle P^+, \underline{P}, \mathcal{S}_L | P^+, \underline{P}, \mathcal{S}_L \rangle}$, so the amplitude ~ $A^+ A^i$

[Dumitru, Mäntysaari, Tawabutr, 2407.08893]

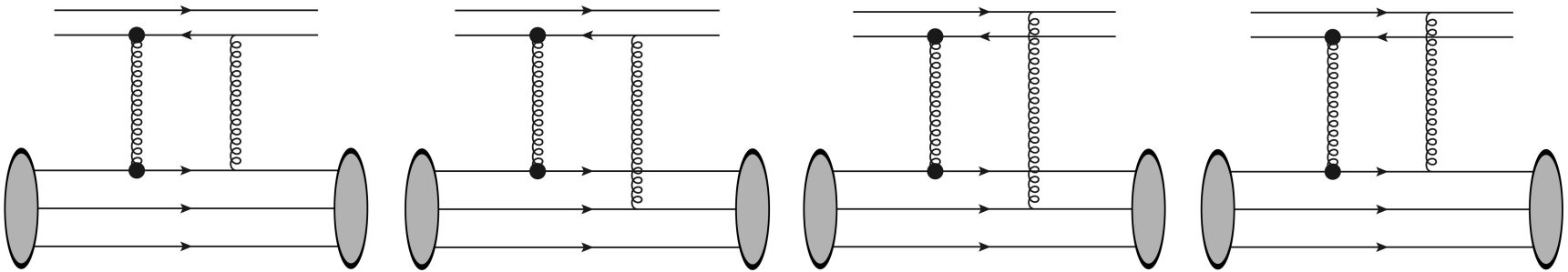
Interaction with the Target: Gluon Exchange

- At the end,

$$Q_f^{\mathrm{G}}(r_\perp, zs) = \frac{1}{3}\,\widetilde{G}^{\mathrm{G}}(r_\perp, zs) = \frac{16\pi^2\alpha_{\mathrm{s}}^2}{9}\int \frac{\mathrm{d}^2\underline{p}}{(2\pi)^2}\,\frac{1}{p_\perp^2}\left[1 - e^{-i\underline{p}\cdot\underline{r}}\right]$$
$$\times \int \frac{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3}{(4\pi)^3}\,4\pi\delta(1-x_1-x_2-x_3)\int \frac{\mathrm{d}^2\underline{q}_1\,\mathrm{d}^2\underline{q}_2\,\mathrm{d}^2\underline{q}_3}{(2\pi)^6}\,(2\pi)^2\delta^2(\underline{q}_1+\underline{q}_2+\underline{q}_3)\,\frac{1}{x_1}$$
$$\times \left[\left|\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right|^2 - \Phi^*(x_1,\underline{q}_1+\underline{p};x_2,\underline{q}_2-\underline{p};x_3,\underline{q}_3)\,\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right]$$

$$G_2(r_\perp, zs) = -\frac{16\pi^2\alpha_{\mathrm{s}}^2}{9}\int \frac{\mathrm{d}^2\underline{p}}{(2\pi)^2}\,\frac{i(\underline{p}\cdot\underline{r})}{p_\perp^4 r_\perp^2}\left[1 - e^{-i\underline{p}\cdot\underline{r}}\right]$$
$$\times \int \frac{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3}{(4\pi)^3}\,4\pi\delta(1-x_1-x_2-x_3)\int \frac{\mathrm{d}^2\underline{q}_1\,\mathrm{d}^2\underline{q}_2\,\mathrm{d}^2\underline{q}_3}{(2\pi)^6}\,(2\pi)^2\delta^2(\underline{q}_1+\underline{q}_2+\underline{q}_3)\,\frac{1}{x_1}$$
$$\times \left[\left|\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right|^2 - \Phi^*(x_1,\underline{q}_1+\underline{p};x_2,\underline{q}_2-\underline{p};x_3,\underline{q}_3)\,\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right]$$

- Gluon exchange terms do not know about flavors



Interaction with the Target: Quark Exchange

- The quark-exchange Wilson line is

$$V_{\underline{x}}^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\,s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\, \left[\gamma^+\gamma^5\right]_{\alpha\beta}\, \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty]$$

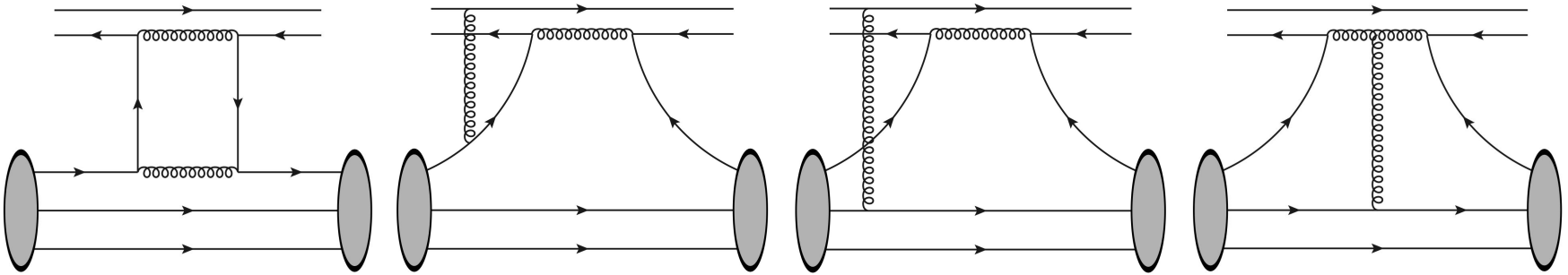
  such that

$$Q_f^{\mathrm{q}}(x_{10}^2, zs) = \frac{zs}{2N_c} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) \mathrm{Re} \left\langle \mathrm{T\,tr} \left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{q}[1]\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{\mathrm{q}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle$$

- Here, the quark fields in the sub-eikonal Wilson line yield creation and annihilation operators that act directly on $|uud\rangle$ or $|uudg\rangle$ terms in the proton's state.

[Dumitru, Mäntysaari, Tawabutr, 2407.08893]

Interaction with the Target: Quark Exchange



- For quark exchanges, we now have flavor dependence:

$$Q_f^{\mathrm{q}}(r_\perp, zs) = Q_f^{\mathrm{NS,q}}(r_\perp, zs) = -\frac{8\pi^2\alpha_s^2}{81}\left[4\delta^{f,u} - \delta^{f,d}\right]\int \frac{\mathrm{d}^2\underline{p}}{(2\pi)^2}\,\frac{1}{p_\perp^2}$$

$$\times \int \frac{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3}{(4\pi)^3}\,4\pi\delta(1-x_1-x_2-x_3)\int \frac{\mathrm{d}^2\underline{q}_1\,\mathrm{d}^2\underline{q}_2\,\mathrm{d}^2\underline{q}_3}{(2\pi)^6}\,(2\pi)^2\delta^2(\underline{q}_1+\underline{q}_2+\underline{q}_3)\,\frac{1}{x_1}$$

$$\times \left[\left(4+e^{-i\underline{p}\cdot\underline{r}}\right)\left|\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right|^2 + \left(1-e^{-i\underline{p}\cdot\underline{r}}\right)\Phi^*(x_1,\underline{q}_1+\underline{p};x_2,\underline{q}_2-\underline{p};x_3,\underline{q}_3)\,\Phi(x_1,\underline{q}_1;x_2,\underline{q}_2;x_3,\underline{q}_3)\right]$$

Impacts to the Fit

- The final results are the sum of gluon- and quark-exchange contributions.
- In term of the Born-inspired initial condition employed in previous fit:

$$F(x_{10}^2, zs) = a \ln \frac{zs}{\Lambda^2} + b \ln \frac{1}{x_{10}^2 \Lambda^2} + c$$

for amplitude $F \in \{Q_u, Q_d, Q_s, \overset{\text{NS}}{Q}_u, \overset{\text{NS}}{Q}_d, \overset{\text{NS}}{Q}_s, \widetilde{G}, G_2\}$, **all a's and b's are fixed up to the factor of α_s^2**. In total, the number of parameters reduces **from 24 to 8**: constant terms, c, for 8 amplitudes, plus the prescription for α_s.

  - The fixed parameters agree with their values from [Adamiak et al, 2308.07461]
- The infrared scale, Λ, can be determined from fit to our numerical integral.

[Adamiak, Mäntysaari, Tawabutr, 2502.16604]

Impacts to the Fit

- With the new IC (24 → 8 free parameters), we perform a new fit with polarized DIS and SIDIS data.
- α_s follows the daughter dipole prescription in the small-x evolution and the dipole size prescription in the free parameter:

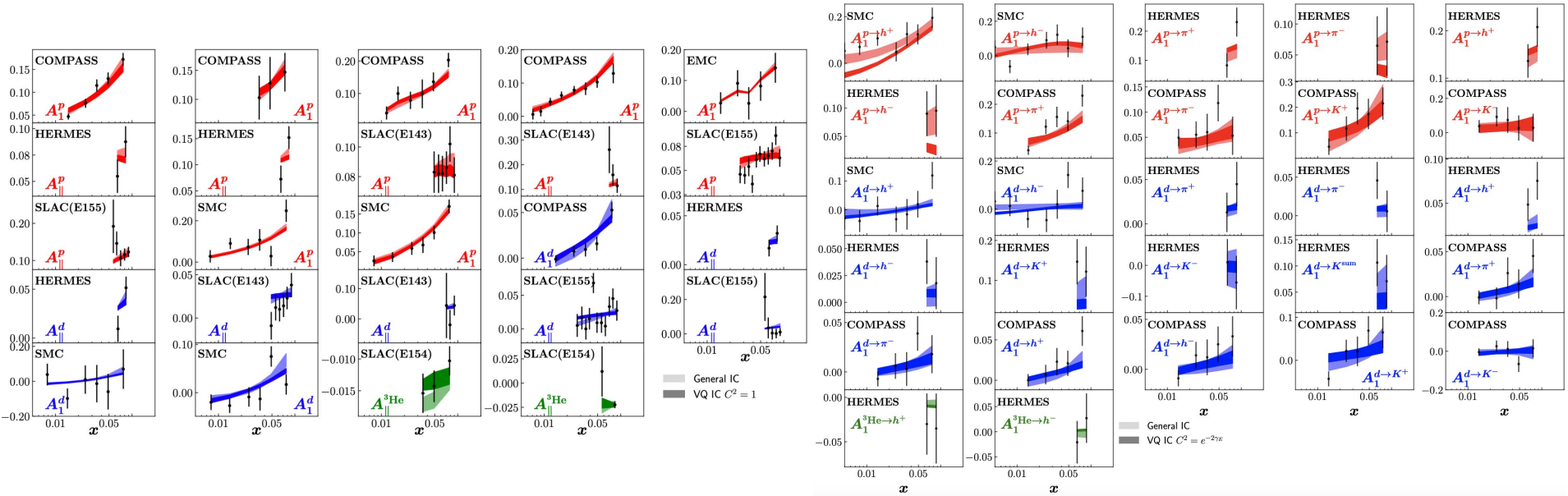
$$\alpha_{\mathrm{s}}(r_\perp) = \frac{12\pi}{(33 - 2N_{\mathrm{f}})\,\ln\left(\frac{4C^2}{r_\perp^2\,\Lambda_{\mathrm{QCD}}^2}\right)}$$

with the same constant under the log, C^2.

[Adamiak, Mäntysaari, Tawabutr, 2502.16604]

Impacts to the Fit

- We get $\chi^2 / N_{\text{pts}} = 1.26$ for $C = 1$ and $\chi^2 / N_{\text{pts}} = 1.28$ for $C = e^{-2\gamma_E}$



[Adamiak, Mäntysaari, Tawabutr, 2502.16604]

Impacts to the Fit

- Physical predictions provide great (even compared to adding pp data):
  - Generalized Born IC: $S_q + S_G \approx \int_{10^{-5}}^{0.1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = -0.64 \pm 0.60$
  - Valence quark IC: $S_q + S_G \approx \int_{10^{-5}}^{0.1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = 0.63 \pm 0.10$
- The fit provides a proof of principle for the valence quark model, but there remains disagreement in the central value of the spins.

Generalizability of the VQ Framework

- **Immediately possible** to implement for single-inclusive jet production in polarized **pp**, which depends on the same polarized dipole amplitudes.
- Possible to apply to **other TMDs**, whose small-x evolutions all involve *different* traces of 1 eikonal and 1 beyond-eikonal Wilson lines (require additional calculations)
- Possible to substitute a different light-front model for a proton.
    - In particular, include the fact that *helicity* is projected onto parton's momentum, which is *not exactly* the light-cone direction: Melosh rotation

Conclusion & Future Works

- The valence quark model provides physical inputs and put additional constraints to the initial conditions at moderate x_0 = 0.1 for polarized dipole amplitudes in helicity evolution at DLA.
- The fit shows promising signs of agreement, but further corrections from the VQ model side should be beneficial.
- Next step: include Melosh rotation and perform another fit with polarized pp.

Backup Slides

[Dumitru, Miller, Venugopalan, 1808.02501]
[Dumitru, Skokov, Stebel, 2001.04516]
[Dumitru, Paatelainen, 2010.11245]

Valence Quark Model

- Polarized dipole amplitudes involve helicity-dependent averaging over (target) proton state:

$$\langle \cdots \rangle = \frac{1}{2} \sum_{\mathcal{S}_L} \mathcal{S}_L \frac{\langle P^+, \underline{P}, \mathcal{S}_L | \cdots | P^+, \underline{P}, \mathcal{S}_L \rangle}{\langle P^+, \underline{P}, \mathcal{S}_L | P^+, \underline{P}, \mathcal{S}_L \rangle}$$

with $\langle K^+, \underline{K}, \mathcal{S}'_L | P^+, \underline{P}, \mathcal{S}_L \rangle = \delta_{\mathcal{S}_L \mathcal{S}'_L} \, 2P^+ \, 2\pi\delta(P^+ - K^+) \, (2\pi)^2 \delta^2(\underline{P} - \underline{K})$

- The proton state can be written as

$$|P^+, \underline{P}, \mathcal{S}_L\rangle = \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{(4\pi)^3 \sqrt{x_1 x_2 x_3}} \, 4\pi\delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 q_1 d^2 q_2 d^2 q_3}{(2\pi)^6} \, (2\pi)^2 \delta^2(\underline{q}_1 + \underline{q}_2 + \underline{q}_3)$$
$$\times \sum_{\{f_1, f_2, f_3\} = \{u, u, d\}} \sum_{\sigma_1, \sigma_2, \sigma_3} \Phi(x_1, \underline{q}_1; x_2, \underline{q}_2; x_3, \underline{q}_3) \, S(\sigma_1, f_1; \sigma_2, f_2; \sigma_3, f_3)$$
$$\times \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} \, |x_1 P^+, x_1 \underline{P} + \underline{q}_1, i_1, \sigma_1, f_1\rangle \, |x_2 P^+, x_2 \underline{P} + \underline{q}_2, i_2, \sigma_2, f_2\rangle \, |x_3 P^+, x_3 \underline{P} + \underline{q}_3, i_3, \sigma_3, f_3\rangle$$

[Dumitru, Miller, Venugopalan, 1808.02501]
[Dumitru, Skokov, Stebel, 2001.04516]
[Dumitru, Paatelainen, 2010.11245]

Valence Quark Model

- We assume that the valence quark wave function is separable: $\psi = \Phi \cdot S$, per the non-relativistic limit of proton wave function [Schlumpf, hep-ph/9212250] (also used in [Dumitru, Mäntysaari, Paatelainen, 2103.11682]).
- The flavor-spin wave function, S, taken to be totally symmetric:

$$
\begin{aligned}
S(\sigma_1, f_1; \sigma_2, f_2; \sigma_3, f_3) = \frac{1}{\sqrt{18}} \Big\{ & \left[2\delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,-\mathcal{S}_L} - \delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,-\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} - \delta_{\sigma_1,-\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} \right] \delta^{f_1,u}\delta^{f_2,u}\delta^{f_3,d} \\
& + \left[2\delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,-\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} - \delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,-\mathcal{S}_L} - \delta_{\sigma_1,-\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} \right] \delta^{f_1,u}\delta^{f_2,d}\delta^{f_3,u} \\
& + \left[2\delta_{\sigma_1,-\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} - \delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,-\mathcal{S}_L}\delta_{\sigma_3,\mathcal{S}_L} - \delta_{\sigma_1,\mathcal{S}_L}\delta_{\sigma_2,\mathcal{S}_L}\delta_{\sigma_3,-\mathcal{S}_L} \right] \delta^{f_1,d}\delta^{f_2,u}\delta^{f_3,u} \Big\}
\end{aligned}
$$

where S_L is the proton's helicity, $\{\sigma_1, \sigma_2, \sigma_3\}$ is the valence quark's helicity and $\{f_1, f_2, f_3\}$ is the valence quark's flavor.

Results

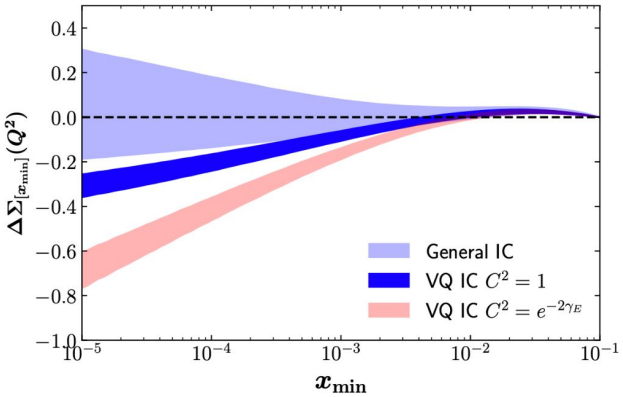
- The values of parameters a's and b's determined here are within the 95% confidence interval of their estimates in the fit [Adamiak et al, 2308.07461].

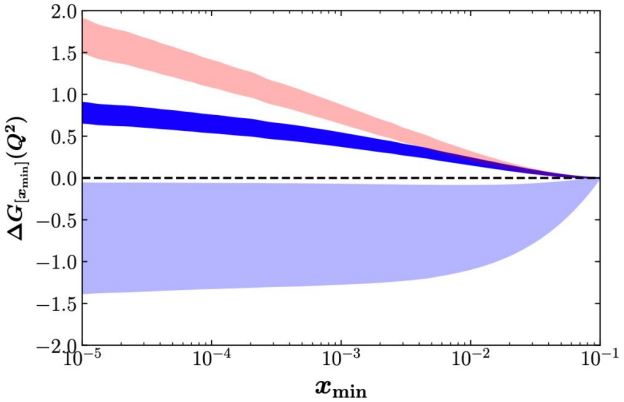
	$\ln(1/r_\perp^2\Lambda^2)$		$\ln(zs/\Lambda^2)$	
	Valence qk	JAM 2023	Valence qk	JAM 2023
Q_u	-0.360	6.822 ± 8.488	0.033	-0.780 ± 4.985
Q_d	-0.278	-7.429 ± 10.818	0.360	6.923 ± 6.479
Q_s	-0.295	-0.547 ± 13.949	0.295	8.785 ± 9.415
Q_u^{NS}	-0.065	4.688 ± 10.600	-0.262	-5.914 ± 6.666
Q_d^{NS}	0.016	2.971 ± 16.022	0.065	-5.735 ± 10.557
Q_s^{NS}	0	-5.183 ± 16.277	0	6.281 ± 9.970
$\widetilde{G}$	-0.810	-4.233 ± 11.079	0.847	2.956 ± 4.416
G_2	0.147	-1.762 ± 3.684	0	-0.203 ± 2.155

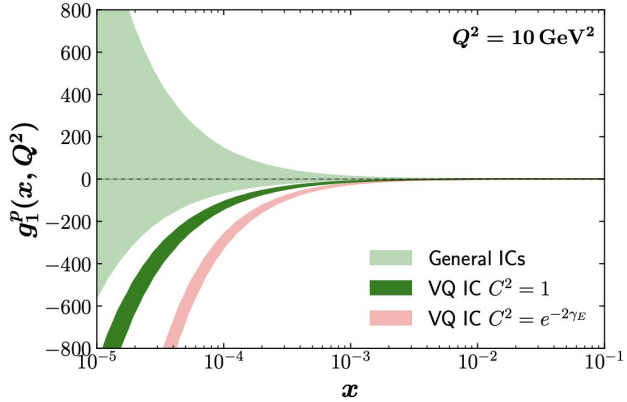
(with α_s = 0.2)

Results

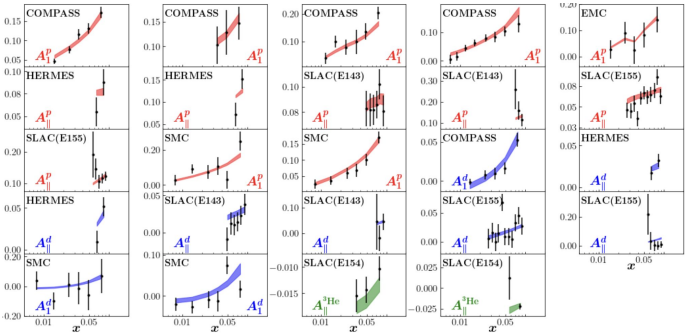
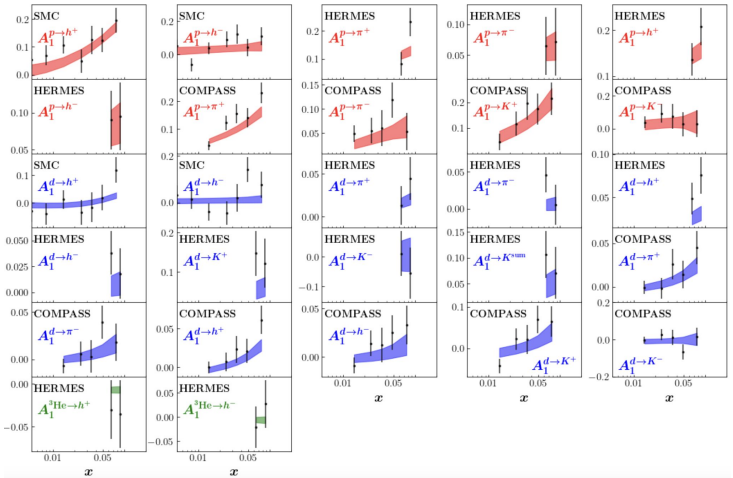
C^2	c_u	c_d	c_s	c_u^{NS}	c_d^{NS}	c_s^{NS}	$\widetilde{c}$	c_2	χ^2/N_{pts}
1	-62.7± 8.1	36.3± 8.2	-15.9± 17.1	19.3± 0.6	-6.4± 1.3	-0.7± 0.9	4.0± 8.2	-4.2± 3.2	1.26
$e^{-2\gamma_E}$	-37.5±5.5	27.4± 4.7	-8.1 ± 10.6	19.5 ± 0.6	-6.5± 1.3	-0.6± 0.8	-12.3 ± 4.1	-5.7± 2.2	1.28







[Adamiak et al, 2308.07461]

Small-x Helicity Phenomenology

- Polarized DIS and SIDIS data (A_1, $A_\parallel$, A_1^h) from SLAC, EMC, SMC, COMPASS and HERMES at $0.005 \leq x \leq 0.1$ and $1.69\ \text{GeV}^2 \leq Q^2 \leq 10.4\ \text{GeV}^2$.
  - Include proton, deuteron and helium-3 targets
  - For SIDIS, include $\pi^\pm$, $K^\pm$ and unidentified charged hadron productions
- In total, $N_{\text{pts}} = 226$ data points
- Overall, $\chi^2 / N_{\text{pts}} = 1.03$