The $q\bar{q}g$ contribution to diffractive DIS at NLO and the soft quark role in it

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Centre of Excellence in Quark Matter

Introduction

The complete NLO result for the structure functions in *diffractive* DIS in the dipole picture has been recently made available in a series of two papers:

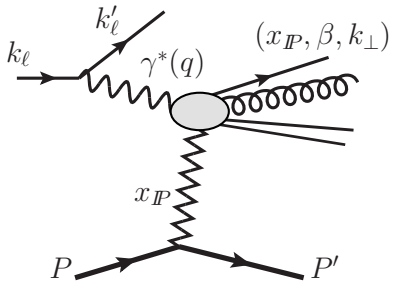
1. Beuf, Hänninen, Lappi, Mulian, Mäntysaari, *Phys. Rev. D* 106 (2022) 9, 094014
2. Beuf, Lappi, Mäntysaari, Paatelainen, Penttala, *JHEP* 05 (2024) 024

In this talk

- I will discuss the **numerical implementation** of the most computationally challenging part of the NLO result: **the $q\bar{q}g$ (aka "trip") contribution**.
- How the results challenged our initial expectations.
- And what we learned about the role soft quarks play in it.

But first, some background on diffraction...

What is Diffraction?



- Coherent: target remains intact
- Incoherent: target gets excited or breaks up

Diffractive processes are characterized by a **rapidity gap**:

- Large separation $Y_{\mathbb{P}} = \ln(1/x_{\mathbb{P}})$ with no particle production.
- **Colorless t-channel strong exchange:** Requires ≥ 2 gluons in pQCD.
- $\sim 15\%$ of HERA DIS events were diffractive.
- Highly sensitive to gluon density and **saturation** at small-x.

Diffraction variables

$$x_{\mathbb{P}} \approx \frac{Q^2 + M_X^2}{W^2 + Q^2} \qquad \beta = \frac{Q^2}{Q^2 + M_X^2} \qquad x_{\mathrm{Bj}} = \beta x_{\mathbb{P}}$$

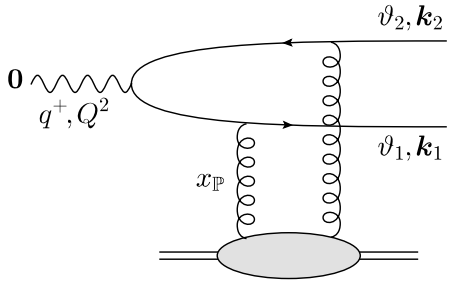
Diffractive DIS structure functions

Diffractive deeply inelastic scattering structure functions $F_2^{D(3)}$ and $F_L^{D(3)}$ parametrize the t-integrated *coherent* diffractive cross-section:

$$\frac{\mathrm{d}\sigma^D_{e+A\to M_X+A}}{\mathrm{d}\beta\,\mathrm{d}Q^2\,\mathrm{d}x_{\mathbb{P}}} = \frac{2\pi\alpha^2_{\mathrm{em}}}{\beta Q^4}\left[1+(1-y)^2\right]\left[F_2^{D(3)}(\beta,Q^2,x_{\mathbb{P}}) - \frac{y^2}{1+(1-y)^2}F_L^{D(3)}(\beta,Q^2,x_{\mathbb{P}})\right]$$

- Momentum transfer: $t = (P-P')^2$
- We assume that the target is a large, homogenous nucleus $\implies$ t-dependence can be approximated by $\delta(t)$

Small-x Diffraction at LO: The Dipole Picture



$$x_{\mathbb{P}} F_{T,q\bar{q}}^{\mathrm{D}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_c Q^4}{16\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z z(1-z) \times \left[\varepsilon^2 (z^2 + (1-z)^2)\Phi_1 + m_f^2 \Phi_0\right] \quad (8)$$

$$x_{\mathbb{P}} F_{L,q\bar{q}}^{\mathrm{D}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_c Q^6}{4\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z z^3 (1-z)^3 \Phi_0$$

LO SFs are proportional to the convolution:

$$\Phi_n = \int \mathrm{d}^2 \mathbf{b}_T \left[\int_0^\infty \mathrm{d}r \, r K_n(\varepsilon r) J_n(kr) \frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2 \mathbf{b}_T} \right]^2$$

Dipole Cross-Section:

- Interaction of dipole with the target:

$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}^p}{\mathrm{d}^2 \mathbf{b}_T} = 2\left[1 - \exp\left(-r^2 F(x, r)\, T_p(\mathbf{b}_T)\right)\right]$$

- Vanishes as $r \to 0$ due to colour transparency

$$\Phi_n \propto (\text{gluon distribution})^2$$

Diffraction at NLO

PHYSICAL REVIEW D **106,** 094014 (2022)

Diffractive deep inelastic scattering at NLO in the dipole picture: The $q\bar{q}g$ contribution

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[2]*Department of Physics, University of Jyväskylä, P.O. Box 35, 40014 University of Jyväskylä, Finland*
[3]*Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland*
[4]*Instituto Galego de Física de Altas Enerxías IGFAE, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Galicia-Spain*

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We calculate the contribution from the $q\bar{q}g$ state production to the diffractive cross sections in deep inelastic scattering at high energy. The obtained cross section is finite by itself and a part of the full next-to-leading order result for the diffractive structure functions. We perform the calculation in exact kinematics in the eikonal limit, and show that the previously known high-Q^2 and large M_X^2 results for the structure functions can be extracted from our results in the appropriate limits. We furthermore discuss the steps required to obtain the full next-to-leading order results for the structure functions.

DOI: 10.1103/PhysRevD.106.094014

$$|\gamma\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + \cdots$$

Diffractive deep inelastic scattering at NLO in the dipole picture

Guillaume Beuf,[a] **Tuomas Lappi**,[b,c] **Heikki Mäntysaari**,[b,c] **Risto Paatelainen**[c] **and Jani Penttala**[d,e,b,c]

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[b]*Department of Physics, University of Jyväskylä, P.O. Box 35, 40014 University of Jyväskylä, Finland*
[c]*Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland*
[d]*Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, U.S.A.*
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Abstract: We compute the transverse and longitudinal diffractive structure functions to full next-to-leading order accuracy in the dipole picture of deep inelastic scattering. Our calculation uses the standard light-cone perturbation theory method for the partonic content of the virtual photon, together with the Color Glass Condensate description of the target color field. Our result includes as a subset the $q\bar{q}g$ contribution calculated earlier. We show that there is a rapidity divergence that can be factorized into the BK/JIMWLK evolution of the target Wilson lines, and that all other divergences cancel.

Keywords: Deep Inelastic Scattering or Small-x Physics, Higher-Order Perturbative Calculations

ArXiv ePrint: 2401.17251

JHEP05(2024)024

Diffraction at NLO: Contributions

NLO calculation not organized according to real and virtual corrections, but rather:

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip\text{-}trip}}$$

- dip: LO contribution from $|q\bar{q}\rangle$ Fock state + vertex corrections to $\gamma \to q\bar{q}$ + gluon exchange or emissions after interaction with shockwave *(both in amplitude and conjugate)*
- trip: contribution from $|q\bar{q}g\rangle$ Fock state interacting wih the shockwave *(both in amplitude and conjugate)* $\longleftarrow$ self contained, no divergences, no subtraction scheme dependence
- dip-trip: interference term: $q\bar{q}g$ interacts with shockwave in amplitude and $q\bar{q}$ interacts with shockwave in the complex conjugate and vice versa. Contains real and virtual corrections.

Diffraction at NLO: Contributions

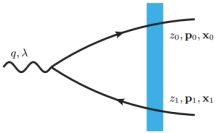


FIG. 2: Leading order amplitude. The blue band represents the interaction with the target color field.

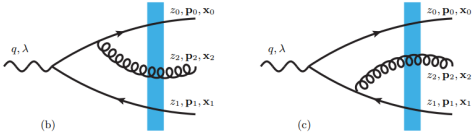


FIG. 3: Gluon emission before the shock.

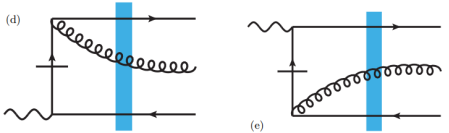


FIG. 4: Instantaneous diagram gluon emission wavefunction with emission before the cut.

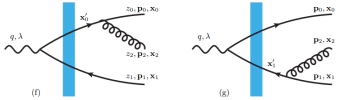


FIG. 5: Gluon emission after shock. Note that the intermediate transverse coordinate of the quark (antiquark) before the gluon emission is not the same as the coordinate at the cut. They are defined by $\mathbf{x}_0' = \frac{z_0\mathbf{x}_0 + z_2\mathbf{x}_2}{z_0 + z_2}$ and $\mathbf{x}_1' = \frac{z_1\mathbf{x}_1 + z_2\mathbf{x}_2}{z_1 + z_2}$.

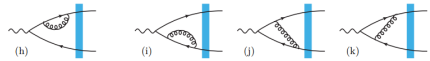
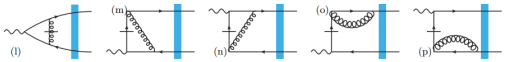


FIG. 6: Propagator and normal vertex correction diagrams calculated in [32, 33]



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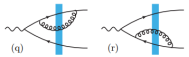


FIG. 8: Propagator correction diagrams where the gluon crosses the shockwave, but is not produced.

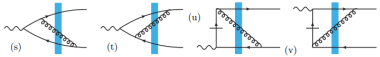
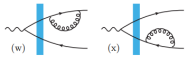


FIG. 9: Gluon emission diagrams where the gluon crosses the shockwave, but is not produced.



Goal of this work: Implement the trip contribution

And check if it shows the expected behaviour

- **Expression for the trip contribution:**

  (contains a challenging 9D oscillatory integral)

$$
\begin{aligned}
x_{\mathbb{P}} F_{T,\,q\bar{q}g}^{\mathrm{D(3)\,NLO}}(x_{\mathrm{Bj}}, Q^2, \beta) = {} & \frac{\alpha_{\mathrm{s}} N_{\mathrm{c}} C_{\mathrm{F}} Q^7 \sqrt{\frac{1}{\beta}-1}}{2\pi^2 \beta (2\pi)^5} \sum_f e_f^2 \int_0^1 \mathrm{d}z_0 \int_0^{1-z_0} \mathrm{d}z_1 \\
& \times \int \mathrm{d}|\mathbf{x}_{20}| \int \mathrm{d}|\mathbf{x}_{21}| \int \mathrm{d}\phi_{21} \int \mathrm{d}|\mathbf{x}_{\overline{20}}| \int \mathrm{d}\phi_{\overline{20}} \int \mathrm{d}|\mathbf{x}_{\overline{21}}| \int \phi_{\overline{21}} \, |\mathbf{x}_{20}||\mathbf{x}_{21}||\mathbf{x}_{\overline{20}}||\mathbf{x}_{\overline{21}}| \\
& \times \frac{\mathrm{J}_1\left(\sqrt{\frac{1}{\beta}-1} Q Y_{012}\right)}{Y_{012}} \frac{z_0 z_1}{X_{012} X_{\bar{0}\bar{1}\bar{2}}} \mathrm{K}_1\left(Q X_{012}\right) \mathrm{K}_1\left(Q X_{\bar{0}\bar{1}\bar{2}}\right) \\
& \times \left\{ \Upsilon_{\text{reg.}}^{(|(b)|^2)} + \Upsilon_{\text{reg.}}^{(|(c)|^2)} + \Upsilon_{\text{inst.}}^{(d)} + \Upsilon_{\text{inst.}}^{(e)} + \Upsilon_{\text{interf.}}^{(b)\times(c)} \right\} \left[1 - S_{\bar{0}\bar{1}\bar{2}}^{\dagger}(\mathbf{x}_{\overline{20}}, \mathbf{x}_{\overline{21}})\right] \left[1 - S_{012}(\mathbf{x}_{20}, \mathbf{x}_{21})\right] \Big|_{\phi_{20}=0} \\
& \times \int \mathrm{d}^2\mathbf{b} \, |T(\mathbf{b})^2| . \qquad (70)
\end{aligned}
$$

- **Expectations:**
  1. At large Q^2: The Golec-Biernat–Wüsthoff (GBW) limit[1]
  2. At large M_X^2 (small β): The Munier–Shoshi limit[2]

[1] K. Golec-Biernat and M. Wüsthoff, *Phys. Rev. D* 60 (1999) 114023.

[2] S. Munier and A. Shoshi, *Phys. Rev. D* 69 (2004) 074022.

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  2. At large M_X^2 (small β): The Munier–Shoshi limit[2]

We will mostly focus on this

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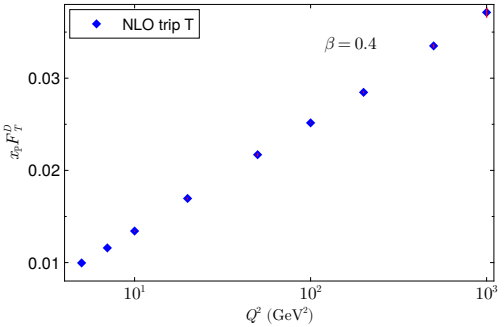
Large Q^2 expectations



- GBW limit shows a log Q^2 enhacement for the transverse polarised SF

$$x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{D\ (GBW)}}(x_{I\!P}, \beta, Q^2) = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{b} \int_0^{Q^2} \mathrm{d}k^2$$
$$\times \int_\beta^1 \mathrm{d}z \left\{ k^4 \ln \frac{Q^2}{k^2} \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \right.$$
$$\left. \times \left[\int_0^\infty \mathrm{d}r\, r \frac{\mathrm{d}\tilde{\sigma}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{I\!P}) \mathrm{K}_2\left(\sqrt{z} k r\right) \mathrm{J}_2\left(\sqrt{1-z} k r\right) \right]^2 \right\}.$$

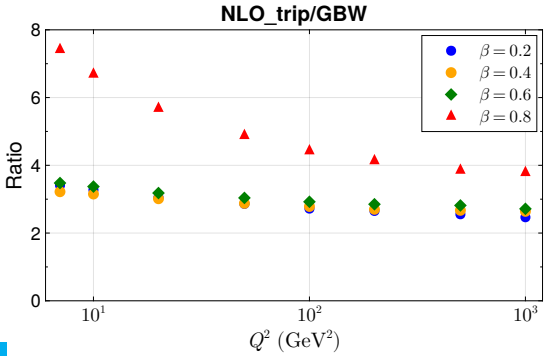
- "trip" numerics also shows a clear log Q^2 enhancement!

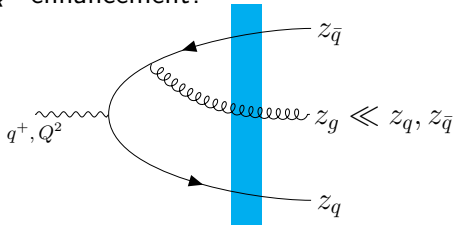
So far so good!

Large Q^2 expectations

But normalization does not match!

- Ratio $\frac{\text{NLO trip}}{\text{GBW}}|_{\text{large } Q^2} \neq 1$
- *NLO trip turns out to be much larger than GBW limit! What went wrong?*
- **What is the GBW limit anyway?** Leading twist behaviour of the $q\bar{q}g$ contribution arises from the *soft gluon* aligned jet configuration.
- Why does this give log Q^2 enhancement?



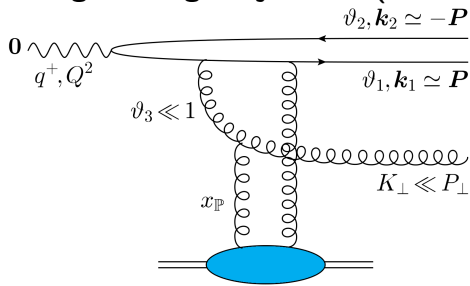


Moving to the target picture

Recent paper established TMD factorization for diffractive jet production in photon-nuclear interactions Hauksson, Iancu, Mueller, Triantafyllopoulos, Wei, *J. High Energ. Phys.* 2024, 180 (2024).

- A soft parton in the projectile can be treated as a struck parton from the target, and hence assigned a PDF (technically a TMD).
- Aligned jet $q\bar{q}g$ configurations correspond to a single step of DGLAP evolution applied to the parton from the target.

Soft gluon aligned jet limit (GBW limit):



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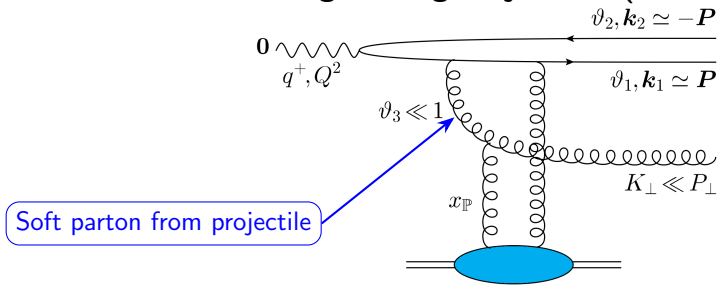

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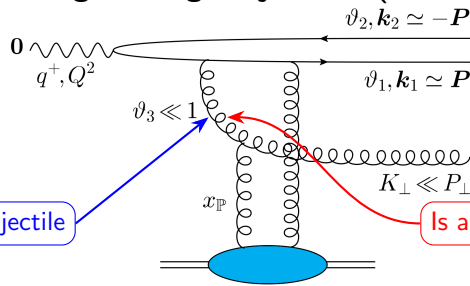

Soft parton from projectile

Is also struck parton form target

Moving to the target picture

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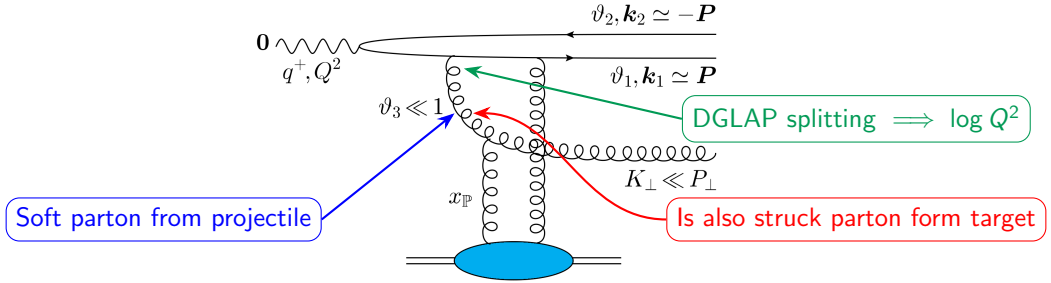
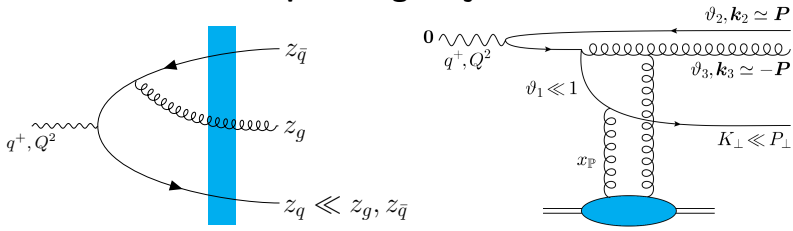


Hauksson, Iancu, Mueller, Triantafyllopoulos, Wei, *J. High Energ. Phys.* 2024, 180 (2024)

Soft quark aligned jet contribution

Iancu *et al.*, point out that *soft quark configurations also contribute to diffractive jet production at leading twist.*

Soft quark aligned jet limit:



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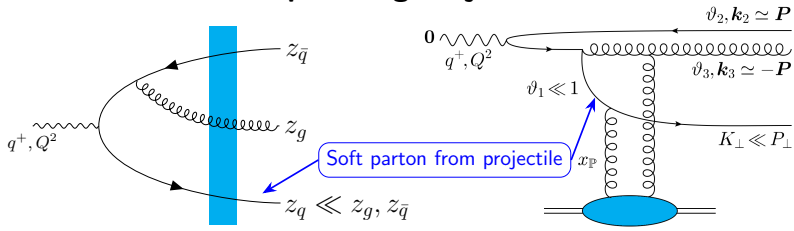

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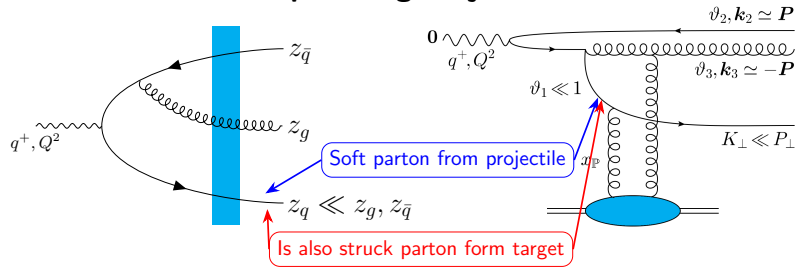

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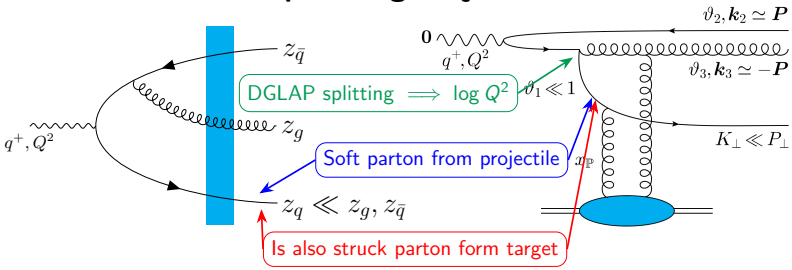

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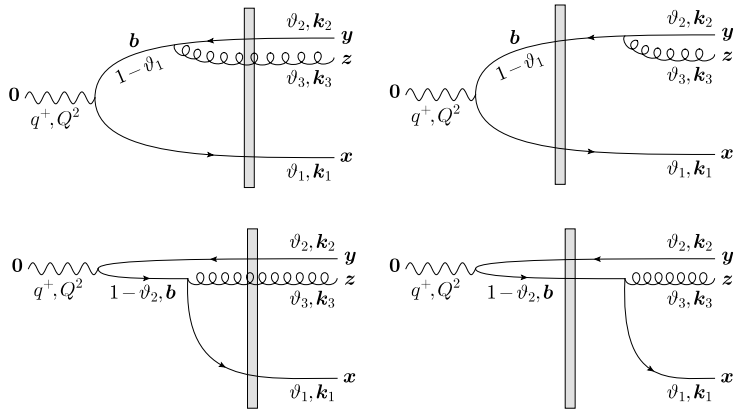

Hauksson, Iancu, Mueller, Triantafyllopoulos, Wei, *J. High Energ. Phys.* 2024, 180 (2024)

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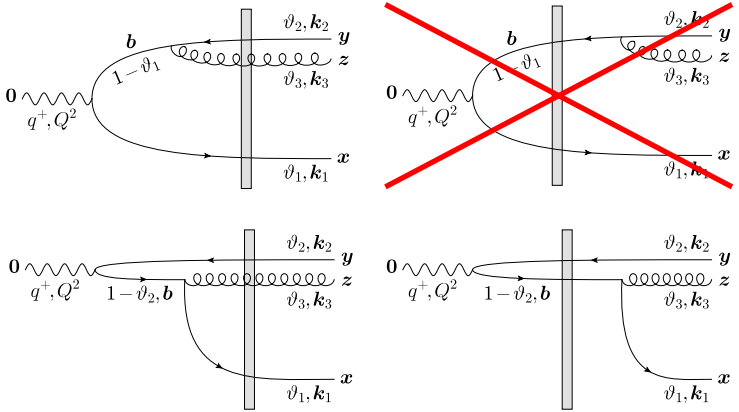

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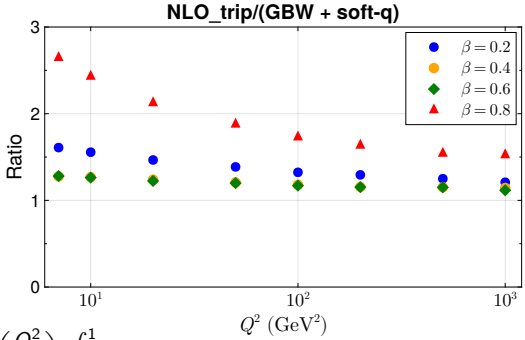
Soft quark contribution to "trip"

- Much better agreement of NLO "trip" contribution with GBW (soft gluon) + soft quark! $\Longrightarrow$



$$x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D\ (soft\text{-}q)}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{\alpha_s C_F S_\perp N_c}{8\pi^4} \left(\sum_f e_f^2 \right) \int_0^{Q^2} \mathrm{d}k^2 \log\left(\frac{Q^2}{k^2}\right) \int_\beta^1 \mathrm{d}x$$

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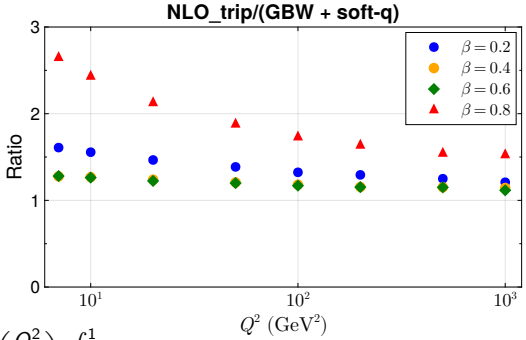

Compare with (5.10) of Iancu et al:

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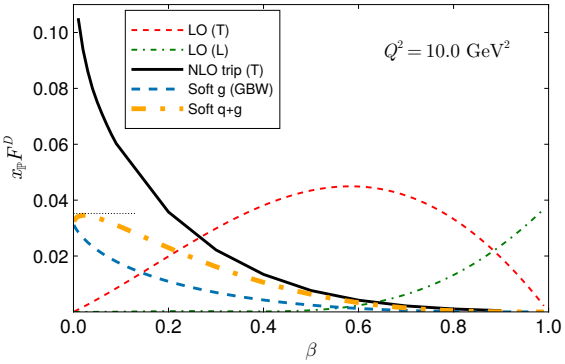

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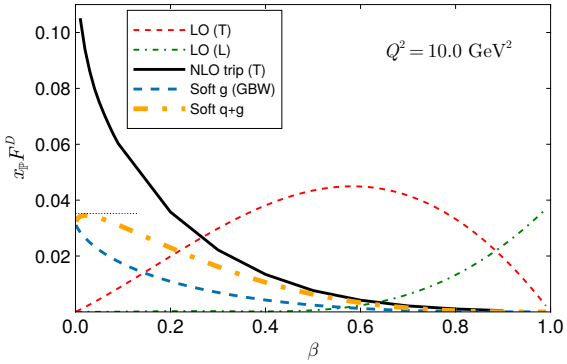

- Addition of **soft quark contribution significantly improves description of $q\bar{q}g$ contribution**
- Sum of the soft parton approximations *coincidentally* seems to be close to the Munier-Shoshi limit, which corresponds to $\beta \to 0$.

Subleading contributions from aligned jet configurations

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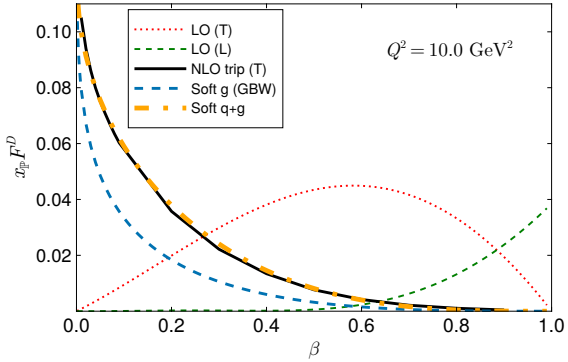

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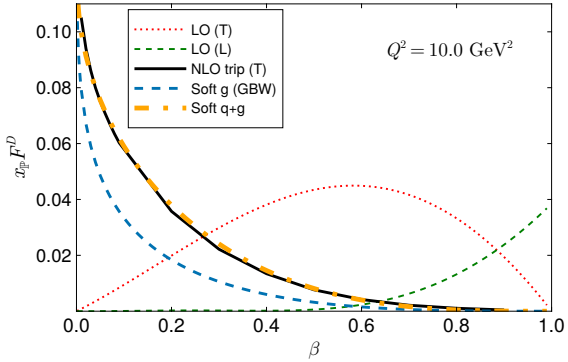
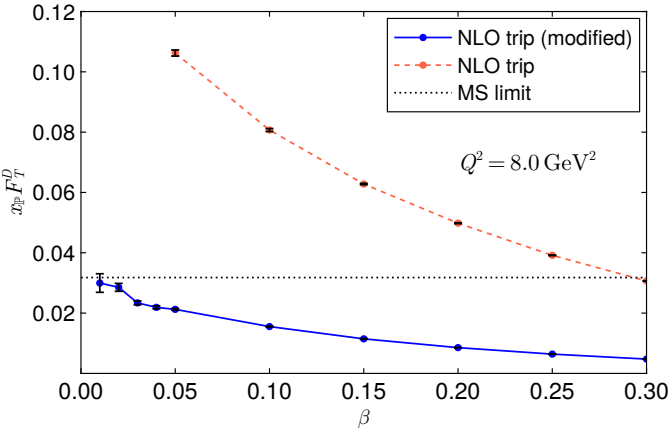


- Even better agreement with NLO "trip"! (but not quite clear why this helps...)

Conclusions

- The GBW result, previously used to estimate NLO corrections, underestimates them by a lot
- Finite contributions from the soft quark turn out to be important!
- Soft quark + GBW (including subleading corrections) provide a good estimate of the NLO corrections
- Very important to use NLO corrections for phenomenology (full NLO coming soon!)

Thank you!

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