DIS2026
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33rd INTERNATIONAL WORKSHOP ON DEEP INELASTIC SCATTERING AND RELATED SUBJECTS

Non-eikonal corrections to dijet production in DIS

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Based on 2603.26526

Introduction

- **Dijet production in Deep Inelastic Scattering off nuclei**:

1. It can potentially reveal non-linear density effects in QCD; i.e. **saturation**.

2. Link between **CGC** calculations and **TMD's** at momentum fraction x=0 in the correlation limit. *T. Altinoluk et al. (2410.00612)*

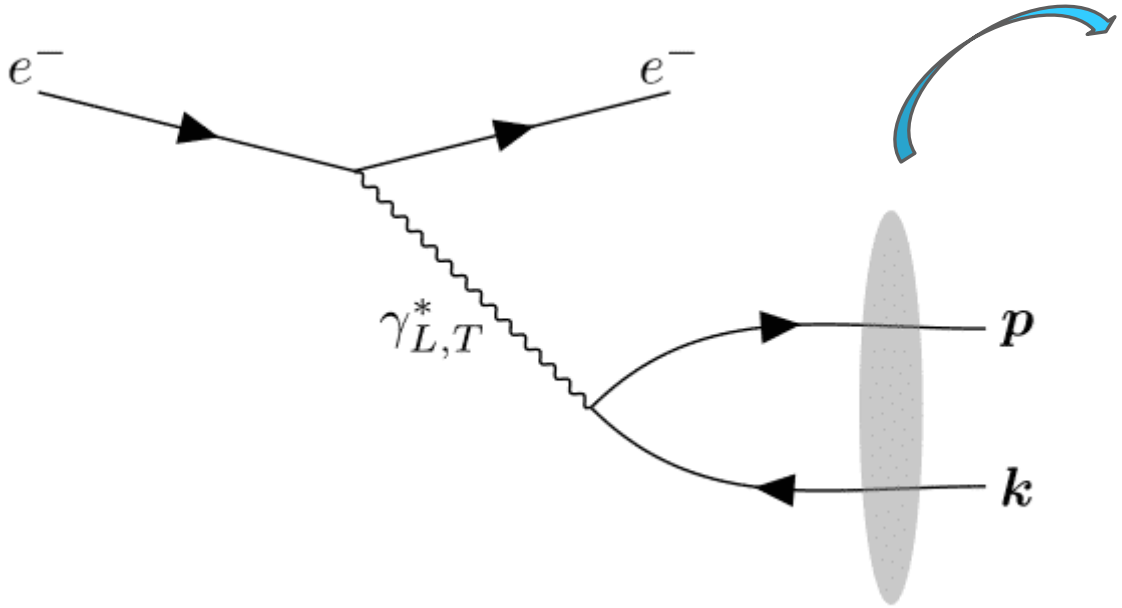
3. How to link **small** (non linear) and **moderate-to-large x** (linear) **QCD evolution**?

4. Laboratory for studying **NLO** and **NEIK** effects. *T. Altinoluk et al. (2212.10484), P. Agostini et al. (2403.04603) P. Caucal et al. (2304.03304), T. Altinoluk et al. (2204.11650)*

5. This observable will be experimentally accessed in the EIC and future colliders.

Introduction

- **Dijet production in Deep Inelastic Scattering off nuclei**:



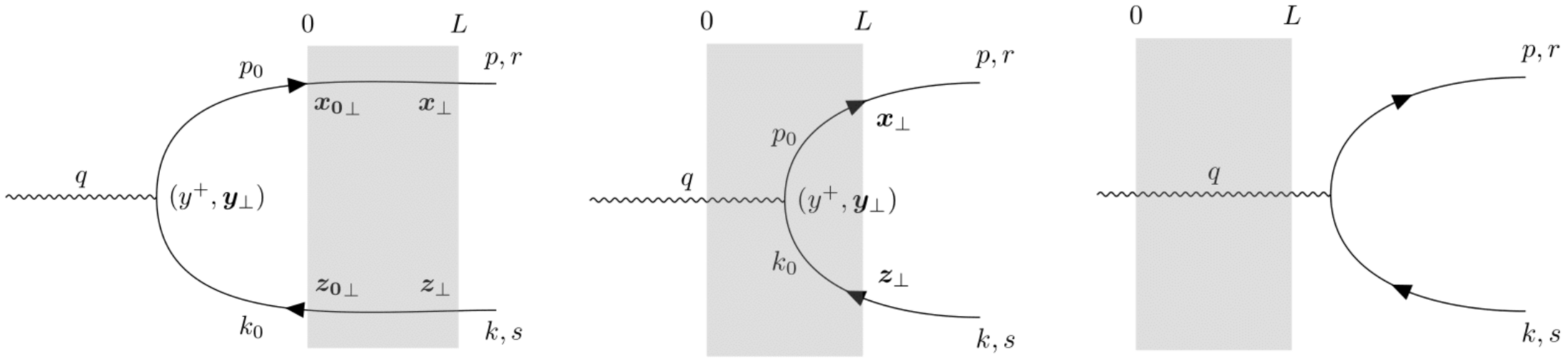
Massless quarks

Eikonal approximation:

- Lorentz contraction: Shockwave
- Time dilatation: $A^\mu \neq A^\mu\left(x^-\right)$
- No transverse fields

Beyond Eikonal

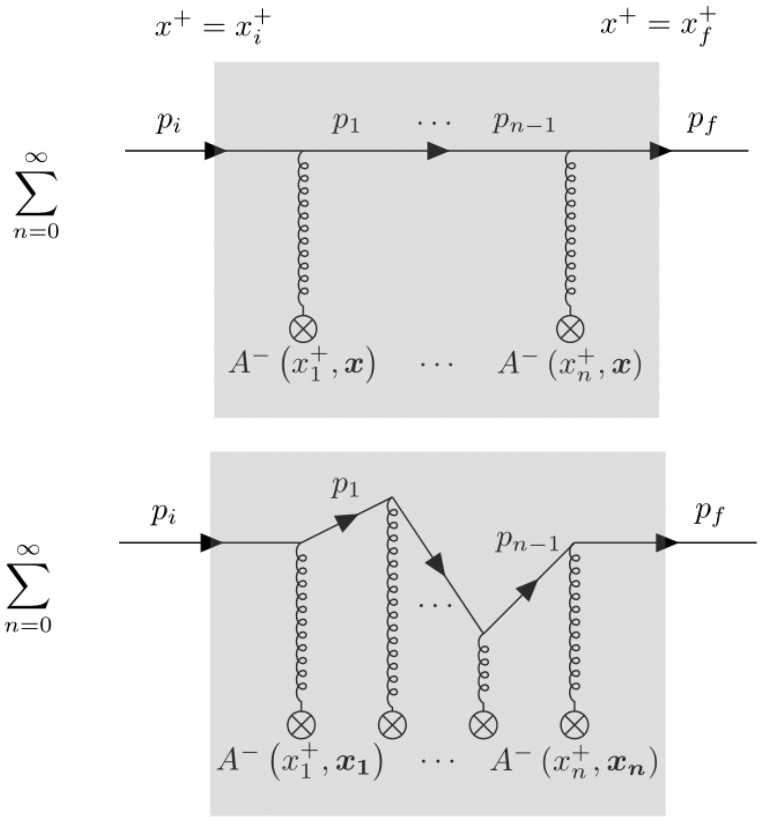
1. The photon can split inside the target



$$\mathcal{M}_{\gamma^*_{L,T} A \to q\bar{q}+X} = \mathcal{M}_{bef} + \mathcal{M}_{in} + \mathcal{M}_{aft}$$

Beyond Eikonal

2. We cannot neglect the transverse propagation of the quark or antiquark inside the target



$$\propto U\left(\mathbf{x}; x_f^+, x_i^+\right) = \mathcal{P}_+ \exp\left[-ig \int_{x_i^+}^{x_f^+} dx^+ A^-\left(x^+, \mathbf{x}\right)\right]$$

B.G. Zakharov.
JETP Lett. 65 (1997) 615-620

R. Baier, Yuri L. Dokshitzer, Alfred H. Mueller, D. Schiff.
Nucl.Phys.B 531 (1998) 403-425

$$\propto G\left(x_f^+, \mathbf{x_f}; x_i^+, \mathbf{x_i}\right) = \int_{\mathbf{r}\left(x_i^+\right)=\mathbf{x_i}}^{\mathbf{r}\left(x_f^+\right)=\mathbf{x_f}} D\mathbf{r}\left(s^+\right) \exp\left(\frac{ip^+}{2} \int_{x_i^+}^{x_f^+} ds^+ \left[\frac{d\mathbf{r}}{ds^+}\right]^2\right) U\left(\mathbf{r}\right)$$

Analogous to jet quenching

CGC average of the medium charges

1. **MV model**: $\left\langle A^{-a}\left(x^{+},\mathbf{x}\right)A^{-b}\left(y^{+},\mathbf{y}\right)\right\rangle=g^{2}n\left(x^{+}\right)\delta^{ab}\delta\left(x^{+}-y^{+}\right)\gamma\left(\mathbf{x}-\mathbf{y}\right)$

2. **Harmonic oscillator approximation** (analytical): $C_{F}g^{4}n\left(x^{+}\right)\left[\gamma\left(\mathbf{0}\right)-\gamma\left(\mathbf{r}\right)\right]\approx\frac{Q_{s}^{2}}{4L}r^{2}$

- **Dipole:** $S_{\mathbf{xy}}^{(2)}\left(t_{f}^{+},t_{i}^{+}\right)=\frac{1}{N_{c}}Tr\left\langle U\left(\mathbf{x}\right)U^{\dagger}\left(\mathbf{y}\right)\right\rangle_{\left(t_{f}^{+},t_{i}^{+}\right)}=\exp\left[-\frac{Q_{s}}{4L}\int_{t_{i}^{+}}^{t_{f}^{+}}d\xi^{+}\ \left|\mathbf{x}\left(\xi^{+}\right)-\mathbf{y}\left(\xi^{+}\right)\right|^{2}\right]$

- **Quadrupole**: $S_{\mathbf{12\bar{2}\bar{1}}}^{(4)}\left(t_{f}^{+},t_{i}^{+}\right)=\frac{1}{N_{c}}Tr\left\langle U\left(\mathbf{r_{1}}\right)U^{\dagger}\left(\mathbf{r_{2}}\right)U\left(\bar{\mathbf{r}}_{\mathbf{2}}\right)U^{\dagger}\left(\bar{\mathbf{r}}_{\mathbf{1}}\right)\right\rangle_{\left(t_{f}^{+},t_{i}^{+}\right)}=S_{\mathbf{1\bar{1}}}^{(2)}\left(t_{f}^{+},t_{i}^{+}\right)S_{\mathbf{2\bar{2}}}^{(2)}\left(t_{f}^{+},t_{i}^{+}\right)$

$$+\int_{t_{i}^{+}}^{t_{f}^{+}}d\xi^{+}\,S_{\mathbf{1\bar{1}}}^{(2)}\left(t_{f}^{+},\xi^{+}\right)S_{\mathbf{2\bar{2}}}^{(2)}\left(t_{f}^{+},\xi^{+}\right)T\left(\xi^{+}\right)S_{\mathbf{12}}^{(2)}\left(\xi^{+},t_{i}^{+}\right)S_{\mathbf{\bar{1}\bar{2}}}^{(2)}\left(\xi^{+},t_{i}^{+}\right)+\mathcal{O}\left(1/N_{c}\right)$$

$$T\left(\xi^{+}\right)=\frac{1}{2}\frac{Q_{s}^{2}}{\left(t_{f}^{+}-t_{i}^{+}\right)}\left(\mathbf{r_{1}}-\bar{\mathbf{r}}_{\mathbf{1}}\right)\cdot\left(\mathbf{r}_{2}-\bar{\mathbf{r}}_{\mathbf{2}}\right)$$

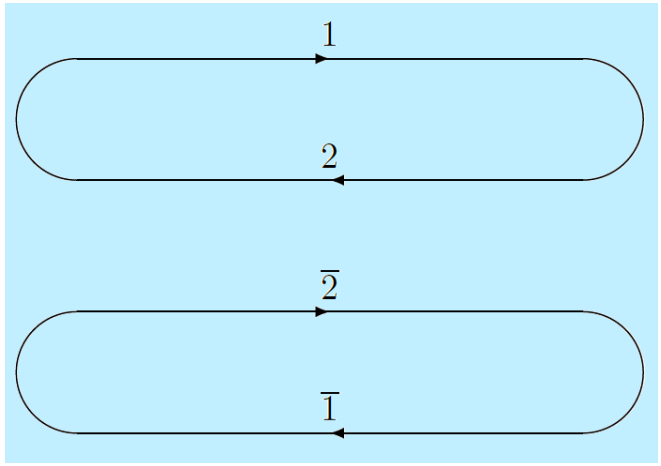
Note: HO identical to the GBW model in this context (P. Agostini, T. Altinoluk, N. Armesto, F. Domínguez, J. Guilherme. *Eur.Phys.J.C* 82 (2022) 1001).

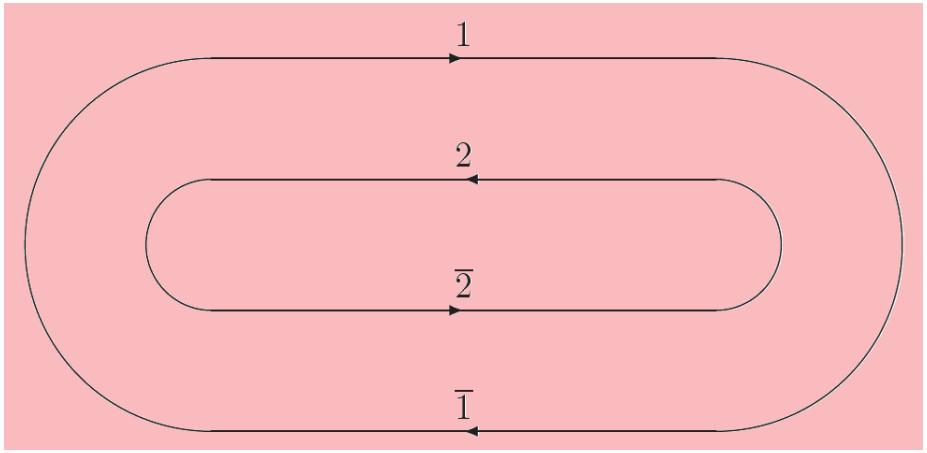
L. Apolinário, N. Armesto, J. Guilherme, C. A. Salgado. *JHEP* 02 (2015) 119

J. Blaizot, F. Domínguez, E. Iancu, Y. Mehtar-Tani. *JHEP* 01 (2013) 143

CGC average of the medium charges

Quark interacts with anti-quark in the (complex conjugate) amplitude: **coherence**





Quark in the amplitude interacts with quark in the complex conjugate: **broadening**

- **Quadrupole**:

$$S^{(4)}_{\mathbf{12\bar{2}\bar{1}}}\left(t_f^+,t_i^+\right)=\frac{1}{N_c}Tr\left\langle U\left(\mathbf{r_1}\right)U^\dagger\left(\mathbf{r_2}\right)U\left(\bar{\mathbf{r}}_\mathbf{2}\right)U^\dagger\left(\bar{\mathbf{r}}_\mathbf{1}\right)\right\rangle_{\left(t_f^+,t_i^+\right)}=S^{(2)}_{\mathbf{1\bar{1}}}\left(t_f^+,t_i^+\right)S^{(2)}_{\mathbf{2\bar{2}}}\left(t_f^+,t_i^+\right)$$

$$+\int_{t_i^+}^{t_f^+}d\xi^+\,S^{(2)}_{\mathbf{1\bar{1}}}\left(t_f^+,\xi^+\right)S^{(2)}_{\mathbf{2\bar{2}}}\left(t_f^+,\xi^+\right)T\left(\xi^+\right)S^{(2)}_{\mathbf{12}}\left(\xi^+,t_i^+\right)S^{(2)}_{\mathbf{\bar{1}\bar{2}}}\left(\xi^+,t_i^+\right)+\mathcal{O}\left(1/N_c\right)$$

$$T\left(\xi^+\right)=\frac{1}{2}\frac{Q_s^2}{\left(t_f^+-t_i^+\right)}\left(\mathbf{r_1}-\bar{\mathbf{r}}_\mathbf{1}\right)\cdot\left(\mathbf{r}_2-\bar{\mathbf{r}}_\mathbf{2}\right)$$

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Path Integral Solutions

- $$\mathcal{J}\left(\mathbf{u}_f, \mathbf{u}_i; \Delta t^+\right) = \int_{\mathbf{u}_i}^{\mathbf{u}_f} D\mathbf{u}\ \exp\left\{\int_{t_i^+}^{t_f^+} dt\left[\frac{iq^+}{2}\zeta\left(1-\zeta\right)\dot{u}^2 - \frac{Q_s^2}{4L^+}u^2\right]\right\}$$
$$= \frac{1}{2\pi i}\frac{q^+\zeta\left(1-\zeta\right)\Omega}{\sin\left(\Omega\Delta t^+\right)}\exp\left\{i\frac{q^+\zeta\left(1-\zeta\right)\Omega}{2\sin\left(\Omega\Delta t^+\right)}\left[\cos\left(\Omega\Delta t^+\right)\left(u_f^2+u_i^2\right) - 2\mathbf{u}_f\cdot\mathbf{u}_i\right]\right\},$$

- $$\int_{\mathbf{u_i}}^{\mathbf{u_f}} D\mathbf{u}\int_{\mathbf{v_i}}^{\mathbf{v_f}} D\mathbf{v}\ \exp\left\{\int_{t_i^+}^{t_f^+} dt^+\left[ik^+\dot{\mathbf{u}}\cdot\dot{\mathbf{v}} - \frac{Q_s^2}{4L^+}u^2\right]\right\}$$
$$= \left(\frac{k^+}{2\pi i\Delta t^+}\right)^2\exp\left\{\frac{ik^+}{\Delta t^+}\Delta\mathbf{u}\cdot\Delta\mathbf{v}\right\}\ \exp\left\{-\frac{Q_s^2}{12L^+}\Delta t^+\left(u_f^2+u_i^2+\mathbf{u_f}\cdot\mathbf{u_i}\right)\right\}$$

After integration of $\mathbf{v_f}, \mathbf{u_i}$

$$\mathcal{P}\left(\mathbf{u_f}; \Delta t^+\right) = \exp\left\{-\frac{Q_s^2}{4L}\Delta t^+ u_f^2\right\}$$

$$\Omega^2 = \frac{-iQ_s}{2Lq^+\zeta\left(1-\zeta\right)} \qquad \zeta = \frac{p^+}{q^+}$$

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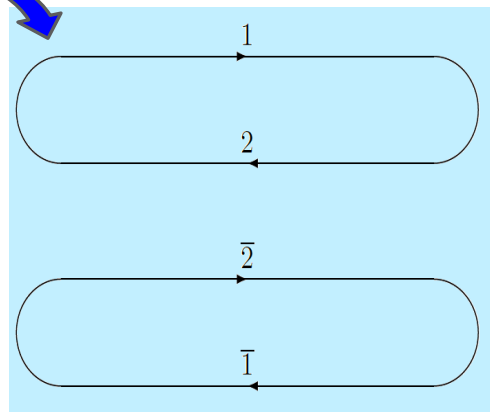

- $$\int_{\mathbf{u_i}}^{\mathbf{u_f}} D\mathbf{u}\int_{\mathbf{v_i}}^{\mathbf{v_f}} D\mathbf{v}\ \exp\left\{\int_{t_i^+}^{t_f^+} dt^+ \left[ik^+\dot{\mathbf{u}}\cdot\dot{\mathbf{v}} - \frac{Q_s^2}{4L^+}u^2\right]\right\}$$
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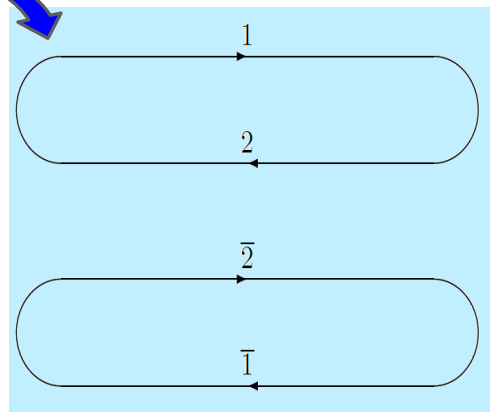

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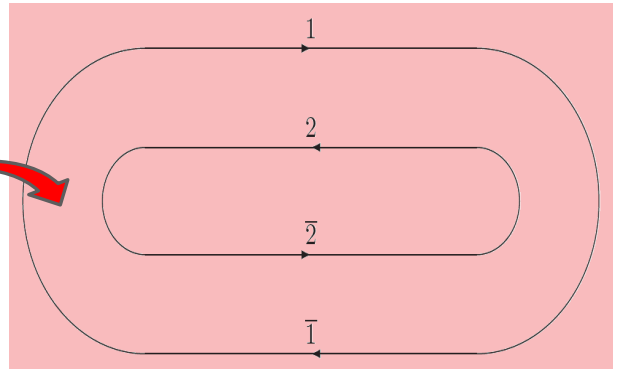

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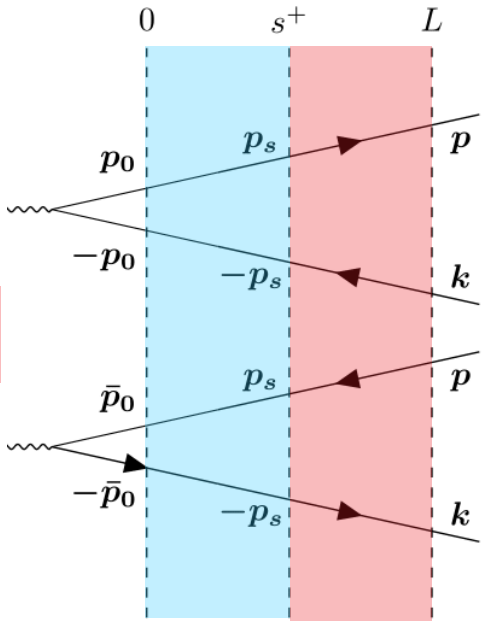
Before-Before Contribution

$$\left\langle \overline{\sum} \left|\mathcal{M}_{bef}\right|^2 \right\rangle = \zeta^2 (1-\zeta)^2 N_c e_q^2 e^2 S_\perp \left[\int \frac{d^2p_0}{(2\pi)^2} \frac{F_{T;L}(\mathbf{p_0}, \mathbf{p_0})}{(p_0^2+\epsilon^2)^2} \tilde{\mathcal{P}}(\mathbf{p}-\mathbf{p_0}; L^+) \tilde{\mathcal{P}}(\mathbf{k}+\mathbf{p_0}; L^+) \right.$$

$$- 2\frac{Q_s^2}{L^+} \int \frac{d^2p_s}{(2\pi)^2} (\mathbf{p}-\mathbf{p_s})\cdot(\mathbf{k}+\mathbf{p_s}) \int_0^{L^+} ds^+ \frac{L^{+2}}{\left[Q_s^2 (L^+ - s^+)\right]^2} \tilde{\mathcal{P}}(\mathbf{p}-\mathbf{p_s}; L^+ - s^+) \tilde{\mathcal{P}}(\mathbf{k}+\mathbf{p_s}; L^+ - s^+)$$

$$\left. \int \frac{d^2p_0}{(2\pi)^2} \frac{d^2\bar{p}_0}{(2\pi)^2} \frac{F_{T;L}(\mathbf{p_0}, \bar{\mathbf{p}}_\mathbf{0})}{(p_0^2+\epsilon^2)(\bar{p}_0^2+\epsilon^2)} \tilde{\mathcal{J}}(\mathbf{p_s}, \mathbf{p_0}; s^+) \tilde{\mathcal{J}}^*(\mathbf{p_s}, \bar{\mathbf{p}}_\mathbf{0}; s^+) \right]$$



$$\epsilon^2 = Q^2 \zeta (1-\zeta)$$

$$F_T(p_0, \bar{p}_0) = \frac{2}{\zeta(1-\zeta)} \left[\zeta^2 + (1-\zeta)^2\right] (p_0 \cdot \bar{p}_0)$$

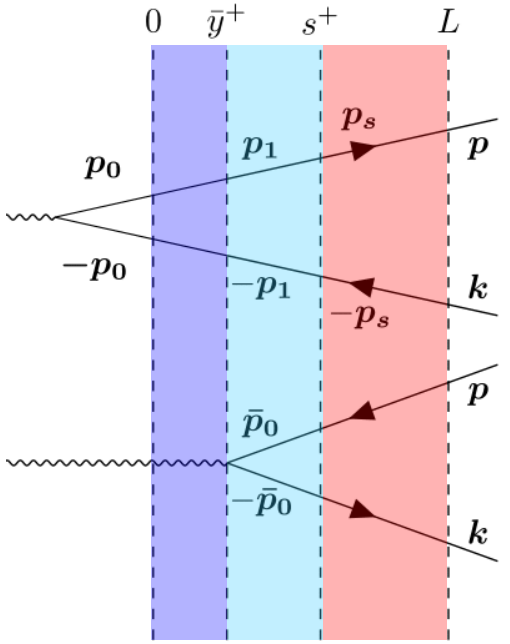
$$F_L(p_0, \bar{p}_0) = \frac{2}{Q^2\zeta(1-\zeta)} \left[p_0^2 - Q^2\zeta(1-\zeta)\right] \left[\bar{p}_0^2 - Q^2\zeta(1-\zeta)\right]$$

Before-Inside Contribution

$$\left\langle \overline{\sum} \mathcal{M}_{bef} \mathcal{M}_{in}^{*} \right\rangle = -\frac{iN_c e_q^2 e^2}{2q^+} \zeta\left(1-\zeta\right) S_\perp \int_0^{L^+} d\bar{y}^+ \; e^{iq^- \bar{y}^+} \int \frac{d^2p_0}{(2\pi)^2} \frac{d^2\bar{p}_0}{(2\pi)^2} \; \frac{F_{T;L}\left(\mathbf{p_0}, \bar{\mathbf{p}}_\mathbf{0}\right)}{p_0^2 + \epsilon^2} \Bigg[\tilde{\mathcal{J}}\left(\bar{\mathbf{p}}_\mathbf{0}, \mathbf{p_0}; \bar{y}^+\right)$$

$$\times \tilde{\mathcal{P}}\left(\mathbf{p} - \bar{\mathbf{p}}_\mathbf{0}; L^+ - \bar{y}^+\right) \tilde{\mathcal{P}}\left(\mathbf{k} + \bar{\mathbf{p}}_\mathbf{0}; L^+ - \bar{y}^+\right) - 2\frac{Q_s^2}{L^+} \int \frac{d^2p_s}{(2\pi)^2} \left(\mathbf{p} - \mathbf{p_s}\right) \cdot \left(\mathbf{k} + \mathbf{p_s}\right) \int \frac{d^2p_1}{(2\pi)^2} \tilde{\mathcal{J}}\left(\mathbf{p_1}, \mathbf{p_0}; \bar{y}^+\right)$$

$$\times \int_{\bar{y}^+}^{L^+} ds^+ \frac{L^{+2}}{\left[Q_s^2\left(L^+ - s^+\right)\right]^2} \tilde{\mathcal{P}}\left(\mathbf{p} - \mathbf{p_s}; L^+ - s^+\right) \tilde{\mathcal{P}}\left(\mathbf{k} + \mathbf{p_s}; L^+ - s^+\right) \tilde{\mathcal{J}}\left(\mathbf{p_s}, \mathbf{p_1}; s^+ - \bar{y}^+\right) \tilde{\mathcal{J}}^*\left(\mathbf{p_s}, \bar{\mathbf{p}}_\mathbf{0}; s^+ - \bar{y}^+\right) \Bigg]$$

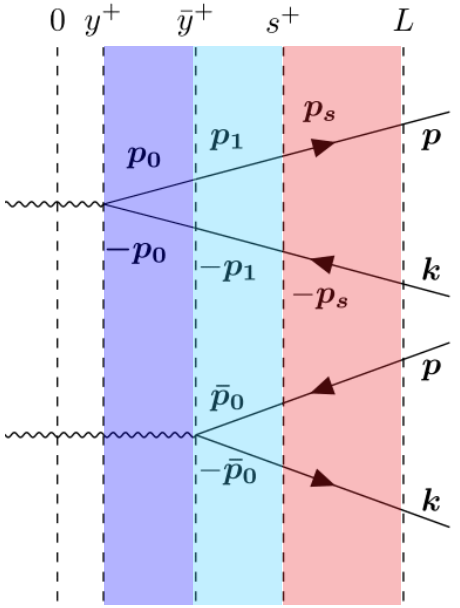


Inside-Inside Contribution

$$\left\langle \overline{\sum} |\mathcal{M}_{in}|^2 \right\rangle = \frac{N_c e_q^2 e^2}{(2q^+)^2} S_\perp 2\mathfrak{Re} \int_0^{L^+} dy^+ \int_{y^+}^{L^+} d\bar{y}^+ \; e^{-iq^-(y^+-\bar{y}^+)} \int \frac{d^2p_0}{(2\pi)^2} \frac{d^2\bar{p}_0}{(2\pi)^2} F_{T;L}\left(\mathbf{p_0}, \bar{\mathbf{p}}_\mathbf{0}\right) \Bigg[\tilde{\mathcal{P}}\left(\mathbf{p} - \bar{\mathbf{p}}_\mathbf{0}; L^+ - \bar{y}^+\right)$$

$$\times \tilde{\mathcal{P}}\left(\mathbf{k} + \bar{\mathbf{p}}_\mathbf{0}; L^+ - \bar{y}^+\right) \tilde{\mathcal{J}}\left(\bar{\mathbf{p}}_\mathbf{0}, \mathbf{p_0}; \bar{y}^+ - y^+\right) - \frac{Q_s^2}{2L^+} 4 \int \frac{d^2p_s}{(2\pi)^2} (\mathbf{p} - \mathbf{p_s}) \cdot (\mathbf{k} + \mathbf{p_s}) \int \frac{d^2p_1}{(2\pi)^2} \tilde{\mathcal{J}}\left(\mathbf{p_1}, \mathbf{p_0}; \bar{y}^+ - y^+\right)$$

$$\times \int_{\bar{y}^+}^{L^+} ds^+ \frac{L^{+2}}{\left[Q_s^2 (L^+ - s^+)\right]^2} \tilde{\mathcal{P}}\left(\mathbf{p} - \mathbf{p_s}; L^+ - s^+\right) \tilde{\mathcal{P}}\left(\mathbf{k} + \mathbf{p_s}; L^+ - s^+\right) \tilde{\mathcal{J}}\left(\mathbf{p_s}, \mathbf{p_1}; s^+ - \bar{y}^+\right) \tilde{\mathcal{J}}^*\left(\mathbf{p_s}, \bar{\mathbf{p}}_\mathbf{0}; s^+ - \bar{y}^+\right) \Bigg]$$



After Contributions

$$\left\langle \overline{\sum} \mathcal{M}_{bef} \mathcal{M}^*_{aft} \right\rangle = -N_c e_q^2 e^2 \zeta^2 (1-\zeta)^2 S_\perp (2\pi)^2 \delta^{(2)}(\mathbf{p}+\mathbf{k}) e^{-i\frac{Q^2}{2q^+}L^+} \int \frac{d^2p_0}{(2\pi)^2} \frac{F_{T;L}(\mathbf{p},\mathbf{p_0})}{(p^2+\epsilon^2)(p_0^2+\epsilon^2)} \tilde{\mathcal{J}}(\mathbf{p},\mathbf{p}_0;L^+).$$

$$\left\langle \overline{\sum} \mathcal{M}_{in} \mathcal{M}^*_{aft} \right\rangle = \frac{-iN_c e_q^2 e^2}{2q^+} \zeta(1-\zeta) S_\perp (2\pi)^2 \delta^{(2)}(\mathbf{p}+\mathbf{k}) e^{-i\frac{Q^2}{2q^+}L^+} \int \frac{d^2p_0}{(2\pi)^2} \frac{F_{T;L}(\mathbf{p},\mathbf{p_0})}{p^2+\epsilon^2} \int_0^{L^+} dy^+ \, e^{i\frac{Q^2}{2q^+}y^+} \tilde{\mathcal{J}}(\mathbf{p},\mathbf{p}_0;L^+-y^+).$$

$$\left\langle \overline{\sum} |\mathcal{M}_{aft}|^2 \right\rangle = (2\pi)^2 \delta^{(2)}(\mathbf{p}+\mathbf{k}) S_\perp N_c e_q^2 e^2 \zeta^2 (1-\zeta)^2 \frac{F_{T;L}(\mathbf{p},\mathbf{p})}{(p^2+\epsilon^2)^2}.$$

‘Shockwave ’ expansion

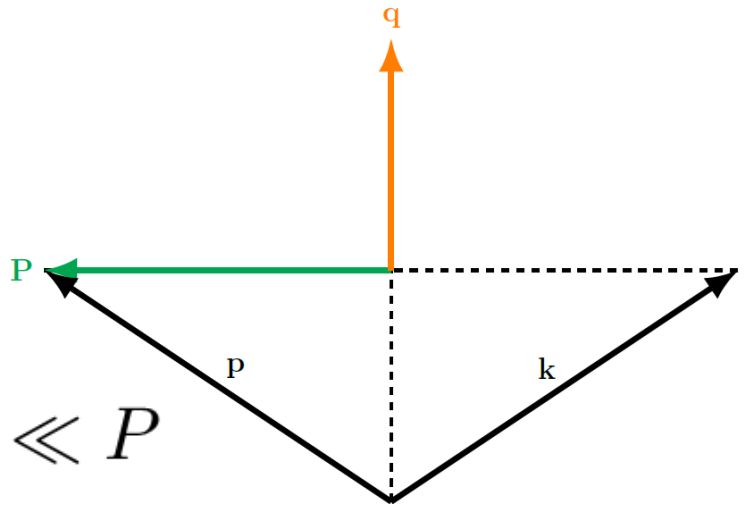
- $Q^2 \frac{L^+}{q^+} \ll 1$
- $Q_s^2 \frac{L^+}{q^+} \ll 1$

$$\left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle = \left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(0)} + \left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(1)} + \left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(2)} + \mathcal{O}\left(\frac{L^{+3}}{q^{+3}}\right)$$

+ **Correlation limit** (hard, back-to-back Jets)

- Relative momentum: $\mathbf{P} = \frac{\mathbf{p} - \mathbf{k}}{2}$
- Imbalance: $\mathbf{q} = \mathbf{p} - \mathbf{k}$

$$q \sim Q_s \ll P$$



‘Shockwave’ expansion: 0th Order (Eikonal)

$$\left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(0)} = H_{\gamma_L^* g \to q\bar{q}} \left[\frac{16\pi N_c}{\alpha_s} S_\perp \int d^2\Delta_b \, \frac{e^{-i\mathbf{q}\cdot\boldsymbol{\Delta_b}}}{\Delta_b^2} \left(1 - e^{-\frac{Q_s^2 \Delta_b^2}{2}} \right) + \mathcal{O}\left(\frac{Q_s^2}{P^2} \right) \right]$$

- Hard partonic cross section: $H_{\gamma_L^* g \to q\bar{q}} = \alpha_s \alpha_{em} e_q^2 \zeta^2 (1-\zeta)^2 \, \frac{8\epsilon^2 P^2}{\left(P^2 + \epsilon^2\right)^4},$

Fabio Domínguez, Cyrille Marquet, Bo-Wen Xiao, Feng Yuan. *Phys.Rev.D* 83 (2011) 105005

- Weizsäcker-Williams gluon distribution:

$$xG^{(1)}(x, q_\perp) = 2 \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P_h^+} e^{ixP_h^+\xi^- - iq_\perp \cdot \xi_\perp} \, Tr \left\langle P_h \left| F^{+i}(\xi^-, \xi) \mathcal{U}^{[+]\dagger} F^{+i}(0) \mathcal{U}^{[+]} \right| P_h \right\rangle$$

‘Shockwave’ expansion: 1st Order (Next-to-Eikonal)

$$\left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(1)} = 0$$

Before taking the Correlation Limit!

Reasons:

- Gaussian distribution of colour sources in the MV model
- Transverse rotational symmetry

Single inclusive gluon production in pA: (T. Altinoluk, N. Armesto, M. Martínez, C. Salgado. *JHEP* 07 (2014) 068)

‘Shockwave’ expansion: 2nd Order (Next(2)-to-Eikonal)

$$\left\langle \overline{\sum} |\mathcal{M}|^2 \right\rangle^{(2)} = H_{\gamma_L^* g \to q\bar{q}} \frac{P^2}{Q_s^2} \left(\frac{L}{q^+}\right)^2 \frac{2}{Q_s^2 Q^2 \zeta^3 (1-\zeta)^3} \left\{ \frac{\pi N_c}{\alpha_s} S_\perp \int d^2\Delta_b \frac{e^{-i\mathbf{q}\cdot\boldsymbol{\Delta_b}}}{\Delta_b^{10}} \right.$$
$$\left. \times \left[1920 \left(1 - e^{-\frac{Q_s^2 \Delta_b^2}{2}}\right) - 960 \Delta_b^2 Q_s^2 + 240 \Delta_b^4 Q_s^4 - 40 \Delta_b^6 Q_s^6 + 5 \Delta_b^8 Q_s^8 - \frac{1}{2} \Delta_b^{10} Q_s^{10} - \frac{\zeta(1-\zeta)}{24} \Delta_b^{10} Q_s^8 Q^2 \right] + \mathcal{O}\left(\frac{Q_s^2}{P^2}\right) \right\}.$$

The expansion of the exponential inside the squared brackets exactly cancels the first terms until order Q_s^{10}

Summary

- We have relaxed the shockwave approximation **to all orders in the medium length** thanks to the path integral formalism.
- We have employed the **MV model** for the medium in order to obtain explicit results.
- Explicit analytic expressions are given in the **harmonic oscillator approximation**.
- We have studied the **correlation limit**.
- We recover the known results for the eikonal case.
- We need to go beyond next-to-eikonal when we relax the shockwave approximation to obtain non-zero contributions.

Thank you for your attention!

Santiago de Compostela, Spain
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https://indico.global/event/16406/