Nucleon energy correlator for the odderon

Xuan-Bo Tong (童炫博)

University of Jyväskylä

Based on **Heikki Mäntysaari, Yossathorn (Josh) Tawabutr, X.B. Tong**

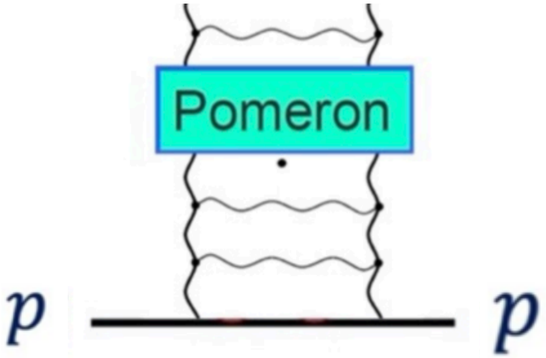
PRD* 112 (2025) 11, 114027**, ***arXiv: 2503.20157

May. 5th DIS 2026, Bologna

Odderon: brother of pomeron

- Colorless small-x gluon exchanges in the high-energy scatterings
- In the dilute region,

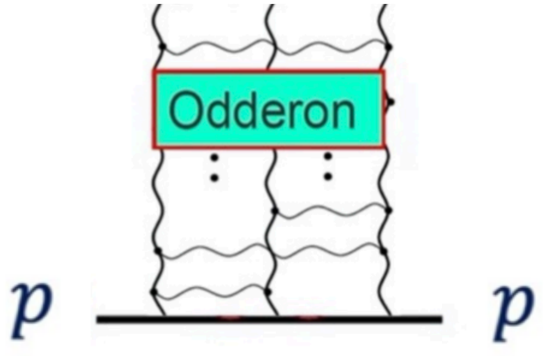
C-even



- Two-gluon exchange

$$\left\langle A^{a,+} A^{a,+} \right\rangle$$

C-odd



- Color symmetric three-gluon exchange

$$\left\langle d^{abc} A^{a,+} A^{b,+} A^{c,+} \right\rangle$$

e.g. [Kovchegov, Szymanowski, Wallon, 2004]

Odderon in the Color Glass Condensate

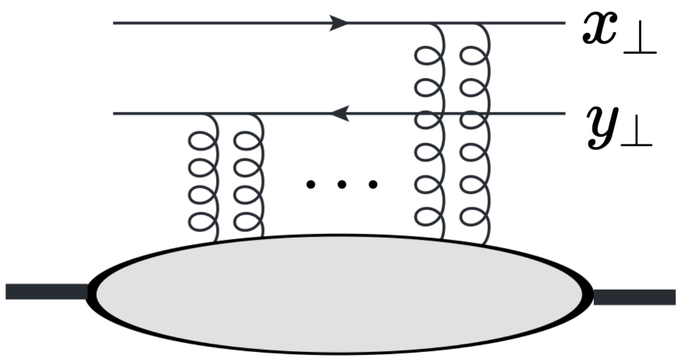
- Odderon from multiple gluon interactions:

  ➡ the imaginary part of the Dipole-proton S-matrix [Hatta, Iancu, Itakura, McLerran, 2005]

$$S(x_\perp, y_\perp) = \frac{1}{N_c}\langle \mathrm{tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]\rangle = P(x_\perp, y_\perp) + iO(x_\perp, y_\perp)$$

Pomeron ⇩ C-even, well-understood

Odderon ⇩ C-odd, flips sign under charge conjugation $x_\perp \leftrightarrow y_\perp$

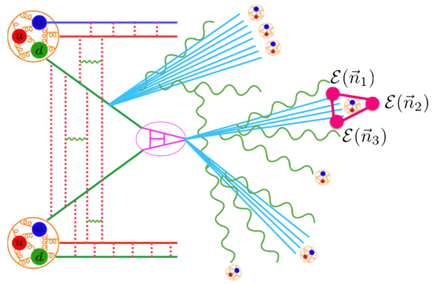


$$S^\dagger(x_\perp, y_\perp) = S(y_\perp, x_\perp)$$
$$\Downarrow$$
$$O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$$

Poorly constraint!

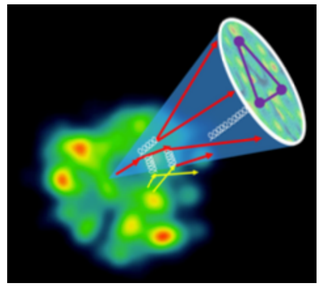
e.g., the normalization and signs

New opportunities for QCD dynamics



Jet substructure

[Chen, Moult, Zhang, Zhu, 20]



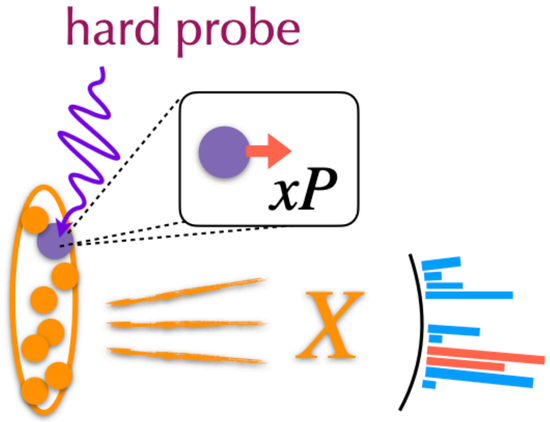
Quark-gluon plasma

[Andres, Dominguez, Elayavalli, Holguin, Marquet, Moult 22; Yang, He, Moult, Wangm, 23;….]

Energy correlator observables

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_k)\rangle$$

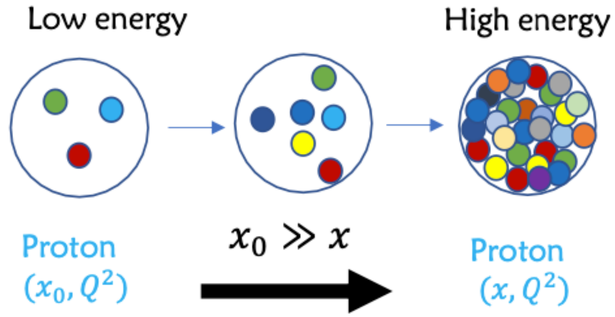
[see a review in Moult, Zhu, 2506.09119]



Nucleon tomography

Nucleon energy correlators (target fragmentation): **[Liu-Zhu, PRL 23]**

TMDs: [Li-Makris-Vitev 21; Kang, Lee, Shao, Fan 24]



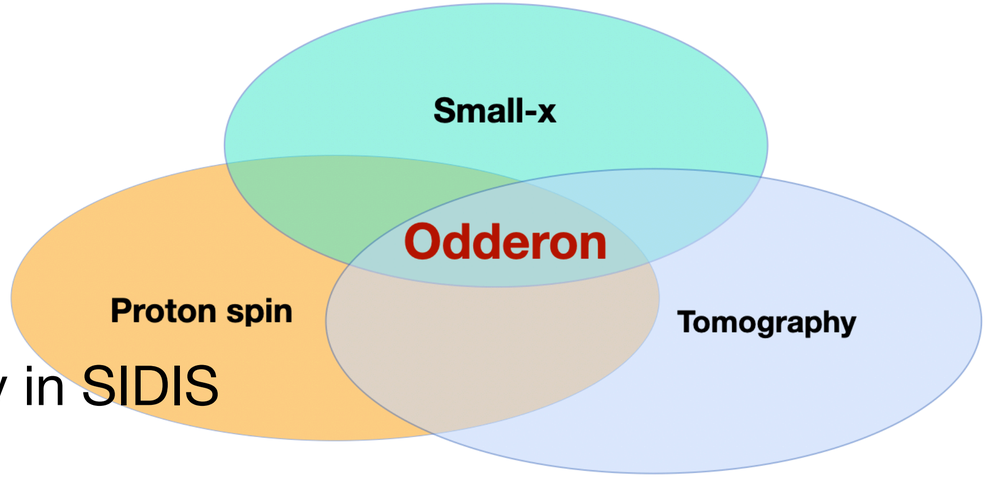
Small-x physics (pomeron & gluon saturation)

[**Liu-Liu-Pan-Yuan-Zhu, PRL 23;** Kang, Penttala, Zhao, Zhou, `23;…]

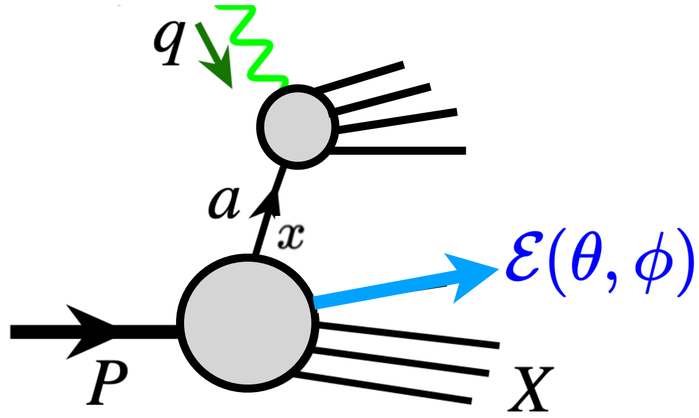
➡ **Odderon?**

Outline

- Spin-dependent odderon $O_{1T}^{\perp} \propto r_{\perp} \times S_{\perp}$
  - Relations to the proton spin
  - Limitations of the observables: single spin asymmetry in SIDIS



- Nucleon energy correlator
  - Energy pattern in the target fragmentation region
  - Connection to spin-dependent odderon
  - A C-odd probe: Incorporating track/charge information



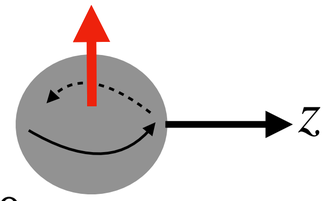
Spin-dependent odderon V.S. Conventional odderon

- C-odd: $O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$ proportional to the vector: $\vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$

- However, the odderon is a scalar, need another 2D vector to contract

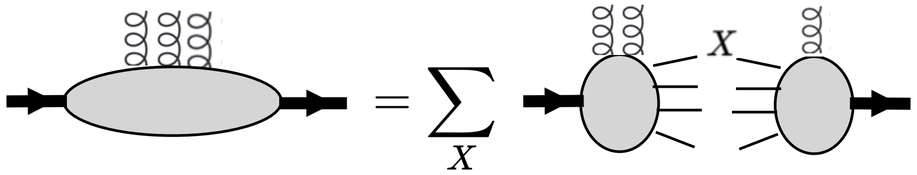
★ Transversely polarized proton spin $S_\perp$

$$O_{1T}^{\perp} \propto r_\perp \times S_\perp$$



Survive in the forward limit, $\Delta_\perp = 0$

Can be measured in an inclusive process:



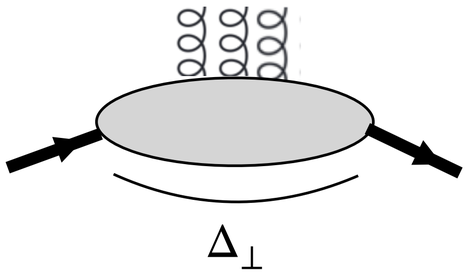
target broken

[Zhou, *PRD* 89 (2014) 7, 074050]

- Target recoil momentum (or impart parameter)

$$O \propto r_\perp \cdot \Delta_\perp$$

Exclusive process: target unbroken



Spin-dependent odderon V.S. Conventional odderon

- C-odd: $O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$ proportional to the vector: $\vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$

- However, the odderon is a scalar, need another 2D vector to contract

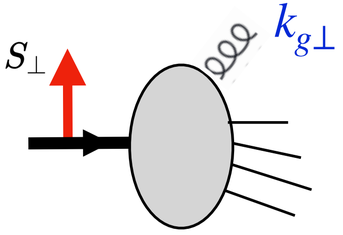
★ Transversely polarized proton spin $S_\perp$

$$O_{1T}^{\perp} \propto r_\perp \times S_\perp$$

Momentum space ⇩

$$\propto k_{g\perp} \times S_\perp$$

$$\propto \sin(\phi_g - \phi_S)$$

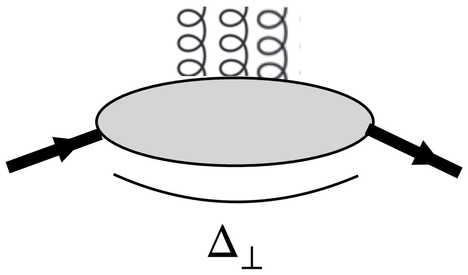


[Zhou, *PRD* 89 (2014) 7, 074050]

- Target recoil momentum (or impart parameter)

$$O \propto r_\perp \cdot \Delta_\perp$$

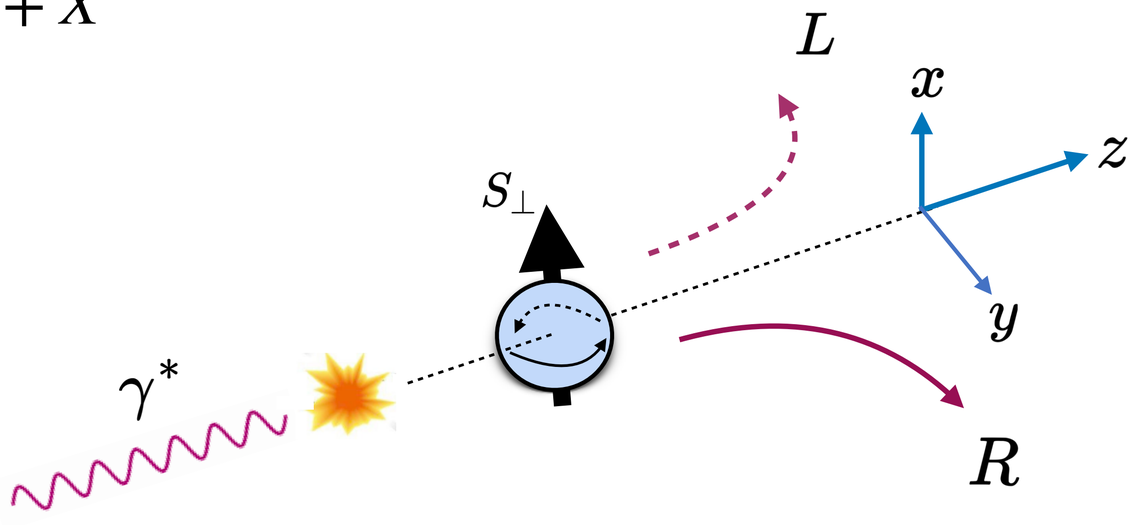
Exclusive process: target unbroken

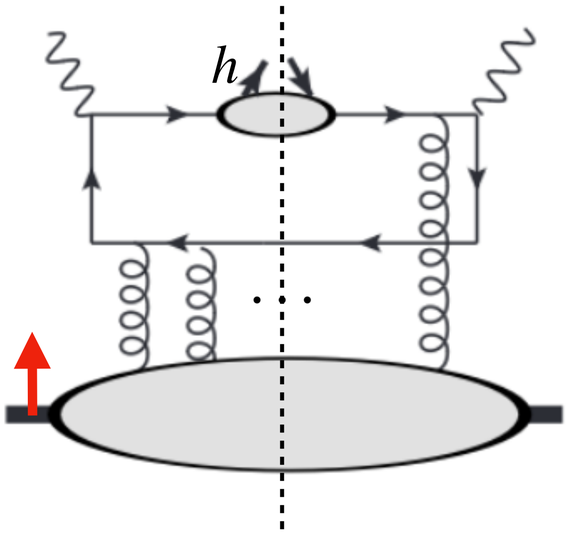


Typical observables: single spin asymmetry (SSA)

Semi-inclusive DIS (SIDIS):

$$\gamma^* + p^\uparrow \to h + X$$





$$d\sigma \propto S_\perp \times P_{h\perp} \propto \sin(\phi_h - \phi_S) \quad \Rightarrow \quad A_{UT} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}$$

- $O_T \propto k_{g\perp} \times S_\perp$: hadron production is left-right asymmetric!
- The spin-dependent odderon is one of the origins of SSAs at small-x

[Zhou `13]

[Yao, Hagiwara, Hatta `18]

[Dong, Zheng, Zhou `19]

[Zhu, Zheng, Zhang `14]

Predicted signature of the odderon & potential background

- Due to the C-odd nature of odderon.

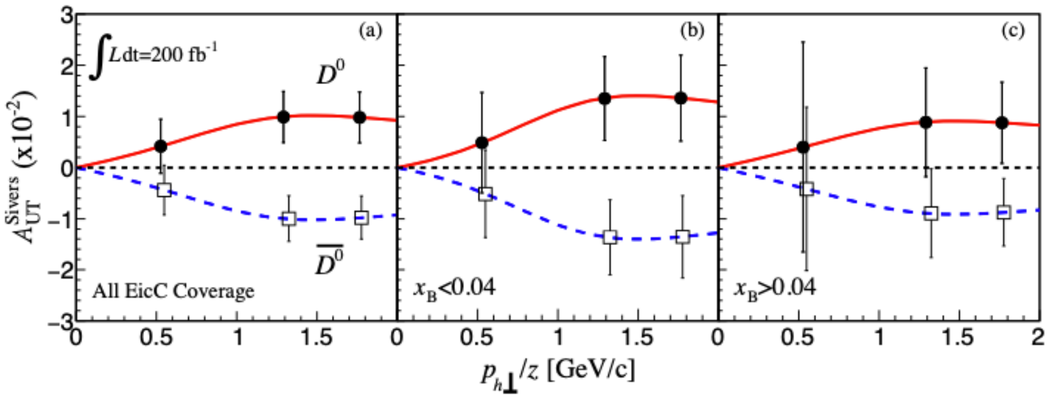
$$A_{UT}^{\bar{h}} = -A_{UT}^{h}$$

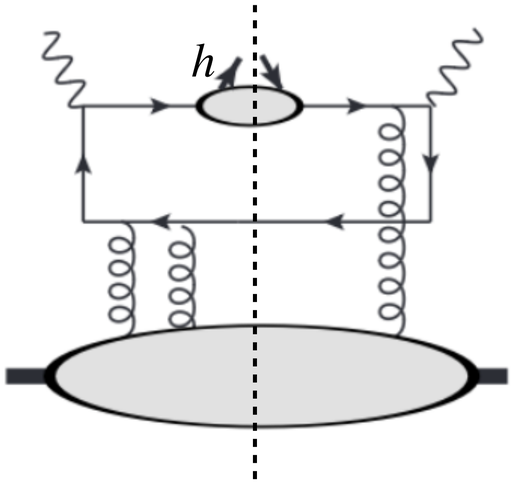
[Yao, Hagiwara, Hatta `18]

[Dong, Zheng, Zhou `19]

➡ Search for the existence at the future EIC

A simulation for EicC: [Zhu et al, 2409.00653]





Predicted signature of the odderon & potential background

- Due to the C-odd nature of odderon.

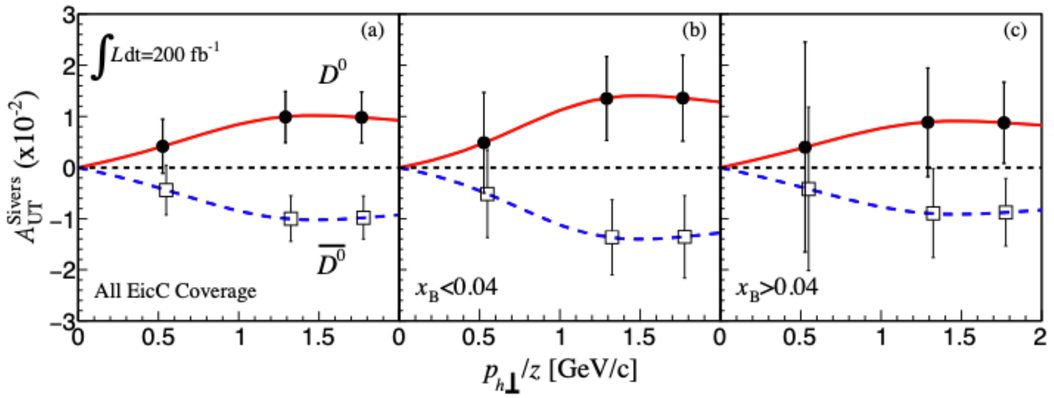
$$A_{UT}^{\bar{h}} = -A_{UT}^{h}$$

[Yao, Hagiwara, Hatta `18]

[Dong, Zheng, Zhou `19]

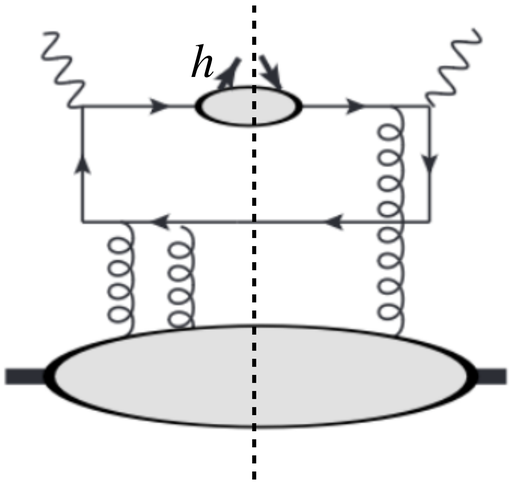
➡ Search for the existence at the future EIC

A simulation for EicC: [Zhu et al, 2409.00653]



- However, the odderon ($\sim d^{abc}A^{+,a}A^{+,b}A^{+,c}$) is not the only possible origin

C-even gluonic correlations can also contribute! e.g., $f^{abc}A^{+,a}A^{+,b}A^{+,c}$



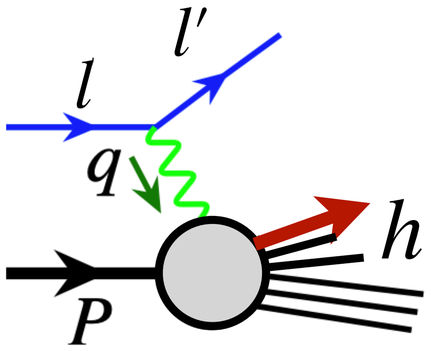
- Twist-3 collinear distributions: [Kang, Qiu PRD 78, 034005 (2008)] [Chen, Ma, Tong 2108.13582] [Schafer,Zhou 1308.4961]
- Sub-eikonal dipole amplitudes: [Kovchegov, Santiago JHEP 11 (2021) 200; JHEP 11 (2022) 098] [Adamiak, Santiago, Tawabutr 2412.14154]

- In the EIC kinematics, these C-even contributions (badly constraint either) might contaminate the signal and the extraction of the spin-dependent odderon

Lack of C-odd constraints in SIDIS

★ Spin-dependent odderon $O_{1T}^{\perp} \propto r_{\perp} \times S_{\perp}$

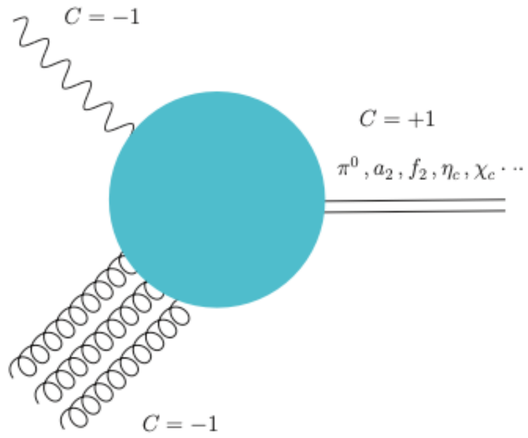
✓SIDIS: $\gamma^* + p^{\uparrow} \to h + X$



- Not measuring everything
- Lose the information of C-parity conservation

• Conventional odderon $O \propto r_{\perp} \cdot \Delta_{\perp}$

✓Exclusive scattering: $\gamma^* + p \to p' + \pi^0$



- C-odd probe by selecting the C-even hadron

e.g., [Benic, Horvatic, Kaushik, Vivoda, Motyka, Stebel PRD 110 1, 014025 (2024)]

➡ How to make a C-odd tag in the inclusive measurements?

Motivations

Spin-dependent odderon

$$O_{1T}^{\perp} \propto r_{\perp} \times S_{\perp}$$

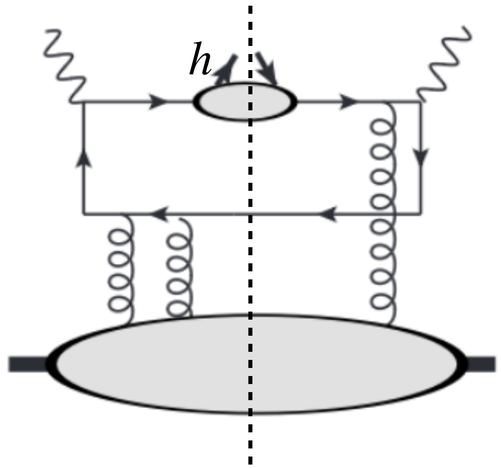
- How to make a C-odd tag in the inclusive measurement ?

- Require identifying a hadron, need PID, involve convolution of fragmentation functions.

  ➡ Is there a way to minimize non-perturbative effects?

- What is the connection between the spin-dependent odderon and the nucleon tomography in the target fragmentation region?

✓Semi-inclusive DIS: $\gamma^* + p^{\uparrow} \to h + X$



[Yao, Hagiwara, Hatta 2019]

[Dong, Zheng, Zhou 2019]

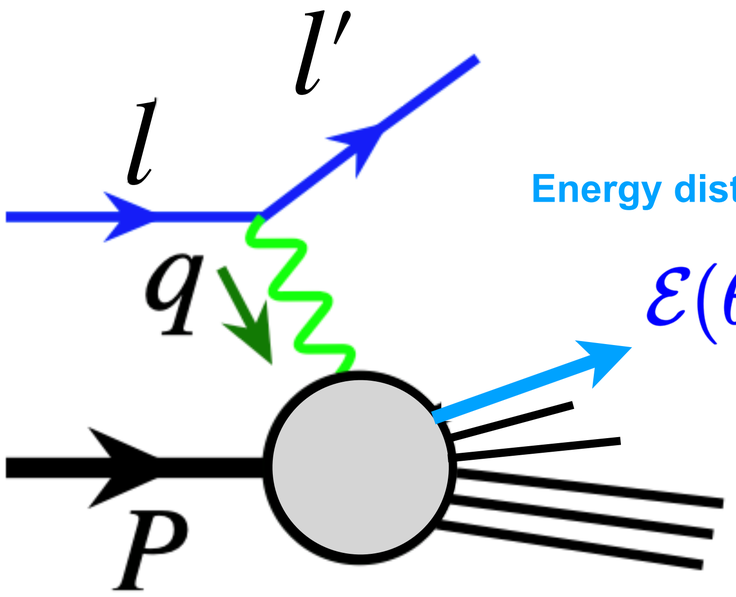
[Zhu, Zheng, Zhang 2024]

Nucleon energy correlator for the odderon

[Mantysaari, Tawabutr, Tong, *PRD* 112 (2025) 11, 114027*]*

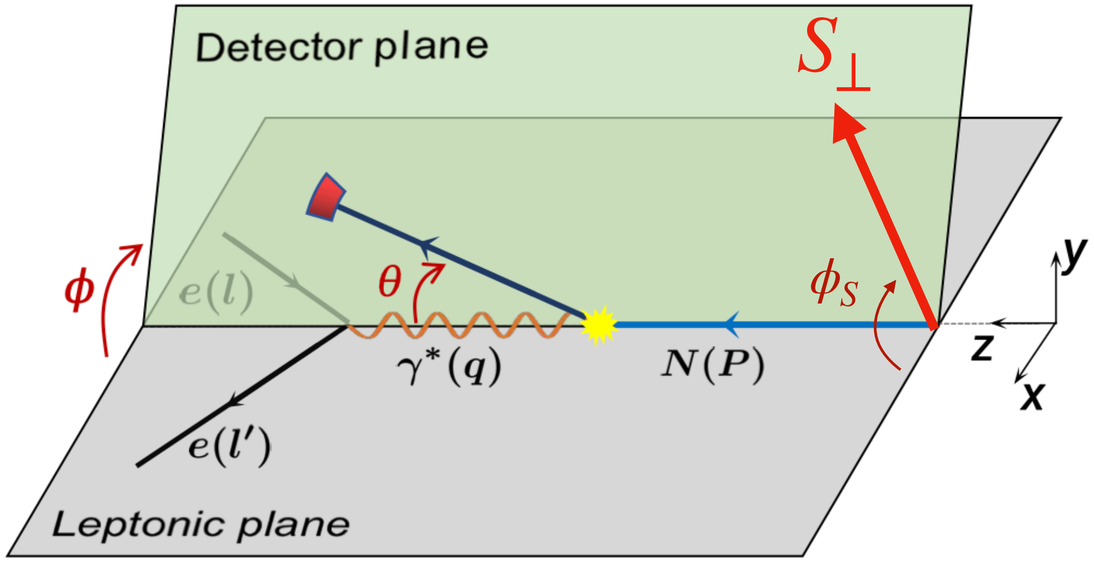
- How to measure: the DIS energy pattern in the target fragmentation region
- Connection to the spin-dependent odderon
- A C-odd probe by incorporating charge information

The DIS energy pattern



Energy distribution

$\mathcal{E}(\theta,\phi)$



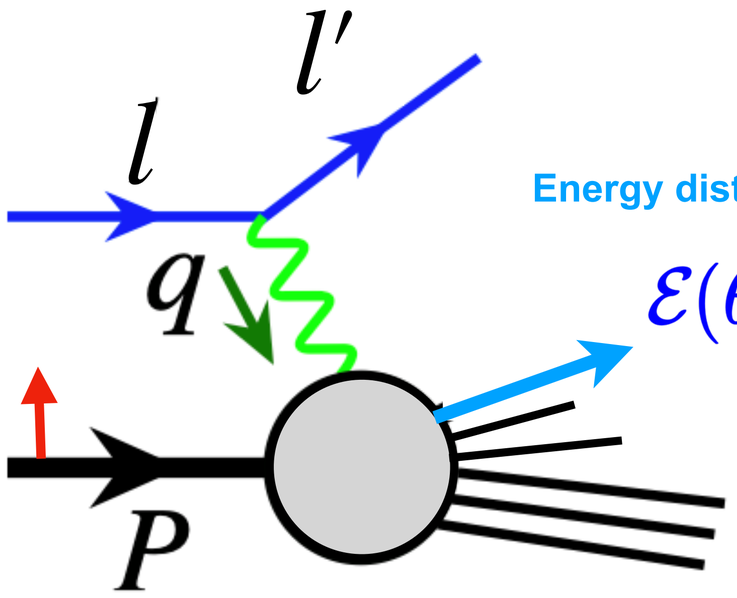
$$\Sigma(\theta,\phi)_{\rm DIS} = \sum_h \int d\sigma^{e+N\to e+h+X} \frac{E_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta(\phi-\phi_h)$$

Inclusive summation Energy weighting

e.g., [Meng-Olness-Soper NPB 371 (1992) 79]
[Li-Makris-Vitev PRD 103 (2021) 094005]
[Chen-Ma-Tong, JHEP 08 (2024) 227]

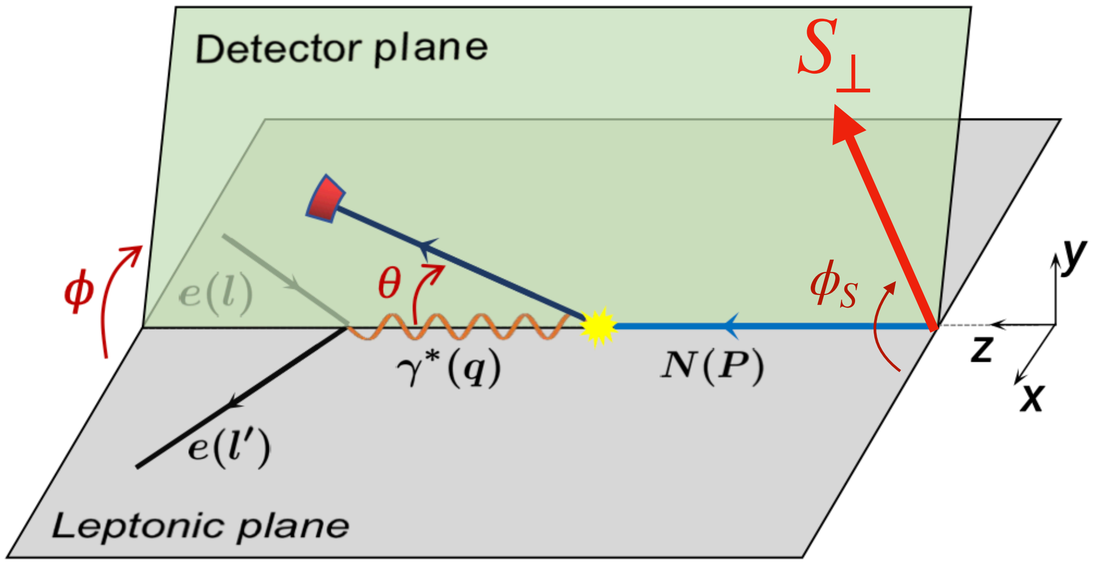
- One-point energy correlator in the DIS
- Cleaner probe ; suppress non-perturbative effects in the final states, e.g. $\sum_h \int_0^1 dz\, z D_{h/a}(z) = 1$

The DIS energy pattern



Energy distribution

$\mathcal{E}(\theta, \phi)$



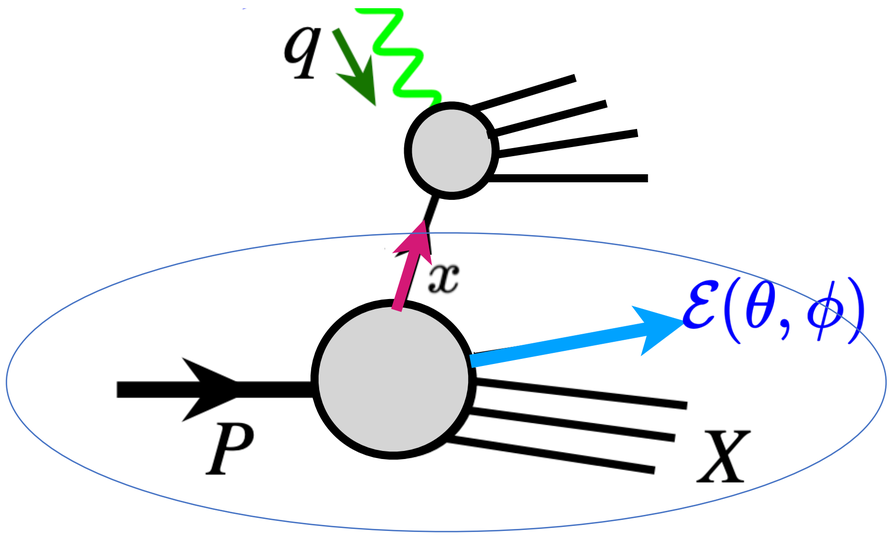
- Single spin asymmetry : a left-right asymmetry of hadronic energy pattern

$$\Sigma(\theta, \phi, \boldsymbol{S}_\perp) = \Sigma_{UU}(\theta) + |\boldsymbol{S}_\perp| \sin(\phi - \phi_S)\, \Sigma_{UT}^{\sin(\phi-\phi_S)}(\theta)$$

[Chen-Ma-Tong, JHEP 08 (2024) 227]

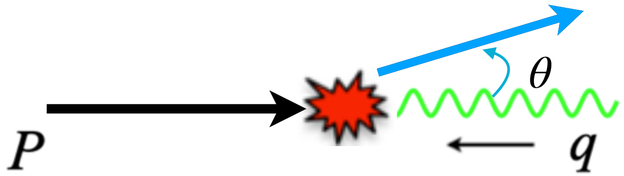
$$\Rightarrow \quad A_{UT} = \frac{\Sigma_{UT}^{\sin(\phi-\phi_S)}}{\Sigma_{UU}}$$

What happens in the target fragmentation region?



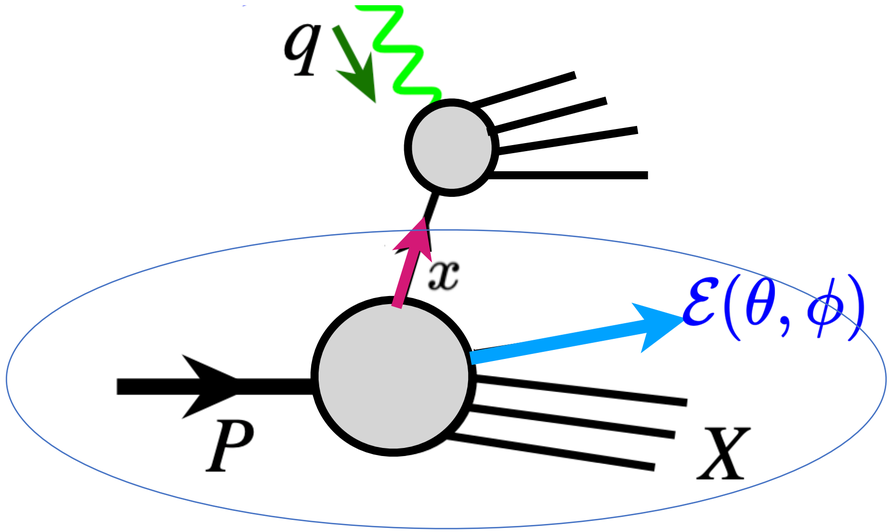
Strong correlations between

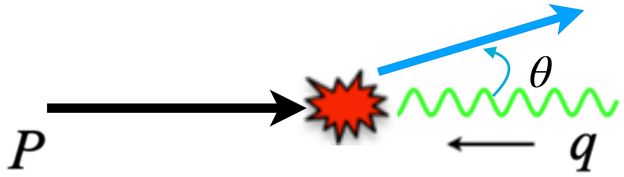
the initial-state parton and the final-state energy flux



small-θ

What happens in the target fragmentation region?





small-θ

✦ **Nucleon energy correlator (NEC)**

[Liu-Zhu PRL 130, 2023]

- The conditional probability of finding a parton with x while observing an energy flux from the target remnants in (θ,ϕ) :

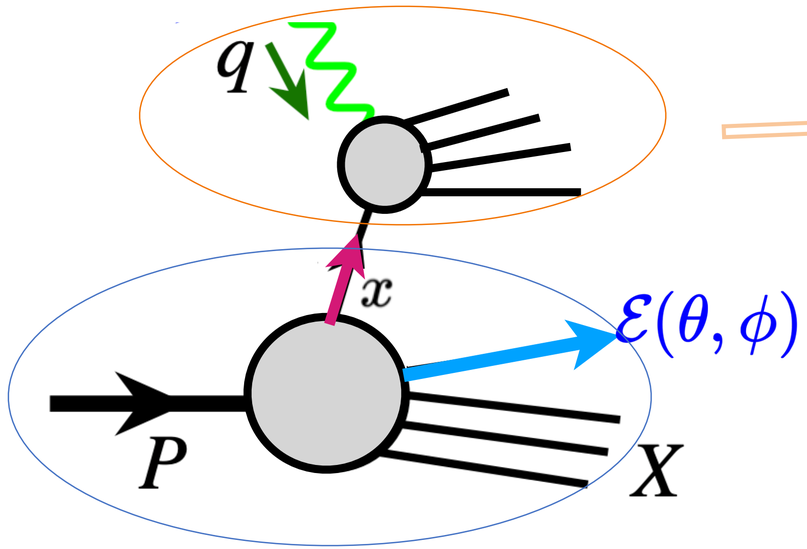
$$f_{\mathrm{NEC}}(x,\theta,\phi)$$

- Collinear distributions (parton kT integrated out)

[See also Yujiao Zhu's talk in WG5

& Yingsheng Huang's talk in WG3]

What happens in the target fragmentation region?



Same as inclusive DIS

Collinear factorization:

$$\Sigma \propto H(Q,\mu,x) \otimes f_{\mathrm{NEC}}(x,\theta,\phi,\mu)$$

DGLAP evolutions

[See also Paul Caucal 's talk in WG2]

✦ **Nucleon energy correlator (NEC)**

[Liu-Zhu PRL 130, 2023]

- The conditional probability of finding a parton with x while observing an energy flux from the target remnants in (θ,ϕ) :

$$f_{\mathrm{NEC}}(x,\theta,\phi)$$

- Collinear distributions (parton kT integrated out)

➡ What kind of NEC is responsible for the SSA?

Nucleon energy correlator (NEC)

Unpolarized quark NEC:

$$f^q_{\mathrm{NEC}}(x,\theta,\phi) = \int \frac{d\eta}{4\pi} e^{-ixP^+\eta^-} \langle PS|\,\bar{\psi}(\eta^-)\,\mathcal{L}^\dagger_n(\eta^-)\,\gamma^+\hat{\mathcal{E}}(\theta,\phi)\mathcal{L}_n(0)\psi(0)\,|PS\rangle$$

- Operator definition: collinear PDF with the insertion of the energy flux operator

$$\mathcal{E}(\theta,\phi) = \sum_h \int \frac{d^3\boldsymbol{P}_h}{2E_h(2\pi)^3} \frac{E_h}{E_N} \delta(\theta^2-\theta_h^2)\delta(\phi-\phi_h) a_h^\dagger a_h$$

Inclusive summation — Energy weighting — Hadronic number density operator

- Classification and parametrization similar to PDF/TMDs

Sivers-type NEC and single-spin asymmetry

Unpolarized quark NEC:

$$f^q_{\mathrm{NEC}}(x,\theta,\phi) = \int \frac{d\eta}{4\pi} e^{-ixP^+\eta^-} \langle PS|\,\bar{\psi}(\eta^-)\,\mathcal{L}^\dagger_n(\eta^-)\,\gamma^+\hat{\mathcal{E}}(\theta,\phi)\mathcal{L}_n(0)\psi(0)\,|PS\rangle$$

- If the proton is transversely polarized

[Chen-Ma-Tong, JHEP 08 (2024) 227]
[Mantysaari, Tawabutr, Tong,2503.20157]

$$f_{\mathrm{NEC}}(x,\theta,\phi,S_\perp) = f_1^q(x,\theta) + \sin(\phi-\phi_S)\,|S_\perp|\,f_{1T}^{t,q}(x,\theta)$$

Unpolarized quark in an unpolarized target

Sivers-type quark NEC

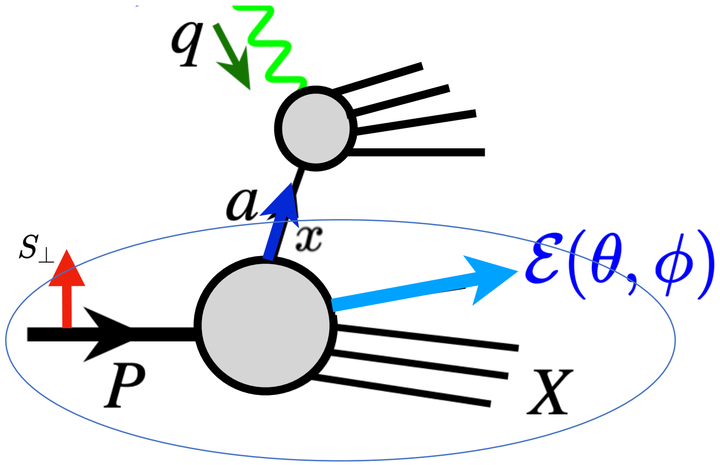
Unpolarized quark when observing an azimuthal correlation between the proton spin and the energy flux

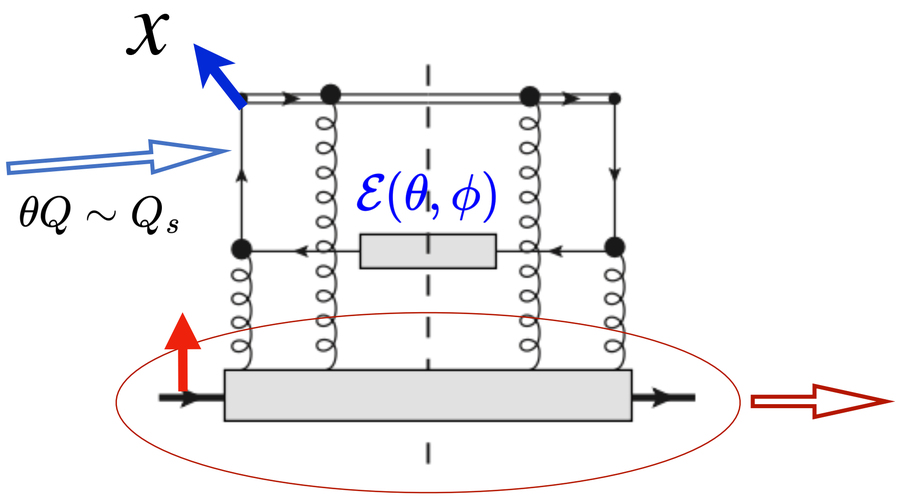
- At LO of α_s, the SSA is given by the ratio between the flavor singlet of the quark NECs

$$A_{UT} = \frac{\sum_{a=q,\bar{q}} e_a^2 f_{1T}^{t,a}(x_B,\theta)}{\sum_{a=q,\bar{q}} e_a^2 f_1^a(x_B,\theta)}$$

What happens at small-x? [Mantysaari, Tawabutr, Tong,2503.20157]

- At small-x, the quark NEC can be computed in the CGC, because of $k_{g\perp} \sim Q_s \sim \theta Q$





The dipole gluon distributions

$$\mathcal{F}_{x_g}(\boldsymbol{k}_{g\perp}, \boldsymbol{S}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{r}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_\perp} \times \frac{1}{N_c}\left\langle \mathrm{tr}\left[U(\boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2})U^\dagger(\boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2})\right]\right\rangle_{x_g}$$

- A quark and anti-quark pair is generated from small-x gluons
  - The quark initiates the hard scattering with γ^*
  - The antiquark fragments into the hadrons that form the energy flux

- The production of the energy flux is IRC safe , due to the energy weighting and the inclusive summation

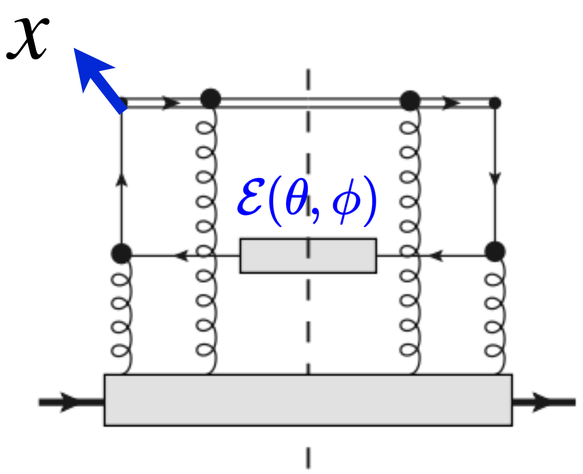
The quark NEC at small-x

[Mantysaari, Tawabutr, Tong,2503.20157]

- The quark NEC is expressed in terms of the dipole gluon distributions:

$$f_1^q(x,\theta) + \sin(\phi-\phi_S)\,|S_\perp|\, f_{1T}^{t,q}(x,\theta)$$

$$= \frac{N_c}{\theta^2(2\pi)^4}\int_0^{1-x} \mathrm{d}\xi \int \mathrm{d}^2\boldsymbol{k}_{g\perp}\boldsymbol{k}_\perp^2 \mathcal{H}_q\left(\xi,\boldsymbol{k}_\perp,\boldsymbol{k}_{g\perp}\right)\mathcal{F}_{x_g}\left(\boldsymbol{k}_{g\perp},\boldsymbol{S}_\perp\right)$$



$$\mathcal{H}_q\left(\xi,\boldsymbol{k}_\perp,\boldsymbol{k}_{g\perp}\right) = \frac{\boldsymbol{k}_\perp^2}{\xi}\left[\frac{\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp}{\epsilon_f^2+\left(\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp\right)^2} - \frac{\boldsymbol{k}_\perp}{\epsilon_f^2+\boldsymbol{k}_\perp^2}\right] \qquad \boldsymbol{k}_\perp \equiv \frac{\xi P^+\theta}{\sqrt{2}}(\cos\phi,\sin\phi)$$

$$\epsilon_f^2 \equiv (x/\xi)\boldsymbol{k}_\perp^2$$

- Pomeron & Spin-dependent odderon:

$$\mathcal{F}_{x_g}\left(\boldsymbol{k}_{g\perp},\boldsymbol{S}_\perp\right) = F_{x_g}(\boldsymbol{k}_{g\perp}^2) + \sin(\phi_g-\phi_S)\frac{|\boldsymbol{k}_g||\boldsymbol{S}_\perp|}{M}O_{1T,x_g}^\perp(\boldsymbol{k}_{g\perp}^2)$$

[See the pomeron as a probe for saturation effects in Liu, Liu, Pan, Yuan, Zhu PRL. 130 (2023) 181901]

➡The quark Sivers NEC is generated by the spin-dependent odderon at small-x (at eikonal accuracy)

However, the odderon is C-odd…

- The anti-quark Sivers NEC has an opposite sign to the quark one

$$f_{1T}^{t,\bar{q}}(x,\theta) = -f_{1T}^{t,q}(x,\theta)$$

- The SSA is sensitive to the flavor singlet of Sivers-type NECs:

$$A_{UT} \propto \sum_{a=q,\bar{q}} e_a^2 f_{1T}^{t,a}\left(x_B,\theta\right)$$

➡ The odderon contributions to the SSA vanish

- The measurement of the energy flux is too inclusive and dose not distinguish the quark and antiquark fragmentation, causing the complete cancelations

➡ Restrict the measurement on positively or negatively charged hadrons (tracks):

$$\mathbb{S} = \left\{h^+\right\}, \left\{h^-\right\}$$

Incorporating track information

$$f^q_{\mathrm{NEC},\mathbb{S}}(x,\theta,\phi) = \int \frac{d\eta}{4\pi} e^{-ixP^+\eta^-} \langle PS|\,\bar{\psi}(\eta^-)\,\mathcal{L}_n^\dagger(\eta^-)\,\gamma^+ \hat{\mathcal{E}}_{\mathbb{S}}(\theta,\phi)\mathcal{L}_n(0)\psi(0)\,|PS\rangle$$

- This introduce fragmentation effects

$$\mathbb{S} = \{h^+\},\{h^-\} \qquad \mathcal{E}_{\mathbb{S}}(\theta,\phi) = \sum_{h\in\mathbb{S}} \int \frac{\mathrm{d}P_h^+ \mathrm{d}^2\boldsymbol{P}_\perp}{2P_h^+(2\pi)^3} \frac{E_h}{E_N} \delta(\theta^2-\theta_h^2)\delta(\phi-\phi_h) a_h^\dagger a_h$$

✦ Simple relation to the inclusive NEC:

$$f^q_{\mathbb{S}}(x,\theta,\phi) = T^{\mathbb{S}}_{\bar{q}} \times f^q(x,\theta,\phi)\ , \qquad T^{\mathbb{S}}_a \equiv \sum_{h\in\mathbb{S}} \int_0^1 z D_{h/a}(z)\mathrm{d}z$$

First moment of FFs

✓ The non-perturbative effects comes in as an overall number, instead of convolution

✓ The SSA becomes non-zero $\quad A_{UT} \propto \sum_q e_q^2 \left(T^{\mathbb{S}}_{\bar{q}} - T^{\mathbb{S}}_q\right) f_{1T}^{t,q}(x,\theta)$

A C-odd tag: Incorporating charge weight

✦ Incorporating charge:

$$\mathcal{E}_{\mathbb{Q}}(\theta,\phi)=\sum_h\int\frac{\mathrm{d}P_h^+\mathrm{d}^2\boldsymbol{P}_\perp}{2P_h^+(2\pi)^3}\frac{E_hQ_h}{E_N}\delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right)a_h^\dagger a_h$$

‣ Relation to the inclusive NEC:

$$f_{1T,\mathbb{Q}}^{t,q}(x,\theta)=D_{\bar{q}}^{\mathbb{Q}}\times f_{1T}^{t,q}(x,\theta)\ ,\qquad D_{\bar{q}}^{\mathbb{Q}}=\sum_h Q_h\int_0^1 zD_{h/\bar{q}}(z)\mathrm{d}z$$

Averaged charge from the anti-quark fragmentation

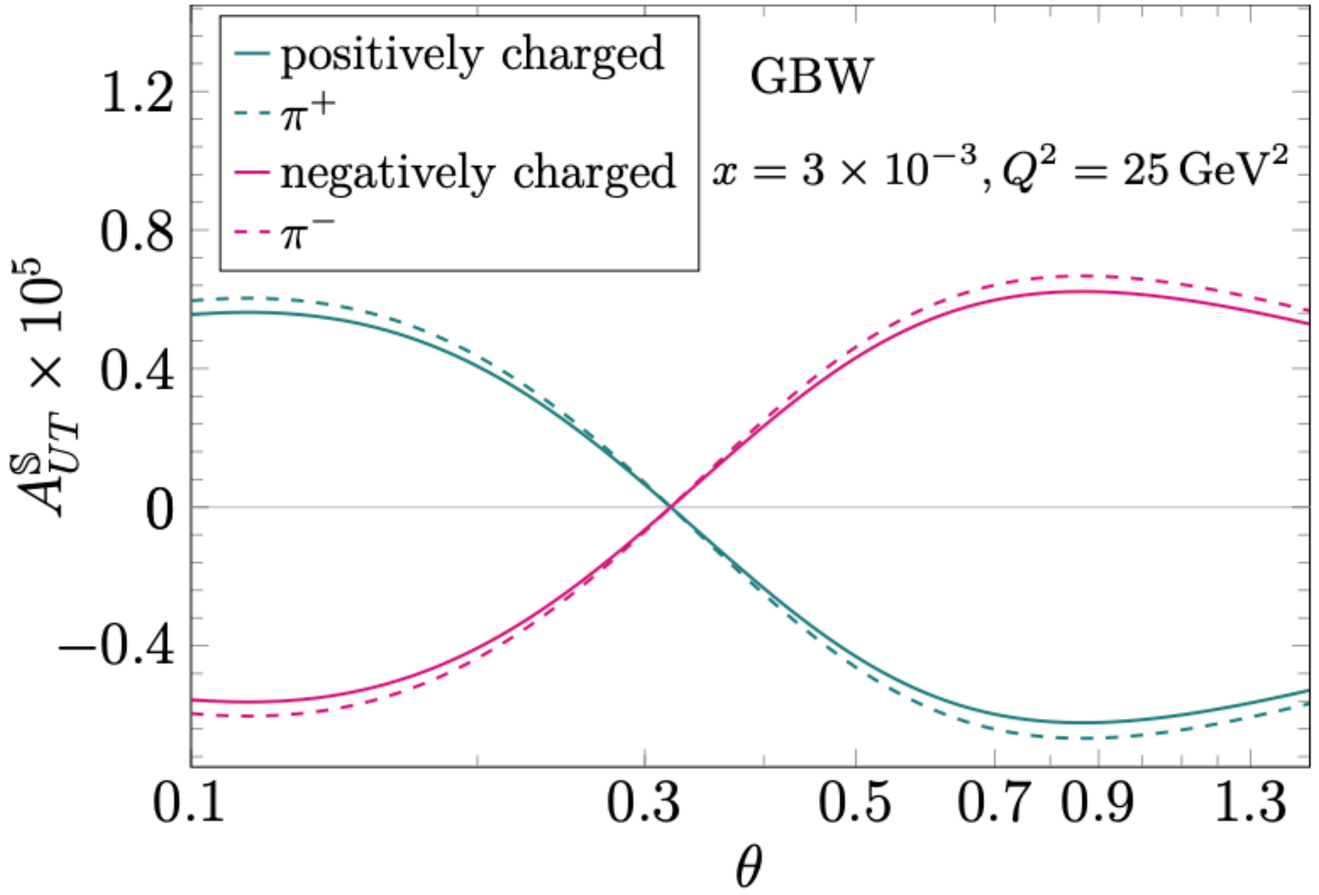
Charge conjugation: $D_q^{\mathbb{Q}}=-D_{\bar{q}}^{\mathbb{Q}}$

✦ A C-odd measurement:

➡If there are C-even contributions, they cancel in the flavors singlet, leaving only the C-odd ones

$$\sum_{a=q,\bar{q}}e_a^2f_{1T,\mathbb{Q}}^{t,a}(x,\theta)=\sum_q e_q^2D_{\bar{q}}^{\mathbb{Q}}\left[f_{1T}^{t,q}(x,\theta)-f_{1T}^{t,\bar{q}}(x,\theta)\right]=2\sum_q e_q^2D_{\bar{q}}^{\mathbb{Q}}f_{1T}^{t,q}(x,\theta)$$

A prediction from the odderon domination



$$A_{UT}^{\mathbb{S}}(\theta) = \mathcal{R}_{\mathbb{S}} \frac{f_T^{t,q}(x_B, \theta)}{f_1^q(x_B, \theta)}$$

$$\mathcal{R}_{\{h^+\}} \approx -\mathcal{R}_{\{h^-\}} = -0.252$$

- Model: assuming the relation between the odderon and the pomeron in the generalized MV model [Jian Zhou 2013]

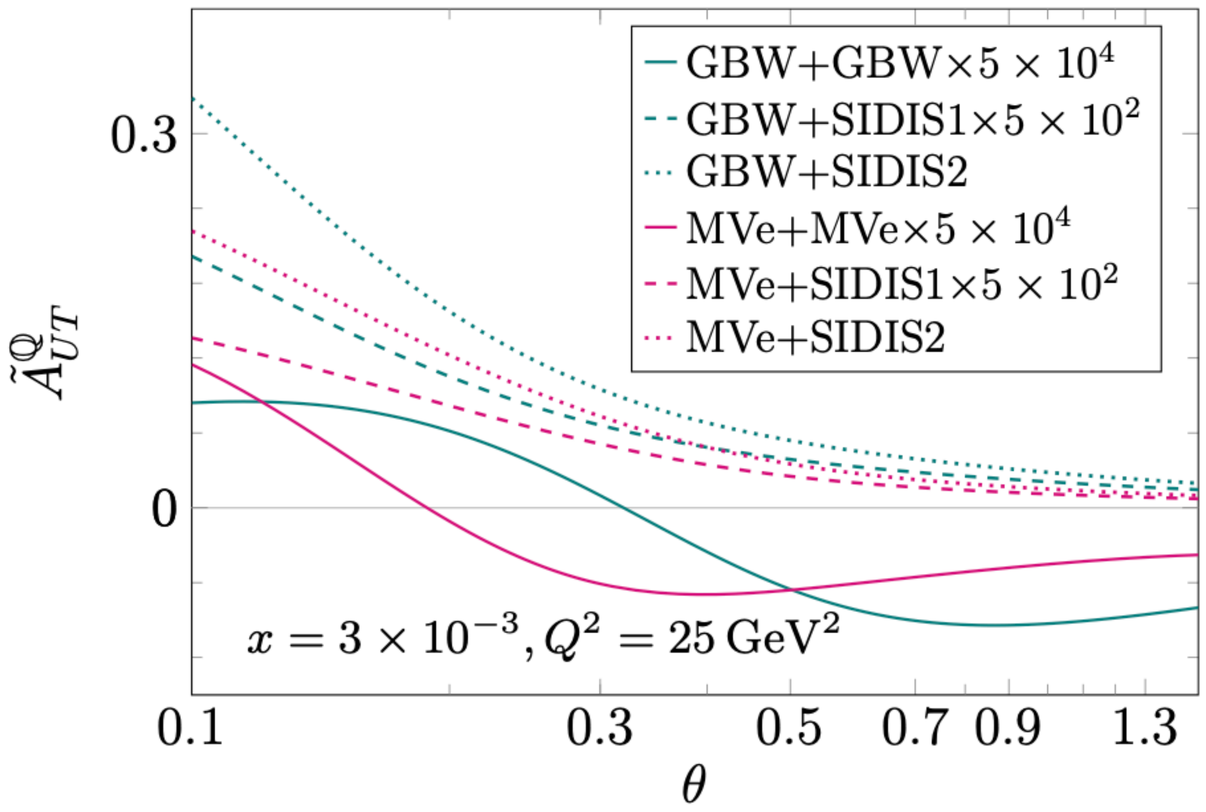
$$O_{1T,x_g}^{\perp}(\boldsymbol{k}_\perp^2) = -\frac{c_0 \alpha_s^3 \left(\kappa_p^u + \kappa_p^d\right)}{4R^4} \frac{\partial}{\partial \boldsymbol{k}_\perp^2} \frac{\partial}{\partial \boldsymbol{k}_\perp^i} \frac{\partial}{\partial \boldsymbol{k}_{\perp i}} F_{x_g}(\boldsymbol{k}_\perp^2)$$

- the GBW pomeron as input

- Signature: the energy-pattern SSA changes sign from $\mathbb{S} = \{h^+\}$ to $\mathbb{S} = \{h^-\}$
- However, may be obscured by the potential C-even contributions

Numerics for the charge-weighed SSA

Pomeron +Odderon



- SSA:

$$\tilde{A}^{\mathbb{Q}}_{UT}(\theta) \equiv \frac{\Sigma^{\mathbb{Q}}_{UT}}{\Sigma^{\mathbb{S}}_{UU}} = \left(\frac{\sum_q e_q^2 D^{\mathbb{Q}}_{\bar{q}}}{\sum_q e_q^2 T^{\mathbb{S}}_{\bar{q}}}\right) \frac{f^{t,q}_{1T}(x,\theta)}{f^q_1(x,\theta)}$$

- GBW/MVe: the odderon with the pomeron as input [Jian Zhou 2013]

$$O^{\perp}_{1T,x_g}(\boldsymbol{k}^2_{\perp}) = -\frac{c_0 \alpha_s^3 \left(\kappa^u_p + \kappa^d_p\right)}{4R^4} \frac{\partial}{\partial \boldsymbol{k}^2_{\perp}} \frac{\partial}{\partial \boldsymbol{k}^i_{\perp}} \frac{\partial}{\partial \boldsymbol{k}_{\perp i}} F_{x_g}(\boldsymbol{k}^2_{\perp})$$

- SIDIS1/2: the odderon from the fitting of gluon Sivers TMDs [Alesio, Murgia,Pisano,JHEP 09 (2015) 119]

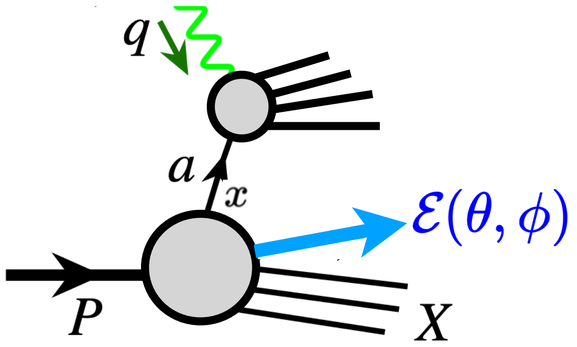
$$O^{\perp}_{1T,x}(\boldsymbol{k}^2_{\perp}) = \frac{4\pi^2 \alpha_s M_p}{\boldsymbol{k}^3_{\perp} N_c} \mathcal{N}_g(x) g(x) h_g(k_{\perp}) \frac{e^{-\boldsymbol{k}^2_{\perp}/\langle \boldsymbol{k}^2_{\perp} \rangle}}{\pi \langle \boldsymbol{k}^2_{\perp} \rangle},$$

$$\mathcal{N}_g(x) = N_g x^{\alpha} (1-x)^{\beta} (\alpha+\beta)^{(\alpha+\beta)} / (\alpha^{\alpha} \beta^{\beta})$$

$$h(\boldsymbol{k}_{\perp}) = \sqrt{2e} e^{-\boldsymbol{k}^2_{\perp}/M_g^2} \boldsymbol{k}_{\perp} / M_g$$

➡ The results heavily reply on the input of the odderon models

Summary



- Nucleon energy correlator for the Odderon

  - Connection between the spin-dependent odderon and the nucleon tomography in the target fragmentation region, complementary to TMDs

  - Incorporating track information

    ➡ Non-vanishing SSA

    ➡ Cleaner probe than SIDIS: non-perturbative effects appear as an overall factor

  - Incorporating charge weight

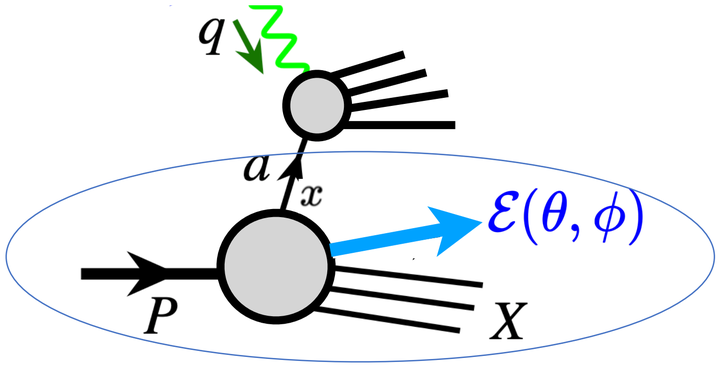
    ➡ A C-odd tag, eliminating the C-even background

- Outlook: Going beyond the eikonal limit, the quark Sivers NEC receives C-even contributions, beside the C-odd odderon. How much can the C-odd tag improve the measurement?

Backup:

Nucleon energy correlator (NEC)

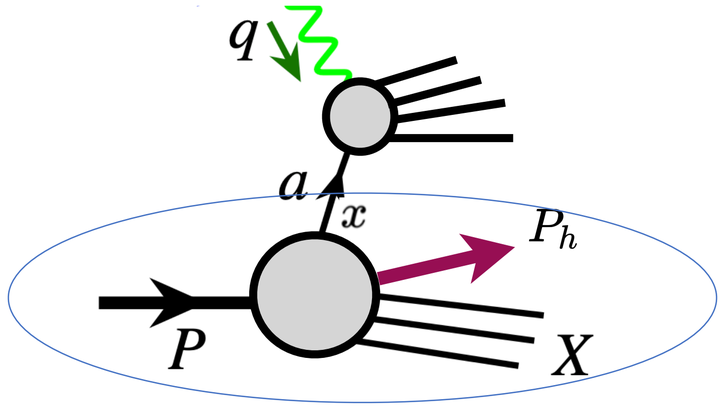
[Liu-Zhu PRL 130, 2023]



➡ The probability of finding a parton while observing an energy flow from target remnants

Fracture functions

[Trentadue-Veneziano *PLB 323 (1994) 201]*



➡ The probability of finding a parton while observing a specific hadron h from target remnants

★ An energy sum rule: [Chen-Ma-Tong, JHEP 08 (2024) 227]

$$\mathcal{M}_{\mathrm{NEC}}(x,\theta,\phi)=\sum_h\int_0^{1-x}\xi_h d\xi_h\int d^2\boldsymbol{P}_{h\perp}\delta\left(\theta^2-\theta_h^2\right)\delta(\phi-\phi_h)\mathcal{M}_{\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h_\perp})$$

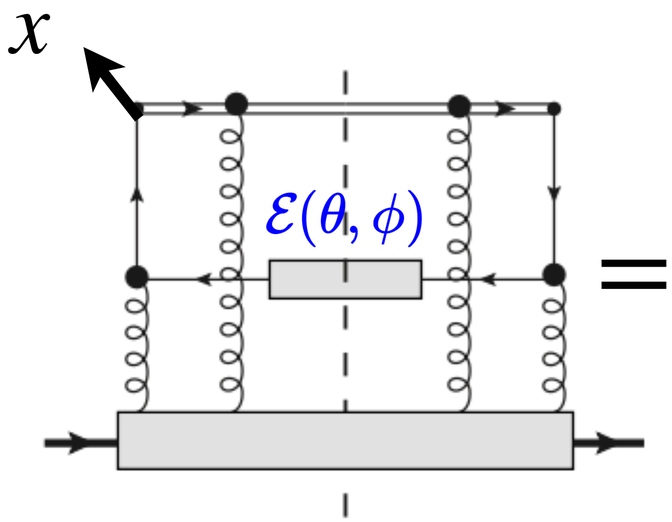
- One-to-one correspondence; Same evolutions
- Fracture functions are the parent functions of nucleon energy correlator

Backup:

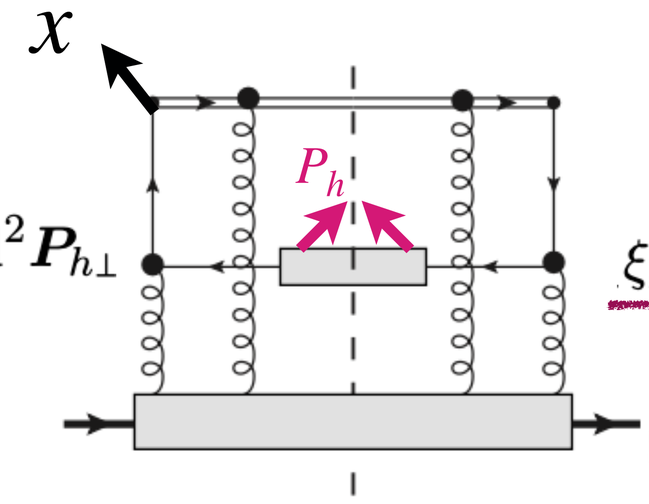
Deriving track-based NECs from Fracture functions

To see how the fragmentation effects come in, one needs to start from the calculation at the hadronic level

An energy sum rule:



$$= \sum_{h\in\mathbb{S}} \int \mathrm{d}\xi_h \, \mathrm{d}^2\boldsymbol{P}_{h\perp}$$



$$\xi_h \delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right)$$

$$\xi_h = \frac{P_h^+}{P^+}$$

$$f_{\mathbb{S}}^q(x,\theta,\phi) \qquad u_h^q\left(x,\xi_h,\boldsymbol{P}_{h\perp}\right)$$

Fracture function at small-x:

Involve the convolution of FFs

$$= \frac{1}{\xi_h 8\pi^4}\int_0^1 \frac{\mathrm{d}z}{z^2} D_{h/\bar{q}}(z)\theta\left(1-x-\xi_h/z\right) \times \int \mathrm{d}^2\boldsymbol{k}_{g\perp}\mathcal{H}_q\left(\frac{\xi_h}{z},\frac{\boldsymbol{P}_{h\perp}}{z},\boldsymbol{k}_{g\perp}\right)\mathcal{F}_{x_g}\left(\boldsymbol{k}_{g\perp}\right)$$

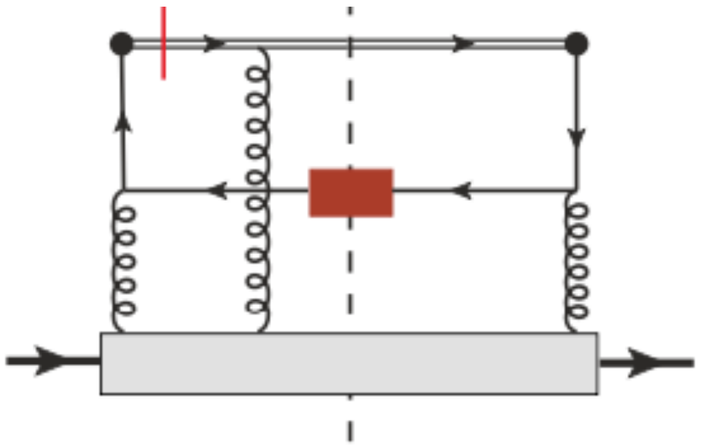
$$= \left(\sum_{h\in\mathbb{S}}\int_0^1 z D_{h/\bar{q}}(z)dz\right) f^q(x,\theta,\phi)$$

Fracture function for a hadron: [Mantysaari, Tawabutr, Tong,2503.20157]

See also the consistency between CGC and jet fracture function: [Caucal, Salazar, 2502.02634]

Backup:

The quark Sivers NEC in the twist-3 collinear formalism



- At large θ, [Chen-Ma-Tong, JHEP 08 (2024) 227]

$$f_{1T}^{t,q}(x,\theta) = \frac{\alpha_s}{x\pi\theta^3\sqrt{2}P^+}\int_x^1 \mathrm{d}z\left[z^2+(1-z)^2\right]$$
$$\left[N\left(x_g,x_g\right)-N\left(x_g,0\right)-O\left(x_g,0\right)-O\left(x_g,x_g\right)\right]$$

C-even, related to sub-eikonal dipole?

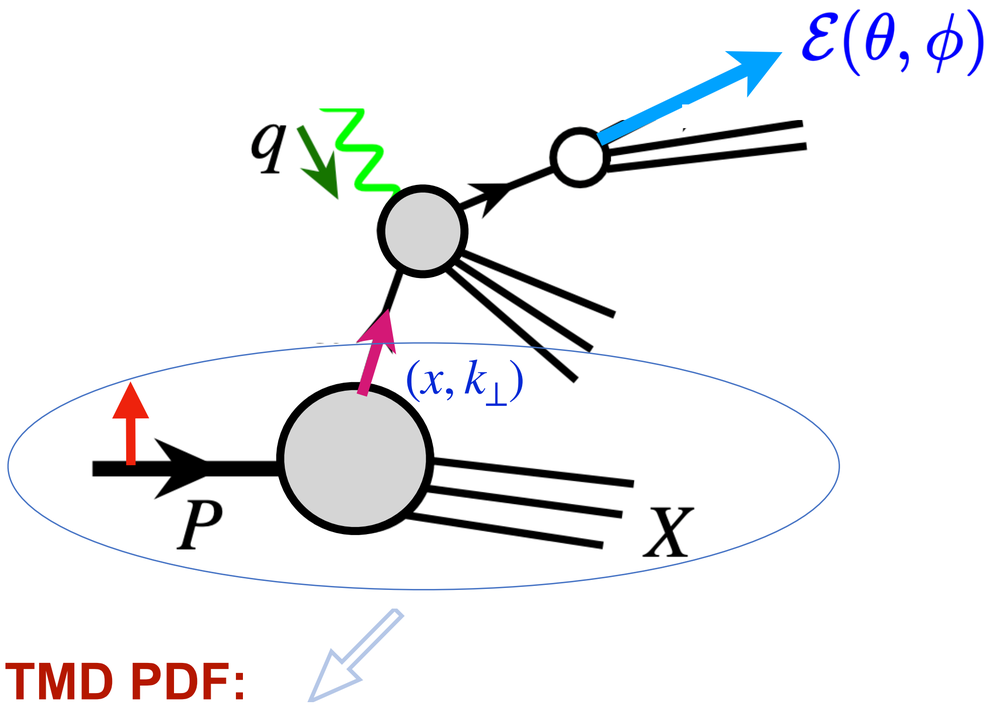
C-odd, related to the spin-dependent odderon

- Twist-3 three-gluon correlation

$$\frac{i^3 g_s}{P^+}\int\frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}e^{i\lambda_1 x_1 P^+ + i\lambda_2(x_2-x_1)P^+}\langle PS|G^{a,+\alpha}(\lambda_1 n)G^{c,+\gamma}(\lambda_2 n)G^{b,+\beta}(0)|PS\rangle$$
$$=\frac{N_c}{(N_c^2-1)(N_c^2-4)}d^{abc}O^{\alpha\beta\gamma}(x_1,x_2)-\frac{i}{N_c(N_c^2-1)}f^{abc}N^{\alpha\beta\gamma}(x_1,x_2)\,,$$

Backup:

Current fragmentation region: connection to TMDs

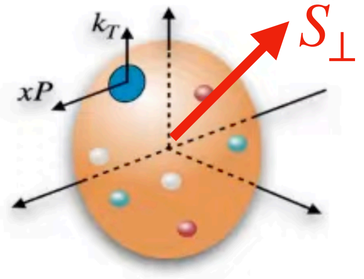


- **TMD PDF:**
  - the probability of finding a parton with $(x, k_\perp)$ in the target
  - TMD factorization derived from SIDIS

    [Li-Makris-Vitev PRD 103 (2021) 094005]

    [Kang-Lee-Shao-Fan JHEP 03 (2024) 153]

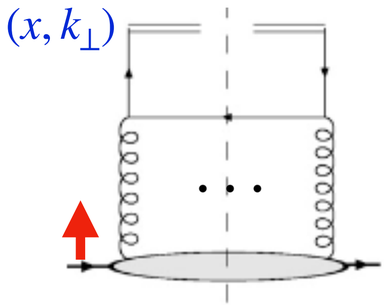
- Single spin asymmetry



➡ Quark Sivers TMD($k_\perp \times S_\perp$)

correlations between unpolarized parton kT and the proton transverse spin

✦ At small-x, generated by the splitting of small-x gluons from the spin-dependent odderon



[Dong- Zheng-Zhou PLB 788 401, 2019]

[Kovchegov, Santiago JHEP 11 (2021) 200]

$$x f_{1T}^{\perp}\left(x,k_\perp^2\right)=\frac{N_c}{8\pi^4}\int d\xi\int d^2l_\perp\frac{k_\perp\cdot l_\perp}{k_\perp^2}O_{1T}^{\perp}\left(x_g,l_\perp^2\right)\left[\frac{l_\perp|l_\perp-k_\perp|}{(1-\xi)l_\perp^2+\xi\left(l_\perp-k_\perp\right)^2}-\frac{l_\perp-k_\perp}{|l_\perp-k_\perp|}\right]^2$$

Transverse EEC& quark Sivers TMD: [Bhattacharya, Kang, Padilla, Penttala, 2504.10475]