Probing Dense Nuclear Matter at Small-x

SURGE Global Analysis WG

GOAL: Sketch a work-flow for a global analysis framework.

Fred Olness

(SMU)

The SMU Team

Fred Olness

Brandon Stevenson grad student

Rebecca Preston grad student

Junaid Khan grad student

Peter Risse Postdoc

with contributions from many others including

C T E Q

nCTEQ nuclear parton distribution functions

xFitter

SURGE COLLABORATION

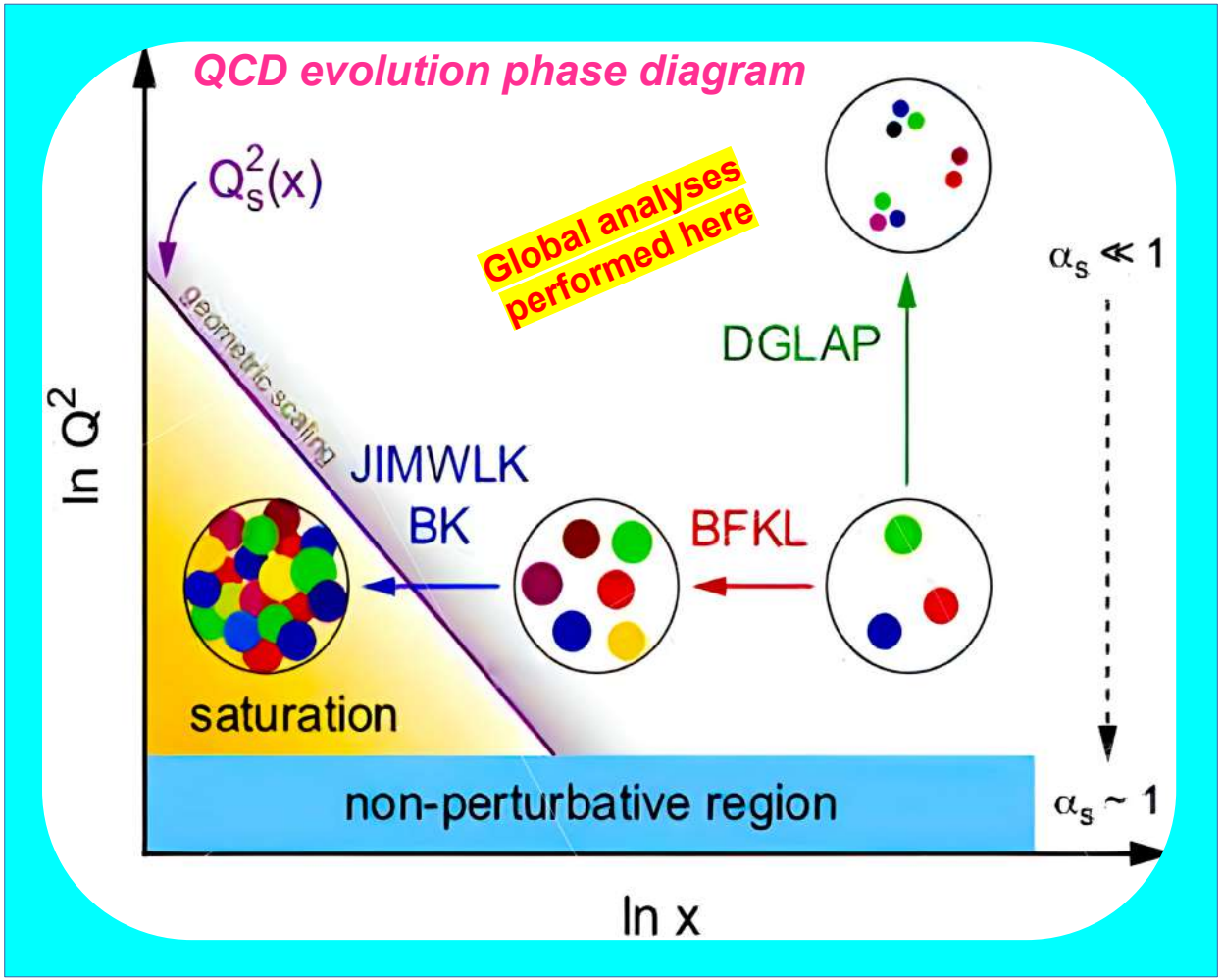
DIS 2026

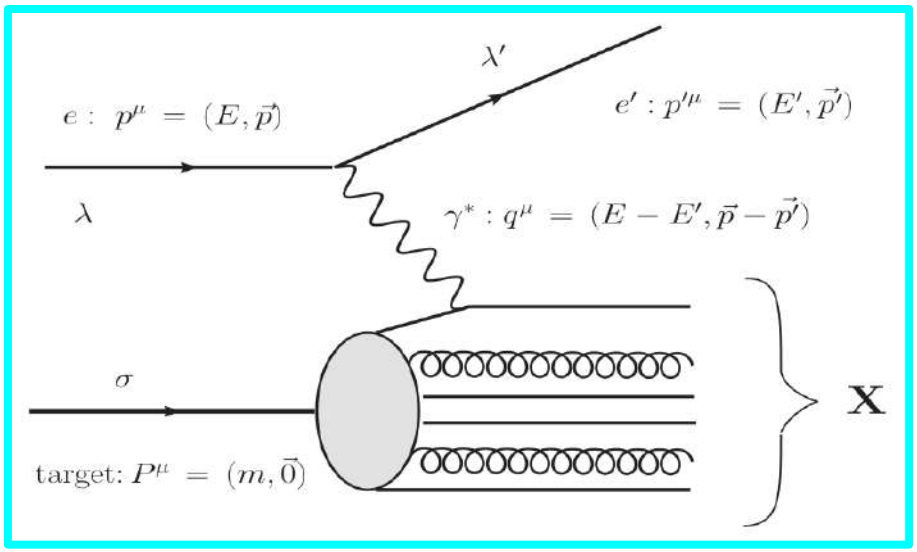
6 May 2026

Bologna, Italy

SURGE Topical Collaboration

Mission statement: Discover and explore the gluon saturation regime of quantum chromodynamics by advancing calculations to high precision and developing a comprehensive framework to compute observables and compare to a wide range of experimental data, including predictions for the Electron Ion Collider (EIC).

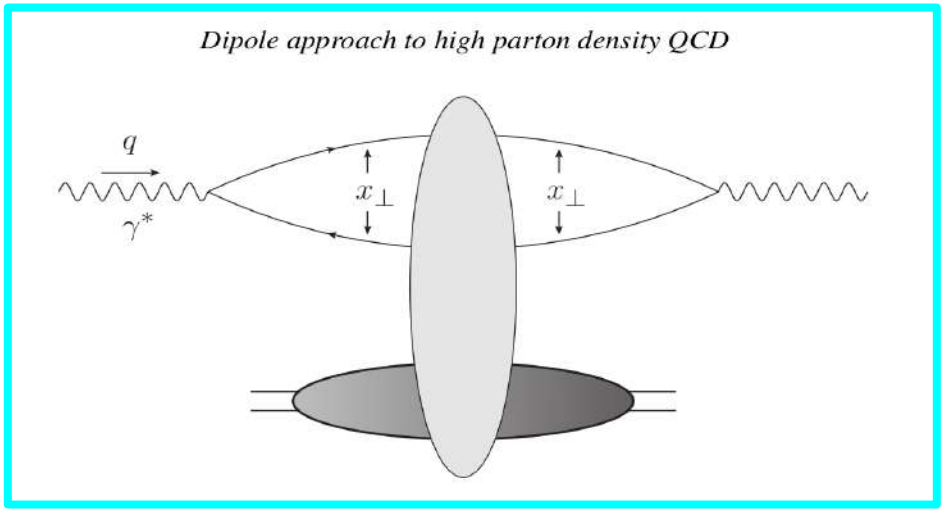




Parton Model

$$\sigma = f_{P\to a}(x, Q) \otimes \widehat{\sigma}_{a\gamma\to X}(x, \hat{s})$$

Implemented in **xFitter**
uses several numerical "tricks"
for efficient computation



Dipole Model

GOAL: Develop flexible
framework to test
initial/final-state, spin, NLO ...

The dipole model excels at small-x and
describes the transition between perturbative
and non-perturbative QCD regime.
Incorporates "shadowing" or "saturation"
which is key for dense gluon configurations.

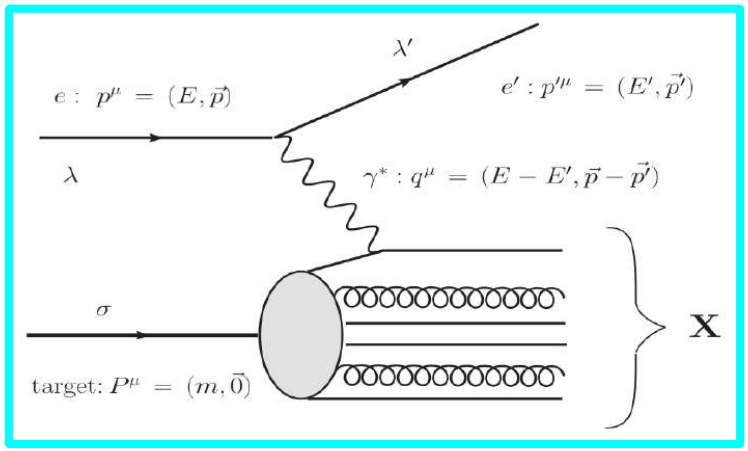
$$\sigma_{tot}^{\gamma^* A}(x, Q^2) \simeq \int\int |\Psi^{\gamma^*\to q\bar{q}}(\vec{x_\perp}, z))|^2 \otimes \sigma_{tot}^{q\bar{q}A}(\vec{x_\perp}, Y)$$

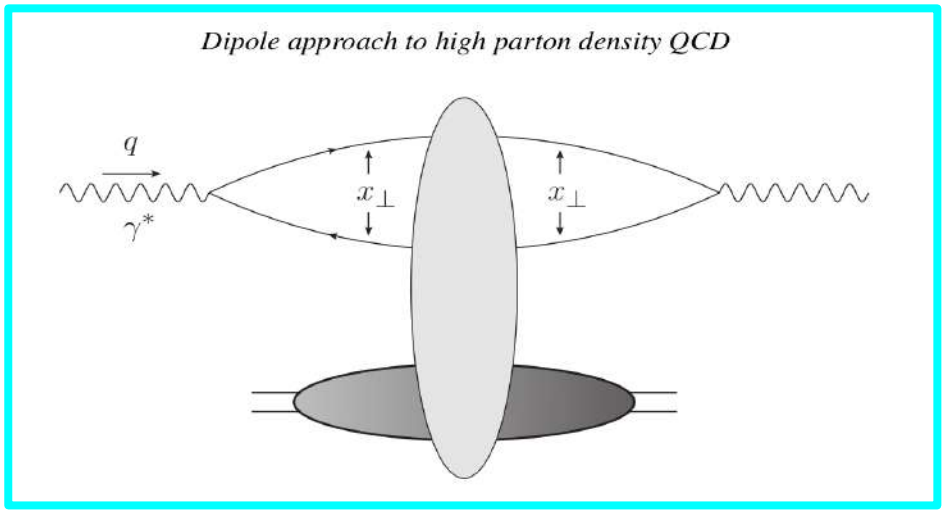
Parton Model & Dipole Model Comparison

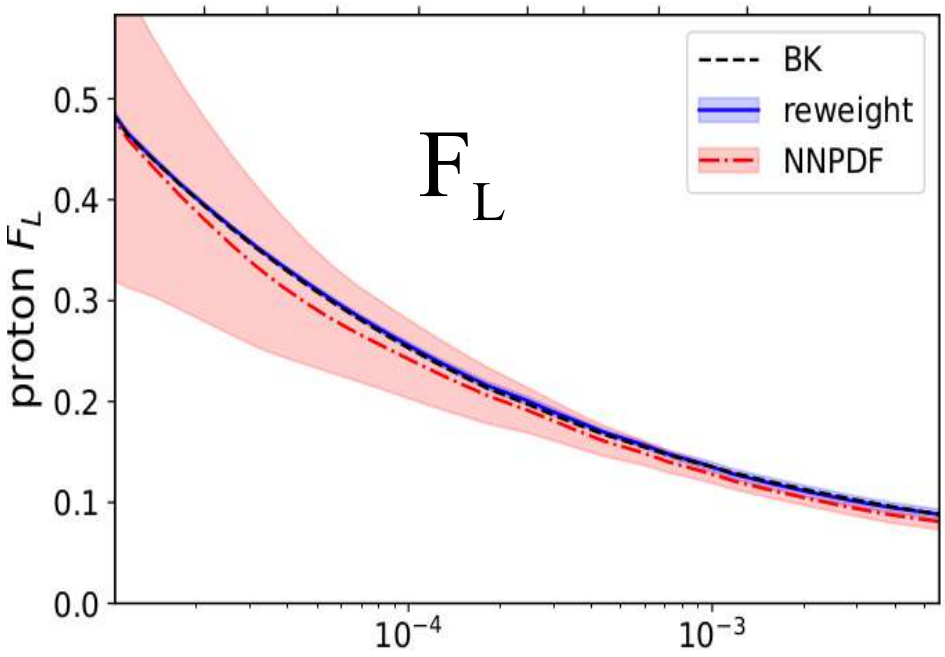
Signatures of gluon saturation from structure-function measurements

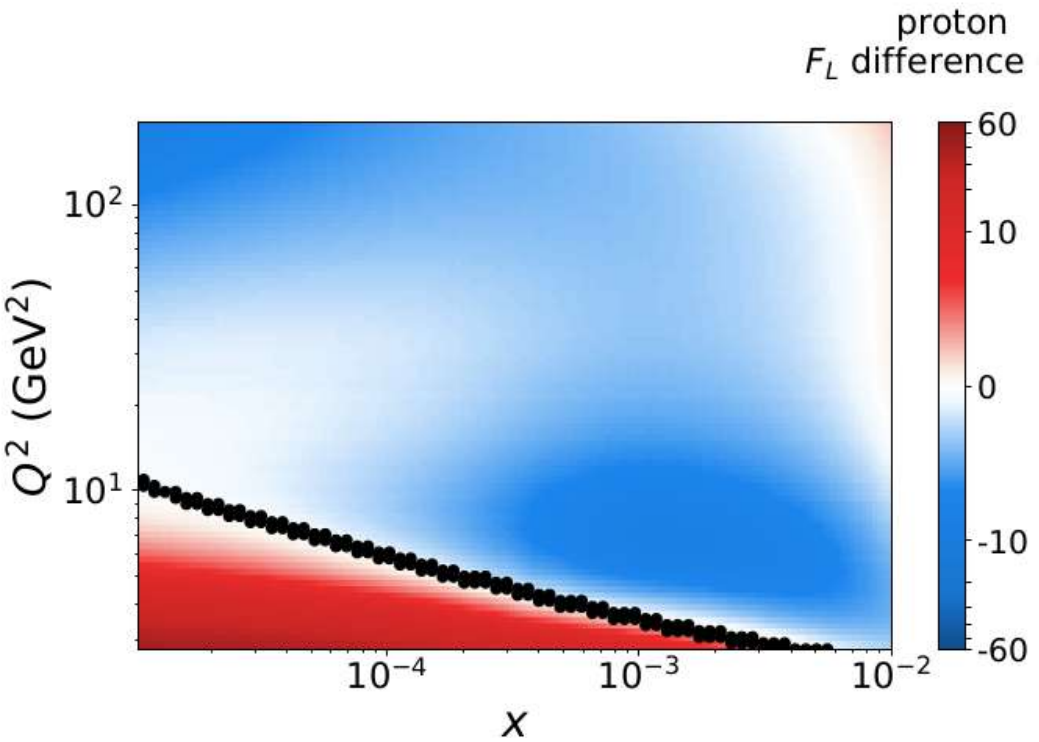
Nestor Armesto,[1,*] Tuomas Lappi,[2,3,†] Heikki Mäntysaari,[2,3,‡] Hannu Paukkunen,[2,3,§] and Mirja Tevio[2,3,¶]

Phys.Rev.D 105 (2022) 11, 114017 • e-Print: 2203.05846 [hep-ph]









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Brookhaven National Laboratory

U.S. DEPARTMENT of ENERGY

SURGE Collaboration

The Saturated Glue (SURGE) Collaboration is a Topical Collaboration in Nuclear Theory, funded by the US Department of Energy, Office of Science, Office of Nuclear Physics for the period 2023-2027.

https://www.bnl.gov/physics/surge/

The SURGE Collaboration aims at the discovery and exploration of the gluon saturation regime in quantum chromodynamics (QCD)

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SURGE: Modular Structure Essential: Input Needed

GOAL: Sketch a work-flow for the global analysis framework.

The calculation must be FAST
to be adaptable for a fitting loop

Initial state WG
Improve the initial conditions for evolution for unpolarized and polarized observables.

Small x evolution + NLO calculations WG
Non-linear evolution at NLO and beyond, computation and implementation of impact factors

Global analysis WG
To establish saturation, perform comprehensive global analysis quantifying and minimizing uncertainties, extracting universal building blocks of high energy factorization.

GitHub

Spin WG
Analyze role saturation in the polarized observables. Elucidate the role of chiral anomaly in small x helicity evolution.

Final states WG Construct a framework for hadronization in a saturated environment, including development of MC generator based on CGC calculations

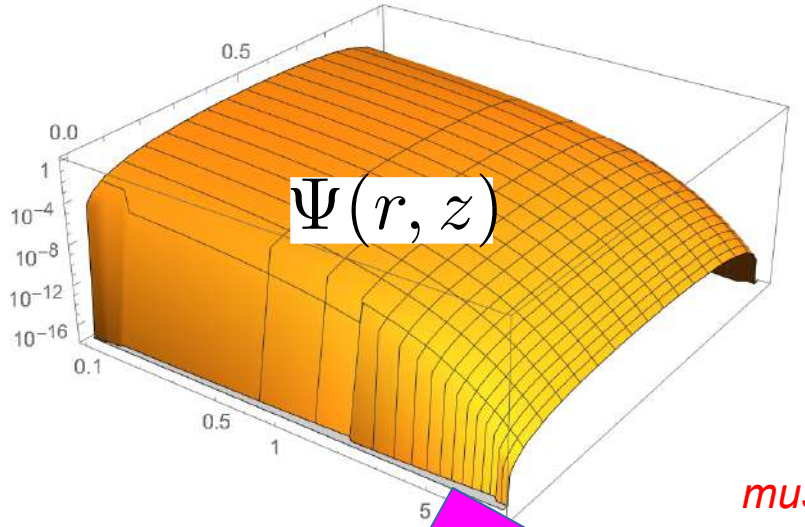
Dipole Model: Global Analysis

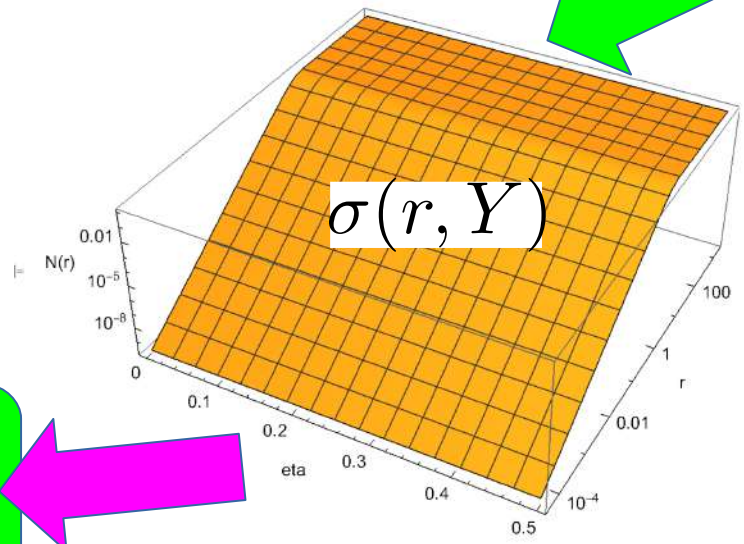
… developing an efficient framework

$$\sigma_{tot}^{\gamma^* A}(x, Q^2) = \int \frac{d^2x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \sigma_{tot}^{q\bar{q}A}(\vec{x}_\perp, Y).$$

Photon light-front wave function

Dipole–nucleus cross section





must be fast!!!

Convolution

2D Grid-based calc

$\sigma_{tot}^{\gamma^* A}(x, Q^2)$

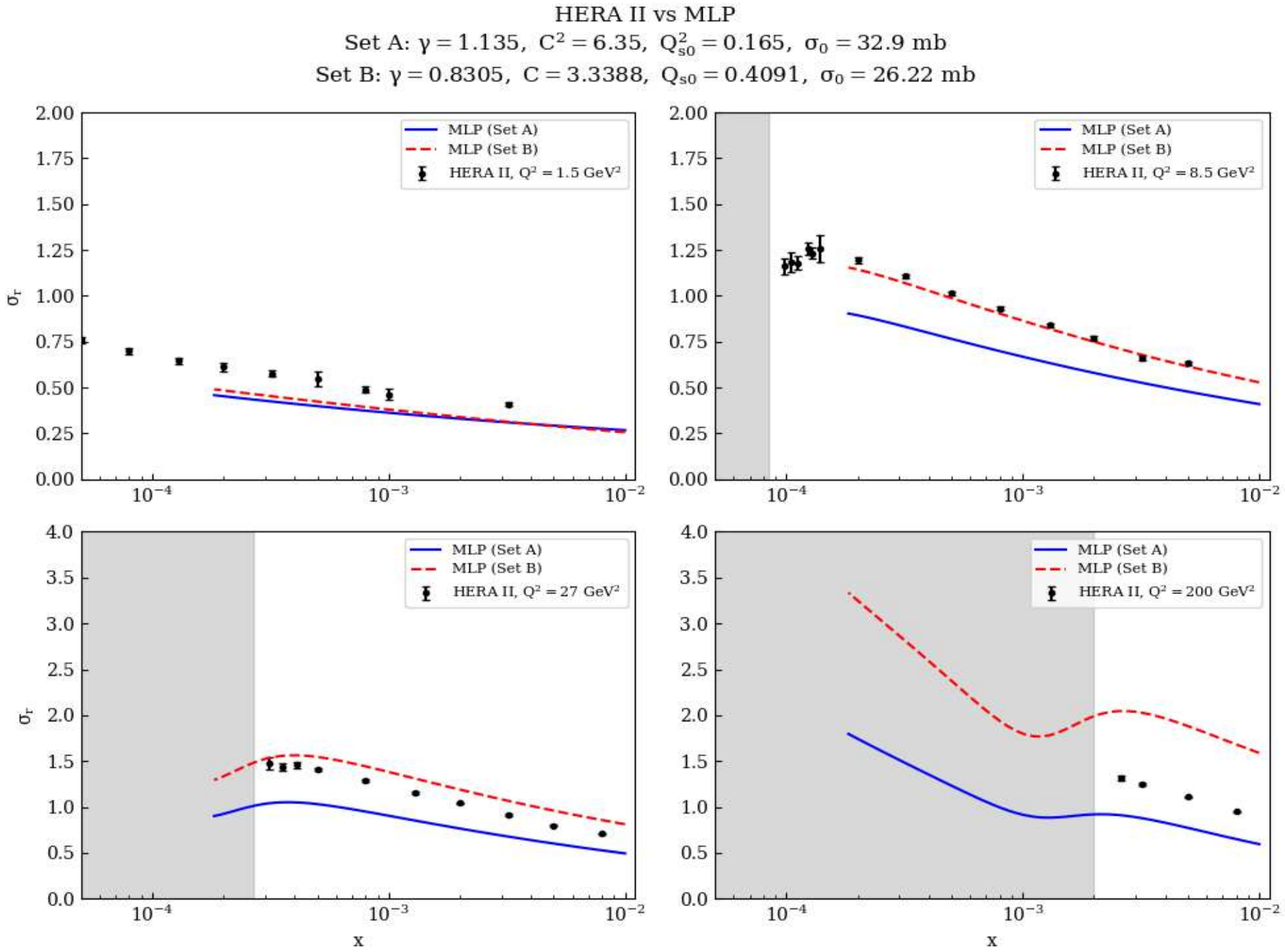
… F_2 & F_L

$$\chi^2 = \frac{(D-T)^2}{\sigma^2}$$

xFitter

The Goal: Global Analysis

… developing an efficient framework

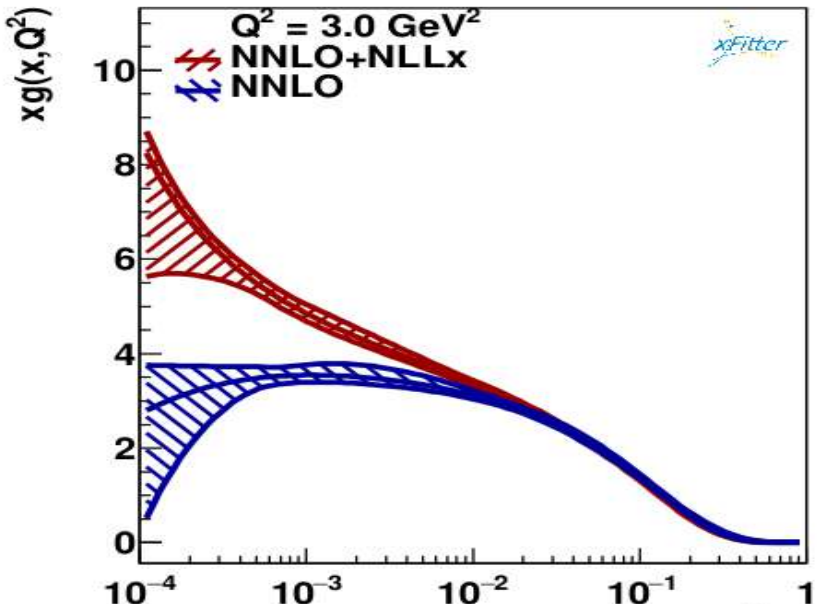


xsect_dipole_proton.c

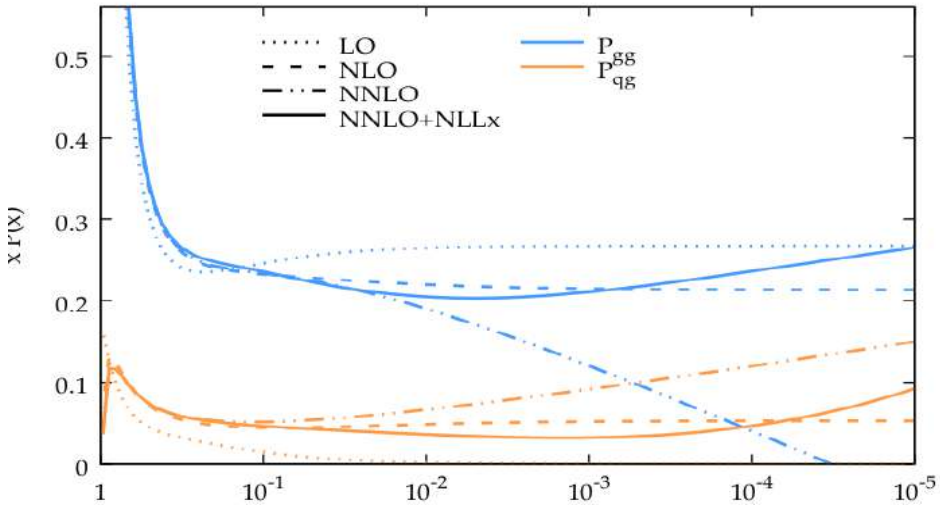
```
#include <math.h>
#include "xsect_dipole_proton.h"

////////////////////////////////////////////////////////
// Parametrisation of the dipole-proton cross-section  //
// as obtained from high-energy QCD with saturation    //
////////////////////////////////////////////////////////
// This file contains two version of the cross-section //
// The first one is the original from                  //
//   [1] E. Iancu, K. Itakura and S. Munier,           //
//       Phys.Lett.B590:199-208,2004 [hep-ph/0310338]  //
// which includes only light quarks.                   //
// The second is from                                  //
//   [2] G. Soyez, in preparation                      //
// which also accounts for heavy quarks.               //
// They both implement eq. (8) of Ref. 1 or eq. (3) of //
// Ref. 2. Only the parameters of the fit vary.        //
//                                                     //
// - Arguments:                                        //
//    r : dipole size                                  //
//    Y : rapidity (Y=log(1/x))                        //
// - Returned value:                                   //
//    the dipole-proton cross-section (in GeV^{-2})    //
//                                                     //
// Additional remarks:                                 //
// - The quark masses are                              //
//     m_{u,d,s}=140 MeV, m_c=1.4 GeV and m_b=4.5 GeV  //
// - When the heavy quarks are considered (2nd model), //
//   the x value for their contribution has to be      //
//   shifted to x(1+4 m^2/Q^2)                         //
////////////////////////////////////////////////////////
```

Impact of low-x resummation on QCD analysis of HERA data

xFitter Developers' team: Hamed Abdolmaleki [1], Valerio Bertone [2,3], Daniel Britzger [4], Stefano Camarda [5], Amanda Cooper-Sarkar [6], Francesco Giuli [6], Alexander Glazov [7], Aleksander Kusina [8], Agnieszka Luszczak [7,9], Fred Olness [10], Andrey Sapronov [11], Pavel Shvydkin [11], Katarzyna Wichmann [7], Oleksandr Zenaiev [7] and Marco Bonvini [12]



$\alpha_s = 0.28,\ n_f = 4$



$$\sigma_{tot}^{\gamma^* A}(x,Q^2) \simeq \iint |\Psi^{\gamma^* \to q\bar{q}}(\vec{x_\perp}, z))|^2 \otimes \sigma_{tot}^{q\bar{q}A}(\vec{x_\perp}, Y)$$

Photon light-front wave function

Dipole target scattering amplitude

Photon light-front wave function

$\Psi(\gamma^* \to q\bar{q})$

LO: Analytic functions

NLO: i) K-Factors

ii) Grid Methods

Fred has experience

Peter has experience

Short-cuts

Example:

LO

$$|\Psi_L^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 = 2N_c \sum_f \frac{\alpha_{EM} Z_f^2}{\pi} 4Q^2 z^3 (1-z)^3 \left[K_0(x_\perp a_f)\right]^2 .$$

We can relate to usual structure functions

$$F_2(x,Q^2) = \frac{Q^2}{4\pi\alpha_{\rm em}} \left(\sigma_T^{\gamma^* A} + \sigma_L^{\gamma^* A}\right),$$

$$F_L(x,Q^2) = \frac{Q^2}{4\pi\alpha_{\rm em}} \sigma_L^{\gamma^* A}.$$

e.g., Extensible to Spin

Photon light-front wave function

Dipole target scattering amplitude

$$\sigma_{tot}^{\gamma^* A}(x,Q^2) \simeq \iint |\Psi^{\gamma^* \to q\bar{q}}(\vec{x_\perp}, z))|^2 \otimes \sigma_{tot}^{q\bar{q}A}(\vec{x_\perp}, Y)$$

$$N(\eta, r) \equiv \sigma_{tot}^{q\bar{q}A}$$

$$\eta = Y = \ln(x_0, x)$$

Balitsky-Kovchegov Equation

$$\frac{\partial N(\eta, r)}{\partial \eta} = \int d^2\mathbf{r}_1 K(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) \Big[N(\eta, r_1) + N(\eta, r_2) - N(\eta, r) - N(\eta, r_1) N(\eta, r_2) \Big]$$

the quadratic term includes the nonlinear saturation effects that tame the growth of gluon densities at small x

Feature	BFKL	BK	JIMWLK
Type	Linear	Non-linear (Dipole)	Non-linear (Functional)
Physics	Gluon emission	Emission + Recombination	Full CGC Color Fields
Limit	Dilute ($x \sim 10^{-2}$)	Large N_c Saturation	General Saturation
Complexity	Simple	Moderate	Very High

Dipole Amplitude Models *(initial conditions)*

There are primarily four dipole amplitude models:

MV$^\gamma$ Model

$$x = \frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(\frac{1}{r\Lambda_{\rm QCD}} + e\right)$$

$$N^{\rm MV^\gamma}(r) = \begin{cases} 1 - \exp(-x), & x > 10^{-6} \\ x, & \text{otherwise} \end{cases}$$

rcMV$^\gamma$ Model

$$x = \frac{r^2 Q_{s0}^2}{4} \cdot \alpha_s(r) \cdot \frac{N_c}{\pi}\left(1 + \ln\left(\frac{\alpha_s^{\rm freeze}}{\alpha_s(r)}\right)\right)$$

$$N^{\rm rcMV}(r) = \begin{cases} [1 - \exp(-x^\gamma)]^{1/\gamma}, & x > 10^{-4} \\ x, & \text{otherwise} \end{cases}$$

GBW Model

$$N^{\rm GBW}(r) = 1 - \exp\left(-\frac{r^2 Q_{s0}^2}{4}\right)$$

GBW$^\gamma$ Model

$$x = \left(\frac{r^2 Q_{s0}^2}{4}\right)^\gamma$$

$$N_0^{\rm GBW^\gamma}(r) = \begin{cases} [1 - \exp(-x)]^{1/\gamma}, & \frac{r^2 Q_{s0}^2}{4} > 10^{-4} \\ \frac{r^2 Q_{s0}^2}{4}, & \text{otherwise} \end{cases}$$

Running coupling constant :

Golec-Biernat–Wusthöff (GBW)
McLerran-Venugopalan (MV) ,

$$\alpha_s = \frac{12\pi}{(33.0 - 2.0N_f)\log\left[\left(\mu_0^{\frac{1}{c}} + \left(\frac{4C^2}{\Lambda_{\rm QCD}^2 r^2}\right)^{\frac{1}{c}}\right)^c\right]}$$

Parameters & Initial Conditions

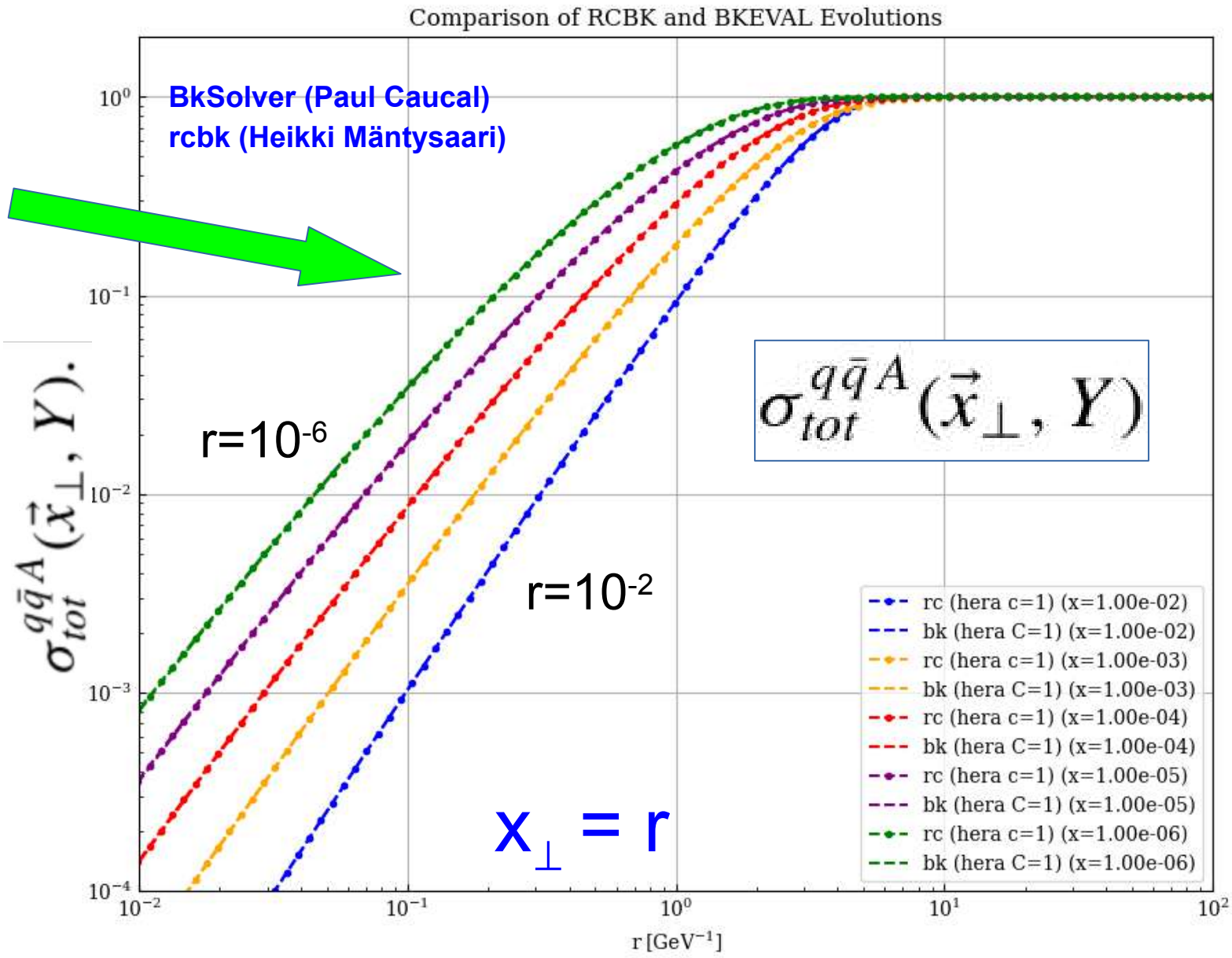
Parameter	bkEval	rcBK	Description
Initial Condition	MV^γ	MV^γ	Dipole amplitude initial model
Evolution Type	Balitsky	Balitsky	BK equation solver type
Model Parameters			
Q_{s0}^2	$Q_{s0} = 0.4062019$ GeV	$Q_{s0}^2 = 0.165$ GeV^2	Initial saturation scale
γ	1.135	1.135	Anomalous dimension in MV^γ model
α_s	Evolving with r	Evolving with r	Strong coupling constant
α_s^{freeze}	0.762	0.762	Freeze value to limit $\alpha_s(r)$ in the IR region
Λ_{QCD}	0.241 GeV	0.241 GeV	QCD scale parameter
C or C^2	$C = 2.51992$	$C^2 = 6.35$	Coefficient in running coupling formula
e_c	1	1	Smoothing constant in running coupling equation
Grid Parameters			
Grid Points	$N_u = 421$, $N_\theta = 21$	150 (step factor 1.10305)	Grid discretization for dipole sizes
Dipole Size Range	$u = \log(r) \in [-10, 4.605]$	$r \in [4.5 \times 10^{-5}, 100]$	Range of dipole size (log-scale for bkEval, linear for rcBK)
Rapidity Range	$\eta_f = 15.0$, $N_\eta = 150$	$y \in [0, 15]$, step 0.1	Rapidity evolution range

TABLE I. Comparison of parameters used in `bkEval` and `rcBK` codes with MV^γ initial condition.

Cross Check: Numerical Solution of BK Evolution

Two sets of lines per color

Excellent agreement



Cross Check: We see we have good agreement across all x values for two different data generators, the RCBK code and BKeval code with all internal settings matching.

$$\sigma_{tot}^{\gamma^* A}(x, Q^2) = \int \frac{d^2x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \sigma_{tot}^{q\bar{q}A}(\vec{x}_\perp, Y).$$

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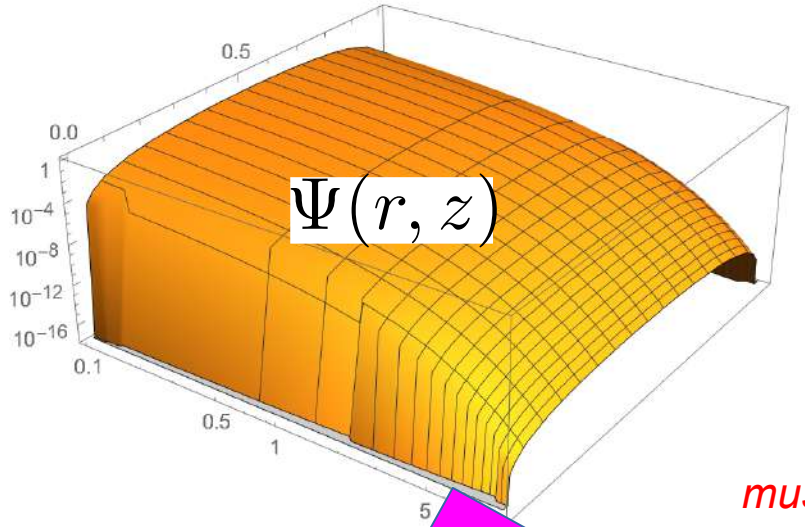
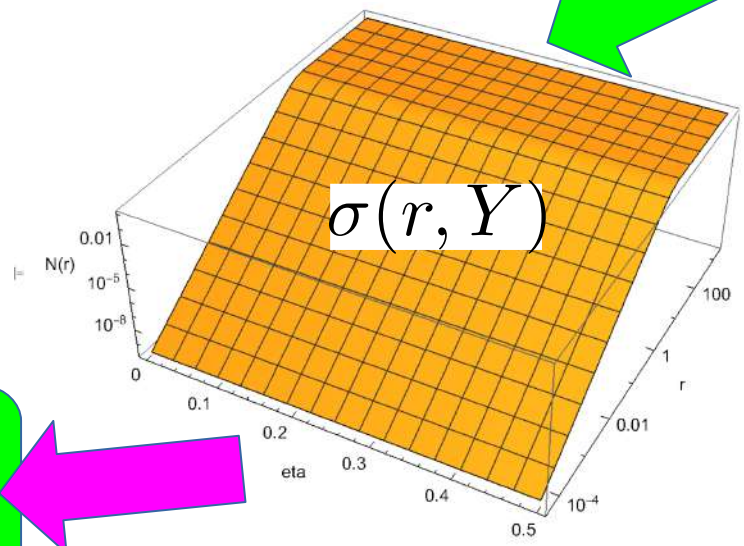
Dipole–nucleus cross section





must be fast!!!

Convolution
2D Grid-based calc

$\sigma_{tot}^{\gamma^* A}(x, Q^2)$
… F_2 & F_L

$$\chi^2 = \frac{(D-T)^2}{\sigma^2}$$

xFitter

Machine Learning Models

- **Need fast evolution**
- Could optimize a program, but ...
  - BFKL, BK, JIMWLK
  - LO, NLO, ...
- **Try Machine Learning**
  - Fast, flexible
  - Improved over interpolation

MLP (Multi-Layer Perceptron) Model

Junaid Khan

Brandon Stevenson

Neural Network Setup

- **Input features**: **x, r, C, Qs0, γ**
- **Input model: McLerran-Venugopalan**
- **Target**: N(x,r) dipole cross section
- **Architecture**: 3 hidden layers (64, 64, 32 neurons) with ReLU activation
- **Output layer**: Linear

Training Details

- Optimizer: **AdamW**
  - AdamW adaptively adjusts the learning rate for each parameter based on estimates of the first and second moments of the gradients. This allows for faster convergence and can handle sparse gradients effectively.
- Learning rate: **Exponentially decaying**
- Loss function: **Custom** (scaled MSE: Mean Absolute Error.)

  Dataset split: **80/10/10** (train/validation/test)

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(y_i^{\mathrm{pred}} - y_i^{\mathrm{true}}\right)^2$$

$$L = \frac{1}{N}\sum_{i=1}^{N}\frac{\mathrm{MSE}}{(0.01\,y_i^{\mathrm{true}} + \varepsilon)^2}$$

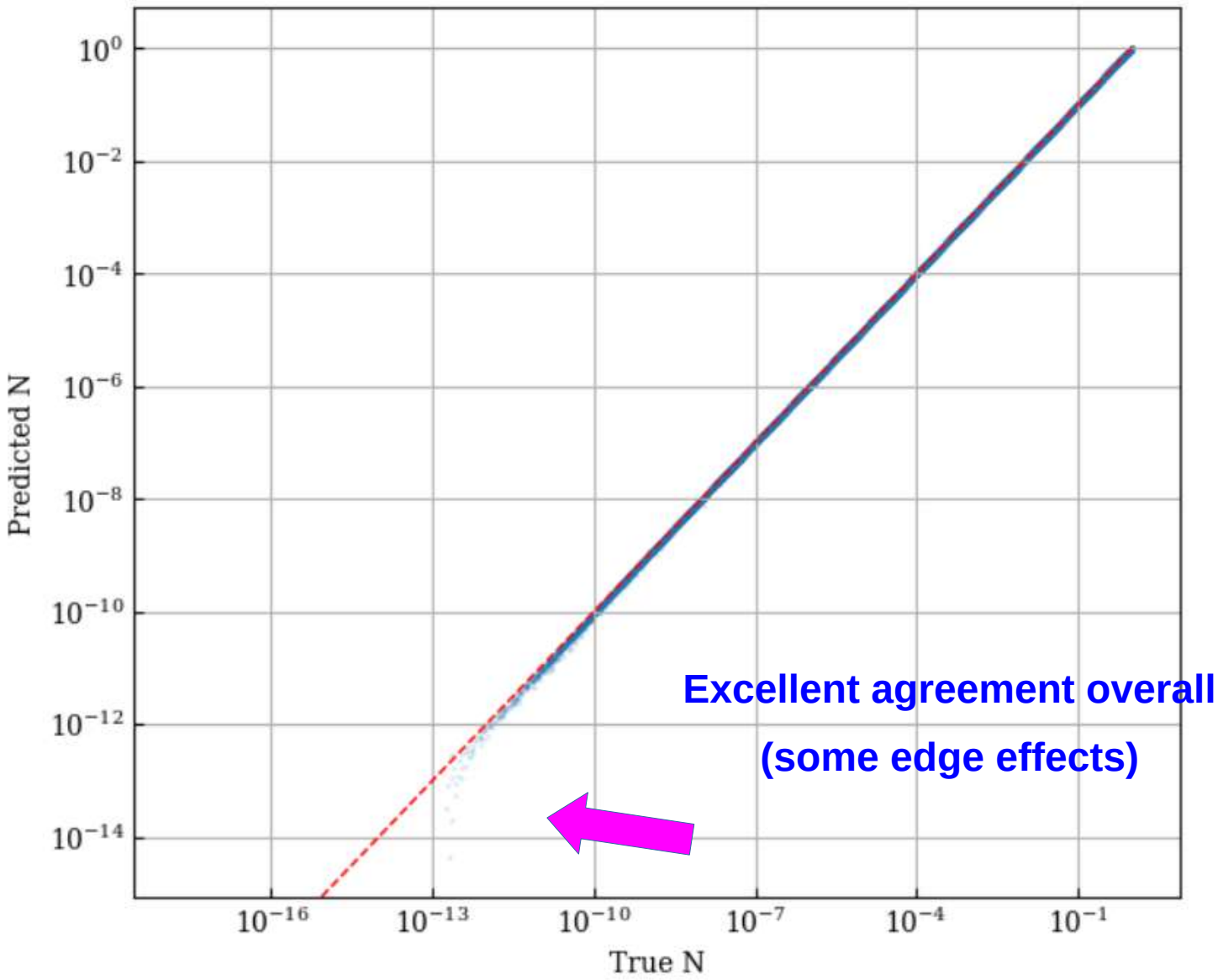
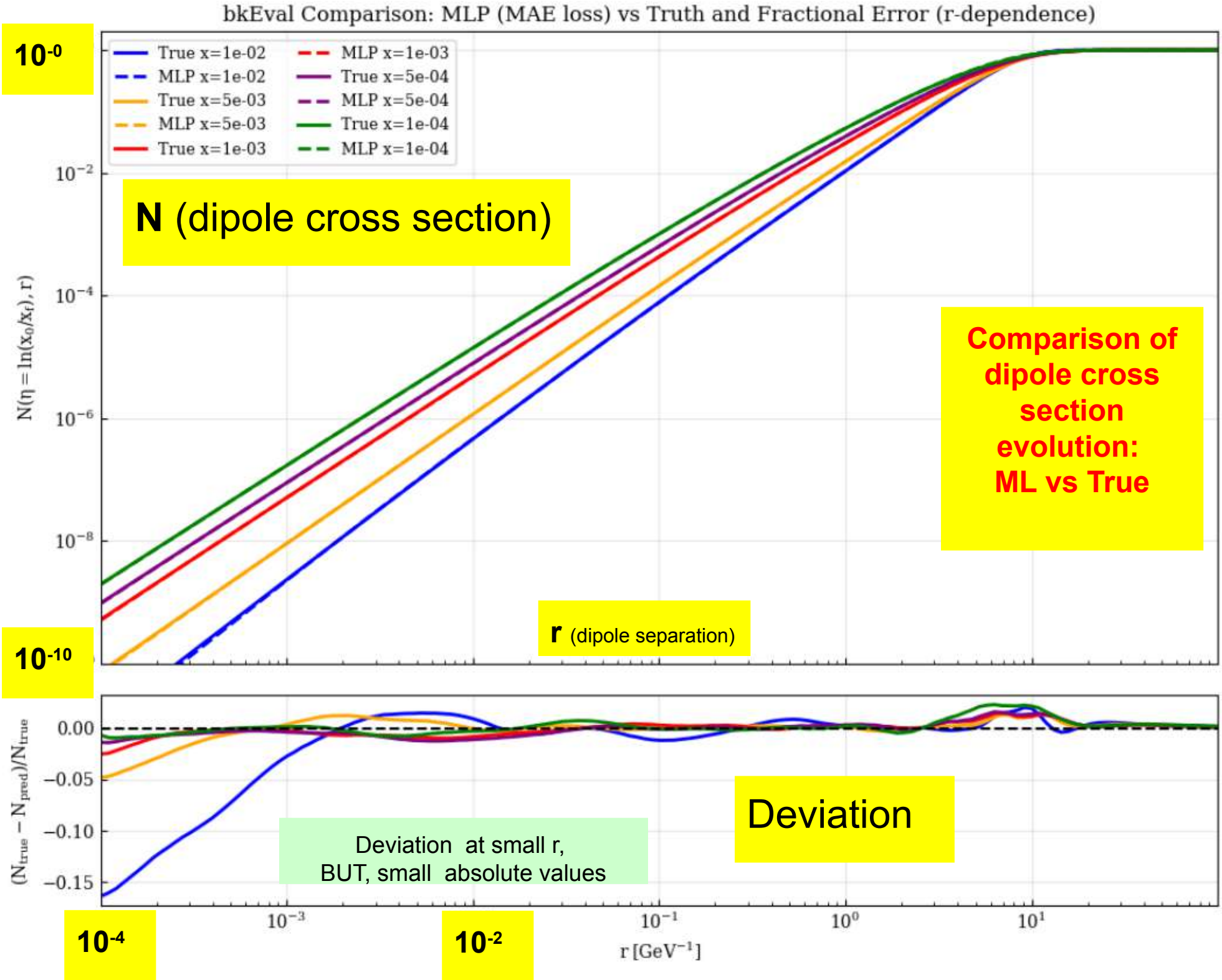


FIG. 1. True vs. predicted N values from the MLP on the test set. The dashed red line indicates perfect agreement.


bkEval Comparison: MLP (MAE loss) vs Truth and Fractional Error (r-dependence)
True x=1e-02
MLP x=1e-02
True x=5e-03
MLP x=5e-03
True x=1e-03
MLP x=1e-03
True x=5e-04
MLP x=5e-04
True x=1e-04
MLP x=1e-04
N (dipole cross section)
Comparison of dipole cross section evolution: ML vs True
r (dipole separation)
Deviation
Deviation at small r,
BUT, small absolute values
10-0
10-10
10-4
10-2
N(η = ln(x0/xf), r)
(Ntrue − Npred)/Ntrue
r [GeV−1]

DeepONet

Brandon Stevenson

Junaid Khan

DeepONet attempts to learn non-linear operators accurately. Many applications to physics.

"DeepONet: Learning nonlinear operators for identifying differential equations based on the universal approximation theorem of operators" https://arxiv.org/pdf/1910.03193

Two sub-networks a branch and trunk. Branch and Trunk network can be any network, MLP (dense connection) chosen for simplicity. Trunk network also uses a simple fourier features layer to give access to high dimensional information.

DeepONet

With test dataset, Varying **C** and **QS0** and 11,998,500 data points:

Training time: ~1 hour, 100 epochs

Size of keras model save: 11mb

Inference time: less than 1 second for full grid

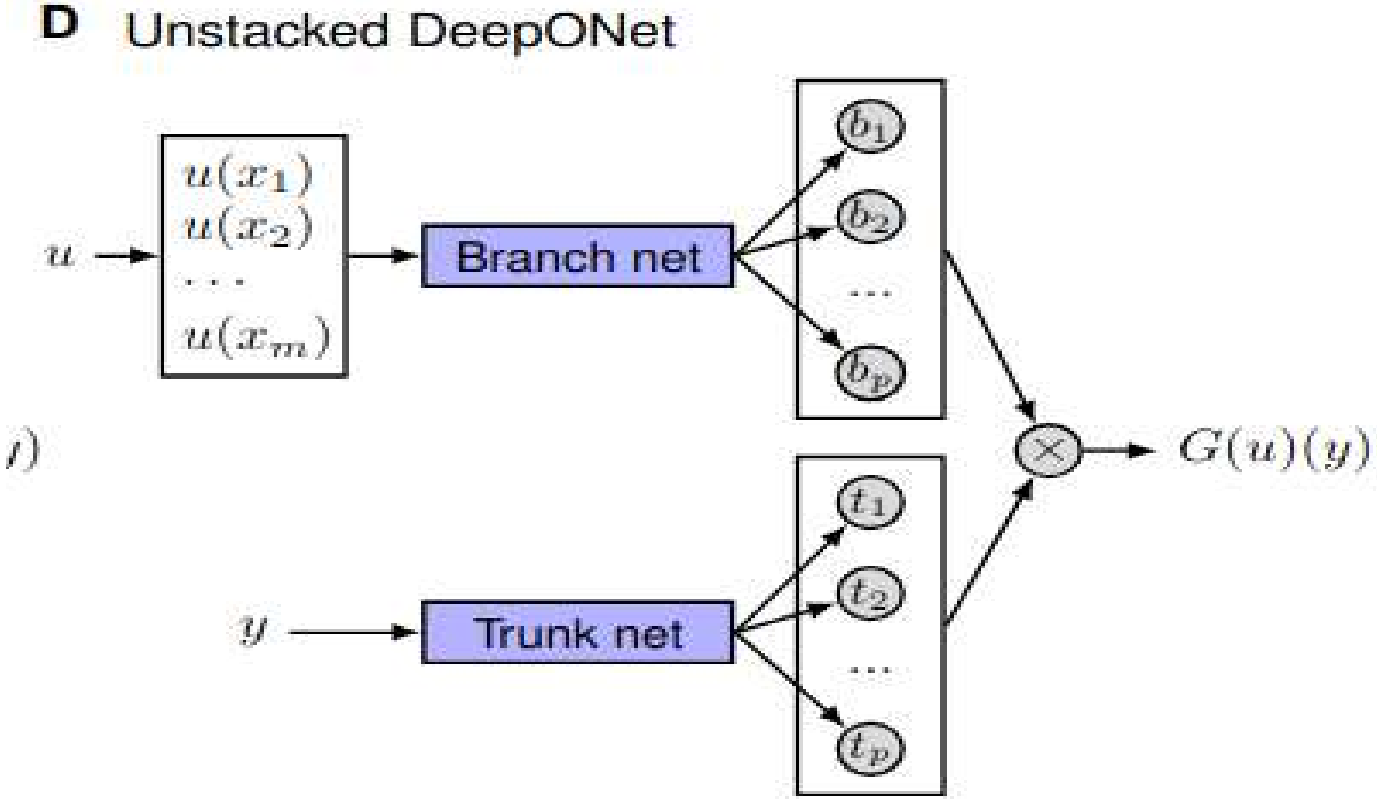
DeepONet

Theorem 1 (Universal Approximation Theorem for Operator). *Suppose that σ is a continuous non-polynomial function, X is a Banach Space, $K_1 \subset X$, $K_2 \subset \mathbb{R}^d$ are two compact sets in X and $\mathbb{R}^d$, respectively, V is a compact set in $C(K_1)$, G is a nonlinear continuous operator, which maps V into $C(K_2)$. Then for any $\epsilon > 0$, there are positive integers n, p, m, constants $c_i^k, \xi_{ij}^k, \theta_i^k, \zeta_k \in \mathbb{R}$, $w_k \in \mathbb{R}^d$, $x_j \in K_1$, $i = 1, \ldots, n$, $k = 1, \ldots, p$, $j = 1, \ldots, m$, such that*

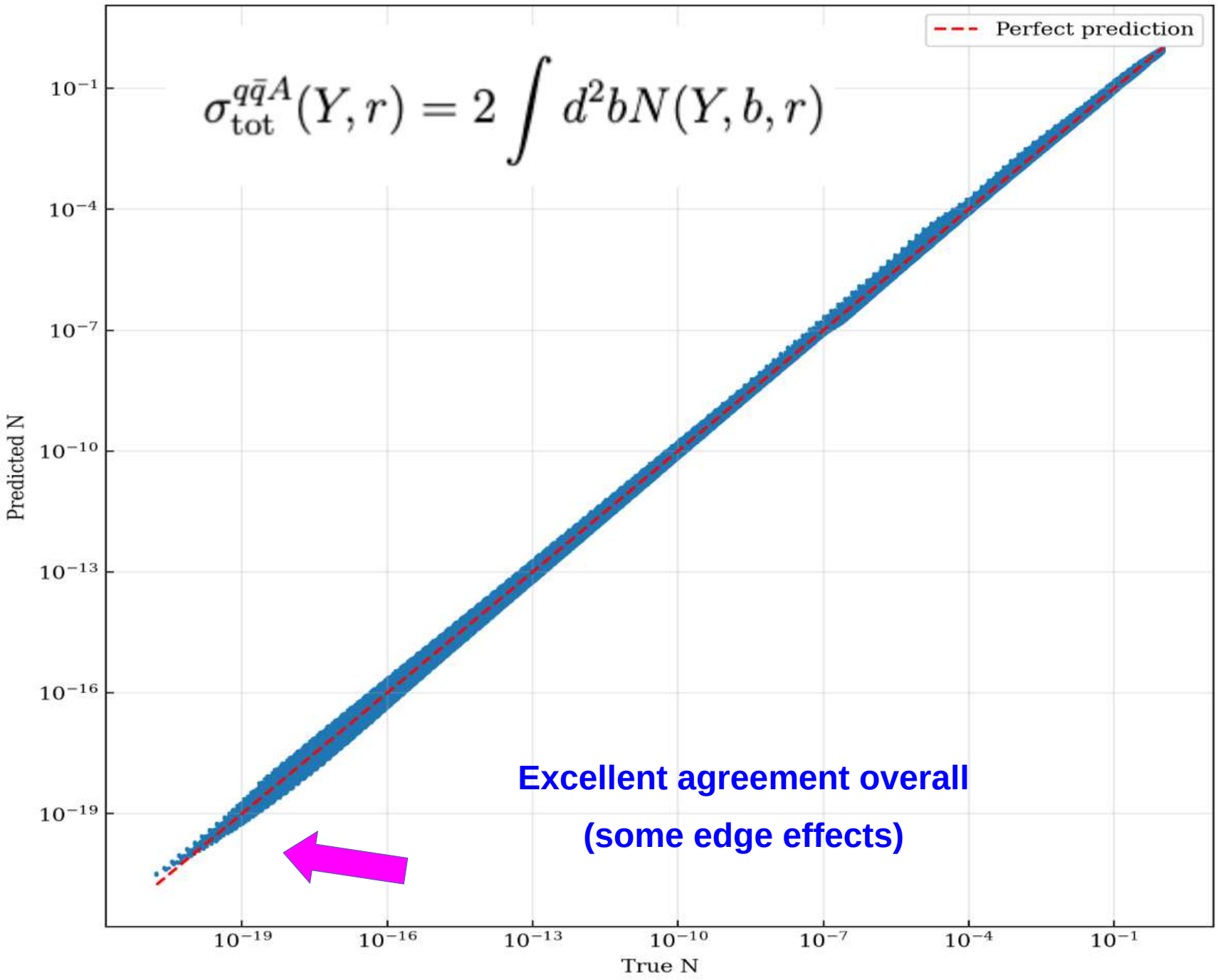
We learn G

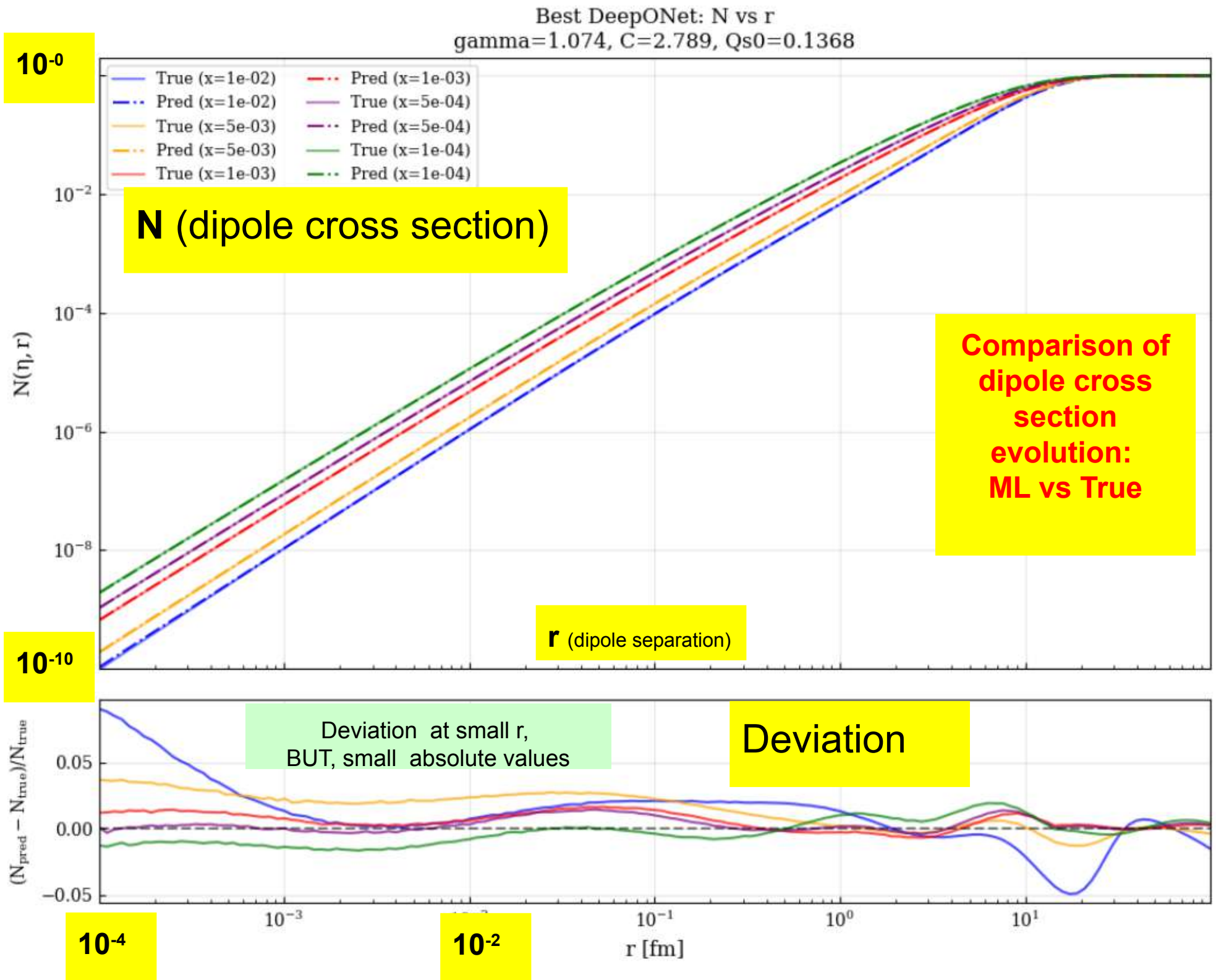
$$\left| G(u)(y) - \sum_{k=1}^{p} \underbrace{\sum_{i=1}^{n} c_i^k \sigma \left(\sum_{j=1}^{m} \xi_{ij}^k u(x_j) + \theta_i^k \right)}_{branch} \underbrace{\sigma(w_k \cdot y + \zeta_k)}_{trunk} \right| < \epsilon \tag{1}$$

holds for all $u \in V$ and $y \in K_2$.



DeepONet: Compare Predicted vs True values




Best DeepONet: N vs r
gamma=1.074, C=2.789, Qs0=0.1368
True (x=1e-02)
Pred (x=1e-02)
True (x=5e-03)
Pred (x=5e-03)
True (x=1e-03)
Pred (x=1e-03)
True (x=5e-04)
Pred (x=5e-04)
True (x=1e-04)
Pred (x=1e-04)
N (dipole cross section)
Comparison of dipole cross section evolution: ML vs True
r (dipole separation)
Deviation at small r, BUT, small absolute values
Deviation
N(η, r)
(N_pred − N_true)/N_true
r [fm]
10-0
10-10
10-4
10-2

Convolutional Neural Network (CNN)

Junaid Khan

Brandon Stevenson

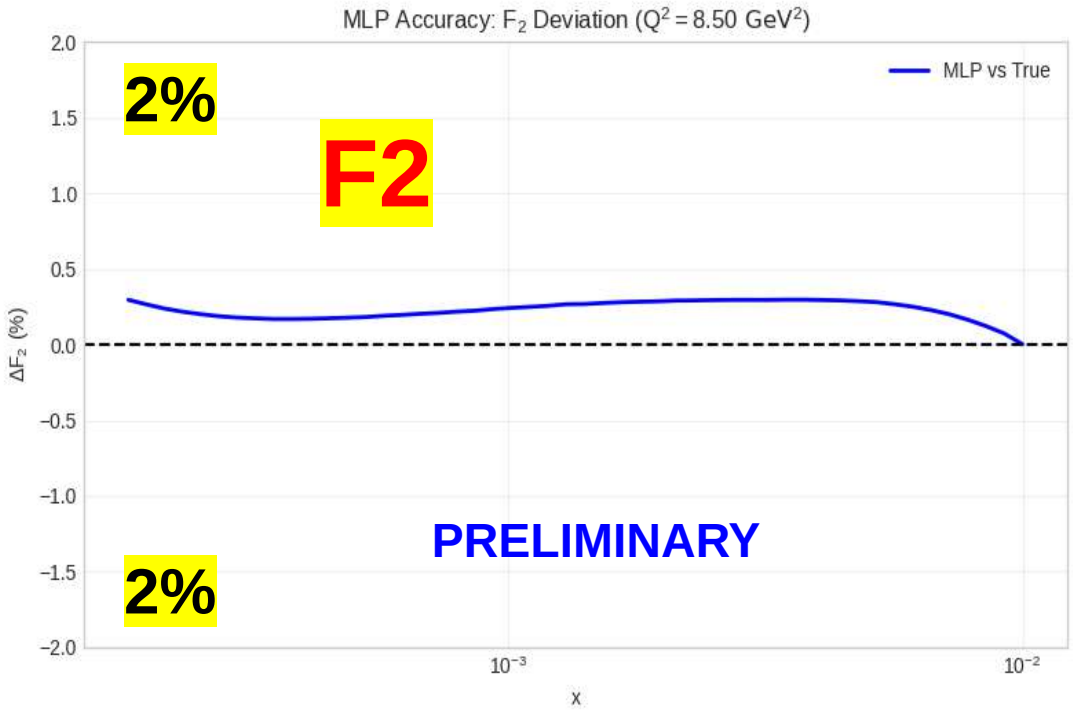
... in progress ...

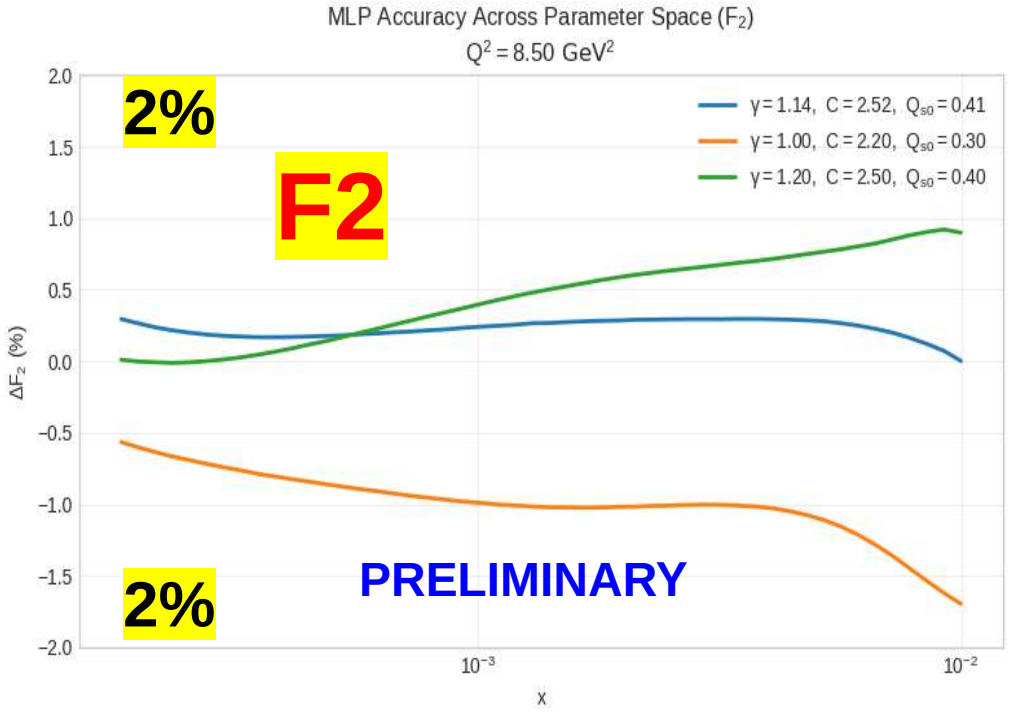
Uncertainties

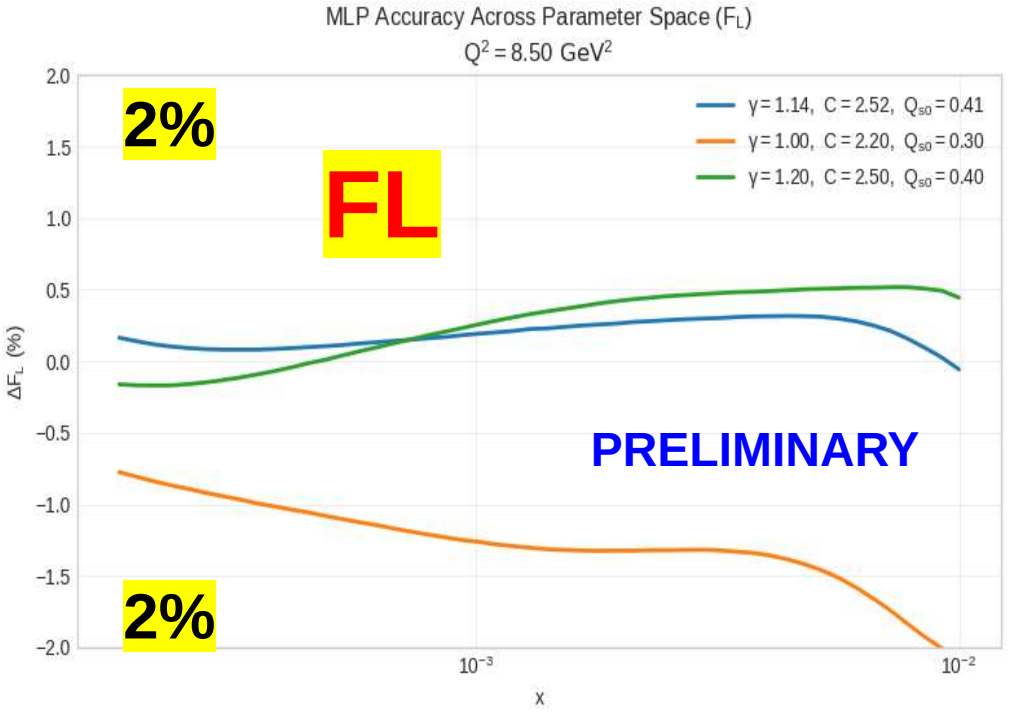
the bottom line

Uncertainties: ML vs. True

Preliminary uncertainties appear to be satisfactory given typical theoretical and experimental uncertainties



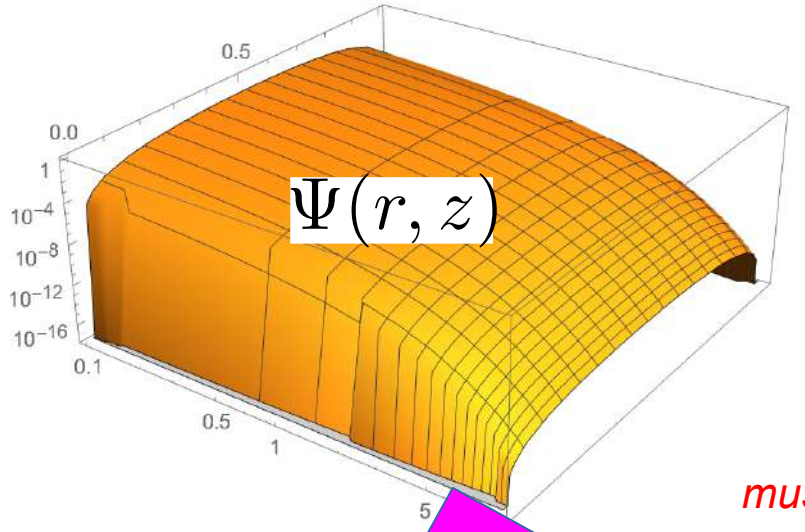


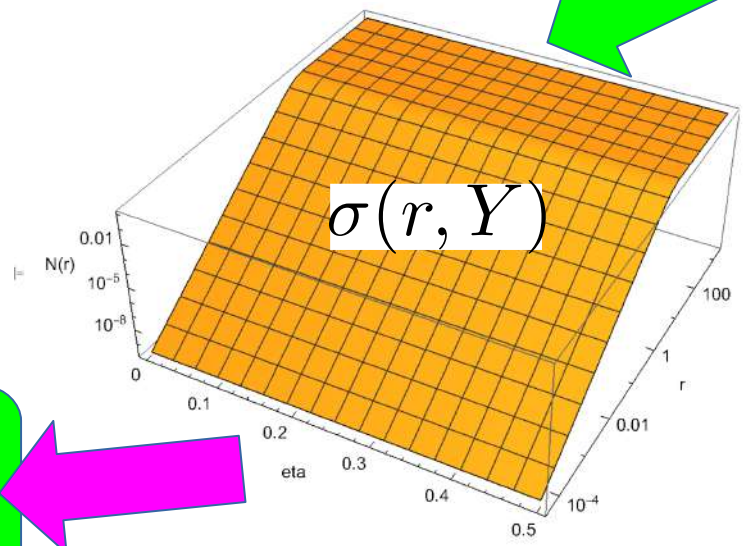


$$\sigma_{tot}^{\gamma^* A}(x, Q^2) = \int \frac{d^2 x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \sigma_{tot}^{q\bar{q}A}(\vec{x}_\perp, Y).$$

Photon light-front wave function

Dipole–nucleus cross section





must be fast!!!

Convolution

2D Grid-based calc

$\sigma_{tot}^{\gamma^* A}(x, Q^2)$ … F_2 & F_L

$$\chi^2 = \frac{(D-T)^2}{\sigma^2}$$

xFitter

Status Summary:

Dipole Calculation:

- Working Python Framework to compute σ^{tot} and F2, FL
- Reasonable agreement with e.g., HERA data *(need to fit)*
- Grid methods enable fast calculation

BK Evolution:

- Full evolution codes: [BKsolver, rcbk] validated
  - Try ML for inner fitting loop

ML approach to BK Evolution

- Reasonable approximation across kinematics [preliminary]
- Fast enough for flexible fits

Other evolution models:

- Could extend to other models: BFKL, JIMWLK, … DGLAP??
- Could ‘warm-up’ running BK with LO & NLO

$$\sigma_{tot}^{\gamma^* A}(x, Q^2) = \int \frac{d^2 x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \sigma_{tot}^{q\bar{q}A}(\vec{x}_\perp, Y).$$

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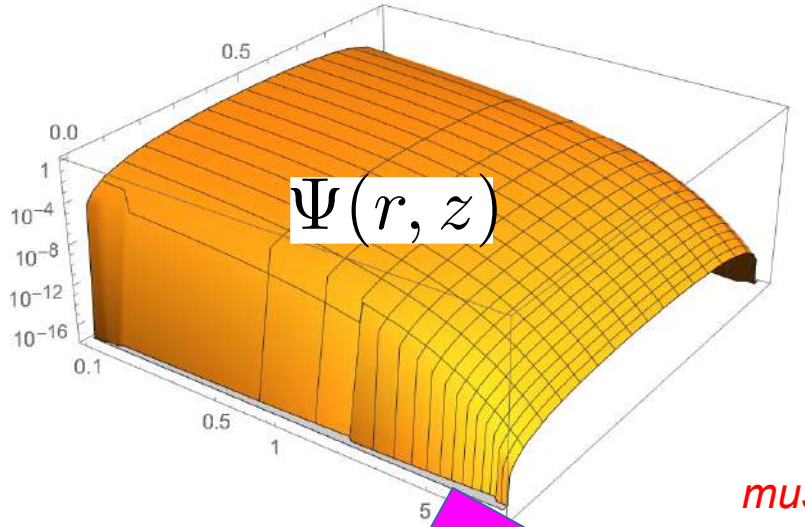

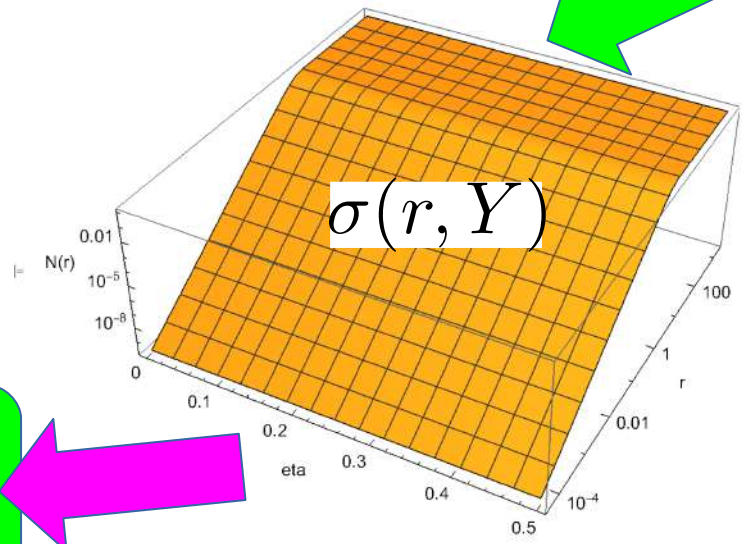

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