Sudakov effects vs saturation dynamics in SIDIS at small x

Jamal Jalilian-Marian

Baruch College

and

City University of New York Graduate Center

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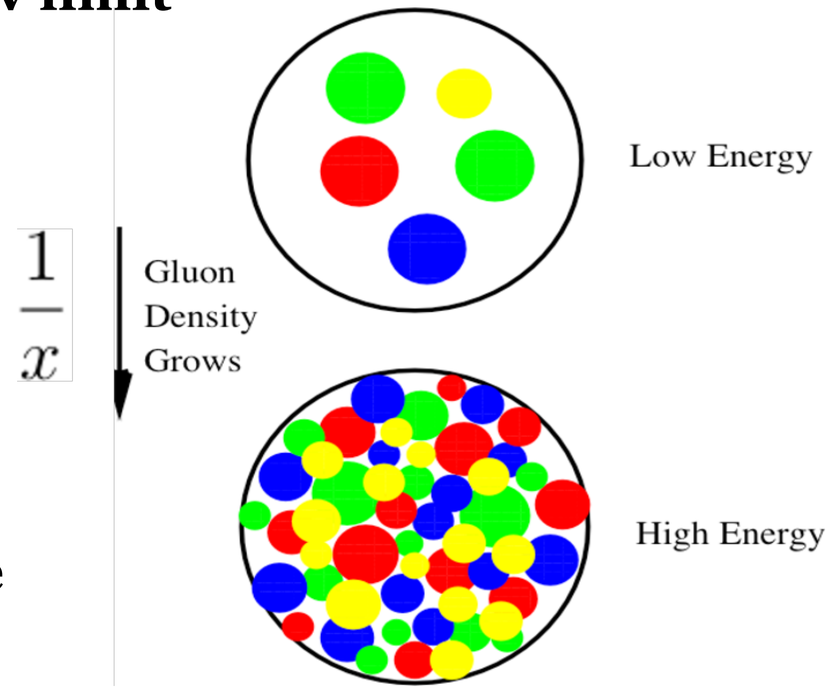
Resolving the nucleus/hadron: Regge-Gribov limit

Gribov-Levin-Ryskin

Q^2 fixed and $\sqrt{S} \to \infty \quad (x \equiv \frac{Q^2}{S} \to 0)$



gluons are radiated into a fixed resolved area

number of gluons increases due to increased longitudinal phase space

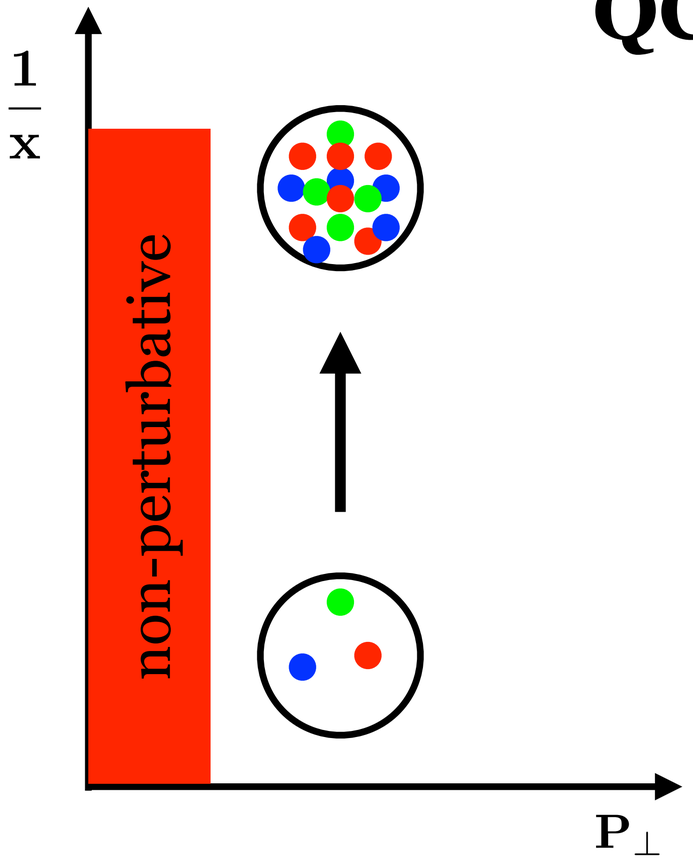
hadron/nucleus becomes a dense state of gluons (CGC) $\frac{\alpha_s}{Q^2}\frac{xG(x,Q^2)}{S_\perp} \sim 1 \qquad Q_s^2(x,A,b_\perp) \sim A^{1/3}\left(\frac{1}{x}\right)^{0.3}$

strong color fields

possible universal properties of QCD observables ?

EIC

QCD at small x: gluon saturation



high gluon density: multiple eikonal scatterings

high energy: evolution in x via BK/JIMWLK

p_t broadening

suppression of single inclusive spectra/away side peak

A systematic formalism for multi-particle production in QCD

nuclear shadowing/modification factor, azimuthal correlations

long range rapidity correlations

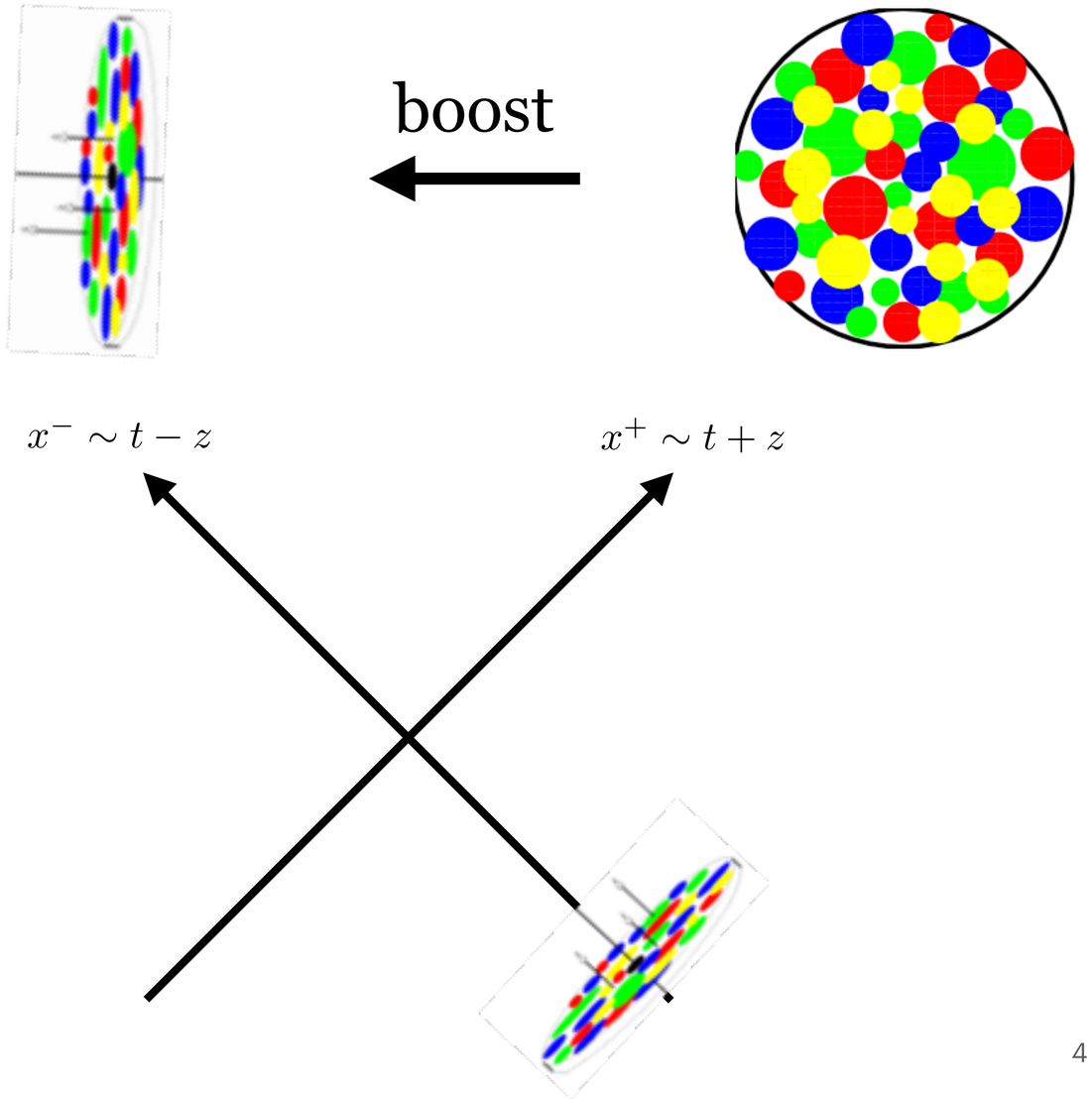
connections to TMD, spin physics, diffraction,....

$$Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$Q_s^2 \sim 1\,\mathrm{GeV}^2$ at

$x \sim 3 \times 10^{-4}$

MV: a high energy hadron/nucleus as a classical color field



$$R \longrightarrow \frac{R}{\gamma}$$

$$\gamma \sim 100 \quad \text{RHIC}$$

$$\gamma \sim 8000 \quad \text{LHC}$$

$$J_a^\mu(x) = \delta^{\mu-}\delta(x^+)\rho_a(\mathbf{x}_\perp)$$

color charge

$$D_\mu F^{\mu\nu} = gJ^\nu$$

solution: classical field $A^\mu(x)$

this is the starting point for CGC

Probing CGC in high energy collisions

nucleus-nucleus collisions: “dense on dense”

significant modeling/QGP

proton-nucleus collisions: “dilute on dense”

DIS: (inclusive/diffractive)

much less modeling

structure functions

particle production

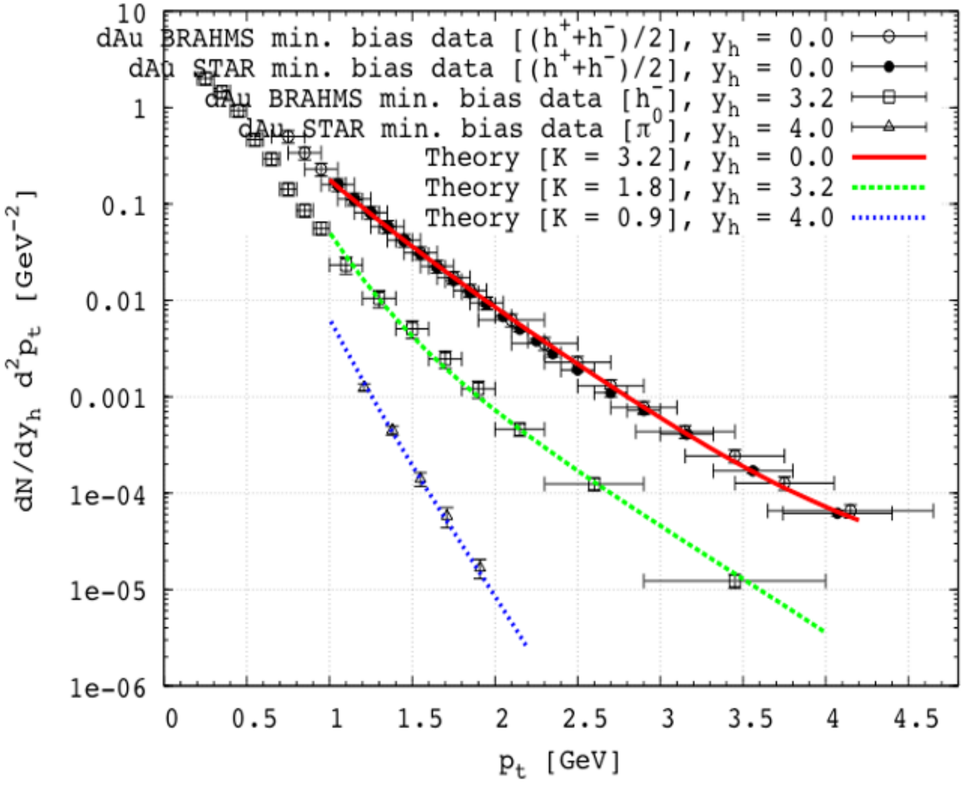
angular correlations

EIC

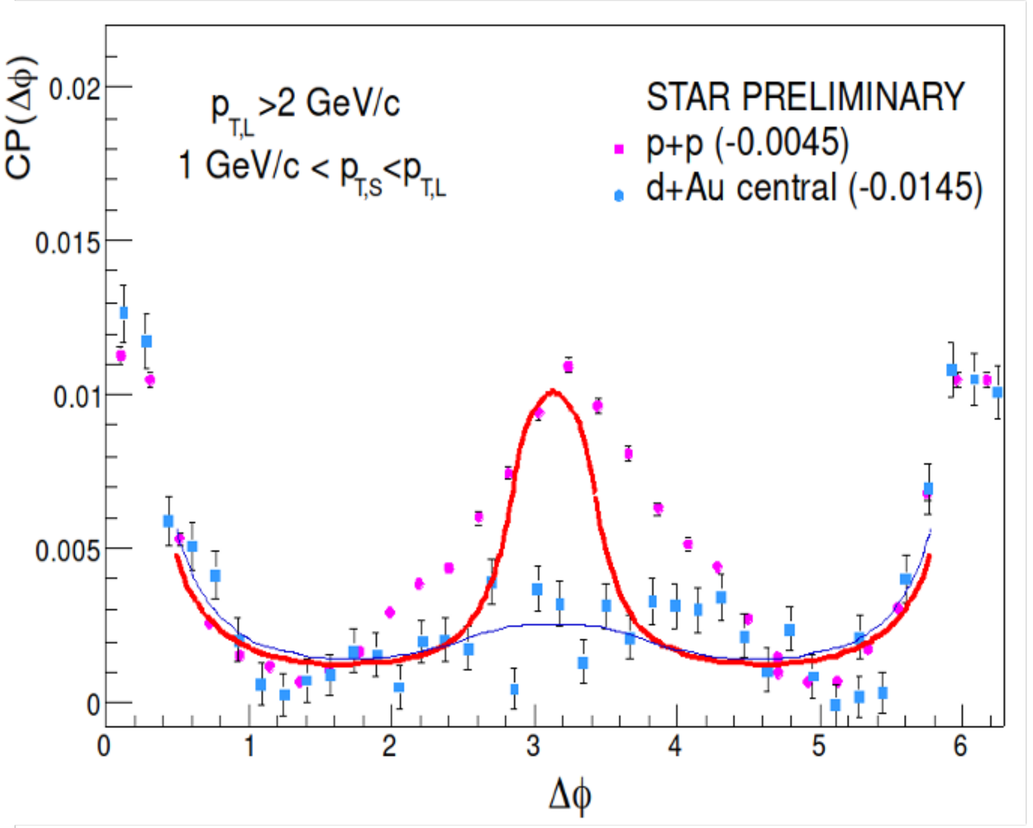
to start in ~ 10 years

CGC at RHIC

Single and double inclusive hadron production in dA collisions

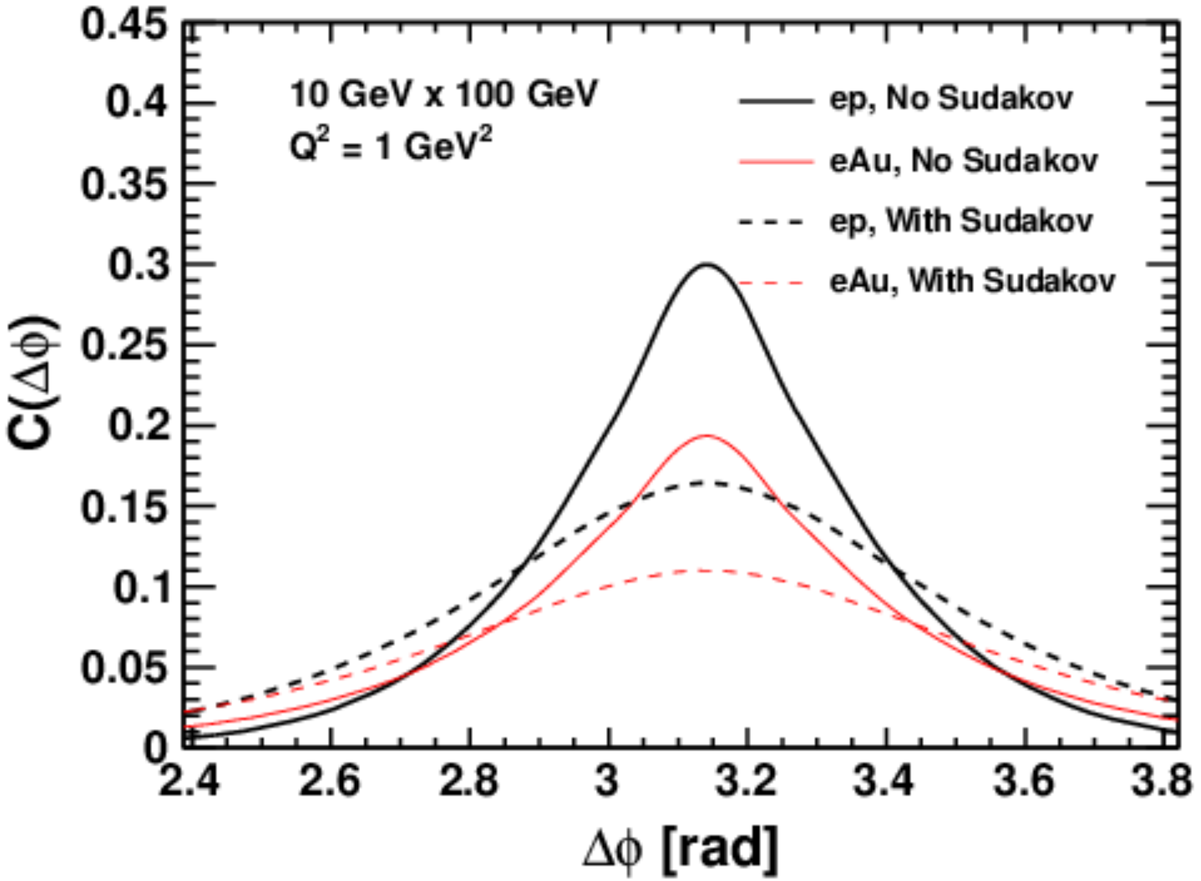


Dumitru,Hayashigaki, JJM, NPA770 (2006) 57



Albacete, Marquet, PRL105 (2010) 162301

Dihadrons at EIC



Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413

SIDIS:

larger kinematic phase space than dihadrons

Sudakov effect can be avoided (in a limited kinematic range)

SIDIS at small x: toward precision

Bergabo, JJM, JHEP 01 (2023) 095, and arXiv:2401.06259

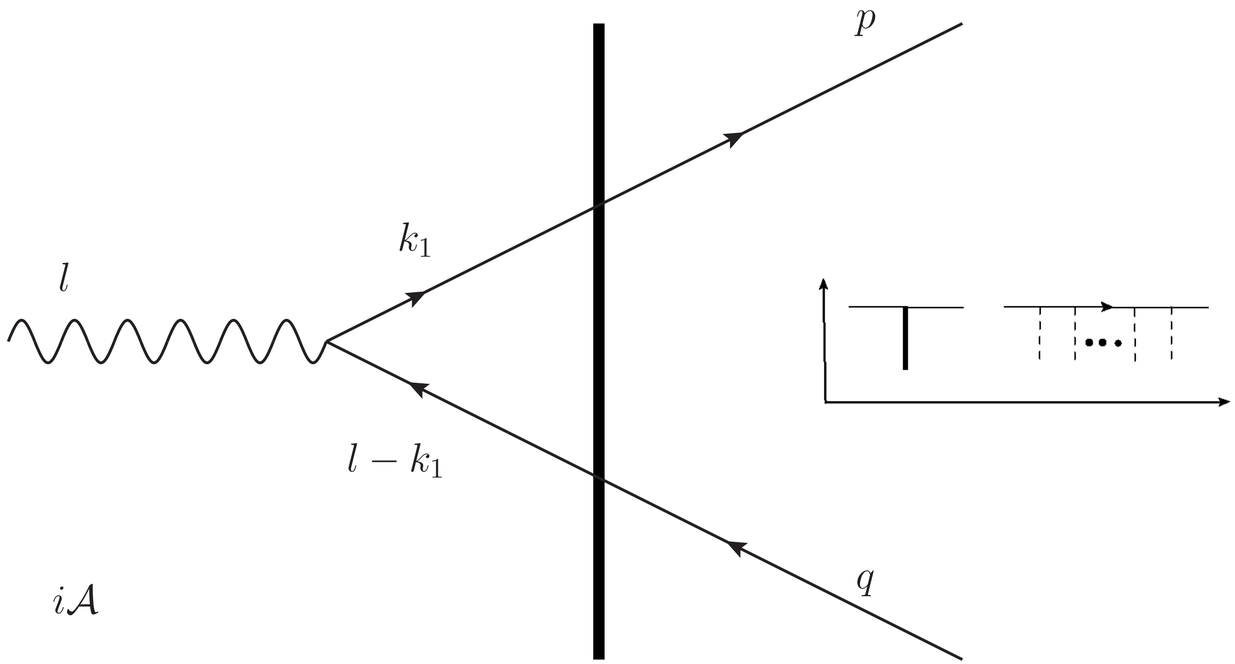
Caucal, Ferrand, Salazar, arXiv:2401.01934

Altinoluk, JJM, Marquet, Shi, arXiv 2505.04557

Forward rapidity

LO:

integrate over final state antiquark



$$\frac{\mathrm{d}\sigma^{\gamma^* p/A\to q(\mathbf{p},y_1)X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2Q^2N_c}{(2\pi)^5}\int \mathrm{d}z_2\,\delta(1-z_1-z_2)\,(z_1^2\,z_2)\int \mathrm{d}^6\mathbf{x}\,[S_{11'}-S_{12}-S_{1'2}+1]\,e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2}|Q_1)+(z_1^2+z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)\right\}$$

with $Q_1^2 \equiv z_1(1-z_1)Q^2$

high Q^2 limit: toward Sudakov double logs

Altinoluk, JJM, Marquet

arXiv:2406.08277

problem at end points: use a trick introduced by Marquet, Xiao, Yuan, PLB682 (2009) 207

$$Q_1^{2n}\, K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\, K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1) = \int_0^{Q^2/4} d\bar{Q}^2\, (\bar{Q}^2)^n K_{(0,1)}(|\mathbf{x}_{12}|\bar{Q}) K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\, \delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right]$$

and

$$\delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right] = \frac{\delta(z_1 - z_+)}{Q^2|1-z_+|} + \frac{\delta(z_1 - z_-)}{Q^2|1-z_-|} \quad \text{with} \quad z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\bar{Q}^2/Q^2}\right)$$

then

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{T,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\, K_1(|\mathbf{x}_{12}|\bar{Q}) K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{L,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{4}{Q^2}\int_0^\infty d\bar{Q}^2\bar{Q}^4 K_0(|\mathbf{x}_{12}|\bar{Q}) K_0(|\mathbf{x}_{1'2}|\bar{Q})$$

contribution of longitudinal photons is suppressed

Quark TMD (LO)

Marquet, Xiao, Yuan 0906.1454

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{T,LP} = \frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}xq(x,\mathbf{p}_h/z_h)$$

$$xq(x,\mathbf{p}) = \frac{2N_c}{(2\pi)^6}\int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\left[S_{11'}-S_{12}-S_{1'2}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

$$xq(x,\mathbf{p}) = \int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\; x\tilde{q}(x,\mathbf{x}_{11'})$$

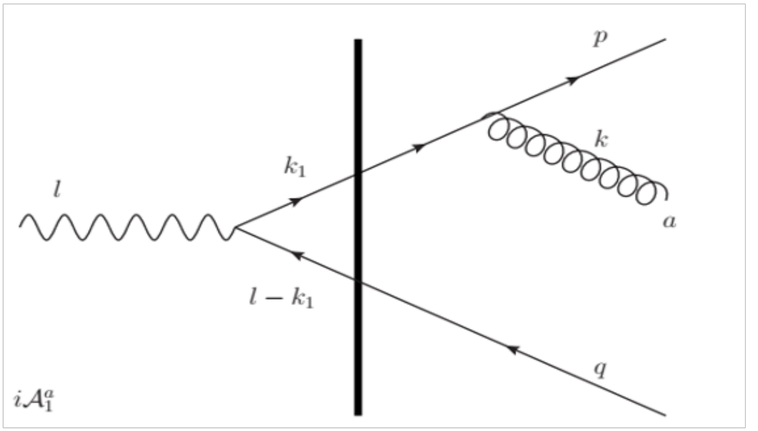
can be rewritten as

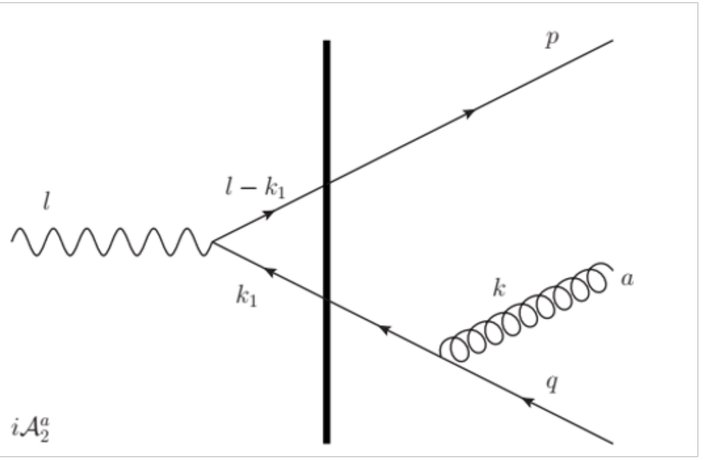
$$xq(x,\mathbf{p}) = \int \mathrm{d}^2\mathbf{k}_g\left(\underbrace{\frac{N_c}{8\pi^4}\frac{\mathbf{k}_g^2}{\alpha_s}\int \mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_{1'}\; S_{11'}\,e^{-i\mathbf{k}_g\cdot\mathbf{x}_{11'}}}\right)\left(\underbrace{\frac{1}{4\pi^2}\frac{\alpha_s}{\mathbf{k}_g^2}\int_0^\infty \mathrm{d}\bar{Q}^2\left|\frac{\mathbf{k}_g-\mathbf{p}}{\bar{Q}^2+(\mathbf{k}_g-\mathbf{p})^2}+\frac{\mathbf{p}}{\bar{Q}^2+\mathbf{p}^2}\right|^2}\right)$$

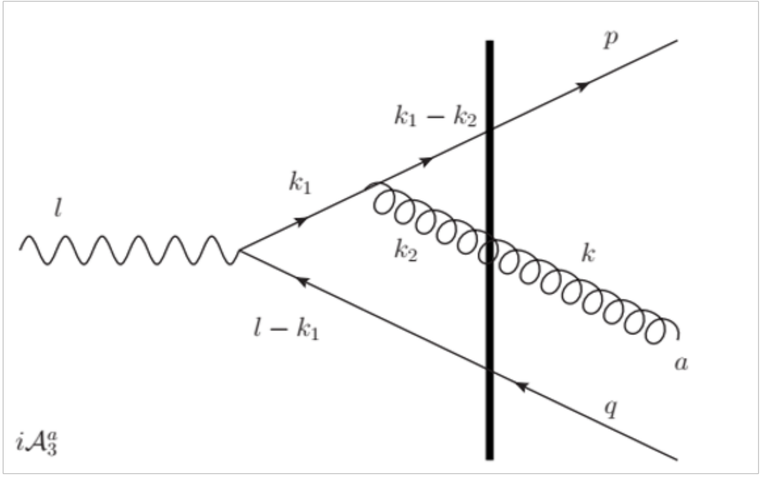
dipole gluon TMD

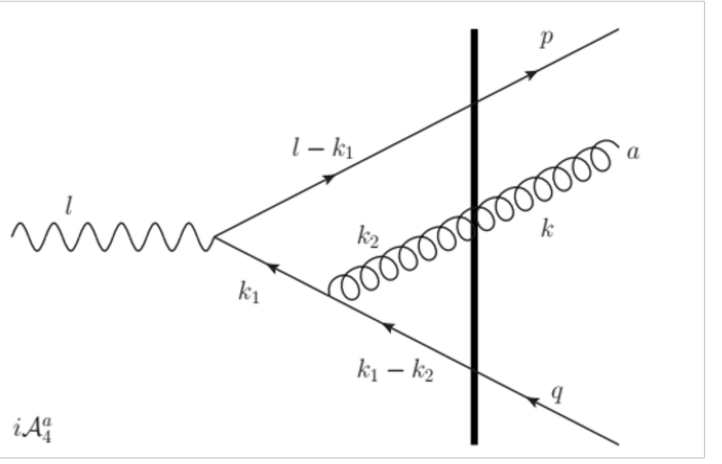
momentum dependent splitting function

One loop corrections - real diagrams





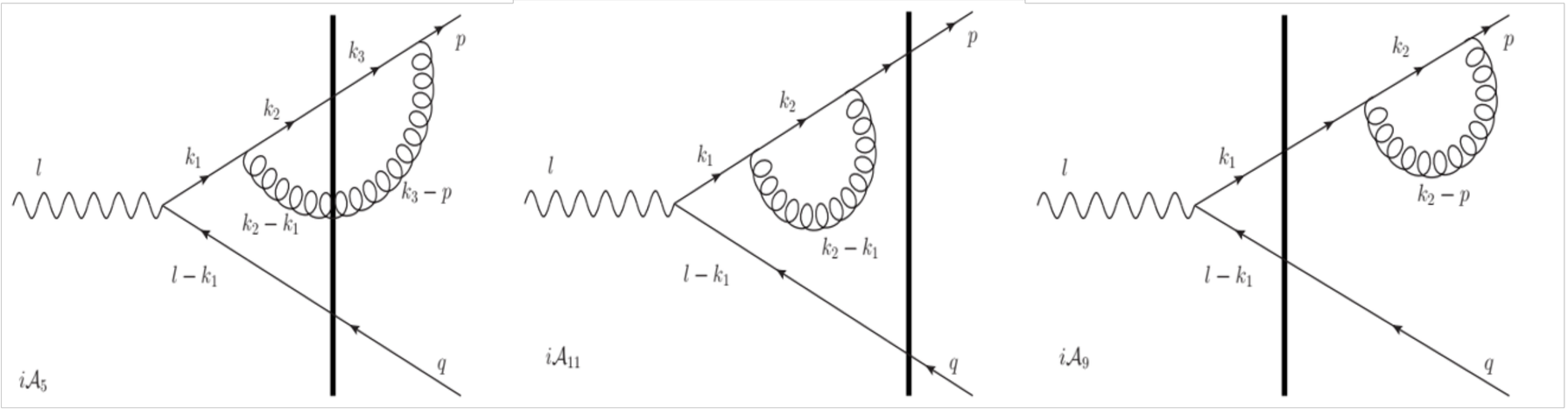


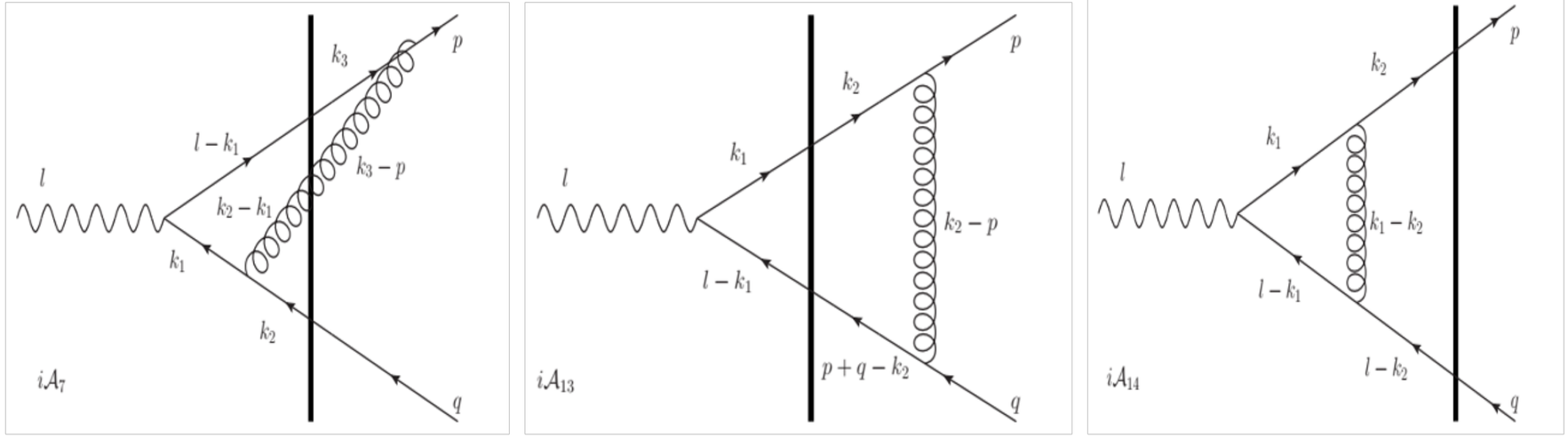


3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

Iancu, Mulian, JHEP 07 (2023) 121

One loop corrections – virtual diagrams



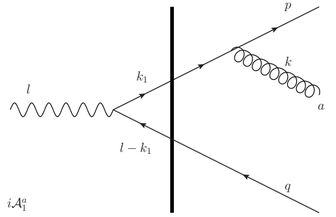


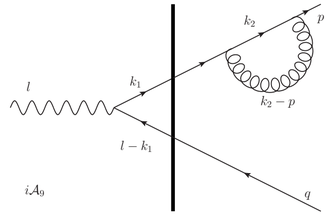
F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

NLO corrections to SIDIS: some of the contributions are

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2g^2Q^2}{(2\pi)^8}\int_0^{1-z_1}\frac{\mathrm{d}z}{z}\frac{(1-z-z_1)(z+z_1)}{z_1}\left[z_1^2(1-z-z_1)^2+\left(z_1^2+(1-z-z_1)^2\right)(z+z_1)^2+(z+z_1)^4\right]$$
$$\int \mathrm{d}^8\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_{1z})K_1(|\mathbf{x}_{1'2}|Q_{1z})N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\,\Delta^{(3)}_{11'}\,e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}$$

$$\frac{\mathrm{d}\sigma^T_{9}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2g^2Q^2}{2(2\pi)^6}\int_0^{z_1}\frac{\mathrm{d}z}{z}(1-z_1)(z_1^2+(1-z_1)^2)\left[z_1^2+(z_1-z)^2\right]\int \mathrm{d}^6\mathbf{x}\,K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_cC_F\left[S_{11'}-S_{12}-S_{21'}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int\frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2}\frac{1}{\mathbf{x}_{31}^2}$$





with $\quad \Delta^{(3)}_{ij} = \dfrac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2\mathbf{x}_{3j}^2}$ $\quad$ and $\quad Q_1^2 \equiv z_1(1-z_1)Q^2$, $\; Q_{1z}^2 \equiv (1-z-z_1)(z+z_1)Q^2$

SIDIS at small x: including Sudakov double logs

Altinoluk, JJM, Marquet, arXiv:2406.08277

taking the high $\mathbf{Q^2}$: introduce a delta function,..., cross section becomes

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$

$$\times\Bigg\{D_{h/q}(z_h)$$

$$+\left[\frac{\alpha_s C_F}{2\pi^2}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}(\frac{z_h}{1-z})-\int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}$$

$$+\left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\left(\int\mathrm{d}^2\mathbf{x}_3\,\Delta^{(3)}_{1'1}-\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\right]D_{h/q}(\frac{z_h}{1-z})\Bigg\}$$

the first two terms are LO (DGLAP) evolution of quark/hadron fragmentation function

in the last term introduce a factorization scale for momentum fraction z $\displaystyle\int_0^{1-z_h}\mathrm{d}z=\int_0^{z_f}\mathrm{d}z+\int_{z_f}^{1-z_h}\mathrm{d}z$

SIDIS at small x: including Sudakov double logs

to get Sudakov logs one must introduce a kinematic constraint

needed for self-consistency of evolution equations (avoid negative cross sections)

at small x we require $k^+ < z_f\, l^+$, this is sufficient at LL accuracy

at NLO accuracy we also need to impose a condition on lifetimes of fluctuations

introduce a cutoff on - component of momenta $k^- > \tilde{z}_f\, l^-$ with $z_f\, \tilde{z}_f \sim 1$

Sudakov single logs may be sensitive to the details of this cutoff

Altinoluk, JJM, Marquet, Yu, in progress

SIDIS at small x: including Sudakov double logs

longitudinal factorization $\int_0^{1-z_h} dz = \int_{z_f}^{1-z_h} dz + \int_0^{z_f} dz\,[1-\theta(kin.const.)] + \int_0^{z_f} dz\theta(kin.const.)$

where we add and subtract the kinematic constraint $\boldsymbol{\Theta}(\text{kin.const.}) = \boldsymbol{\Theta}\left(\mathbf{z_f}\dfrac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)$

$$\int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\left[\mathbf{1} - \boldsymbol{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)\right] = \int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\boldsymbol{\Theta}\left(\mathbf{z} - \mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}}\right) = \boldsymbol{\Theta}\left(\mathbf{Q^2} - \mathbf{k^2}\right)\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k^2}}\right)$$

and dipoles satisfy kinematically-constrained BK evolution

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}(z_f)\otimes D_{h/q}(z_h,Q^2) + d\sigma_{NLO-rap-finite} +$$

$$\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$

$$\times\left\{\frac{\alpha_s C_F}{\pi^2}\int^{Q^2}\frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left(e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}}-1\right)\ln\left(\frac{Q^2}{\mathbf{k}^2}\right)\right\}$$

Sudakov double logs in SIDIS

using

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left[e^{-i\mathbf{k}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{k}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right)$$

$$= -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right)$$

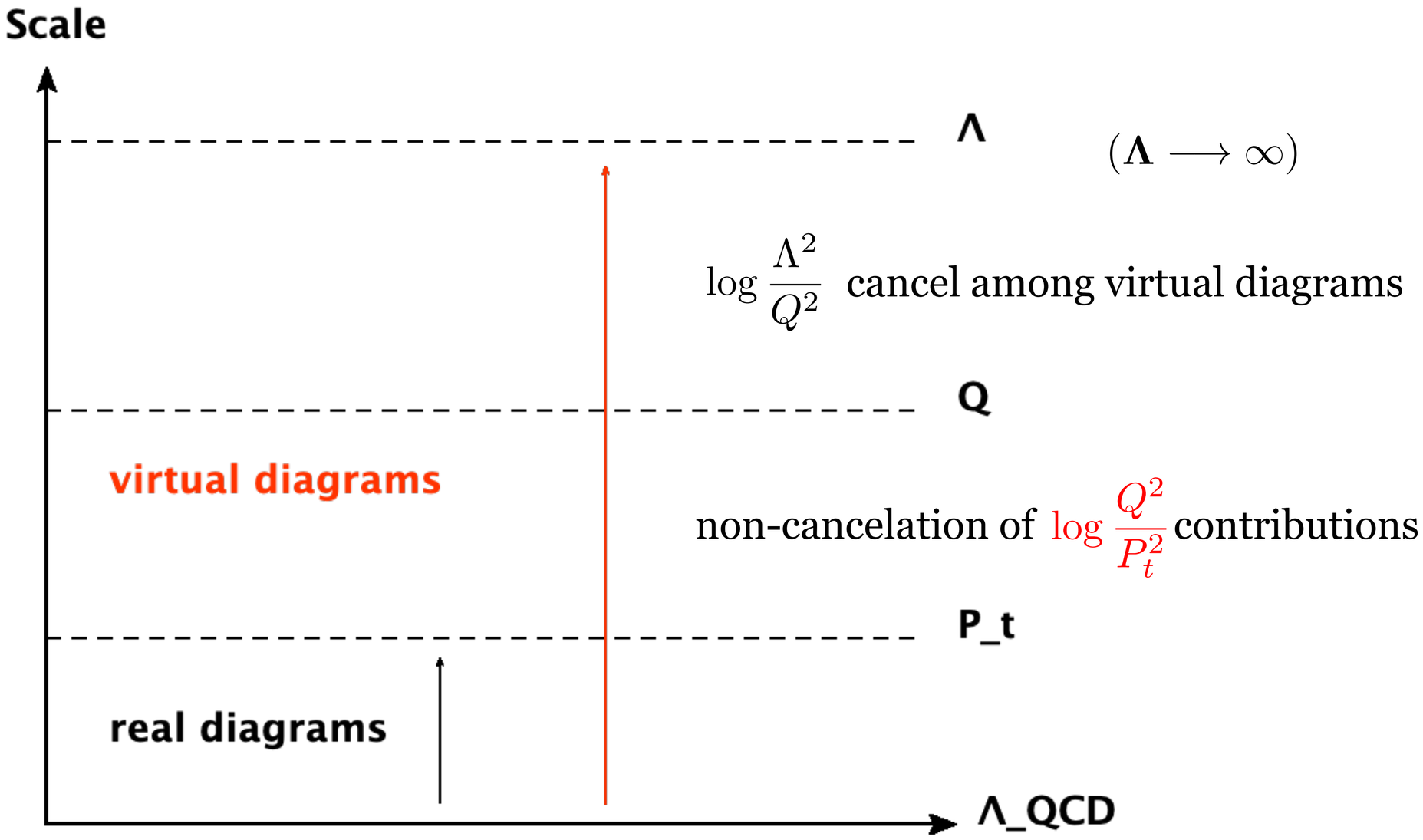
we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{NLO-rap-finite}$$

$$+\frac{\pi e^2}{Q^2}\frac{D_{h/q}(z_h,Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})}$$

with

$$S_{sud}(\mathbf{x}_{11'}) \equiv \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)$$


Scale
Λ
$(\Lambda \longrightarrow \infty)$
$\log \frac{\Lambda^2}{Q^2}$ cancel among virtual diagrams
Q
virtual diagrams
non-cancelation of $\log \frac{Q^2}{P_t^2}$ contributions
P_t
real diagrams
Λ_QCD

SIDIS at small x : can one avoid Sudakov logs?

EIC will have a reasonably large window in $\mathbf{Q^2}$ where $\mathbf{Q^2} >> \mathbf{p_{t,h}^2}$ so that $\alpha_s \log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \sim \mathbf{1}$

CGC contains <u>*no*</u> Sudakov double logs in SIDIS <u>*unlike*</u> dihadron production (wrong sign)

avoid large Sudakov logs: $\mathbf{p_{t,h}^2} \sim \mathbf{Q^2}$ so that $\log\left(\frac{\mathbf{Q^2}}{\mathbf{p_{t,h}^2}}\right) \simeq \mathbf{0}$

Summary

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,…

to be probed precisely at EIC

toward precision: NLO, beyond-eikonal corrections, …

deep connections to TMD, Sudakov physics, …

missing a unified approach to large and small x!