Jet definition and TMD factorisation in SIDIS

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with E. Iancu, A. H. Mueller, and F. Yuan [based on 2408.03129, PRL 134 (2025) 6]

Goals and outline

This talk, in a nutshell:

A small-x perspective on TMD factorisation for single-inclusive **jet** production in DIS.

- First calculation of Sudakov logs in SIDIS with **jets** and their dependence on the jet algorithm.
- New asymmetric jet distance measure which ensures TMD factorisation.
  See also Centauro algorithm, Arratia, Makris, Neill, Ringer, Sato, PRD 104 (2021)
- Emergence of the DGLAP and CSS evolutions from the small x approach.
  $\Rightarrow$ combined high-energy, CSS and DGLAP evolution within the TMD formalism.

Single inclusive jet production in DIS

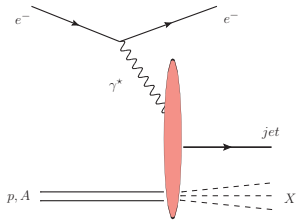
[PC, Iancu, Mueller, Yuan, 2408.03129]

⇒ Measure **jets** in DIS events and bin in terms of $P_\perp$ measured in Breit (or dipole) frame:

$$\frac{\mathrm{d}\sigma^{eA\to \mathrm{e'+jet}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}P_\perp}$$

⇒ In the case of a hadron measurement, see Altinoluk, Jalilian-Marian, Marquet, 2406.08277. Talk by Jamal Jalilian-Marian, 07/05, 11.55am

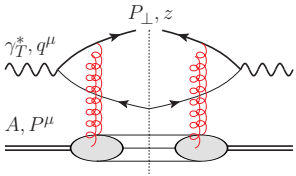
⇒ Also accesses the sea quark TMD at small x in the limit $Q^2 \gg \boldsymbol{P}_\perp^2$.



TMD factorisation in SIDIS at LO from the dipole picture

- $\gamma^* \to q\bar{q}$ splitting + interaction with the "shockwave" (CGC EFT).
  [Mueller (1990), Nikolaev and Zakharov (1991)]
- CGC result depends on the dipole correlator

$$\mathcal{D}(x, \boldsymbol{r}) = \frac{1}{N_c}\langle V(0)V^\dagger(\boldsymbol{r})\rangle_x$$



- For $Q^2 \gg P_\perp^2, Q_s^2$, factorization in terms of the (sea) quark TMD $x\mathcal{F}_q(x, P_\perp^2)$
  [Marquet, Xiao, Yuan, PLB 682, 207 (2009)]

$$\left.\frac{\mathrm{d}\sigma^{\gamma_T^*+A\to\mathrm{jet}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2} \times \underbrace{\frac{N_c}{\pi^2}\int_{\boldsymbol{b}_\perp}\int\frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2}\,\mathcal{D}(x,\boldsymbol{q}_\perp)\left[1-\frac{\boldsymbol{P}_\perp\cdot(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)}{(P_\perp^2-(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2)}\ln\frac{P_\perp^2}{(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2}\right]}_{\text{sea quark TMD}}$$

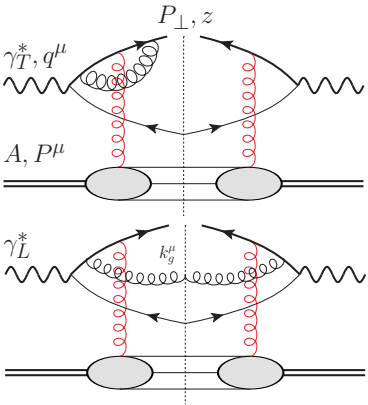
$$\underset{P_\perp\gg Q_s}{\to} \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^1\mathrm{d}\xi P_{qg}(\xi)\frac{x}{\xi}G\left(\frac{x}{\xi},P_\perp^2\right)$$

Outline of the NLO computation in the dipole picture

- NLO impact factor depends on the jet definition.

- Consider the limit $Q^2 \gg P_\perp^2 \gg Q_s^2$ in the NLO impact factor

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star + A \to \mathrm{jet} + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star + A \to \mathrm{jet} + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \Big[1 + \alpha_s \mathcal{I}(\boldsymbol{P}_\perp, Q, R, ...)\Big]$$



Examples of NLO diagrams

Jet sequential recombination algorithms

- Popular jet definitions nowadays use sequential recombination algorithms.
  (Unlike cone-based jet definitions)

- Example with jets in e^+e^-: JADE, k_t algorithms,...
  JADE, Z.Phys.C 33 (1986), Catani, Dokshitzer, Olsson, Turnock, Webber, PLB 269, 432 (1991)

- Distance measure d_{ij} between particles i, j.
  Ex: $d_{ij} = M_{ij}^2/Q^2$ for JADE def.

- Sequential clustering of particles.
  - $\rightarrow$ For each pair of particles (i, j), work out the distance d_{ik}.
  - $\rightarrow$ Find the minimum of all d_{ij}.
  - $\rightarrow$ If the min is $< d_{\rm cut}$, recombine i and j and repeat from step 1.
    Otherwise, terminate the iteration.

Jet definition in DIS: summary

- Not all jet definitions ensure factorisation of the fully inclusive **jet** cross section.
  [Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)]
- Does the same phenomenon arise for TMD factorisation?

	Clustering condition A	Clustering condition B
	$M_{ij}^2 z^2/(z_i z_j P_\perp^2) \leq R^2$	$M_{ij}^2 z^\beta/(z_i z_j Q^2) \leq R^2$
Examples	LI generalized k_t	SI C/A if $\beta = 2$ DIS k_t if $\beta = 1$

- $M_{ij}^2 = (k_i + k_j)^2$, $z_i = (k_i \cdot P)/(P \cdot q) = k_i^+/q^+$.
- Goal: find a jet definition which ensures TMD factorisation of the single inclusive jet cross-section.

Sudakov logarithms in the NLO impact factor

- NLO Sudakov logs $L \sim \ln(Q^2/\boldsymbol{P}_\perp^2)$ depend on the jet definition!

1. Clustering condition A (e.g. LI C/A or anti-k_t):

$$\left.\frac{\mathrm{d}\sigma^{\gamma^*_{\mathrm{T}}+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma^*_{\mathrm{T}}+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{3}{4}L^2 + \left(\frac{3}{4} - \ln(R)\right)L + \mathcal{O}(1)\right]$$

2. CLustering condition B:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^*_{\mathrm{T}}+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma^*_{\mathrm{T}}+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \left(\frac{3(1-\beta/2)}{4} + \ln(R)\right)L + \mathcal{O}(1)\right]$$

- From CSS evolution of the quark TMD alone, we expect the log structure

$$\frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \frac{3}{4}L\right]$$

$\Rightarrow$ consistency with quark TMD evolution implies $\beta = 0$.
New LI jet definition in DIS suitable for TMD factorisation with jets.

Heuristic derivation of the Sudakov double log

- To DLA, Sudakov comes the virtual gluon emissions in the phase space forbidden to the real ones:

$$S_{\mathrm{DL}} = -\frac{\alpha_s C_F}{2\pi} \int_{P_\perp^2}^{Q^2} \frac{\mathrm{d}k_{g\perp}^2}{k_{g\perp}^2} \int_{k_{g\perp}^2/Q^2}^{z_{\max}} \frac{\mathrm{d}z_g}{z_g},$$

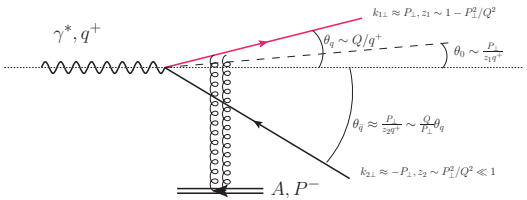
- $k_{g\perp} \gg P_\perp$ forbidden due to the constraint $P_\perp^2 \ll Q^2$.
- Virtual emissions are effectively cut in the UV by the hard scale Q^2.
- Lower limit on $z_g \Leftrightarrow \tau_g \sim 1/k_g^- \gg \tau_\gamma \sim 1/q^-$: excludes gluons contributing to (collinearly improved) high energy evolution.
- $z_{\max}$ depends on the jet constraint $\Rightarrow$ a real gluon is forbidden only if it is **not** in the jet.

For a hadron measurement, $z_{\max} = 1$ and $S_{\mathrm{DL}} = -\frac{\alpha_s C_F}{2\pi} \ln(Q^2/P_\perp^2)$. Altinoluk, Jalilian-Marian, Marquet, 2406.08277

Physical interpretation of the jet constraint

- Our new clustering condition equivalent to $\theta_{ij} \leq R\theta_{\text{jet}}$ with $\theta_{\text{jet}} \sim Q/q^+$.
- Angle of the jet set by its **virtuality** rather by its transverse momentum.
  (Naively, $\theta_{\text{jet}} \sim \frac{P_\perp}{zq^+}$.)
- Soft gluons contributing to Sudakov must have $\theta_g \gg \theta_{\text{jet}} \Leftrightarrow z_g \leq k_{g\perp}/Q$.

  Stronger constraint than $\theta_g \gg \frac{P_\perp}{zq^+} \Leftrightarrow z_g \leq k_{g\perp}/P_\perp$!
- Using $z_{\max} = k_{g\perp}/Q$ gives $S_{\text{DL}} = -\frac{\alpha_s C_F}{4\pi} \ln(Q^2/P_\perp^2)$.



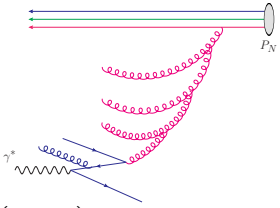
⇑

Aligned jet configuration in dipole frame.

Complete NLO result for $Q^2 \gg P_\perp^2 \gg Q_s^2$

- The complete NLO corrections are organized as

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} + \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V}$$



- Term where $P_\perp$ is given by a $q \to qg$ hard collinear splitting

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{1-\frac{RP_\perp}{Q}}\mathrm{d}\xi P_{qq}(\xi)\frac{x}{\xi}f_q\left(\frac{x}{\xi},P_\perp^2\right)$$

- Sudakov logs and finite pieces, for our asymmetric jet clustering definition.

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}\ln^2\left(\frac{Q^2}{P_\perp^2}\right) + \left(\frac{3}{4}+\ln(R)\right)\ln\left(\frac{Q^2}{P_\perp^2}\right) - \frac{3}{2}\ln(R) + \frac{11}{4} - \frac{3\pi^2}{4} + \frac{3}{4}\ln^2(x_\star) + \frac{3}{8}\ln(x_\star) + \mathcal{O}(R^2)\right]$$

- $x_\star$ factorisation scale: gluons with $z_g \leq x_\star P_\perp^2/Q^2$ resummed with high energy evolution.

Conclusion

- Importance of the jet definition (clustering algorithm) for jet production in SIDIS in the TMD limit $Q^2 \gg P_\perp^2$.
- Calculation of the Sudakov effect for jets in SIDIS.
- NLO calculation in the high-energy formalism (dipole picture, CGC).
  Conclusions remain valid at moderate x.
- Emergence of the DGLAP and CSS evolutions of the quark TMD from the small-x approach.
- **Outlook:** provide complete result also in the saturation regime $P_\perp \sim Q_s$, MonteCarlo studies of jet production at the EIC with our new asymmetric jet clustering condition.

THANK YOU!

Back-up slides

DGLAP+CSS resummation

- Beyond LO, the quark TMD also depends upon the photon virtuality Q^2.
- $x\mathcal{F}_q(x, P_\perp^2, Q^2)$ = number of quarks in the target with given x and $P_\perp$, as probed with a longitudinal resolution fixed by Q^2.
- Taking derivative w.r.t. $\ln(Q^2)$ and assuming Markovian evolution:

$$\frac{\partial \mathcal{F}_q(x, P_\perp^2, Q^2)}{\partial \ln Q^2} = \frac{C_F}{2\pi}\left\{\frac{\alpha_s(P_\perp^2)}{P_\perp^2}\int_{\Lambda^2}^{P_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_q(x, \ell_\perp^2, Q^2) - \int_{P_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\alpha_s(\ell_\perp^2)\mathcal{F}_q(x, P_\perp^2, Q^2)\right\}$$
$$+ \frac{3}{2}\frac{\alpha_s(P_\perp^2)C_F}{\pi}\mathcal{F}_q(x, P_\perp^2, Q^2)$$

  $\Rightarrow$ "diagonal" version of the CSS equation for the quark TMD from the dipole picture.

- Integrating our 1-loop result up to Q^2 yields:

$$xf_q(x, Q^2) = xf_q^{(0)}(x, Q^2) + \int_{\Lambda^2}^{Q^2}\frac{\mathrm{d}P_\perp^2}{P_\perp^2}\frac{\alpha_s(P_\perp^2)}{2\pi}\int_x^1 \mathrm{d}\xi\, \mathcal{P}_{qq}(\xi)\,\frac{x}{\xi}f_q\left(\frac{x}{\xi}, P_\perp^2\right)$$

  $\Rightarrow$ DGLAP evolution of the quark pdf from the dipole picture.

Similar results obtained for the WW gluon TMD in PC, Iancu, 2406.04238

Jet definitions in DIS

- Jet definitions designed to ensure factorisation of **inclusive** jet cross sections in terms of universal pdf. Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)
- Longitudinally invariant "generalized-k_t" algorithms.

$$d_{ij} = \min(p_{t,i}^{2k}, p_{t,j}^{2k}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{t,i}^{2k}$$

Catani, Dokshitzer, Seymour, Webber, NPB, 406 (1993), Cacciari, Salam, Soyez, JHEP 0804:063,2008

$\Rightarrow$ Many jet analysis at HERA chose longitudinally invariant k_t algorithm in the Breit frame. Ex: α_s extraction from jet cross-sections, ZEUS, PLB 547 (2002), H1 PLB 653, 134 (2007), ...

- e^+e^- spherically invariant jet definitions in the Breit frame.

$$d_{ij} = \min(E_i^{2k}, E_j^{2k}) \frac{1 - \cos(\theta_{ij})}{1 - \cos(R)}, \quad d_{iB} = E_i^{2k}$$

$\Rightarrow$ Recent studies on TMD factorisation with jets in DIS at moderate x use this definition with WTA scheme.
Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi, PRL 121, (2018), JHEP 10, 031 (2019)

Known issues with previous options

- Spherically invariant jet definitions in the Breit frame are not boost invariant.
  $\Rightarrow$ Hard to distinguish beam remnant from backward jets.

- Longitudinally invariant jet definitions in Breit frame fail to cluster hadrons in the forward region.

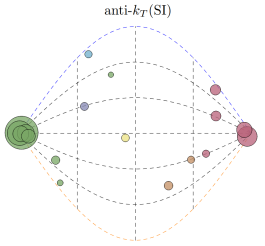



Fig. from Arratia, Makris, Neill, Ringer, Sato, PRD 104 (2021)