Matching relations for gluon TMDs up to one-loop accuracy

Alessio Carmelo Alvaro

In collaboration with N. Kato, B. Pasquini, C. Pisano, S. Rodini

DIS International Workshop, Bologna, 05-06-2026

Paper soon available on the arXiv

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Matching relations

In the small-b regime:

$$f_i(x,b) = \sum_n \alpha_s^n \sum_j \left[C_{ij}^{(n)} \otimes f_j \right](x) + O(b^2)$$

TMD

Matching Coefficient

PDF

Matching relations

In the small-b regime:

$$f_i(x,b) = \sum_n \alpha_s^n \sum_j \left[C_{ij}^{(n)} \otimes f_j \right](x) + O(b^2)$$

Constraints on TMD functional form $\rightarrow$ Parameterization $f_i(x,b) \propto \sum_j \left(C_{ij} \otimes f_j \right)(x)$

Predictions for TMD observables

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Actual Status of Matching Relations

Quark TMDs

Distribution	Tw2	Tw3	Accuracy
f_1	$f_1\,, f_g$	-	$\mathrm{N^3LO}$
g_1	$g_1\,, \Delta f_g$	-	$\mathrm{N^3LO}$
g_{1T}	$g_1\,, \Delta f_g$	$\mathcal{T}_g\,, \mathcal{G}$	NLO
h_1	h_1	-	$\mathrm{N^3LO}$
h_{1L}	h_1	$\mathcal{T}_h$	NLO
$h_{1T}^{\perp}$	-	$\mathcal{T}_h$	LO
$f_{1T}^{\perp}$	-	$T\,, \mathcal{G}$	NLO
$h_1^{\perp}$	-	E	NLO

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h_1	h_1	-	$\mathrm{N^3LO}$
h_{1L}	h_1	$\mathcal{T}_h$	NLO
$h_{1T}^{\perp}$	-	$\mathcal{T}_h$	LO
$f_{1T}^{\perp}$	-	$T\,, \mathcal{G}$	NLO
$h_1^{\perp}$	-	E	NLO

Gluon TMDs

Distribution	Tw2	Tw3	Accuracy
f_1^g	$f_g\,, f_1$	-	$\mathrm{N^3LO}$
$h_1^{\perp g}$	$f_g\,, f_1$	-	$\mathrm{N^3LO}$
g_{1L}^g	$\Delta f_g\,, g_1$		$\mathrm{N^3LO}$
g_{1T}^g			
$f_{1T}^{\perp g}$	-		
h_{1T}^g	-		
$h_{1L}^{\perp g}$			
$h_{1T}^{\perp g}$			

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Tree level computation

$$G^{\mu\nu}(x,b) = \sum_{n=0}^{\infty} \frac{1}{n!} b_{\mu_1} \dots b_{\mu_n} \left[\partial^{\mu_1} \dots \partial^{\mu_n} G^{\mu\nu}(x,b) \right]_{|b=0}$$

Each term is a sum of operators with geometrical twist $2 \leq k \leq n+2$

Issue: each operator needs to be treated separately

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Issue: each operator needs to be treated separately

Does a unified treatment exist?
Moos, Vladimirov, JHEP 12 (2020) 145
Rodini, Alvaro, Pasquini, PLB 845 (2023) 138163

Example of results: unpolarized

$$f_1^g(x,b) = f_g(x)$$

$$+\sum_{k=1}^{\infty} \frac{1}{k!(k-1)!}\left(\frac{x^2M^2b^2}{4}\right)^k \int_0^1 du \int dy\, \delta(x-uy)\left(\frac{\bar{u}}{u}\right)^{k-1} f_g(y)$$

Leading Term

Mass series

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NLO: Parton-in-parton

$$\Phi_i(x,b) \overset{NLO}{=} a_s \int_0^1 du \int dy\, \delta(x-uy) \sum_j C_{ij}(u)\, \Phi_j(y)$$

$$C_{ij} \Longleftrightarrow \langle p_j\,, s_j \,|\, F^{\mu+}\, \Gamma^i_{\mu\nu}\, F^{\nu+} \,|\, p_j\,, s_j \rangle$$

$\Gamma_{\mu\nu}$ proper projector

$p_j\,, s_j$ gluon/quark final state

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Main idea

Question: can we extend to higher-twist? and how?

$$p_j^\mu = (p^+ , 0 , \mathbf{0}) \Longrightarrow p_j^\mu = (p^+ , \frac{\boldsymbol{p_T^2}}{2p^+} , \boldsymbol{p_T})$$

$\left(\boldsymbol{b} \cdot \boldsymbol{p_T}\right)^n$ associated to higher-twist terms

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$\left(\boldsymbol{b} \cdot \boldsymbol{p_T}\right)^n$ associated to higher-twist terms

LO computation: $C_{std}^{(0)} \rightarrow C_{ext}^{(0)} = C_{std}^{(0)} e^{i(bp_T)}$

NLO computation: $C_{std}^{(1)} \rightarrow C_{ext}^{(1)} = C_{std}^{(1)} e^{iu(bp_T)}$

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Formalization

$$p_T^\mu \to -i\,\partial_T^\mu(0) \Longrightarrow e^{iub\cdot p_T} \to e^{ub\cdot\partial_T(0)}$$

$$\Phi_i(x,b) \overset{NLO}{=} \int_0^1 du \int dy\,\delta(x-uy)\,\sum\nolimits_j C_{ij}(u)\,\left(e^{ub\cdot\partial_T(0)}\,\Phi_j(y,b)\right)$$

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Inclusion of hadron-mass corrections

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Inclusion of hadron-mass corrections

At the moment only twist 2 PDFs

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Summary

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g_{1L}^g	$\Delta f_g\,, g_1$	-	$\mathrm{N^3LO}$
g_{1T}^g	$\Delta f_g\,, g_1$	$\mathcal{F}\,, \mathcal{T}$	NLO/LO
$f_{1T}^{\perp g}$	-	$2F_2^+ + F_4^+$	LO
h_{1T}^g	-	$2F_2^+ - 2F_4^+$	LO
$h_{1L}^{\perp g}$	-	$2F_2^+ - 2F_4^+$	LO
$h_{1T}^{\perp g}$	-	-	LO

Summary

LO: matching up to twist 3 PDFs (T-Odd effects)

NLO Wandzura-Wilczek for g_{1T}^{g}

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‘Extended’ parton-in-parton method

Inclusion of hadron mass corrections at NLO

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Outlook 1: higher-twist PDFs

Outlook 2: NLP TMDs

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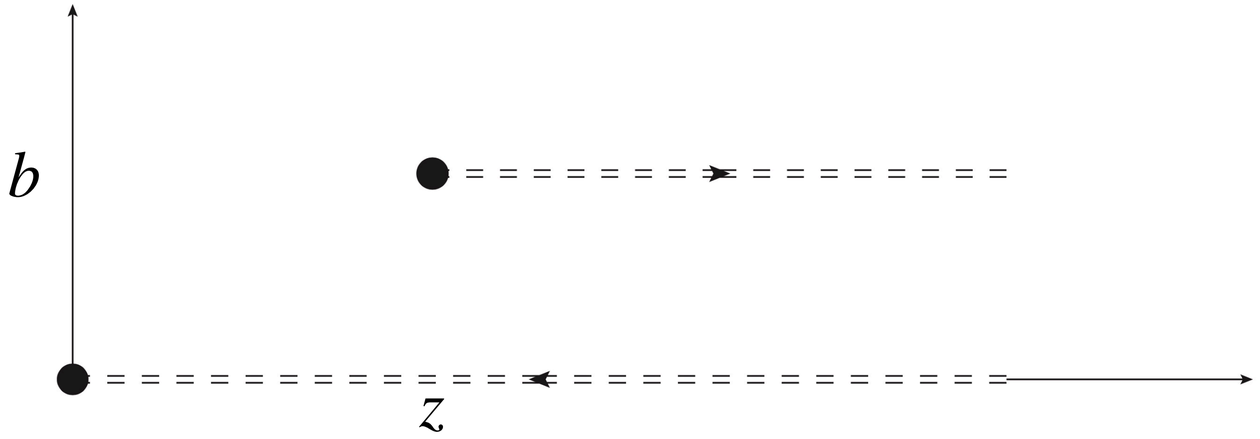
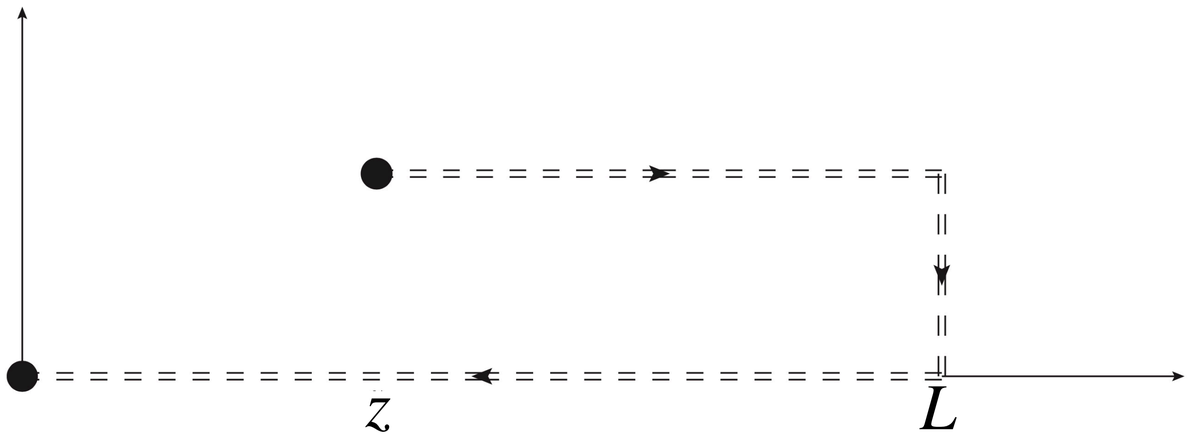
Backup

Tree level computation



$$F^{\mu+}(zn+b)\,[zn+b\,,\infty n+b]$$
$$[\infty n+b\,,\infty n]\,[\infty n\,,0]\,F^{\nu+}(0)$$



$$F^{\mu+}(zn+b)\,[zn+b\,,Ln+b]$$
$$[Ln+b\,,Ln]\,[Ln\,,0]\,F^{\nu+}(0)$$

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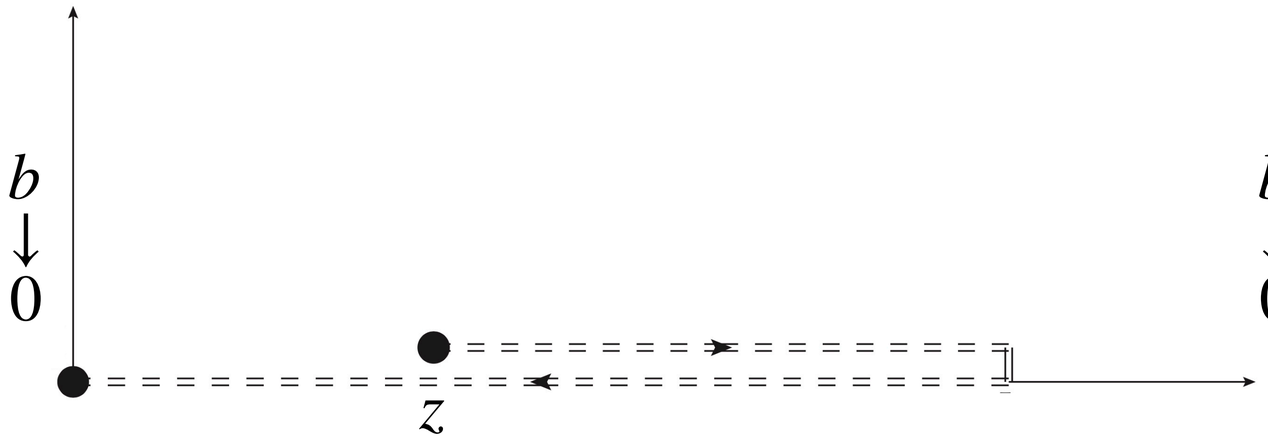
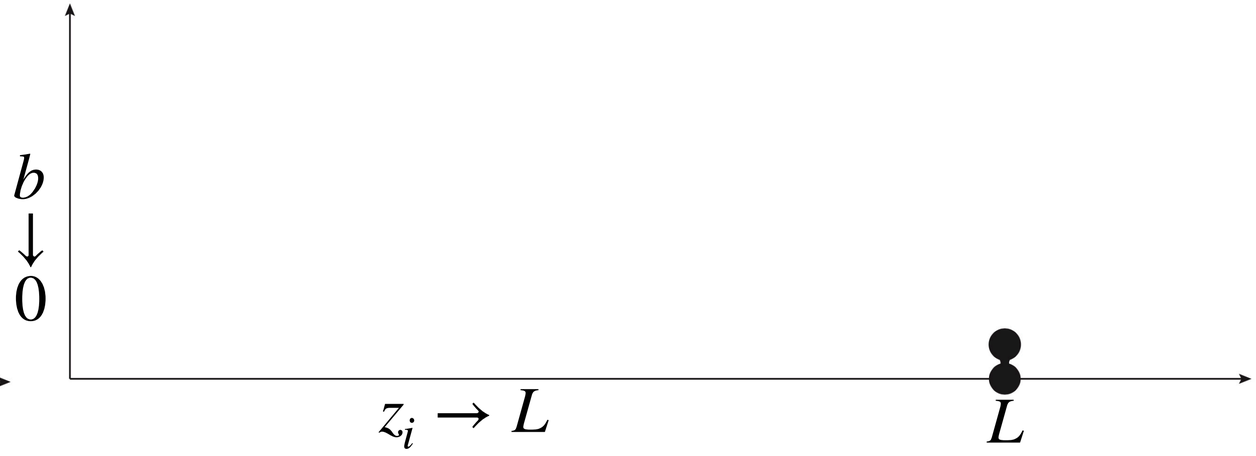
Tree level computation



$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(F^{\mu+}(zn) \, [zn \, , Ln] \right) \left(b \cdot \overleftarrow{D} \right)^n [Ln \, , 0] \, F^{\nu+}(0)$$



$$\sum_{n=0}^{\infty} \frac{1}{n!} \sum_{s,t=0}^{\infty} \frac{(z-L)^s (-L)^t}{s!\,t!} F^{\mu+} \overleftarrow{D}_+^s \left(b \cdot \overleftarrow{D} \right)^n D_+^t \, F^{\nu+}$$

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Tree level computation

Spinor Formalism: $x_\mu \rightarrow x_{\alpha\dot{\alpha}} = x_\mu \sigma^\mu_{\alpha\dot{\alpha}}$

Twist decomposition $\rightarrow$ (anti)symmetrization of spinor indeces

Hadronic matrix element

$$L \rightarrow \mp \infty$$

Fourier transform to x-space

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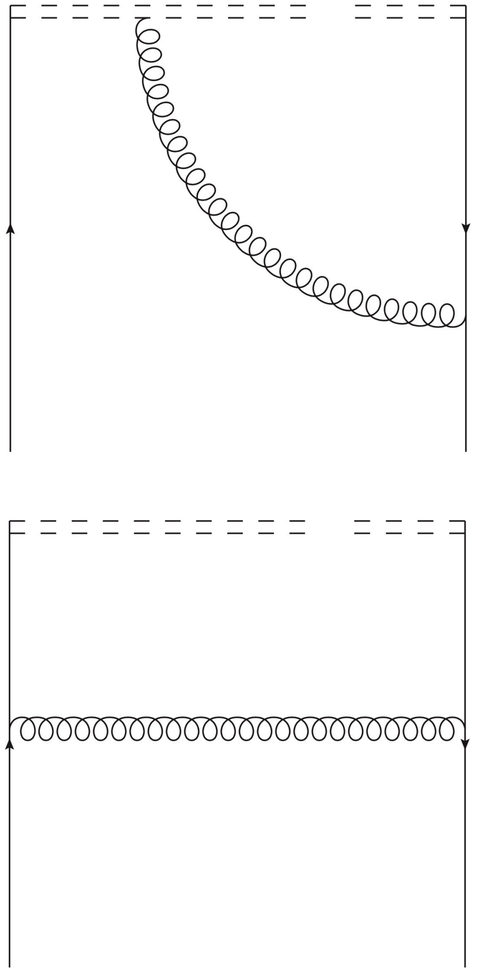
Check: quark worm-gears (one-loop)

Comparison with position-space computation
(Feynman gauge)

For each diagram:

LT $\rightarrow$ LP + NLP

NLT $\rightarrow$ LP



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Check: quark worm-gears (one-loop)

Position space:

$$\text{LT} \rightarrow \gamma^+\gamma^5 + (b\partial)\gamma^+\gamma^5 + \ldots \sim O(b^0) + O(b^1) + \ldots$$

$$\text{NLT} \rightarrow \gamma_T^\mu\gamma^5 \sim O(b^1) + \ldots$$

Parton-in-parton:

Global phase $e^{iub\cdot p_T} = 1 + iub \cdot p_T + \ldots$

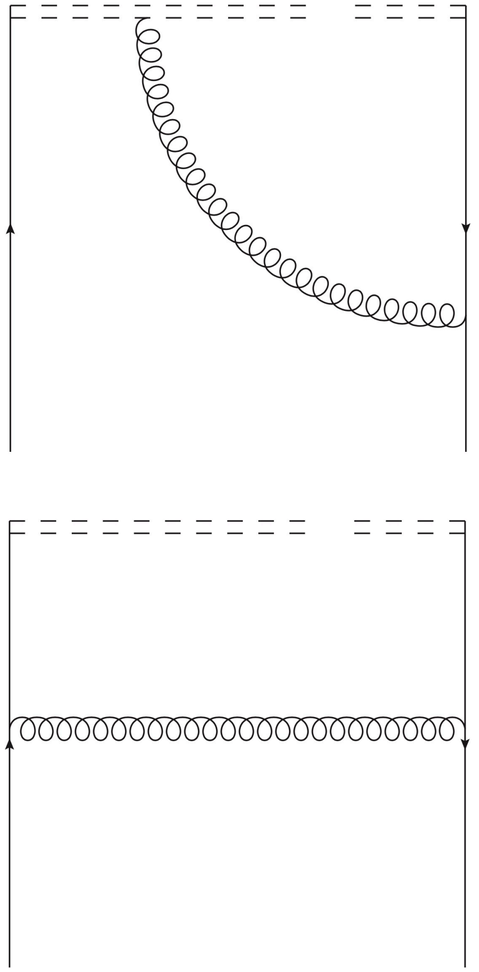
Projector $\gamma^5\gamma^\alpha P_\alpha = \gamma^5\gamma^- p^+ + \gamma^5\gamma_T^\alpha p_{T,\alpha} + \gamma^5\gamma^+ p^-$

$$\text{LT} \rightarrow O(b^0) + O(b^1) + \ldots$$

$$\text{NLT} \rightarrow O(b^1) + \ldots$$

$$O(b^0) \sim tw2 \rightarrow g_1$$

$$O(b^1) \sim tw2 + tw3 \rightarrow g_{1T}$$



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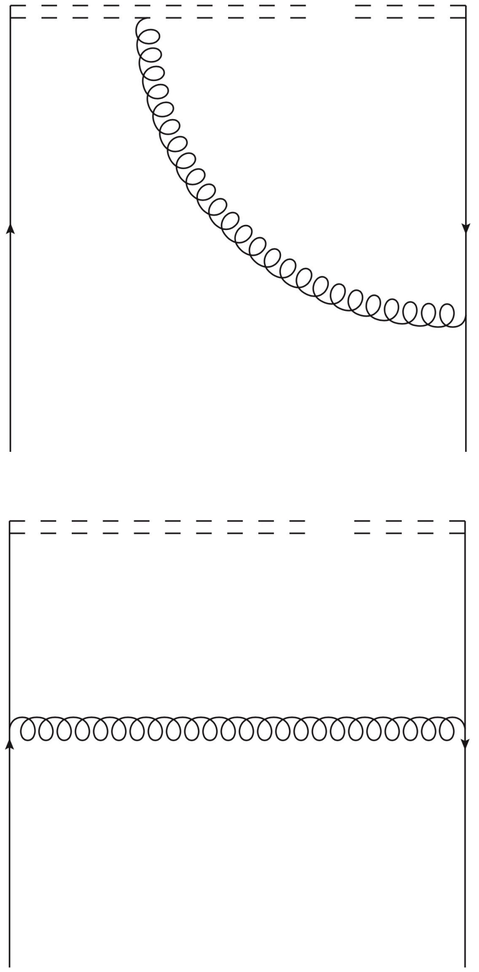
Exact correspondence!



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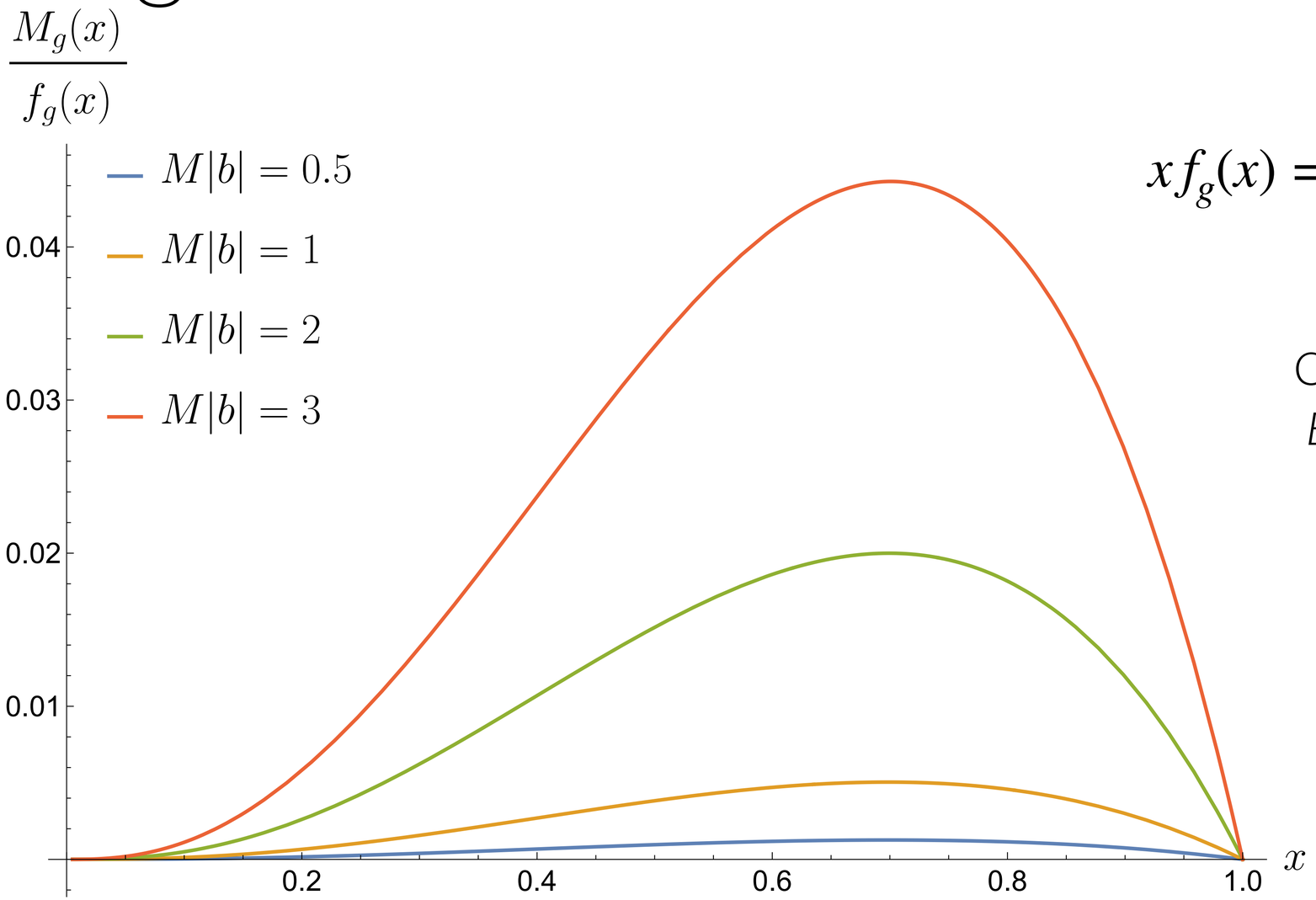
Magnitude of mass corrections: unpolarized

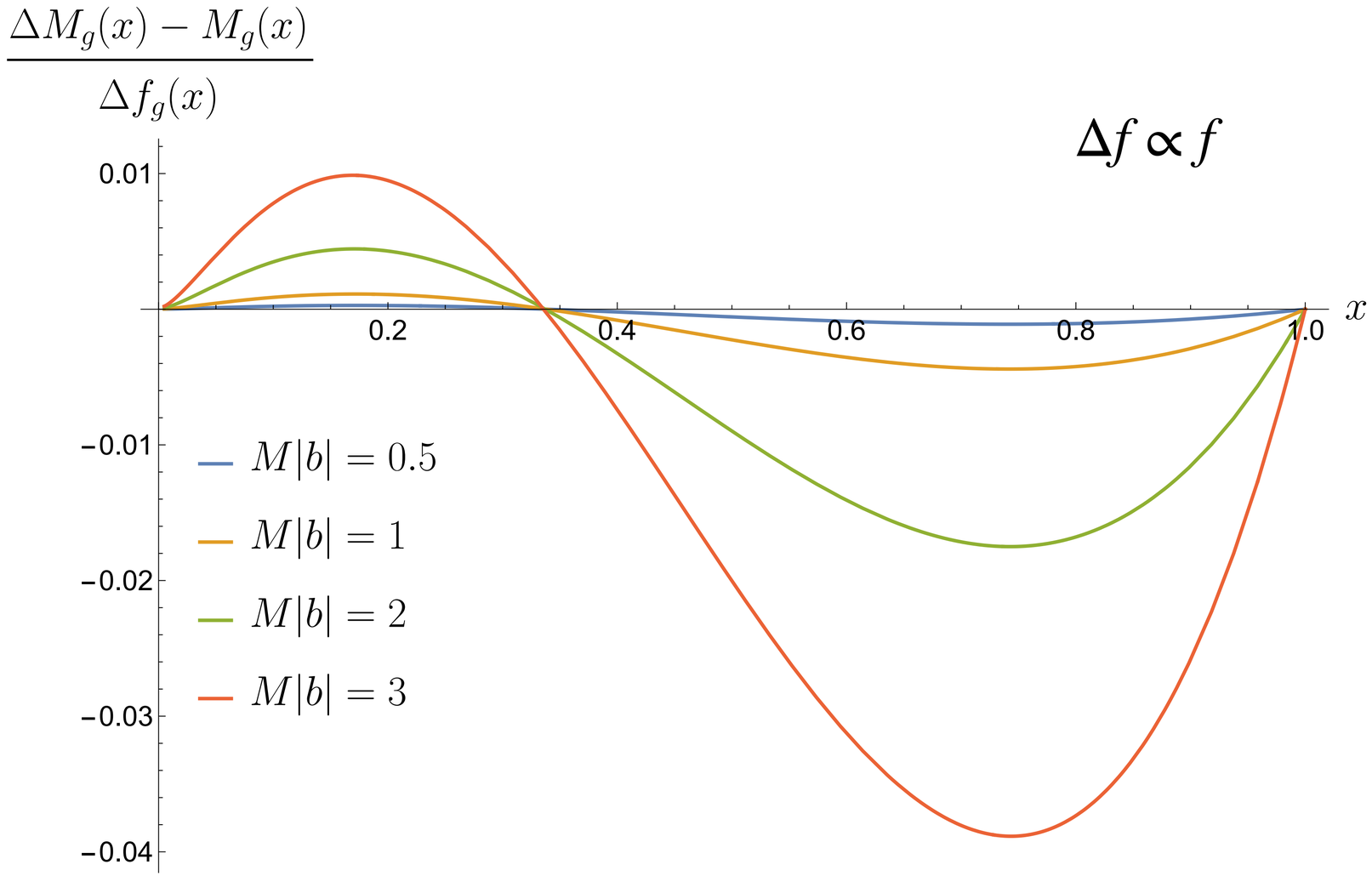


$$x f_g(x) = A_g\, x^{\alpha_g} (1-x)^{\beta_g} - A'_g\, x^{\alpha'_g} (1-x)^{\beta'_g}$$

$$\mu^2 = 1.9\,\mathrm{GeV}^2$$

Central values of HeraPDF2.0
Eur.Phys.J.C 75 (2015) 12, 580

Magnitude of mass corrections: helicity



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