Extraction of twist-three distributions from global data

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Outline:

I present the extraction of ***genuine twist-three distributions*** from the analyses of the global data.

- It **the first** analyses of such kind
  - Simultanious extraction of d_2, g_2, $f_{1T}^{\perp}$ (Sivers), and $g_{1T}^{\perp}$ (wgt) functions
- It **opens new exciting perspectives** for studies of hadron structure

Disclamer

The presentation is based on [2511.04294].
However, the results are already confirmed and updated in a newer fit, which is in preparation.

What are genuine twist-three distributions?

$\frac{1}{Q^0}$: $q, \Delta q, \delta q$ (twist-2) $\longrightarrow$ probability densities parton-"static" $\longleftrightarrow$ $\left| q|P\rangle \right|^2$

What are genuine twist-three distributions?

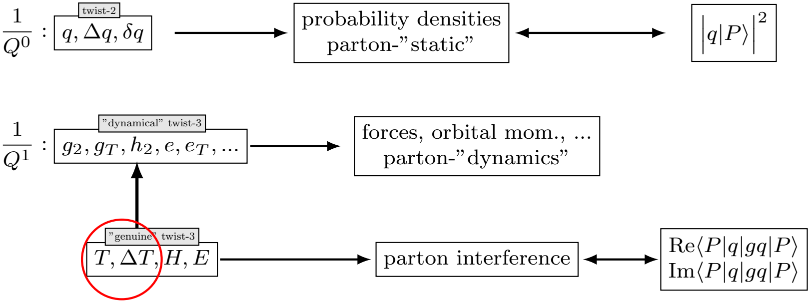
$\frac{1}{Q^0}$: twist-2 $q, \Delta q, \delta q$ → probability densities parton-"static" ↔ $\left| q|P\rangle \right|^2$

$\frac{1}{Q^1}$: "dynamical" twist-3 $g_2, g_T, h_2, e, e_T, \ldots$ → forces, orbital mom., ... parton-"dynamics"

↑

"genuine" twist-3 $T, \Delta T, H, E$ → parton interference ↔ $\mathrm{Re}\langle P|q|gq|P\rangle$ $\mathrm{Im}\langle P|q|gq|P\rangle$

What are genuine twist-three distributions?



$$\langle p,S|g\,\overline{q}(z_1 n)F^{\mu+}(z_2 n)\gamma^+ q(z_3 n)|p,S\rangle = 2\epsilon_T^{\mu\nu} s_\nu (p^+)^2 M \int [dx]\, T_f(x_1,x_2,x_3)\, e^{-ip^+ x_i z_i}$$

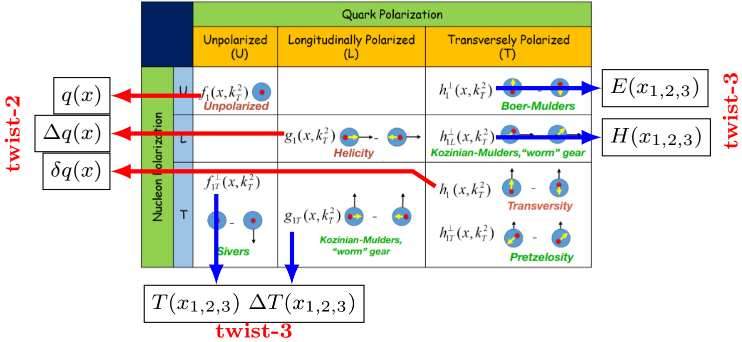
$$\langle p,S|ig\,\overline{q}(z_1 n)F^{\mu+}(z_2 n)\gamma^+\gamma^5 q(z_3 n)|p,S\rangle = -2s_T^{\mu}(p^+)^2 M \int [dx]\, \Delta T_f(x_1,x_2,x_3)\, e^{-ip^+ x_i z_i}$$

Genuine twist-3 distributions are
irreducible basis for all other twist-3 distributions

Twist-three PDFs are everywhere in polarized physics

Most known "connections" to twist-3

- DIS structure function $g_2(x,Q)$ (accompany helicity $g_1(x,Q)$)
- d_2 moment (local $\bar{q}Fq$-operator)
- TMDs ($f_{1T}^{\perp}$, g_{1T}, etc)
- Spin-symmetries, Jet-asymmetries (A_N, ..)
- (Twist-three) GPDs
- ...

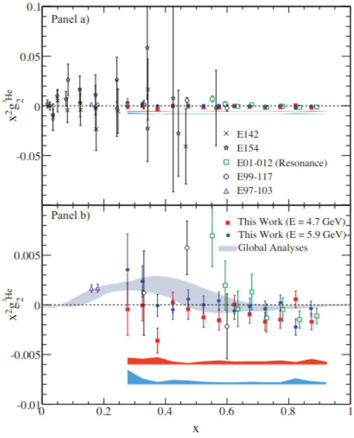


The theory of twist-3 PDFs is studied sufficiently well in 80's-00's. We know

- Symmetry relations, limiting properties, sum rules
- Evolution at LO
- Many coefficient function are computed at NLO (1-loop)
- Many model computations

However, **all this was never applied in practice** (to the real data), because

- Twist-3 PDFs are 2D-functions (too complicated object)
- Data is 1D (cross-section), i.e. it integrates 2D→1D
- Twist-3 PDFs are small
- Data is often very bad...



The main idea:
fit distinct observables altogether

- increases the number of available data points,
- fixes various parts of 2D PDFs.

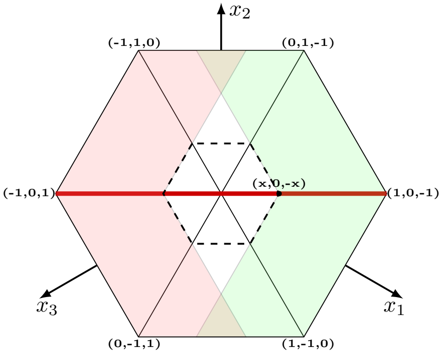
Should be done at least at LO (one-loop)

List of observables:

- g_2 (DIS structure function)
- d_2 (moment)
- $f_{1T}^{\perp}$ (Sivers TMD distribution)
- $g_{1T}^{\perp}$ (worm-gear-T TMD distribution)

- A_N (SSA)
- ...

Different observables are sensitive to different parts of the hexagon

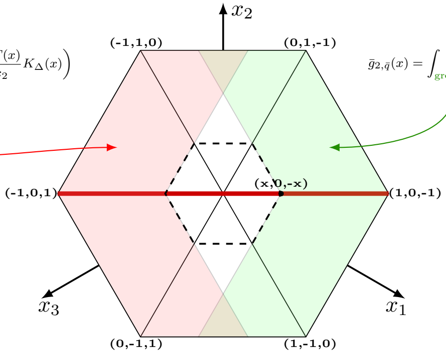


Different observables are sensitive to different parts of the hexagon

NLO
[Braun,Korechemsky, Manashov, 00-01]
[Kodaira, Tanaka, et al, 96-97]
[Geyer, Mueller, Robaschik, 96]
etc

$$\bar{g}_{2,q}(x) = \int_{\text{red}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_\Delta(x) \right)$$

$$\bar{g}_{2,\bar{q}}(x) = \int_{\text{green}} [dx] \left(\frac{dT(x)}{dx_2} K(x) + \frac{d\Delta T(x)}{dx_2} K_\Delta(x) \right)$$



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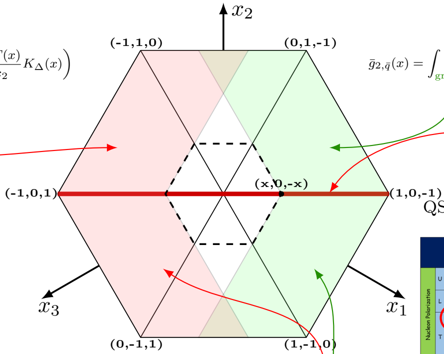
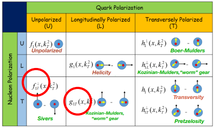


QS function $= T(-x,0,x)$



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ Unpolarized		$h_1^\perp(x,k_T^2)$ Boer-Mulders
	L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$ Kozinian-Mulders,"worm" gear
	T	$f_{1T}^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Kozinian-Mulders, "worm" gear	$h_1(x,k_T^2)$ Transversity $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

$$\int [dx] \left(T(x) K_T(x) + \Delta T(x) K_{T\Delta}(x) \right)$$

NLO

[Scimemi, Tarasov, AV, 1901.04519]
[Rein, Rodini, AV, 2209.00962]

Different observables are sensitive to different parts of the hexagon

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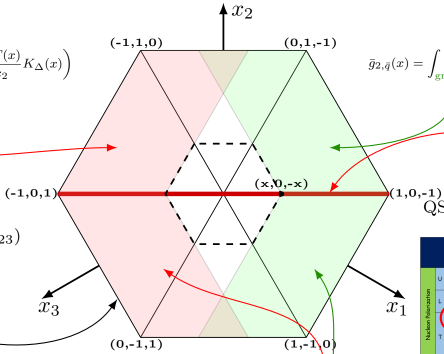
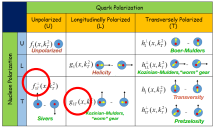


QS function $= T(-x, 0, x)$

$$d_2 = \int_{\text{hexagon}} [dx] T(x_{123})$$



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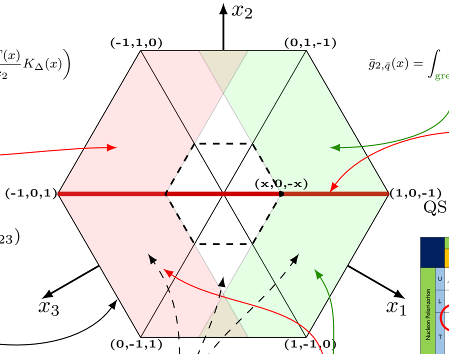
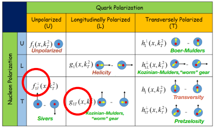


QS function $= T(-x,0,x)$

$d_2 = \int_{\text{hexagon}} [dx] T(x_{123})$



$\int [dx] \left(T(x) K_T(x) + \Delta T(x) K_{T\Delta}(x) \right)$

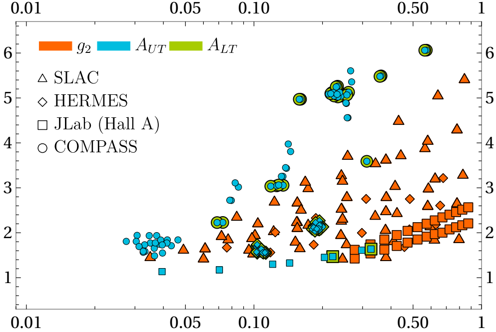
Evolution: $\frac{dT(x_{123},\mu)}{d\ln\mu} = \int_{\text{outer hex}} [dy] \left[P(x,y) T(y,\mu) + P_\Delta(x,y) \Delta T(y,\mu) \right]$

[Bukhvostov,Frolov,Lipatov,Kuraev, 85] ... [Braun,Manashov,Pirnay, 0909.3410]

Data

d_2
SANE
...
RQCD
$N_{\text{pt}} = 7$

g_2
SLAC
JLab
HERMES
$N_{\text{pt}} = 103$



$f_{1T}^{\perp}$
HERMES
COMPASS
JLab
$N_{\text{pt}} = 63$

$g_{1T}^{\perp}$
HERMES
COMPASS
JLab
$N_{\text{pt}} = 70$

$N_{\text{tot}} = 243$

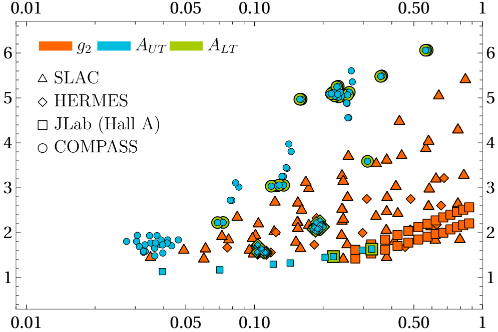
[AV,G.Portela,S.Rodini,2512.04294]

The largerst collection of tw3 data ever considered
The first simultanious extraction of TMD and PDF

Data

d_2
SANE ... RQCD
$N_{pt} = 7$
$N_{pt} = 13$

g_2
SLAC JLab HERMES
$N_{pt} = 103$
$N_{pt} = 168$



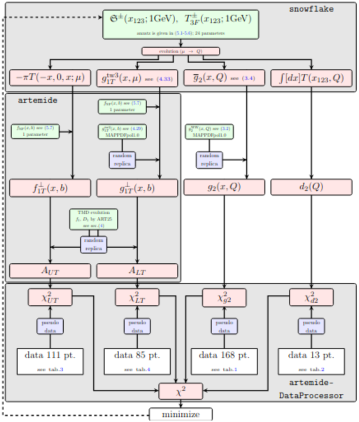
$f_{1T}^{\perp}$
HERMES COMPASS JLab
$N_{pt} = 63$
$N_{pt} = 117$

$g_{1T}^{\perp}$
HERMES COMPASS JLab
$N_{pt} = 70$
$N_{pt} = 74$

$N_{tot} = 243$
[AV,G.Portela,S.Rodini,2512.04294]

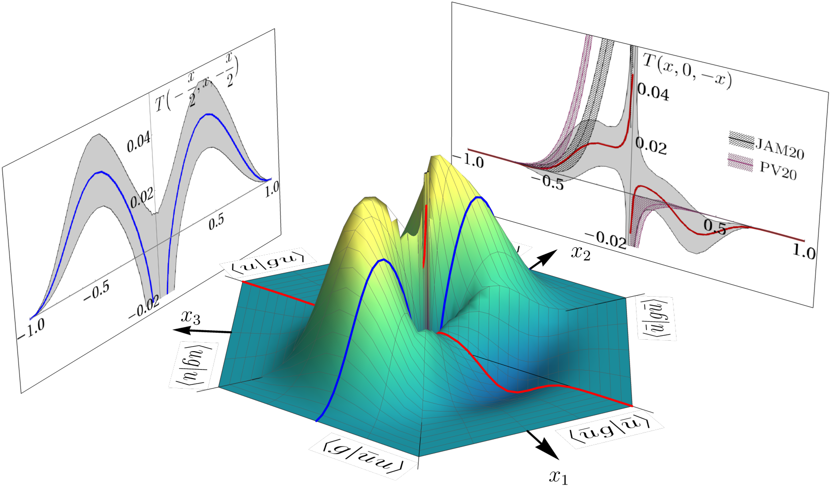
$N_{tot} = 372$
[in preparation]

The largerst collection of tw3 data ever considered
The first simultanious extraction of TMD and PDF



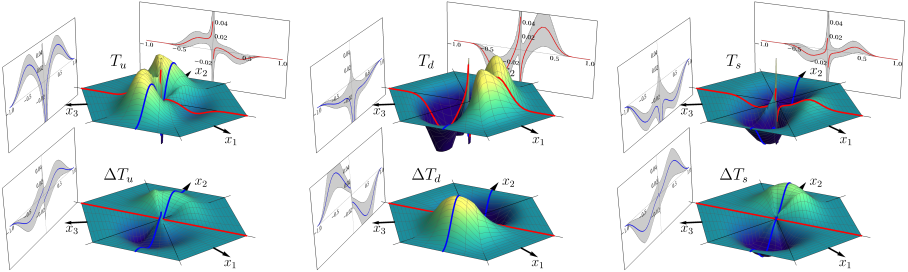
Technical details

- **artemide+snowflake**
- Complete **LO** twist-3 part (i.e. LO evolution) [Rodini, Rossi, AV, 2405.01162]
- N^3LO TMD-evolution
- State-of-the-art other inputs
  - TMD part ⇒ ART25
  - unpol part ⇒ MSHT20
  - WW-helicity part ⇒ MAPPDFpol
- Uncertainly propagation
  - From data
  - From theory input

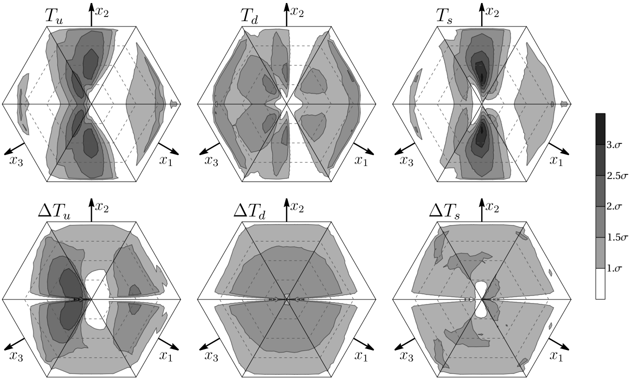


$T(-\frac{x}{2}, x, -\frac{x}{2})$
0.04
0.02
−0.02
−1.0
−0.5
0.5
1.0
$T(x, 0, -x)$
JAM20
PV20
x_1
x_2
x_3
$\langle u|gu\rangle$
$\langle ug|u\rangle$
$\langle \bar{u}|g\bar{u}\rangle$
$\langle \bar{u}g|\bar{u}\rangle$
$\langle u\bar{u}|g\rangle$

Early days of twist-three phenomenology



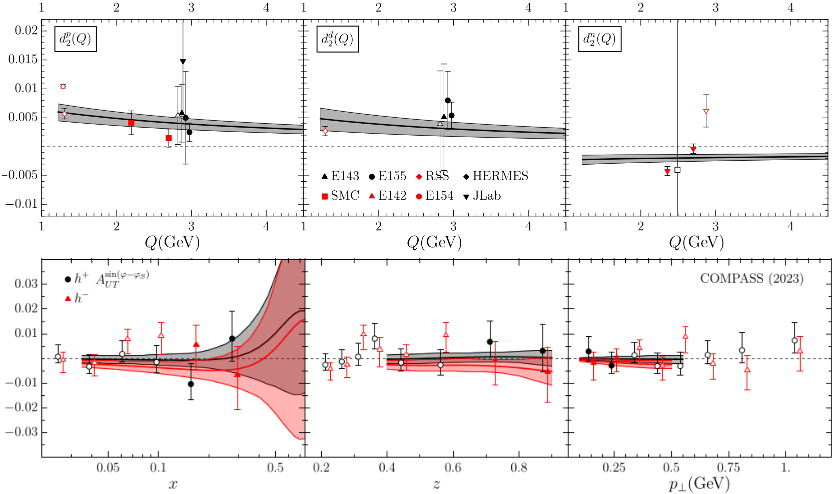
- General agreement with earlier fits/models
- "quark"∼ "anti/sea-quark"
- Uncertainty are reasonable (better in the updated fit!), and larger than by other groups
- Evolution is important (especially for TMDs)
- Gluon distribution is $\sim 1 \pm 1.3$



Uncertanties:

- Experimental
- Theoretical (PDF = MSHT20, MAPpol1.0; TMD=ART25)

Example of data



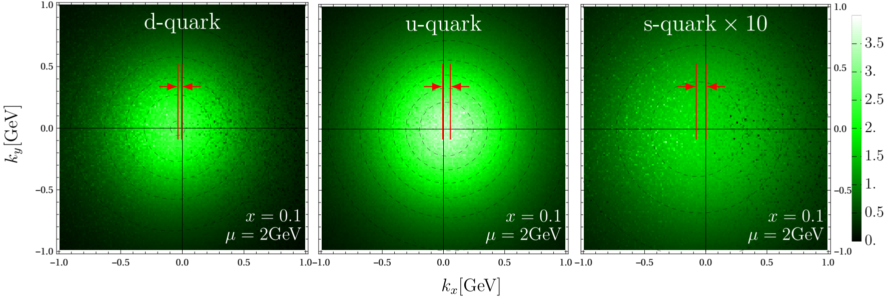
Why is that interesting?

Apart of multiple observables that we are able to compute, we also get access to multiple properties of proton

- Avarage transverse momenum shift
- Color-magnetic and color-electic forces
- Orbital momentum (Ji sum rule; qg-component)
- ...

Avarage transverse momentum shift

$$\langle \mathbf{k}_\mu \rangle = -\pi \epsilon_{\mu\nu} s_T^\nu M \int_0^1 dx T(-x,0,x)$$



$\langle k_x \rangle_u = 9.5^{+7.0}_{-7.1}$MeV, $\langle k_x \rangle_d = -18.8^{+18.4}_{-17.7}$MeV,

$\langle k_x \rangle_s = -5.0^{+6.4}_{-7.4}$MeV, $\langle k_x \rangle_c = 2.3^{+3.2}_{-3.3}$MeV,

$\langle k_x \rangle_{\bar u} = -2.5^{+6.3}_{-6.4}$MeV, $\langle k_x \rangle_{\bar d} = 16.2^{+17.9}_{-19.0}$MeV,

$\langle k_x \rangle_{\bar s} = 2.2^{+7.2}_{-7.2}$MeV, $\langle k_x \rangle_{\bar c} = -0.7^{+3.5}_{-3.8}$MeV,

$\langle k_x \rangle_g = -36.9^{+36.4}_{-39.8}$MeV,

↑ Gluon Sivers!

Budkard sum rule

[M.Budkardt, hep-ph/0311013]

$$\sum_{f=q,\bar q,g} \langle k_x \rangle_f = -33.^{+34.}_{-32.}\text{MeV}$$

Conclusion

The first determination of *genuine* twist-three distribution
and (by-product) of d_2, g_2, $f_{1T}^{\perp}$, $g_{1T}^{\perp}$

Novel direction of studies

- "Unification" of data from multiple polarized experiments
  (SLAC, JLab, HERMES, COMPASS, lattice, ...)
  - "first" joined fit of TMD and collinear distributions
- Exceptional test of QCD universality
  - "first" implimentation and test of proper twist-3 evolution
- Novel "vector-like" information about hadron structure
  (forces, orbital momentum, transverse momentum,..)

Nearest future

All figures and results are taken from the **proof-of-concept** study
[AV, G.Portela, S.Rodini, 2511.04294]
the updared extraction with many additional studies are to be appear soon

Backup

How the global data works together?

$\chi^2/N_{\rm pt}$	
d_2	g_2
1.05	0.94
10.1	3.6
1.15	0.99

Fit! (1.05 / 0.94) → 1.95 / 14.7

Fit! (1.15 / 0.99)

$\chi^2/N_{\rm pt}$	
A_{UT}	A_{LT}
1.95	14.7
1.02	0.90
1.08	0.92

Fit! (1.02 / 0.90) → 10.1 / 3.6

Fit! (1.08 / 0.92)

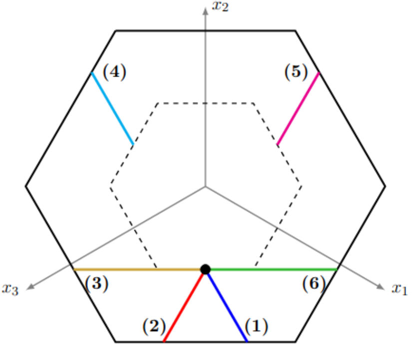
Evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} \mathfrak{S}^{\pm}_{\mathrm{NS}_i} = -a_s(\mu) \mathbb{H}_{\mathrm{NS}} \mathfrak{S}^{\pm}_{\mathrm{NS}_i} \ ,$$

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \mathfrak{S}^{\pm}_{S} \\ \mathfrak{F}^{\pm} \end{pmatrix} = -a_s(\mu) \begin{pmatrix} \mathbb{H}^{\pm}_{qq} & \mathbb{H}^{\pm}_{qg} \\ \mathbb{H}^{\pm}_{gq} & \mathbb{H}^{\pm}_{gg} \end{pmatrix} \begin{pmatrix} \mathfrak{S}^{\pm}_{S} \\ \mathfrak{F}^{\pm} \end{pmatrix},$$

Compare to DGLAP

$$\mu^2 \frac{d}{d\mu^2} q(x) = \int_x^1 \frac{dy}{y} P(\frac{x}{y}) q(y)$$



- Evolution involves all area with $> ||x||$
- Mixes $(T, \Delta T, T^{+}_{3F}, T^{-}_{3F})$ and flavors
- Two independent implementations
  `snowflake`:
  https://github.com/VladimirovAlexey/Snowflake
  `honeycomb`:
  https://github.com/slrodini/honeycomb2

How reliable is this determination?

- Many data sets have very large uncertainties $\Rightarrow$ no restrictions, but low χ^2
- If we set $T = \Delta T = 0$ we get $\boxed{\chi^2/N = 1.4}$ (243 pt.)

But T and ΔT are not positive definite,
and thus, $T = \Delta T = 0$ is correct for a portion of data.

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- So we observe that if we set $T = \Delta T = 0$
  - some experiments almost do not change χ^2.
  - many experiments get $\chi^2/N > 1.7$ (154 pt.)

    E155, HERMES (d_2, $A_{UT}^{\pi^+}$, $A_{UT}^{K^+}$), JLab (d_2, g_2), RQCD, COMPASS (F_{UT}^{h+}, F_{LT}^{h-})
- For these, experiments χ^2 reduces to $\boxed{\chi^2/N = 1.2}$
- Furthermore, all experiments have good $\chi^2/N < 1.1$ except HERMES K^+-channel ($A_{LT}^{K^+}$ [1.5] and $A_{UT}^{K^+}$ [2.3] both 11pt). Without these 22 pt. the reduction is more spectacular $1.55 \to 1.08$

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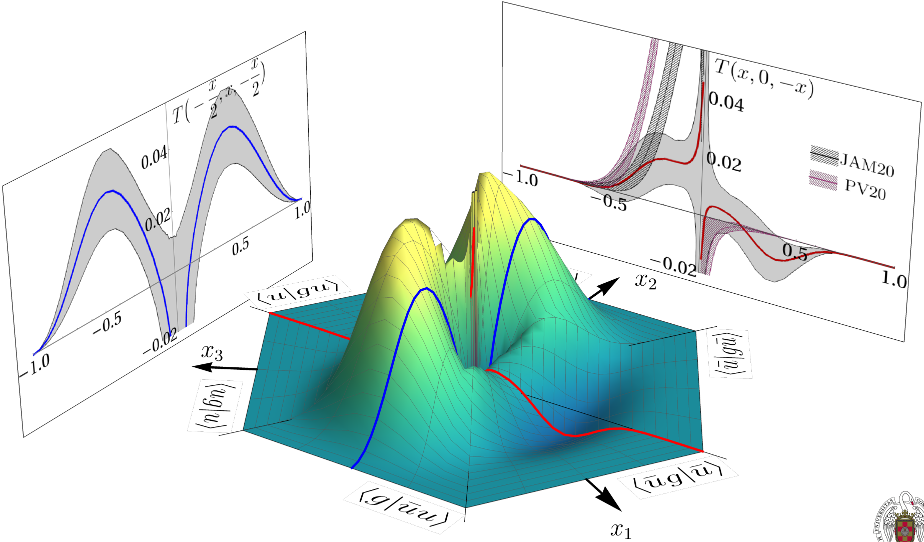
Based on this, we believe that
we do **observe the signal of tw3 at the declared level**
2-3σ's in some points.

2025-fit



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JAM20
PV20
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$\langle \bar{u}|g\bar{u}\rangle$
$\langle ug|u\rangle$
$\langle \bar{u}g|\bar{u}\rangle$
$\langle g|\bar{u}u\rangle$
x_1
x_2
x_3

2026-fit (30% more data, different parametrization)