Stony Brook University Center for Nuclear Theory FWF Austrian Science Fund

Probing the Eikonal Spin-Dependent Odderon

Challenges and Opportunities for the Gluon Sivers Function

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Based on 2603.09630

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DIS 2026
May 6, 2026

Outline

1. Introduction
   - The Theory Perspective
   - The Phenomenology Perspective
2. Pomeron and Odderon in pQCD
3. Light-Cone Constituent Quark Model
4. Numerical Results & Phenomenological Implications
5. Summary and Outlook

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From PDFs to TMDs

Collinear PDF

$$f_1^g(x) = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} f_1^g(x, \boldsymbol{k}_\perp)$$

- Probability density for gluon carrying longitudinal fraction x of P^+
- $k_\perp$ integrated out $\Rightarrow$ **1D nucleon picture**
- Probed in inclusive DIS

TMD PDF

$$f_1^g(x, \boldsymbol{k}_\perp) \overset{\text{F.T.}}{\longleftrightarrow} f_1^g(x, \boldsymbol{r}_\perp)$$

- Retains $k_\perp$ dependence $\Rightarrow$ **3D nucleon tomography**
- Encodes spin–orbit correlations: $\vec{S}_\perp \times \vec{k}_\perp$
- Probed in SIDIS, Drell–Yan

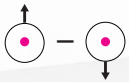
Gluon TMD Correlator (unsubtracted)

$$f_g^{ij}(x, \boldsymbol{k}_\perp) = \frac{2}{P^+} \int \frac{\mathrm{d}v^- \mathrm{d}^2 \boldsymbol{v}_\perp}{(2\pi)^3} e^{ixP^+v^-} e^{-i\boldsymbol{k}_\perp \boldsymbol{v}_\perp} \langle P\Lambda | F^{i+} W_{\sqsubset}^{[+]} F^{j+} W_{\sqsubset}^{[-]\dagger} | P\Lambda \rangle$$

- $W_{\sqsubset}$: staple Wilson line encodes process dependence (SIDIS $\neq$ DY), T-odd TMDs (Sivers), and f-type/d-type
- Tensor decomposition of $f_g^{\mu\nu}$ $\Rightarrow$ all leading-twist gluon TMDs

The TMD Landscape at Leading Twist

Leading Gluon TMDPDFs

Nucleon Polarization	Gluon Operator Polarization: Un-Polarized	Gluon Operator Polarization: Helicity 0 antisymmetric	Gluon Operator Polarization: Helicity 2
U	f_1^g = Unpolarized		$h_1^{\perp g}$ = Linearly Polarized
L		g_{1L}^g = Helicity	$h_{1L}^{\perp g}$ =
T	$f_{1T}^{\perp g}$ = 	$g_{1T}^{\perp g}$ =	h_{1T}^g = Transversity $h_{1T}^{\perp g}$ =

[Boussarie et al., 2023]

The Sivers Function

Physical Interpretation

$$\frac{\mathrm{d}N}{\mathrm{d}^2 k_\perp} \propto f_1(x, k_\perp^2) + \frac{\vec{S}_\perp \times \vec{k}_\perp}{M_p} f_{1T}^\perp(x, k_\perp^2)$$

- Azimuthal modulation $\sin(\phi_k - \phi_S)$ in the transverse plane.
  (E.g. in Single Spin Asymmetries (SSAs): $A_N = \frac{\mathrm{d}\sigma(\vec{S}_\perp) - \mathrm{d}\sigma(-\vec{S}_\perp)}{\mathrm{d}\sigma(\vec{S}_\perp) + \mathrm{d}\sigma(-\vec{S}_\perp)}$)
- **T-odd**: changes sign under naive time reversal
- Non-zero only due to **gauge links** (Wilson lines)

Key Properties

- Sign change (quark and f-type GSF): SIDIS $\leftrightarrow$ Drell-Yan (universality test)
- Connects to orbital angular momentum (OAM) of partons
- Requires **non-zero** $k_\perp$: genuinely 3D observable

Experimental Landscape: Status

Existing Data

- HERMES, COMPASS: quark Sivers
- PHENIX, BRAHMS, STAR: A_N in pp
- Gluon Sivers: **no clean extraction yet**
- Model estimates: [Zhou, 2014, D'Alesio et al., 2015, Yao et al., 2019]

Theoretical Uncertainty

- Gaussian ansatz in $k_\perp$
- No/loose connection to Odderon/small-x physics
- Not driven by QCD

Need for Theory Input

A theoretically motivated model valid at small x, rooted in QCD color charge correlators, is required as starting point for evolution and phenomenology.

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Dipole Scattering and Charge Conjugation

The Dipole S-matrix

[Mueller, 1994, Iancu and Venugopalan, 2003, Kovchegov and Levin, 2013]

$$S_{\Lambda'\Lambda}(r_\perp, b_\perp) = \frac{1}{N_c}\langle P'\Lambda'|\,\mathrm{tr}\,V(x_\perp)V^\dagger(y_\perp)|P\Lambda\rangle = 1 - P_{\Lambda'\Lambda}(r_\perp, b_\perp) - iO_{\Lambda'\Lambda}(r_\perp, b_\perp)$$

- $V(x_\perp) = \mathcal{P}\exp\left[-ig\int \mathrm{d}x^-\, A^{+a}(x^-, x_\perp)\, t^a\right]$, $r_\perp = x_\perp - y_\perp$, $b_\perp = \frac{x_\perp + y_\perp}{2}$
- $P_{\Lambda'\Lambda}\ldots$ Pomeron (C-even) $\qquad$ $O_{\Lambda'\Lambda}\ldots$ Odderon (C-Odd)

Spin Structure of the Odderon

[Boussarie et al., 2020]:

$$O_{\Lambda'\Lambda}(r_\perp, \Delta_\perp) = \underbrace{\delta_{\Lambda\Lambda'}\, i\cos(\phi_{r\Delta})\,\mathcal{O}}_{\text{spin-indep.}} + \underbrace{\delta_{\Lambda,-\Lambda'}\, i\Lambda e^{i\Lambda\phi_r}\,\mathcal{O}_S}_{\text{Sivers!}} + \underbrace{\delta_{\Lambda,-\Lambda'}\, i\Lambda e^{i\Lambda\phi_\Delta}\cos(\phi_{r\Delta})\,\mathcal{O}_S^\perp}_{\text{helicity-flip}}$$

- $\mathcal{O}_S(r_\perp, \Delta_\perp)$: **spin-dependent Odderon** $\Rightarrow$ Sivers function in forward limit

The Spin-Dependent Odderon and the GSF

Key Relation

$$x f_{1T}^{g\perp}(x, k_\perp) = \frac{N_c k_\perp M_p}{2\pi^3 g^2}\, e^{-i\phi_k}\, O_{-+}(k_\perp)$$

- The gluon Sivers function **is** the Fourier transform of the forward, helicity-flip Odderon amplitude: $O_{-+}(k_\perp) = \int \mathrm{d}^2 b_\perp\, \mathcal{O}_S(r_\perp, b_\perp)$
  [Boer et al., 2016, Zhou, 2014, Yao et al., 2019]

3-Gluon Exchange Amplitude

$$O_{\Lambda'\Lambda}(k_\perp) = -\frac{g^6 (N_c^2-4)(N_c^2-1)}{16 N_c^2} (2\pi)^2 \int_{q_\perp} \frac{G^3_{\Lambda'\Lambda}(k_\perp, q_\perp, -k_\perp - q_\perp)}{k_\perp^2\, q_\perp^2\, (k_\perp + q_\perp)^2}$$

where $G^3_{\Lambda'\Lambda}$ is the **C-odd cubic color charge correlator**:

$$d^{abc} \langle P'\Lambda' | \rho^a(q_1) \rho^b(q_2) \rho^c(q_3) | P\Lambda \rangle = N_c C_{3F}\; G^3_{\Lambda'\Lambda}(q_1, q_2, q_3)$$

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Light-Front Fock Space Expansion

Proton State at Moderate $x \sim 0.1$

$$|P,\Lambda\rangle = \int [dx_i]\,[d^2k_{i\perp}] \sum_{\{j_i,\lambda_i\}} \frac{\epsilon^{j_1 j_2 j_3}}{\sqrt{N_c!}}\, \Psi_\Lambda(x_i,k_{i\perp},\lambda_i)\, |\{x_i P^+, k_{i\perp},\lambda_i, j_i\}\rangle,$$

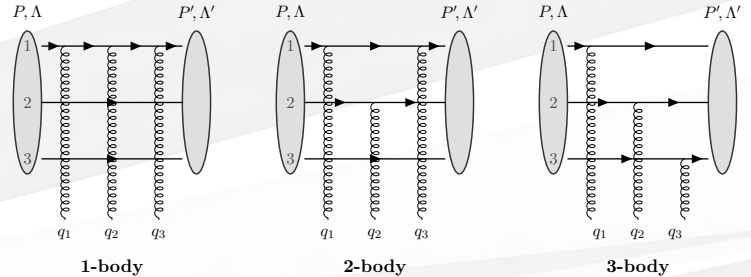
with product wave function $\Psi_\Lambda(x_i,k_{i\perp},\lambda_i) = \psi(x_i,k_{i\perp}) \times \Phi_\Lambda(k_{i\perp},\lambda_i),\ i=1,2,3$

Wave Function

- Gaussian $\psi_{\rm exp} \propto e^{-M^2/2\beta^2}$ or power-law $\psi_{\rm pwr} \propto (1+M^2/\beta^2)^{-p}$
- $M^2 = \sum_i (k_{i\perp}^2 + m^2)/x_i$
- Parameters: m, β fitted to EM form factors
- $x \sim 0.1$, $k_\perp \lesssim 1\ {\rm GeV}$, flavor blindness, no sea quarks, no gluons in Fock space
- Additional spin structure from Melosh rotation (Rest frame $\Rightarrow$ LF spinors)

Computing G^3_{-+} in the LC Model

$$G^3_{\Lambda'\Lambda} = \sum_{\lambda_i} \int_{x_i,\, k_{i\perp}} \Big[\Psi^*_{\Lambda'} \cdot (\text{3 gluon attachments}) \cdot \Psi_{\Lambda} \Big]$$



- WF overlap involves both S-wave⊗P-wave and P-wave⊗D-wave interferences
- G^3_{-+} is **odd** under simultaneous sign flip of all $q_{i\perp}$ (as required by C-parity)

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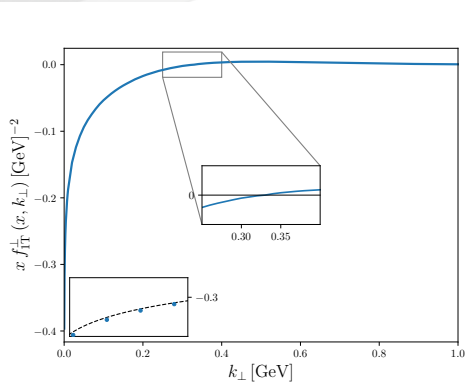
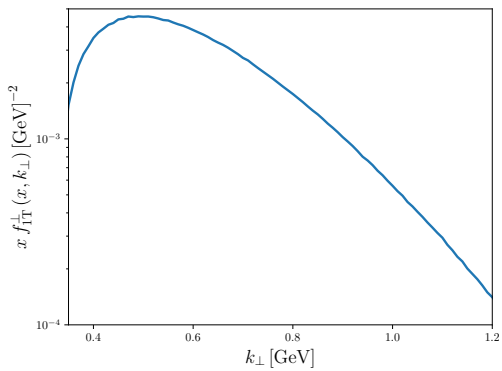
The Gluon Sivers Function $xf_{1T}^{g\perp}(x, k_\perp)$





Novel Features

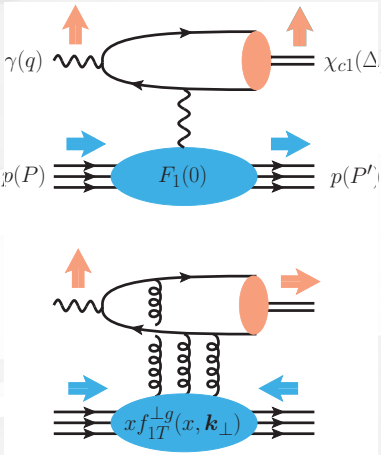
- **Log divergence** at low $k_\perp$: $xf_{1T}^{g\perp} \sim \log(k_\perp/Q_b)$; origin: $G^3_{-+}(k_\perp, q, -q) \sim k_L\, q^2$
- **Peak** at $k_\perp \simeq 0.5$ GeV: magnitude (4–5) $\times 10^{-3}$ GeV^{-2} ($\alpha_s = 0.25$)
- Perturbative tail: $\sim 1/k_\perp^4$ (three-gluon exchange)

χ_{c1} Photoproduction: Odderon vs. Primakoff

$$r(x,\mu^2) = \frac{4\pi^2}{q_c^2 N_c^2}\frac{\alpha_s^2}{\alpha^2}\frac{M_p^2}{M_\chi^2}\left|x f_{1T}^{g\perp(1)}(x,\mu^2)\right|^2$$
$$\approx 10^{-6} \text{ at } x \simeq 0.1$$

- r: ratio of Odderon to Primakoff X-section
- χ_{c1}: axial-vector meson; Landau-Yang theorem suppresses Coulomb singularity

[Benic et al., 2025]



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Summary

- Parametrized three-quark LC wave function of the proton (Schlumpf model + Melosh rotation) using up-to-date EM FF fit.
- Explicit computation of G^3_{-+}: only 3-different-quark topology contributes at $\Delta_\perp = 0$
- **Novel**: logarithmic divergence of $xf_{1T}^{g\perp}$ at low $k_\perp$

$$k_\perp xf_{1T}^{g\perp}(x, k_\perp) = \frac{24 M_p}{\pi} \alpha_s^2 \mathcal{N} \frac{k_\perp}{Q_b} \log\left(\frac{k_\perp}{Q_b}\right) e^{-k_\perp^2/4Q_c^2}$$

  $\mathcal{N} = 0.036\ \mathrm{GeV}^{-2}$, $Q_b = 0.331\ \mathrm{GeV}$, $Q_c = 0.220\ \mathrm{GeV}$.
- Sum rule $\int dk_\perp\, k_\perp\, xf_{1T}^{g\perp} = 0 \Rightarrow$ strong cancellations in all $k_\perp$-moments
- Tri-gluon PDF suppressed: $r \approx 10^{-6}$ in χ_{c1} photoproduction (Primakoff dominated)
- BFKL evolution: anomalous dimension $\gamma(Y) \approx 0.8$ at $Y = 1/\alpha_s \Rightarrow xf_{1T}^{g\perp} \sim k_\perp^{-3.3}$

Thank You

Probing the Eikonal Spin-Dependent Odderon:
Challenges and Opportunities for the Gluon Sivers Function

S. Benić, A. Dumitru, F. Hechenberger, T. Stebel

arXiv:2603.09630 [hep-ph]

Code and Data

Numerical code and results publicly available:
Zenodo: 10.5281/zenodo.18703045 **GitHub**: HechenMountain/nucleon-lccqm

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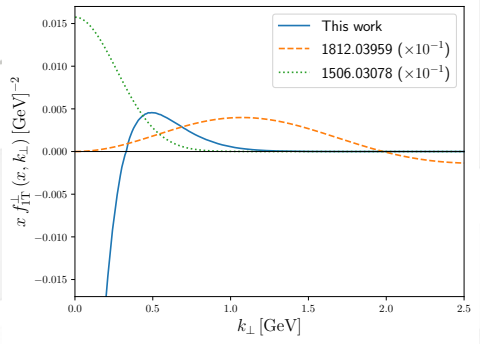
Backup Slides

Comparison to Existing Models



- [D'Alesio et al., 2015]: Gaussian ansatz, peaks at $k_\perp = 0$
- [Yao et al., 2019]: Saturation (ext. MV) model, peak at finite $k_\perp \sim Q_s$
- **This work**: LC quark model, peak at $k_\perp \simeq 0.5\ \mathrm{GeV}$

Key Differences

- Log divergence at small $k_\perp$ (unique to this work)
- Sources in fundamental rep. of color and spin
- Suitable as initial condition for BFKL evolution