DIS2026 - Bologna

The Impact of Drell-Yan Data on Four-Fermion SMEFT Operators

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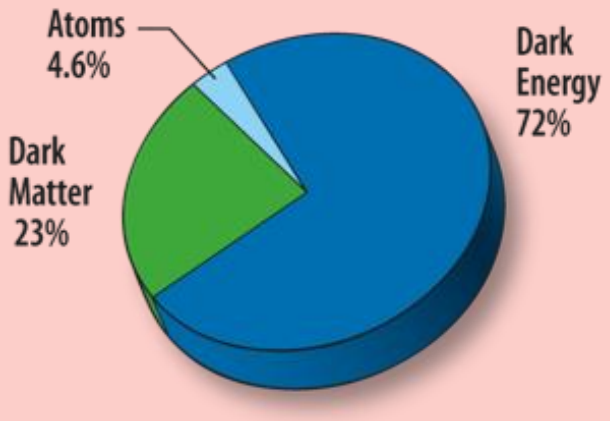
Standard Model Effective Field Theory (SMEFT)

- SM is the most successful theory in particle physics

- **However**, it cannot explain several observed phenomena
  - Matter-Antimatter asymmetry
  - Neutrino Oscillations
  - Gravity
  - Dark Matter
  - Dark Energy
  - …



Standard Model Effective Field Theory (SMEFT)

- Many SM extensions predict heavy new particles
- SMEFT extends the SM in a model-independent way

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

- Λ is the energy scale of new physics (here fixed at 1 TeV)
- C_i are the so-called ***Wilson coefficients***
- $O_i^{(5)}$ and $O_i^{(7)}$ are not considered because they violate either leptonic or baryonic number

Standard Model Effective Field Theory (SMEFT)

- We can search for SMEFT effects in precise measurements of SM processes
- Computing the matrix element squared:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\mathrm{SM}}|^2 + 2\,\mathrm{Re}\left(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}^{(6)}_{\mathrm{SMEFT}}\right) + |\mathcal{M}^{(6)}_{\mathrm{SMEFT}}|^2 + 2\,\mathrm{Re}\left(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}^{(8)}_{\mathrm{SMEFT}}\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Linear / Interference

$(c_i, \ldots)$

$(\sim 1/\Lambda^2)$

Squared / 'Pure SMEFT'

$(c_i^2, c_i c_j)$

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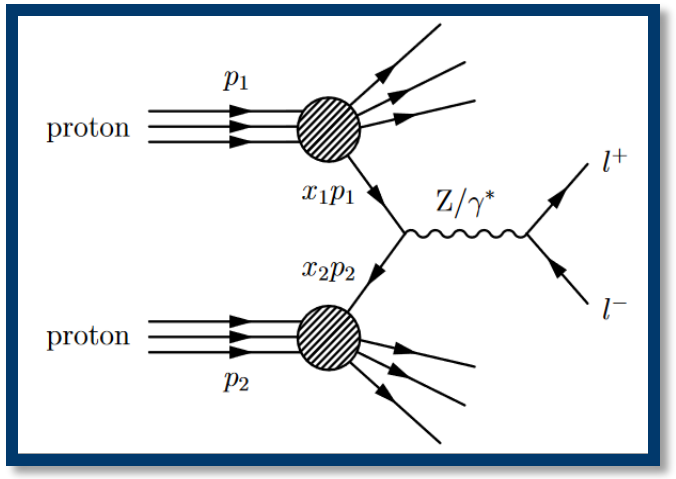
The Drell-Yan (DY) Process

- DY has a simple final state
- Many measurements of DY have been performed at colliders
- Only 10 relevant four-fermion operators
- Varying chirallity, quark flavour, Lorentz structures
- Many free parameters ➡ include multiple measurements



Operators $(LL)(LL)$

$$Q_{\ell q}^{(1)} = (\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$$

$$Q_{\ell q}^{(3)} = (\bar{\ell}_p \gamma_\mu \tau^I \ell_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$

Operators $(RR)(RR)$

$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$

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Operators $(LR)(LR)$ and $(LR)(RL)$

$$Q_{ledq} = (\bar{\ell}_p^j e_r)(\bar{d}_s q_t^j)$$

$$Q_{lequ}^{(1)} = (\bar{\ell}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$$

$$Q_{lequ}^{(3)} = (\bar{\ell}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$$

[+] *Dimension-Six Terms in the Standard Model Lagrangian - arXiv:1008.4884v3*

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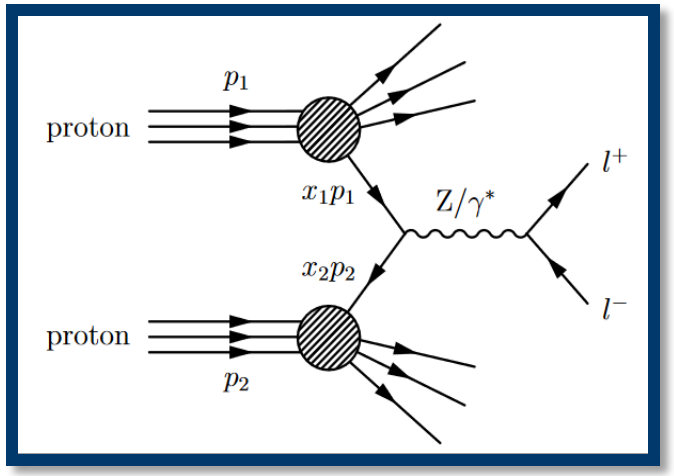

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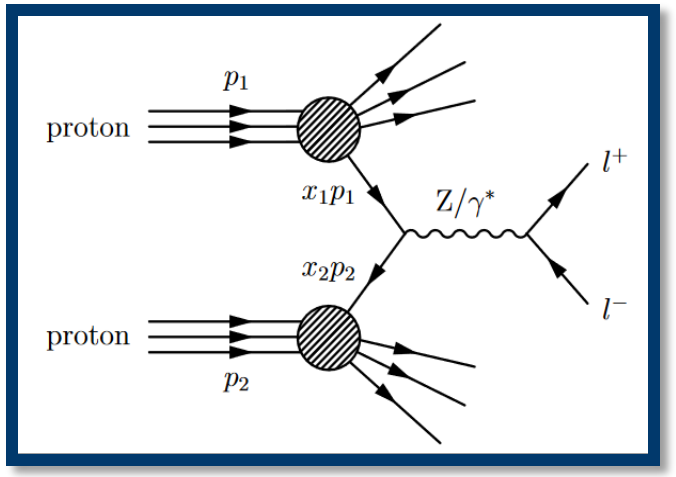

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- All four-fermion operators grow with mll
- Up-type operators and down-quark operators have different y dependency
- Right and left operators have different cosθ* dependency

Input Observables and Measurements

Exp	√s (TeV)	Type	Observable	Mass range (GeV)	L (fb^{-1})	PDFs	Ref.
ATLAS	8	NC	$d^2\sigma / (dm_{ll}\, dy)$	116–1500	20.3	CT10	1606.01736
ATLAS	8	NC	$d^3\sigma / (dm_{ll}\, dy\, d\cos\theta^*)$	46–200	20.2	CT10	1710.05167
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Considering only the electron channel and assuming flavour-universality

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Considering only the electron channel and assuming flavour-universality

Fitting strategy

- For the fit, we use the *signal strength μ*:
- And minimize the χ^2:

$$\mu \equiv \frac{\sigma_{\rm exp}}{\sigma_{\rm SM}}$$

$$\chi^2(\vec{C}) = \left(\vec{\mu}_{\rm data} - \vec{\mu}_{\rm th}(\vec{C})\right)^T \mathbf{V}^{-1} \left(\vec{\mu}_{\rm data} - \vec{\mu}_{\rm th}(\vec{C})\right)$$

- Where:
  - C is a vector of the Wilson coefficients
  - V is the full covariance matrix
  - μ_{th} is defined as:

$$\mu_{\rm th}(\vec{C}) = 1 + \sum_i C_i\, \mu_i^{\rm INT} + \sum_{i \le j} C_i C_j\, \mu_{ij}^{\rm SQR}$$

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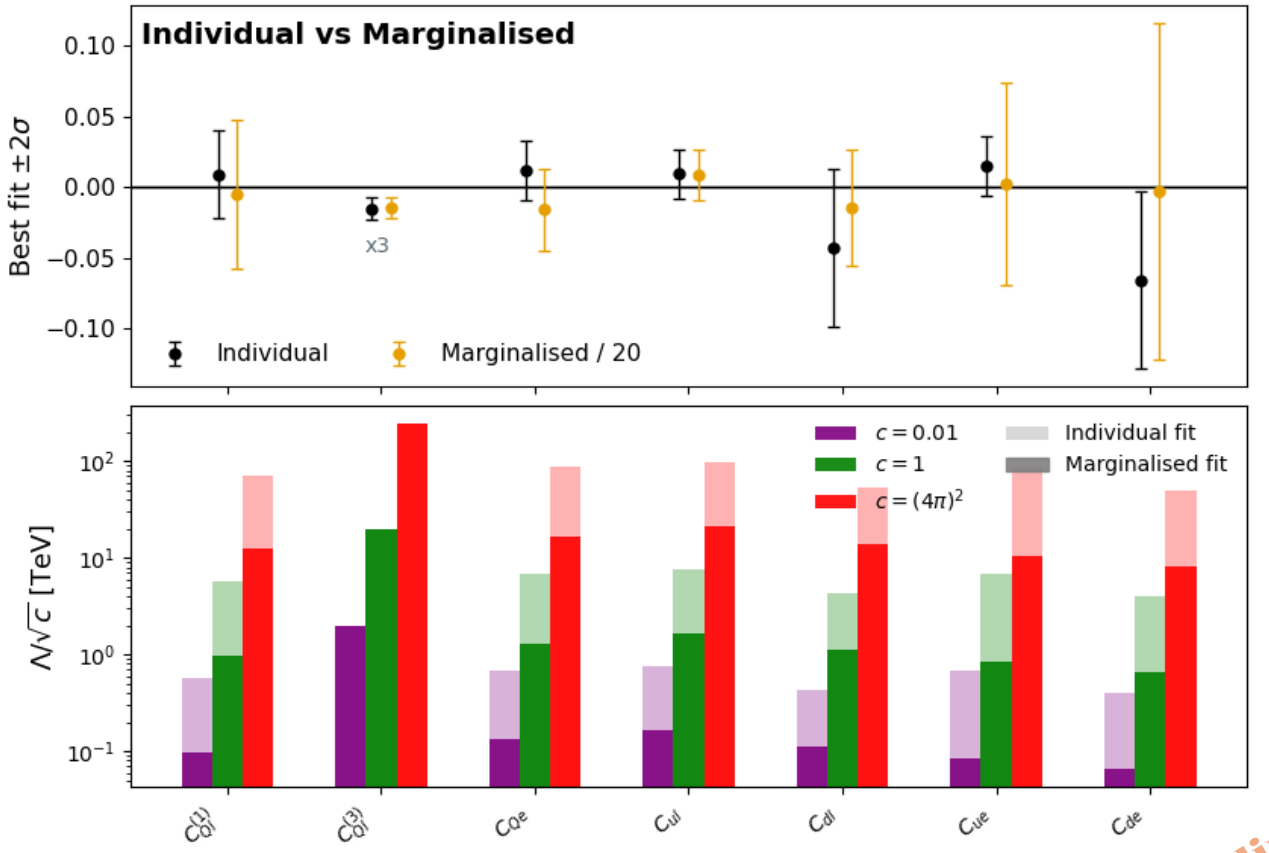
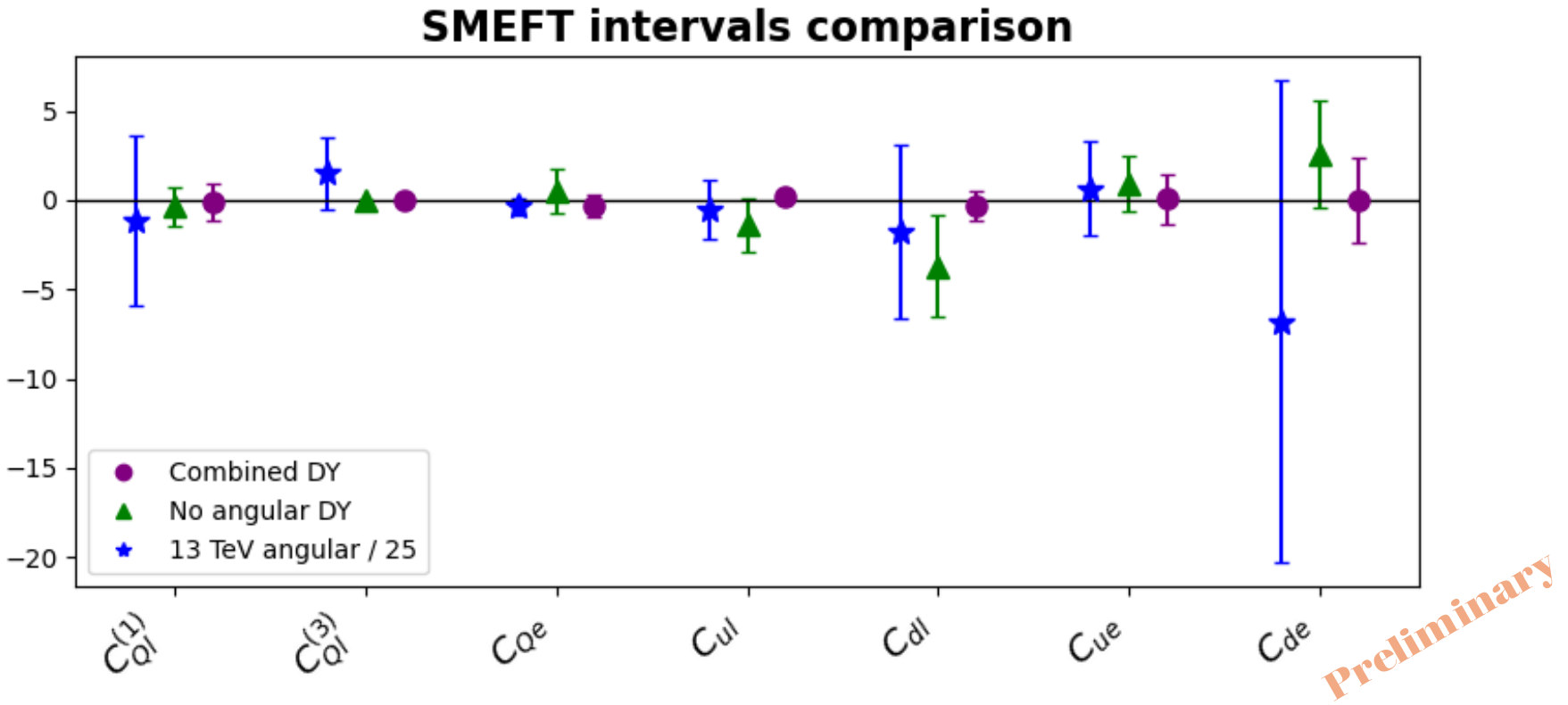
Results

- Complete **linear** fit:
  - Individual vs marginalised
- New physics scale:
  - Individual vs marginalised
  - Different values of the coupling



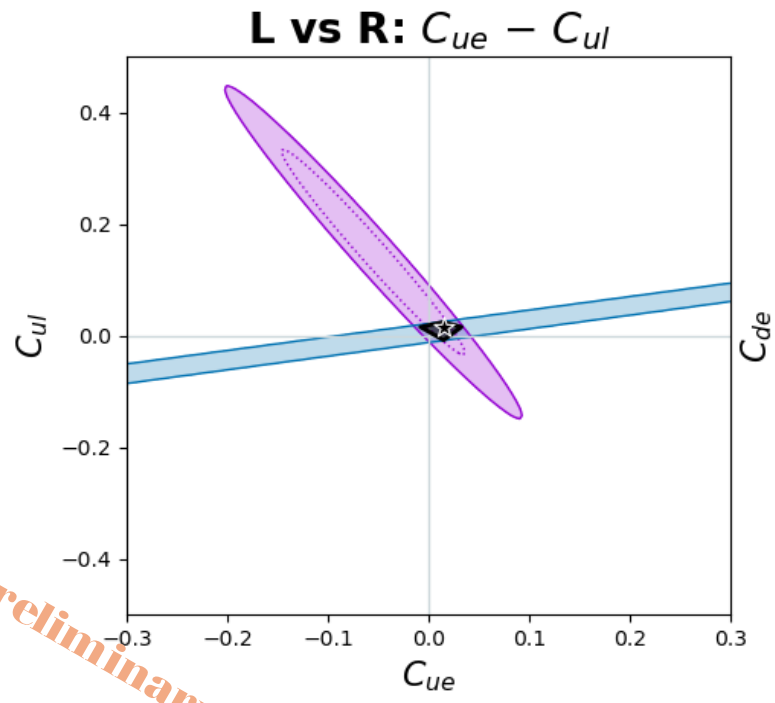
Preliminary

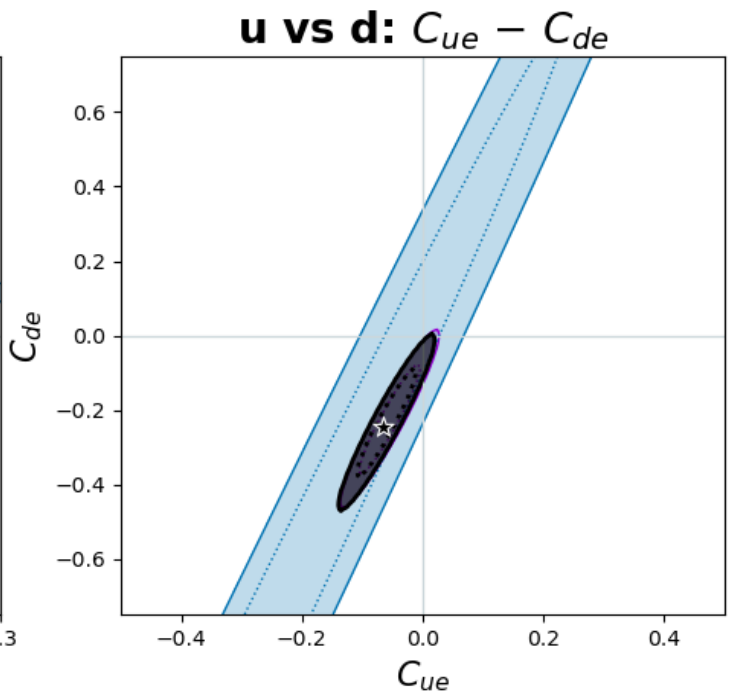


Results

- We can compare the fit results with and without the dataset 'angular 13 TeV'
- The 13 TeV angular data improves the marginalized fit substantially

Results





Preliminary

- Using differential observables helps to solve flat directions
- This improves constraints significantly
- 13 TeV angular is not differential in y_{ll}

Conclusion & Next Steps

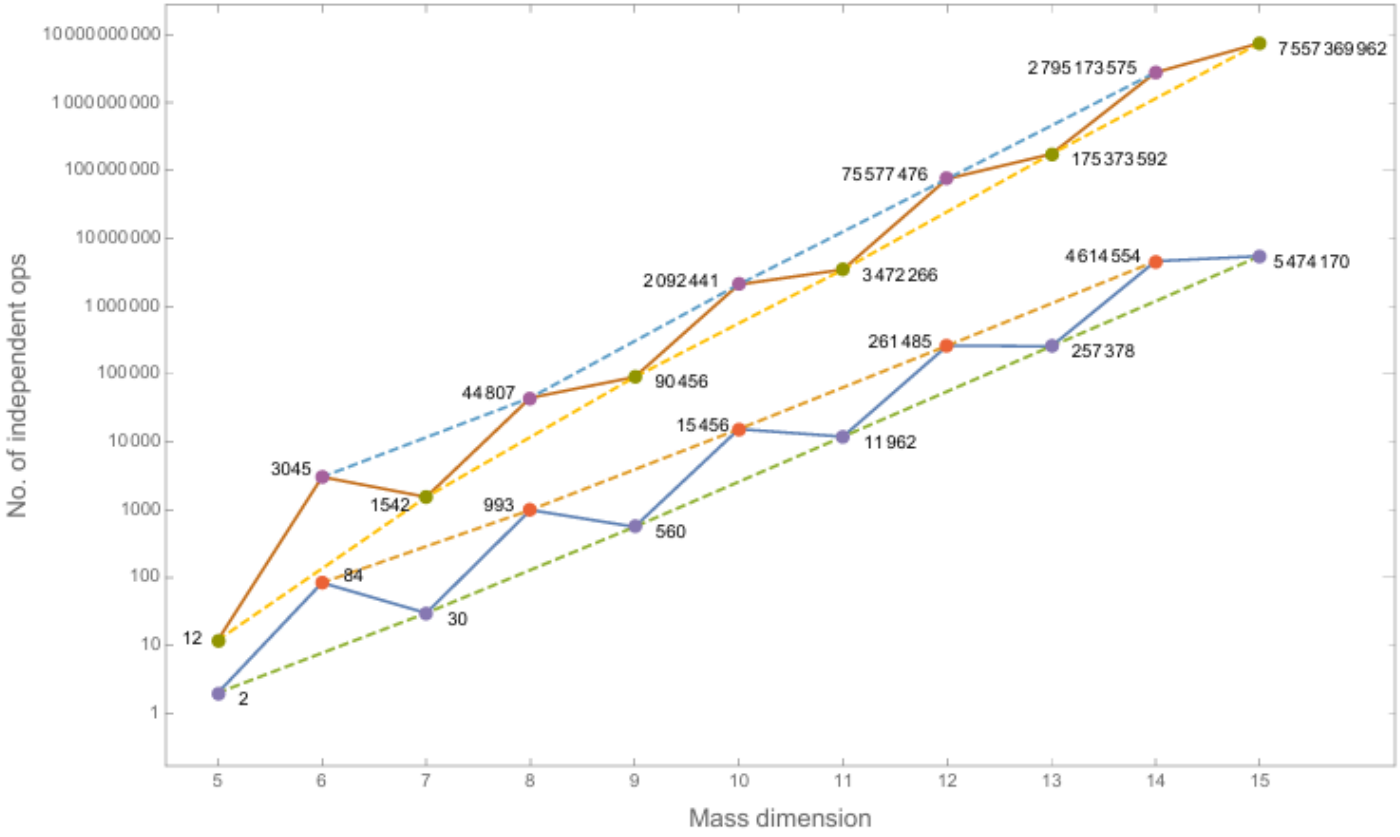
- I have presented results from a SMEFT fit to **8 TeV and 13 TeV Drell–Yan data**
- Obtained individual and marginalised constraints at **interference level**
- Highlighted the role of these observable in solving flat directions

Outlook

- Broaden the analysis including squared terms and more operators
- Quantify the impact of NLO SMEFT terms
- Address flat directions further by including:
  - Low-energy observables (data from e+e− colliders, neutrino scattering on electron or nucleon target)
  - More LHC measurements

Backup Slides

How many SMEFT operators?



Growth of the number of independent operators in the SM EFT up to mass dimension 15. Points joined by the lower solid line are for one fermion generation; those joined by the upper solid line are for three generations. Dashed lines are to guide the eye to the growth of the even and odd mass dimension operators in both cases.

Ref: arXiv:1512.03433v2

Helicity Amplitudes of DY Subprocesses

$\bar{u}u \to e^+e^-$

$$\{\bar{R}L\bar{L}L\}, \{\bar{R}L\bar{R}R\}, \{\bar{L}L\bar{L}R\}, \{\bar{R}R\bar{R}L\}, \quad 0$$
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$$\{\bar{R}L\bar{L}R\} \quad -\frac{4ic_{ulS}^{1111}\, s\, \delta_{Col1,Col2}}{\Lambda^2}$$

$$\{\bar{L}L\bar{L}L\} \quad \frac{is(c_{Ql1x}^{1111}-c_{Ql3x}^{1111})(1+\cos\theta)\,\delta_{Col1,Col2}}{\Lambda^2} + \frac{ie^2(1+\cos\theta)\,\delta_{Col1,Col2}(3c_w^4 s+4c_w^2 s_w^2(s-2M_Z^2)+ss_w^4)}{12c_w^2 s_w^2(M_Z^2-s)}$$

$$\{\bar{L}L\bar{R}R\} \quad \frac{ie^2(1-\cos\theta)\,\delta_{Col1,Col2}(c_w^2(4M_Z^2-s)-ss_w^2)}{6c_w^2(M_Z^2-s)} - \frac{2ic_{Qe}^{1111}s(1-\cos\theta)\,\delta_{Col1,Col2}}{\Lambda^2}$$

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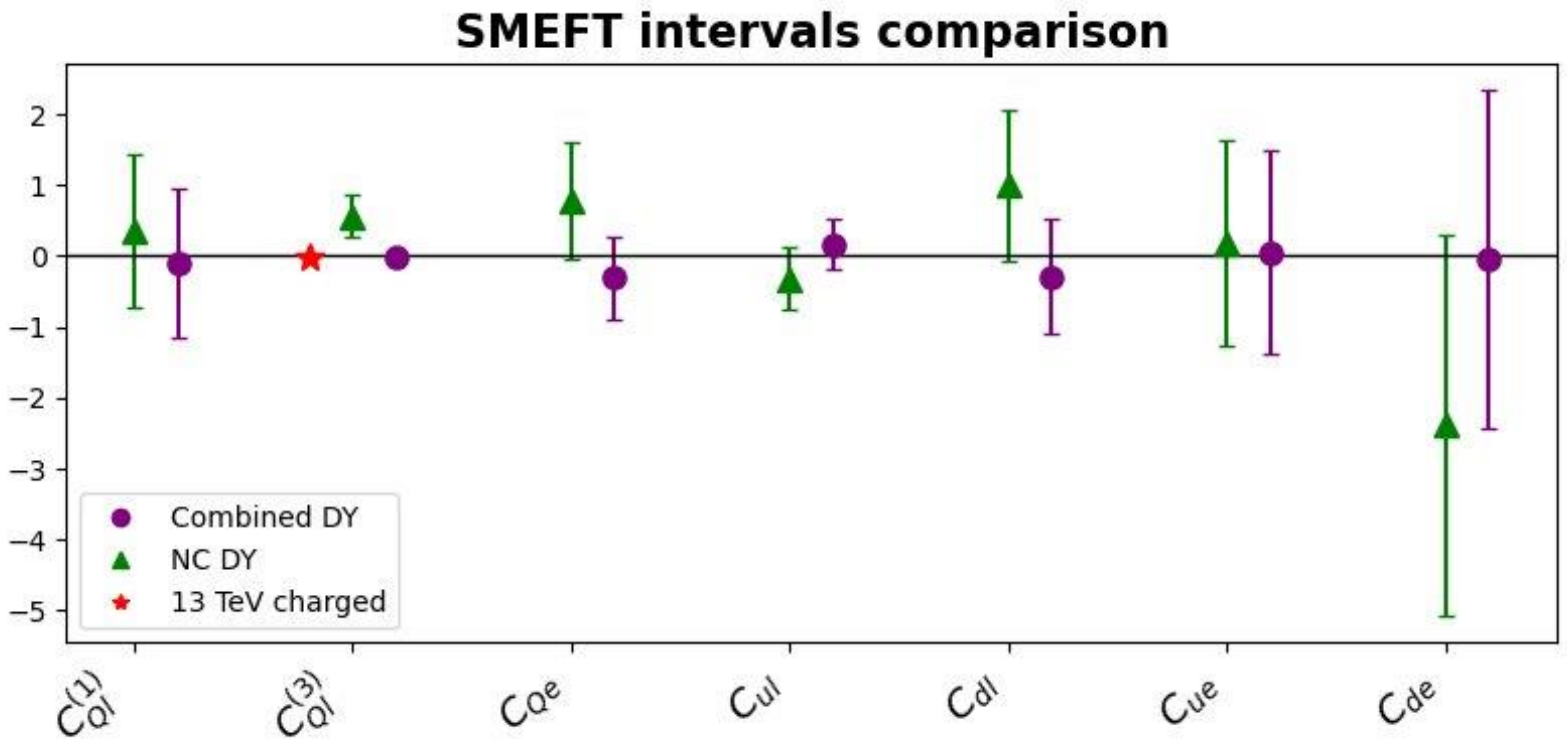
$$\{\bar{R}R\bar{R}R\} \quad \frac{2}{3}ie^2\left(\frac{1+\cos\theta}{2}\right)\delta_{Col1,Col2}\left(\frac{ss_w^2}{c_w^2(M_Z^2-s)}-1\right) - \frac{2ic_{de}^{1111}s\left(\frac{1+\cos\theta}{2}\right)\delta_{Col1,Col2}}{\Lambda^2}$$

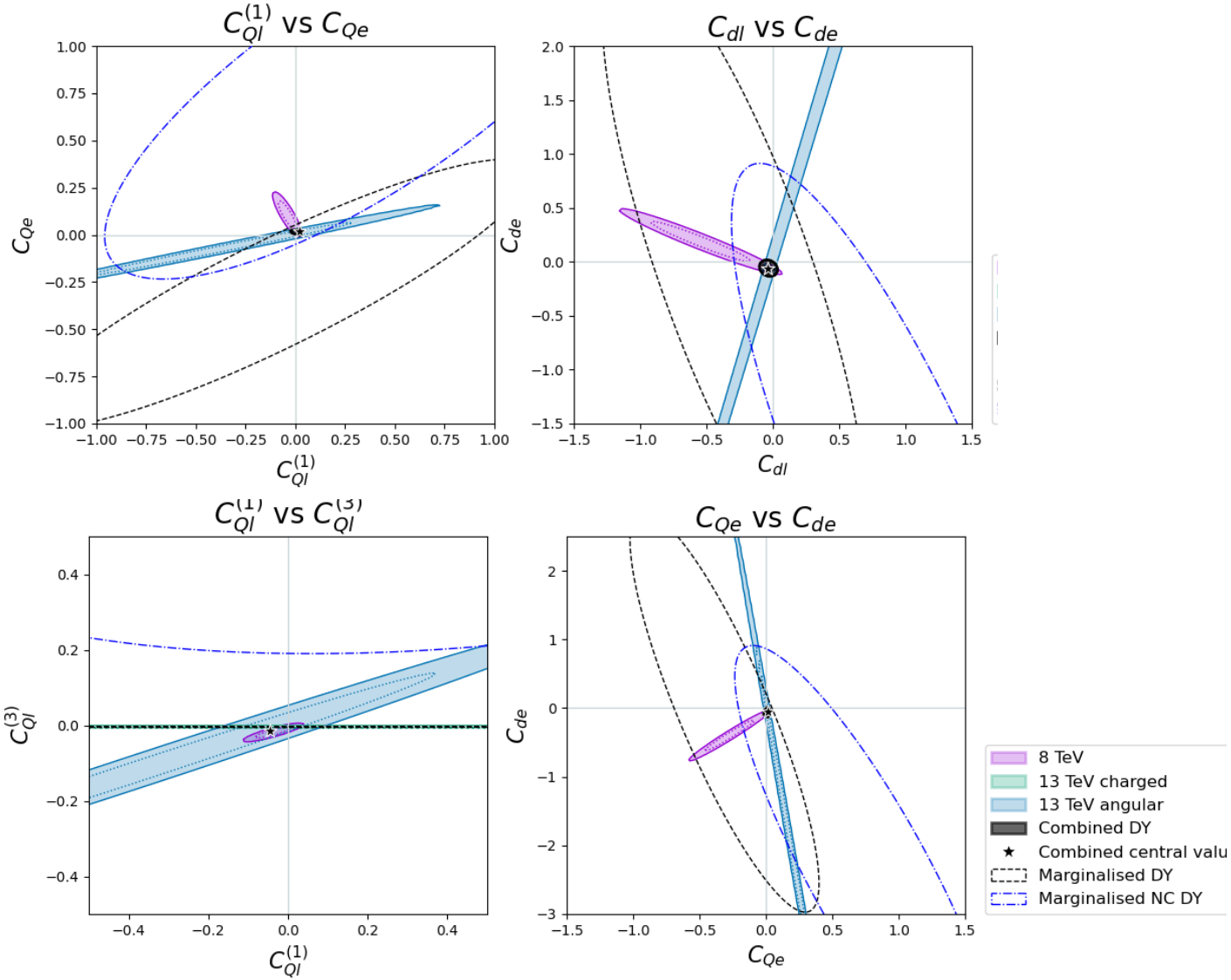
$$\{\bar{L}R\bar{L}R\} \quad \frac{4ic_{dls}^{1111}\, s\, \delta_{Col1,Col2}}{\Lambda^2}$$

These are the subprocesses helicity amplitudes at order ~$1/\Lambda^2$
(SM is marked in *black*, interference terms are in *red*)

Impact of 13 TeV charged on the marginalized fit



Results



- Using different observables helps to solve flat directions
- This improves constraints significantly

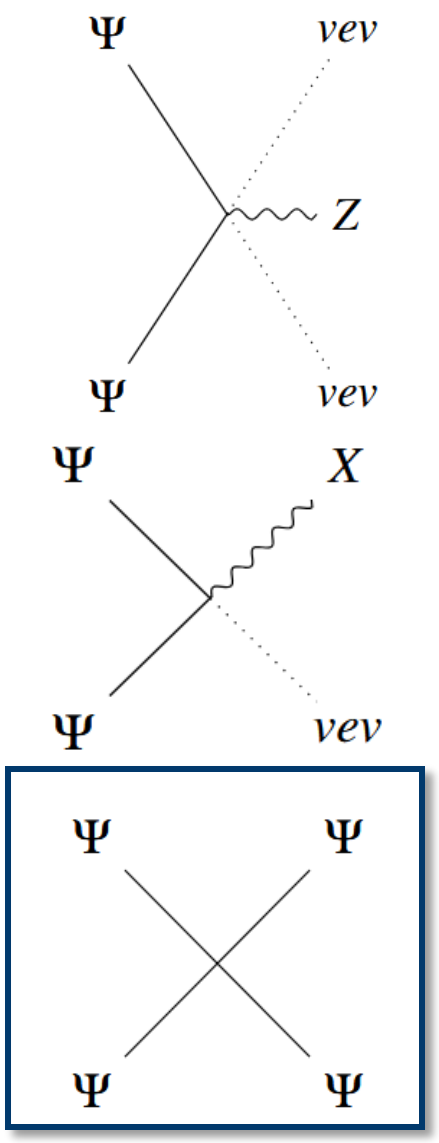
Dim-6 SMEFT Operators in DY

There are three classes of dim-6 operators for DY:

- $\Psi^2\phi^2D$: these operators include a single derivative, a Higgs doublet and two fermions
  - They shift the SM coupling of the fermions to gauge bosons
  - *They have already been constrained at LEP*

- $\Psi^2X\phi$: two fermions coupled to a gauge boson (X = $B_{\mu\nu}$, $G_{\mu\nu}$, $W_{\mu\nu}$) and a Higgs.
  - *They don't interfere with the SM*

- **Ψ_4: four-fermion operators**
  - **10 different operators (7 interfere with the SM)**
  - **They grow with E^2/Λ^2**



Lam-Tung Relation

$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1\sin(2\theta)\cos\phi + \frac{A_2}{2}\sin^2\theta\cos(2\phi)$$
$$+ A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big]$$

- At LO, for the Drell-Yan process:

$A_0 = A_2 = 0$

- The only non-zero SMEFT contributions are:

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Analytical Fit

$$\mu(c) = \mu_{\mathrm{SM}} + H \cdot c$$

$$\chi^2(c) = \chi^2_{\mathrm{SM}} - 2\,c^\top \cdot \omega + c^\top \cdot F \cdot c$$

$$\omega = H^\top \cdot V^{-1} \cdot (y - \mu_{\mathrm{SM}}) \qquad F = H^\top \cdot V^{-1} \cdot H$$

$$\nabla_c \chi^2(c) = -2\,\omega + 2\,F \cdot c = 0$$

$$\Rightarrow \quad \hat{c} = F^{-1} \cdot \omega = \left(H^\top \cdot V^{-1} \cdot H\right)^{-1} \cdot H^\top \cdot V^{-1} \cdot (y - \mu_{\mathrm{SM}})$$

Analytical Fit

$$\mu(c) = \mu_{\mathrm{SM}} + H \cdot c$$

$$\chi^2(c) = \chi^2_{\mathrm{min}} + (c - \hat{c})^\top \cdot \hat{w} + (c - \hat{c})^\top \cdot F \cdot (c - \hat{c})$$

$$\hat{w} = \nabla_c \chi^2(c)\big|_{c=\hat{c}} = 0$$

$$F = \frac{1}{2}(\nabla_c \otimes \nabla_c)\, \chi^2(c) = H^\top \cdot \tilde{V} \cdot H$$

$$\chi^2_{\mathrm{min}} = \chi^2(\hat{c}) = \chi^2_{\mathrm{SM}} - 2\,\hat{c}^\top \cdot \omega + \hat{c}^\top \cdot F \cdot \hat{c} = \chi^2_{\mathrm{SM}} - \hat{c}^\top \cdot F \cdot \hat{c}$$

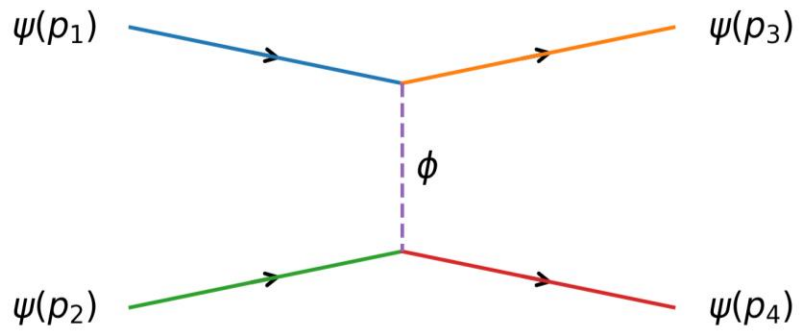
$$F^{-1} \equiv U$$

A Toy Effective Field Theory[+]

Let's consider this toy model:

$$S = \int d^4x \left\{ -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}M^2\phi^2 + \bar{\Psi}\,(i\gamma^\mu \partial_\mu - m)\Psi + iy\,\phi\,\bar{\Psi}\gamma^5\Psi - \frac{1}{4!}\,\lambda\phi^4 \right\}$$

- Ψ is a fermionic field with small mass
- Φ is a scalar field with high mass m << M
- *y*, λ << 1 so that we can work in perturbation theory



The matrix element for an elastic scattering of two fermions

$$i\mathcal{M} = (-y)^2 \left\{ \bar{u}_{\lambda_3,p_3}\gamma^5\, u_{\lambda_1,p_1}\, \frac{-i}{(p_3-p_1)^2 + M^2}\, \bar{u}_{\lambda_4,p_4}\gamma^5\, u_{\lambda_2,p_2} - (3 \leftrightarrow 4) \right\}$$

[+]*An Introduction to Effective Field Theories - arXiv:2006.16285v1*

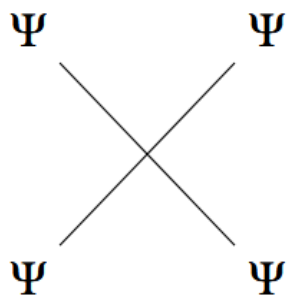
A Toy Effective Field Theory

The scalar propagator can be expanded in powers of momenta (considering $m^2 \ll p^2 \ll M^2$).
The first term is:

$$i\mathcal{M} \simeq -\frac{i\,y^2}{M^2}\left[\bar{u}_{\lambda_3,p_3}\gamma^5 u_{\lambda_1,p_1}\;\bar{u}_{\lambda_4,p_4}\gamma^5 u_{\lambda_2,p_2} - (3 \leftrightarrow 4)\right] \;+\; \mathcal{O}\left(\frac{E^2}{M^2}\right)$$

We would have obtained the same result with the effective theory:

$$S_{\text{eff}} = \int d^4x \left\{\bar{\Psi}(i\;\partial\!\!\!/ - m)\Psi \;-\; \frac{y^2}{2M^2}\left(\bar{\Psi}\gamma^5\Psi\right)^2\right\}$$



- At low energy ($E \ll M$) the two theories are *indistinguishable*
- The effective theory does **not** postulate a scalar boson