Multi-scale improved predictions for pp->ttW$^+$(j)

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in collaboration with
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Outline

- Description of the MiNLO method
- Differences between MiNLO and standard NLO
- Comparison between MiNLO and standard NLO for the fully off-shell pp -> ttW^+ + jets processes at the integrated and differential cross section level
- Merged predictions and comparison with the fully off-shell NLO pp -> ttW^+ sample

The MiNLO method in a nutshell

- The Multi-scale improved NLO (MiNLO) method was first proposed in 2012 by *Hamilton, Nason, Zanderighi '12*
- Scales are determined based on the *most likely branching history* via the CKKW procedure (*Catani, Krauss, Webber, Kuhn '02*) based on an inverse k_T-clustering algorithm → *nodal scales*
- Inclusion of *Sudakov form factors* of the form $\Delta_f(q_1^2, q_2^2)$ to resum large logarithms
- The nodal scales are used as arguments for the a_s couplings and as inputs for the Sudakov form factors
- Proper subtraction at NLO to ensure the NLO accuracy of the calculation
- Implemented in HELAC-NLO software package *G. Bevilacqua, M. Czakon, M.V. Garzelli, A. van Hameren, A. Kardos, C.G. Papadopoulos, R. Pittau, M. Worek '13*

The MiNLO method at LO - MiLO

- The following weight is applied in the Matrix element:

$$[\alpha_s(q_{core})]^m \times \prod_{i=1}^{n} \alpha_s(q_i) \times \prod_i \Delta_{f_i}^{\text{ext}}(q_{res}^2; q_i^2) \times \prod_{ij} \Delta_{f_{ij}}^{\text{int}}(q_{res}^2; q_i^2, q_j^2)$$

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user-defined scale assigned to the primary system

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Sudakov form factors to all external and internal lines

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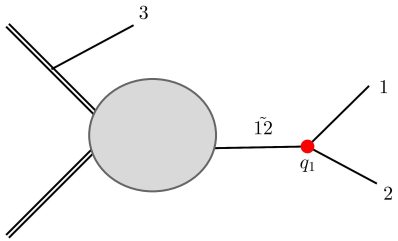
Example:



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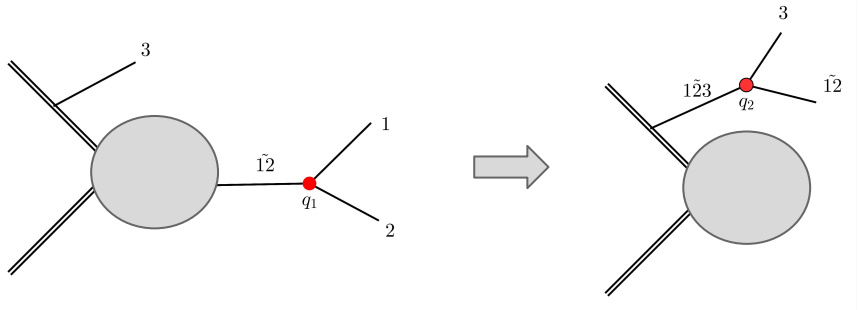

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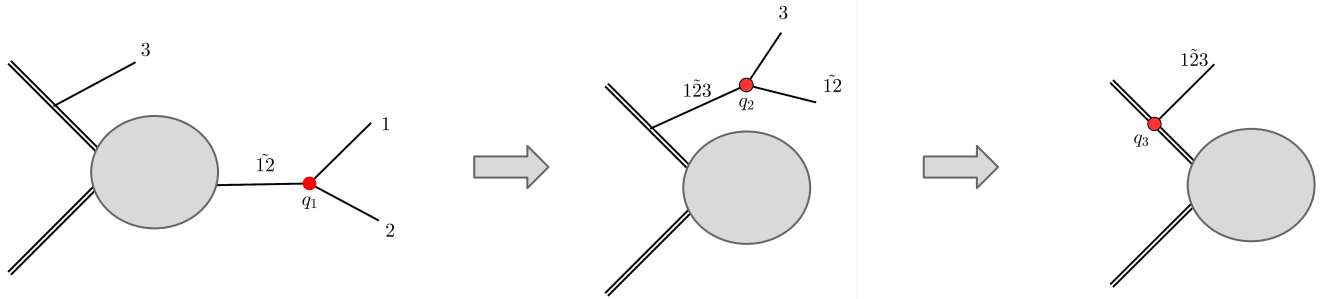

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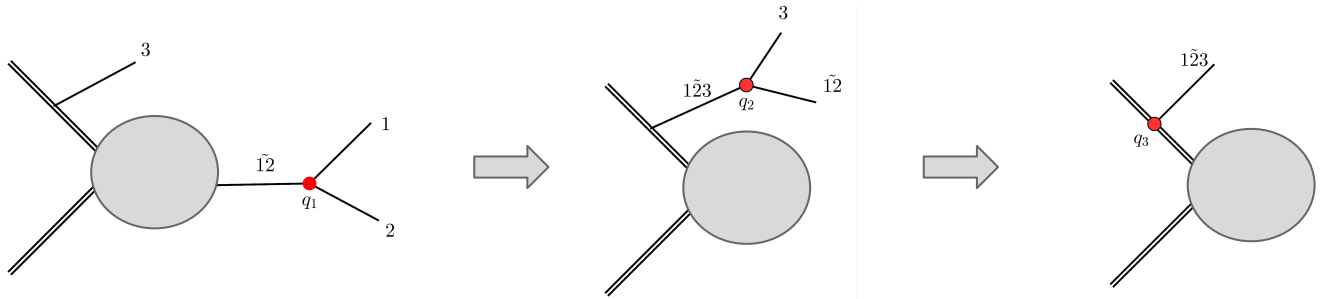

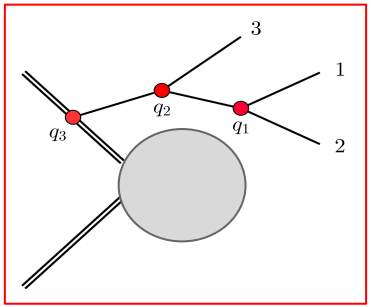

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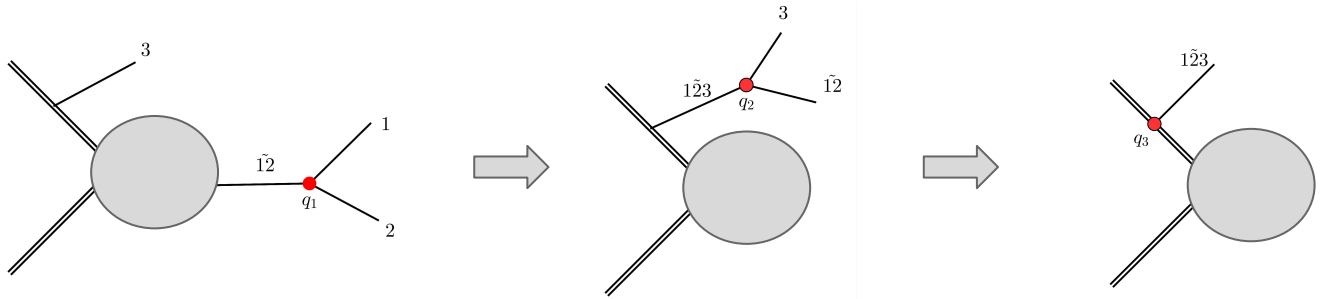
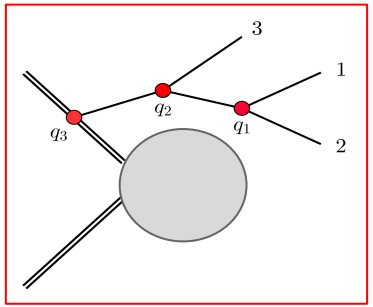




- We require *ordered clusterings*:

$$q_1 < \ldots < q_n \leq q_{core}$$

The MiNLO method at NLO

$$\sigma^{\rm NLO} = \int d\Phi_B \left[\mathcal{B}(\Phi_B;\mu_F,\mu_R) + \mathcal{V}(\Phi_B;\mu_F,\mu_R) + \mathcal{I}^{(S)}(\Phi_B;\mu_F,\mu_R)\right]$$
$$+ \int d\Phi_R \left[\mathcal{R}(\Phi_R;\mu_F,\mu_R) - \mathcal{S}(\Phi_R;\mu_F,\mu_R)\right].$$

- Terms that live in the Born phase space are treated in the same manner as at LO
- The smallest nodal scale q_0 in $\mathcal{R}$ is excluded from the calculation to ensure proper cancellation of the IR poles
- The following replacement is applied to the Born contribution to avoid double counting of the $\mathcal{O}(\alpha_s)$ corrections that are included in the Virtual part

$$\mathcal{B} \to \mathcal{B}\times\left(1 - \sum_i \left[\Delta^{(1)}_{f_i}(q^2_{res},q^2_i) - \Delta^{(1)}_{f_i}(q^2_{res},q^2_1)\right] - \sum_{ij}\left[\Delta^{(1)}_{f_{ij}}(q^2_{res},q^2_i) - \Delta^{(1)}_{f_{ij}}(q^2_{res},q^2_j)\right]\right)$$

Scale variation in the MiNLO method

- There are various ways for performing scale variation

Scale Variation	
$q_i \to \xi_R q_i, \quad q_{core} \to \xi_R q_{core}$ $\mu_R = \left((\xi_R q_{core})^m \times \prod_{i=1}^{n} (\xi_R q_i) \right)^{\frac{1}{m+n}}$	$\alpha_s(q_{eff}) = \left([\alpha_s(q_{core})]^m \times \prod_{i=1}^{n} \alpha_s(q_i) \right)^{\frac{1}{m+n}}$ $\mu_R = \xi_R q_{eff}$
$\alpha_s^{(n+m+1)} = \frac{1}{n+m} \left(\sum_{i=1}^{n} \alpha_s(\xi_R q_i) + m \alpha_s(\xi_R q_{core}) \right)$	$\alpha_s^{(n+m+1)} = \alpha_s(\mu_R)$
$\mu_F = \xi_F q_1$	$\mu_F = \xi_F q_1$
Hamilton, Nason, Zanderighi '12	Höche, Maierhoefer, Moretti, Pozzorini, Siegert '17

- The scales that enter the Sudakov form factors for the outgoing lines are not varied
- For incoming lines we make the following replacement

$$\Delta(q_1, q_i) \to \Delta(\xi_F q_1, q_i)$$

Fully off-shell ttbarW^{+} + jets

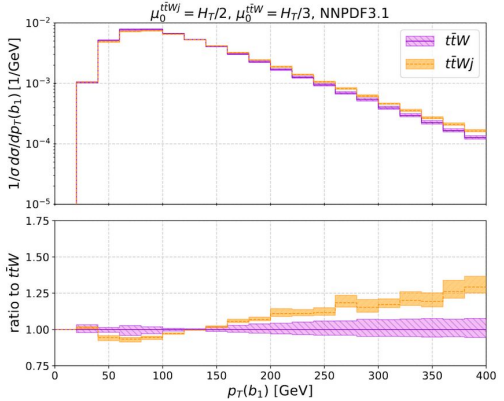
in the multi-lepton channel

Motivation

- Full off-shell predictions for the pp -> ttW $^+$ and pp -> ttW $^+$j processes at NLO in QCD for the fully leptonic channel are available in the literature
  Bevilacqua, Bi, Hartanto, Kraus, Worek '20 , Denner, Pelliccioli '20 and Bi, Kraus, Reinartz, Worek '23

- NLO QCD corrections with an extra jet are crucial for accurate modeling of pp -> ttW $^+$

	$\sigma^{t\bar{t}W^+}_{H_T/3}$ [ab]	$\sigma^{t\bar{t}W^+j}_{H_T/2}$ [ab]
LO	$216.6\,^{+24\%}_{-18\%}$	$115.8\,^{+38\%}_{-26\%}$
NLO	$254.6\,^{+2.8\%}_{-5.9\%}$	$142.3\,^{+1.4\%}_{-8.1\%}$



- Main goal: Assess scale sensitivity by determining the scale for the strong coupling of the extra jets using the MiNLO method

- To improve full off-shell predictions, merging ttW $^+$ and ttW $^+$j samples is necessary

PRELIMINARY

Integrated comparison MiNLO vs NLO

$$H_T = p_T(e^+) + p_T(\mu^-) + p_T(\tau^+) + p_T^{miss} + p_T(b_1) + p_T(b_2) + \sum_j p_T(j)$$

$$pp \to t\bar{t}W^+j$$

μ_0	σ^{NLO} [ab]	σ^{MiNLO} [ab]	$\frac{\sigma^{\text{MiNLO}}}{\sigma^{\text{NLO}}}$
$m_t + m_W/2$	$144.7^{+0.3\%}_{-14.0\%}$	$140.9^{+1.6\%}_{-11.2\%}$	0.97
$E_T/2$	$140.1^{+3.8\%}_{-10.0\%}$	$136.5^{+6.1\%}_{-9.6\%}$	0.97
$E_T/3$	$144.9^{+1.3\%}_{-7.0\%}$	$141.7^{+1.4\%}_{-5.9\%}$	0.98
$H_T/2$	$142.6^{+1.5\%}_{-8.2\%}$	$139.5^{+2.0\%}_{-6.8\%}$	0.98
$H_T/3$	$144.3^{+1.0\%}_{-12.5\%}$	$138.4^{-3.5\%}_{-17.5\%}$	0.96

$$E_T = \sqrt{m_t^2 + p_T^2(t)} + \sqrt{m_t^2 + p_T^2(\bar{t})} + \sqrt{m_W^2 + p_T^2(W)} + \sum_j p_T(j)$$

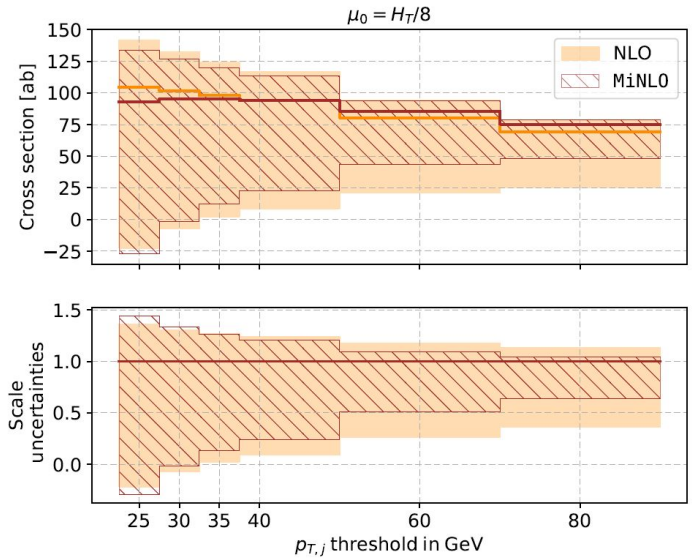
$$pp \to t\bar{t}W^+jj$$

μ_0	σ^{LO} [ab]	σ^{MiLO} [ab]	$\frac{\sigma^{\text{MiLO}}}{\sigma^{\text{LO}}}$
$m_t + m_W/2$	$80.3^{+58.8\%}_{-34.6\%}$	$72.0^{+48.8\%}_{-30.3\%}$	0.90
$E_T/2$	$47.7^{+50.9\%}_{-31.9\%}$	$41.3^{+46.5\%}_{-29.3\%}$	0.87
$E_T/3$	$60.4^{+54.3\%}_{-32.9\%}$	$57.9^{+48.2\%}_{-29.9\%}$	0.96
$H_T/2$	$53.7^{+52.7\%}_{-32.4\%}$	$50.8^{+47.6\%}_{-29.7\%}$	0.95
$H_T/3$	$68.5^{+55.9\%}_{-33.6\%}$	$69.2^{+49.3\%}_{-30.5\%}$	1.01

- MiNLO and standard NLO are compatible within uncertainties
- MiNLO exhibits an overall reduced scale sensitivity with smaller theoretical errors
- Differences in scale uncertainties increase with jet multiplicity

PRELIMINARY

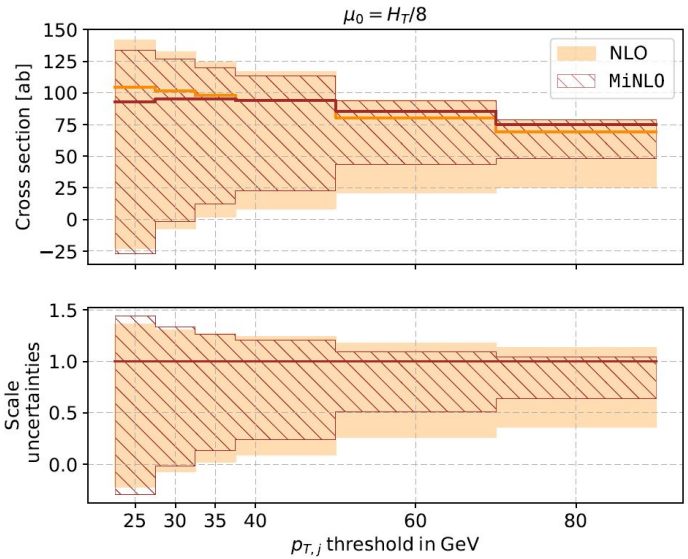
Subomtimal scales and various $p_{T,j}$ thresholds for ttW$^+$j

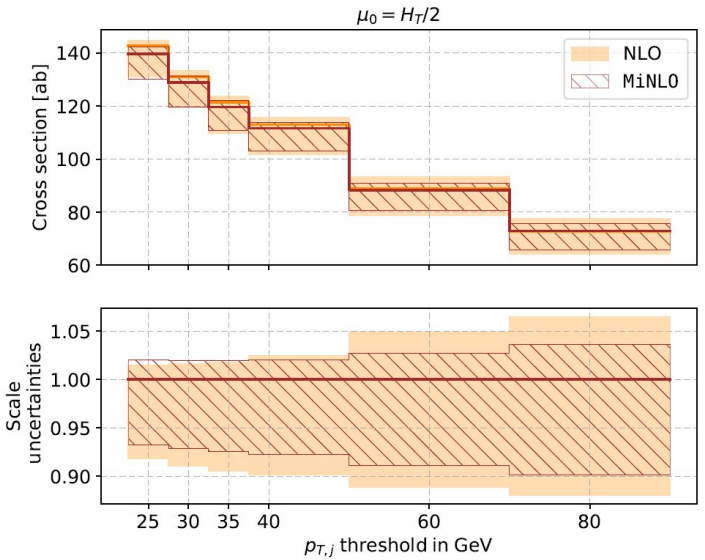


- For poorly-chosen scale choices:
  1. MiNLO improves scale uncertainties for moderate-high $p_{T,j}$
  2. reduction increases with the $p_{T,j}$ threshold
  3. nearly half in size compared to NLO at $p_{T,j}$ = 80 GeV

PRELIMINARY

Subomtimal scales and various $p_{T,j}$ thresholds for ttW$^+$j





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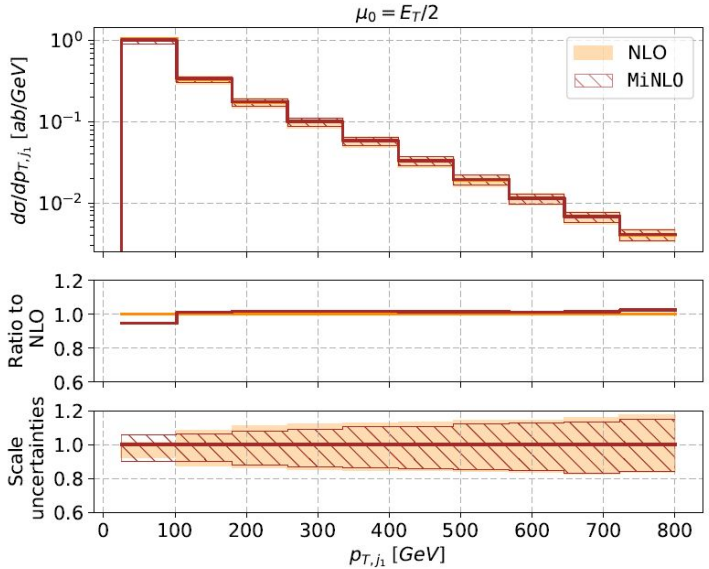
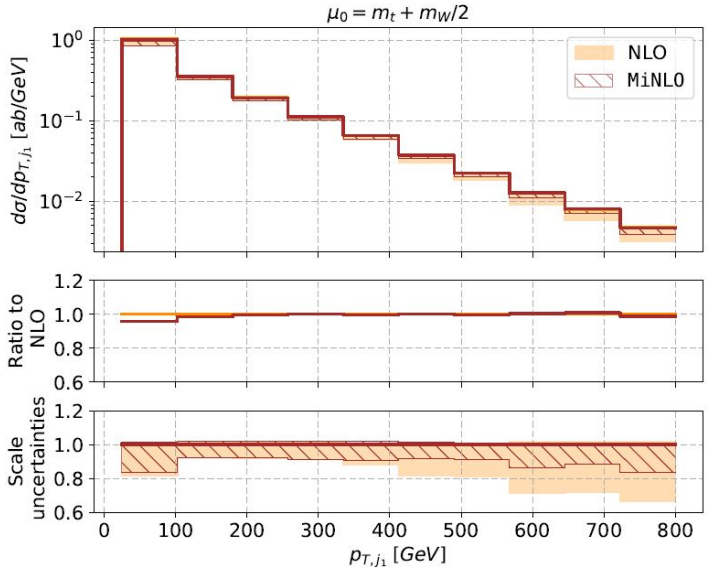
  3. nearly half in size compared to NLO at $p_{T,j}$ = 80 GeV
- This behaviour persists for well-behaved scale choices but with smaller differences

PRELIMINARY

Differential comparison MiNLO vs NLO for ttW⁺j



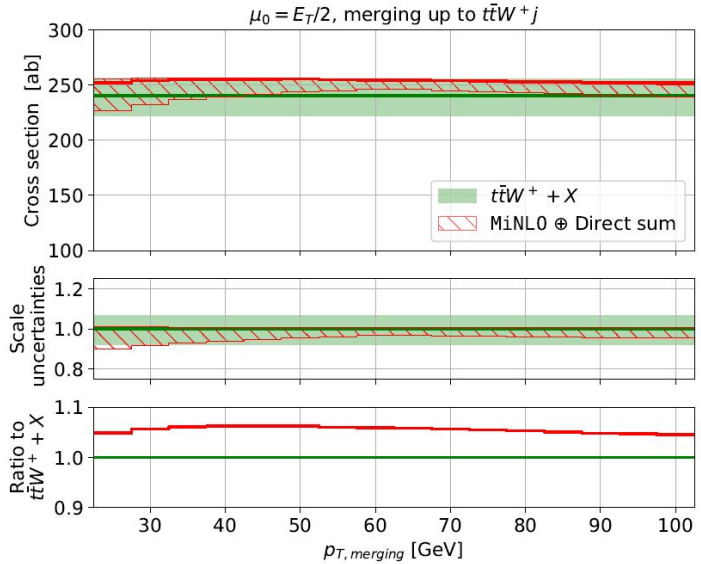


- MiNLO and NLO are fully compatible within uncertainties
- Scale uncertainties are comparable for well-behaved scale choices
- With a fixed scale choice, MiNLO shows significant improvement
  - reduced uncertainties in the tail -> 17% (MiNLO) vs 34% (NLO)

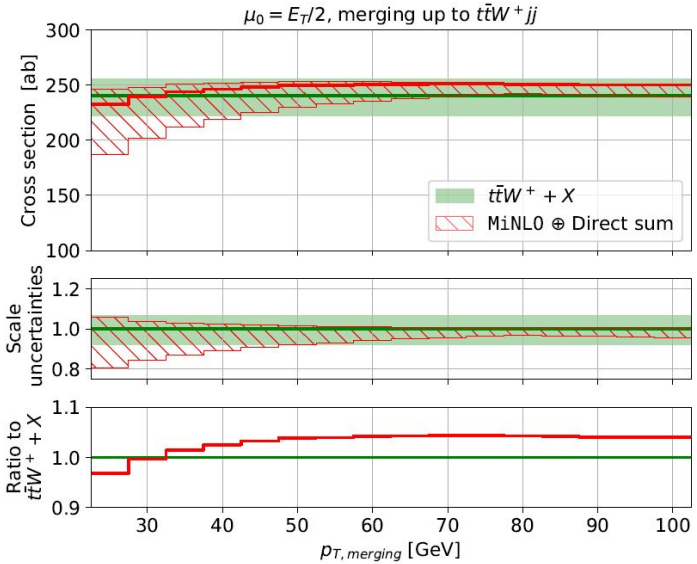
PRELIMINARY

Merged predictions

$$\left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+ + jets}\right)^{\text{up to 1 jet}}_{\text{merged}} = \left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+}\right)_{\text{excl}} + \left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+ j}\right)_{\text{incl}}$$



$$\left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+ + jets}\right)^{\text{up to 2 jets}}_{\text{merged}} = \left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+}\right)_{\text{excl}} + \left(\sigma^{\text{MiNLO}}_{pp\to t\bar{t}W^+ j}\right)_{\text{excl}} + \left(\sigma^{\text{MiLO}}_{pp\to t\bar{t}W^+ jj}\right)$$

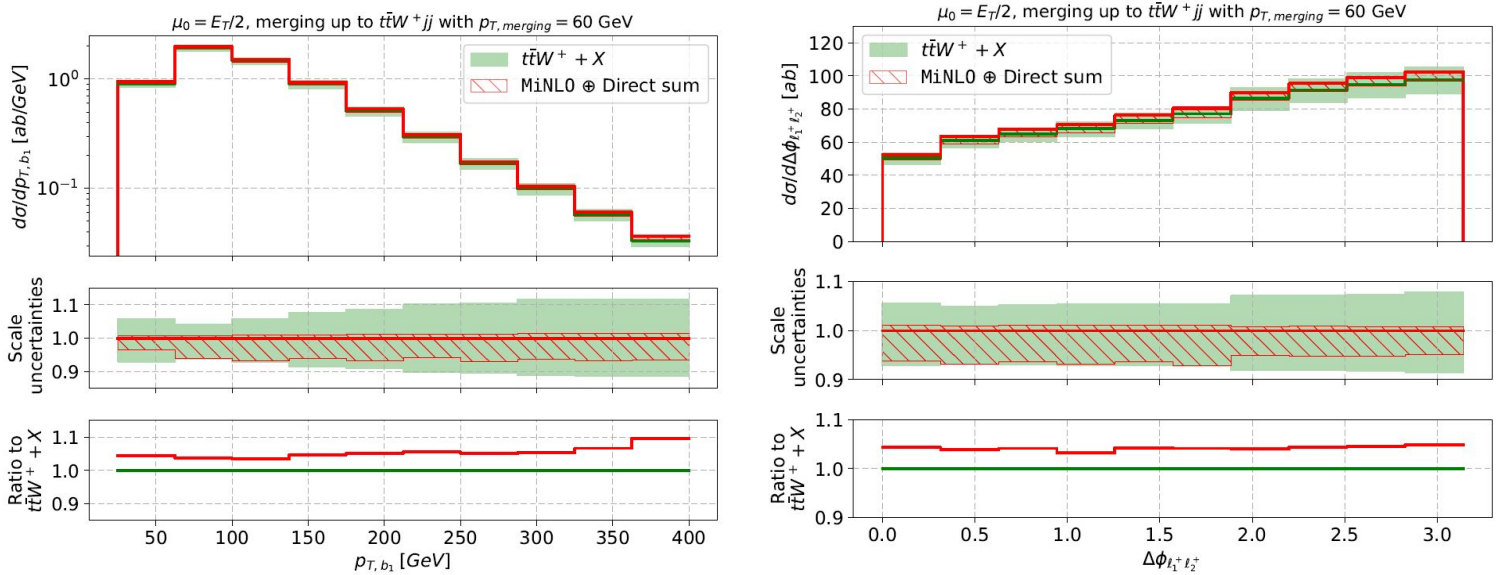


- Merged uncertainties depend on the merging scale
  1. larger at small merging scales
  2. improved predictions at larger merging scales, roughly above 40 GeV-50 GeV

- Ratio to inclusive ttW NLO within 0.97–1.07, depending on jet multiplicity and merging scale

PRELIMINARY

Merged predictions



- Reduced scale uncertainties in the merged sample
  1. pTb1: 7% (merged) vs 12% (NLO)
  2. Dphil1l2: 5% (merged) vs 8% (NLO)

- Agreement with inclusive ttW⁺ NLO
  - ~10% for these observables, up to ~25% in general, still within uncertainties

Conclusions & Outlook

- Standard NLO and MiNLO are in good agreement within uncertainties
- MiNLO shows reduced scale uncertainties
  → differences increase with jet multiplicity
- MiNLO provides more stable predictions especially for suboptimal scales and large $p_{T,j}$ thresholds
- Merging improves ttW^+ modelling
  → for merging scales above roughly 40 GeV
  → with reduced theoretical uncertainties
- Future goal
  → Extend merging to ttW^+jj at NLO (full off-shell)

Thanks for your attention!

BACKUP

Inverse k_T-clustering algorithm

- For each final-final (FF) branching we consider:

$$y_{ij} = \frac{min(p_{T,i}^2, p_{T,j}^2)\Delta R_{ij}^2}{R^2}$$

- For each final-initial (FI) branching we consider:

$$y_{ij} = p_{T,i}^2$$

$$q = \sqrt{min\{y_{ij}\}}$$

nodal scale

- A branching is allowed only if is compatible in flavor:

$$\begin{aligned}
&\text{FF:} \quad gg \to g, \quad gq \to q, \quad g\bar{q} \to \bar{q}, \quad q\bar{q} \to g \\
&\text{FI:} \quad gg \to g, \quad gq \to q, \quad qg \to \bar{q}, \quad qq \to g \\
&\text{FI:} \quad g\bar{q} \to \bar{q}, \quad \bar{q}g \to q, \quad \bar{q}\bar{q} \to g,
\end{aligned}$$

where q=u,d,s,c,b.

Sudakov form factors

- In line with [Höche, Maierhoefer, Moretti, Pozzorini, Siegert '17] we employ the following Sudakov form factors:

$$\Delta_f(t_0,t_1)=\exp\left\{-\int_{t_0}^{t_1}dt\frac{\alpha_s(t)}{2\pi t}\sum_{a=q,g}\int_0^{1-\sqrt{t/t_1}}dz\left(zP_{af}(z)+\delta_{af}\frac{2C_f}{1-z}\Lambda_2(t)\right)\right\}$$

$$\Delta^{\rm ext}_{f_j}(q^2_{res};q^2_j)=\frac{\Delta_{f_j}(q^2_{res},q^2_j)}{\Delta_{f_j}(q^2_{res},q^2_1)}$$

$$\Delta^{\rm int}_{f_{ij}}(q^2_{res};q^2_i,q^2_j)=\frac{\Delta_{f_{ij}}(q^2_{res},q^2_j)}{\Delta_{f_{ij}}(q^2_{res},q^2_i)}$$

where:

$$\Lambda_2(t)=\frac{\alpha_s(t)}{2\pi}\left[\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A-\frac{10}{9}T_RN_f(t)\right]$$

$$q_{res}=\begin{cases}q_1, & \text{if the line is final}\\ \xi_F q_1, & \text{if the line is initial}\end{cases}$$

- This approach ensures that Sudakov form factors do not exceed unity allowing for their interpretation as no-branching probabilities
- These Sudakovs provide NLL resummation for soft and collinear logarithms that might appear in processes where there is a big separation of scales

Sudakov form factors

- We want to ensure that:

$$\mathcal{P}(q_i^2, q_k^2) = \mathcal{P}(q_i^2, q_j^2) \times \mathcal{P}(q_j^2, q_k^2)$$

where $\mathcal{P}(q_i^2, q_k^2)$ represents the probability of no emission between q_i and q_k

- This is guaranteed only if $\Delta^{\text{int}}_{f_{ij}}(q^2_{res}; q_i^2, q_j^2) = \dfrac{\Delta_{f_{ij}}(q^2_{res}, q_j^2)}{\Delta_{f_{ij}}(q^2_{res}, q_i^2)}$ because otherwise we would have:

$$\Delta^{\text{int}}_{f_{ij}}(q_i^2, q_j^2) \times \Delta^{\text{int}}_{f_{ij}}(q_j^2, q_k^2) = \exp\left\{-\int_{q_i^2}^{q_j^2} dt\, \Gamma(t, q_j^2)\right\} \times \exp\left\{-\int_{q_j^2}^{q_k^2} dt\, \Gamma(t, q_k^2)\right\}$$
$$\neq \exp\left\{-\int_{q_i^2}^{q_k^2} dt\, \Gamma(t, q_k^2)\right\} = \Delta^{\text{int}}_{f_{ij}}(q_i^2, q_k^2)$$

where:

$$\Gamma(t, q_i^2) = \frac{\alpha_s(t)}{2\pi t} \sum_{b=q,g} \int_0^{1-\sqrt{t/q_i^2}} dz \left(z P_{ab}(z) + \delta_{ab} \frac{2C_f}{1-z} \Lambda_2(t) \right)$$

NLO subtraction in the Born term

- The Sudakov form factors in the Born term include some higher-order effects

$$\mathcal{B} \times \left(\prod_k \frac{\Delta_{f_k}(q^2_{res}, q^2_k)}{\Delta_{f_k}(q^2_{res}, q^2_1)} \right) \times \left(\prod_{ij} \frac{\Delta_{f_{ij}}(q^2_{res}, q^2_i)}{\Delta_{f_{ij}}(q^2_{res}, q^2_j)} \right) =$$

$$\mathcal{B} \times \left(1 + \sum_i \left[\Delta^{(1)}_{f_k}(q^2_{res}, q^2_k) - \Delta^{(1)}_{f_k}(q^2_{res}, q^2_1) \right] + \mathcal{O}(\alpha_s^2) \right)$$

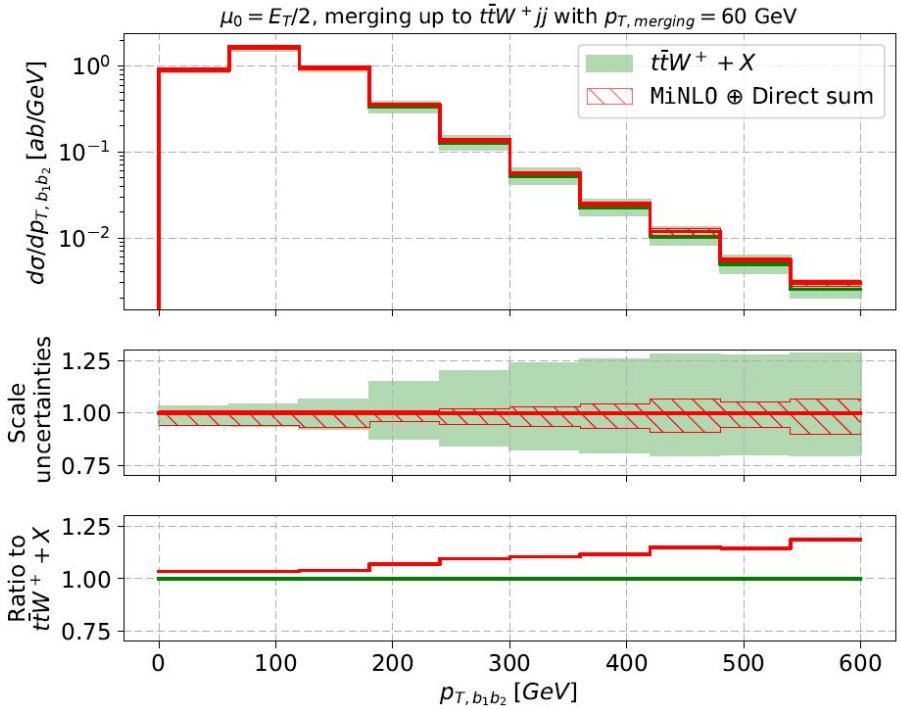
$$\times \left(1 + \sum_i \left[\Delta^{(1)}_{f_{ij}}(q^2_{res}, q^2_i) - \Delta^{(1)}_{f_{ij}}(q^2_{res}, q^2_j) \right] + \mathcal{O}(\alpha_s^2) \right) =$$

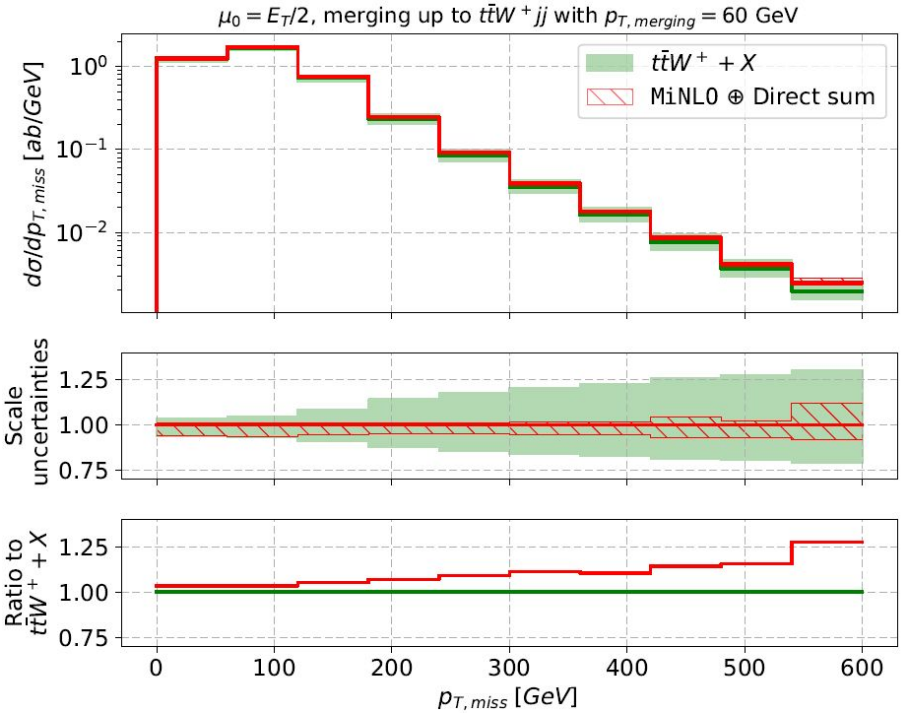
$$\mathcal{B} \times \left(1 + \sum_i \left[\Delta^{(1)}_{f_k}(q^2_{res}, q^2_k) - \Delta^{(1)}_{f_k}(q^2_{res}, q^2_1) \right] + \sum_i \left[\Delta^{(1)}_{f_{ij}}(q^2_{res}, q^2_i) - \Delta^{(1)}_{f_{ij}}(q^2_{res}, q^2_j) \right] + \mathcal{O}(\alpha_s^2) \right)$$

NLO part that needs to be subtracted since it is already included in the virtual and real-emission contributions

PRELIMINARY

Merged predictions

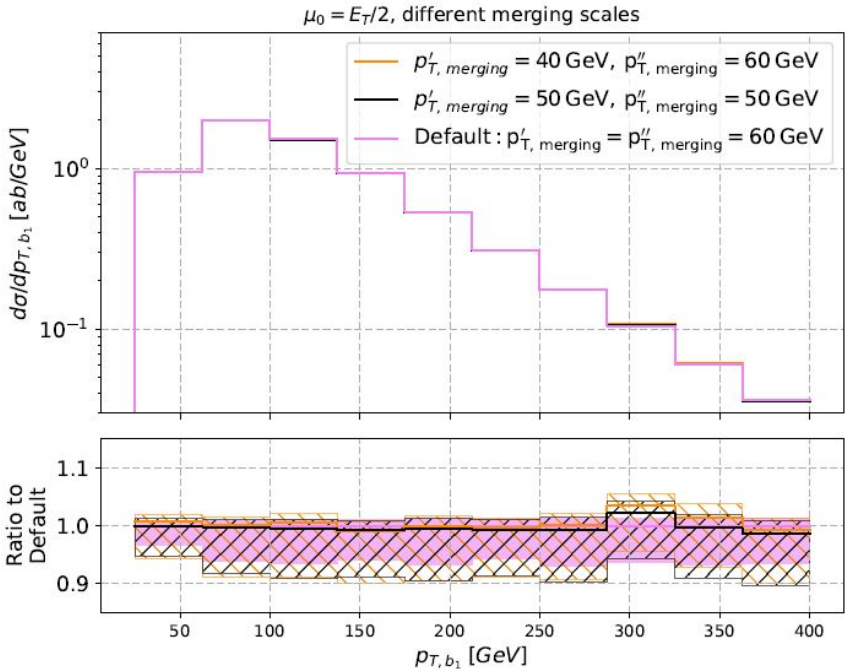


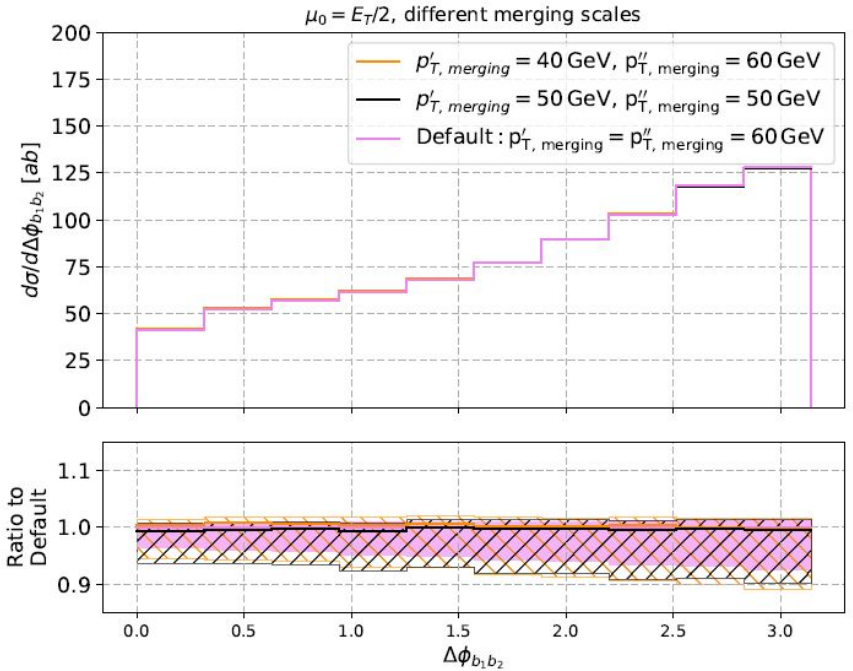


- Differences up to 25% between the merged predictions and the inclusive ttW⁺ NLO sample

PRELIMINARY

Different merging-scale parameters





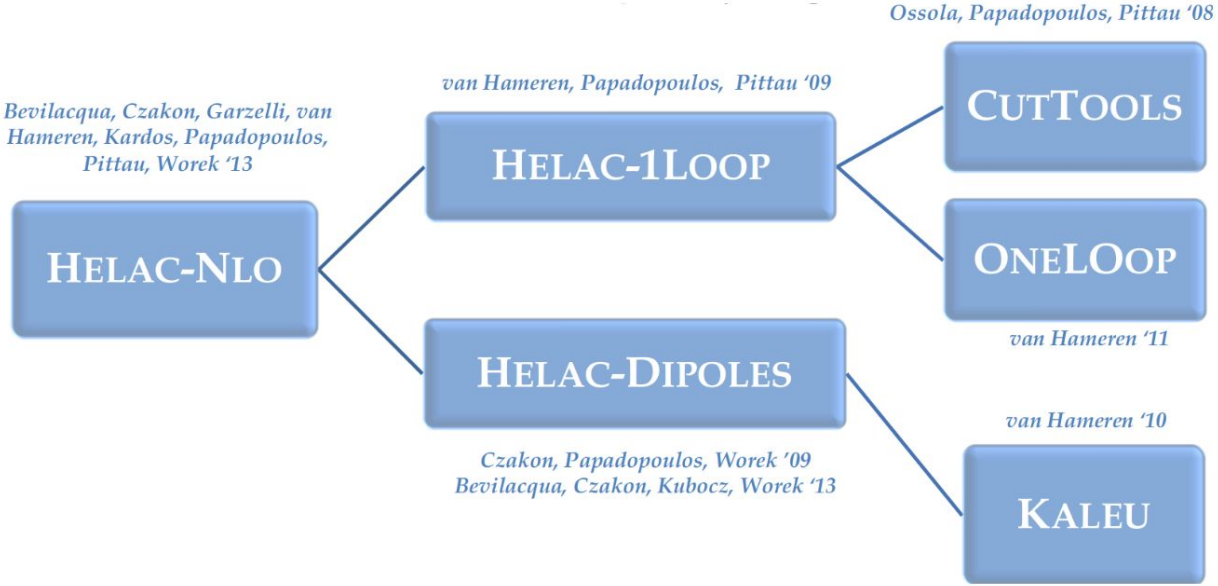
- Differences between different merging-scale parameters reflect the ones already observed at the integrated level

Input parameters and fiducial cuts

$$G_\mu = 1.166378 \times 10^{-5}\ \text{GeV}^{-2}, \qquad m_t = 172.5\ \text{GeV},$$
$$m_W = 80.379\ \text{GeV}, \qquad \Gamma_W^{\text{NLO}} = 2.0972\ \text{GeV},$$
$$m_Z = 91.1876\ \text{GeV}, \qquad \Gamma_Z^{\text{NLO}} = 2.5074\ \text{GeV},$$
$$\Gamma^{\text{LO}}_{t,\,\text{off-shell}} = 1.45766\ \text{GeV}, \qquad \Gamma^{\text{NLO}}_{t,\text{off-shell}} = 1.33254\ \text{GeV},$$

$$p_T(\ell) > 25\ \text{GeV}, \quad p_T(b) > 25\ \text{GeV}, \quad p_T(j) > 25\ \text{GeV},$$
$$|y(\ell)| < 2.5, \quad |y(b)| < 2.5, \quad |y(j)| < 2.5,$$
$$\Delta R_{\ell\ell} > 0.4, \quad \Delta R_{bb} > 0.4, \quad \Delta R_{b\ell} > 0.4,$$
$$\Delta R_{j\ell} > 0.4, \quad \Delta R_{bj} > 0.4, \quad \Delta R_{jj} > 0.4.$$

Helac-Nlo framework



- The output is saved in Les Houches & ROOT Ntuple files
  arXiv:hep-ph/0609017, arXiv:1310.7439 [hep-ph]

- It can be further analysed by adding new cuts, changing the renormalization and factorization scales, using different PDF set

PRELIMINARY

Cross-checks with Sherpa

- The following processes have been cross-checked with the Sherpa code using the $E_T/2$ dynamical scale

$$\text{Sherpa}: \sigma^{\text{MILO}}_{\text{t}\bar{\text{t}}\text{j}} = 262.14(6)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MILO}}_{\text{t}\bar{\text{t}}\text{j}} = 262.12(5)\ \text{pb}$$

$$\text{Sherpa}: \sigma^{\text{MINLO}}_{t\bar{t}j} = 381.26(67)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MINLO}}_{t\bar{t}j} = 381.08(52)\ \text{pb}$$

+ 1jet

$$\text{Sherpa}: \sigma^{\text{MILO}}_{t\bar{t}jj} = 113.41(4)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MILO}}_{t\bar{t}jj} = 113.40(2)\ \text{pb}$$

$$\text{Sherpa}: \sigma^{\text{MINLO}}_{t\bar{t}jj} = 162.0(2.2)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MINLO}}_{t\bar{t}jj} = 160.1(1.3)\ \text{pb}$$

+ 2jets

$$\text{Sherpa}: \sigma^{\text{MILO}}_{t\bar{t}jjj} = 41.153(23)\ \text{pb}$$
$$\text{HEPlot}: \sigma^{\text{MILO}}_{t\bar{t}jjj} = 41.126(14)\ \text{pb}$$

+ 3jets

Sherpa 3:
Bothmann, Flower, Gütschow, Höche, Hoppe, Isaacson, Knobbe, Krauss, Meinzinger, Napoletano, Price, Reichelt, Schönherr, Schumann, Siegert '24

HEPlot:
Bevilacqua, "unpublished," '19