α_{QED} with small angle Bhabha scattering

Luca Trentadue - Universita' di Parma and INFN
Woring Group on Radiative Corrections and Generators for Low Energy Hadronic Cross-Section and Luminosity - Frascati - October 16th 2006

$\Delta\alpha(q^2)$

Vacuum polarization

$\Delta\alpha = \Delta\alpha_{lept} + \Delta\alpha_{had}$

arises from quantum loop contribution to the photon propagator receiving contributions from quarks (hadrons), leptons and gauge bosons

The hadronic contribution is estimated in the s channel with a dispersion integral
from the cross-section e+e- to hadrons cross-section

S. Eidelman and F. Jegerlehner: Z. Phys. **C67** (1995) 602
F. Jegerlehner: hep-ph/0308117
M. Davier and A. Höcker: Phys. Lett. **B435** (1998) 427
M. Davier, S. Eidelman, A. Höcker and Z. Zhang: Eur. Phys. J. **C27** (2003) 497

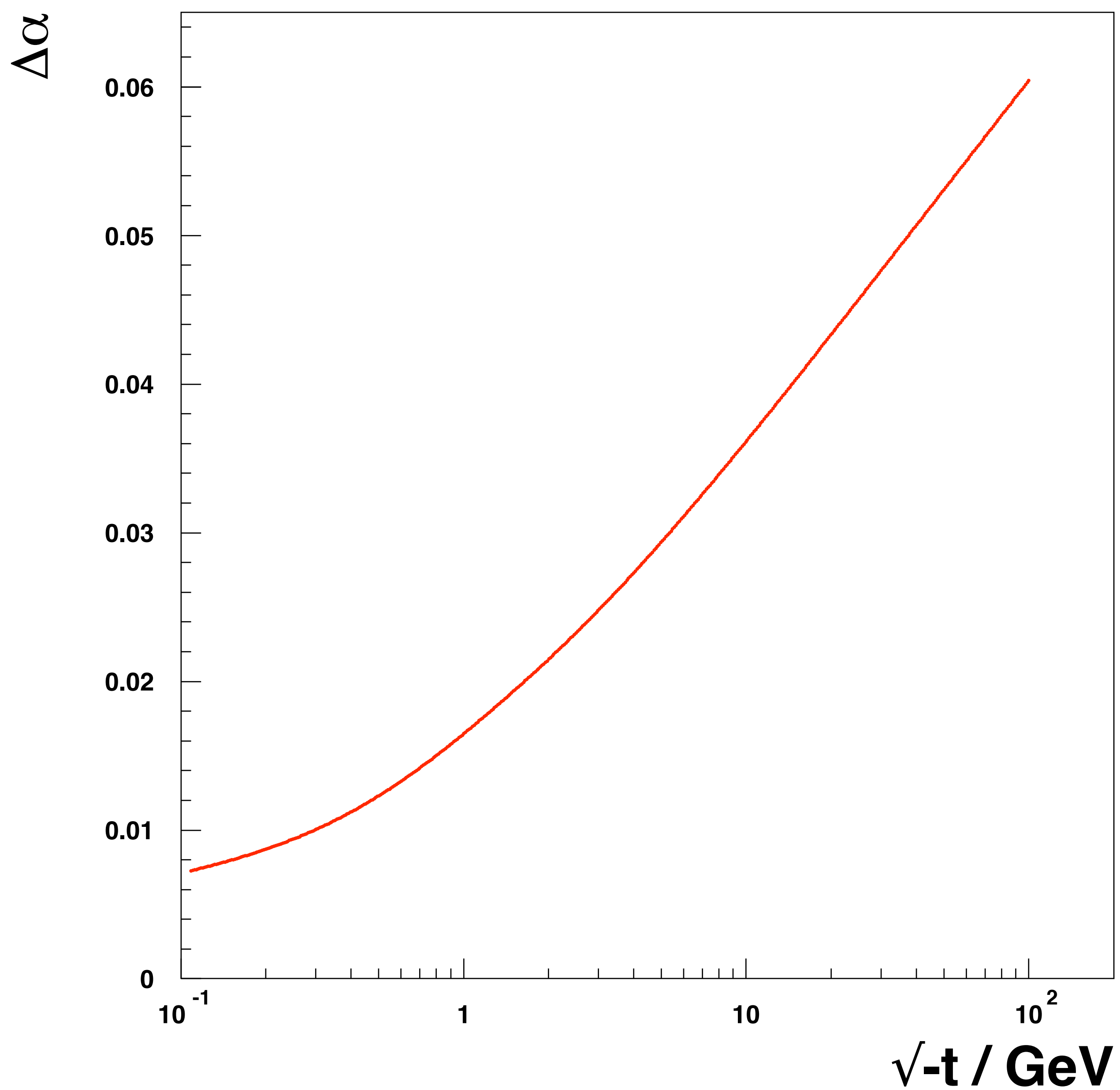
Here we follow an alternative approach:

- the running of α is studied using small-angle Bhabha scattering
- This process provides unique information on the QED coupling constant α at low space-like momentum transfer $t = -|q^2|$ in the t channel, with $t = -(1/2)s(1-\cos\theta)$ for example for

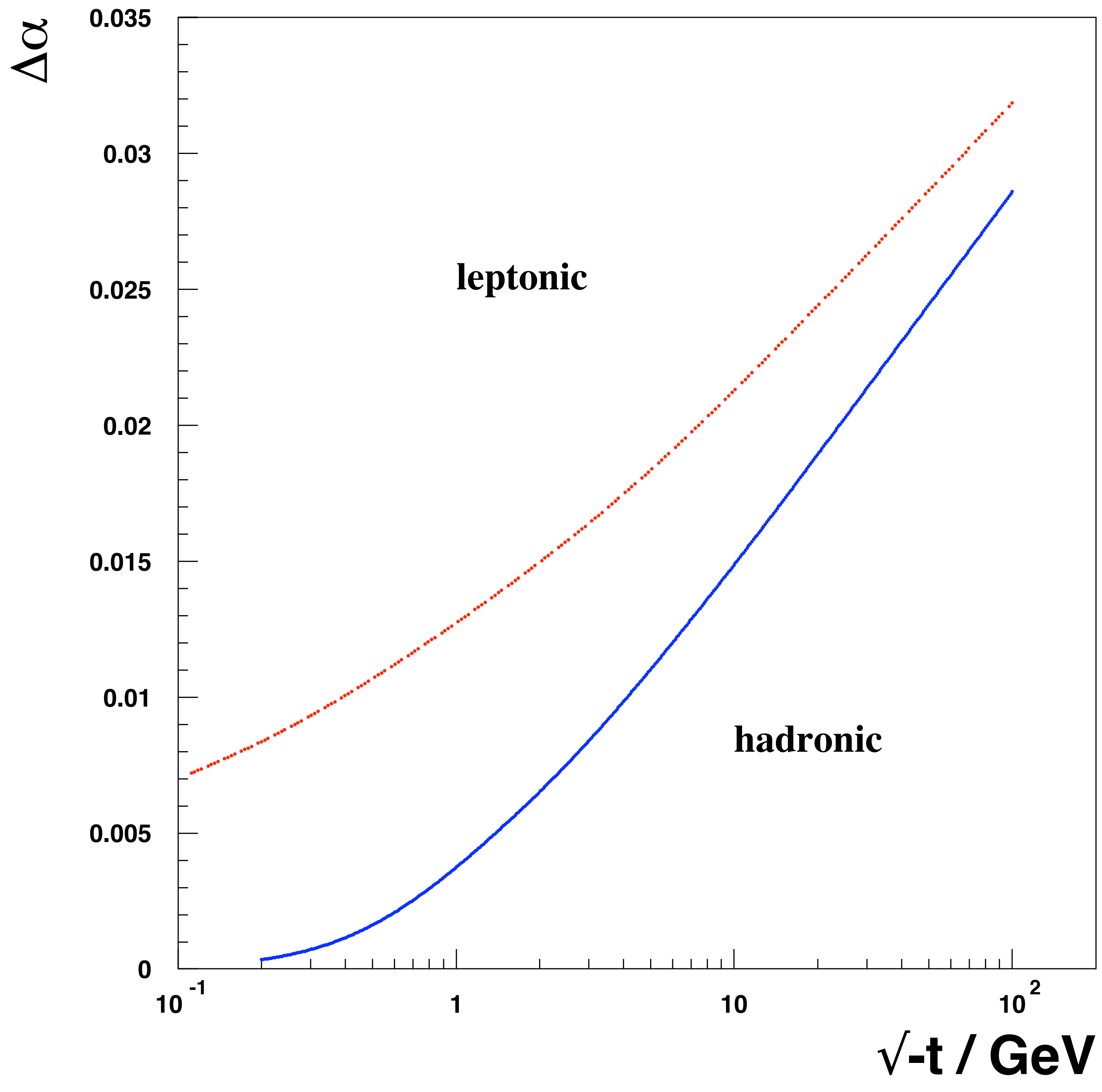
$$\sqrt{s} = 91.1 GeV$$

$$\theta = 30\ mrad \qquad t = 2.2\ GeV^2$$

$$\theta = 150\ mrad \qquad t = 30\ GeV^2$$



by using alphaQED F. Jegerlehner



by using alphaQED F. Jegerlehner

The method

A.Arbuzov D.Haidt C.Matteuzzi M.Paganoni L.T.

European Physics Journal C35, 267 (2004)

exploits the fact that the cross section for the process e+e–→e+e– can be conveniently decomposed into three factors:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\mathrm{d}\sigma^0}{\mathrm{d}t}\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 (1+\Delta r(t))$$

- The Bhabha Born cross section, including soft and virtual photons is precisely known and accounts for the strongest dependence on t
- The vacuum-polarization effect in the leading photon t-channel exchange is incorporated in the running of α and gives rise to the squared factor
- Δr(t) collects all the remaining real (in particular collinear) and virtual radiative effects not incorporated in the running of α
- The experimental data (after correction for detector effects) have to be compared with this distribution

The Luminosity measurement

The precise determination of the luminosity at e+e- colliders is a crucial ingredient to obtain an accurate evaluation of all the physically relevant cross sections.

They necessarily have to rely on some reference process, which is usually taken to be the small-angle Bhabha scattering.
Given the high statistical precision provided by the LEP collider, an equally precise knowledge of the theoretical small-angle Bhabha cross section is mandatory. In the 1990's the substantial progress in measuring the luminosity reached by the LEP machine has prompted several groups to make a theoretical effort aiming

at a **0.1%** accuracy.

An even better accuracy can be reached once the complete two-loop Bhabha (including α constants) cross-section will be computed

This goal has indeed been achieved by developing a dedicated strategy.
For the first time small-angle Bhabha scattering was evaluated analytically, following a new calculation technique that yields the required precision

Arbuzov,Fadin,Lipatov,Merenkov,Kuraev,T.(1995)

Nucl.Phys.B485(1997)457

Analytical calculations have been combined with Monte Carlo programs in order to simulate realistically the conditions of the LEP experiments

LABSMC NLLBHA SAMBHA

BHLUMI

theory update

http://www-conf.kek.jp/radcor05/

talks by Riemann, Penin, Bonciani

The cross section $\mathrm{d}\sigma^0/\mathrm{d}t$

does not depend on alpha

$$\frac{\mathrm{d}\sigma^0}{\mathrm{d}t} = \frac{\mathrm{d}\sigma^B}{\mathrm{d}t}\left(\frac{\alpha(0)}{\alpha(t)}\right)^2$$

$$\frac{\mathrm{d}\sigma^B}{\mathrm{d}t} = \frac{\pi\alpha_0^2}{2s^2}\mathrm{Re}\{B_t + B_s + B_i\},$$

$$\Pi(t) = \Delta\alpha(t)$$

$$\begin{aligned}
B_t &= \left(\frac{s}{t}\right)^2\left\{\frac{5+2c+c^2}{(1-\Pi(t))^2} + \xi\frac{2(g_v^2+g_a^2)(5+2c+c^2)}{(1-\Pi(t))}\right.\\
&+ \left.\xi^2\left(4(g_v^2+g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2)\right)\right\}\\
B_s &= \frac{2(1+c^2)}{|1-\Pi(s)|^2} + 2\chi\frac{(1-c)^2(g_v^2-g_a^2)+(1+c)^2(g_v^2+g_a^2)}{1-\Pi(s)}\\
&+ \chi^2\left[(1-c)^2(g_v^2-g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2)\right]\\
B_i &= 2\frac{s}{t}(1+c)^2\left\{\frac{1}{(1-\Pi(t))(1-\Pi(s))}\right.\\
&+ (g_v^2+g_a^2)\left(\frac{\xi}{1-\Pi(s)} + \frac{\chi}{1-\Pi(t)}\right)\\
&+ \left.(g_v^4+6g_v^2g_a^2+g_a^4)\xi\chi\right\}
\end{aligned}$$

$$\begin{aligned}
\chi &= \frac{s}{s-m_z^2+im_Z\Gamma_Z}\cdot\frac{1}{\sin 2\theta_w}\\
\xi &= \frac{t}{t-m_Z^2}\cdot\frac{1}{\sin 2\theta_w},\\
g_a &= -\frac{1}{2}, \quad g_v = -\frac{1}{2} + 2\sin^2\theta_w),\\
t &= (p_1-q_1)^2 = -\frac{1}{2}\,s\,(1-c),\\
c &= \cos\theta, \qquad \theta = \widehat{\boldsymbol{p}_1\boldsymbol{q}_1}.
\end{aligned}$$

The running of α

$$\boxed{\Pi(t) = \Delta\alpha(t)}$$

$$\begin{aligned}
\Pi(t) &= \frac{\alpha_0}{\pi}\left(\delta_t + \frac{1}{3}L - \frac{5}{9}\right) \\
&+ \left(\frac{\alpha_0}{\pi}\right)^2\left(\frac{1}{4}\,L + \zeta(3) - \frac{5}{24}\right) \\
&+ \left(\frac{\alpha_0}{\pi}\right)^3 \Pi^{(3)}(t) + \mathcal{O}\left(\frac{m_e^2}{t}\right),
\end{aligned}$$

$$L = \ln\frac{Q^2}{m_e^2}, \qquad Q^2 = -t, \qquad \zeta(3) = 1.202$$

The radiative factor $1 + \Delta r(t)$ and neglected terms

For the present investigation of the small-angle Bhabha cross section only the correction consistently needed to maintain the required accuracy are kept

All these corrections are included in the new code SAMBHA

All the following contributions have been proved to be negligible and are dropped:

- Any electroweak effect beyond the tree level, for instance appearing in boxes or vertices with Z^0 and W bosons, running weak coupling, etc.

- Box diagrams at order α^2 and larger

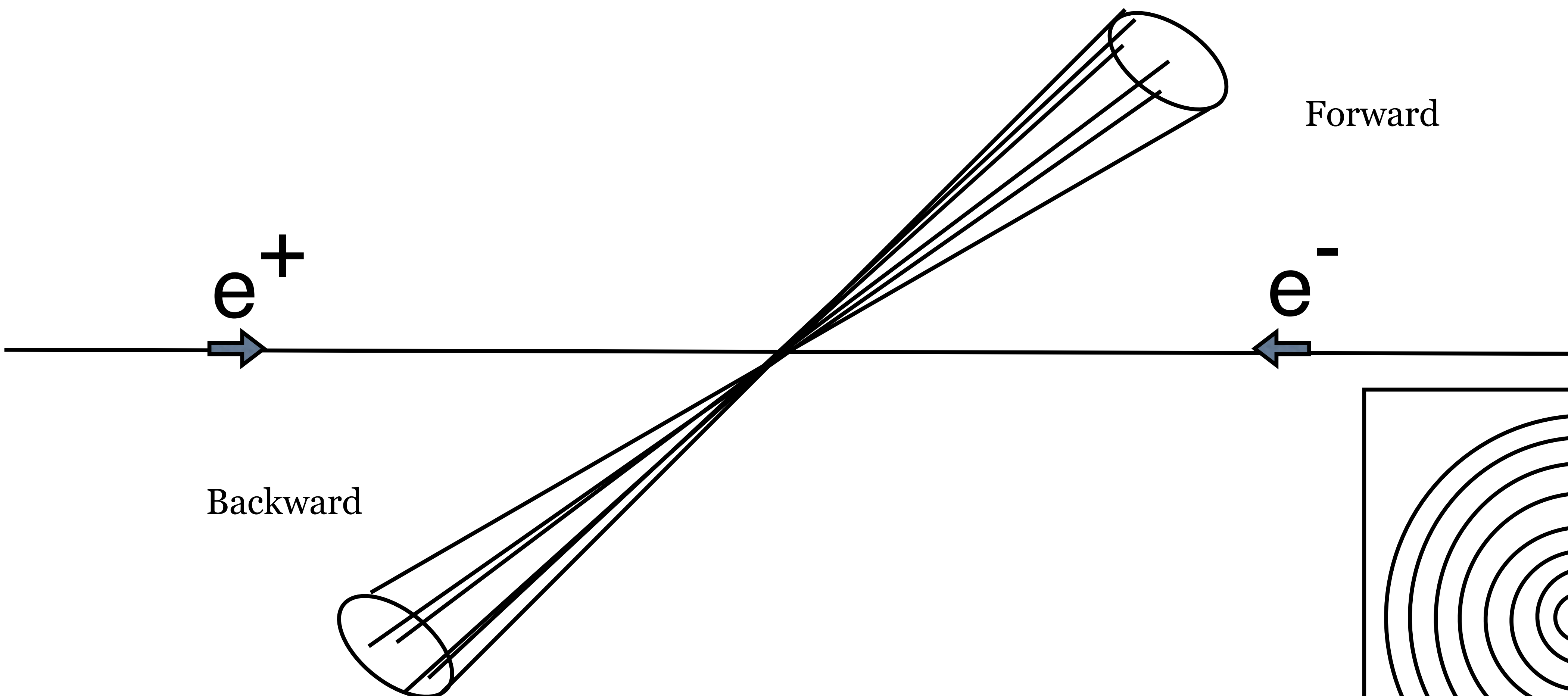
- Contributions of order α^2 without large logarithms, leading from order α^4 (i.e. $\alpha^4 L^4$) and subleading higher order ($\alpha^3 L^2$, $\alpha^4 L^3$, ...)

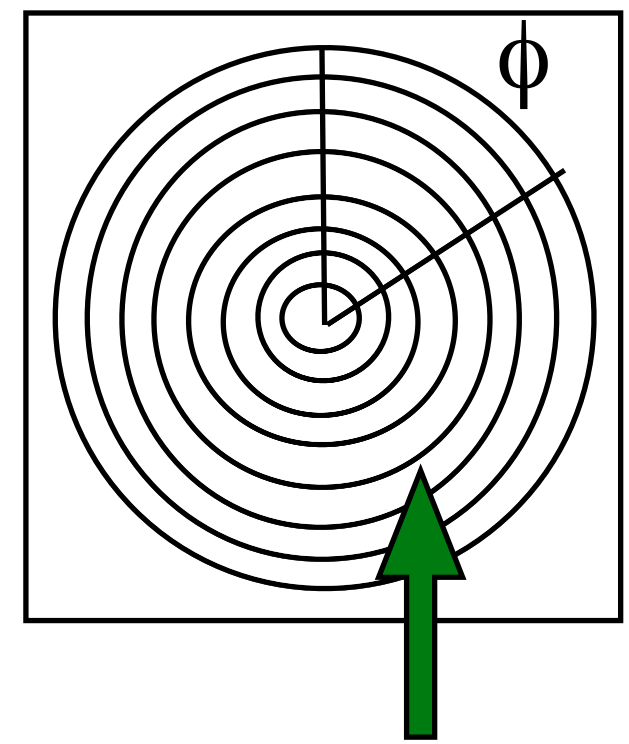
- Contributions from pair-produced hadrons, muons, taus and the corresponding virtual pair corrections to the vertices (estimated to be of the order of 0.5×10^{-4})

$\sqrt{s}$ (GeV)	91.187	91.2	189	206	500	1000	3000
	45 mrad $<$ θ $<$ 110 mrad						
$\sqrt{\langle -t \rangle}$ (GeV)	3.4	3.4	7.1	7.7	18.8	37.5	112.6
QED	51.428	51.413	11.971	10.077	1.7105	0.42763	0.047514
QED$_t$	51.484	51.469	11.984	10.088	1.7124	0.42809	0.047566
EW	51.436	51.413	11.965	10.072	1.7105	0.42871	0.049507
EW+VP$_t$	54.041	54.018	12.743	10.745	1.8590	0.47303	0.055748
EW+VP	54.036	54.013	12.742	10.744	1.8588	0.47296	0.055742
	5 mrad $<$ θ $<$ 50 mrad						
$\sqrt{\langle -t \rangle}$ (GeV)	1.1	1.1	2.2	2.4	5.8	11.6	34.8
QED	4963.4	4962.0	1155.4	972.54	165.08	41.271	4.5857
QED$_t$	4963.5	4962.1	1155.4	972.57	165.09	41.272	4.5858
EW	4963.4	4962.0	1155.4	972.53	165.08	41.272	4.5885
EW+VP$_t$	5075.0	5073.5	1190.6	1003.3	172.51	43.647	4.9603
EW+VP	5075.0	5073.5	1190.6	1003.3	172.51	43.646	4.9605

Table 1: Various cross sections in nb as a function of the centre-of- mass energy in GeV integrated over the two angular ranges 45–110 mrad and 5–50 mrad. The index t denotes the contribution of the corresponding t channel Feynman diagrams alone. The last columns are of interest for furture Linear Colliders.

Calorimetric type measurement





rings

1.The cluster is reconstructed

2. Find the cluster center

3. The highest energy cluster in each emisphere F, B

4. Energy Cuts (suppresses the Initial State Radiation) min(E_F,E_B)>0.65 E_{beam}

Max(E_F,E_B)>0.95 E_{beam}

5.Take the angle of the center of the most energetic cluster: this defines $\Rightarrow$ θ

6.Repeat this procedure for several "rings" segmented detector to reproduce the differential distribution

Comparison and evaluation

Table 3: Numbers of events generated with BHLUMI

$\sqrt{s}$(GeV)	91.2	189	200
$\int \mathcal{L}\mathrm{d}t$ (pb^{-1})	75	150	200
Ring 2	1844850	863571	1028210
Ring 3	907754	425586	506131
Ring 4	513696	240550	286994
Ring 5	313218	146731	174740
Ring 6	201893	94033	112168

$$\sigma_i = \sigma_i^0 \left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 (1 + \Delta r_i),$$

$$\begin{aligned}
\sigma_i &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma}{\mathrm{d}t} \\
\sigma_i^0 &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \\
\left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 &= \int^{R_i} \frac{\mathrm{d}t}{t_{\mathrm{max}} - t_{\mathrm{min}}} \left(\frac{\alpha(t)}{\alpha(0)} \right)^2, \\
1 + \Delta r_i &= \left(\frac{\alpha(0)}{\alpha(t_i)} \right)^2 \frac{\sigma_i}{\sigma_i^0}
\end{aligned}$$

$$\left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1 + \Delta r_i},$$

$$\left(\frac{\alpha(t)}{\alpha(0)} \right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle}$$

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)}, \qquad \left(\frac{\alpha(t)}{\alpha(0)}\right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log\frac{-t}{\langle -t\rangle}$$

$$\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1 + \Delta r_i},$$

$$\frac{N_i}{\sigma_i^0} \frac{1}{1 + \Delta r_i} = n_0 + n_1 \log \frac{-t_i}{\langle -t\rangle}$$

$$\begin{aligned} n_0 &= \int \mathcal{L}\mathrm{d}t \cdot \Big(1 + 2\Delta\alpha(\langle t\rangle)\Big) \\ n_1 &= \int \mathcal{L}\mathrm{d}t \cdot \left(\frac{\mathrm{d}}{\mathrm{d}\log(-t)} 2\Delta\alpha(t)\right). \end{aligned}$$

$$\frac{\mathrm{d}}{\mathrm{d}\log(-t)}\Delta\alpha = \frac{n_1}{2n_0}\Big(1 + 2\Delta\alpha(\langle t\rangle)\Big)$$

$$\int \mathcal{L}\mathrm{d}t = \frac{n_0}{1 + 2\Delta\alpha(\langle t\rangle)}$$

Table 4: Theoretical predictions for each ring of the three factors of eq. 7. For the conditions defined in sect. 5.1 the angular boundary of ring i is θ_i=arctan (7+3(i-1))/220).

No. of ring	1	2	3	4	5	6	7
	$\sqrt{s} = 91.2$ GeV						
σ_i^0	63.077	24.728	12.170	6.8694	4.2517	2.8120	1.9552
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0425	1.0475	1.0516	1.0551	1.0582	1.0609	1.0634
$1 + \Delta r_i$	0.9426	0.9440	0.9412	0.9395	0.9240	0.8915	0.7982
	$\sqrt{s} = 189$ GeV						
σ_i^0	14.685	5.7563	2.8324	1.5984	0.9889	0.6537	0.4542
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0554	1.0613	1.0661	1.0702	1.0736	1.0767	1.0794
$1 + \Delta r_i$	0.9377	0.9390	0.9360	0.9329	0.9165	0.8858	0.7898
	$\sqrt{s} = 200$ GeV						
σ_i^0	13.115	5.1406	2.5295	1.4274	0.8831	0.5838	0.4057
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0565	1.0625	1.0673	1.0714	1.0749	1.0780	1.0807
$1 + \Delta r_i$	0.9376	0.9387	0.9352	0.9330	0.9158	0.8847	0.7896
	$\sqrt{s} = 1000$ GeV						
σ_i^0	0.5248	0.2059	0.1014	0.0573	0.0356	0.0236	0.0165
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0921	1.0994	1.1050	1.1096	1.1135	1.1169	1.1199
$1 + \Delta r_i$	0.8622	0.8620	0.8590	0.8545	0.8398	0.8084	0.7205
	$\sqrt{s} = 3000$ GeV						
σ_i^0	0.0590	0.0234	0.0117	0.0067	0.0042	0.0028	0.0020
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.1192	1.1267	1.1325	1.1373	1.1414	1.1448	1.1479
$1 + \Delta r_i$	0.8467	0.8457	0.8422	0.8381	0.8253	0.7956	0.6975

Final formula:

$$\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1+\Delta r_i},$$

Which can be transformed in a linear fit defining the t dependence of α:

$$\boxed{\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle}}$$

Table 5: Table of fit results; the uncertainties δu_0 and δu_1 are uncorrelated.

$\sqrt{s}$	91.2 GeV	189 GeV	200 GeV
u_0	1.0573±0.0005	1.0698±0.0008	1.0703±0.0007
u_1	0.0242±0.0028	0.0284±0.0041	0.0318±0.0038
$\langle -t \rangle$	8.5 GeV^2	36.6 GeV^2	40.9 GeV^2

$$\int \mathcal{L}\mathrm{d}t = \frac{n_0}{1+2\Delta\alpha(\langle t \rangle)}$$

$$\frac{\delta n_0}{n_0} = 10^{-3}$$

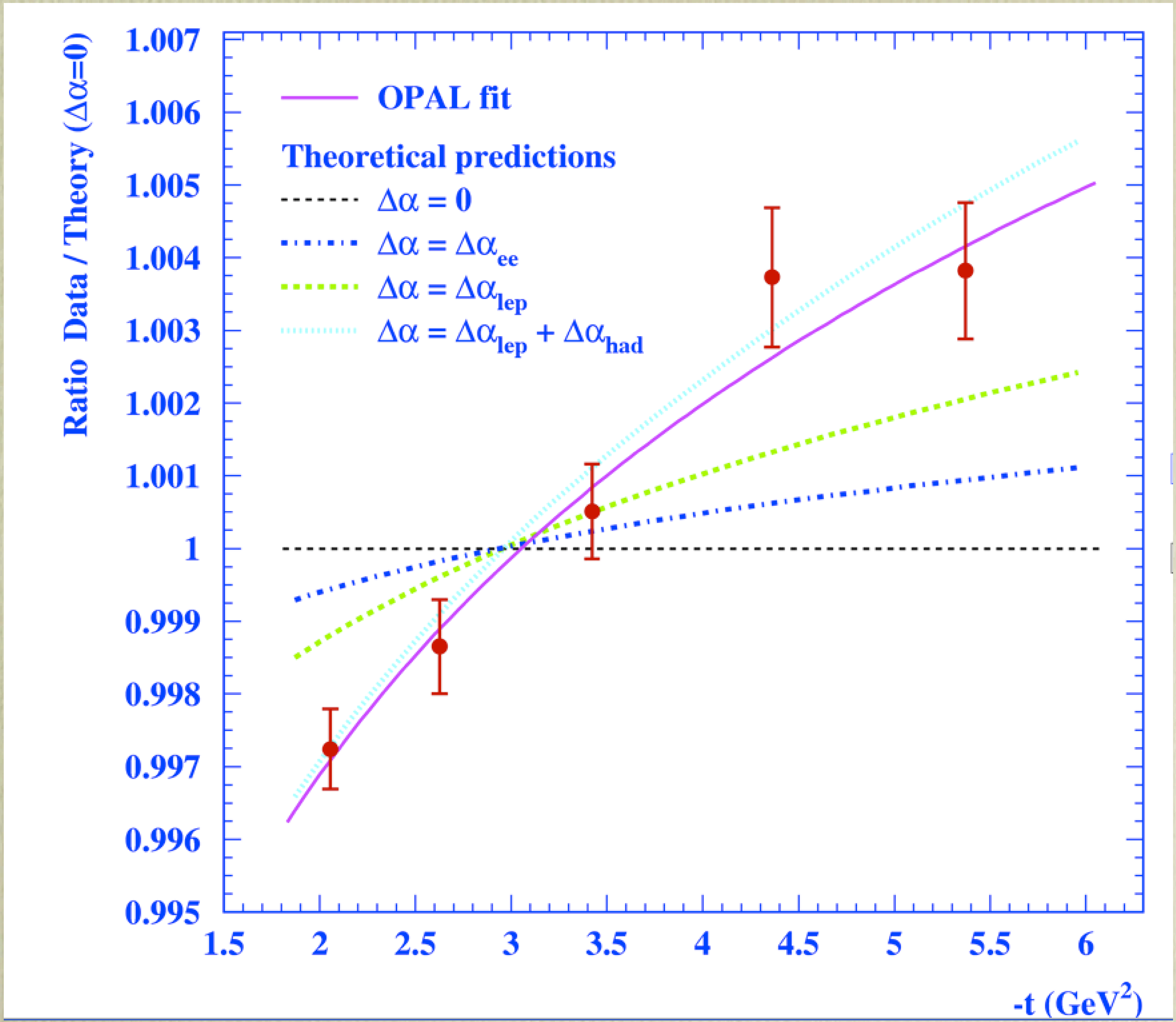
statistical precision

More formulae:

$$
\begin{aligned}
n_0 &= \int \mathcal{L}\mathrm{d}t \cdot \Big(1 + 2\Delta\alpha(\langle t \rangle)\Big) \\
n_1 &= \int \mathcal{L}\mathrm{d}t \cdot \left(\frac{\mathrm{d}}{\mathrm{d}\log(-t)} 2\Delta\alpha(t)\right).
\end{aligned}
$$

$$n_i = u_i \cdot \int \mathcal{L}\mathrm{d}t$$

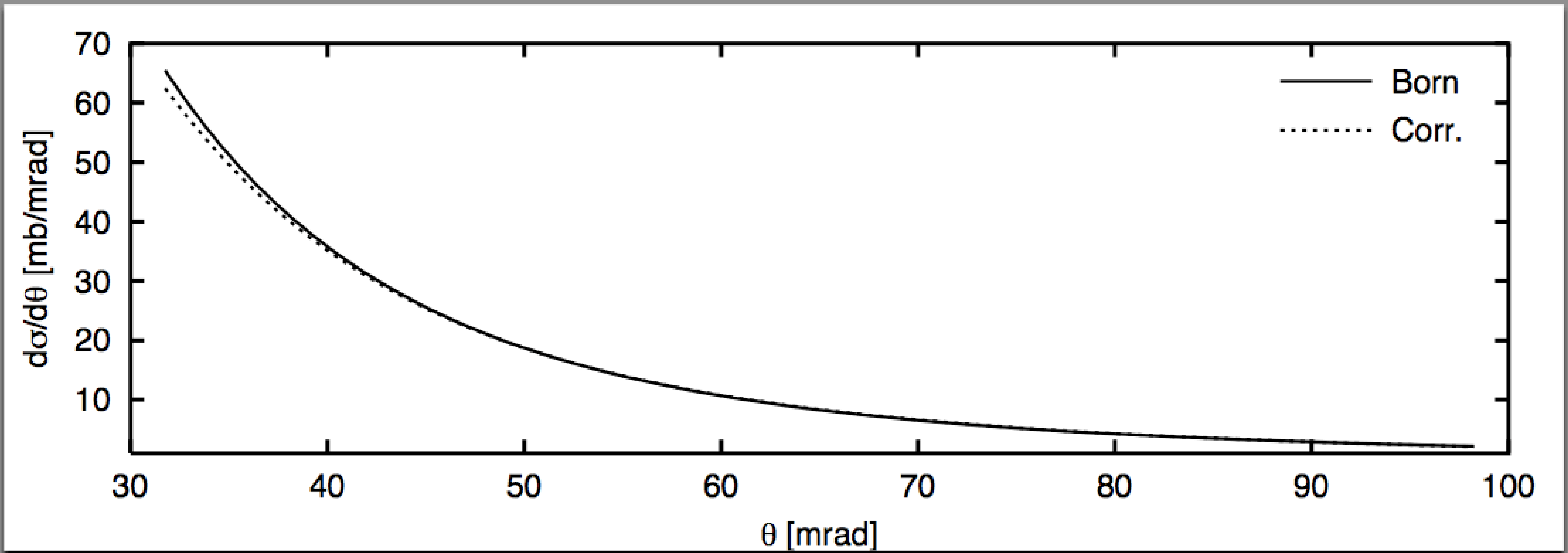
$$\frac{\mathrm{d}}{\mathrm{d}\log(-t)}\Delta\alpha = \frac{n_1}{2n_0}\Big(1 + 2\Delta\alpha(\langle t \rangle)\Big)$$



Ratio Data / Theory (Δα=0)
OPAL fit
Theoretical predictions
Δα = 0
Δα = Δα_ee
Δα = Δα_lep
Δα = Δα_lep + Δα_had
1.007
1.006
1.005
1.004
1.003
1.002
1.001
1
0.999
0.998
0.997
0.996
0.995
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
-t (GeV^2)

DAPHNE-2

Small angle Bhabha cross-section



$E_{CM} = 2\ GeV$ small-angle $20 \leq \theta \leq 50\ mrad$ large -angle $30 \leq \theta \leq 90\ degrees$

approximation	small-angle		large-angle	
t-channel QED Born	0.54706 mbarn	+0.02%	0.72532 mkbarn	+11.31%
QED Born	0.54695 mbarn	+0.00%	0.65161 mkbarn	+0.00%
EW Born	0.54695 mbarn	+0.00%	0.65161 mkbarn	+0.00%
Born + vac. pol.	0.55114 mbarn	+0.77%	0.67081 mkbarn	+2.95%
Born + LLA	0.52415 mbarn	-4.17%	0.63515 mkbarn	-2.53%
Born + $\mathcal{O}(\alpha)$	0.53059 mbarn	-2.99%	0.62908 mkbarn	-3.46%
Born + rad. corr.	0.53739 mbarn	-1.75%	0.65131 mkbarn	-0.05%

EW corrections are negligible (small and large angles)
Radiative Corrections are important (small and large angles)
Vacuum Polarization has a sizeable impact (large angles expecially 2.95%)

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EW Born means taking into account Z-boson exchange, while it is not relevant for the given energy.

Born + vac. pol. means the Born-level cross section plus the effect of vacuum polarization taken into account in the photon propagator.

Born + LLA means the Born-level cross section plus higher order photonic leading logarithmic corrections (without vacuum polarization).

Born + $\mathcal{O}(\alpha)$ means the Born-level cross section plus $\mathcal{O}(\alpha)$ photonic corrections (without vacuum polarization).

Born + rad. corr. means the corrected cross sections where we took into account: vacuum polarization, full $\mathcal{O}(\alpha)$, and LLA in higher orders.

addition:

- 2006-08 INTAS project approved (g-2,hadr. x-sect.) Novisibirsk-Dubna-Kharkov-Kracow-Pavia-Bologna-Parma