Remarks on the precision of Radiative Corrections

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Outline of Talk:

① **Status as summarized by previous speakers A.A. G.M,...**

② **Disentangling the many channels**

③ **Extra problems in radiative return**

④ **Outlook**

Issues:

Complete Order by Order calculations

Order by Order leading, subleading,.... corrections

Errors: in calculations, independent calculations, crosschecks

Errors: in numerical evaluations, stability,...

Limits: approximations may break down

Models for RC in hadroproduction:

pions: scalar QED, resonance extended Chiral Perturbation Theory (CHPT): RPT

What if higher order corrections are relevant?

How to use such models:

Correct implementation of VDM in accord with low energy symmetries of QCD:

- **vector meson extended CHPT: RPT model**
- **hidden local symmetry (HLS) model**
- **extended NJL (ENJL) model**

to large extend equivalent

Problem: radiative corrections matching L.D. with S.D. $\Rightarrow$ results depend on matching

cut off $\Lambda \Rightarrow$ model dependence (non-renormalizable low energy effective theory vs. renormalizable QCD)

① Status as summarized by previous speakers A.A. G.M,...

- weak corrections < 0.1% for $E < 2$ GeV
- 2–loop non-log terms (missing mass effects): $(\frac{\alpha}{\pi})^2\; C \sim 10^{-4}$ for $C \lesssim 10 \Rightarrow \lesssim$ 0.1%
  exceptions: kinematically singular regions, possible factorization scale dependences
- uncertainty of hadronic VP: 1% shift in $\sigma_{\rm had} \Rightarrow$ 0.04% in $\sigma_{\rm leptons}$
- 0.1% energy dependence in hadronic form factor
  this may be vastly underestimated for resonance regions like the ρ e.g. shift in rho mass by 2.5 MeV $\Rightarrow$ 10% effect in tails of resonance, note: in a_μ integral strongly energy dependent kernel!
- up to 0.1% technical precision (cut parameters soft Δ_0, collinear Θ_0 dependence)
- $\lesssim$ 0.05% - pairs- cuts. MC simulation required
  usually other background to be considered
- My comment:
  Unaccounted $O(\alpha^2)$ from one–loop $\otimes$ real photon radiation ?? should be calculated urgently !
- In distributions: known at 1% only, I think full 2–loop including mass effects are still important

Can we get control of FSR by hadrons??

What is usually done:

- sQED for low energy
- if QED corrections not UV finite (QED on top of weak charged current four fermion interaction): calculate leading (singular) SD term using quark parton model $\longrightarrow S$–factor
- True QED corrections not known!

Neutral current: in inclusive quantities KLN theorem guarantees no logs!

Inclusive correction $C_i\,\frac{\alpha}{\pi}$, C_i model dependent; in scalar QED correction 0.2%, in real world ???

Charged currents: e.g. tau spectral functions model dependence in $C_i\,\frac{\alpha}{\pi}\ln\frac{m_pi}{M_Z}$; known $C_{\rm sQED}\sim -C_{\rm QPM}$ large model dependence expected

② Disentangling the many channels

At energies above about 1 GeV many multiharon channels open.

A word on exclusive vs. inclusive strategy; e.g. VEPP 2000 detector allows exclusive strategy only.

DAFNE II should go for inclusive strategy

in scan factorization of ISR (see `hep-ph/0212386`) allows to extract

$\sigma_{\rm had}$ **incl FSR very precisely!**

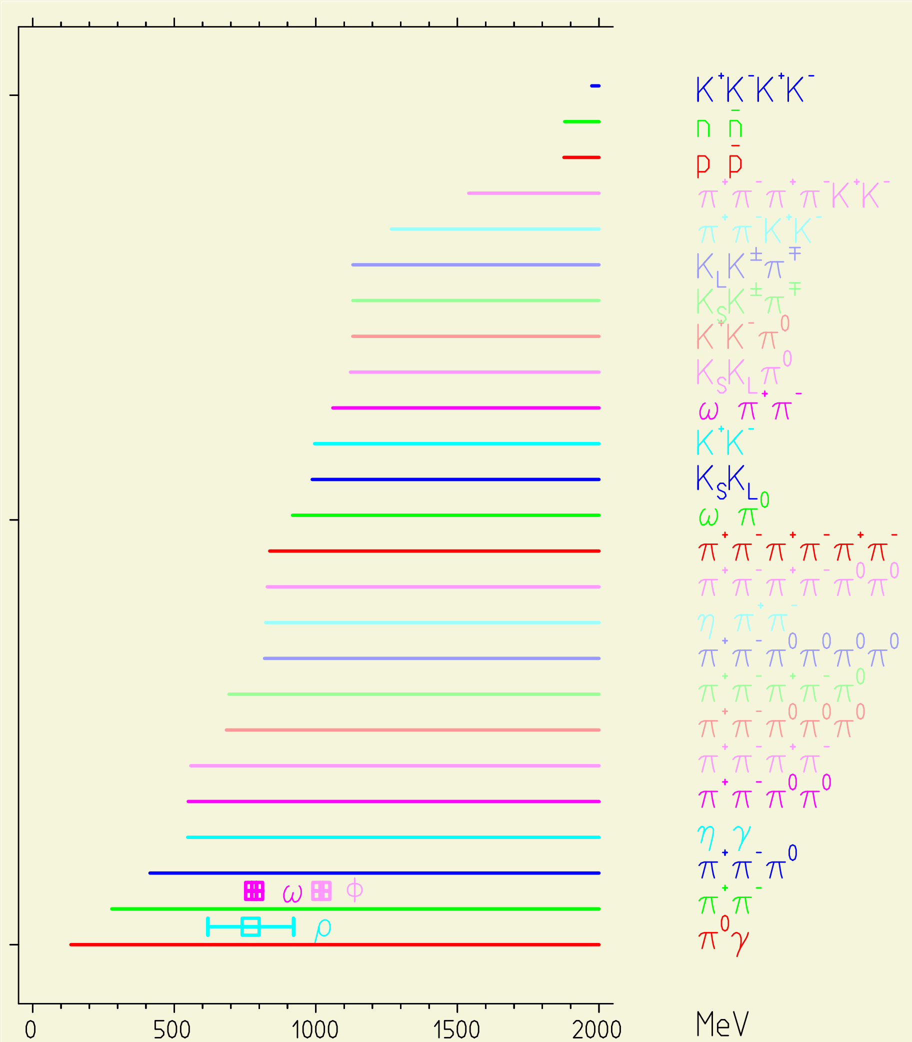

Figure 1: Thresholds for exclusive multi particle channels below 2 GeV

channel X	a_μ^X	%	$\Delta\alpha^X$	%
$\pi^0\gamma$	0.04	0.04	0.00	0.03
$\pi^+\pi^-$	11.99	11.66	1.59	9.64
$\pi^+\pi^-\pi^0$	9.22	8.98	1.25	7.55
$\eta\gamma$	0.45	0.44	0.05	0.30
$\pi^+\pi^-2\pi^0$	19.27	18.75	3.79	22.93
$2\pi^+2\pi^-$	13.99	13.62	2.80	16.92
$\pi^+\pi^-3\pi^0$ $_{iso}$	1.17	1.14	0.26	1.56
$2\pi^+2\pi^-\pi^0$	1.94	1.88	0.43	2.60
$\pi^+\pi^-4\pi^0$ $_{iso}$	0.08	0.08	0.02	0.12
$\eta^*\pi^+\pi^-$	0.26	0.25	0.05	0.31
$2\pi^+2\pi^-2\pi^0$	1.70	1.65	0.42	2.54
$3\pi^+3\pi^-$	0.32	0.31	0.08	0.49
$\omega^*\pi^0$	0.77	0.75	0.13	0.78
K^+K^-	21.99	21.39	2.64	15.94
$K_S^0K_L^0$	13.17	12.82	1.49	8.99
$\omega^*\pi^+\pi^-$	0.09	0.08	0.02	0.12
$K^+K^-\pi^0$	0.35	0.34	0.08	0.49
$K_S^0K_L^0\pi^0$ $_{iso}$	0.35	0.34	0.08	0.49
$K_S^0K^\pm\pi^\mp$	1.08	1.05	0.25	1.49
$K_L^0K^\pm\pi^\mp$ $_{iso}$	1.08	1.05	0.25	1.49
$K^+K^-\pi^+\pi^-$	1.08	1.05	0.28	1.70
$K\bar{K}\pi\pi$ $_{iso}$	2.22	2.16	0.54	3.23
$p\bar{p}$	0.07	0.07	0.02	0.12
$n\bar{n}$	0.08	0.07	0.02	0.13
$\phi\to$ missing	0.03	0.03	0.00	0.02

Contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{(5)}_{\rm had}(-s_0)$

from the energy region $2M_K < E < 2$ GeV.

$X^* = X(\to \pi^0\gamma)$,

$_{iso}$**=evaluated using iospin relations.**

③ Extra problems in radiative return

Photon tagging measurements: KLOE/Frascati, BaBar/SLAC, Belle/KeK

Normally "observed" cross section (C-invariant cuts, one–loop $\longrightarrow$ no initial-final state interference):

$$\begin{aligned}\sigma^{\rm obs}(s) &= \sigma_0(s)\ \left[1+\delta_{\rm ini}(\Lambda_{\rm IR})+\delta_{\rm fin}(\Lambda_{\rm IR})\right] \\ &+ \int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda_{\rm IR}} ds'\ \sigma_0(s')\rho_{\rm ini}(s,s') \\ &+ \sigma_0(s)\int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda_{\rm IR}} ds'\ \rho_{\rm fin}(s,s')\ ,\end{aligned}$$

unfolding problem to get $\sigma_0(s)$. Here additional problem: $\rho_{\rm fin}(s,s')$ model-dependent (only soft photon part known)!

Experimentally: acceptance cuts, efficiencies etc. in addition

Note: in higher orders multiple convolutions $\Rightarrow$ iterative disentanglement required.

Radiative return measurement: look at $\pi^+\pi^-$ invariant mass s' distribution $\left(\frac{d\sigma}{ds'}\right)$ plus anything (photon). s' fixed $\longrightarrow$ missing energy fixed $\longrightarrow$ "automatic" unfolding. Pion form factor ansatz:

$$\left(\frac{d\sigma}{ds'}\right)_{\mathrm{sym-cut}} = |F_\pi(s')|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{ini,\,sym-cut}} + |F_\pi(s)|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{fin,\,sym-cut}}$$

and hence we may resolve for the pion form factor as

$$|F_\pi(s')|^2 = \frac{1}{\left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{ini,\,sym-cut}}} \left\{ \left(\frac{d\sigma}{ds'}\right)_{\mathrm{sym-cut}} - |F_\pi(s)|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{fin,\,sym-cut}} \right\} .$$

Limitation: in higher orders again convoluted $\Rightarrow$ again an unfolding problem!

- Lowest order radiative return is one order in α higher than scan
- I claim: to get same precision as in scan radiative return required one order higher in theory concerning photonic corrections
- It is true the calculation by itself is the same for scan and for radiative return, however, the application in RR is more sensitive to small effects!
- Definitively: including higher order effects remains an urgent project particularly, also in future radiative return measurements will play an important role in many places

④ Outlook

- **Interest in HO corrections still very important. Driven manly be LEP and preLEP=ILC projects**
- **Such calculations are tedious and usually not honered sufficiently and if they are applied only 10 years later is not very motivating**
- **However: at many low energy facilities still very important input. I would like to encourage a lot these activities they are an indispesible input for a better determination of fundamental parameters like the effective fine structure constant** $\alpha(E)$**, very important in many places at te high precision frontier, like** $g-2$ **.**

Improvements possible by new strateies: measure inclusive FSR in scan, measure FSR whereever possible, $e^+e^- \to \gamma\gamma$ **,** $e^+e^- \to \mu^+\mu^-$ **as normalizing processes**
Note also $e^+e^- \to \gamma\gamma \to \pi^{0*}, \eta^*, ..., f_1^*, ...(\mathrm{hadrons}) \to \gamma\gamma$ **includes hadronic effects which do not factorize**
$e^+e^- \to \mu^+\mu^-$ **to be corrected for mass effects (only to lowest order this is trivial: may be sufficient)**