The muon g-2: where we are?

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Low Energy Hadronic Cross Sections and Luminosity

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Outline of Talk:

① **The Anomalous Magnetic Moment of the Muon**

② **Standard Model Prediction for a_μ**

③ **Hadronic Light-by-Light Scattering Contribution to $g-2$**

④ **Outlook**

Recent progress:

KLOE, SND and CMD-2 data

Recent BABAR data on exclusice channels in range 1 GeV to 4 GeV

New measurement of $a_e \longrightarrow$ improved value of α

Questions staiyng with us:

τ vs e^+e^- dicrepany: large of size of RC's to be applied in e^+e^- (SND); wating for τ spectral function results from Belle and BaBar

KLOE vs. CMD-2/SND

some discrepancies not understood, RC correct? (e.g. persisting discrepancies in Bhabha), sufficient precision (full two-needed), model dependencies (pions)?

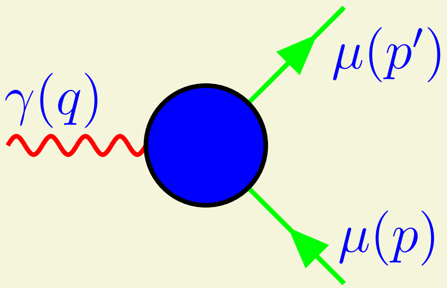
Full weak contributions: (Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05) Light-by-light scattering contribution: (... Knecht, Nyffeler 02, Melnikov, Vaninhstein 03)

① The Anomalous Magnetic Moment of the Muon

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;\; ; \quad g_\mu = 2\,(1+a_\mu)$$

Dirac: $g_\mu = 2$, a_μ muon anomaly

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$



$$= (-ie)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)$$

$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

a_μ responsible for the Larmor precession

directly proportional at magic energy $\sim$ 3.1 GeV

CERN, BNL g-2 experiments

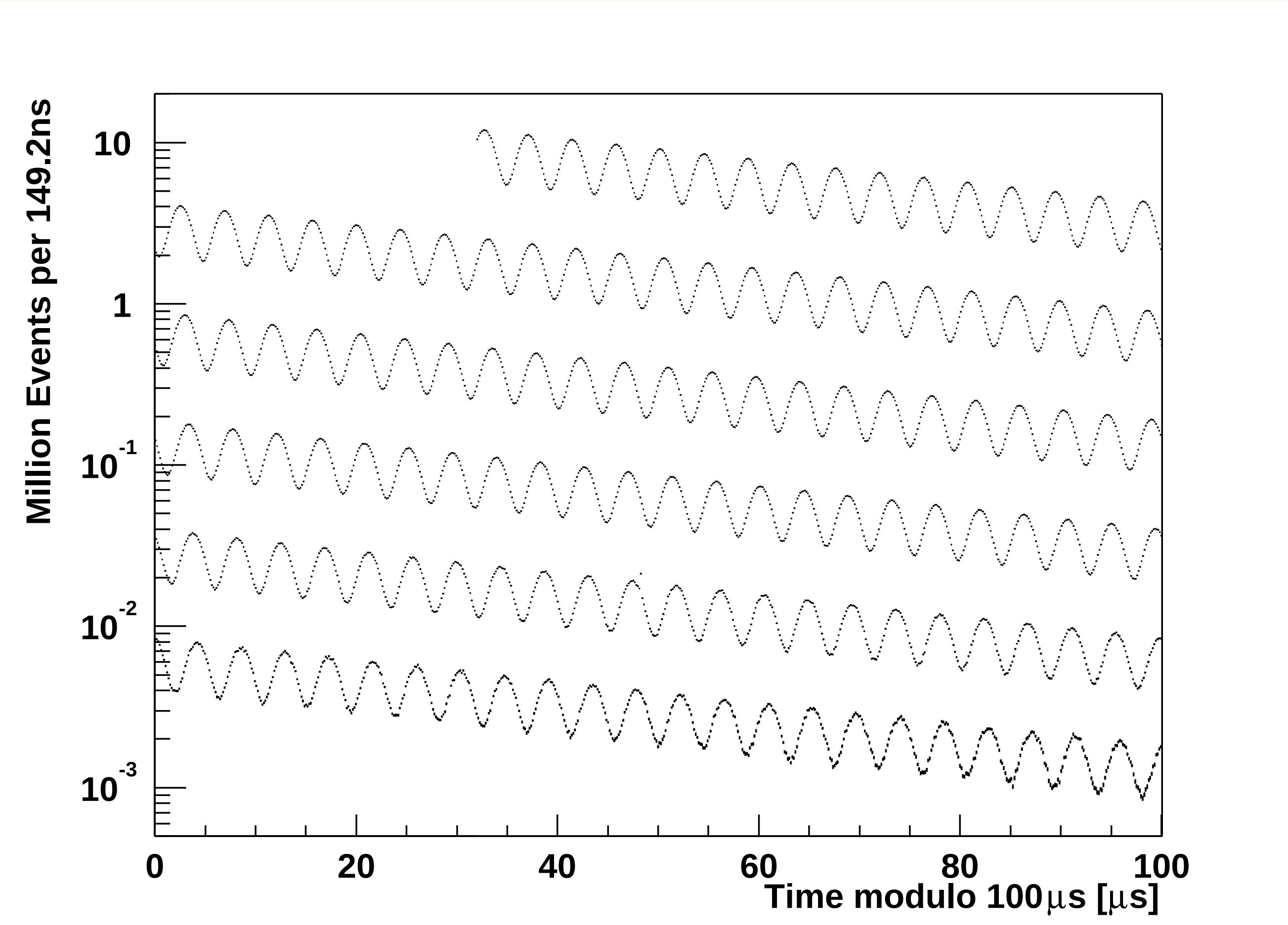
$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\,\vec{\beta}\times\vec{E}\right]_{\text{at "magic } \gamma\text{"}}^{E\sim 3.1\,\text{GeV}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

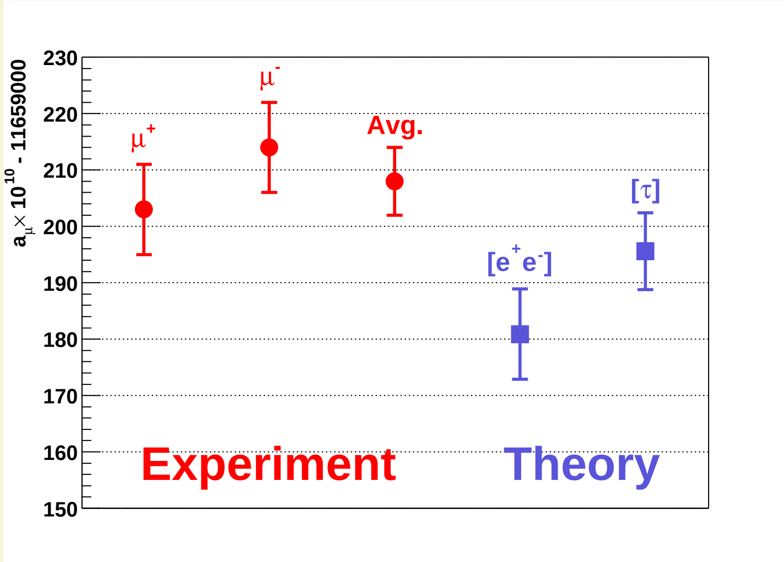
The BNL muon storage ring

Measured: ratio $\bar{R} = \omega_s/\omega_c$, ω_s = precession frequency, ω_c = cyclotron frequency; also need ratio μ_μ/μ_p between muon magnetic moment and proton magnetic moment $\Rightarrow$

$$a_\mu = \frac{\bar{R}}{\mu_\mu}\mu_p - \bar{R}$$

Distribution of counts versus time for the 3.6 billion decays in the 2001 negative muon data-taking period





$$a_\mu = 116592080(63) \cdot 10^{-11}$$

2.9 σ (e^+e^-) [1.4 σ (τ)]

BNL - E821

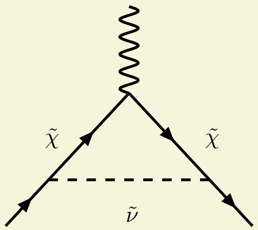
Muon (g-2) Collaboration

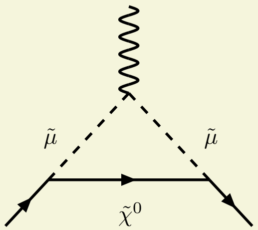
hep-ex/0401008/0602035

Uncertainties:	experiment	0.5ppm	=	6×10^{-10}	[5×10^{-10} stat, 4×10^{-10} syst]
	theory	0.6ppm	=	7×10^{-10}	(e^+e^-) [0.6ppm = 7×10^{-10} (τ)]
	new BNL 969 prop.	0.2ppm	=	2.4×10^{-10}	

New physics sensitivity: (example)

$$\Delta a_\mu^{\mathrm{SUSY}}/a_\mu \simeq 1.25\mathrm{ppm} \left(\frac{100\mathrm{GeV}}{\tilde{m}}\right)^2 \tan\beta$$





$\tilde{m}$ lightest SUSY particle; SUSY requires two Higgs dublets

$$\tan\beta = \frac{v_1}{v_2},\ v_i = < H_i > \quad ; \quad i = 1, 2$$

$$\tan\beta \sim m_t/m_b \sim 40 \quad [4 - 40]$$

most NP models yield

$$|a_\mu(\text{New Physics})| \simeq m_\mu^2/M^2 \ \frac{\alpha}{\pi} \Rightarrow \boxed{M \simeq 1-2 \text{ TeV}}$$

② Standard Model Prediction for a_μ

❏ QED Contribution

The QED contribution to a_μ has been computed (or estimated) through 5 loops

Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops.

New: $a_e = 0.001\,159\,652\,180\,85(76)$ **Gabrielse et al. 2006**

$$\alpha^{-1}(a_e) = 137.035\,999\,710\,(96)$$ **[0.70 ppb]**

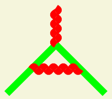
based on work of Kinoshita, Nio 04

$$a_\mu^{\rm QED} = 116\,584\,718.20\;\underbrace{(0.03)}_{\alpha^4}\underbrace{(1.15)}_{\alpha^5}\underbrace{(0.08)}_{\alpha_{\rm inp}}\times 10^{-11}$$

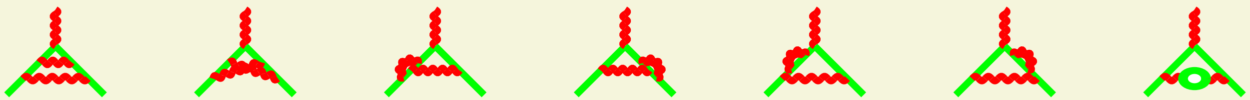
The current uncertainty is well below the $\pm 60 \times 10^{-11}$ experimental error from E821

# n of loops	C_i [$(\alpha/\pi)^n$]	$a_\mu^{\rm QED} \times 10^{11}$
1	+0.5	116140972.87 (0.44)
2	+0.765 857 376(27)	413217.60 (0.02)
3	+24.050 508 98(44)	30141.90 (0.00)
4	+126.07(41)	367.01 (1.19)
5	+930.0(170)	6.29 (1.15)
tot		116584705.66 (2.80)

➊ 1 diagram — Schwinger 1948



➋ 7 diagrams — Peterman 1957, Sommerfield 1957

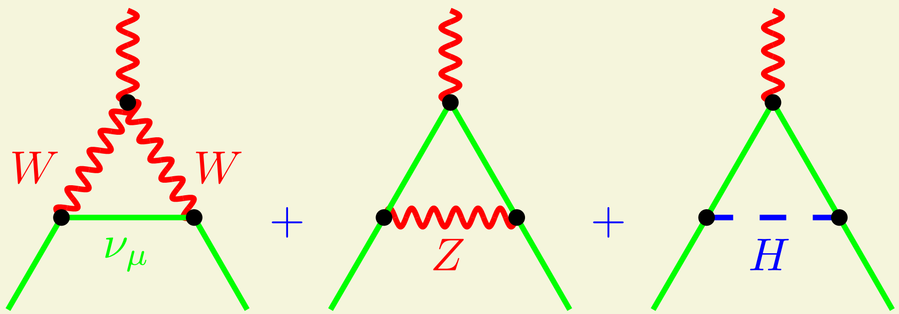


➌ 72 diagrams — Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

➍ about 1000 diagrams — Kinoshita 1999, Kinoshita, Nio 2004

➎ estimate of leading terms Karshenboim 93, Czarnecki, Marciano 00, Kinoshita, Nio 05

❑ Weak Contributions

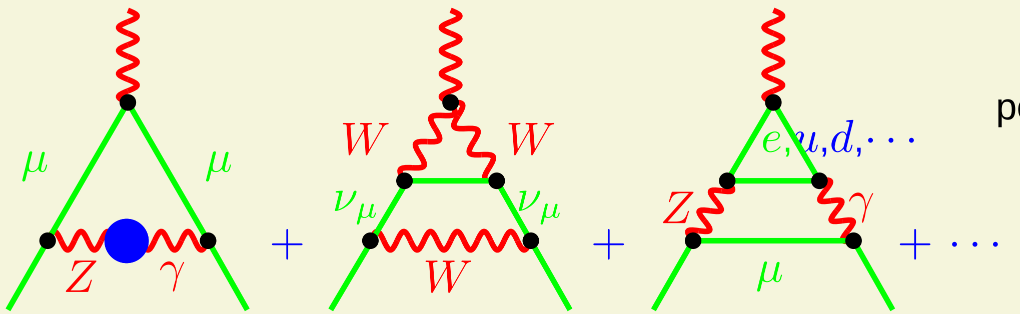


$$a_\mu^{\rm weak(1)} = (195 \pm 0) \times 10^{-11}$$

Brodsky, Sullivan 67, ...,

Bardeen, Gastmans, Lautrup 72

Higgs contribution tiny!



$$a_\mu^{\rm weak(2)} = -(44 \pm 4) \times 10^{-11}$$

Kukhto et al 92

potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$

Peris, Perrottet, de Rafael 95

quark-lepton (triangle anomaly) partial

Czarnecki, Krause, Marciano 96

Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05 final full 2–loop result known

Most recent evaluations: improved hadronic part (beyond QPM)

$a_\mu^{\rm weak} = (152 \pm 1[{\rm had}] \pm ?) \times 10^{-11}$ **(Knecht, Peris, Perrottet, de Rafael 02)**

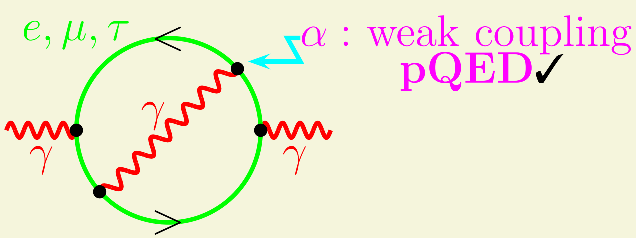
$a_\mu^{\rm weak} = (154 \pm 1[{\rm had}] \pm 2[{\rm m_H, m_t, 3-loop}]) \times 10^{-11}$ **(Czarnecki, Marciano, Vainshtein 02)**

❑ Hadronic Contributions

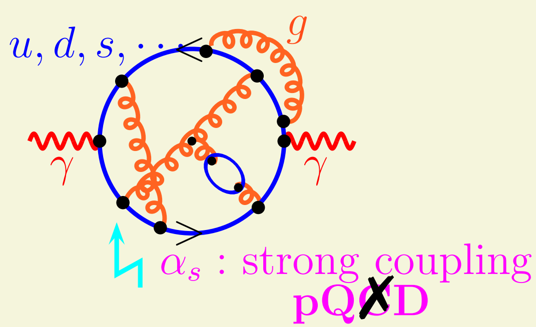
General problem in electroweak precision physics:

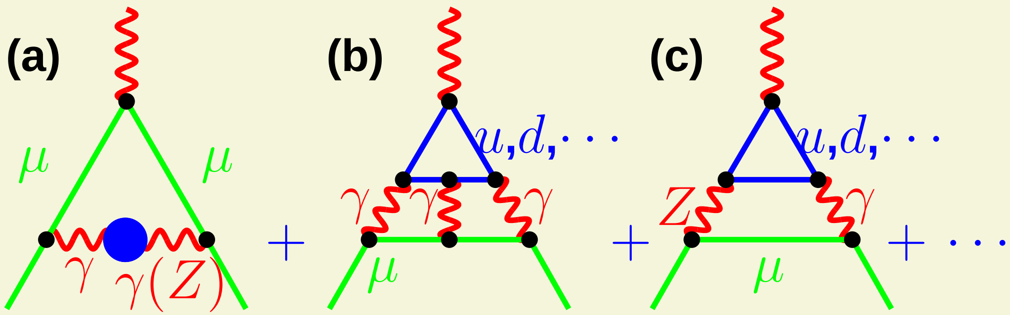
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks





(a) **Hadronic vacuum polarization** $O(\alpha^2), O(\alpha^3)$

(b) **Hadronic light-by-light scattering** $O(\alpha^3)$

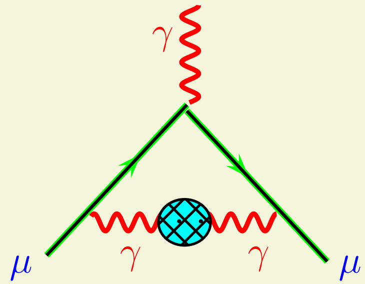
(c) **Hadronic effects in 2-loop EWRC** $O(\alpha G_F m_\mu^2)$

Light quark loops $\longrightarrow$ **Hadronic "blob"**

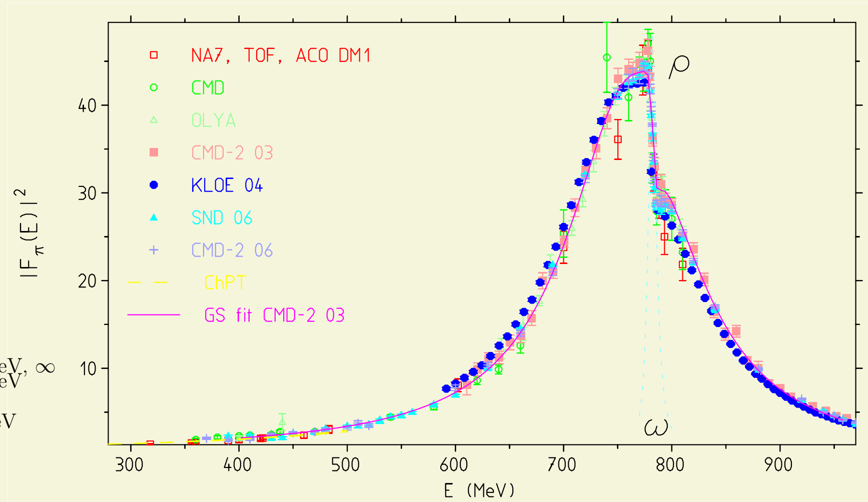
❑ Evaluation of $a_\mu^{\rm had}$

Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds \, \frac{R_\gamma^{\rm data}(s)\;\hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\;\hat{K}(s)}{s^2} \right)$$



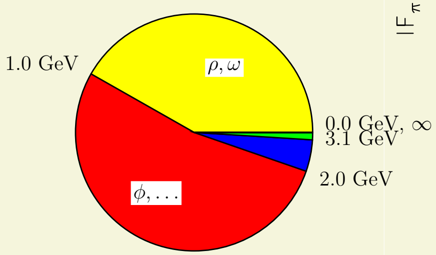
Data:...,CMD2,KLOE,SND
Key role: VEPP-2M/Novosibirsk, DAFNE/Frascati

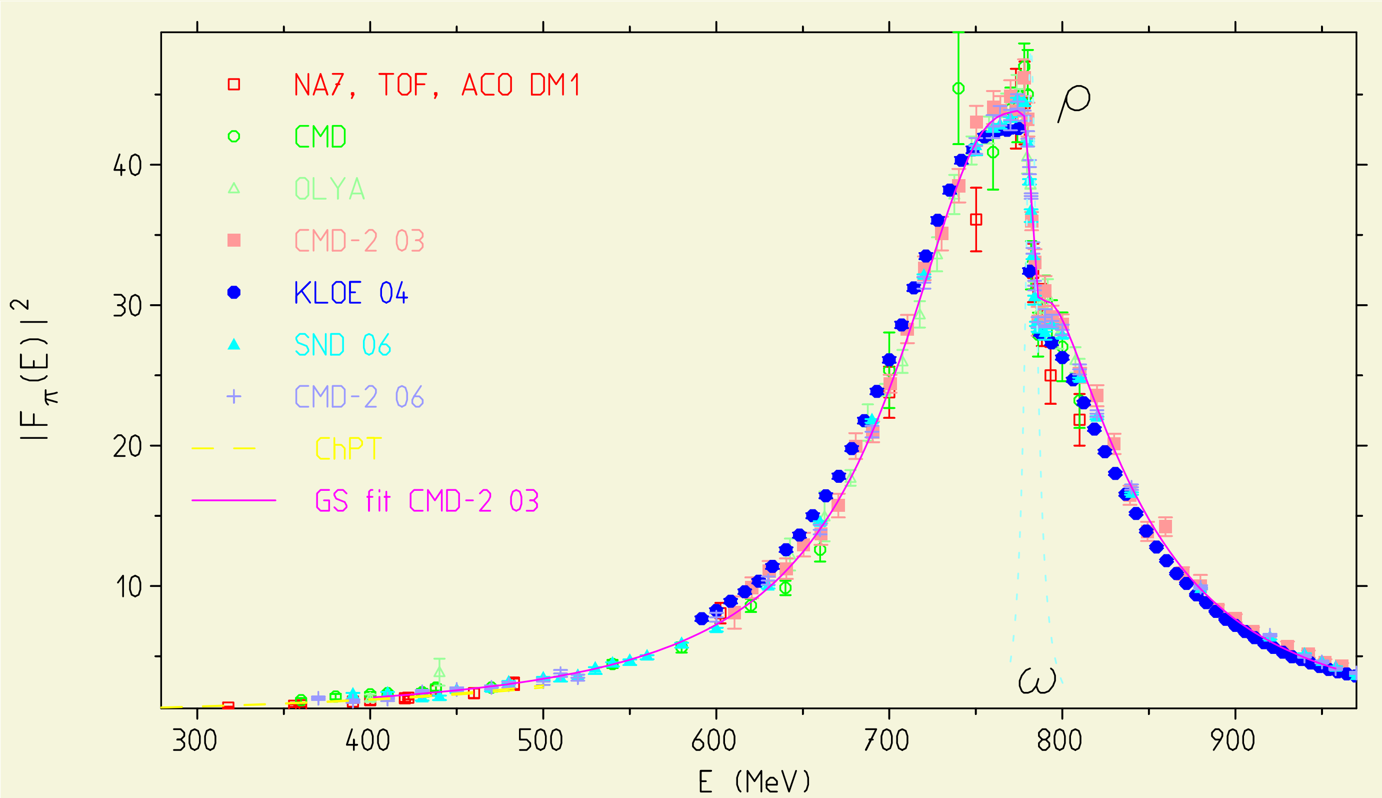


- **Experimental error implies theoretical uncertainty!**
- **Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$**

$$a_\mu^{\rm had(1)} = (692.0 \pm 6.0)\;10^{-10}$$

e^+e^-**–data based**

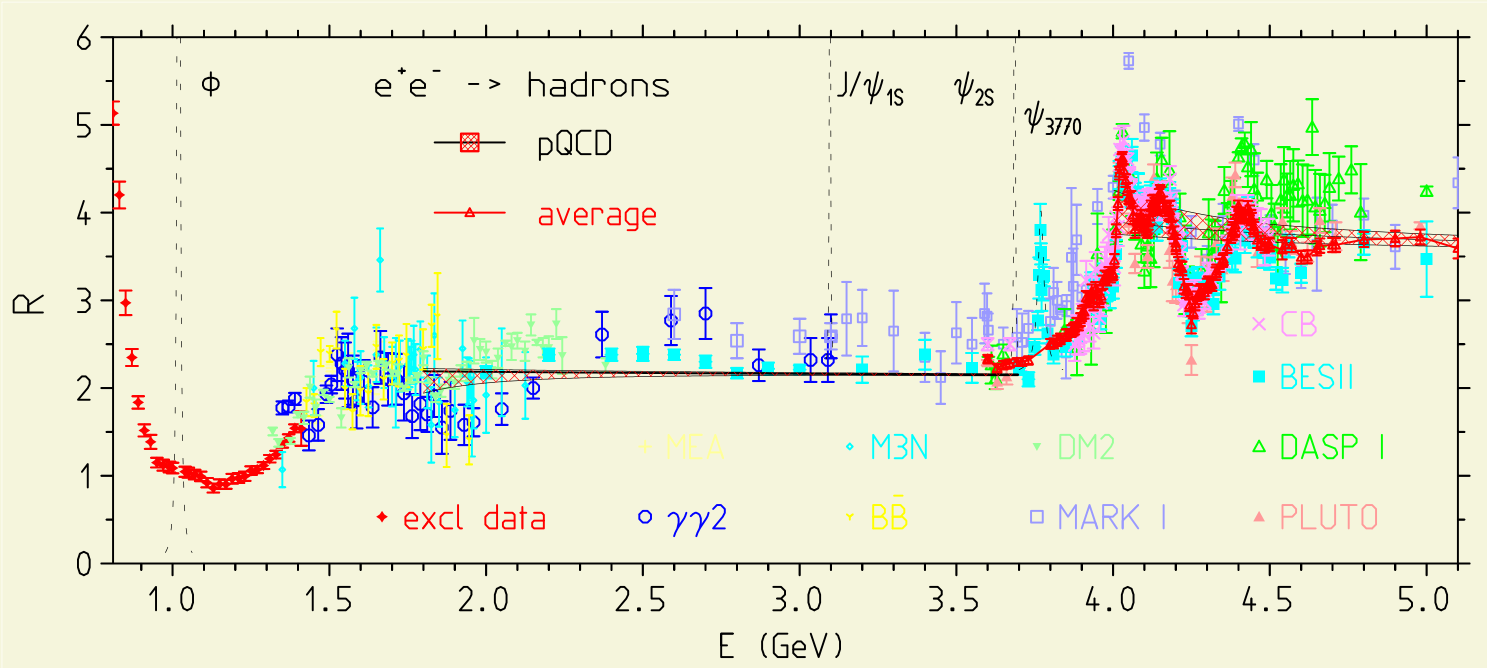




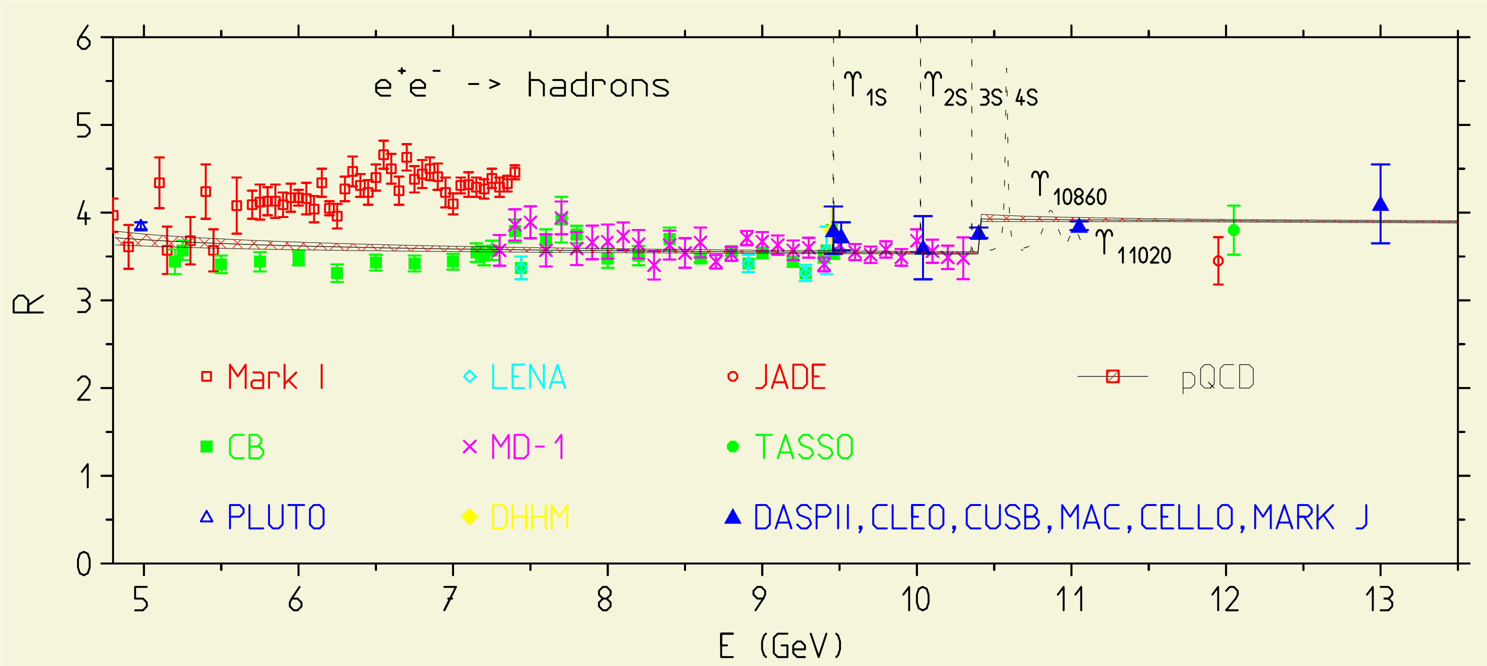
Novosibirsk – Frascati battle fields

after CMD-2 e^+e^- (direct) contra ALEPH τ (indirect); KLOE (new technique: radiative return) clearly is on CMD-2 side (3% discrepancy vs 10 to 20%), SND joined, at the moment τ–data not included since indirect and probably corrections not fully understood or experimental problem (Belle preliminary)

Soon: BABAR and Belle will join the battle new KLOE result awaited!



e+e- -> hadrons
pQCD
average
φ
J/ψ1S
ψ2S
ψ3770
excl data
γγ2
MEA
BB̄
M3N
DM2
MARK I
DASP I
PLUTO
CB
BESII
R
E (GeV)



e+e- -> hadrons
ϒ1S
ϒ2S
3S
4S
ϒ10860
ϒ11020
Mark I
LENA
JADE
pQCD
CB
MD-1
TASSO
PLUTO
DHHM
DASPII,CLEO,CUSB,MAC,CELLO,MARK J
R
E (GeV)

Recent evaluations of $a_\mu^{\rm had}$

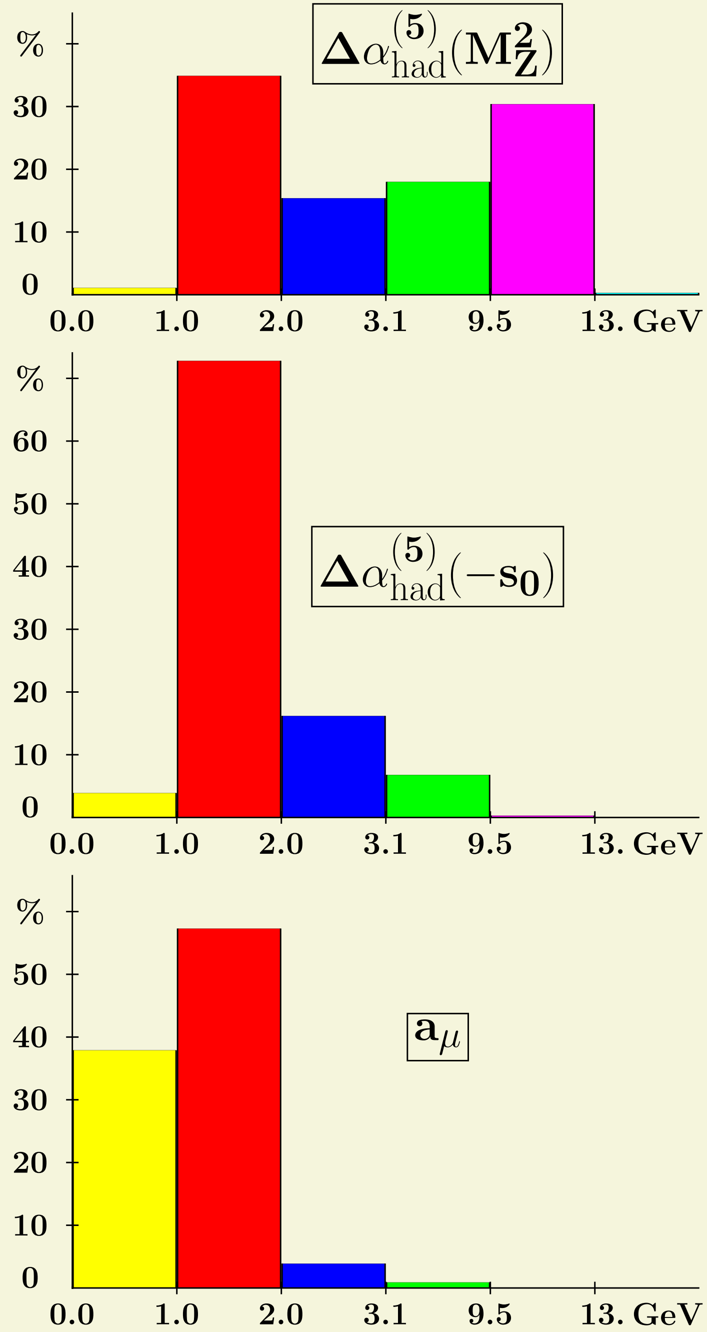
data	$a_\mu^{\rm had(1)} \times 10^{10}$	Ref.	hep-ph
e^+e^-	$694.8[8.6]$	FJ,GJ 03	0310181
e^+e^-	$692.0[6.0]$	FJ 06	0608xxx**
e^+e^-	$696.3[7.2](6.2)_{\rm exp}(3.6)_{\rm rad}$	DEHZ 03	0308213
e^+e^-	$690.9[4.4](3.9)_{\rm exp}(1.9)_{\rm rad}(0.7)_{\rm QCD}$	DEHZ 06	ICHEP06**
e^+e^-	$692.4[6.4](5.9)_{\rm exp}(2.4)_{\rm rad}$	HMNT 03	0312250
e^+e^-	$699.6[8.9](8.5)_{\rm exp}(1.9)_{\rm rad}(2.0)_{\rm proc}$	ELZ 03	0312114
τ	$711.0[5.8](5.0)_{\rm exp}(0.8)_{\rm rad}(2.8)_{SU(2)}$	DEHZ 03	0308213
e^+e^-	$693.5[5.9](5.0)_{\rm exp}(1.0)_{\rm rad}(3.0)_{\ell\times\ell}$	TY 04	0402285
$e^+e^- + \tau$	$701.8[5.8](4.9)_{\rm exp}(1.0)_{\rm rad}(3.0)_{\ell\times\ell}$	TY 04	0402285

Differences in errors mainly by utilizing more or less theory: pQCD, SR, low energy QCD methods

** **most recent data SND, CMD-2, BaBar included**

Contributions to $a_\mu^{\rm had} \times 10^{10}$ with relative (rel) and absolute (abs) error in percent.

Energy range	$a_\mu^{\rm had}$[%]$(\rm error) \times 10^{10}$	rel [%]	abs [%]
ρ, ω ($E < 2M_K$)	538.58 [77.8](3.84)	0.7	37.9
$2M_K < E < 2$ GeV	102.33 [14.8](4.72)	4.6	57.3
2 GeV $< E < M_{J/\psi}$	22.13 [3.2](1.23)	5.6	3.9
$M_{J/\psi} < E < M_\Upsilon$	26.39 [3.8](0.59)	2.2	0.9
$M_\Upsilon < E < E_{\rm cut}$	1.40 [0.2](0.09)	6.2	0.0
$E_{\rm cut} < E$ pQCD	1.53 [0.2](0.00)	0.1	0.0
$E < E_{\rm cut}$ data	690.83 [99.8](6.23)	0.9	100.0
total	692.36 [100.0](6.23)	0.9	100.0



Comparison of error profiles between

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

$\Delta\alpha^{(5)}_{\rm had}(-s_0)$

and

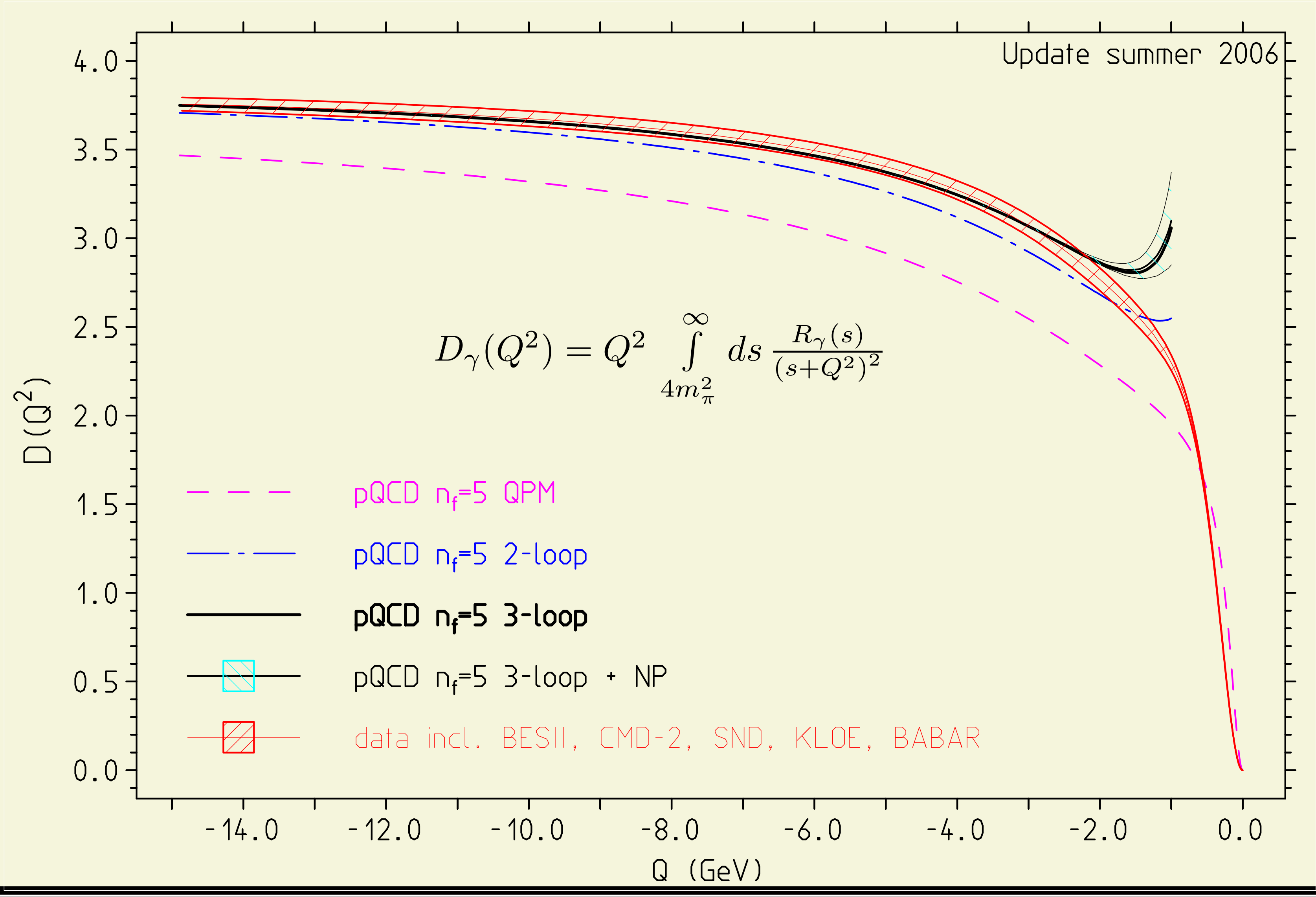
a_μ

Note high improvement potential

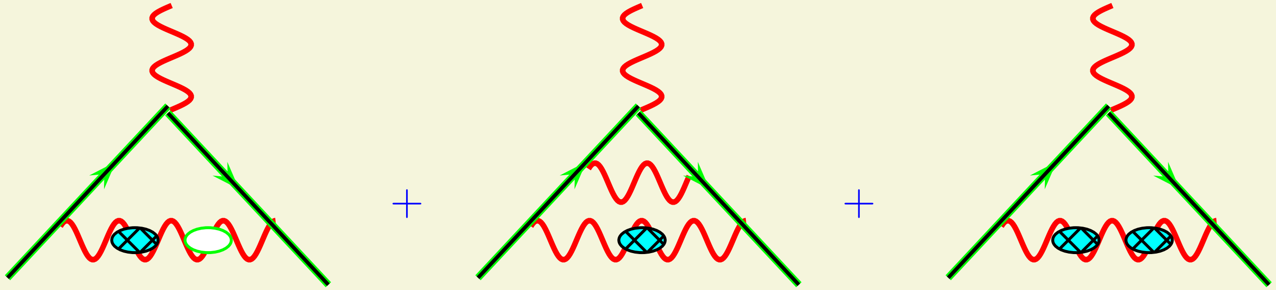
for

VEPP-2000 and DANAE/KLOE-2

"Experimental" Adler-function versus theory (pQCD + NP) in the low energy region ((EJKV98)). Note that the error includes both statistical and systematic ones, in contrast to R-data plots where only statistical errors are shown !!!.



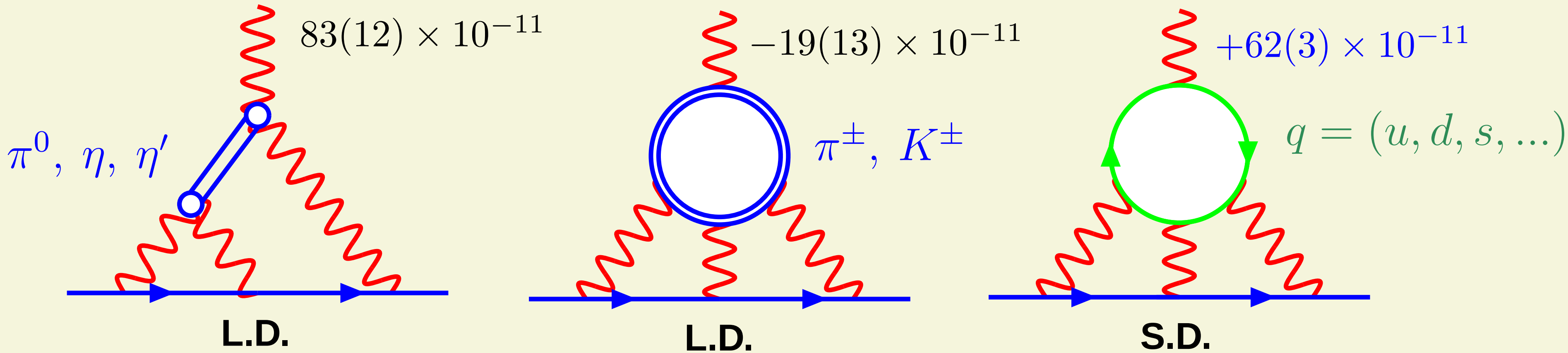
- **Higher order hadronic contributions** $a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$ **(Krause 96)**
  $a_\mu^{\mathrm{had}(2)} = -(\,98 \pm 1) \times 10^{-11}$ **(Hagiwara et al. 03)**



③ Hadronic Light-by-Light Scattering Contribution to $g-2$

- **Hadronic light–by–light scattering** $a_\mu^{\rm lbl} = (\,80 \pm 40)\ \times 10^{-11}$ **(Knecht&Nyffeler 02)**
  $a_\mu^{\rm lbl} = (136 \pm 25)\ \times 10^{-11}$ **(Melnikov & Vainshtein 03)**

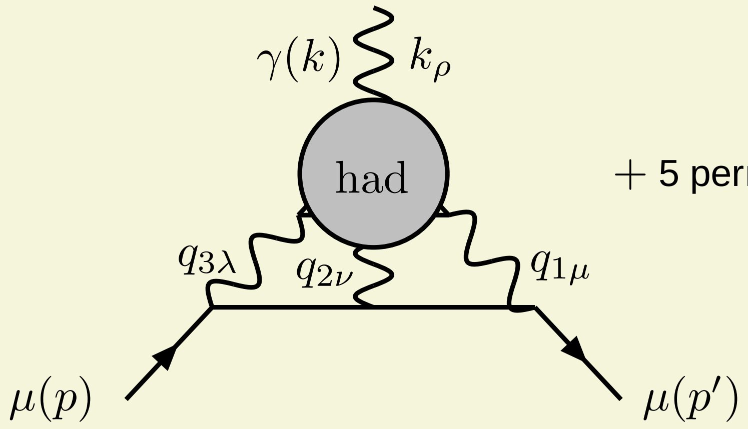
shift by $+56 \times 10^{-11}$



Low energy effective theory: e.g. ENJL

Kinoshita-Nizic-Okamoto 85, Hayakawa-Kinoshita-Sanda 95, Bijnens-Pallante-Prades 95

Hadrons in $< 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0 >$



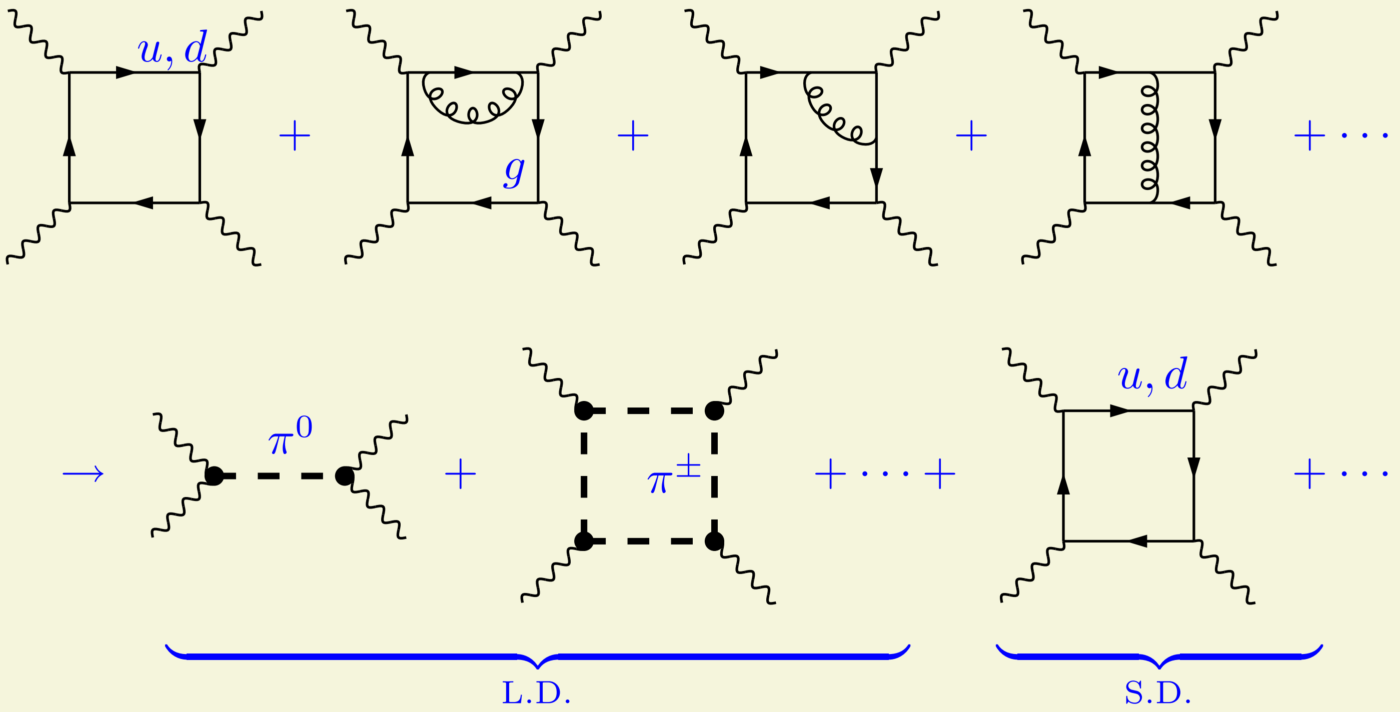
$+$ 5 permutations of the q_i

Key object full rank-four hadronic vacuum polarization tensor

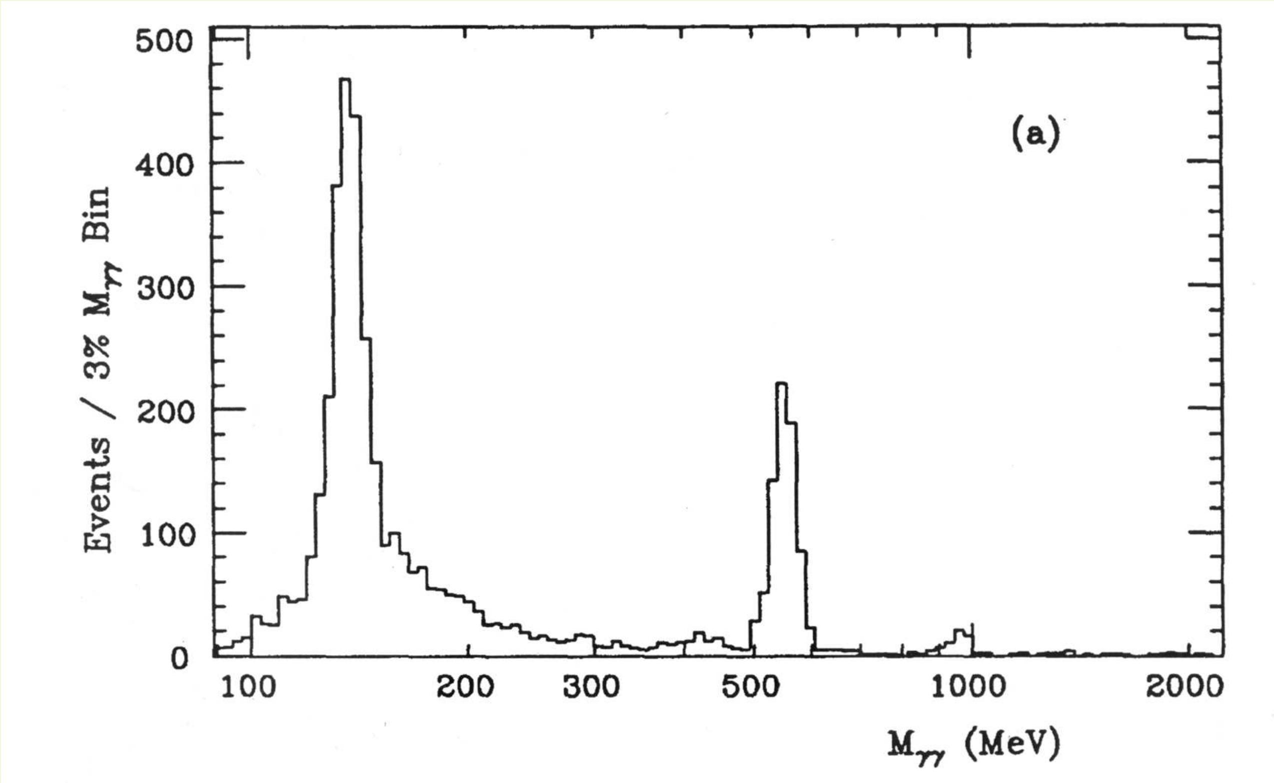
$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{e}^{\mathrm{i}\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\,0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\ .$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$

- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', \ldots$ described by the effective Wess-Zumino Lagrangian
- generally, pQCD useful to evaluate the short distance (S.D.) tail
- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons play key role

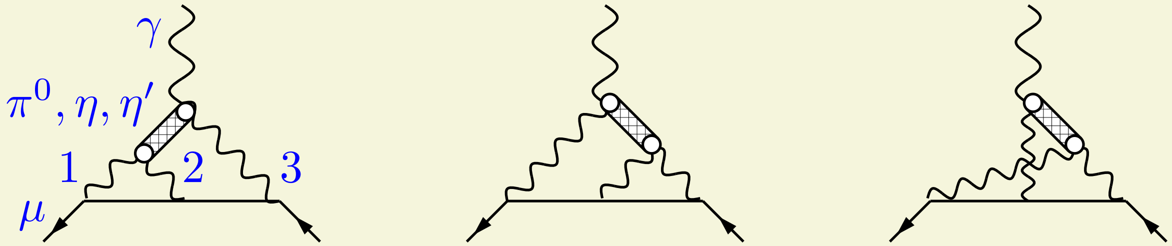


Hadronic light–by–light scattering is dominated by π^0 exchange in the odd parity channel, pion loops etc. at long distances (L.D.) and quark loops incl. hard gluonic corrections at short distances (S.D.)



The spectrum of invariant $\gamma\gamma$ masses obtained with the Crystal Ball detector. The three rather pronounced spikes seen are the $\gamma\gamma \to$ pseudoscalar (PS) $\to \gamma\gamma$ excitations: PS=π^0, η, η'

Pion-pole contribution **dominating hadronic contributions = neutral pion exchange**



Leading hadronic light–by–light scattering diagrams

The key object here is the $\pi^0\gamma\gamma$ **form factor** $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2)$ **which is defined by the matrix element**

$$\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{j_\mu(x)j_\nu(0)\}|\pi^0(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha p^\beta\, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q^2, (p-q)^2) \ .$$

Properties:

- **bose symmetric** $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, q_1^2, q_2^2) = \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, q_2^2, q_1^2)$
- **need it off-shell in integrals; if** $(s, q_1^2, q_2^2) \neq (m_\pi^2, 0, q^2)$ **in fact not known**
- **pion pole approximation in mind (pion pole dominance)** $s = m_\pi^2$
- **for generality use vertex function (not matrix element)**

$$\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{j_\mu(x)\, j_\nu(0)\, \tilde{\varphi}_{\pi^0}(p)\}|0\rangle = \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha p^\beta\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p^2, q^2, (p-q)^2) \times \frac{\mathrm{i}}{p^2 - m_\pi^2} \ ,$$

with $\tilde{\varphi}(p) = \int \mathrm{d}^4 y\, \mathrm{e}^{\mathrm{i}px} \varphi(y)$ the Fourier transformed π^0–field. $\Rightarrow$ representation in terms of form factors

To compute $a_\mu^{\mathrm{LbL};\pi^0} \equiv F_{\mathrm{M}}(0)|_{\text{pion pole}}$, we need $\mathrm{i}\,\frac{\partial}{\partial k^\rho} \Pi^{(\pi^0)}_{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2)$ at $k = 0$ where $p_3 = -(p_1 + p_2)$. Computing the Dirac traces yields

$$
\begin{aligned}
a_\mu^{\mathrm{LbL};\pi^0} \quad = \quad & -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]} \\
& \times \left[\frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2, q_1^2, q_3^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2, q_2^2, 0)}{q_2^2 - m_\pi^2}\, T_1(q_1, q_2; p) \right. \\
& \left. + \frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_3^2, q_3^2, 0)}{q_3^2 - m_\pi^2}\, T_2(q_1, q_2; p) \right] ,
\end{aligned}
$$

where $T_1(q_1, q_2; p)$ and $T_2(q_1, q_2; p)$ are scalar kinematics factors; two terms unified by bose symmetry.

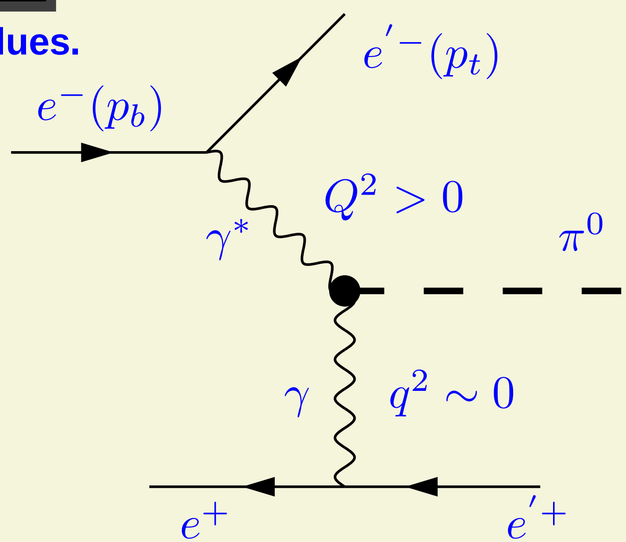
The pion-photon-photon transition form factor

- **Form factor function $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown**
- **Fortunately some experimental data is available:**
- **The constant $\boxed{\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)}$ is well determined by the $\pi^0 \to \gamma\gamma$ decay rate**

The invariant matrix element reads (follows from Wess-Zumino Lagrangian)($f_\pi \sim F_0$)

$$M_{\pi^0\gamma\gamma} = e^2\, \mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025 \text{ GeV}^{-1}\ ,$$

Information on $\boxed{\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)}$ comes from experiments $e^+e^- \to e^+e^-\pi^0$ where the electron (positron) gets tagged to high Q^2 values.



Measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2

Data for $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is available from CELLO and CLEO. Brodsky–Lepage interpolating formula

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\,\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)}$$

gives an acceptable fit to the data.

Assuming the pole approximation this FF has been used by all authors (HKS,BPP,KN) in the past, but has been criticized recently (MV).

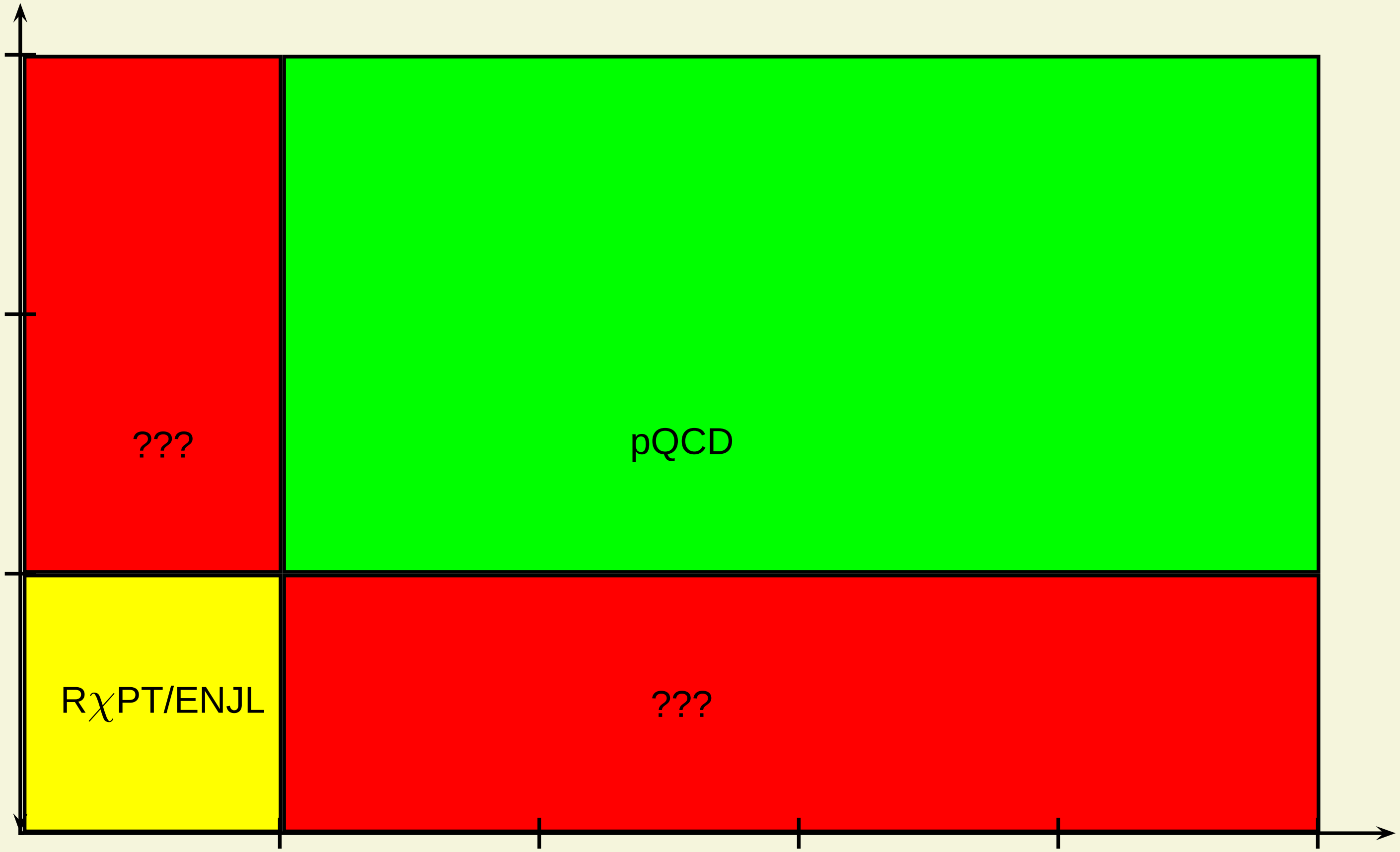
In fact in $g-2$ we are at zero momentum such that only the FF $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0) \neq \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable.

An alternative way to look at the problem is to use the anomalous PCAC relation and to relate $\pi^0\gamma\gamma$ to directly the ABJ anomaly.

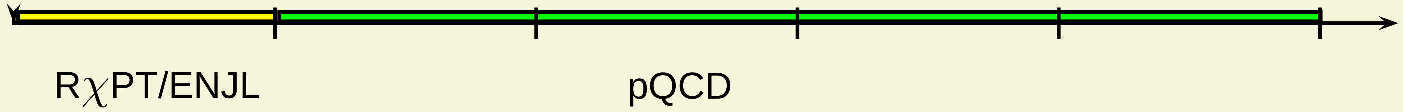
SD constraint from OPE:

Melnikov, Vainshtein: vertex with external photon must be non-dressed! i.e. no VDM damping $\Rightarrow$ result increases by **30%** !

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



Compare VP: s-axis



Originally VMD model (Kinoshita et al 85):

- ρ mesons play an important role in the game (see hadronic VP) $\Rightarrow$ looks natural to apply a vector meson dominance (VDM) model
- Naive VDM

$$\frac{\mathrm{i}}{q^2} \to \frac{\mathrm{i}}{q^2}\,\frac{m_\rho^2}{q^2-m_\rho^2} = \frac{\mathrm{i}}{q^2} - \frac{\mathrm{i}}{q^2-m_\rho^2} \quad .$$

- provides a damping at high energies, ρ mass as an effective cut-off (physical version of a Pauli-Villars cut-off) $\Rightarrow$ photons $\longrightarrow$ dressed photons!
- naive VDM violates electromagnetic WT-identity

Correct implementation in accord with low energy symmetries of QCD:

- vector meson extended CHPT (EχPT) model (Ecker et al 89)
- hidden local symmetry (HLS) model (Bando et al 85)
- extended NJL (ENJL) model (Dhar et al 85, Ebert, Reinhardt 86, Bijnens 96)

to large extend equivalent

Problem: matching L.D. with S.D. $\Rightarrow$ results depend on matching cut off Λ $\Rightarrow$ model dependence (non-renormalizable low energy effective theory vs. renormalizable QCD)

Novel approach: refer to quark–hadron duality of large-N_c QCD, hadrons spectrum known, infinite series of narrow spin 1 resonances ('t Hooft 79) $\Rightarrow$ no matching problem (resonance representation has to match quark level representation) (De Rafael 94, Knecht, Nyffeler 02)

⇒ *lowest meson dominance (LMD) plus one vector state (V)* approximation to large-N_c QCD allows for correct matching

⇒ "LMD+V" parametrization of $\pi^0\gamma\gamma$ form–factor (see below)

Summary of most recent results

$10^{11}\, a_\mu$	BPP	HKS	KN	MV
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10
π, K loops	-19 ± 13	-4.5 ± 8.1		0 ± 10
axial vector	2.5 ± 1.0	1.7 ± 0.0		22 ± 5
scalar	-6.8 ± 2.0	-	-	-
quark loops	21 ± 3	9.7 ± 11.1	-	-
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25

Note: MV and KN utilize the same model LMD+V form factor:

$$F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{4\pi^2F_\pi^2}{N_c}\,\frac{q_1^2q_2^2(q_1^2+q_2^2)-h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+(N_cM_1^4M_2^4/4\pi^2F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_1^2)(q_2^2+M_2^2)},$$

where $M_1 = 769\ \mathrm{MeV}$, $M_2 = 1465\ \mathrm{MeV}$, $h_5 = 6.93\ \mathrm{GeV}^4$.

with two modifications:

- form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2,q_2^2,0)=1$: undressed soft photon (non-renormalization of ABJ) Note: to have anomaly correct doesn not imply that there is no damping! PVV anomaly quark loop is counter example; it has correct $\pi\gamma\gamma$ in chiral limit (anomaly) and goes like $1/q_i^2$ up to logs in all directions
- $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2,q_1^2,q_3^2) \simeq \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2,q_1^2,q_3^2)$ = KN
with $h_2=0\pm 20$ GeV^2 (KN) vs. $h_2=-10$ GeV^2 (MV) fixed by twist 4 in OPE $(1/q^4)$
- $a_1[f_1,f_1^*]$ different mixing scheme; axial vector meson production requires one far off shell photon; again experimental result does not appy directly to g-2 for kinematical resons!

Criticism: KN Ansatz only covers $(0,q_1^2,q_2^2)$–plane, with consistent kinematics depends on 3 variables $\longrightarrow$ 2–dim integral representation no longer valid.

Is this the final answer? How to improve? A limitation to more precise $g-2$ tests?

Looking for new ideas to get ride of model dependence

In principle lattice QCD could provide an answer [far future (“yellow” region only)]

Theoretical models: Behavior of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ in the CQM=quark triangular loop

$$F^{\rm CQM}_{\pi^{0*}\gamma^*\gamma^*}(q^2,p_1^2,p_2^2) = 2m_q^2\,C_0(m_q,m_q,m_q;q^2,p_1^2,p_2^2) \equiv \int [{\rm d}\alpha]\,\frac{2m_q^2}{m_q^2-\alpha_2\alpha_3 p_1^2-\alpha_3\alpha_1 p_2^2-\alpha_1\alpha_2 q^2}\;,$$

where $[{\rm d}\alpha]={\rm d}\alpha_1{\rm d}\alpha_2{\rm d}\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$ and m_q constituent quark mass ($q=u,d,s$). For $p_1^2=p_2^2=q^2=0$ we obtain $F^{\rm CQM}_{\pi^{0*}\gamma^*\gamma^*}(0,0,0)=1$. Note the symmetry of C_0 under permutations of the arguments (p_1^2,p_2^2,q^2). For our purpose it is sufficient to calculate at one of the square momentum set to zero, one finds

$$F^{\rm CQM}_{\pi^{0*}\gamma^*\gamma^*}(0,p_1^2,p_2^2)=\frac{-m_q^2}{p_1^2-p_2^2}\left\{\ln^2\frac{\sqrt{4m_q^2-p_1^2}-\sqrt{-p_1^2}}{\sqrt{4m_q^2-p_1^2}+\sqrt{-p_1^2}}-(p_1^2\to p_2^2)\right\}.$$

For large p_1^2 at $p_2^2\sim 0,\ q^2\sim 0$ the asymptotic behavior is given by

$$F^{\rm CQM}_{\pi^{0*}\gamma^*\gamma^*}(0,p_1^2,0)\sim\frac{m_q^2}{-p_1^2}\;\left\{\ln^2\left(\frac{-p_1^2}{m_q^2}\right)\right\}\quad.$$

For large $p_1^2\sim p_2^2$ at $q^2\sim 0$ we have

$$F^{\rm CQM}_{\pi^{0*}\gamma^*\gamma^*}(0,p_1^2,p_1^2)\sim 2\,\frac{m_q^2}{-p_1^2}\;\left\{\ln\left(\frac{-p_1^2}{m_q^2}\right)\right\}$$

and the same behavior follows for $q^2 \sim p_1^2$ at $p_2^2 \sim 0$. Note that in all cases we have the same power behavior $\sim 1/p_1^2$ modulo logarithms. It is important to note that in the chiral limit $F^{\mathrm{CQM}}_{\pi^{0*}\gamma^*\gamma^*} \stackrel{m_q\to 0}{\to} 0$ if $(q^2,p_1^2,p_1^2) \neq (0,0,0)$. Thus our consideration seems to be not quite relevant, as it says that the chiral corrections at high energies are damped by a $1/Q^2$ behavior in all the relevant directions. The dominant terms come from the chiral limit, but surprisingly the CQM calculation also sheds light on this leading contribution as we shall discuss now. Actually, the singular behavior of $F^{\mathrm{CQM}}_{\pi^{0*}\gamma^*\gamma^*}$ under exchange of limits:

$$\lim_{m_q\to 0} \quad F^{\mathrm{CQM}}_{\pi^{0*}\gamma^*\gamma^*}(q^2,p_1^2,p_1^2) \equiv 0 \;\; \text{for all} \;\; (q^2,p_1^2,p_1^2) \neq (0,0,0)$$

$$\lim_{(q^2,p_1^2,p_1^2)\to(0,0,0)} \quad F^{\mathrm{CQM}}_{\pi^{0*}\gamma^*\gamma^*}(q^2,p_1^2,p_1^2) \equiv 1 \;\; \text{for all} \;\; m_q \neq 0$$

implies that the chiral limit is either zero or unity,

$$\lim_{m_q\to 0} \lim_{(q^2,p_1^2,p_1^2)\to(0,0,0)} F^{\mathrm{CQM}}_{\pi^{0*}\gamma^*\gamma^*}(q^2,p_1^2,p_1^2) \equiv 1 \;,$$

depending on whether $(q^2,p_1^2,p_1^2) \neq (0,0,0)$ and $(q^2,p_1^2,p_1^2) = (0,0,0)$, respectively. This singular behavior is an alternative form of expressing the ABJ anomaly and the non-renormalization theorem. For the pseudoscalar vertex the latter just means that the last identity to all orders of perturbation theory yields a pure number, which always may be renormalized to unity by a renormalization of the axial current, the divergence of which being the interpolating field of the pseudoscalar Goldstone mode

involved[a]. Amazingly, the pseudoscalar vertex (at one loop) is UV finite the two vector currents are trivially conserved, because of the $\varepsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta$ tensor structure, and we obtain the ABJ anomaly as a IR phenomenon and not as a UV renormalization effect as it appears if one looks at the VVA matrix element. Since the anomaly is exact to all orders and at all energy scales, it is not surprising that it may be obtained from the IR region as well. In addition, the way it is obtained as an IR effect, namely, as the $(q^2,p_1^2,p_1^2)=(0,0,0)$ chiral limit piece of the form factor, everything else being suppressed chiral symmetry breaking effects, it is clear that we are dealing with a constant at all scales.

Strong indication:

- m_q^2/Q^2 in all directions of phase space: dressing by chiral SB effects (quark measses); in pQCD (relevant for HE behavior) permutation symmetry.
- Effective theory approach (properly implemented VDM) all far off shell legs dressed.

[a] The anomaly cancellation required by renormalizability of a gauge theory here just would mean the absence of a non-smooth chiral limit

Summary of contributions:

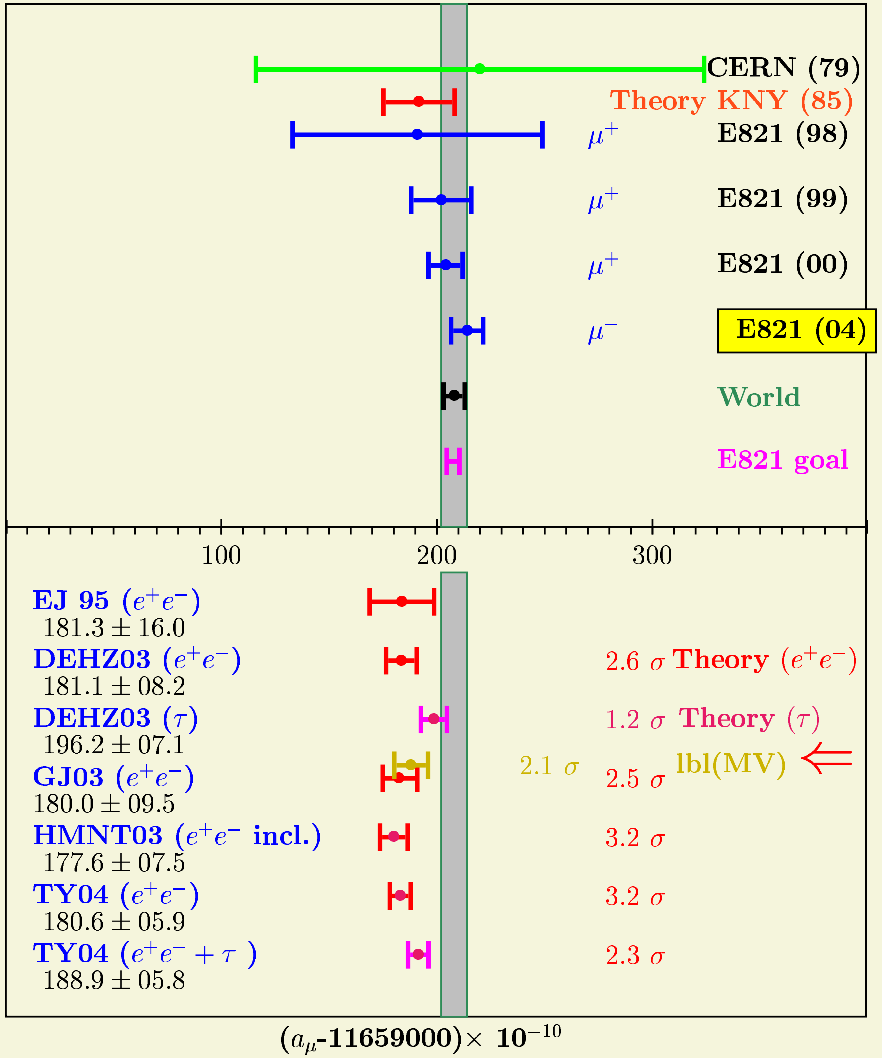
a_μ in units 10^{-6}, ordered according to their size (L.O. lowest order, H.O. higher order, LBL. light–by–light); ($\alpha^{-1} = 137.0359991100$)

L.O. universal	$1161.409\,73\ \ (0)$
e–loops	$6.194\,57\ \ (0)$
H.O. universal	$-1.757\,55\ \ (0)$
L.O. hadronic	$0.069\,20\ \ (60)$
L.O. weak	$0.001\,95\ \ (0)$
H.O. hadronic	$-0.000\,98\ \ (1)$
LBL. hadronic	$0.001\,10\ \ (40)$
τ–loops	$0.000\,43\ \ (0)$
H.O. weak	$-0.000\,41\ \ (2)$
e+τ–loops	$0.000\,01\ \ (0)$
theory	$1165.918\,03\ \ (72)$
experiment	$1165.920\,80\ \ (60)$

"New Physics" ???

$$a_\mu^{\rm exp} - a_\mu^{\rm the} = 277 \pm 94 \cdot 10^{-11} \quad [2.9\,\sigma]$$

a_μ Summary: experiment vs. theory



Given theory results only differ by $a_\mu^{\mathrm{had}(1)}$!

④ Outlook

- BNL $g-2$ experiment 14–fold improvement (vs CERN 1979) to 0.54ppm

$$a_\mu = 116592080(54)(33) \cdot 10^{-11}$$

- SM Test:
  - substantial improvement of CPT test
  - confirms for the first time weak contribution: $2\sigma - 3\sigma$
  - very sensitive to precise value of the hadronic contribution: limits theoretical precision $2\sigma \to 1\sigma$, now $\sim \delta$LBL
  - Light-by-Light (model-dependent) not far from being as important as the weak contribution, may become the limiting factor for future progress (theory?)

- New Physics ??? 2.9 [3.4] σ

$$|a_\mu^{\rm exp} - a_\mu^{\rm SM}| = (28 \pm 9)[(27 \pm 8)] \times 10^{-10}$$

Small discrepancy persists, however, established deviation from the SM, in contrary: very strong constraint on many NP scenarios

- On “prediction” side:
  - big experimental challenge: attempt cross-section measurements at 1% level up to J/ψ[Υ] !!! crucial for $g-2$ and $\alpha_{\rm QED}(M_Z)$ at GigaZ
  - new ideas on how to get model–independent hadronic LbL contribution
  - theory of pion FF below 1 GeV allows for further improvement (Colangelo, Gasser, Leutwyler); constraints from $\pi\pi$ scattering phase shifts
- Plans for new $g-2$ experiment!

Present:	BNL E821	0.5 ppm
Future:	BNL E969	0.2 ppm
	J-PARC	0.1 ppm

History of sensitivity to various contributions