

# PHOKHARA:

where we are and what is to come



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Working group on radiative corrections and  
generators for low energy hadronic cross-section  
and luminosity, 16-17 October 2006, LNF Frascati

# PHOKHARA 5.1 (July 2006)

## Fixed order radiative corrections: NLO accuracy

### Hadronic channels

$$\pi^+\pi^-$$

*+ radiative phi decays*

$$\mu^+\mu^-$$

*background and normalization*

$$2\pi^0\pi^+\pi^-, 2\pi^+2\pi^-$$

$$p\bar{p}, n\bar{n}$$

*Pauli and Dirac Form Factors*

$$\pi^0\pi^+\pi^-, K^+K^-, K^0\bar{K}^0$$

*new channels*

### Tagged or untagged photons

### Modular structure: easy replacement of hadronic form factors



# PHOKHARA

radiative return at meson factories

## Physics

Electron--positron annihilation into hadrons plus an energetic photon from initial state radiation (ISR) allows the hadronic cross-section to be measured over a wide range of energies at high luminosity meson factories [[DAPHNE](#), [CESR](#), [PEP-II](#), [KEK-B](#)].

## Content

**PHOKHARA** is a Monte Carlo event generator which simulates this process at the next-to-leading order (NLO) accuracy. This includes virtual and soft photon corrections to one photon emission events and the emission of two real hard photons.

## Downloads

**VERSION 5.1 (July 2006):** New parameters and functional form of the **three-pion** form factor.

- manual [[Postscript](#), [PDF](#)], source [[uuencoded](#)]

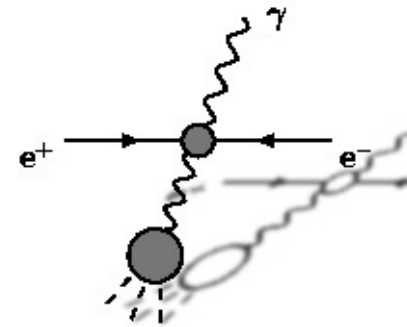
## Forthcoming features

- Full one-loop radiative corrections for muon production
- Simulation of narrow resonances ( $J/\psi$  and  $\psi(2S)$ )
- Simulation of other exclusive hadronic channels
- FSR for three pion production

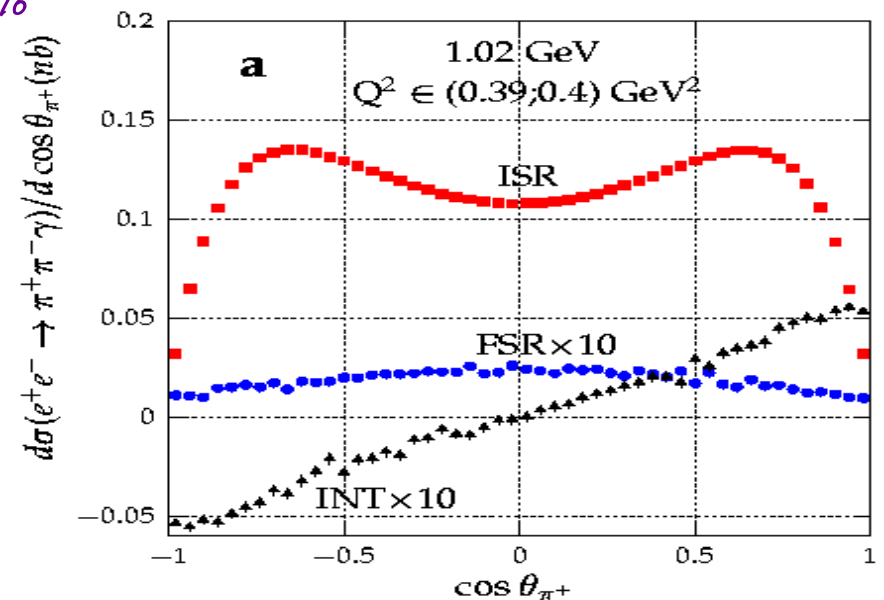
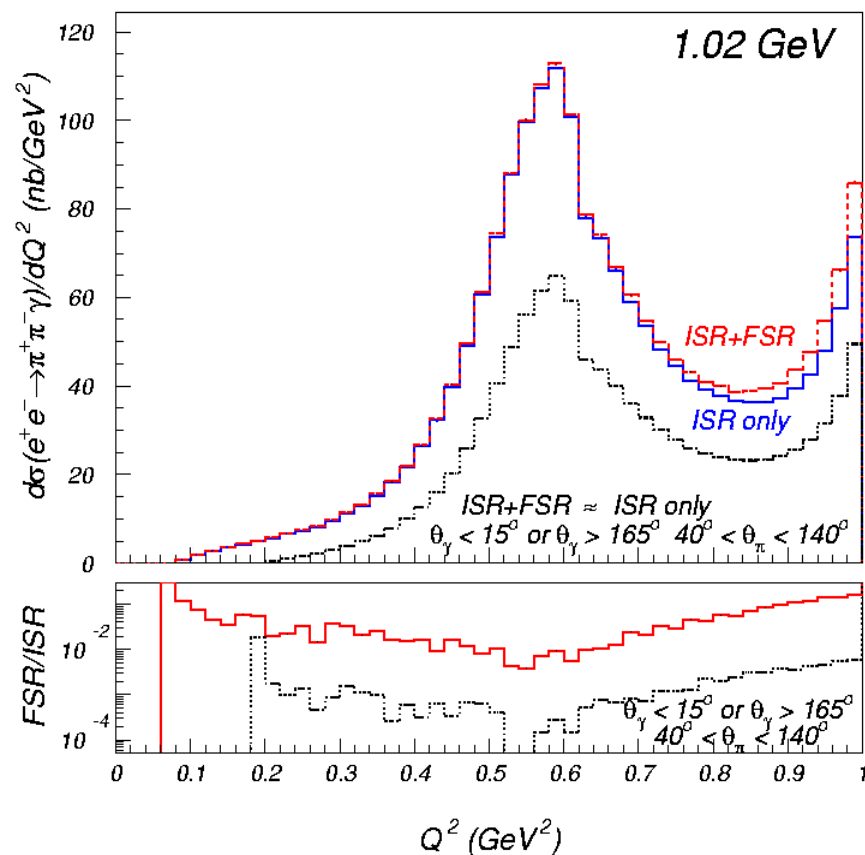
# FSR

suppressed at **B-factories**:

very hard photons for low hadronic invariant masses



**DAPHNE**: ISR dominates for untagged photons (small angle), but suppress threshold tail  
tagged photons (large angle) 10-20%



$$A_{FB}(Q^2) = \frac{N(\theta_{\pi^+} > \pi/2) - N(\theta_{\pi^+} < \pi/2)}{N(\theta_{\pi^+} > \pi/2) + N(\theta_{\pi^+} < \pi/2)}(Q^2)$$

$$A_C(\theta_\pi) = \frac{N(\pi^+) - N(\pi^-)}{N(\pi^+) + N(\pi^-)}(\theta_\pi)$$

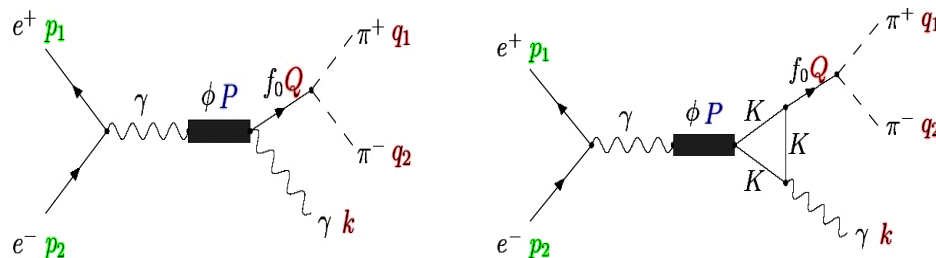
# radiative phi decays

Czyż, Grzelinka, Kühn, PLB611(05)116,  
KLOE PLB634(05)148

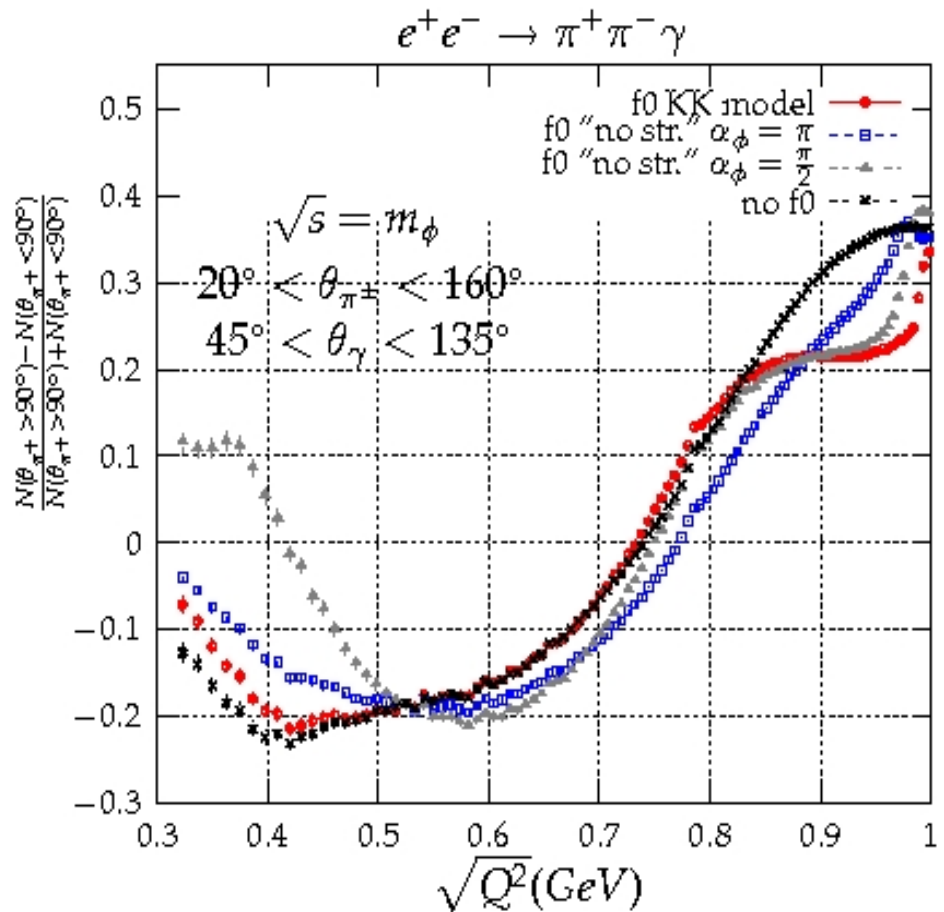
$e^+ e^- \rightarrow \phi \rightarrow \pi^+ \pi^- \gamma$  pollutes the extraction of  $\sigma(e^+ e^- \rightarrow \pi^+ \pi^-)$   
close to the phi mass

charge asymmetry allows to discriminate  
between different models of the radiative  
decay  $\phi \rightarrow \pi^+ \pi^- \gamma$ ,  $\pi^0 \pi^0 \gamma$ ,

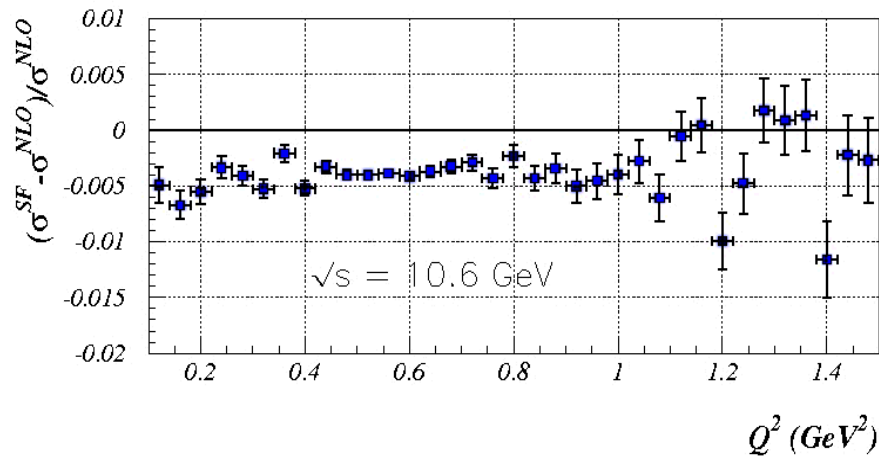
$$\phi \rightarrow (f_0(980) + f_0(600)) \gamma \rightarrow \pi \pi \gamma$$



other contributions (beyond sQED +  
VMD + radiative phi decays) might be  
important in the threshold region  
[Pancheri, Shekhovtsova, Venanzoni]



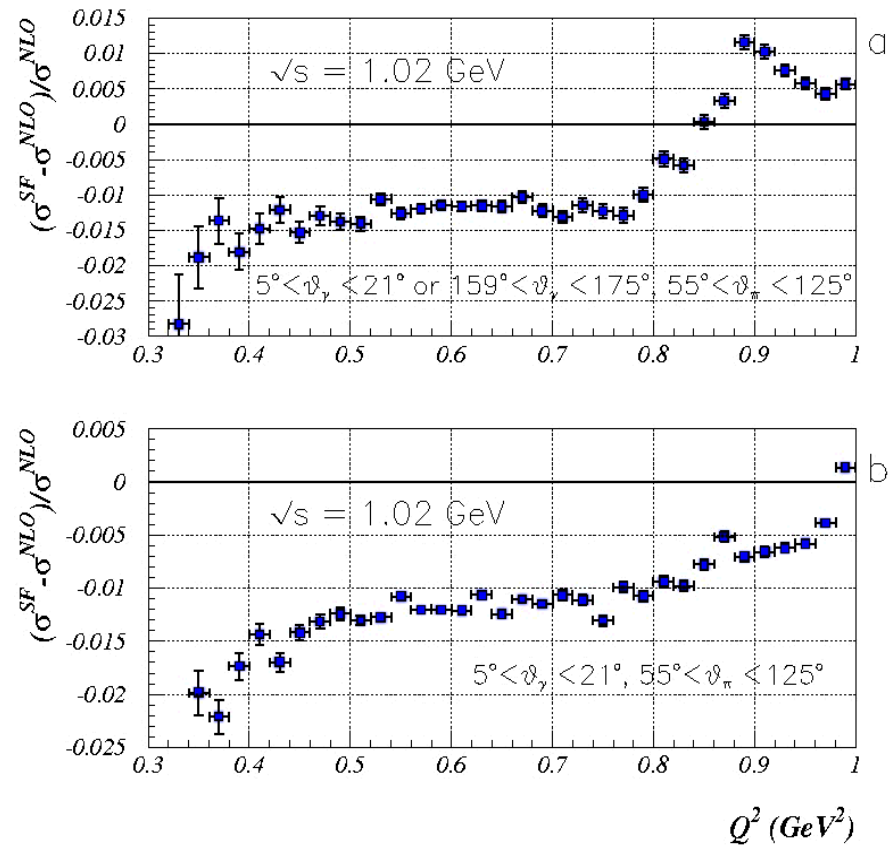
# NLO vs SF



LL: EVA [Binner, Melnikov, Kühn]

EVA4 $\pi$  [Czyz, Kühn]

- resums big logs  $L = \text{Log}(s/m_e^2)$  to all orders
- Extra collinear emission integrated out: no momentum conservation
- Untagged photon: double counting

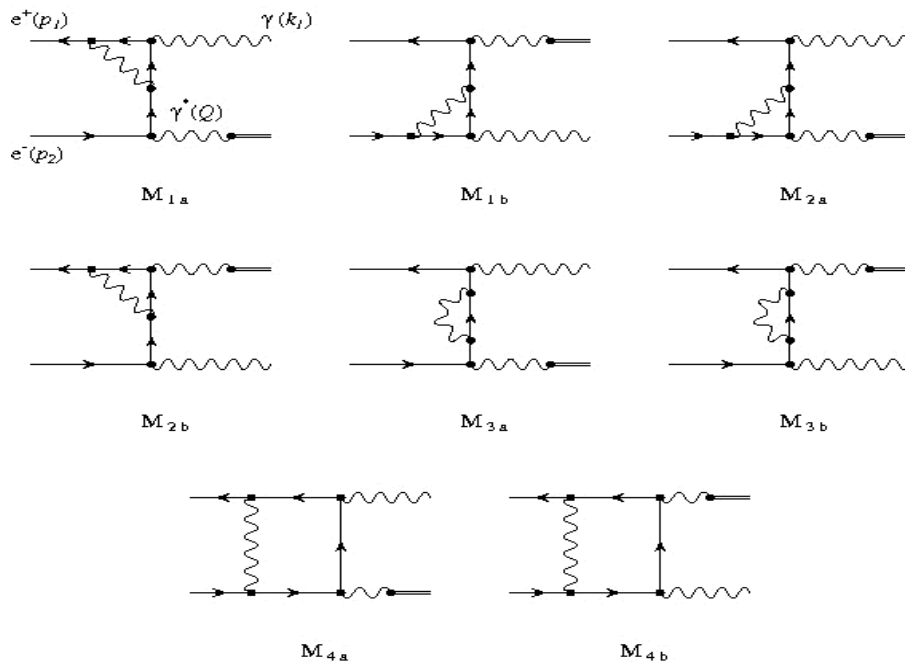


NLO: PHOKHARA

- LL+subleading terms (1%)
- Full angular dependence
- Momentum conservation
- Tagged or untagged photons
- ISR accuracy 0.5%



# Leptonic Tensor



ISR virtual+soft corrections to  
 $e^+e^- \rightarrow \text{hadrons} + \gamma$   
 factorizable

$$\sigma = \int L_{ISR}^{\mu\nu} H_{\mu\nu}$$

independent of the hadronic  
 channel

assuming conserved currents  $Q_\mu J^\mu = 0$   
 the **leptonic tensor** has the following form

$$L_{ISR}^{\mu\nu} = \alpha^2 [a_{00} g^{\mu\nu} + a_{11} p_1^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{12} (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) + i\pi a_{-1} (p_1^\mu p_2^\nu - p_2^\mu p_1^\nu)]$$

where at NLO

$$a_{ij} = a_{ij}^{(0)} + \frac{\alpha}{\pi} a_{ij}^{(1)}, \quad a_{-1} = \frac{\alpha}{\pi} a_{-1}^{(1)}$$



real corrections in **PHOKHARA** through helicity amplitudes

# From Leptonic Tensors to Helicity amplitudes

The one-loop helicity amplitude

$$|A\rangle = |A\rangle_{ISR} + |A\rangle_{FSR} + |A\rangle_{2\gamma^*}$$

$$|A\rangle_{ISR} = \frac{-i}{Q^2} (L_{ISR}^{(1)\mu} M_{\mu}^{(0)} + L_{ISR}^{(0)\mu} M_{\mu}^{(1)})$$

$$|A\rangle_{FSR} = \frac{-i}{S} (L_{FSR}^{(0)\mu} M_{FSR\mu}^{(1)} + L_{FSR}^{(1)\mu} M_{FSR\mu}^{(0)})$$

imposing QED Ward identity ( $L_{ISR}^{\mu} |_{\epsilon_1 \rightarrow k_1} = 0$ ) and conservation of vector currents ( $L_{ISR}^{\mu} Q_{\mu} = 0$ ): **14 independent** spinorial chains at one-loop

$$L_{ISR}^{\mu} = \frac{-ie^2}{S} \sum_{i=1}^{14} V_i(p_1, p_2, k_1) \bar{v}(p_1) S_{ISR}^{(i)\mu} u(p_2) ,$$

$$L_{ISR}^{\mu\nu} = \frac{-1}{Q^2} \overline{L_{ISR}^{\mu}} L_{ISR}^{\nu}$$

$$S_{ISR}^{(1)\mu} = \frac{2\epsilon_1 \cdot p_1 - \epsilon_1 k_1}{2 p_1 \cdot k_1} \gamma^{\mu} - \gamma^{\mu} \frac{2\epsilon_1 \cdot p_2 - k_1 \epsilon_1}{2 p_2 \cdot k_1}$$

$$S_{ISR}^{(2)\mu} = \frac{1}{2} (k_1 \epsilon_1 \gamma^{\mu} - \gamma^{\mu} \epsilon_1 k_1)$$

$$S_{ISR}^{(3,4)\mu} = k_1 \epsilon_1 (p_1 \pm p_2)^{\mu} \quad S_{ISR}^{(5)\mu} = [\epsilon_1, k_1]$$

$$S_{ISR}^{(6,7)\mu} = [(p_1 \pm p_2) \cdot (k_1^{\mu} \epsilon_1 - \epsilon_1^{\mu} k_1), \gamma^{\mu}] \quad \dots$$

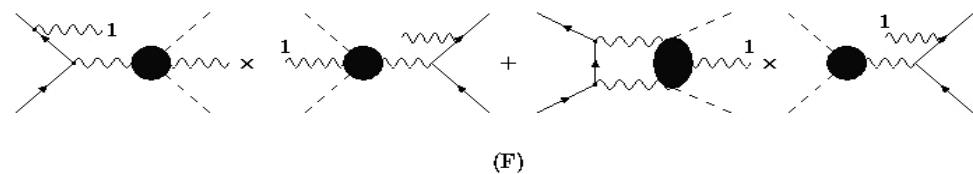
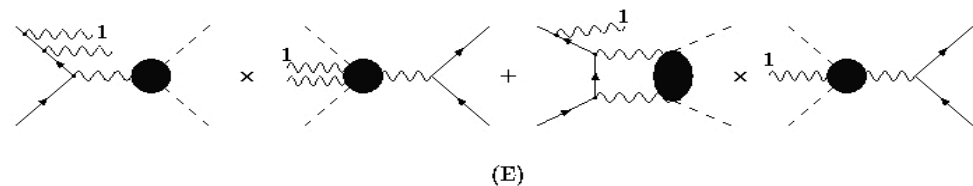
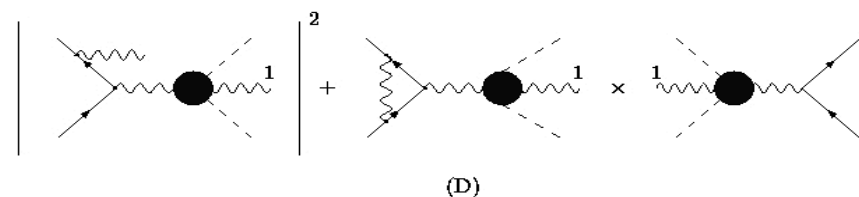
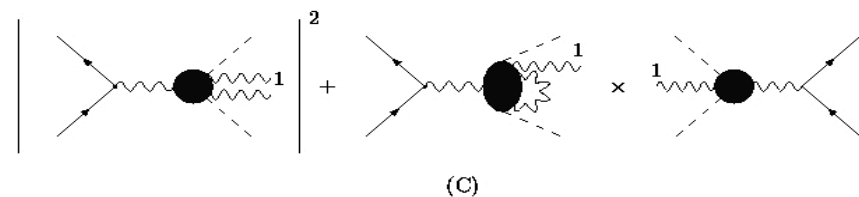
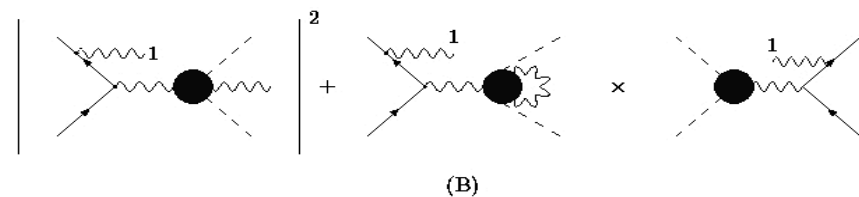
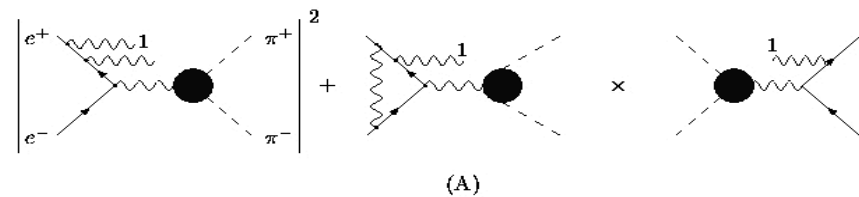
Coefficients  $V_i$  evaluated including terms of order  $m_e^2/s$  and  $m_e^4/s^2$  enhanced by singularities  $1/\theta_{\gamma}^4$  and  $1/\theta_{\gamma}^6$   
similar for FSR but muon mass dependence kept exact



# FSR @ NLO

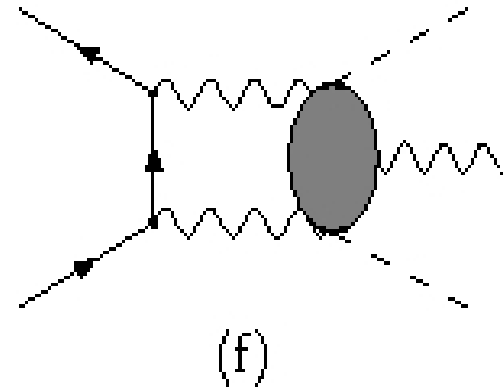
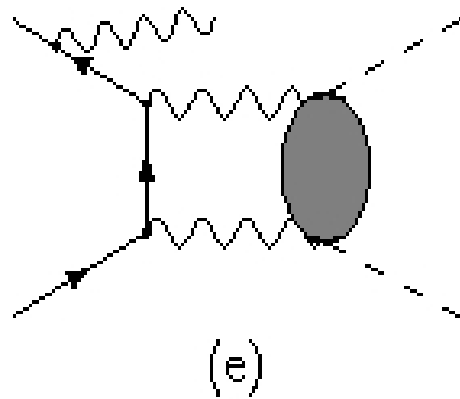
FSR @ NLO dominated by  
simultaneous emission of one  
photon from FSR and another one  
from ISR (+ virtual corrections)

**PHOKHARA** includes at present  
gauge invariant sets of amplitudes  
which lead to infrared-finite  
charge-even combinations for  
 $\pi^+\pi^-$  and  $\mu^+\mu^-$



# Two photon exchange

non factorizable



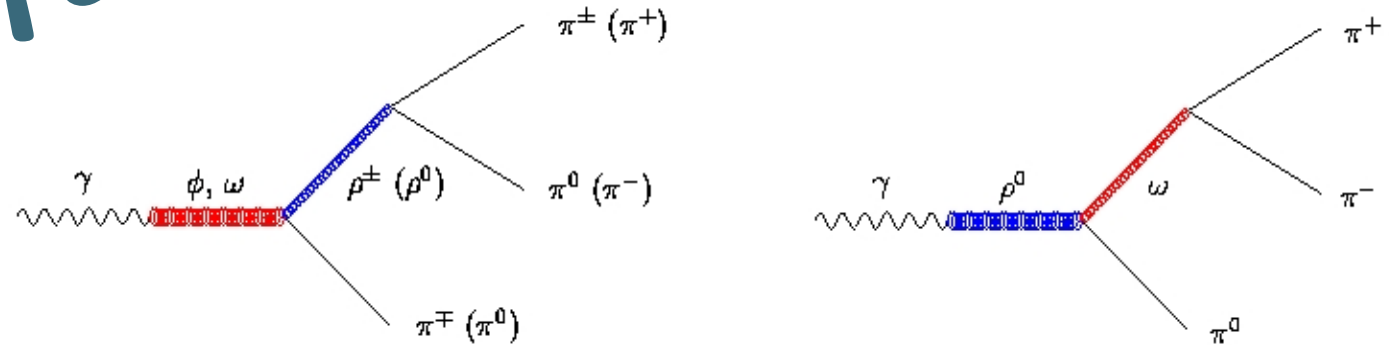
pentagon integrals with two different mass scales:  $m_e \ll m_\mu$

cumbersome with standard loop techniques (Gramm determinants ...)



**twistor** inspired methods (still few results with massive particles)

# $\pi^+ \pi^- \pi^0$



Model for the form factor based on **generalized vector dominance**,  $l=0,1$  components

$$J_v^{em,3\pi} = \langle \pi^+(q_+) \pi^-(q_-) \pi^0(q_0) | J_v^{em} | 0 \rangle = \epsilon_{\nu\alpha\beta\gamma} q_+^\alpha q_-^\beta q_0^\gamma \sum F_{3\pi}^{I=0,1}(q_+, q_-, q_0)$$

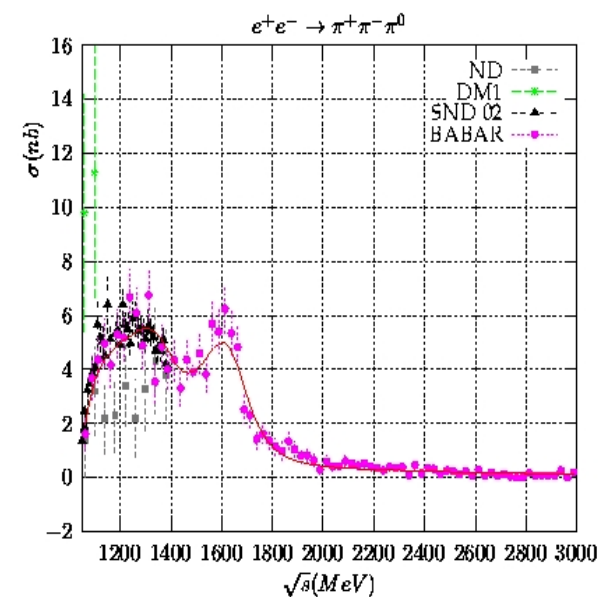
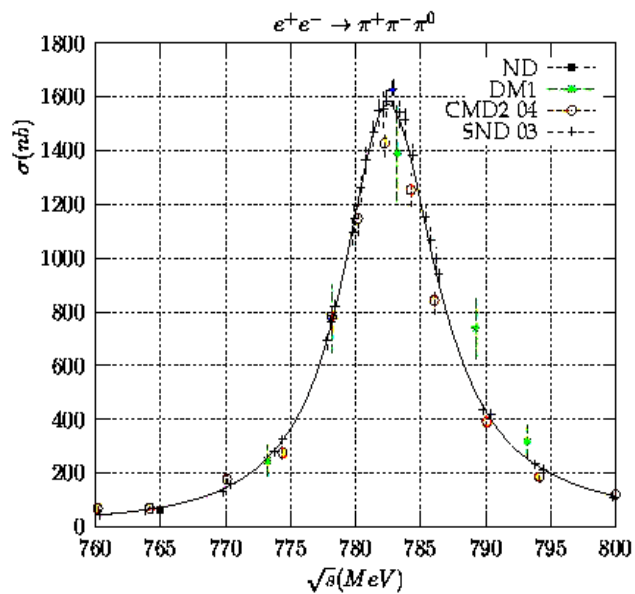
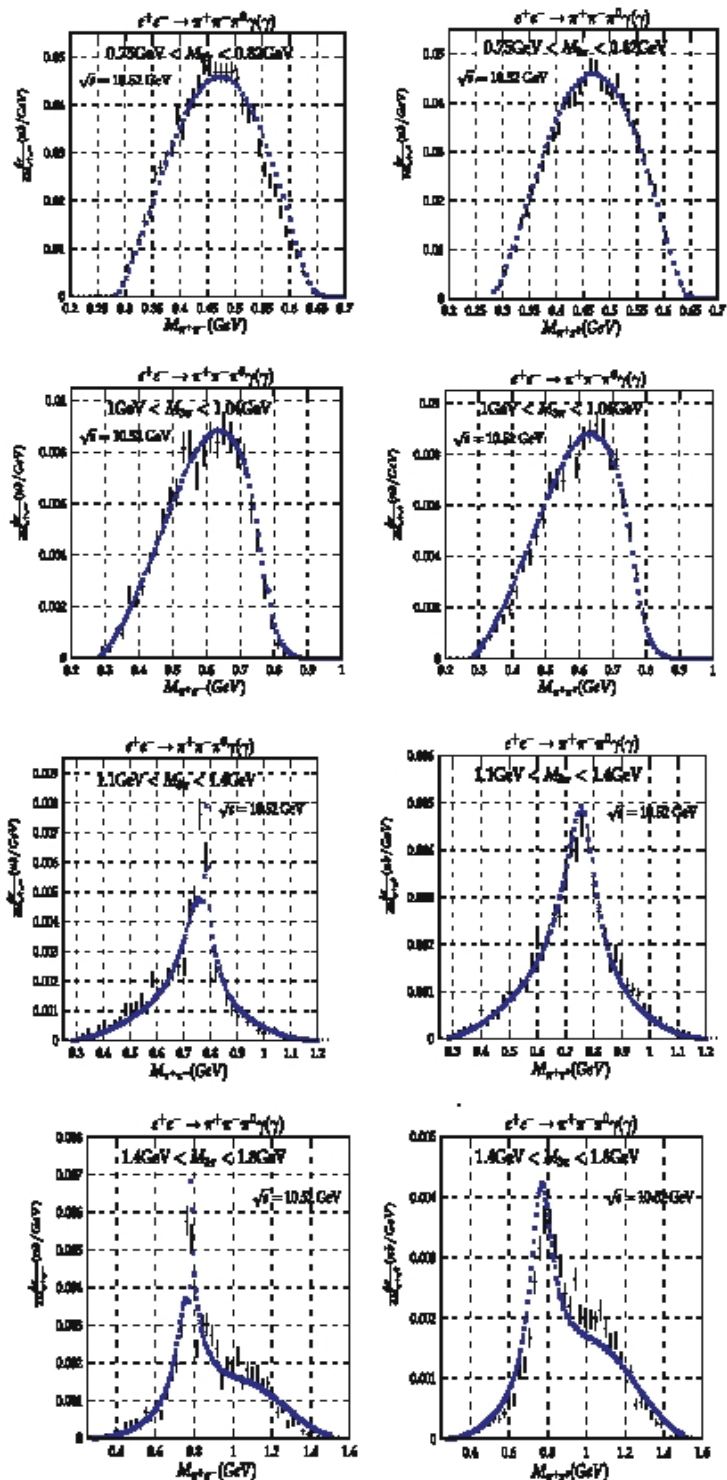
$$F_{3\pi}^{I=0} = \sum a_{ij} BW_{V_i}(Q^2) H_{\rho_j}(Q_+^2, Q_-^2, Q_0^2) \quad V_i = \phi, \omega_i$$

$$F_{3\pi}^{I=1} = BW_\omega(Q_0^2) \sum b_j BW_{\rho_j}(Q)$$

$l=1$  taken from 4pi current [Decker et al., 96]

data fitted with:

$\omega(782), \omega'=\omega(1420), \omega''=\omega(1650), \Phi(1020), \rho(770), \rho'=\rho(1450), \rho''=\rho(1700)$



good description of the **total cross section**  
(DM2 results excluded)

information on **subdistributions** (invariant mass  
of pion pairs or angular distributions) needed for  
a better determination of couplings

predict:  $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ , the slope parameter  $\pi^0 \rightarrow \gamma\gamma^*$   
and radiative vector meson decays  $\rho \rightarrow \pi^0\gamma$ ,  $\phi \rightarrow \pi^0\gamma$   
but in conflict with  $\omega \rightarrow \pi^0\gamma$

# Nucleon form factors

Dirac ( $F_1$ ) and Pauli ( $F_2$ ) form factors

$$J_\mu = -ie \bar{u}(q_2) \left( F_1^N(Q^2) \gamma_\mu - \frac{F_2^N(Q^2)}{4m_N} [\gamma_\mu, Q] \right) v(q_1)$$

Electric and magnetic Sachs FF

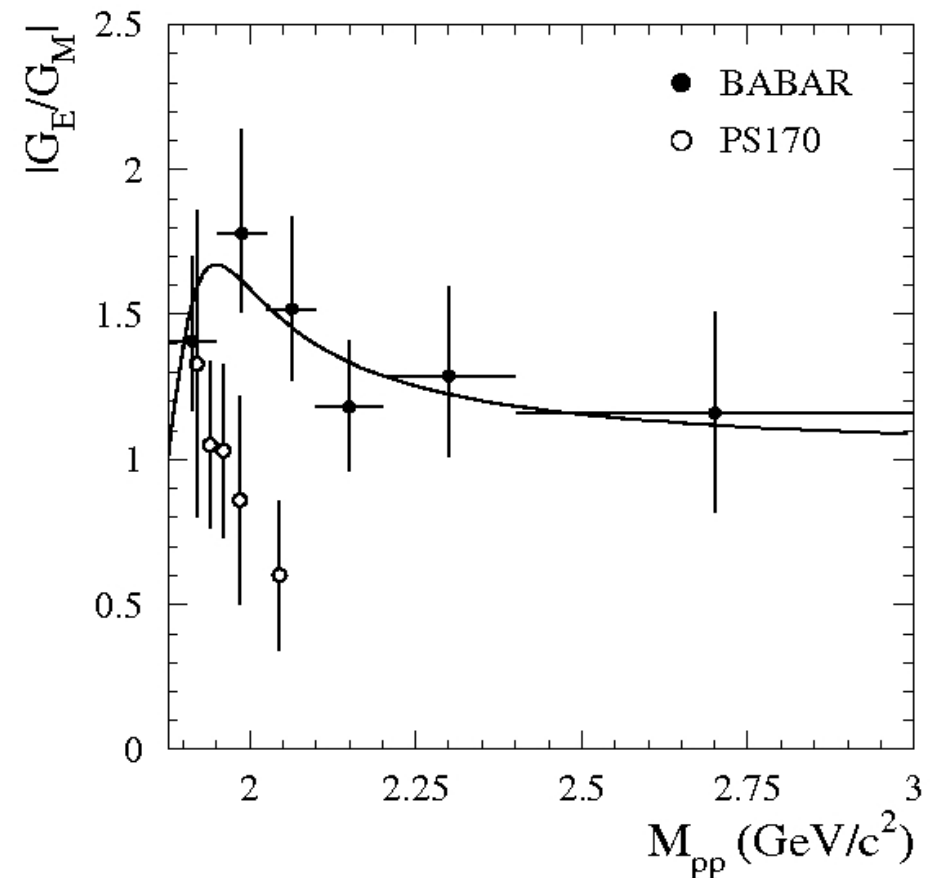
$$G_M^N = F_1^N + F_2^N, \quad G_E^N = F_1^N + \frac{Q^2}{4m_N^2} F_2^N$$

disagreement in the space-like region on the ratio  $G_E/G_M$  between Rosenbluth and recoil polarization methods

radiative return in the **time-like** region

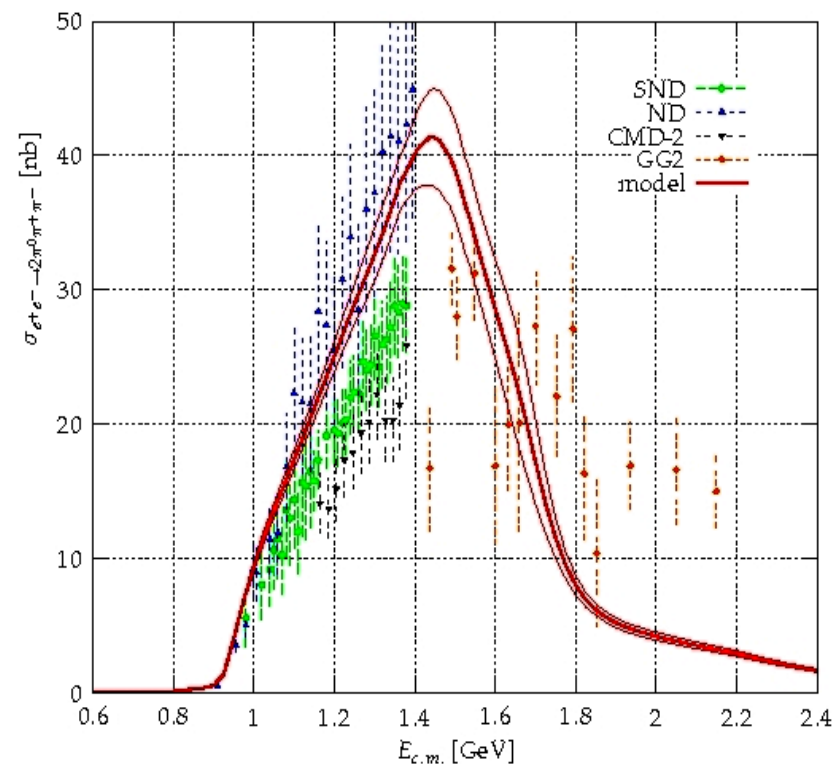
Czyż, Kühn, Nowak, GR EPJC35(04)527

relative phase between  $G_E$  and  $G_M$  requires access to Nucleon spin



# PHOKHARA: near future developments

- ▶ full one-loop radiative corrections to  
$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$
- ▶  $4\pi$  revisited [Kühn,Czyż,Wapienik]
- ▶  $J/\psi$ ,  $\psi(2S)$  with the radiative return
- ▶ **PHOKHARA** for energy scan





# Conclusions

- **radiative return:** not only hadronic cross-section and  $(g-2)_\mu$  but also valuable information on hadronic physics
- **PHOKHARA 5.1:** new developments, improvements, and new channels
- further developments within the **FLAVIANet MRTN network**  
<http://ific.uv.es/flavianet/>



http://ific.uv.es/flavianet/

**Marie Curie Actions**  
Human Resources and Mobility Activity

# FLAVIANet

## Flavour Dynamics of Fundamental InterActions

RTN network

[Overview](#) | [Scientific Objectives](#) | [Training](#) | [Working Groups](#) | [Teams](#)

### Entering the high-precision era of flavour physics through the alliance of lattice simulations, effective field theories and experiment

One of the most profound open questions in particle physics is to understand the pattern of fermion masses and mixings, and the source of fermion replication. The origin of CP violation is intimately related and has deep implications for cosmology. CP violation is a crucial ingredient to understand the surprising fact that the universe contains more matter than antimatter. Large-scale experimental efforts have begun to illuminate the above questions and new facilities such as LHC-B should start soon collecting data. Interpreting the experimental results in terms of the fundamental dynamics is often difficult owing to the uncertainties arising from hadronic contributions to measured quantities. This is an area where close collaboration between theory and experiment is essential. Such collaboration is an aim of the network, which puts together the existing European expertise in those theoretical areas which are relevant for the data analysis. A multidisciplinary approach, combining lattice technologies, dispersive methods, effective field theories (ChPT, HQET, NRQCD, SCET), higher-order perturbative tools and Monte Carlo event generators, should allow a more efficient use of the experimental data to improve our current understanding of the flavour dynamics, and possibly guide us towards a more fundamental theory, valid at higher energy scales. FLAVIANet is a Research Training Network within the 6th framework program addressing these problems.

The Flavianet contract No. **MRTN-CT-2006-035482** has been already signed by the European Commission (11-8-2006). The official starting date of the project will be October 1st, 2006.

Inaugural FLAVIANet Meeting EUROPLAVOUR06  
Barcelona, November 2-4