Understanding the real accuracy of radiative corrections for e^+e^- annihilation into lepton & hadron pair

Andrej Arbuzov
JINR, Dubna
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1. Motivation:

$(g-2)_\mu$, M_{Higgs}, etc. $\rightarrow$ vacuum polarization by hadrons

$\rightarrow$ R $\rightarrow$ $\sigma(e^+e^- \rightarrow \mu^+\mu^-, \pi^+\pi^-, K^+K^-, \ldots)$

Important: high precision of the order $\boxed{\ldots}$ should be reached for each process individually, because of different experimental conditions (acceptances etc.) for each channel

2. The aim:

- estimate the present level of the theoretical accuracy,
- define the critical points to be improved

Bhabha scattering

$e^+e^- \to e^+e^- (n\gamma, \text{pairs})$

Let's consider the accuracy of MCGPJ

[A. A., G. Fedotovich, F. Ignatov, E. Kuraev, A. Sibidanov, Eur. Phys. J. C'06 ; A.A. et al. JHEP'97]

The master formula for this (and other) channel is based on the factorization formalism in renormalization group approach at the LO.

We have:

1) Born : $O(1)$
2) Born + vac. pol. $O(1) \times O(\alpha)$
3) $O(\alpha)$ complete photonic RC with exact kinematics (except collinear cones) + vacuum pol.
4) up to 4 collinear photon jets in LLA (including corresponding Soft+Virt parts): $O(\alpha^2L^2, \alpha^3L^3, \alpha^4L^4)$

$\Rightarrow$ Complete $O(1) + O(\alpha) + O(\alpha^2L^2, \alpha^3L^3)$

$+ O(\alpha^4L^4)^{\leftarrow \text{main part}} + O(\alpha^2L)^{\leftarrow \text{incomplete}}$

$- O(\alpha^2L^2, \alpha^2L)$ pairs

↖ see LABSMC for some numbers

↑ small: $(\lesssim \frac{1}{5})$ from $O(\alpha^2L^2)$ photonic

Technical precision in theoretical predictions

Auxiliary parameters: 1) $\Delta \frac{\sqrt{s}}{2}$ – maximal energy of a soft γ

$\Delta \sim 10^{-4}$

2) $\theta_0 \sim 10^{-2}$ rad – $\frac{1}{2}$ opening angle of collinear cones

Some terms of the order $O\left(\frac{\alpha}{\pi}\Delta\left[\ln\Delta, L, 1\right], \frac{\alpha}{\pi}\frac{\theta_0^2}{2}\left[\ln\frac{\theta_0^2}{4}, L, 1\right]\right)$

are omitted, but they are all $\lesssim 10^{-4}$

MCGPJ has shown plots with dependence of the result on Δ & θ_0 and see fluctuations below 0.1%. But those look random and mainly come from the lack of statistics or/and numerical instabilities.

$\Rightarrow$ technical precision of MC generators is one of the points we should take into account...

Tuned comparisons between different codes help a lot.

Deviations of MCGPJ from BHWIDE in VEPP2M region do not exceed 0.1%.

But looking in some corners of differential distributions we see up to 0.5% shifts.

Comparison with MC containing only exact $O(\alpha)$ RC [Berends et al.]

$\Rightarrow$ estimate the size of higher order contributions:

up to 1% difference for $\Delta\theta < 0.1$ rad (acollinearity cut) but $< 0.2\%$ for $\Delta\theta < 0.25$ rad.

$\Rightarrow$ for the conditions of interest for the experiment, the inclusive effect of $O(\alpha^2 L^2)$ is $\sim 0.2\%$.

For differential distributions it can be several times larger in particular regions.

Comparison with data is the best check!

Good agreement of MCGPJ with data in the shape of distributions. $O(\alpha)$ MC is much less accurate here.

Estimate the size of missing terms

We have some complete analytical calculations for $O(\alpha^2)$ RC in e^+e^- annihilation and Bhabha

Let's look at Virtual+Soft ($\Delta=1$) RC in $O(\alpha^2)$ to Bhabha [A.Penin'05]

$\sqrt{s}=1$ GeV	$O(\alpha^2L^2)$	$O(\alpha^2L)$	$O(\alpha^2\cdot 1)$
$L=\ln\left(\frac{s}{m_e^2}\right)$	1.5%	$0.1\div 0.5\%$	$\sim 0.1\%$
$L=\ln\left(\frac{-t}{m_e^2}\right)$	$0.5\div 1.5\%$	$\lesssim 0.1\%$	$<0.01\%$

(see A.A., E.Scherbakova '06)

$\Rightarrow \frac{\alpha^2}{\pi^2}\cdot C$, $C\lesssim 10$ except corners (like $\theta\to\pi$)

$\Rightarrow$ take care on the factorization scale.

$e^+e^- \to \mu^+\mu^- \, (n\gamma)$

In addition to what we had in Bhabha:

$O(\alpha \frac{m_\mu^2}{s})$ is taken into account

„Good" agreement between MCGPJ & KKMC:

$\pm 0.2\%$ for $400 \div 1400$ GeV

– due to similar content included.

Note, $O(\alpha^3 L^3)$ versus exponentiation $\to$ nothing

Unfortunately, comparison with data here suffers from low statistics of the latter.

$e^+e^- \to \pi^+\pi^- (n\gamma)$

Approximation: point-like pions $\Rightarrow$ scalar QED

N.B. Nature doesn't like fundamental scalars.

But scalar QED is not a wrong (bad) theory, it is as „bad" as the spinor QED.

Absence of UV divergencies in $e^+e^- \to \pi^+\pi^- (\gamma)$ justifies applicability of scalar QED here.

Effects of pion structure (polarizability etc.) are known to be small (at given energies), see also $\pi^+ \to e^+\nu\gamma$ etc.

Sakharov-Sommerfeld factor

is important at threshold

$$f(z) = \frac{z}{1-\exp(-z)} - \frac{z}{2}$$

subtract $O(\alpha)$ part to avoid double counting

$$z = \frac{2\pi\alpha}{v}$$

v is the relative velocity:

$$v = 2\sqrt{\frac{s-4m^2}{s}}\left(1+\frac{s-4m^2}{s}\right)^{-1}$$ [A.A. '94]

Sources of uncertainties

- weak interactions $< \underline{0.1\%}$ for $2E < 10$ GeV
- $\left(\frac{\alpha}{\pi}\right)^2 L \sim 10^{-4} \cdot C$, $C \lesssim 10 \Rightarrow \underline{\lesssim 0.1\%}$,
  but corners & factorization scale dependence
- uncertainty in hadronic vac. pol.:
  1% shift in σ(hadr.) $\longrightarrow$ $\underline{0.04\%}$ in σ(lept.)
- $\underline{0.1\%}$ - energy dependence in form factors for hadronic cross-sections
- up to $\underline{0.1\%}$ - technical precision (including $\Delta\,(\theta_0)$ dependence)
- $\underline{\lesssim 0.05\%}$ - pairs - require specific consideration involving MC simulations.

To do list

1. pushing technical precision of MC generators to desirable 0.01% level

2. $O(\alpha^2 L)$ photonic RC – known for inclusive cases, but non-trivial for distributions. Still possible by means of NLO factorisation formalism.

3. Pair RC $O(\alpha^2 L^2, \alpha^2 L)$ may be in some approximation, taking into account typical experimental conditions.

4. Organization of tuned comparison working groups – only in this way we may do a conclusion about the real accuracy of RC to processes under consideration.