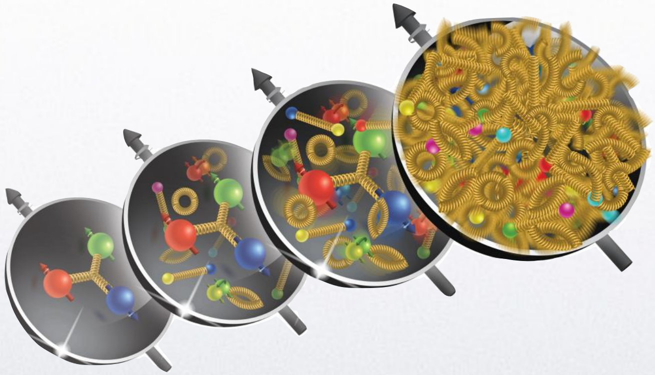
Istituto Nazionale di Fisica Nucleare
Sezione di Pavia

Marco Radici

Pedagogical overview of SIDIS and TMD phenomenology

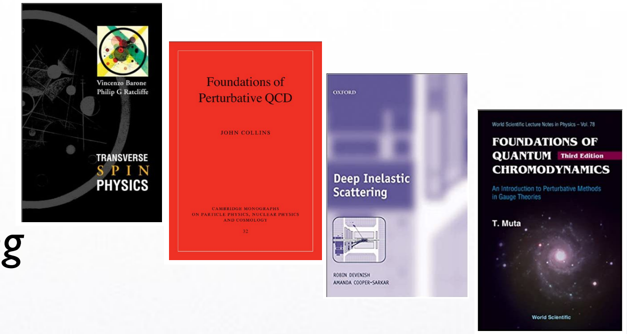


Useful references

Lecture notes

- V. Barone - Cabeo School https://www.fe.infn.it/cabeo_school/2010/cabeo_school_2010.pdf
- A. Bacchetta - Trento School https://www2.pv.infn.it/~bacchett/teaching/Bacchetta_Trento2012.pdf
- R. Jaffe - Erice School https://arxiv.org/pdf/hep-ph/9602236.pdf
- P. Mulders - GGI School http://www.nat.vu.nl/~mulders/tmdreview-vs3.pdf

Books



- V. Barone, P. Ratcliffe - *Transverse Spin Physics*
- J. Collins - *Foundations of perturbative QCD*
- R. Devenish, A. Cooper-Sarkar - *Deep Inelastic Scattering*
- T. Muta - *Foundations of Quantum Chromodynamics*

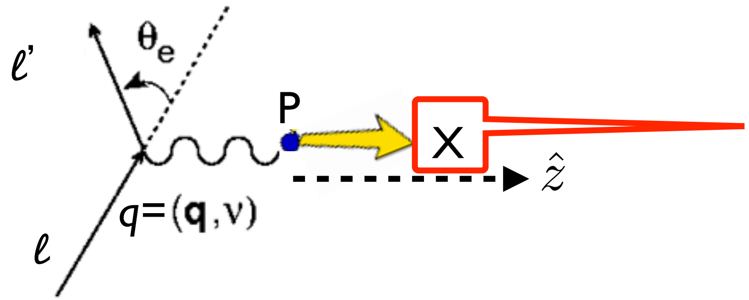
Papers

- EPJ-A topical issue: The 3D structure of the nucleon https://link.springer.com/journal/10050/topicalCollection/AC_628286e999d9a60c9a780398df15f93d
- M. Diehl - *Introduction to GPDs and TMDs* https://inspirehep.net/literature/1408303
- A. Metz, A. Vossen - *Parton fragmentation functions* https://inspirehep.net/literature/1475000

Outline

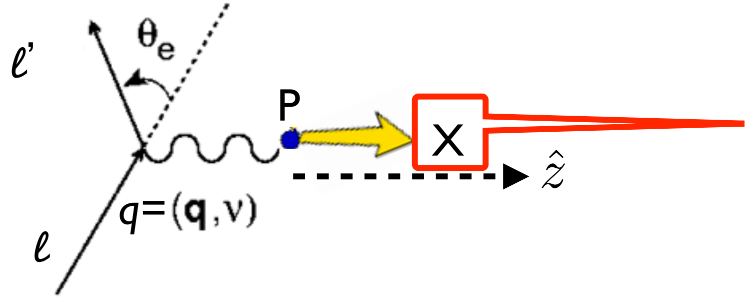
- Why SIDIS?

SIDIS: 1) access fragmentation

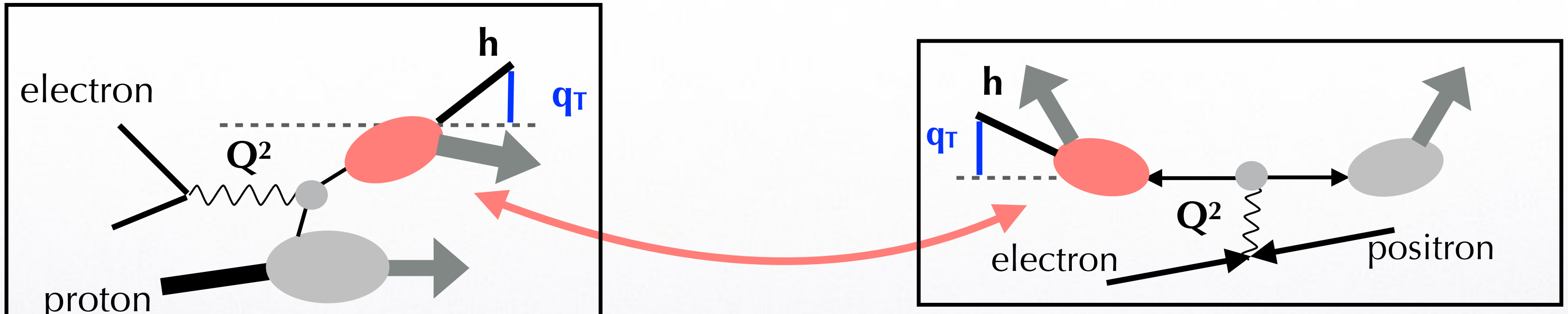


inclusive DIS: no sensitivity to fragmentation

SIDIS: 1) access fragmentation



inclusive DIS: no sensitivity to fragmentation



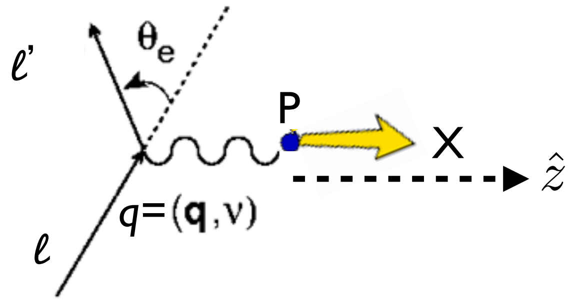
SIDIS

e⁺e⁻ annihilation

check universality of FFs

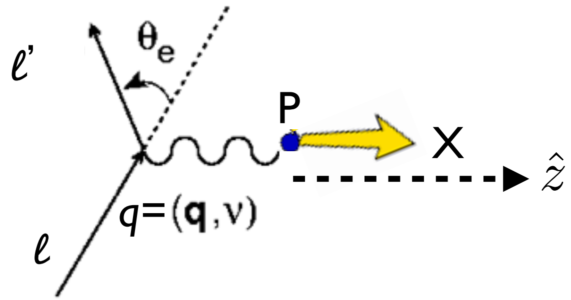
(simpler picture than hadronic collisions)

SIDIS: 2) access intrinsic partonic ⊥ motion

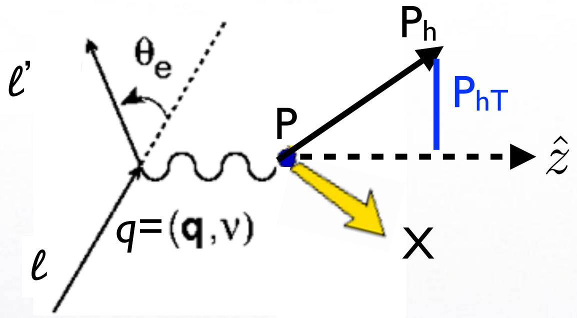


inclusive DIS: - hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- no further scale to probe proton interior

SIDIS: 2) access intrinsic partonic ⊥ motion



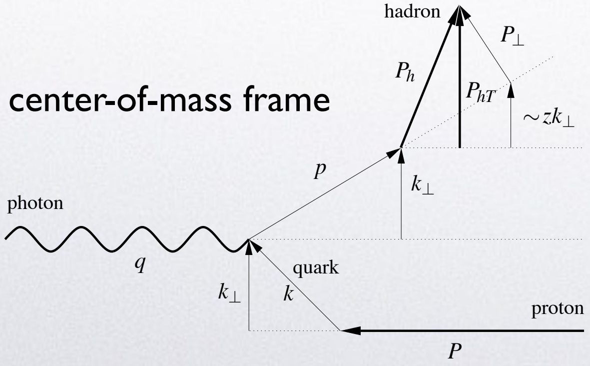
inclusive DIS: - hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- no further scale to probe proton interior



semi-inclusive DIS (SIDIS):

- hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- soft scale: detect hadron *h* with $P_{hT}^2 \sim M^2 \ll Q^2$

center-of-mass frame



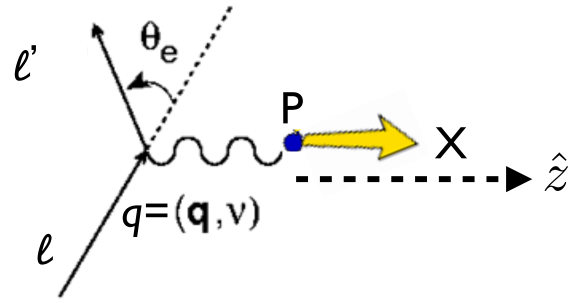
with these two scales, the process is factorizable into a hard photon-quark vertex and a quark→hadron fragmentation

$$\mathbf{P}_{hT} = z\mathbf{k}_\perp + \mathbf{P}_\perp + \mathcal{O}(\mathbf{k}_\perp^2/Q^2)$$

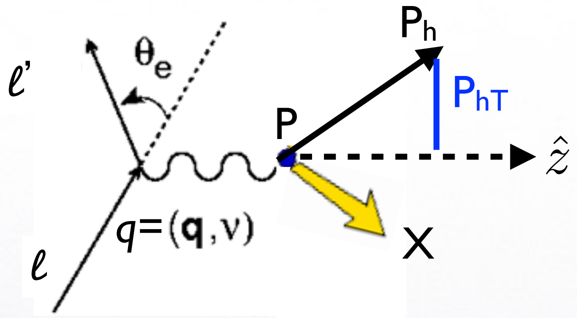
z = fractional energy of *h*
(analogous of *x*)

hadron $\mathrm{P_{hT}}$ arises from struck quark $\mathrm{k_\perp}$ and transverse momentum $\mathrm{P_\perp}$ generated during fragmentation

SIDIS: 2) access intrinsic partonic ⊥ motion

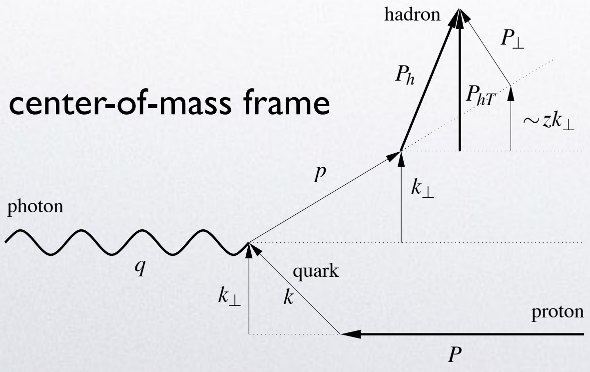


inclusive DIS: - hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- no further scale to probe proton interior



semi-inclusive DIS (SIDIS):

- hard scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- soft scale: detect hadron *h* with $P_{hT}^2 \sim M^2 \ll Q^2$



with these two scales, the process is factorizable into a hard photon-quark vertex and a quark→hadron fragmentation

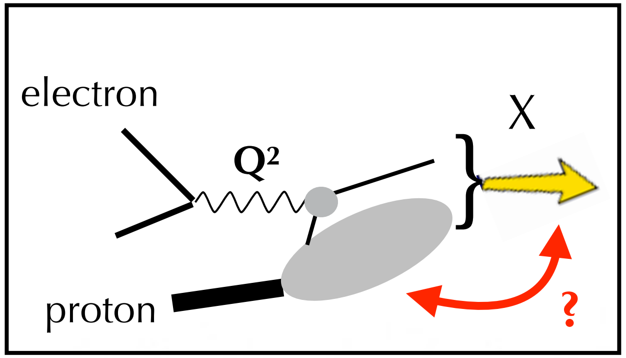
$$\mathbf{P}_{hT} = z\mathbf{k}_\perp + \mathbf{P}_\perp + \mathcal{O}(\mathbf{k}_\perp^2/Q^2)$$

z = fractional energy of *h* (analogous of *x*)

hadron P_{hT} arises from struck quark $k_\perp$ and transverse momentum $P_\perp$ generated during fragmentation

measure P_{hT} → get to $k_\perp$

SIDIS: 3) access chiral-odd structures



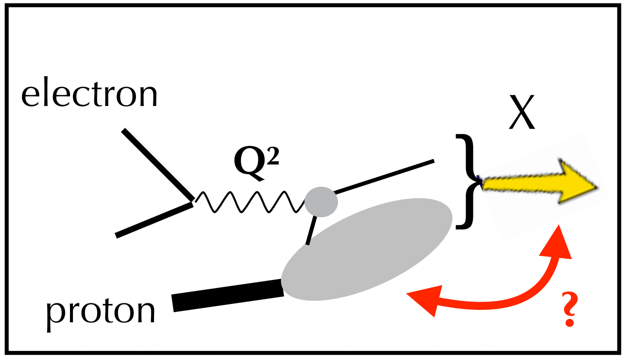
DIS

chirality = helicity for a spin-1/2 object
chiral-odd structures mix quark helicities:

$$\langle + | .. | - \rangle , \langle - | .. | + \rangle$$

hence, chiral-odd structures can appear only paired to another chiral-odd structure because cross section is chiral even
chiral-odd structures suppressed in DIS

SIDIS: 3) access chiral-odd structures

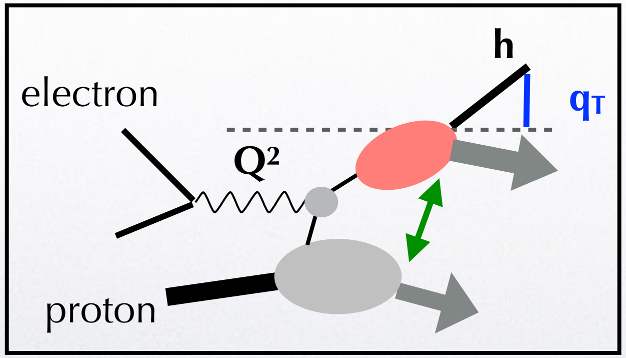


DIS

chirality = helicity for a spin-1/2 object
chiral-odd structures mix quark helicities:

$$\langle + | .. | - \rangle , \langle - | .. | + \rangle$$

hence, chiral-odd structures can appear only paired to another chiral-odd structure because cross section is chiral even
chiral-odd structures suppressed in DIS



SIDIS

chiral-odd structures possible by pairing with a chiral-odd fragmentation function

Outline

- A short recap of inclusive DIS and collinear factorization

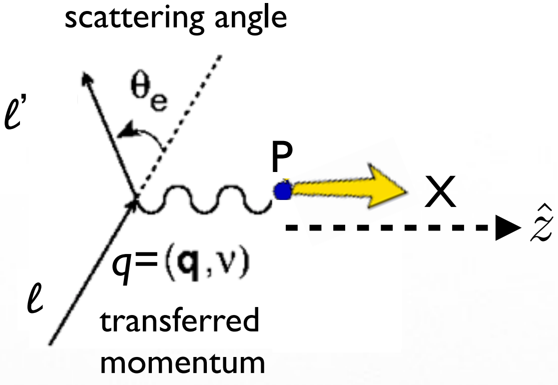
“Deep-Inelastic” kinematics

Internal hadron structure is best explored with a powerful “microscopic lense”
need a process with a hard scale; example: inclusive lepton-proton scattering

ℓ + N(P) → ℓ’ + X

Kinematic invariants

$$
\begin{aligned}
P^2 &= M^2 \\
Q^2 &= -q^2 \approx 2EE'(1-\cos\theta_e) = 4EE'\sin^2\theta_e/2 \\
\nu &= \frac{P\cdot q}{M} \overset{\mathrm{TRF}}{=} E-E' && \text{transferred energy} \\
y &= \frac{P\cdot q}{P\cdot \ell} \overset{\mathrm{TRF}}{=} \frac{E-E'}{E} && \text{fraction of ” ” } 0\leq y\leq 1 \\
x_{_B} &= \frac{Q^2}{2P\cdot q} \overset{\mathrm{TRF}}{=} \frac{Q^2}{2M\nu} && \text{inelastic } 0<x\leq 1 \text{ elastic} \\
W^2 &= (P+q)^2 = M^2 + Q^2(1/x-1) \geq M^2 && \text{invariant mass}
\end{aligned}
$$



$P = (M,0,0,0)$ initially at rest

$q = (\nu,0,0,|\mathbf{q}|)$

$P' = (\sqrt{M^2 + P_z'^2},0,0,P_z')$

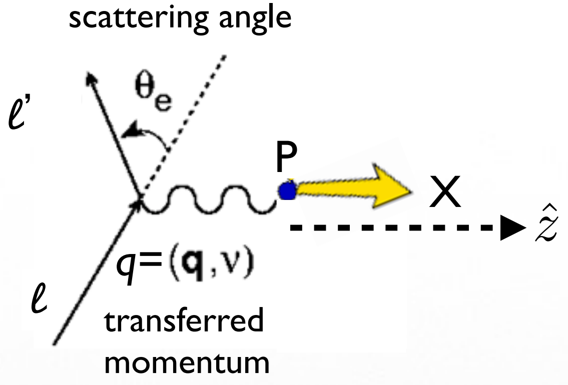
“Deep-Inelastic” kinematics

Internal hadron structure is best explored with a powerful “microscopic lense”
need a process with a hard scale; example: inclusive lepton-proton scattering

$\ell + \mathrm{N}(P) \rightarrow \ell' + X$

Kinematic invariants

$$
\begin{aligned}
P^2 &= M^2 \\
Q^2 &= -q^2 \approx 2EE'(1-\cos\theta_e) = 4EE'\sin^2\theta_e/2 \\
\nu &= \frac{P\cdot q}{M} \overset{\mathrm{TRF}}{=} E-E' && \text{transferred energy} \\
y &= \frac{P\cdot q}{P\cdot \ell} \overset{\mathrm{TRF}}{=} \frac{E-E'}{E} && \text{fraction of '' '' } 0\leq y\leq 1 \\
x_B &= \frac{Q^2}{2P\cdot q} \overset{\mathrm{TRF}}{=} \frac{Q^2}{2M\nu} && \text{inelastic } 0<x\leq 1 \text{ elastic} \\
W^2 &= (P+q)^2 = M^2 + Q^2(1/x-1) \geq M^2 && \text{invariant mass}
\end{aligned}
$$



$P = (M,0,0,0)$ initially at rest

$q = (\nu,0,0,|\mathbf{q}|)$

$P' = (\sqrt{M^2+P_z'^2},0,0,P_z')$

$P_z' = |\mathbf{q}| \gg M$

$P' \approx (|\mathbf{q}|,0,0,|\mathbf{q}|)$

$P' \approx (P_+' = \sqrt{2}\,|\mathbf{q}|, P_-' \approx 0, \mathbf{0}_\perp)$

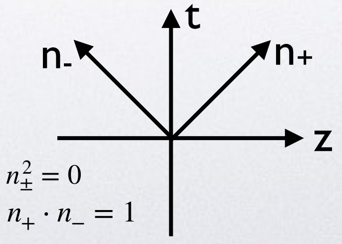
in general $P_+' \gg P_-'$

Deep-Inelastic regime: $\begin{cases} Q^2 \rightarrow \infty \\ x_B = \dfrac{Q^2}{2P\cdot q} \quad \text{finite} \end{cases}$

Light-Cone coordinates:

$$a^\mu = (a_0, a_1, a_2, a_3) = (a_+, a_-, \mathbf{a}_\perp)$$

$$a_\pm = \frac{a_0 \pm a_3}{\sqrt{2}} \qquad \mathbf{a}_\perp = (a_1, a_2)$$



$n_\pm^2 = 0$

$n_+ \cdot n_- = 1$

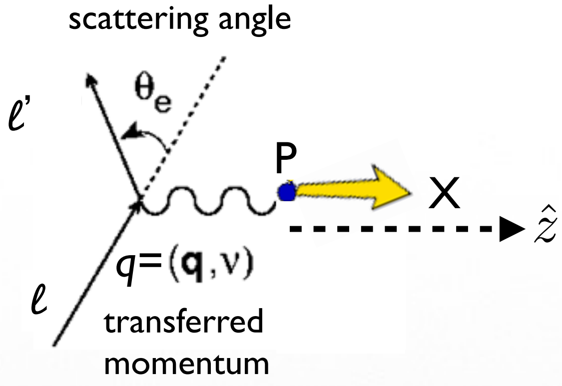
“Deep-Inelastic” kinematics

Internal hadron structure is best explored with a powerful “microscopic lense”
need a process with a hard scale; example: inclusive lepton-proton scattering

ℓ + N(P) → ℓ’ + X

Kinematic invariants

$$
\begin{aligned}
P^2 &= M^2 \\
Q^2 &= -q^2 \approx 2EE'(1-\cos\theta_e) = 4EE'\sin^2\theta_e/2 \\
\nu &= \frac{P\cdot q}{M} \overset{\text{TRF}}{=} E-E' && \text{transferred energy} \\
y &= \frac{P\cdot q}{P\cdot \ell} \overset{\text{TRF}}{=} \frac{E-E'}{E} && \text{fraction of ” ” } 0\le y\le 1 \\
x_B &= \frac{Q^2}{2P\cdot q} \overset{\text{TRF}}{=} \frac{Q^2}{2M\nu} && \text{inelastic } 0<x\le 1 \text{ elastic} \\
W^2 &= (P+q)^2 = M^2 + Q^2(1/x-1) \ge M^2 && \text{invariant mass}
\end{aligned}
$$



$P = (M,0,0,0)$ initially at rest

$q = (\nu,0,0,|\mathbf{q}|)$

$P' = (\sqrt{M^2 + P_z'^2},0,0,P_z')$

$P_z' = |\mathbf{q}| \gg M$

Deep-Inelastic regime:
$$\begin{cases} Q^2 \to \infty \\ x_B = \dfrac{Q^2}{2P\cdot q} \quad \text{finite} \end{cases}$$

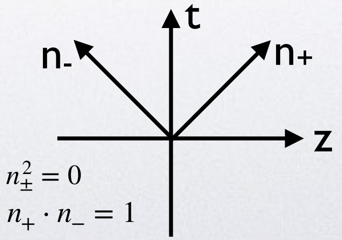
$P' \approx (|\mathbf{q}|,0,0,|\mathbf{q}|)$

In this regime, only one single dominant component of proton momentum, P₊

Light-Cone coordinates:

$$a^\mu = (a_0, a_1, a_2, a_3) = (a_+, a_-, \mathbf{a}_\perp)$$

$$a_\pm = \frac{a_0 \pm a_3}{\sqrt{2}} \qquad \mathbf{a}_\perp = (a_1, a_2)$$



$n_\pm^2 = 0$
$n_+ \cdot n_- = 1$

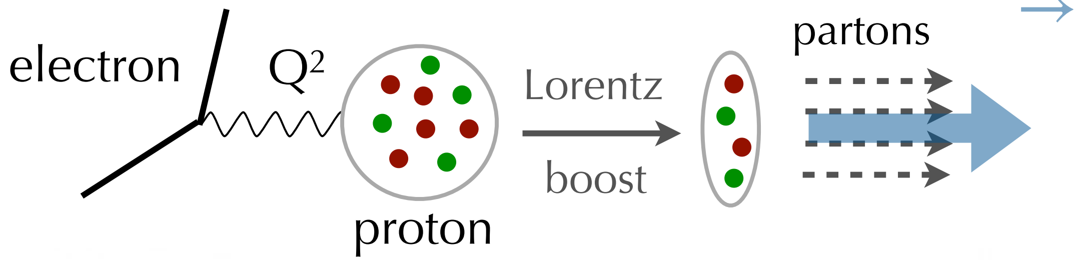
$P' \approx (P_+' = \sqrt{2}|\mathbf{q}|, P_-' \approx 0, \mathbf{0}_\perp)$

in general $P_+' \gg P_-'$

collinear framework

inclusive Deep-Inelastic Scattering (DIS): **1 dominant direction of momenta**

→ all partons collinear to proton



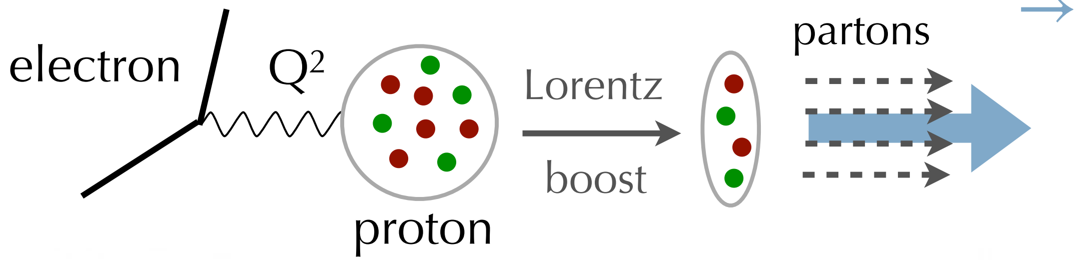
Basics of Feynman parton model:

- DIS regime and relativistic corrections: the virtual photon probes a frozen ensemble of partons
- **factorisation** between hard collision and proton structure

collinear framework

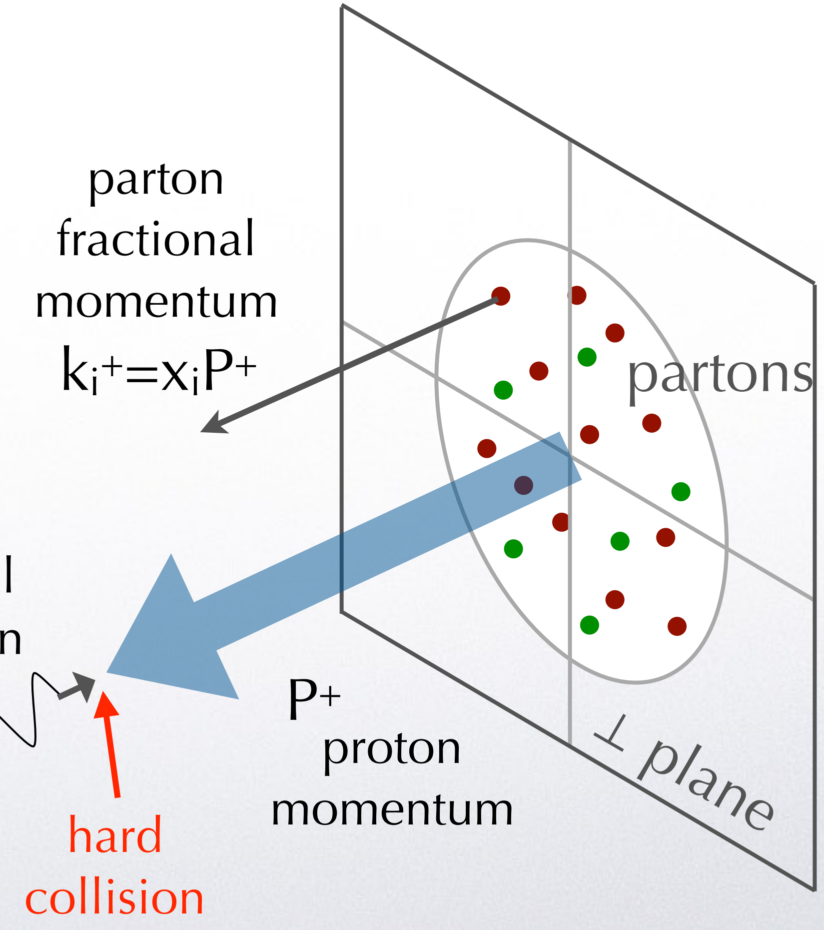
inclusive Deep-Inelastic Scattering (DIS): **1 dominant direction of momenta**

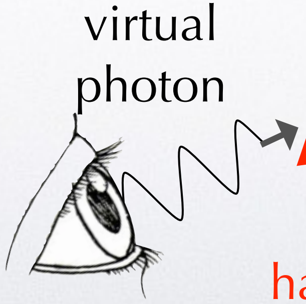
→ all partons collinear to proton



Basics of Feynman parton model:

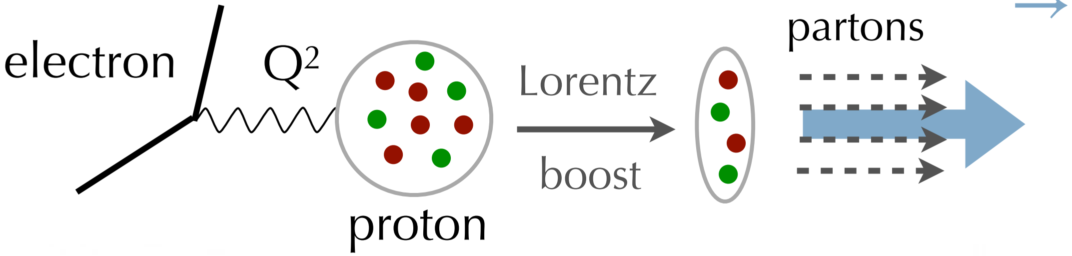
- DIS regime and relativistic corrections: the virtual photon probes a frozen ensemble of partons
- **factorisation** between hard collision and proton structure





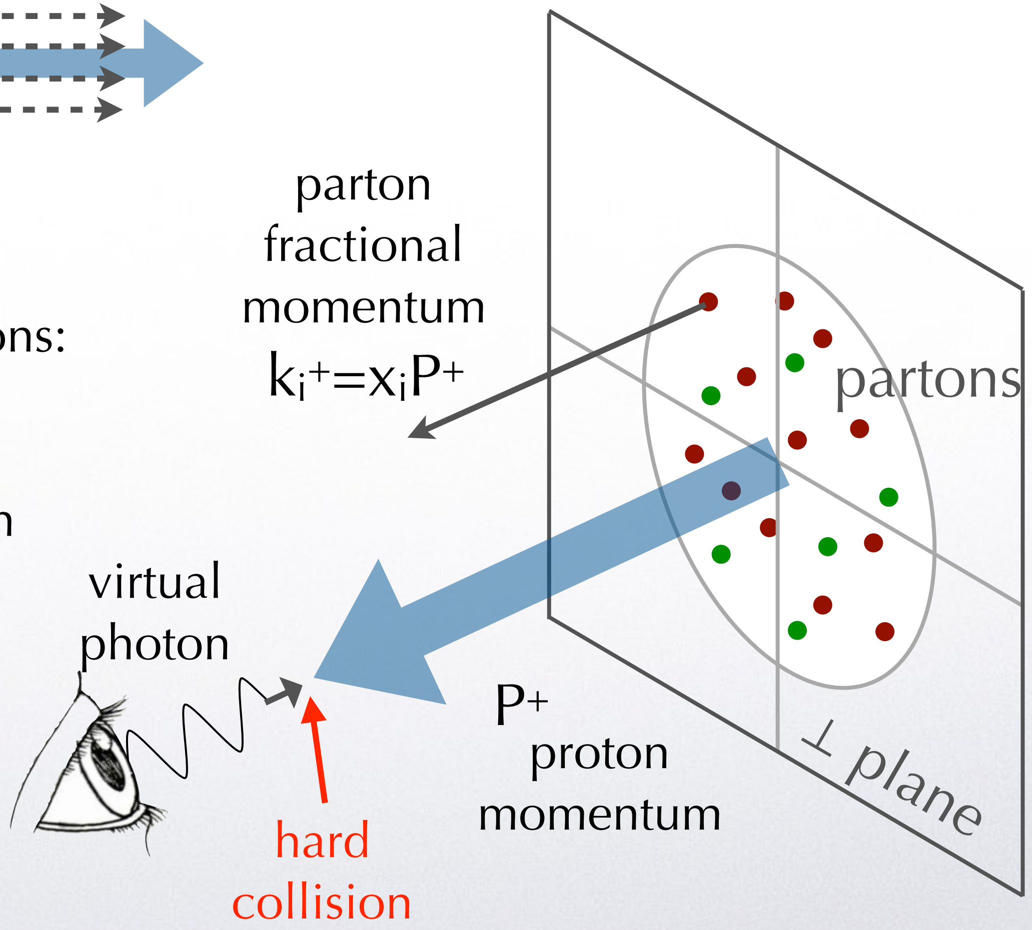
collinear framework

inclusive Deep-Inelastic Scattering (DIS): **1 dominant direction of momenta**
→ all partons collinear to proton



Basics of Feynman parton model:

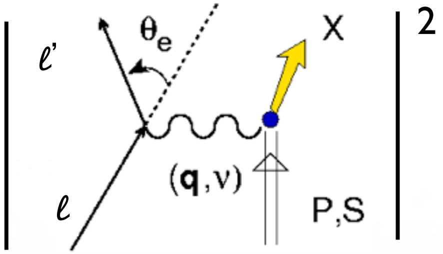
- DIS regime and relativistic corrections: the virtual photon probes a frozen ensemble of partons
- **factorisation** between hard collision and proton structure
- **1D imaging of proton structure**, parametrised by collinear Parton Distribution Functions **PDF(x,Q^2)**

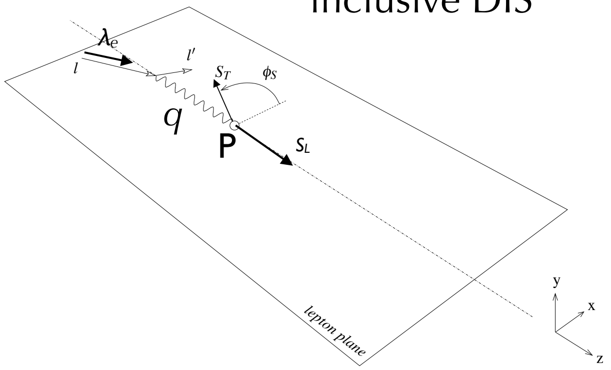


inclusive DIS

More rigorously:

one photon-exchange approximation

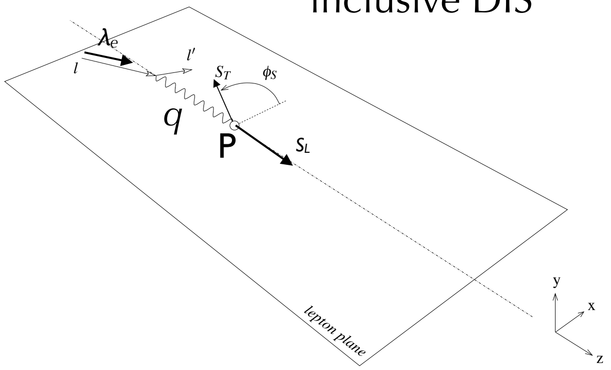



inclusive DIS

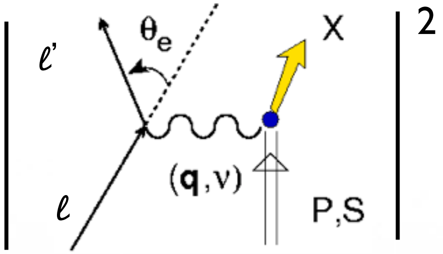
inclusive DIS

More rigorously:

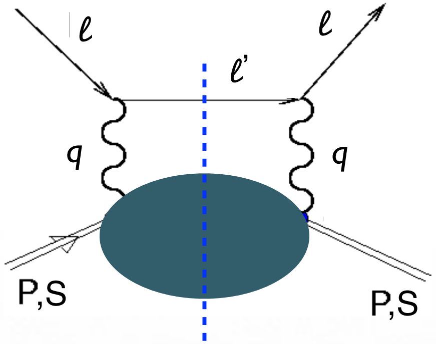
inclusive DIS



one photon-exchange approximation



optical theorem

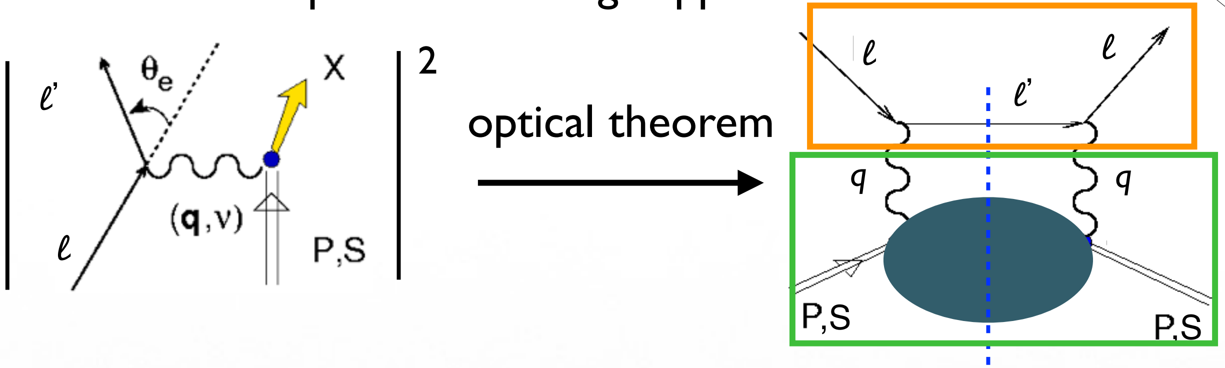


cut-diagram notation:
cross section = product of two amplitudes
particles entering cut are on-shell

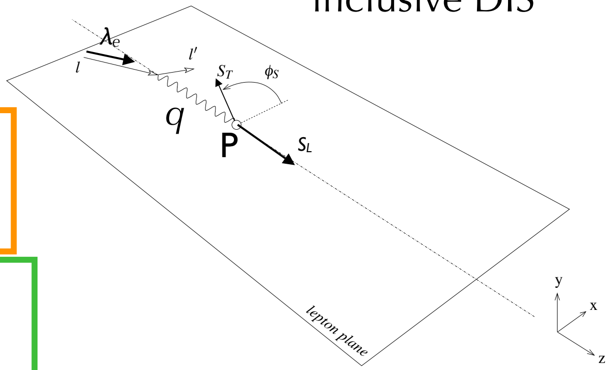
inclusive DIS

More rigorously:

one photon-exchange approximation



inclusive DIS



cut-diagram notation:
cross section = product of two amplitudes
particles entering cut are on-shell

$$\frac{d\sigma}{dx_B\,dy\,d\phi_S} = \frac{\alpha^2 y}{2Q^4}\, L_{\mu\nu}(\ell,\ell',\lambda_e)\, W^{\mu\nu}(q,P,S)$$

leptonic tensor

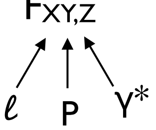
hadronic tensor

calculable in QED

linear combination of all tensor structures with q, P, S, subject to Hermiticity, gauge-, parity- and time reversal- invariance
→ parametrised with **four** structure functions

Collinear factorization theorem

$$\frac{d\sigma}{dx_B dy d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \left\{ A(y) F_{UU,T} + B(y) F_{UU,L} + \lambda_e S_L C(y) F_{LL} + \lambda_e |\mathbf{S}_T| D(y) \cos\phi_S F_{LT} \right\}$$



each F.. (x,Q^2)

$F_{XY,Z}$

Collinear factorization theorem

$$\frac{d\sigma}{dx_B dy d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \left\{ A(y) F_{UU,T} + B(y) F_{UU,L} + \lambda_e S_L C(y) F_{LL} + \lambda_e |\mathbf{S}_T| D(y) \cos\phi_S F_{LT} \right\}$$

each F.. (x,Q²)

$F_{XY,Z}$

ℓ P γ*

connection to standard notation

$$F_{UU,T} = 2x_B F_1$$

$$F_{UU,L} = F_2 - 2x_B F_1 + \mathcal{O}(\gamma^2) F_2$$

$$F_{LL} = 2x_B g_1 + \mathcal{O}(\gamma^2) g_2$$

$$F_{LT} = \mathcal{O}(\gamma)(g_1 + g_2)$$

$$\gamma = \frac{2xM}{Q}$$ target mass correction

Collinear factorization theorem

$$\frac{d\sigma}{dx_B dy d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \left\{ A(y) F_{UU,T} + B(y) F_{UU,L} + \lambda_e S_L C(y) F_{LL} + \lambda_e |\mathbf{S}_T| D(y) \cos\phi_S F_{LT} \right\}$$

each F.. (x,Q²)

$F_{XY,Z}$

ℓ P γ*

connection to standard notation

$$F_{UU,T} = 2x_B F_1$$

$$F_{UU,L} = F_2 - 2x_B F_1 + \mathcal{O}(\gamma^2) F_2$$

$$F_{LL} = 2x_B g_1 + \mathcal{O}(\gamma^2) g_2$$

$$F_{LT} = \mathcal{O}(\gamma)(g_1 + g_2)$$

$\gamma = \frac{2xM}{Q}$ target mass correction

unpolarized: $\lambda_e = S_L = 0$

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[A(y)\, F_2(x, Q^2) - y^2\, F_L(x, Q^2) \right] \qquad Q^2 = sxy$$

$$F_i(x, Q^2) = \sum_f e_f^2 \int_x^1 \frac{d\xi}{\xi}\, d\hat{\sigma}_{i,f} \left(\alpha_s, \frac{x}{\xi}, \frac{Q^2}{\mu_F^2} \right) \phi_f(\alpha_s, \xi, \mu_F) \equiv \sum_f e_f^2\, d\hat{\sigma}_{i,f} \otimes \phi_f$$

usually $\mu_R^2 = \mu_F^2 = Q^2$

$$d\hat{\sigma}_{i,f} = d\hat{\sigma}_{i,f}^{(0)} + \frac{\alpha_s}{4\pi}\, d\hat{\sigma}_{i,f}^{(1)} + \ldots$$

Collinear factorization theorem

$$\frac{d\sigma}{dx_B dy d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{A(y)F_{UU,T} + B(y)F_{UU,L} + \lambda_e S_L C(y)F_{LL} + \lambda_e |\mathbf{S}_T| D(y)\cos\phi_S F_{LT}\right\}$$

each F.. (x,Q²)

$$F_{XY,Z}$$

$$\ell \quad P \quad \gamma^*$$

connection to standard notation

$$F_{UU,T} = 2x_B F_1$$

$$F_{UU,L} = F_2 - 2x_B F_1 + \mathcal{O}(\gamma^2)F_2$$

$$F_{LL} = 2x_B g_1 + \mathcal{O}(\gamma^2)g_2$$

$$F_{LT} = \mathcal{O}(\gamma)(g_1 + g_2)$$

$\gamma = \frac{2xM}{Q}$ target mass correction

unpolarized: $\lambda_e = S_L = 0$

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[A(y)\, F_2(x,Q^2) - y^2\, F_L(x,Q^2)\right] \qquad Q^2 = sxy$$

$$F_i(x,Q^2) = \sum_f e_f^2 \int_x^1 \frac{d\xi}{\xi}\, d\hat{\sigma}_{i,f}\left(\alpha_s, \frac{x}{\xi}, \frac{Q^2}{\mu_F^2}\right)\, \phi_f(\alpha_s, \xi, \mu_F) \equiv \sum_f e_f^2\, d\hat{\sigma}_{i,f} \otimes \phi_f$$

usually $\mu_R^2 = \mu_F^2 = Q^2$

$$d\hat{\sigma}_{i,f} = d\hat{\sigma}_{i,f}^{(0)} + \frac{\alpha_s}{4\pi}\, d\hat{\sigma}_{i,f}^{(1)} + \dots$$

QCD collinear factorization theorem at scale μ_F , valid at all orders

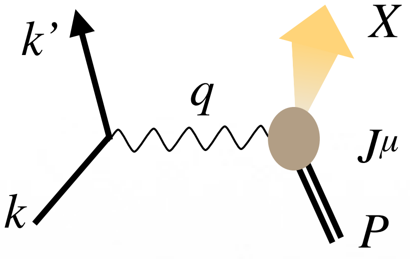
Physics does not depend on fictitious scale μ_F : DGLAP evolution equations

Operator Product Expansion

Factorization from another point of view: the OPE

1. Justification

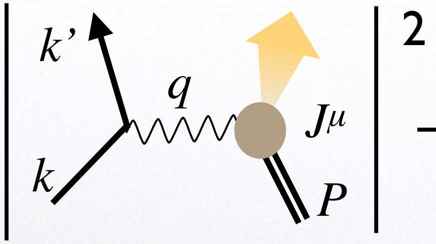
consider the inclusive DIS



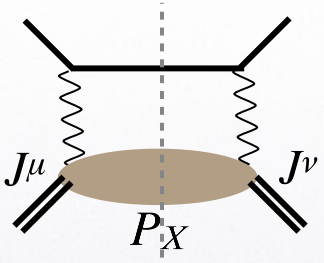
scattering amplitude $\mathscr{M} = \bar{u}(k')\,\gamma_\mu\,u(k)\,\frac{e^2}{Q^2}\,\langle P_X\,|\,J^\mu(0)\,|\,P\rangle$

phase space $dR = (2\pi)^4\,\delta(P + q - P_X)\,d^4P_X\,d^4k'$

cross section $d\sigma \;\propto\; \int_X |\,\mathscr{M}\,|^2\,dR \;=\; \frac{e^4}{Q^4}\,L_{\mu\nu}\,W^{\mu\nu}\,d^4k'$



optical theorem



$L_{\mu\nu} = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2\,g_{\mu\nu}$ leptonic tensor

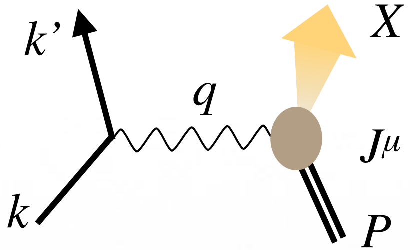
$$W^{\mu\nu} = \int d^4P_X\,(2\pi)^4\,\delta(P_X - P - q) \times \langle P\,|\,\hat{J}^\mu(0)\,|\,P_X\rangle\,\langle P_X\,|\,J^\nu(0)\,|\,P\rangle$$

hadronic tensor

Operator Product Expansion

Factorization from another point of view: the OPE

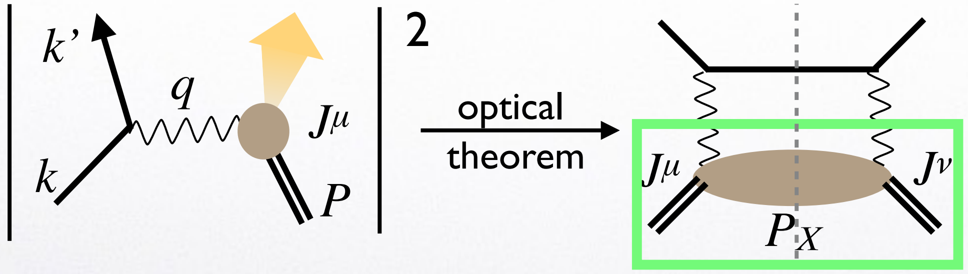
1. Justification consider the inclusive DIS



scattering amplitude $\mathcal{M} = \bar{u}(k')\,\gamma_\mu\, u(k)\,\frac{e^2}{Q^2}\,\langle P_X\,|\,J^\mu(0)\,|\,P\rangle$

phase space $dR = (2\pi)^4\,\delta(P+q-P_X)\,d^4P_X\,d^4k'$

cross section $d\sigma \;\propto\; \int_X |\,\mathcal{M}\,|^2\,dR \;\;=\frac{e^4}{Q^4}\,L_{\mu\nu}\,W^{\mu\nu}\,d^4k'$



$L_{\mu\nu} = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2\,g_{\mu\nu}$ leptonic tensor

$$W^{\mu\nu} = \int d^4P_X\,(2\pi)^4\,\delta(P_X - P - q) \times \langle P\,|\,\hat{J}^\mu(0)\,|\,P_X\rangle\,\langle P_X\,|\,J^\nu(0)\,|\,P\rangle$$

hadronic tensor

matrix element of bilocal operator $= \int d\xi\,e^{iq\cdot\xi}\,\langle P\,|\left[\hat{J}^\mu(\xi)\,,\hat{J}^\nu(0)\right]|\,P\rangle$

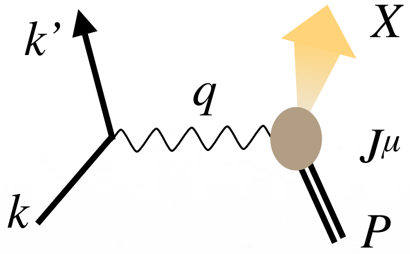
$\hat{J}^\mu = \bar{\psi}\,\gamma^\mu\,\psi$
parton e.m. current
check $\hat{J}^{\mu\dagger} = \hat{J}^\mu$

Operator Product Expansion

Factorization from another point of view: the OPE

1. Justification

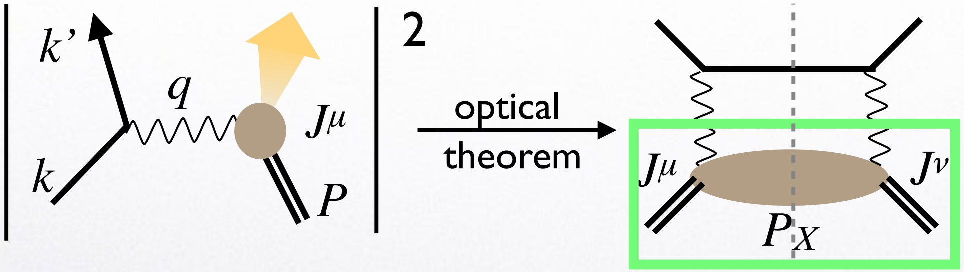
consider the inclusive DIS



scattering amplitude $\mathscr{M} = \bar{u}(k')\,\gamma_\mu\, u(k)\,\frac{e^2}{Q^2}\,\langle P_X\,|\,J^\mu(0)\,|\,P\rangle$

phase space $dR = (2\pi)^4\,\delta(P+q-P_X)\,d^4P_X\,d^4k'$

cross section $d\sigma \,\propto\, \int_X |\,\mathscr{M}\,|^2\,dR \;\;= \frac{e^4}{Q^4}\,L_{\mu\nu}\,W^{\mu\nu}\,d^4k'$



$L_{\mu\nu} = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2\,g_{\mu\nu}$ leptonic tensor

$$W^{\mu\nu} = \int d^4P_X\,(2\pi)^4\,\delta(P_X - P - q) \times \langle P\,|\,\hat{J}^\mu(0)\,|\,P_X\rangle\,\langle P_X\,|\,J^\nu(0)\,|\,P\rangle$$

hadronic tensor

matrix element of bilocal operator $= \int d\xi\; e^{iq\cdot\xi}\,\langle P\,|\,\Big[\hat{J}^\mu(\xi)\,,\hat{J}^\nu(0)\Big]\,|\,P\rangle$

dominated by time-like short distances $\xi^2 \rightarrow 0$, but ill defined !

$\hat{J}^\mu = \bar{\psi}\,\gamma^\mu\,\psi$

parton e.m. current

check $\hat{J}^{\mu\dagger} = \hat{J}^\mu$

Operator Product Expansion

DIS regime:

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \, \langle P | \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] | P \rangle$$

$Q^2 \to \infty$

$x = \frac{Q^2}{2P \cdot q}\Big|_{\mathrm{TRF}} = \frac{Q^2}{2M\nu}$ fixed

Target Rest Frame $\Rightarrow \ \nu \to \infty$

Operator Product Expansion

DIS regime:

$Q^2 \to \infty$

$x = \frac{Q^2}{2P \cdot q}\Big|_{\mathrm{TRF}} = \frac{Q^2}{2M\nu}$ fixed

Target Rest Frame $\Rightarrow \nu \to \infty$

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \langle P | \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] | P \rangle$$

Riemann - Lebesgue theorem:
for $|q \cdot \xi| \to \infty$, large oscillations and cancelations;
integral is dominated by terms with $|q \cdot \xi| \leq K$ constant

Operator Product Expansion

DIS regime:

$Q^2 \to \infty$

$x = \left. \frac{Q^2}{2P \cdot q} \right|_{\mathrm{TRF}} = \frac{Q^2}{2M\nu}$ fixed

Target Rest Frame $\Rightarrow \nu \to \infty$

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \langle P | \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] | P \rangle$$

Riemann - Lebesgue theorem:

for $|q \cdot \xi| \to \infty$, large oscillations and cancelations;

integral is dominated by terms with $|q \cdot \xi| \le K$ constant

Then, $(q \cdot \xi)|_{\mathrm{cm}} = \nu \, \xi^0 \le K \quad \Rightarrow \quad \xi^0 \le \frac{K}{\nu} \xrightarrow{\nu \to \infty} 0$

space-like distances $\xi^2 < 0$ are forbidden by causality;

for time-like distances $\xi^2 \ge 0$, $\xi^2 = (\xi^0)^2 - \vec{\xi}^2 \ge 0 \;\Rightarrow\; (\xi^0)^2 \ge \vec{\xi}^2 \xrightarrow{\nu \to \infty} 0$

The integral is dominated by short time-like distances $\xi^2 \to 0$, but in this limit the bilocal operator is ill defined. Example: free neutron scalar field $\phi(x)$ with propagator $\Delta(x\text{-}y)$

Operator Product Expansion

DIS regime:

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \langle P| \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] |P\rangle$$

$Q^2 \to \infty$

$x = \left.\frac{Q^2}{2P\cdot q}\right|_{\mathrm{TRF}} = \frac{Q^2}{2M\nu}$ fixed

Target Rest Frame $\Rightarrow \nu \to \infty$

Riemann - Lebesgue theorem:
for $|q\cdot\xi| \to \infty$, large oscillations and cancelations;
integral is dominated by terms with $|q\cdot\xi| \le K$ constant

Then, $(q\cdot\xi)|_{\mathrm{cm}} = \nu\,\xi^0 \le K \quad \Rightarrow \quad \xi^0 \le \frac{K}{\nu} \xrightarrow{\nu\to\infty} 0$

space-like distances $\xi^2 < 0$ are forbidden by causality;
for time-like distances $\xi^2 \ge 0$, $\xi^2 = (\xi^0)^2 - \vec{\xi}^2 \ge 0 \;\Rightarrow\; (\xi^0)^2 \ge \vec{\xi}^2 \xrightarrow{\nu\to\infty} 0$

The integral is dominated by short time-like distances $\xi^2 \to 0$, but in this limit the bilocal operator is ill defined. Example: free neutron scalar field $\phi(x)$ with propagator $\Delta(x\text{-}y)$

$$\langle 0|\mathcal{T}[\phi(x)\,\phi(y)]|0\rangle = -\,i\,\Delta(x-y) = i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot(x-y)}}{p^2-m^2+i\varepsilon}$$

for $x \to y$, the integral is divergent :

$$= \frac{m}{4\pi^2}\,\frac{K_1\left(m\sqrt{-(x-y)^2+i\varepsilon}\right)}{\sqrt{-(x-y)^2+i\varepsilon}} - \frac{i}{4\pi}\,\delta\left((x-y)^2\right) \xrightarrow{x\to y} \infty$$

K_1 modified Bessel funct. of 2º kind

Operator Product Expansion

2. Definition

$$\hat{A}(x)\,\hat{B}(y) = \sum_{i=0}^{\infty} C_i(x-y)\,\hat{O}_i\left(\frac{x+y}{2}\right)$$

local operators,
regular for $x \to y$,
typically $\hat{O}_0 = \mathbf{I}$

Wilson coefficients, singular for $x \to y$,
ordered in decreasing singularity

Operator Product Expansion

2. Definition

$$\hat{A}(x)\,\hat{B}(y) = \sum_{i=0}^{\infty} C_i(x-y)\,\hat{O}_i\left(\frac{x+y}{2}\right)$$

local operators, regular for $x \to y$, typically $\hat{O}_0 = \mathbf{I}$

Wilson coefficients, singular for $x \to y$, ordered in decreasing singularity

Example: the Wick theorem

$$\begin{aligned}\lim_{x\to y} \mathscr{T}\left[\phi(x)\,\phi(y)\right] &= :\phi(x)\,\phi(y): + \langle 0|\,\mathscr{T}\left[\phi(x)\,\phi(y)\right]|0\rangle \\ &= 1\cdot\hat{O}_1 + C_0(x-y)\,\mathbf{I}\end{aligned}$$

Operator Product Expansion

2. Definition

$$\hat{A}(x)\,\hat{B}(y) = \sum_{i=0}^{\infty} C_i(x-y)\,\hat{O}_i\left(\frac{x+y}{2}\right)$$

local operators,
regular for $x \to y$,
typically $\hat{O}_0 = \mathbf{I}$

Wilson coefficients, singular for $x \to y$,
ordered in decreasing singularity

Example: the Wick theorem

$$\lim_{x\to y} \mathscr{T}\left[\phi(x)\,\phi(y)\right] = :\phi(x)\,\phi(y): + \langle 0|\,\mathscr{T}\left[\phi(x)\,\phi(y)\right]|0\rangle$$
$$= 1\cdot\hat{O}_1 + C_0(x-y)\,\mathbf{I}$$

3. Application to inclusive DIS

$$W^{\mu\nu} = \int d\xi\, e^{iq\cdot\xi}\,\langle P|\left[\hat{J}^{\mu}(\xi)\,,\hat{J}^{\nu}(0)\right]|P\rangle \;\; = \sum_{\{\alpha\}} C^{\mu\nu}_{\{\alpha\}}\left(\frac{M}{Q}\right)^{t-2}$$

twist t = canonical dimension - spin
of operator $\hat{O}_i$

Operator Product Expansion

2. Definition

$$\hat{A}(x)\,\hat{B}(y) = \sum_{i=0}^{\infty} C_i(x-y)\,\hat{O}_i\left(\frac{x+y}{2}\right)$$

local operators,
regular for $x \to y$,
typically $\hat{O}_0 = \mathbf{I}$

Wilson coefficients, singular for $x \to y$,
ordered in decreasing singularity

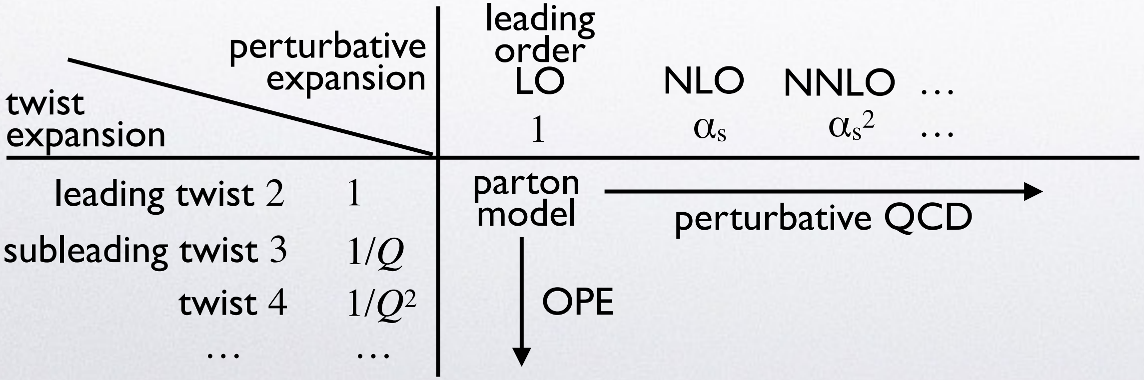
Example: the Wick theorem

$$\lim_{x\to y} \mathscr{T}\left[\phi(x)\,\phi(y)\right] = :\phi(x)\,\phi(y): + \langle 0|\,\mathscr{T}\left[\phi(x)\,\phi(y)\right]|0\rangle$$
$$= 1\cdot\hat{O}_1 + C_0(x-y)\,\mathbf{I}$$

3. Application to inclusive DIS

$$W^{\mu\nu} = \int d\xi\, e^{iq\cdot\xi}\,\langle P|\left[\hat{J}^{\mu}(\xi)\,,\hat{J}^{\nu}(0)\right]|P\rangle \;\; = \sum_{\{\alpha\}} C^{\mu\nu}_{\{\alpha\}}\left(\frac{M}{Q}\right)^{t-2}$$

twist t = canonical dimension - spin
of operator $\hat{O}_i$



twist expansion \ perturbative expansion		leading order LO 1	NLO α_s	NNLO α_s^2	… …
leading twist 2	1	parton model	perturbative QCD →		
subleading twist 3	$1/Q$	OPE ↓			
twist 4	$1/Q^2$				
…	…				

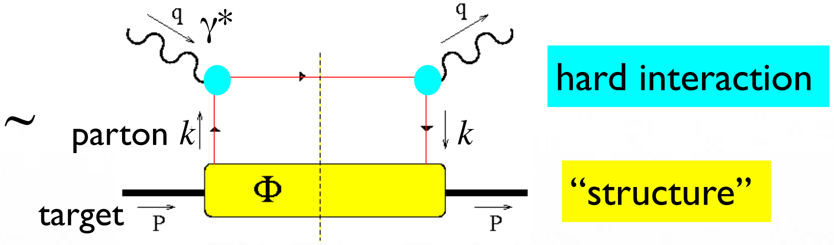
OPE rigorous proof only for inclusive processes. It can be effectively extended to all semi-inclusive hard processes.

Operator Product Expansion

4. Factorization

By applying the same technique of Wick theorem, it can be shown that the dominant contribution to the hadronic tensor of inclusive DIS comes from the so-called "handbag" diagram:

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \langle P | \left[\hat{J}^{\mu}(\xi) \, , \hat{J}^{\nu}(0) \right] | P \rangle \qquad \sim$$

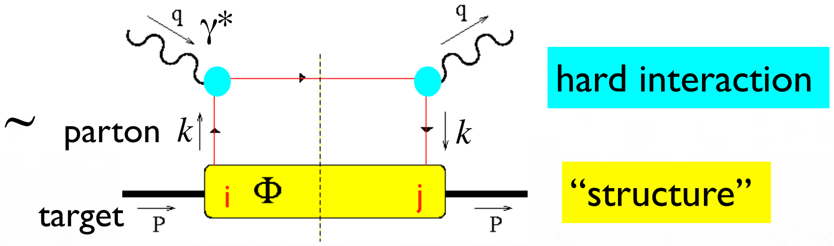


Operator Product Expansion

4. Factorization

By applying the same technique of Wick theorem, it can be shown that the dominant contribution to the hadronic tensor of inclusive DIS comes from the so-called "handbag" diagram:

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \langle P | \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] | P \rangle \quad \sim$$



Φ bilocal quark-quark correlator:

$$\underset{\text{ij}}{\Phi_f}(k, P, S) = \int d^4P_X \, \delta(P - k - P_x) \, \langle P, S | \bar{\psi}_f(0) | P_X \rangle \, \langle P_X | \psi_f(0) | P, S \rangle$$

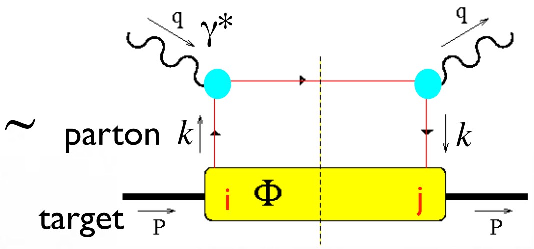
$$= \int \frac{d^4\xi}{(2\pi)^4} \, e^{-i\,k\cdot\xi} \langle P, S | \underset{\text{j}}{\bar{\psi}_f(\xi)} \, \underset{\text{i}}{\psi_f(0)} | P, S \rangle$$

Operator Product Expansion

4. Factorization

By applying the same technique of Wick theorem, it can be shown that the dominant contribution to the hadronic tensor of inclusive DIS comes from the so-called "handbag" diagram:

$$W^{\mu\nu} = \int d\xi \, e^{iq\cdot\xi} \, \langle P | \left[\hat{J}^\mu(\xi) \, , \hat{J}^\nu(0) \right] | P \rangle \quad \sim$$



hard interaction

"structure"

Φ bilocal quark-quark correlator:

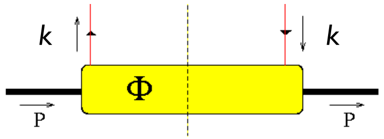
$$\underset{\text{ij}}{\Phi_f}(k,P,S) = \int d^4P_X \, \delta(P-k-P_x) \, \langle P,S | \bar{\psi}_f(0) | P_X \rangle \, \langle P_X | \psi_f(0) | P,S \rangle$$

$$= \int \frac{d^4\xi}{(2\pi)^4} \, e^{-ik\cdot\xi} \, \langle P,S | \underset{\text{j}}{\bar{\psi}_f(\xi)} \, \underset{\text{i}}{\psi_f(0)} | P,S \rangle$$

By taking suitable projections, one can extract from the "structure" the leading-twist part, the subleading part at twist 3, at twist 4, etc..

parton-parton correlator

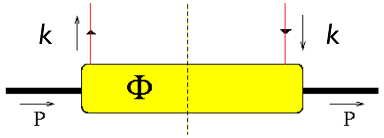
$$\Phi_f(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i\,k\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_f(\xi)\,\psi_f(0)\,|\,P,S\rangle$$



non-local correlator

parton-parton correlator

$$\Phi_f(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i\,k\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_f(\xi)\,\psi_f(0)\,|\,P,S\rangle$$

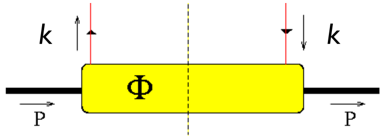


non-local correlator

- Φ(*k*, *P*, *S*) = linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

parton-parton correlator

$$\Phi_f(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i\,k\cdot\xi}\, \langle P,S\,|\,\bar\psi_f(\xi)\,\psi_f(0)\,|\,P,S\rangle$$



non-local correlator

- $\Phi(k, P, S)$ = linear combination of all tensor structures with k, P, S, subject to Hermiticity and parity-invariance (see later about time reversal)

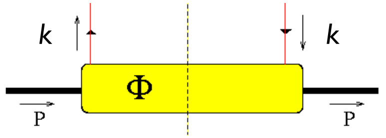
- DIS regime $\to P^+$ dominant component; OPE on $\Phi(k, P, S) \to$ expansion in powers of M/P₊

Caveat

canonical OPE on local operators $\hat{\mathcal{O}}$; expansion in twist = dim($\hat{\mathcal{O}}$) - spin($\hat{\mathcal{O}}$)
Here, Φ is non-local, but can be expanded in local operators of same twist
"working" definition of twist = 2 + powers of M/P_+

parton-parton correlator

$$\Phi_f(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i\,k\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_f(\xi)\,\psi_f(0)\,|\,P,S\rangle$$



non-local correlator

- Φ(*k*, *P*, *S*) = linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

- DIS regime $\to P^+$ dominant component; OPE on Φ(*k*, *P*, *S*) → expansion in powers of M/P₊

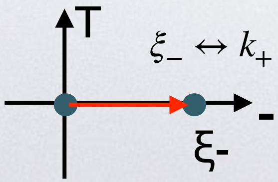
  Caveat

  canonical OPE on local operators $\hat{\mathcal{O}}$; expansion in twist = dim($\hat{\mathcal{O}}$) - spin($\hat{\mathcal{O}}$)
  Here, Φ is non-local, but can be expanded in local operators of same twist
  "working" definition of twist = 2 + powers of M/P₊

- P^+ dominant component → take partons collinear, $k^+ = xP^+$, and integrate other components:

$$\Phi(x,S) = \int dk_+ dk\_d\mathbf{k}_T\, \delta(k_+ - xP_+) \int \frac{d^4\xi}{(2\pi)^4}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}(\xi)\,\psi(0)\,|\,P,S\rangle$$

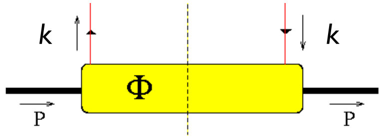
$$= \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$



non-locality along "-" direction

parton-parton correlator

$$\Phi_f(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i\,k\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_f(\xi)\,\psi_f(0)\,|\,P,S\rangle$$



non-local correlator

- Φ(*k*, *P*, *S*) = linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

- DIS regime → P^+ dominant component; OPE on Φ(*k*, *P*, *S*) → expansion in powers of M/P_+

  Caveat

  canonical OPE on local operators $\hat{\mathcal{O}}$; expansion in twist = dim($\hat{\mathcal{O}}$) - spin($\hat{\mathcal{O}}$)
  Here, Φ is non-local, but can be expanded in local operators of same twist
  “working” definition of twist = 2 + powers of M/P_+

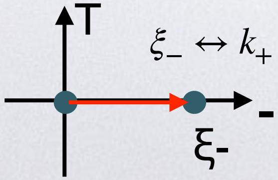
- P^+ dominant component → take partons collinear, $k^+ = xP^+$, and integrate other components:

$$\Phi(x,S) = \int dk_+ dk_- d\mathbf{k}_T\, \delta(k_+ - xP_+) \int \frac{d^4\xi}{(2\pi)^4}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}(\xi)\,\psi(0)\,|\,P,S\rangle$$

$$= \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

Φ(*k*, *P*, *S*)

expanded in powers of M/P_+



non-locality along “-“ direction

OPE → PDFs

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P₊ → keeping only leading twist

$$\Phi(x,S) = \int dk_+ dk_- d\mathbf{k}_T \, \delta(k_+ - xP_+) \, \Phi(k,P,S)$$

$$= \frac{1}{2}\Big[f_1(x)\,\gamma_- +$$

$$g_1(x)\, S_L \gamma_5 \gamma_- +$$

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu] \qquad h_1(x)\, i\sigma_{-\nu}\, \gamma_5 S_T^\nu \Big]$$

OPE → PDFs

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P_+ → keeping only leading twist

$$\Phi(x,S) = \int dk_+ dk_- d\mathbf{k}_T \, \delta(k_+ - xP_+) \, \Phi(k,P,S)$$

$$= \frac{1}{2}\Big[f_1(x)\, \gamma_- +$$

$$g_1(x)\, S_L \gamma_5 \gamma_- +$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu , \gamma^\nu\right] \qquad h_1(x)\, i\sigma_{-\nu}\, \gamma_5 S_T^\nu \Big]$$

$$f_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\, \gamma_+\big] \equiv \Phi^{[\gamma_+]}$$

$$S_L\, g_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\, \gamma_+ \gamma_5\big] \equiv \Phi^{[\gamma_+ \gamma_5]}$$

$$(S_T)_i\, h_1(x) = \frac{1}{2}\text{Tr}\big[\Phi\, i\sigma_{+i}\, \gamma_5\big] \equiv \Phi^{[i\sigma_{+i}\, \gamma_5]}$$

Let's define

$$\Phi^{[\Gamma]}(x) = \frac{1}{P^+}\int dk^-\, d\mathbf{k}_T \,\text{Tr}\left[\big(\Phi(k,P,S)\big)_{ji}\, (\Gamma)_{ij}\right]\Big|_{k^+ = xP^+}$$

OPE → PDFs

OPE on $\Phi(k, P, S)$ → expansion in powers of M/P$_+$ → keeping only leading twist

$$\Phi(x,S) = \int dk_+ dk\_ d\mathbf{k}_T \, \delta(k_+ - xP_+) \, \Phi(k,P,S)$$

$$= \frac{1}{2}\Big[f_1(x)\,\gamma_- +$$ unpolarized Parton Distribution Function (PDF)

$$g_1(x)\, S_L \gamma_5 \gamma_- +$$ longitudinally polarized PDF (requires hadron long. pol. S_L)

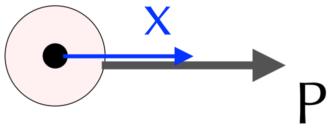
$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu , \gamma^\nu\right]$ $$h_1(x)\, i\sigma_{-\nu}\,\gamma_5 S_T^\nu \Big]$$ transversely polarized PDF (requires hadron transv. pol. S_T)

$$f_1(x) = \frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma_+\right] \equiv \Phi^{[\gamma_+]}$$ (fractional) momentum distribution

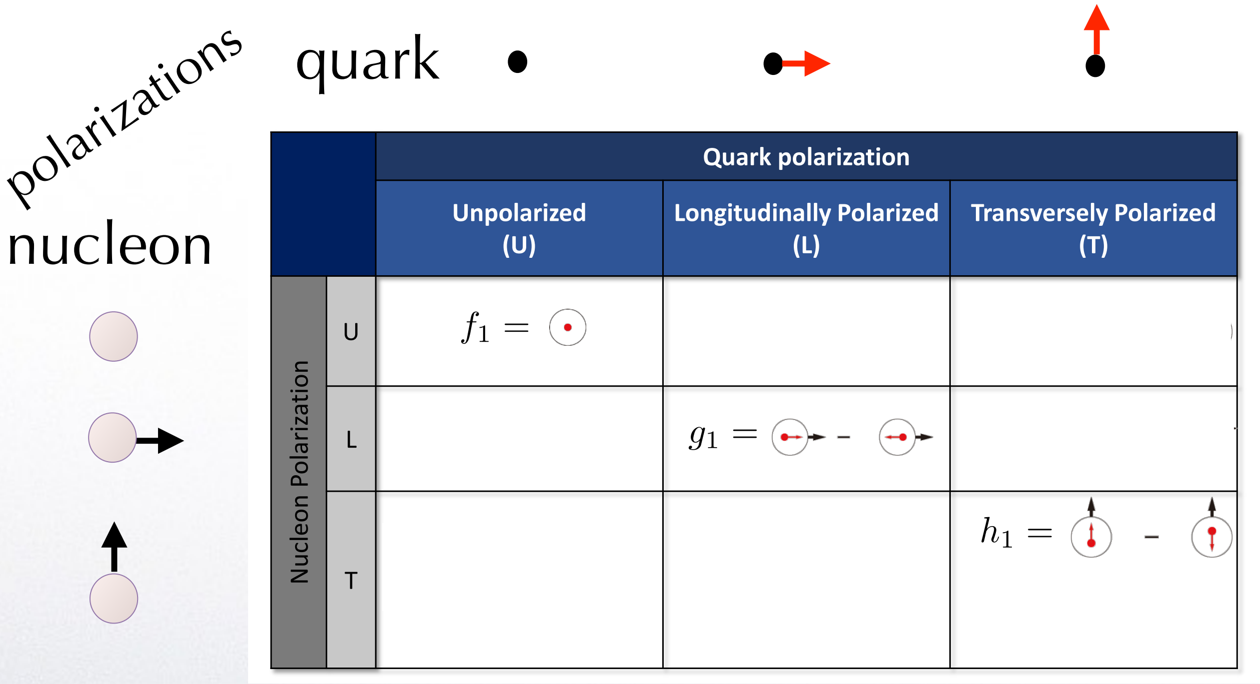
$$S_L\, g_1(x) = \frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma_+\gamma_5\right] \equiv \Phi^{[\gamma_+\gamma_5]}$$ helicity distribution

$$(S_T)_i\, h_1(x) = \frac{1}{2}\mathrm{Tr}\left[\Phi\, i\sigma_{+i}\,\gamma_5\right] \equiv \Phi^{[i\sigma_{+i}\,\gamma_5]}$$ transversity distribution

The PDF table

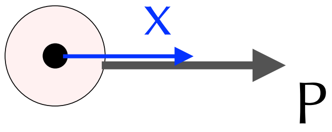


PDFs (x; Q^2) at leading twist for a spin-1/2 hadron (Nucleon)

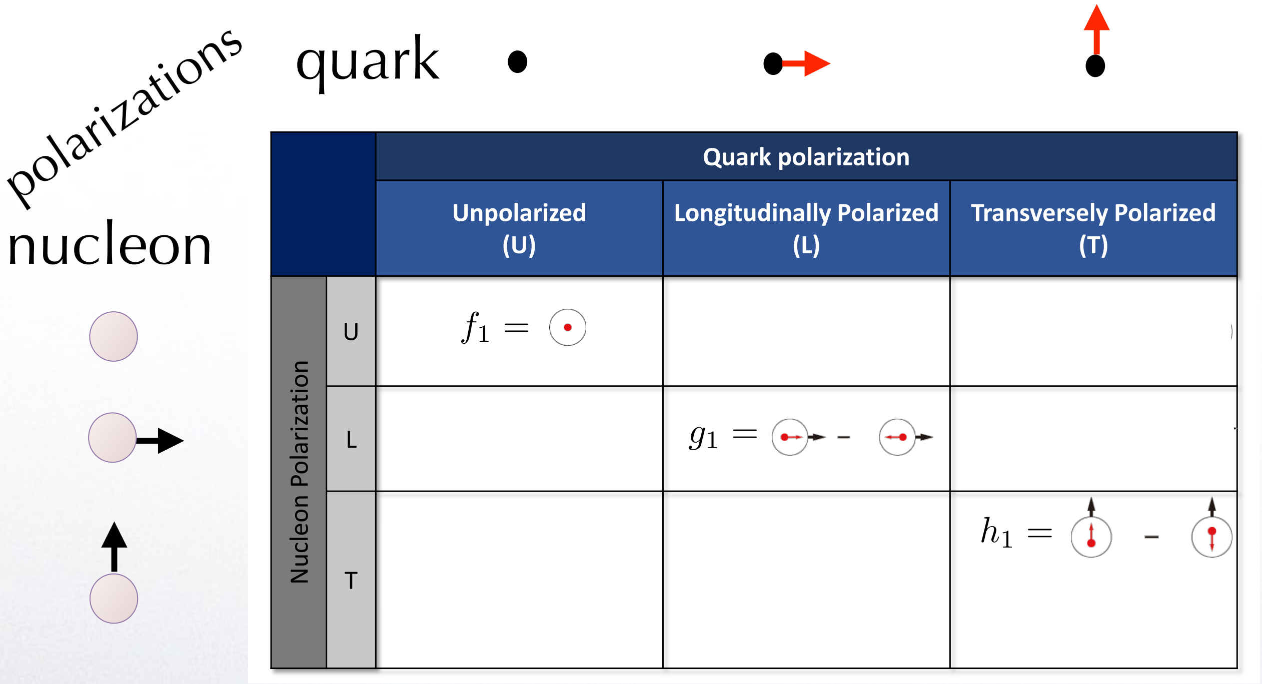


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		
	L		$g_1 =$ ⊙→ - ⊙→	
	T			$h_1 =$ ⊙↑ - ⊙↑

The PDF table



PDFs (x; Q²) at leading twist
for a spin-1/2 hadron (Nucleon)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		
	L		$g_1 =$ -	
	T			$h_1 =$ -

probabilistic interpretation

probability density of finding
an unpol. quark in an unpol. nucleon

probability density of finding a
long. pol. quark in a long. pol. nucleon

probability density of finding a
transv. pol. quark in a transv. pol. nucleon

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist PDFs, but no probabilistic interpretation

“observable” PDFs

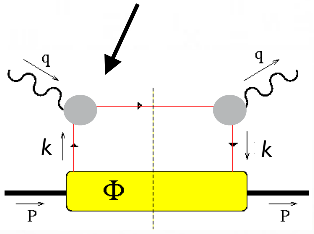
connection of PDFs with measurable structure functions

at leading order $\mathcal{O}(\alpha_s^0)$ and leading twist

$$F_{UU,T}(x_B, Q^2) = x_B \sum_q e_q^2 \, f_1^q(x_B, Q^2) \qquad F_{UU,L}(x_B, Q^2) \approx 0$$

$$F_{LL}(x_B, Q^2) = x_B \sum_q e_q^2 \, g_1^q(x_B, Q^2) \qquad F_{LT}(x_B, Q^2) \approx 0$$

hard cross section $d\hat{\sigma} = 1 + c_1 \alpha_s + \dots$
produce F_L , F_{LT} $\neq 0$



“observable” PDFs

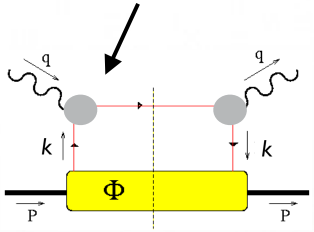
connection of PDFs with measurable structure functions

at leading order $\mathcal{O}(\alpha_s^0)$ and leading twist

$$F_{UU,T}(x_B, Q^2) = x_B \sum_q e_q^2\, f_1^q(x_B, Q^2) \qquad F_{UU,L}(x_B, Q^2) \approx 0$$

$$F_{LL}(x_B, Q^2) = x_B \sum_q e_q^2\, g_1^q(x_B, Q^2) \qquad F_{LT}(x_B, Q^2) \approx 0$$

hard cross section $d\hat{\sigma} = 1 + c_1\alpha_s + \dots$
produce F_L , F_{LT} $\neq 0$



Transversity PDF does not appear in inclusive DIS cross section!

It happens because transverse polarization mixes quark helicities:

$$\langle \uparrow | \ldots | \uparrow \rangle \propto \langle + | \ldots | - \rangle , \langle - | \ldots | + \rangle$$

chirality = helicity for a spin-1/2 object; hence, $h_1(x)$ is a chiral-odd PDF and can appear in the cross section only paired to another chiral-odd structure.

Transversity is not suppressed (as expected in perturbative QCD as m_q/Q), it can be extracted in processes with at least two hadrons

The gauge link

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

this non-local operator is not color-gauge invariant under $\psi(x) \to e^{i\alpha^a(x)\,t^a}\,\psi(x) \equiv U(x)\,\psi(x)$

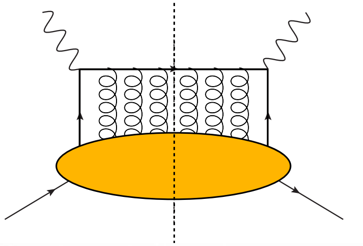
The gauge link

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

this non-local operator is not color-gauge invariant under $\psi(x) \to e^{i\alpha^a(x)\,t^a}\,\psi(x) \equiv U(x)\,\psi(x)$

$$\downarrow$$

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,U_{[\xi,0]}\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$



gauge-link operator $U_{[a,b]} = \mathscr{P}\exp\left[-ig\int_a^b d\eta_\mu\, A^\mu(\eta)\right]$

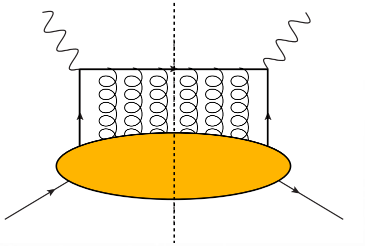
it transforms as $U_{[\xi,0]} \to U(\xi)\,U_{[\xi,0]}\,U^\dagger(0)$ so that Φ(*k*, *P*, *S*) is invariant

The gauge link

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

this non-local operator is not color-gauge invariant under $\psi(x) \to e^{i\alpha^a(x)\,t^a}\,\psi(x) \equiv U(x)\,\psi(x)$

$\downarrow$



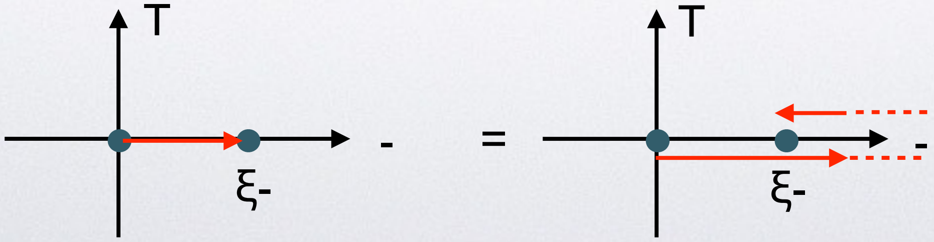
$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(\xi)\,U_{[\xi,0]}\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

gauge-link operator $U_{[a,b]} = \mathscr{P}\exp\Big[-ig\int_a^b d\eta_\mu\, A^\mu(\eta)\Big]$

it transforms as $U_{[\xi,0]} \to U(\xi)\,U_{[\xi,0]}\,U^\dagger(0)$ so that Φ(*k*, *P*, *S*) is invariant

Φ(*x*, *S*) involves only the LC "-" direction: $\int d\xi_- \,\dots\,|_{\xi_+,\xi_T=0}$

trick: $\Phi(x,S) \propto \langle P,S\,|\,\bar{\psi}(\xi_-)\,U_{[\xi_-,0]}\,\psi(0)\,|\,P,S\rangle = \langle P,S\,|\,\bar{\psi}(\xi_-)\,U_{[\xi_-,\infty_-]}\,U_{[\infty_-,0]}\,\psi(0)\,|\,P,S\rangle \equiv \langle P,S\,|\,\{\bar{\psi}(\xi_-)\}\,\{\psi(0)\}\,|\,P,S\rangle$



$\{\psi(x_-)\} = U_{[\infty_-,x_-]}\,\psi(x_-)$

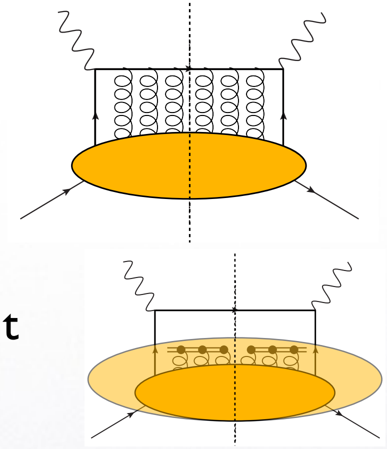
The gauge link

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar\psi_j(\xi)\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

this non-local operator is not color-gauge invariant under $\psi(x)\to e^{i\alpha^a(x)\,t^a}\,\psi(x)\equiv U(x)\,\psi(x)$

↓

$$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar\psi_j(\xi)\,U_{[\xi,0]}\,\psi_i(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

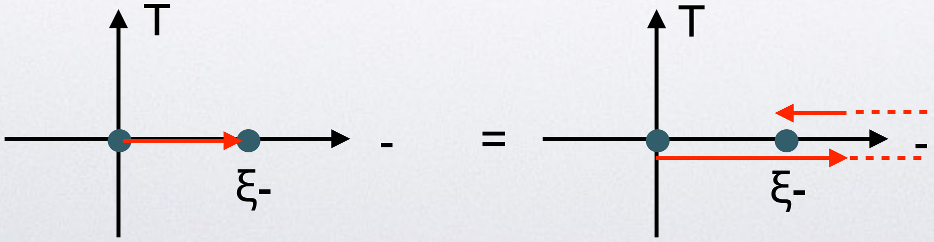


gauge-link operator $U_{[a,b]} = \mathscr{P}\exp\left[-ig\int_a^b d\eta_\mu\, A^\mu(\eta)\right]$

it transforms as $U_{[\xi,0]}\to U(\xi)\,U_{[\xi,0]}\,U^\dagger(0)$ so that Φ(*k*, *P*, *S*) is invariant

Φ(*x*, *S*) involves only the LC "-" direction: $\int d\xi_-\ \dots|_{\xi_+,\xi_T=0}$

trick: $\Phi(x,S)\propto\langle P,S\,|\,\bar\psi(\xi_-)\,U_{[\xi_-,0]}\,\psi(0)\,|\,P,S\rangle = \langle P,S\,|\,\bar\psi(\xi_-)\,U_{[\xi_-,\infty_-]}\,U_{[\infty_-,0]}\,\psi(0)\,|\,P,S\rangle \equiv \langle P,S\,|\,\{\bar\psi(\xi_-)\}\,\{\psi(0)\}\,|\,P,S\rangle$

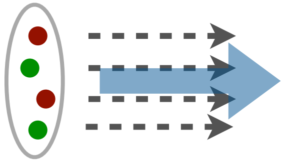


factorisation is preserved

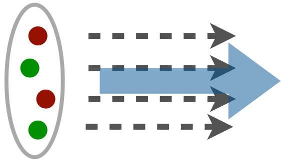
$\{\psi(x_-)\} = U_{[x_-,\infty_-]}\,\psi(x_-)$

Recap

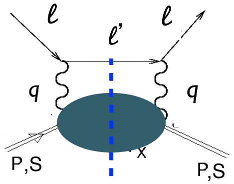
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



Recap



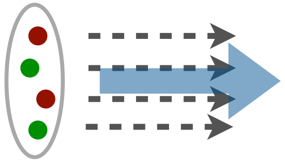
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



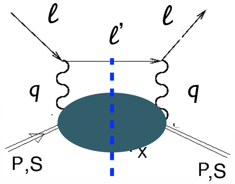
- Example: inclusive DIS, cross section $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$ can be parametrised in terms of 4 structure functions (including polarization)

Recap

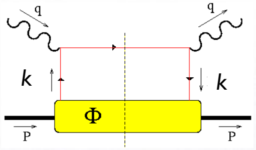
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



- Example: inclusive DIS, cross section $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$ can be parametrised in terms of 4 structure functions (including polarization)

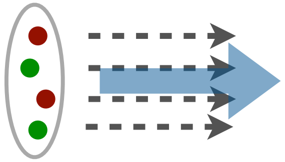


- OPE on $W^{\mu\nu}$ → factorisation of hadron structure in parton-parton non-local correlator Φ. It can be made color-gauge invariant by inserting proper gauge link

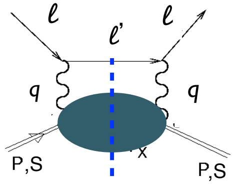


Recap

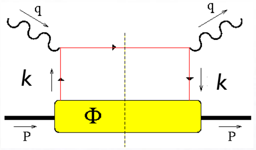
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



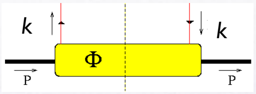
- Example: inclusive DIS, cross section $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$ can be parametrised in terms of 4 structure functions (including polarization)



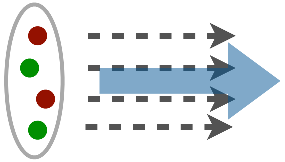
- OPE on $W^{\mu\nu}$ → factorisation of hadron structure in parton-parton non-local correlator Φ. It can be made color-gauge invariant by inserting proper gauge link



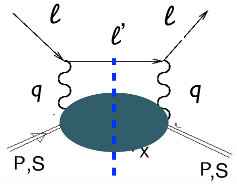
- Expansion of Φ in powers of M/Q (effective twist) contains operator-definition of collinear PDFs, that can be extracted by suitable projections



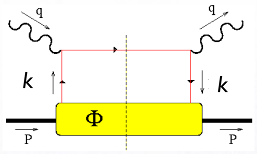
Recap



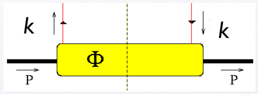
- hadron structure better explored in processes with a hard scale (much bigger than involved masses, $Q^2 \gg M^2$) ; on the Light-Cone, it implies one dominant direction → collinear framework natural choice



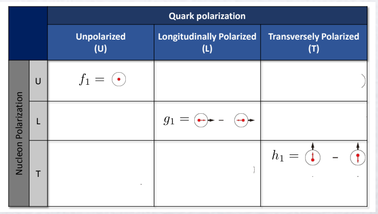
- Example: inclusive DIS, cross section $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$ can be parametrised in terms of 4 structure functions (including polarization)



- OPE on $W^{\mu\nu}$ → factorisation of hadron structure in parton-parton non-local correlator Φ. It can be made color-gauge invariant by inserting proper gauge link



- Expansion of Φ in powers of *M/Q* (effective twist) contains operator-definition of collinear PDFs, that can be extracted by suitable projections



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		
	L		$g_1 =$ ⊙ - ⊙	
	T			$h_1 =$ ⊙ - ⊙

- Leading-twist PDFs have nice probabilistic interpretations, and can be connected to structure functions (except the chiral-odd transversity PDF)

Outline

- Why TMDs ?

Evidences to go beyond collinear

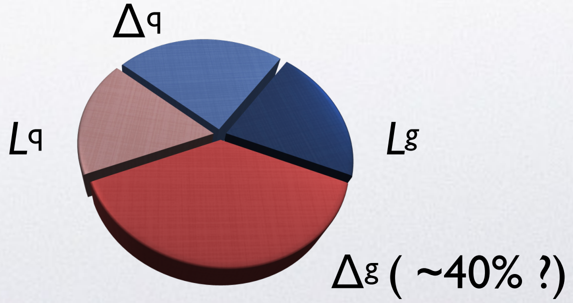
Evidences of going beyond the collinear framework

Example #1: the "Spin Crisis"

Ashmann et al. (EMC),
P.L. ***B206*** *(88) 364*

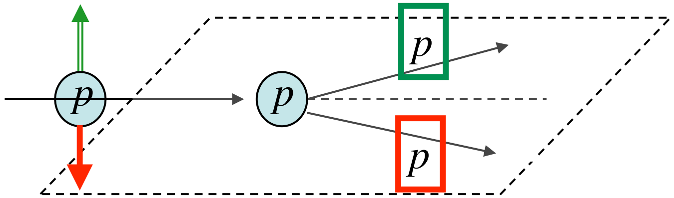
- In 1988, the EMC Collaboration at CERN measures the F_{LL} structure function in the polarized inclusive DIS process $\vec{\mu} + \vec{p} \to \mu' + X$. Surprisingly, the sum of quark helicities Δq contributes at most 25% of spin 1/2 of the proton (depending on Q^2).
- There has been an intense activity to measure the gluon helicity Δg, which is currently known with a large error → there is room for contribution from the orbital motion of partons
- Contribution from the orbital angular momentum of partons L^q, L^g → need to be sensitive also to intrinsic transverse components of parton momentum

$$\frac{1}{2} = \sum_q \left(\frac{1}{2} \Delta q(Q^2) + L^q(Q^2) \right) + \Delta g(Q^2) + L^g(Q^2)$$



Evidences to go beyond collinear

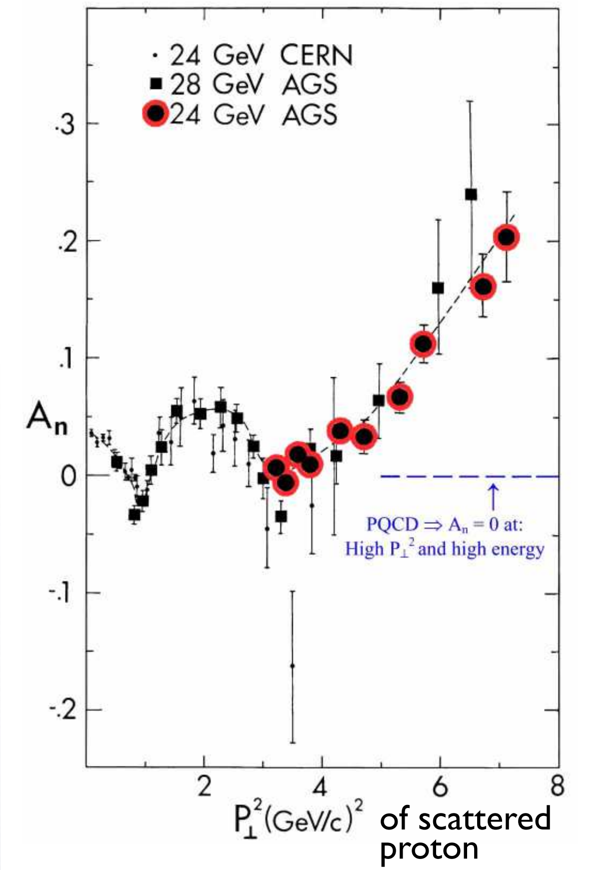
Example #2: elastic p-p scattering



$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$p^{\uparrow} p \to p\, p$ versus $p^{\downarrow} p \to p\, p$

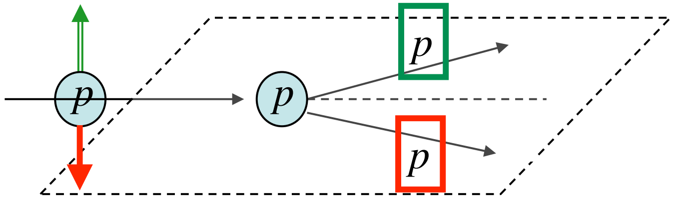
for a review, see
Krisch, E.P.J. **A31** (07) 417



of scattered proton

Evidences to go beyond collinear

Example #2: elastic p-p scattering



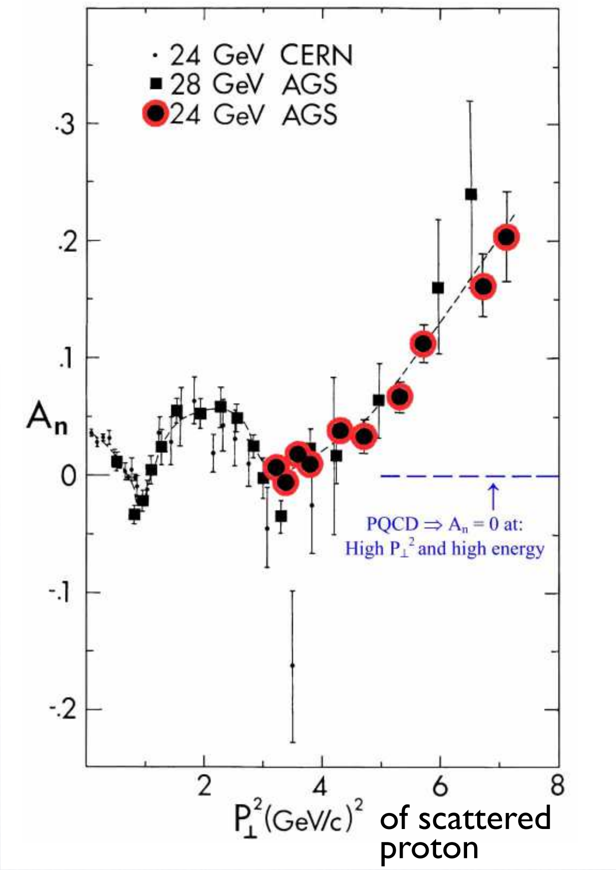
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$p^{\uparrow} p \to p\, p$ versus $p^{\downarrow} p \to p\, p$

correlation between spin of the proton and k_T of partons

$\leftrightarrow$ orbital motion

for a review, see
Krisch, E.P.J. **A31** (07) 417

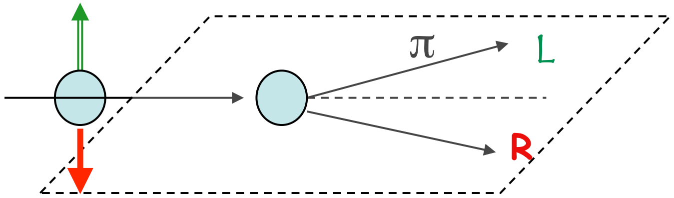


of scattered proton

Evidences to go beyond collinear

Example #3: semi-inclusive p-p collisions

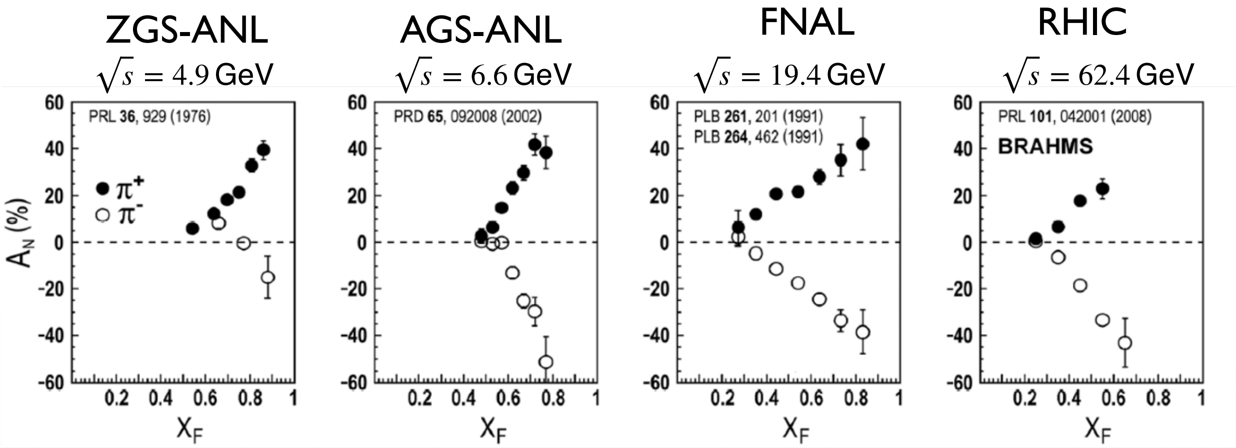
$$p^{\uparrow}\, p \longrightarrow \pi\, X$$

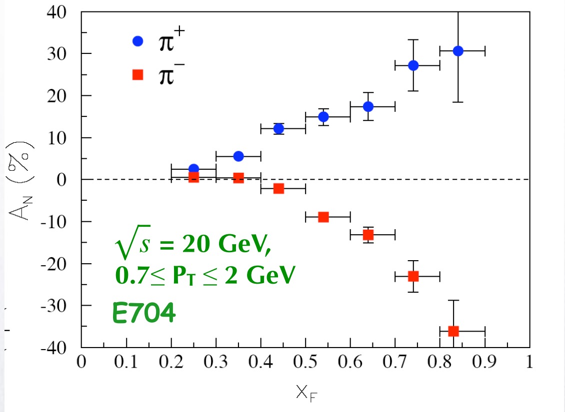


$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$ single-spin asymmetry

perturbative QCD $\propto \frac{m_q}{p_T}\alpha_s \sim \mathcal{O}(10^{-3})$

Kane, Pumplin, Repko,
P.R.L. **41** ('78) 1689





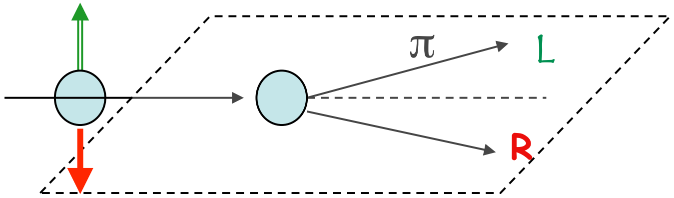
Persisting also up to
$\sqrt{s} = 200$ GeV

Adams *et al.* (STAR),
PRL **92** (04) 171801

Evidences to go beyond collinear

Example #3: semi-inclusive p-p collisions

$$p^{\uparrow}\, p \longrightarrow \pi\, X$$

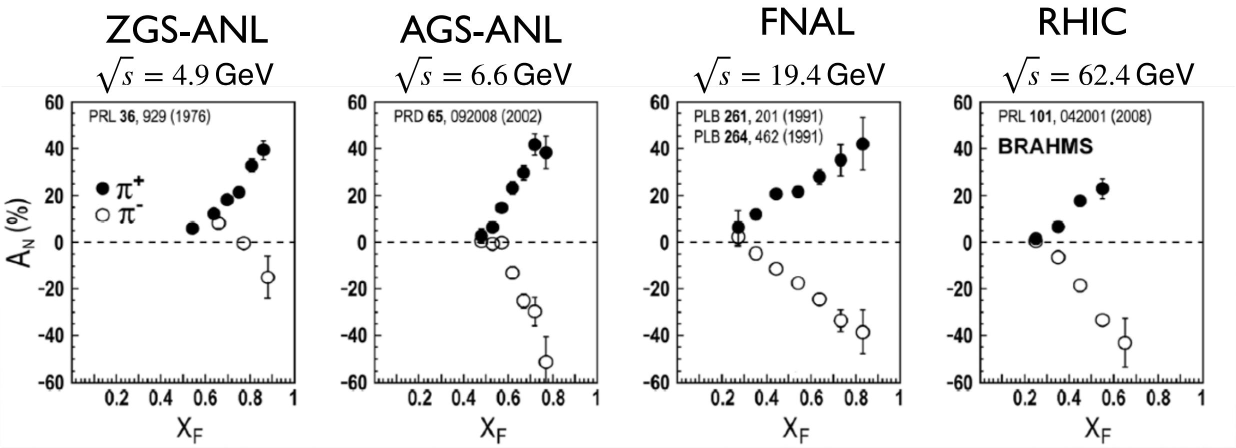


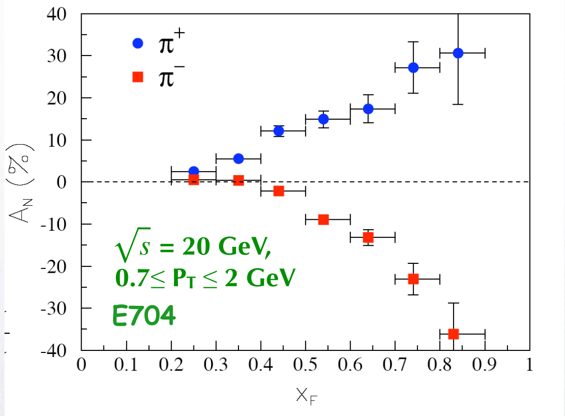
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$ single-spin asymmetry

perturbative QCD $\propto \frac{m_q}{p_T}\alpha_s \sim \mathcal{O}(10^{-3})$

Kane, Pumplin, Repko,
P.R.L. **41** ('78) 1689

Instead, large asymmetries observed.
Evidence of correlation between
spin of the proton and
k_T and flavor of partons





Persisting also up to
$\sqrt{s}$ = 200 GeV

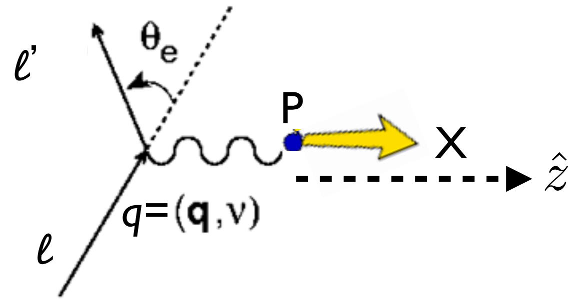
STAR

Adams *et al.* (STAR),
PRL **92** (04) 171801

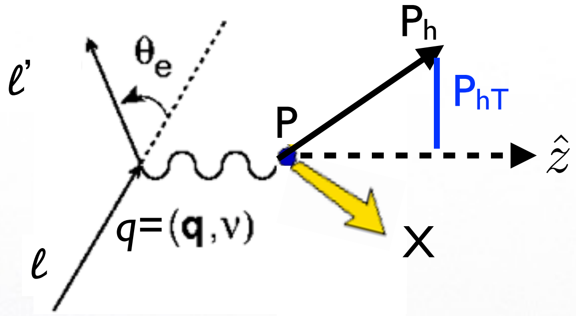
Outline

- The "TMD zoo"
  - factorisation theorem and general properties (generalising same steps that lead to PDFs)
  - specific properties

Need semi-inclusive process



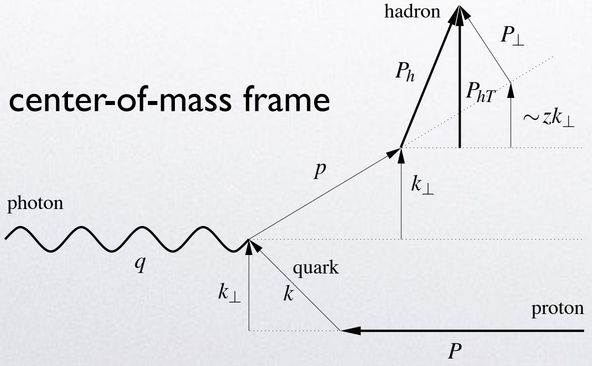
inclusive DIS: - **hard** scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- factorisation → isolate PDFs
- no further scale to probe proton interior



semi-inclusive DIS (SIDIS):
- **hard** scale $Q^2 = -q^2 \gg M^2$ to "see" partons
- **soft** scale: detect hadron *h* with $P_{hT}^2 \sim M^2 \ll Q^2$
- factorisation → isolate TMDs

*Ji, Yuan, Ma, P.R. D**71** (05)*
*Rogers & Aybat, P.R. D**83** (11)*
Collins, "Foundations of Perturbative QCD" (11)
*Echevarria, Idilbi, Scimemi, JHEP **1207** (12)*

center-of-mass frame



with these **two** scales, the process is factorizable into a hard photon-quark vertex and a quark→hadron fragmentation

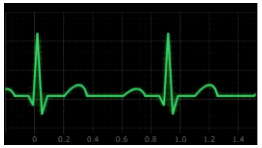
$$\mathbf{P}_{hT} = z\mathbf{k}_\perp + \mathbf{P}_\perp + \mathcal{O}(\mathbf{k}_\perp^2/Q^2)$$

z = fractional energy of *h*
(analogous of *x*)

hadron P_{hT} arises from struck quark $k_\perp$ and transverse momentum $P_\perp$ generated during fragmentation

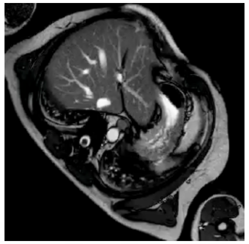
measure P_{hT} → get to $k_\perp$

The TMD framework



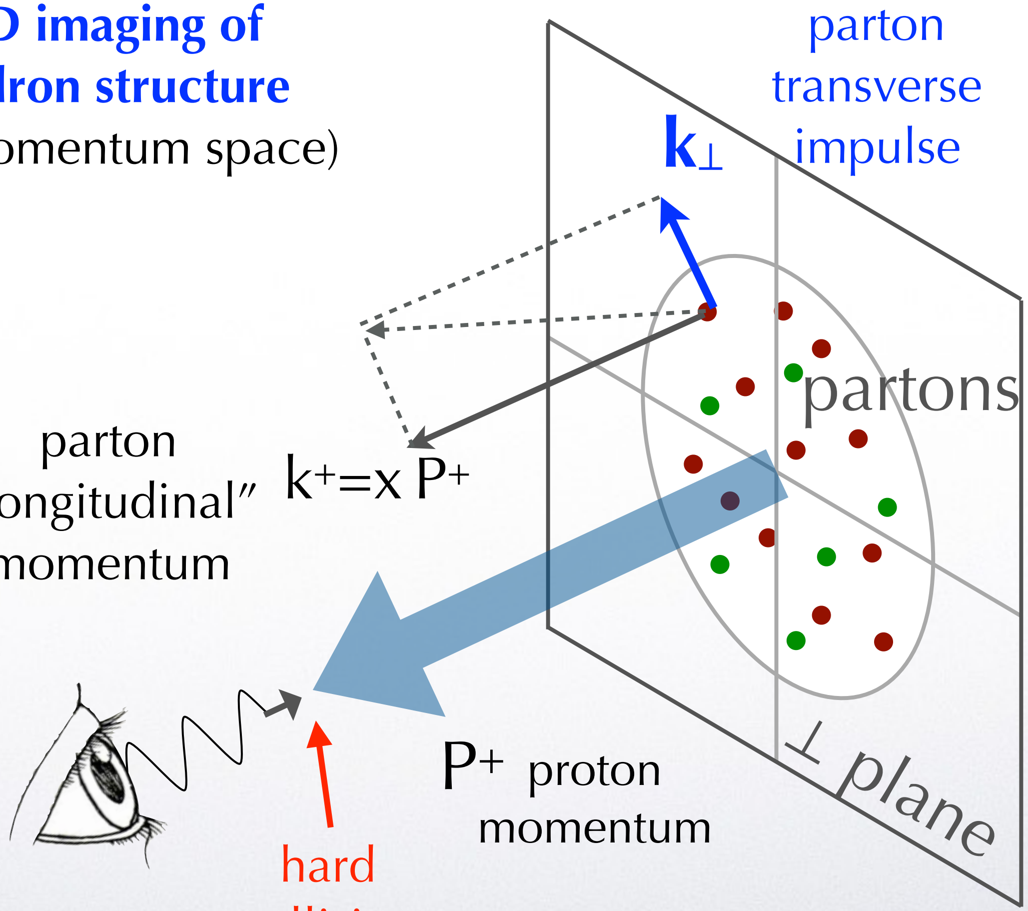
ECG

↓



cardio
MR

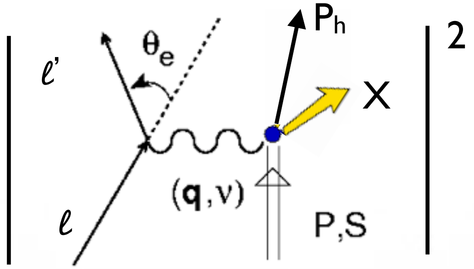
A new paradigm:
3D imaging of
hadron structure
(in momentum space)



parametrised by
Transverse-Momentum
Dependent PDFs
TMD PDF(x, $k_\perp$; Q^2)

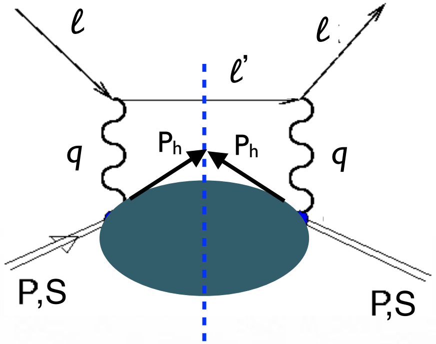
SIDIS

one photon-exchange approximation

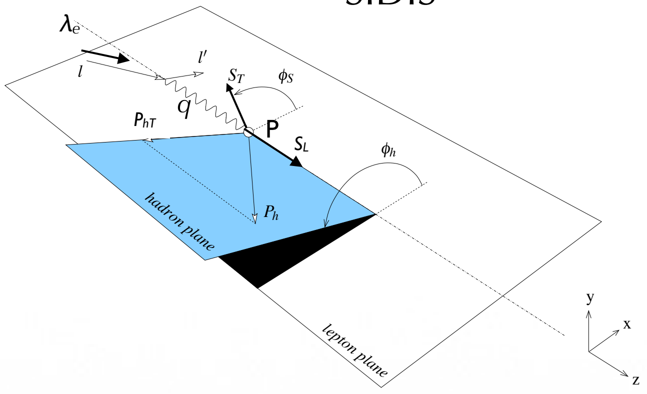


optical theorem

$\longrightarrow$



SIDIS



same invariants as inclusive DIS plus

$z_h = \frac{P \cdot P_h}{P \cdot q}$ "energy fraction" of fragmenting parton carried by final hadron

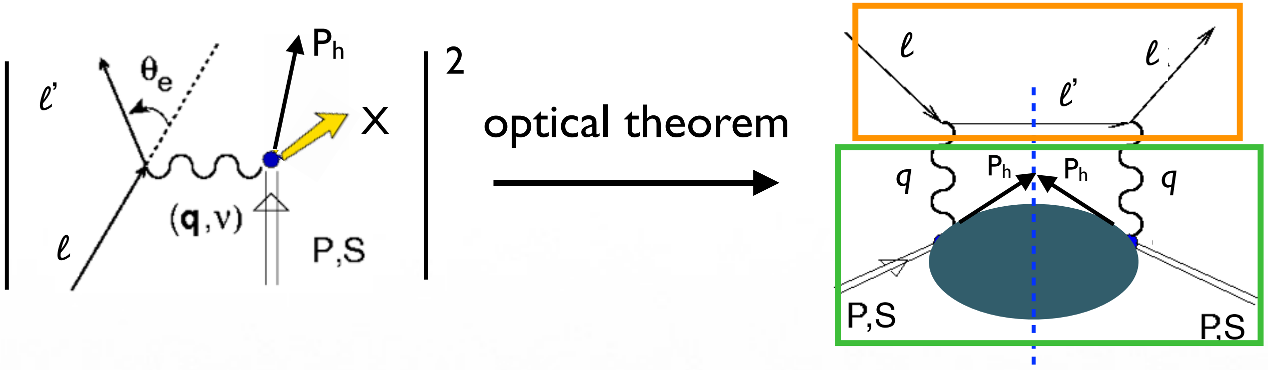
$$\frac{d\sigma}{dx_B dy d\phi_S \underline{dz_h d\phi_h dP_{hT}^2}} = \frac{\alpha^2 y}{2 z_h Q^4} L_{\mu\nu}(\ell, \ell', \lambda_e) W^{\mu\nu}(q, P, S, P_h)$$

new dependence

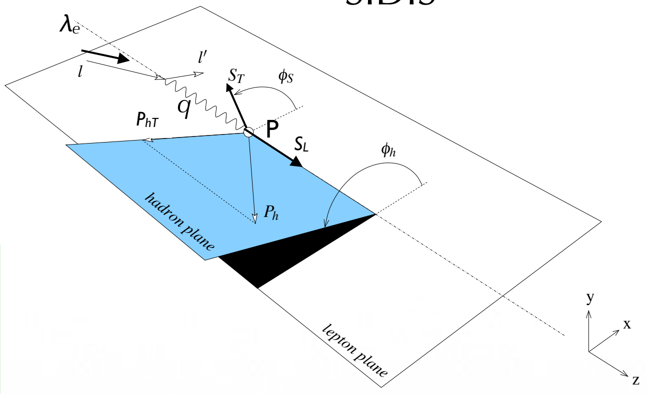
(for unpolarized hadron, S_h=0)

Example : SIDIS

one photon-exchange approximation



SIDIS



same invariants as inclusive DIS plus

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

"energy fraction" of fragmenting parton carried by final hadron

$$\frac{d\sigma}{dx_B dy d\phi_S \underbrace{dz_h d\phi_h dP_{hT}^2}} = \frac{\alpha^2 y}{2 z_h Q^4} L_{\mu\nu}(\ell, \ell', \lambda_e) W^{\mu\nu}(q, P, S, P_h)$$

new dependence

(for unpolarized hadron, S_h=0)

leptonic tensor

hadronic tensor

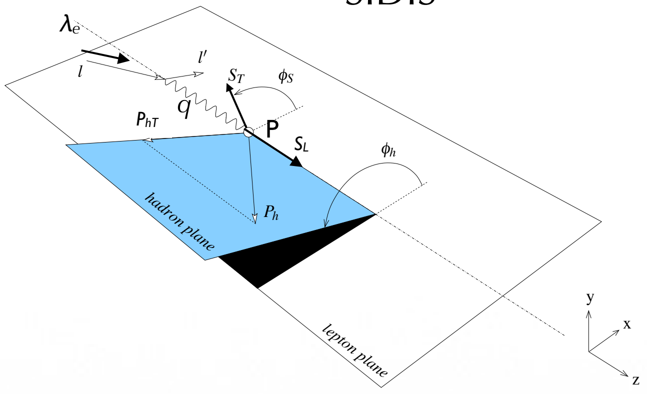
parametrised with

8 structure functions at leading twist

(**18** including subleading twist)

SIDIS cross section

SIDIS



$$\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{hT}^2} =$$

$$= \frac{\alpha^2}{x_B y Q^2} \Big[A(y)\, F_{UU,T} + B(y)\,\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h}$$

$$+ \lambda_e\, S_L\, C(y)\, F_{LL}$$

$$+ S_T \big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h - \phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)} + B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h - \phi_S)} \big]$$

$$+ \lambda_e\, S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h - \phi_S)} \Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

each

$F_{\ldots}(x_B, z_h, P_{hT}^2, Q^2)$

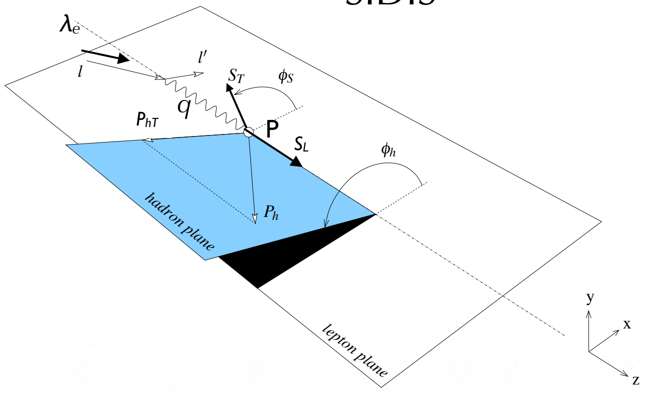
$F_{XY,Z}$

↗ ↑ ↖

ℓ P γ^*

SIDIS cross section

SIDIS



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, dP_{hT}^2} =$$

$$= \frac{\alpha^2}{x_B y Q^2}\Big[A(y)\, F_{UU,T} + B(y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\, \sin 2\phi_h\, F_{UL}^{\sin 2\phi_h}$$

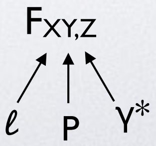
$$+ \lambda_e S_L\, C(y)\, F_{LL}$$

$$+ S_T\big[A(y)\, \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h-\phi_S)} + B(y)\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + B(y)\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}\big]$$

$$+ \lambda_e S_T\, C(y)\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)}\Big] \quad + \mathcal{O}\left(\frac{M}{Q}\right)$$

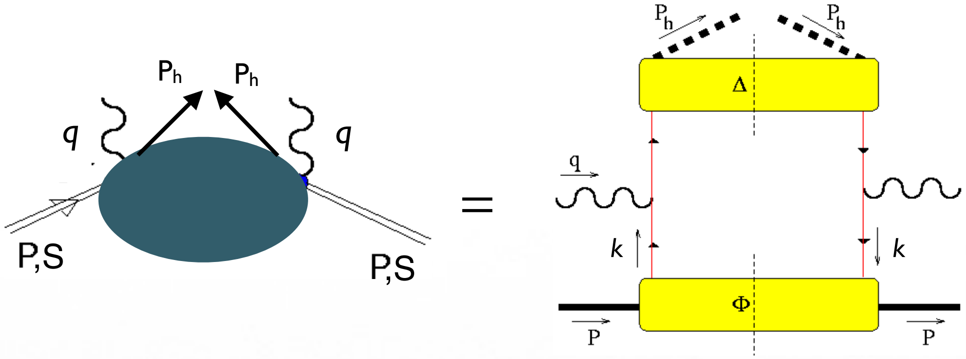
each

$F_{\dots}(x_B, z_h, P_{hT}^2, Q^2)$



SIDIS : factorisation

OPE not possible, use diagrammatic approach (select dominant diagram by counting powers of divergences)



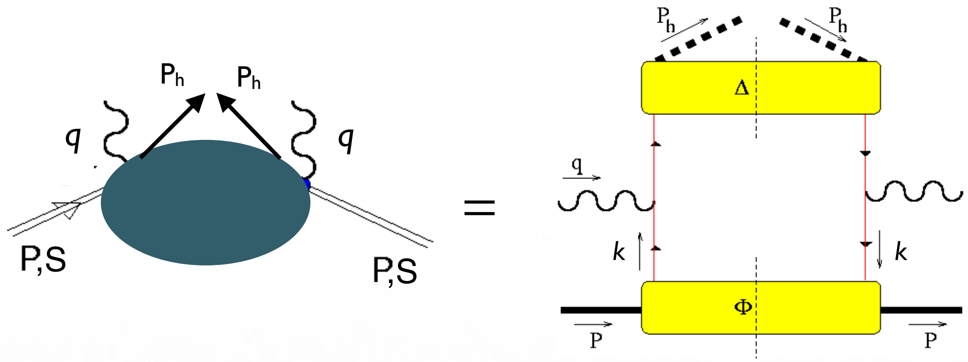
+ higher twists (suppressed)

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathscr{C}\Big[\mathrm{Tr}\big[\Phi(x_B,\mathbf{k}_\perp,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{P}_\perp)\,\gamma^\nu\big]\Big]$$

$$\mathscr{C}\big[\ldots\big] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\big[\ldots\big]$$

SIDIS : factorisation

OPE not possible, use diagrammatic approach (select dominant diagram by counting powers of divergences)



+ higher twists (suppressed)

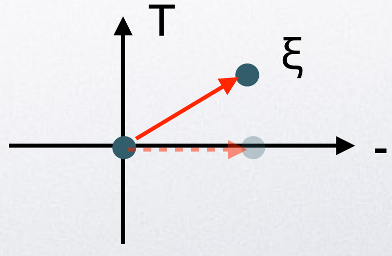
$$2MW^{\mu\nu}(q,P,S,P_h) = \; 2z_h\,\mathscr{C}\Big[\mathrm{Tr}\big[\Phi(x_B,\mathbf{k}_\perp,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{P}_\perp)\,\gamma^\nu\big]\;\Big]$$

$$\mathscr{C}\Big[\dots\Big] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\Big[\dots\Big]$$

non-local correlator:

from collinear $$\Phi(x,S) = \int \frac{d\xi_-}{2\pi}\,e^{-ik\cdot\xi}\,\langle P,S\,|\,\bar\psi(\xi)\,U_{[\xi,0]}\,\psi(0)\,|\,P,S\rangle_{\xi_+=\xi_T=0}$$

to $$\Phi(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\,e^{-ik\cdot\xi}\,\langle P,S\,|\,\bar\psi(\xi)\,U_{[\xi,0]}\,\psi(0)\,|\,P,S\rangle_{\xi_+=0}$$



$x_B \sim x = \dfrac{k_+}{P_+}$

$\xi_- \leftrightarrow k_+$

$\xi_T \leftrightarrow k_\perp$

SIDIS : factorisation

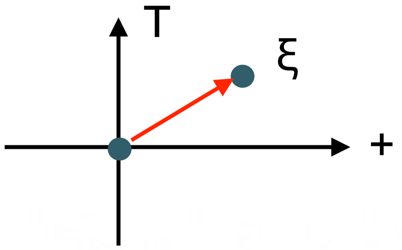
non-local correlators

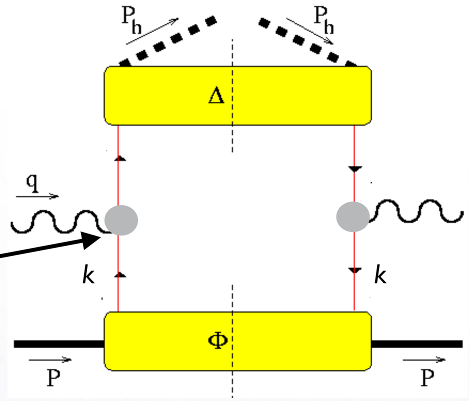
$z_h \sim z = \frac{P_{h-}}{k_-}$

$\xi_+ \leftrightarrow k_-$

$\xi_T \leftrightarrow k_\perp$

$$\Delta(z, \mathbf{k}_\perp) = \sum_X \int \frac{d\xi_+ d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi} \langle 0 \,|\, \psi(0) \,|\, X, P_h \rangle \langle X, P_h \,|\, \bar{\psi}(\xi) \,|\, 0 \rangle_{\xi_-=0}$$

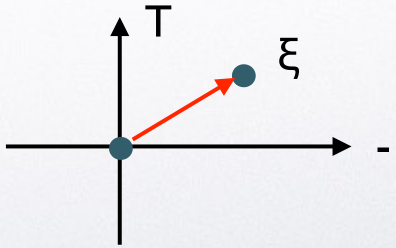




hard cross section

$d\hat{\sigma} = 1 + c_1\alpha_s + \dots$

$$\Phi(x, \mathbf{k}_\perp, S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi} \langle P, S \,|\, \bar{\psi}(\xi)\, U_{[\xi,0]}\, \psi(0) \,|\, P, S \rangle_{\xi_+=0}$$



$x_B \sim x = \frac{k_+}{P_+}$

$\xi_- \leftrightarrow k_+$

$\xi_T \leftrightarrow k_\perp$

SIDIS : factorisation

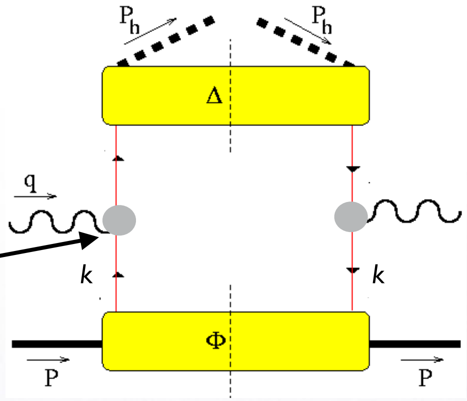
non-local correlators

$z_h \sim z = \frac{P_{h-}}{k_-}$

$\xi_+ \leftrightarrow k_-$

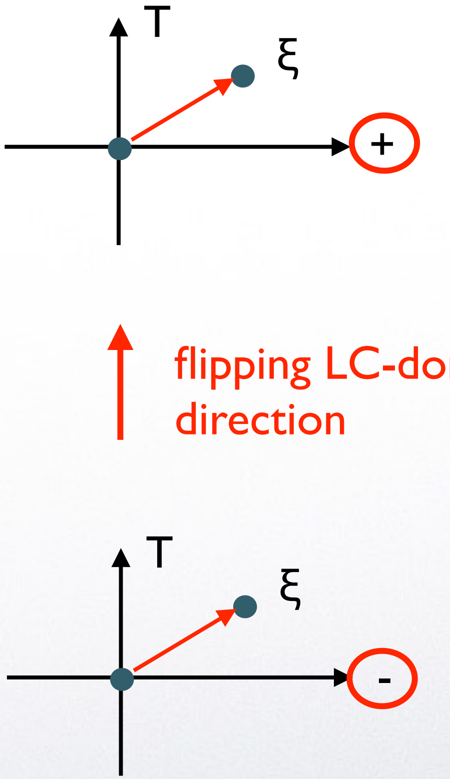
$\xi_T \leftrightarrow k_\perp$

$$\Delta(z,\mathbf{k}_\perp) = \sum_X \int \frac{d\xi_+ d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi} \langle 0|\psi(0)|X,P_h\rangle\langle X,P_h|\bar{\psi}(\xi)|0\rangle_{\xi_-=0}$$



hard cross section

$d\hat{\sigma} = 1 + c_1\alpha_s + \ldots$



flipping LC-dominant direction

$$\Phi(x,\mathbf{k}_\perp,S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi} \langle P,S|\bar{\psi}(\xi)\, U_{[\xi,0]}\, \psi(0)|P,S\rangle_{\xi_+=0}$$

$x_B \sim x = \frac{k_+}{P_+}$

$\xi_- \leftrightarrow k_+$

$\xi_T \leftrightarrow k_\perp$

SIDIS : factorisation

flipping LC-dominant direction

Definitions: $x = \frac{k^+}{P^+}$ $\qquad x_B = \frac{-q^2}{2P \cdot q} = \frac{-2q^+q^-}{2P^+q^-} = -\frac{q^+}{P^+}$

Parton model → elastic kinematics → $x \approx x_B$

SIDIS : factorisation

flipping LC-dominant direction

Definitions: $x = \frac{k^+}{P^+} \qquad x_B = \frac{-q^2}{2P \cdot q} = \frac{-2q^+q^-}{2P^+q^-} = -\frac{q^+}{P^+}$

Parton model → elastic kinematics → $x \approx x_B$

initial parton $$k = \{k^+, k^-, \mathbf{k}_\perp\} = \left\{ xP^+, \frac{k^2 + \mathbf{k}_\perp^2}{2xP^+}, \mathbf{k}_\perp \right\} \approx \{xP^+, 0, \mathbf{k}_\perp\}$$

$k^2 = 2k^+k^- - \mathbf{k}_T^2$

+ component dominant

SIDIS : factorisation

flipping LC-dominant direction

Definitions: $x = \frac{k^+}{P^+}$ $\qquad x_B = \frac{-q^2}{2P \cdot q} = \frac{-2q^+q^-}{2P^+q^-} = -\frac{q^+}{P^+}$

Parton model → elastic kinematics → $x \approx x_B$

initial parton $$k = \{k^+, k^-, \mathbf{k}_\perp\} = \left\{ xP^+, \frac{k^2 + \mathbf{k}_\perp^2}{2xP^+}, \mathbf{k}_\perp \right\} \approx \{xP^+, 0, \mathbf{k}_\perp\}$$

$$\searrow \quad k^2 = 2k^+k^- - \mathbf{k}_T^2 \quad \nearrow$$

+ component dominant

momentum transfer $$q = \{q^+, q^-, \mathbf{0}_T\} = \left\{ -xP^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T \right\}$$

$$\searrow \quad q^2 = 2q^+q^- \quad \nearrow$$

SIDIS : factorisation

flipping LC-dominant direction

Definitions: $x = \frac{k^+}{P^+}$ $\qquad x_B = \frac{-q^2}{2P\cdot q} = \frac{-2q^+q^-}{2P^+q^-} = -\frac{q^+}{P^+}$

Parton model → elastic kinematics → $x \approx x_B$

initial parton $$k = \{k^+, k^-, \mathbf{k}_\perp\} = \left\{ xP^+, \frac{k^2 + \mathbf{k}_\perp^2}{2xP^+}, \mathbf{k}_\perp \right\} \approx \{xP^+, 0, \mathbf{k}_\perp\}$$

$k^2 = 2k^+k^- - \mathbf{k}_T^2$

+ component dominant

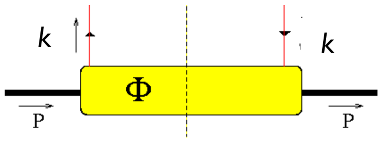
momentum transfer $$q = \{q^+, q^-, \mathbf{0}_T\} = \left\{ -xP^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T \right\}$$

$q^2 = 2q^+q^-$

final parton $$k' = k + q = \left\{ 0, \frac{Q^2}{2xP^+}, \mathbf{k}_\perp \right\} \approx \{0, Q, \mathbf{k}_\perp\}$$

- component dominant

parton-parton correlator



linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

expansion of Φ in powers of M/P+ . At leading twist:

$$\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[f_1\,\gamma_- - f_{1T}^\perp \frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\,\gamma_-$$

$$+g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,\gamma_5\gamma_-$$

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu,\gamma^\nu]$$

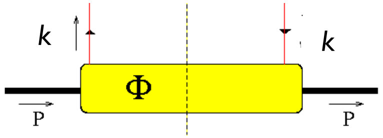
$$+h_{1T}\,i\sigma_{-\nu}\,\gamma_5\,S_T^\nu + h_{1L}^\perp\,i\sigma_{-\nu}\,\gamma_5\,S_L\,\frac{k_\perp^\nu}{M}$$

$$+h_{1T}^\perp\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\,\gamma_5\,\frac{k_\perp^\nu}{M} - h_1^\perp\,\sigma_{-\nu}\,\frac{k_\perp^\nu}{M}\Big]$$

Notations:

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)



expansion of Φ in powers of M/P₊ . At leading twist:

$$\begin{aligned}\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[& f_1\,\gamma_- - f_{1T}^\perp\,\frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\,\gamma_- \\ & + g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,\gamma_5\gamma_- \\ & + h_{1T}\,i\sigma_{-\nu}\,\gamma_5\,S_T^\nu + h_{1L}^\perp\,i\sigma_{-\nu}\,\gamma_5\,S_L\,\frac{k_\perp^\nu}{M} \\ & + h_{1T}^\perp\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\,\gamma_5\,\frac{k_\perp^\nu}{M} - h_1^\perp\,\sigma_{-\nu}\,\frac{k_\perp^\nu}{M}\Big]\end{aligned}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]$$

Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$

waited by $k_\perp^i$

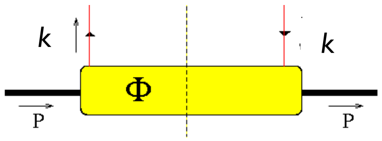
leading twist

X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)



expansion of Φ in powers of M/P₊ . At leading twist:

$$\Phi(x,\mathbf{k}_\perp,S) = \frac{1}{2}\Big[f_1\,\gamma_- - f_{1T}^\perp\,\frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\,\gamma_-$$

$$+ g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,\gamma_5\gamma_-$$

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu,\gamma^\nu]$$

$$+ h_{1T}\,i\sigma_{-\nu}\,\gamma_5\,S_T^\nu + h_{1L}^\perp\,i\sigma_{-\nu}\,\gamma_5\,S_L\,\frac{k_\perp^\nu}{M}$$

$$+ h_{1T}^\perp\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\,\gamma_5\,\frac{k_\perp^\nu}{M} - h_1^\perp\,\sigma_{-\nu}\,\frac{k_\perp^\nu}{M}\Big]$$

$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \equiv \Phi^{[\gamma_+]}$ → 2 TMDPDFs for unpol. parton

Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$

waited by $k_\perp{}^i$

leading twist

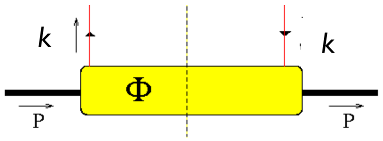
X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

parton-parton correlator

linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

expansion of Φ in powers of M/P₊ . At leading twist:



$$\Phi(x,\mathbf{k}_\perp,S)=\frac{1}{2}\Big[f_1\,\gamma_- - f_{1T}^{\perp}\,\frac{(\mathbf{k}_\perp\times\mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}\,\gamma_-$$

$$+g_{1L}\,S_L\,\gamma_5\gamma_- + g_{1T}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,\gamma_5\gamma_-$$

$$\sigma^{\mu\nu}=\frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]\qquad +h_{1T}\,i\sigma_{-\nu}\,\gamma_5\,S_T^\nu + h_{1L}^{\perp}\,i\sigma_{-\nu}\,\gamma_5\,S_L\,\frac{k_\perp^\nu}{M}$$

$$+h_{1T}^{\perp}\,\frac{\mathbf{k}_\perp\cdot\mathbf{S}_T}{M}\,i\sigma_{-\nu}\,\gamma_5\,\frac{k_\perp^\nu}{M} - h_1^{\perp}\,\sigma_{-\nu}\,\frac{k_\perp^\nu}{M}\Big]$$

$\frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma_+\right]\equiv\Phi^{[\gamma_+]}$ → 2 TMDPDFs for unpol. parton

$\frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma_+\gamma_5\right]\equiv\Phi^{[\gamma_+\gamma_5]}$ → 2 TMDPDFs for long. pol. parton

$\frac{1}{2}\mathrm{Tr}\left[\Phi\,i\sigma_{+i}\,\gamma_5\right]\equiv\Phi^{[i\sigma_{+i}\,\gamma_5]}$ → 4 TMDPDFs for transv. pol. parton along *i*

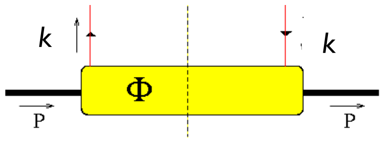
Notations: $t_{1X}^{(\perp)}(x,\mathbf{k}_\perp^2)$ waited by $k_\perp^i$

leading twist

X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

parton-parton correlator



linear combination of all tensor structures with *k*, *P*, *S*, subject to Hermiticity and parity-invariance (see later about time reversal)

expansion of Φ in powers of M/P₊ . At leading twist:

$$\Phi(x, \mathbf{k}_\perp, S) = \frac{1}{2}\Big[\boxed{f_1} \gamma_- - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot \hat{\mathbf{P}}}{M} \gamma_-$$

$$+ \boxed{g_{1L}} S_L \gamma_5 \gamma_- + g_{1T} \frac{\mathbf{k}_\perp \cdot \mathbf{S}_T}{M} \gamma_5 \gamma_-$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \qquad + \boxed{h_1} i\sigma_{-\nu}\gamma_5 S_T^\nu + h_{1L}^\perp i\sigma_{-\nu}\gamma_5 S_L \frac{k_\perp^\nu}{M}$$

$$+ h_{1T}^\perp \frac{\mathbf{k}_\perp \cdot \mathbf{S}_T}{M} i\sigma_{-\nu}\gamma_5 \frac{k_\perp^\nu}{M} - h_1^\perp \sigma_{-\nu} \frac{k_\perp^\nu}{M} \Big]$$

$\frac{1}{2}\mathrm{Tr}[\Phi\, \gamma_+] \equiv \Phi^{[\gamma_+]}$ → 2 TMDPDFs for unpol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\, \gamma_+ \gamma_5] \equiv \Phi^{[\gamma_+ \gamma_5]}$ → 2 TMDPDFs for long. pol. parton

$\frac{1}{2}\mathrm{Tr}[\Phi\, i\sigma_{+i}\gamma_5] \equiv \Phi^{[i\sigma_{+i}\gamma_5]}$

→ 4 TMDPDFs for transv. pol. parton along *i*

Notations: $t_{1X}^{(\perp)}(x, \mathbf{k}_\perp^2)$ waited by $k_\perp{}^i$

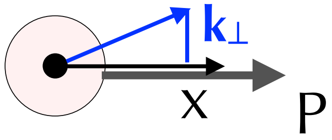
leading twist

X = *L* longitudinally polarized hadron
X = *T* transversely polarized hadron

t = *f* unpolarized parton
t = *g* longitudinally polarized parton
t = *h* transversely polarized parton

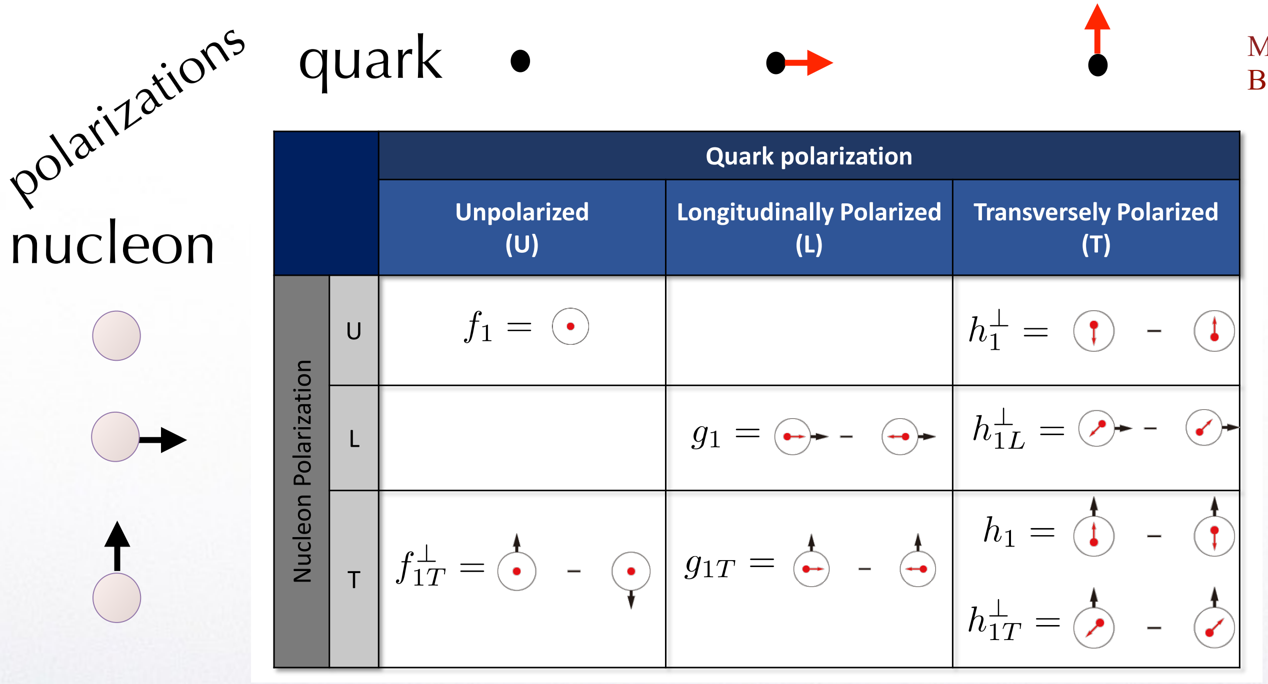
survive upon $\int d\mathbf{k}_\perp$ → collinear PDF

The TMD PDF table



TMD PDFs (x, k⊥; Q²) at leading twist for a spin-1/2 hadron (Nucleon)

Mulders & Tangerman, N.P. **B461** (96)
Boer & Mulders, P.R. D57 (98)

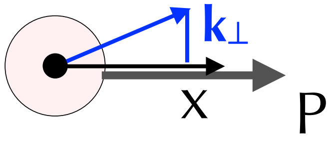


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

Each entry has a nice probabilistic interpretation

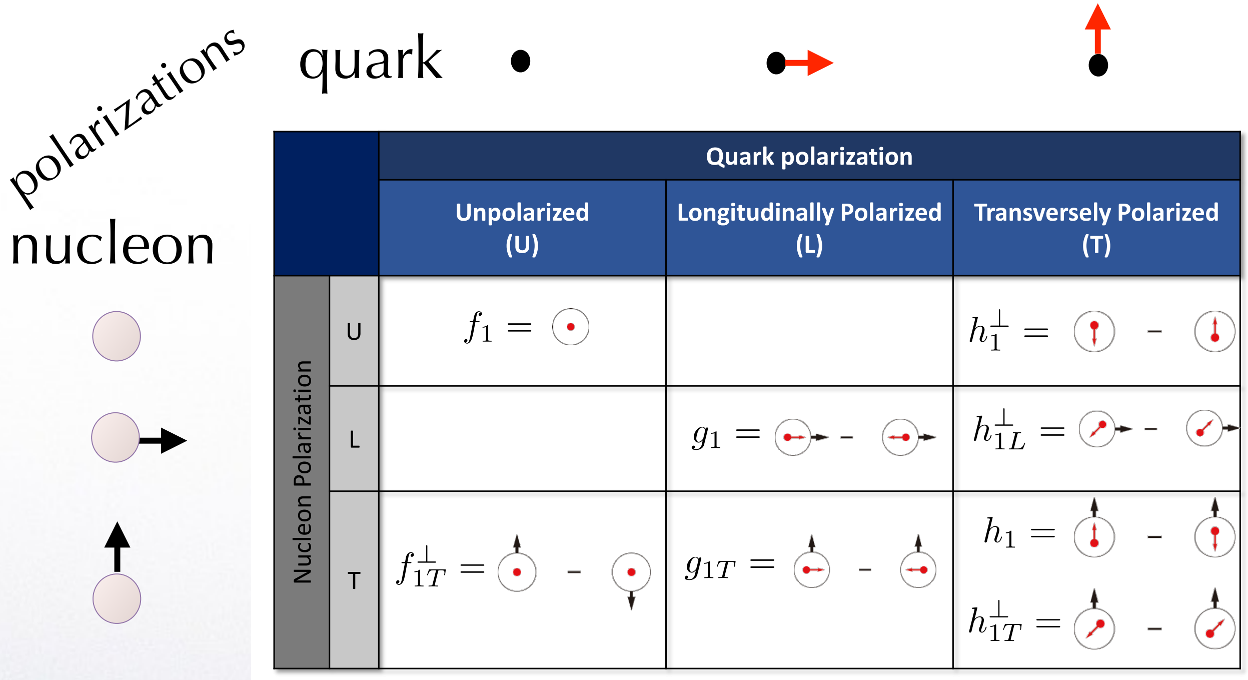
N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD PDFs, but no probabilistic interpretation

The TMD PDF table



TMD PDFs (x, k⊥; Q²) at leading twist for a spin-1/2 hadron (Nucleon)

Mulders & Tangerman, N.P. **B461** (96)
Boer & Mulders, P.R. **D57** (98)



Nucleon Polarization		Quark polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
	U	$f_1 =$ ⊙		$h_1^{\perp} =$ ○ − ○
	L		$g_1 =$ ○ − ○	$h_{1L}^{\perp} =$ ○ − ○
	T	$f_{1T}^{\perp} =$ ⊙ − ⊙	$g_{1T} =$ ○ − ○	$h_1 =$ ○ − ○ $h_{1T}^{\perp} =$ ○ − ○

nomenclature

no-name Boer-Mulders

helicity Kotzinian-Mulders

transversity

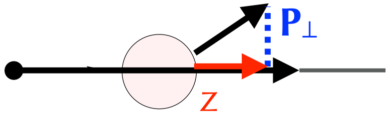
pretzelocity

Sivers worm gear

Each entry has a nice probabilistic interpretation

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD PDFs, but no probabilistic interpretation

The TMD FF table



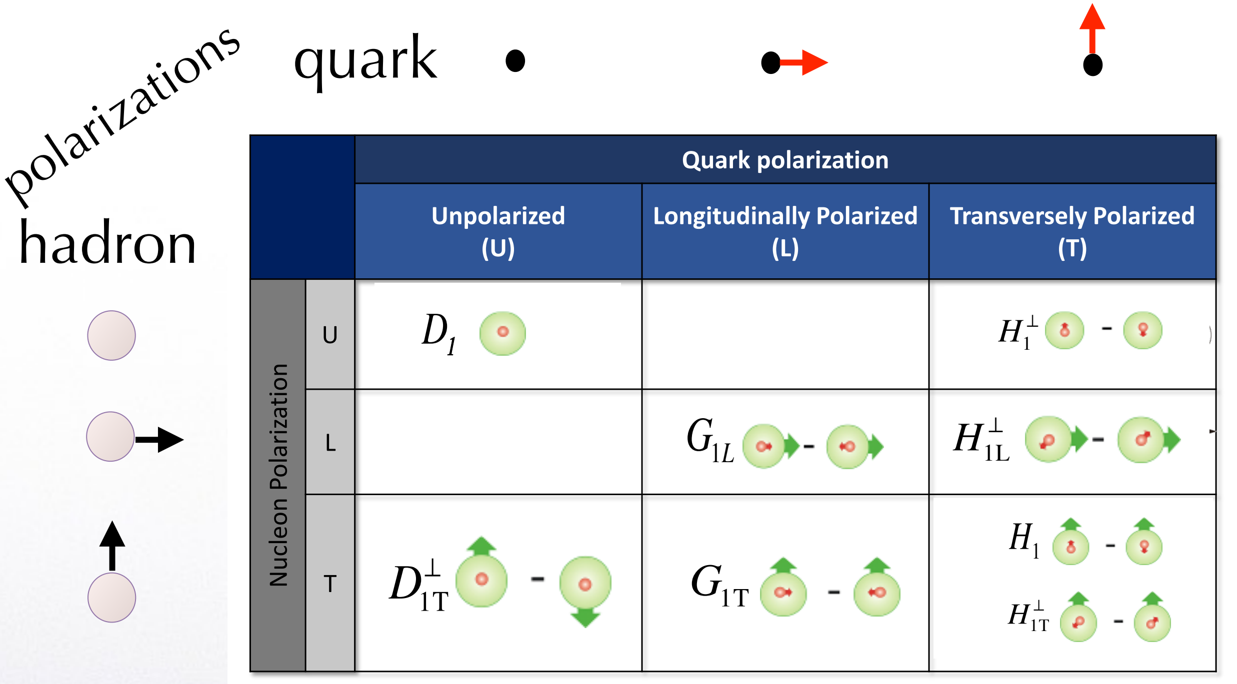
TMD FFs (z, $P_\perp$; Q^2) at leading twist (and $S_h \leq 1/2$)

polarizations

quark

hadron

Mulders & Tangerman, N.P. **B461** (96)
Boer & Mulders, P.R. **D57** (98)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1		$H_1^\perp$ -
	L		G_{1L} -	$H_{1L}^\perp$ -
	T	$D_{1T}^\perp$ -	G_{1T} -	H_1 - $H_{1T}^\perp$ -

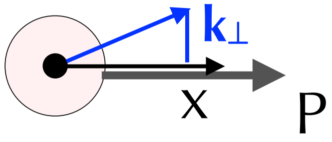
polarising FF …

nomenclature	
no-name	Collins
…	….
…	….

Each entry has a nice probabilistic interpretation

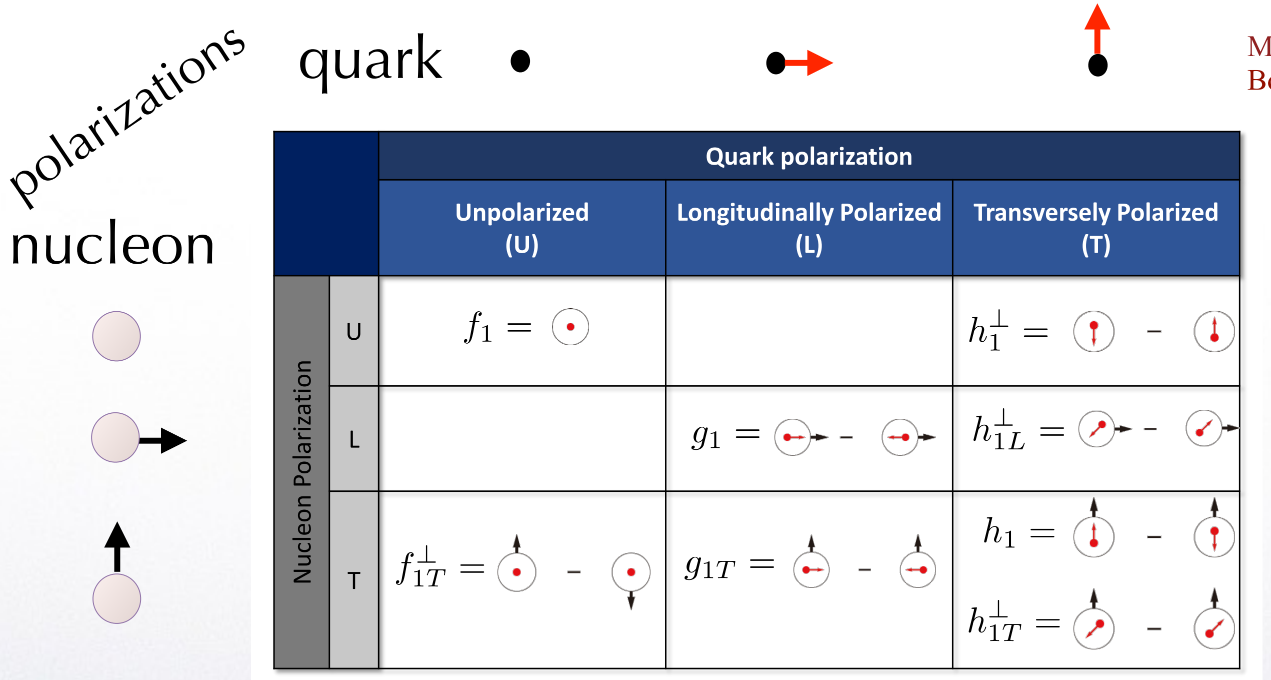
N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD FFs, but no probabilistic interpretation

The TMD PDF table



TMD PDFs (x, k⊥; Q²) at leading twist for a spin-1/2 hadron (Nucleon)

Mulders & Tangerman, N.P. **B461** (96)
Boer & Mulders, P.R. **D57** (98)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} = \quad - $
	L		$g_1 = \quad - $	$h_{1L}^{\perp} = \quad - $
	T	$f_{1T}^{\perp} = \quad - $	$g_{1T} = \quad - $	$h_1 = \quad - $ $h_{1T}^{\perp} = \quad - $

Sivers
worm gear

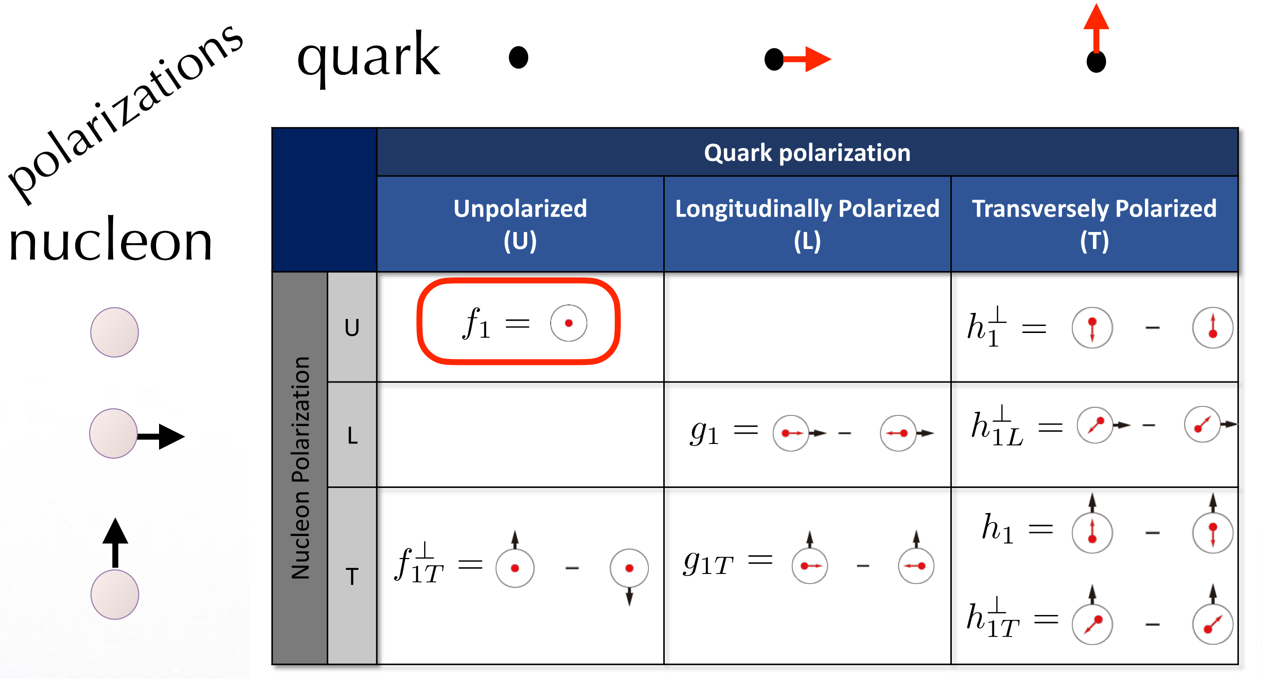
nomenclature

no-name	Boer-Mulders
helicity	Kotzinian-Mulders
transversity	
pretzelocity	

Each entry has a nice probabilistic interpretation

N.B. Using appropriate projections $\Phi^{[\Gamma]}$ one can extract also subleading-twist TMD PDFs, but no probabilistic interpretation

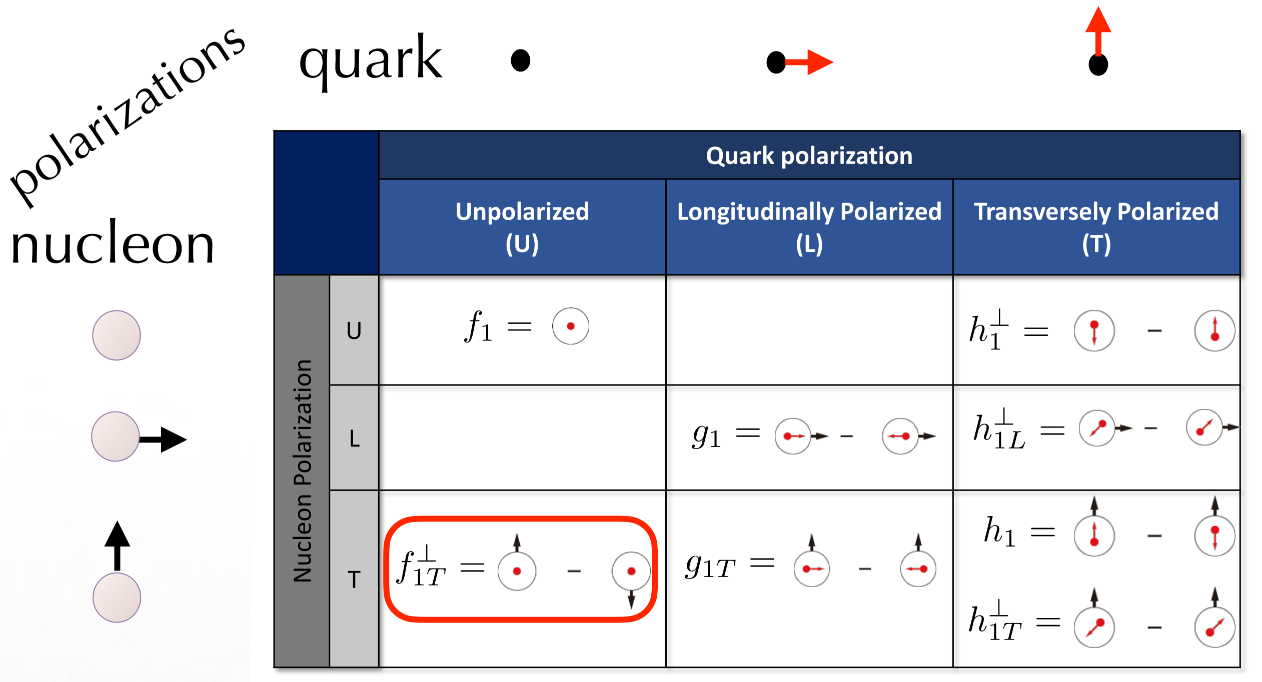
The unpolarized TMD PDF



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

$f_1^q(x, \mathbf{k}_\perp^2)$ probability density of finding a quark *q* with “longitudinal” (along “+” LC direction) fraction *x* of nucleon momentum, and transverse momentum $\mathbf{k}_\perp$

The Sivers TMD PDF



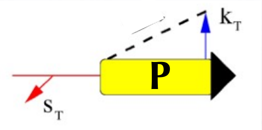
polarizations

quark

nucleon

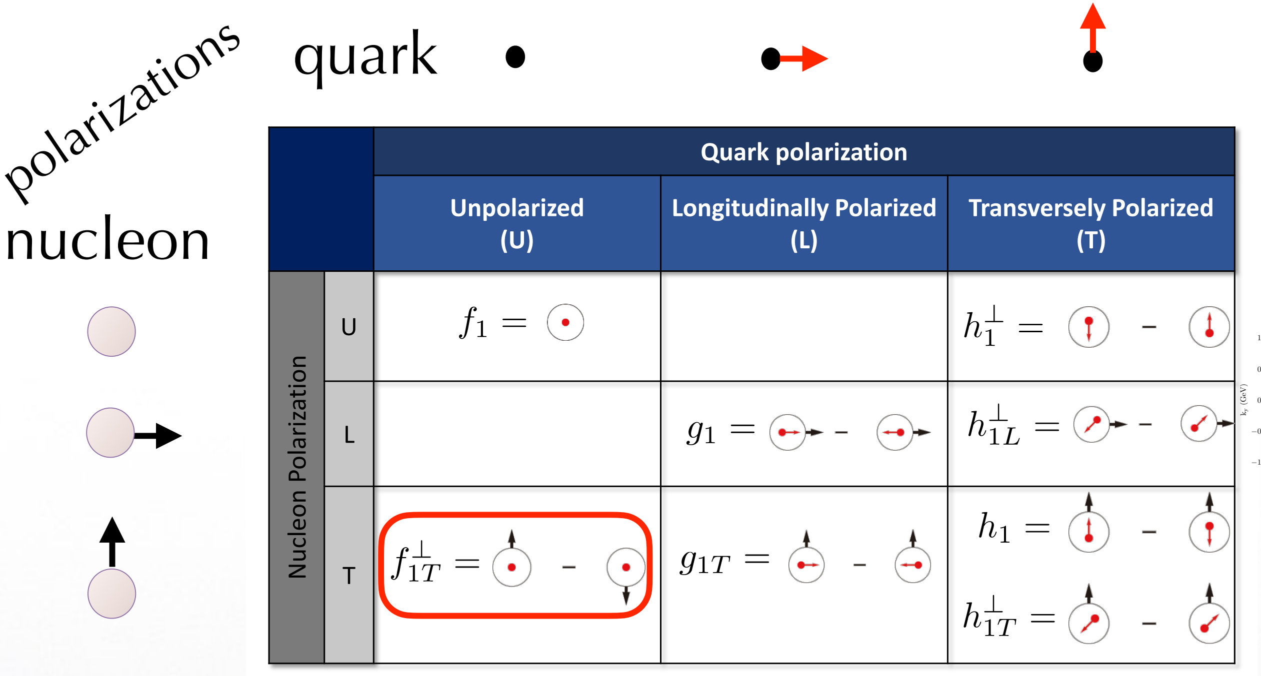
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

$$\frac{1}{2}\mathrm{Tr}[\Phi\gamma_+] \to f_1 - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}$$



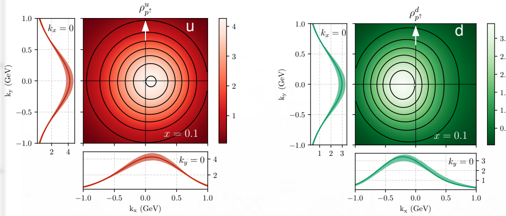
$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

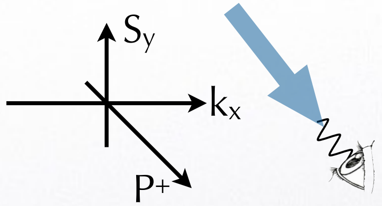
The Sivers TMD PDF



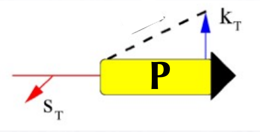
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ − ⊙
	L		$g_1 =$ ⊙ − ⊙	$h_{1L}^\perp =$ ⊙ − ⊙
	T	$f_{1T}^\perp =$ ⊙ − ⊙	$g_{1T} =$ ⊙ − ⊙	$h_1 =$ ⊙ − ⊙ $h_{1T}^\perp =$ ⊙ − ⊙

Bacchetta et al.,
P.L. ***B827*** *(22) 136961,*
arXiv:2004.14278





$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \to f_1 - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}$$



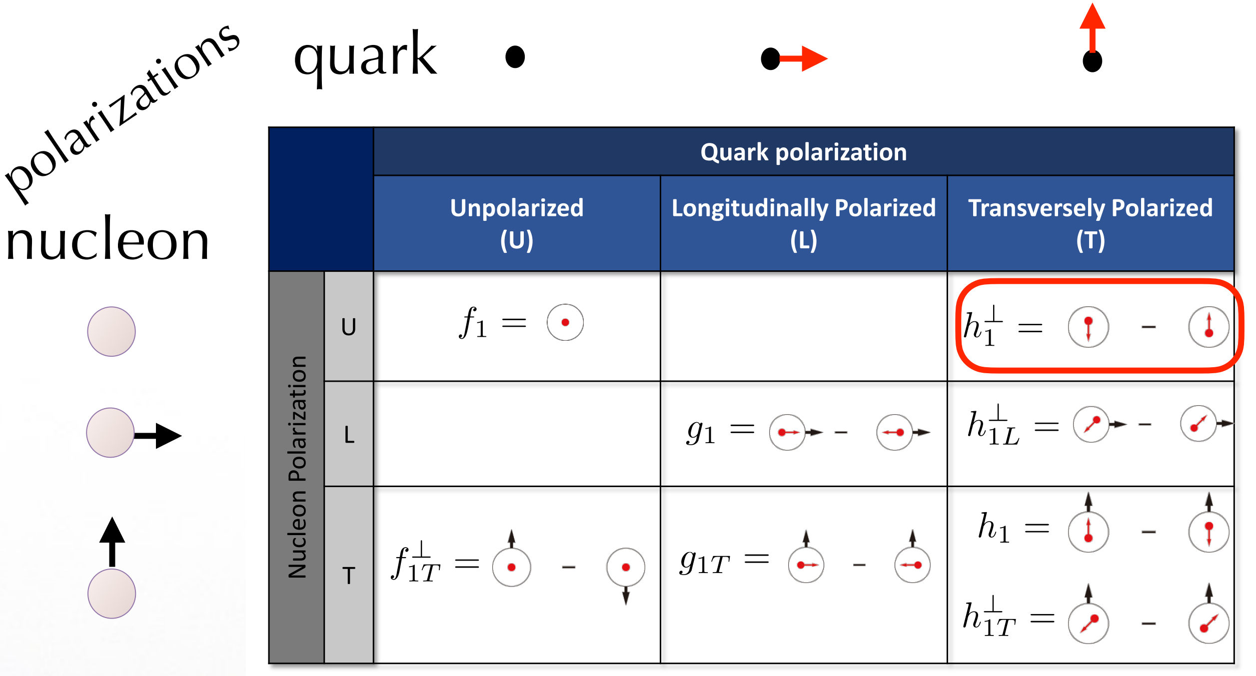
$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

Sivers effect: how the momentum distribution of quarks is distorted by the transverse polarization of parent nucleon ("spin-orbit" correlation)

Sivers $f_{1T}^\perp$ → indirect access to quark orbital angular momentum

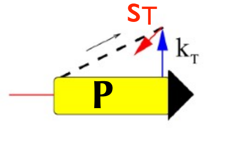
Burkardt, P.R. D**66** (2002) 114005;
N.P. **A735** (2004) 185
Bacchetta & Radici, P.R.L. **107** (2011) 212001
Ji et al., N.P. **B652** (2003) 383

The Boer-Mulders TMD PDF



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^\perp =$ ⊙ - ⊙
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

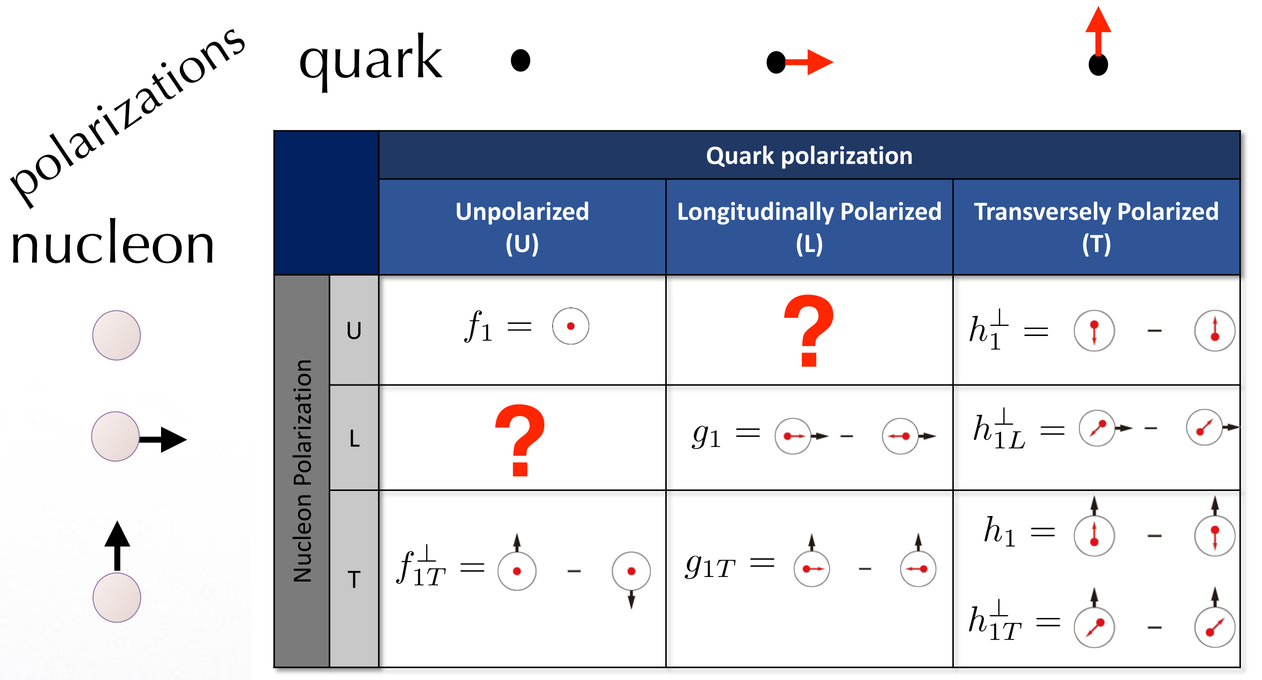
$$\frac{1}{2}\mathrm{Tr}[\Phi\, i\sigma_{+i}\gamma_5] \rightarrow \ \ldots + h_1^\perp \frac{(\mathbf{k}_\perp \times \mathbf{s}_T)\cdot \hat{\mathbf{P}}}{M}$$



$\mathbf{s}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

Boer-Mulders effect: “spin-orbit” correlation at partonic level

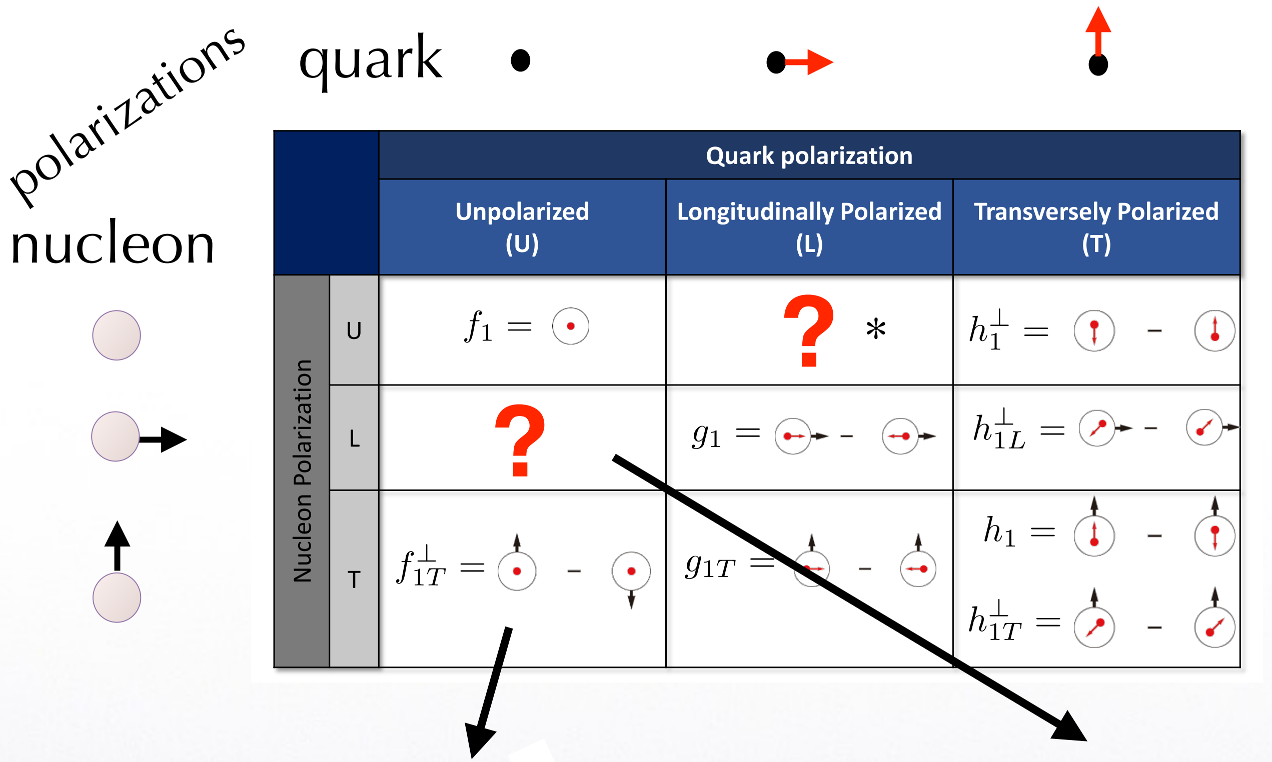
Forbidden combinations



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$	?	$h_1^\perp =$ -
	L	?	$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

Why?

Forbidden combinations



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$	? *	$h_1^\perp =$ -
	L	?	$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

Why?

prohibited by parity invariance

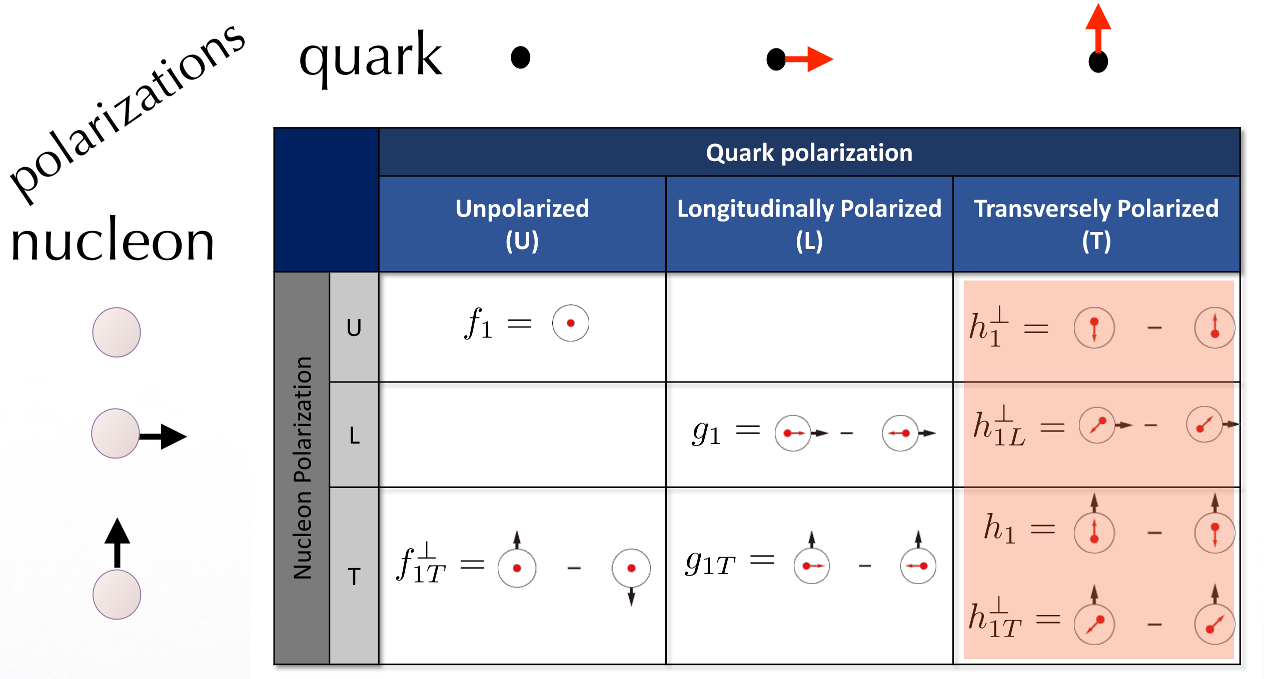
$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

$\mathbf{S}_L \cdot \mathbf{k}_\perp \times \mathbf{P} = 0$

not enough vectors for $f_{1L}^\perp$!

* similarly for "swapped" combination

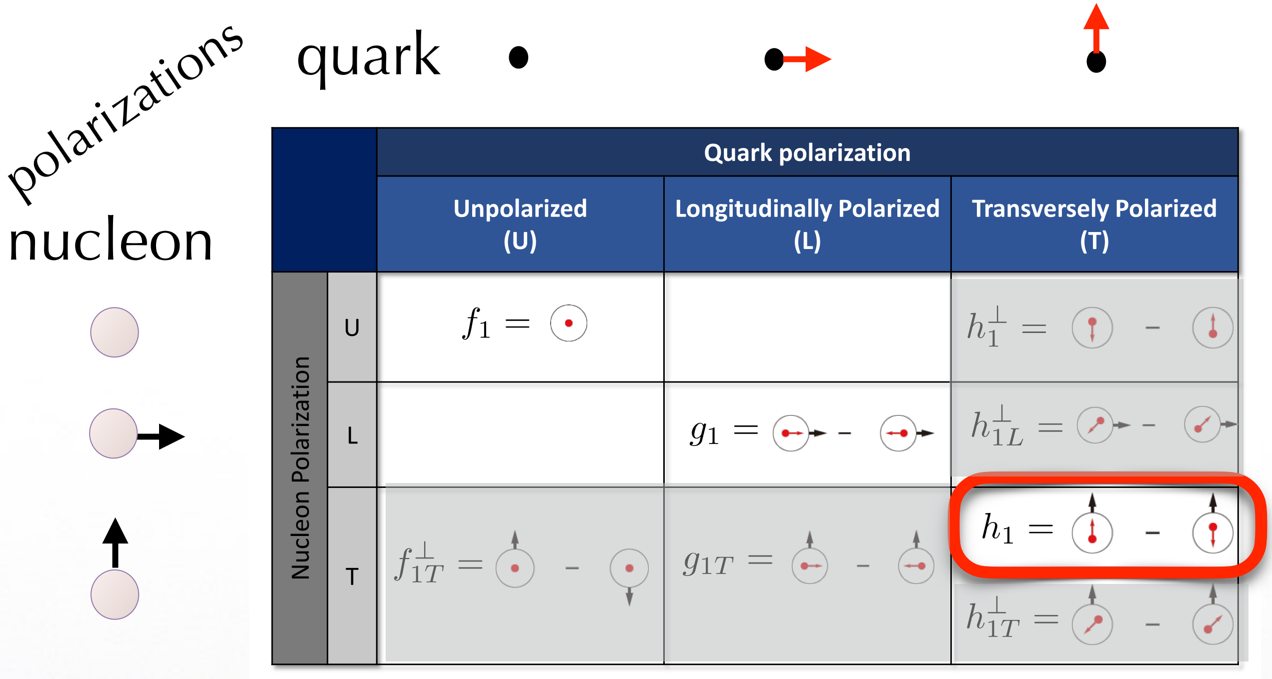
The chiral-odd TMD PDFs



	Quark polarization		
	Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ − ⊙
L		$g_1 =$ ⊙ − ⊙	$h_{1L}^\perp =$ ⊙ − ⊙
T	$f_{1T}^\perp =$ ⊙ − ⊙	$g_{1T} =$ ⊙ − ⊙	$h_1 =$ ⊙ − ⊙ $h_{1T}^\perp =$ ⊙ − ⊙

all TMD PDFs belonging to right column involve transverse polarization of quarks, hence they are "chiral-odd" and are suppressed in perturbative QCD as m_q/Q.
Similarly to transversity h_1, they can appear in the cross section at leading twist if paired to another chiral-odd structure. For SIDIS, they must be paired to a chiral-odd TMD FF.

Transversity



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^\perp =$ ⊙ - ⊙
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

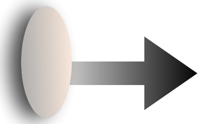
- transversity is the prototype of chiral-odd structures

- the only chiral-odd structure that survives in collinear kinematics

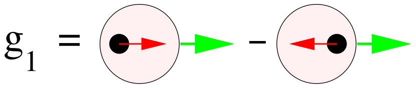
- only way to determine the tensor charge $\delta^q(Q^2) = \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$

Transversity properties

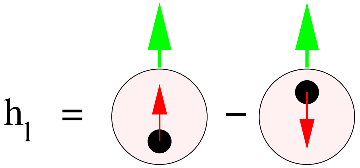
both defined in
Infinite Mom. Frame



boosted Nucleon



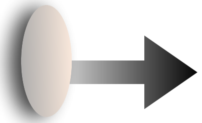
helicity



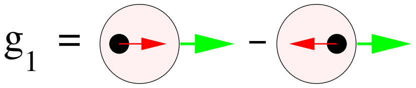
transversity

Transversity properties

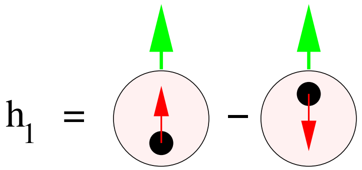
both defined in
Infinite Mom. Frame



boosted Nucleon



helicity



transversity

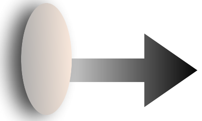
Non-relativistic theory:
boosts & rotations commute

=

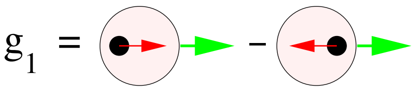
Differences
=> info on relativistic motion of quarks

Transversity properties

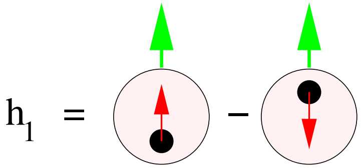
both defined in
Infinite Mom. Frame



boosted Nucleon



helicity



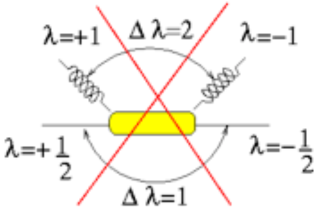
transversity

=

Non-relativistic theory:
boosts & rotations commute

Differences
=> info on relativistic motion of quarks

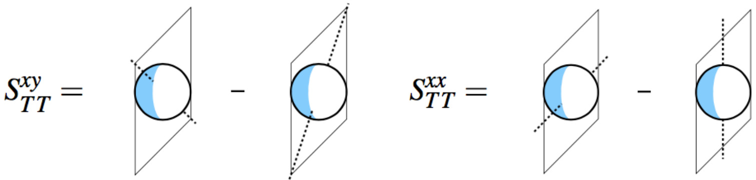
In a spin-1/2 hadron,
no transversity of gluons



singlet and
non-singlet
evolution

only
non-singlet
evolution

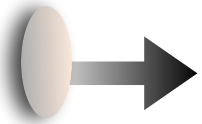
In a spin-1 hadron, gluon transversity possible
because transverse tensor polariz. => Δλ=2
but $h^g_{1,TT} \equiv h^g_1$ is only a TMD and T-odd



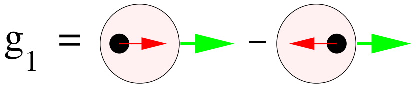
Jaffe & Manohar (1989), Artru & Mekhfi (1990), Bacchetta & Mulders (2000)

Transversity properties

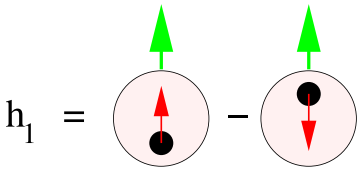
both defined in
Infinite Mom. Frame



boosted Nucleon



helicity



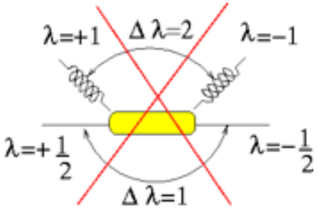
transversity

$$=$$

Non-relativistic theory:
boosts & rotations commute

Differences
=> info on relativistic motion of quarks

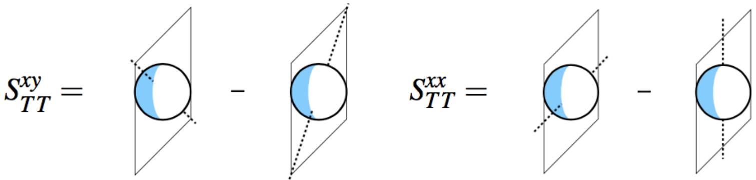
In a spin-1/2 hadron,
no transversity of gluons



singlet and
non-singlet
evolution

only
non-singlet
evolution

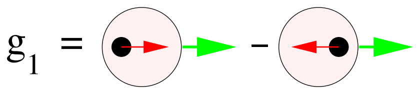
In a spin-1 hadron, gluon transversity possible
because transverse tensor polariz. => Δλ=2
but $h^g_{1,TT} \equiv h^g_1$ is only a TMD and T-odd



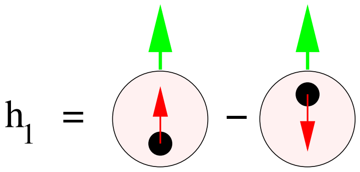
Jaffe & Manohar (1989), Artru & Mekhfi (1990), Bacchetta & Mulders (2000)

Soffer bound: $|h_1| \le \frac{1}{2}(f_1 + g_1)$ for any (x,Q^2)

Transversity properties



helicity



transversity

charges connected to hadronic matrix elements of local operators (calculable on lattice)

$$\langle P, S_L |\, \bar{q}\, \gamma^\mu \gamma_5\, q \,| P, S_L \rangle = S_L P^\mu\, g_A^q$$

axial current <=> axial charge

$$= S_L P^\mu \int_0^1 dx\, g_1^{q+\bar{q}}(x, Q^2)$$

connected to C-even structure

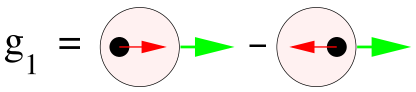
$$\langle P, S |\, \bar{q}\, \sigma^{\mu\nu}\, q \,| P, S \rangle = P^{[\mu} S^{\nu]}\, \delta^q(Q^2)$$

tensor current <=> tensor charge

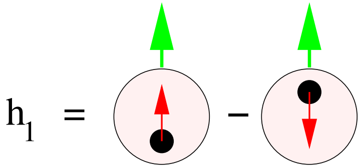
$$= P^{[\mu} S^{\nu]} \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$$

connected to C-odd structure

Transversity properties



helicity



transversity

charges connected to hadronic matrix elements of local operators (calculable on lattice)

$$\langle P, S_L | \bar{q}\, \gamma^\mu \gamma_5\, q | P, S_L \rangle = S_L P^\mu\, g_A^q$$

axial current <=> axial charge

$$= S_L P^\mu \int_0^1 dx\, g_1^{q+\bar{q}}(x, Q^2)$$

connected to C-even structure

anomalous dim. $\Delta\gamma^{(1)} = 0$

=> $g_A{}^q$ is constant

$$\langle P, S | \bar{q}\, \sigma^{\mu\nu}\, q | P, S \rangle = P^{[\mu} S^{\nu]}\, \delta^q(Q^2)$$

tensor current <=> tensor charge

$$= P^{[\mu} S^{\nu]} \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$$

connected to C-odd structure

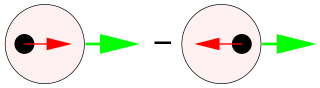
anomalous dim. $\delta\gamma^{(1)} = -\, C_F/2$

$$C_F = \frac{N_c^2 - 1}{2N_c}$$

=> δ^q scales with Q^2

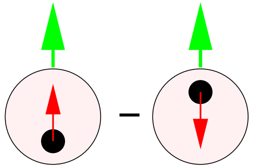
Transversity properties

g_1 =



$\neq$

h_1 =



helicity

transversity

charges connected to hadronic matrix elements of local operators (calculable on lattice)

$$\langle P, S_L | \bar{q}\, \gamma^\mu \gamma_5\, q | P, S_L \rangle = S_L P^\mu\, g_A^q$$

axial current <=> axial charge

$$= S_L P^\mu \int_0^1 dx\, g_1^{q+\bar{q}}(x, Q^2)$$

connected to C-even structure

$$\langle P, S | \bar{q}\, \sigma^{\mu\nu}\, q | P, S \rangle = P^{[\mu} S^{\nu]}\, \delta^q(Q^2)$$

tensor current <=> tensor charge

$$= P^{[\mu} S^{\nu]} \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$$

connected to C-odd structure

anomalous dim. $\Delta\gamma^{(1)} = 0$

=> $g_A{}^q$ is constant

anomalous dim. $\delta\gamma^{(1)} = -\, C_F/2$

$$C_F = \frac{N_c^2 - 1}{2N_c}$$

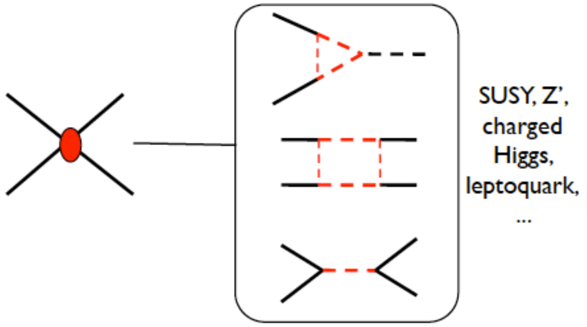
=> δ^q scales with Q^2

helicity and transversity are very different !

Potential for BSM discovery ?

Tensor (and chiral-odd) structures do not appear in the Standard Model Lagrangian at tree level.

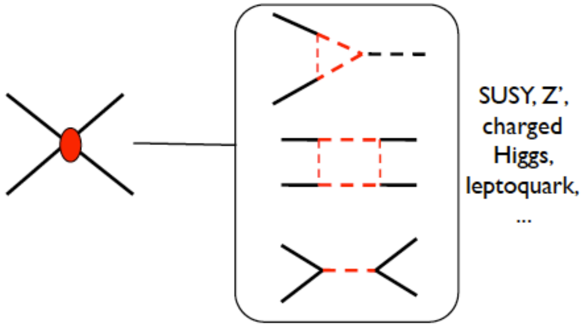
Is it a possible low-energy footprint of BSM physics at higher scale ?



Potential for BSM discovery ?

Tensor (and chiral-odd) structures do not appear in the Standard Model Lagrangian at tree level.

Is it a possible low-energy footprint of BSM physics at higher scale ?



neutron β-decay

$n \rightarrow p\, e^- \bar{\nu}_e$

$$\mathcal{L}_{\rm SM} \sim G_F\, V_{ud}\, \bar{e}\gamma^\mu(1-\gamma_5)\nu_e\, \bar{p}\gamma_\mu(1-\gamma_5)n$$

$$+\mathcal{L}_{\rm eff} \sim G_F\, V_{ud}\, g_T\, \varepsilon_T\, \bar{e}\sigma^{\mu\nu}\nu_e\, \bar{p}\sigma_{\mu\nu}n$$

precision => BSM scale $\frac{M_W^2}{M_{\rm BSM}^2} \approx g_T\, \varepsilon_T$

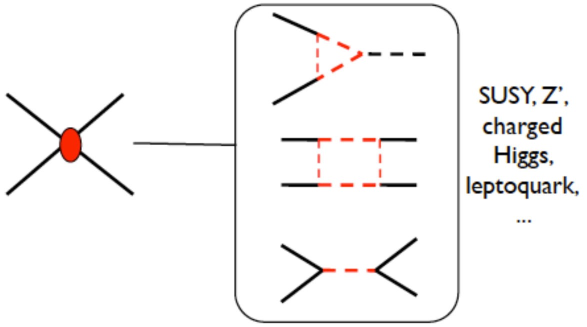
BSM coupling ?

$g_T = \delta u - \delta d$ isovector tensor charge

Potential for BSM discovery ?

Tensor (and chiral-odd) structures do not appear in the Standard Model Lagrangian at tree level.

Is it a possible low-energy footprint of BSM physics at higher scale ?



<u>neutron β-decay</u>

n → p e⁻ $\bar{\nu}_e$

$$\mathcal{L}_{\mathrm{SM}} \sim G_F\, V_{ud}\, \bar{e}\gamma^{\mu}(1-\gamma_5)\nu_e\, \bar{p}\gamma_{\mu}(1-\gamma_5)n$$

$$+\mathcal{L}_{\mathrm{eff}} \sim G_F\, V_{ud}\, g_T\, \varepsilon_T\, \bar{e}\sigma^{\mu\nu}\nu_e\, \bar{p}\sigma_{\mu\nu}n$$

precision => BSM scale $\frac{M_W^2}{M_{\mathrm{BSM}}^2} \approx g_T\, \varepsilon_T$ ← BSM coupling ?

$g_T = \delta u - \delta d$ isovector tensor charge

<u>SMEFT with strong CP violation</u>

permanent Electric Dipole Mom.

$$\mathcal{L}_{\mathrm{SMEFT}} \to \sum_{f=u,d,s,c} d_f\, \bar{\psi}_f\, \sigma_{\mu\nu}\gamma_5\, \psi_f\, F^{\mu\nu} \quad ?$$

↖ quark EDM

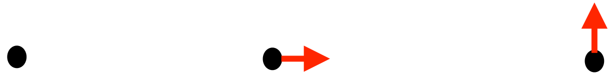
neutron EDM $d_n = \delta u\, d_u + \delta d\, d_d + \delta s\, d_s + \ldots$

exp. data + tensor charge => constrain amount of CP violation

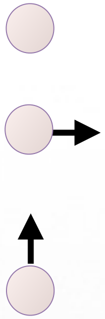
The Sivers TMD PDF

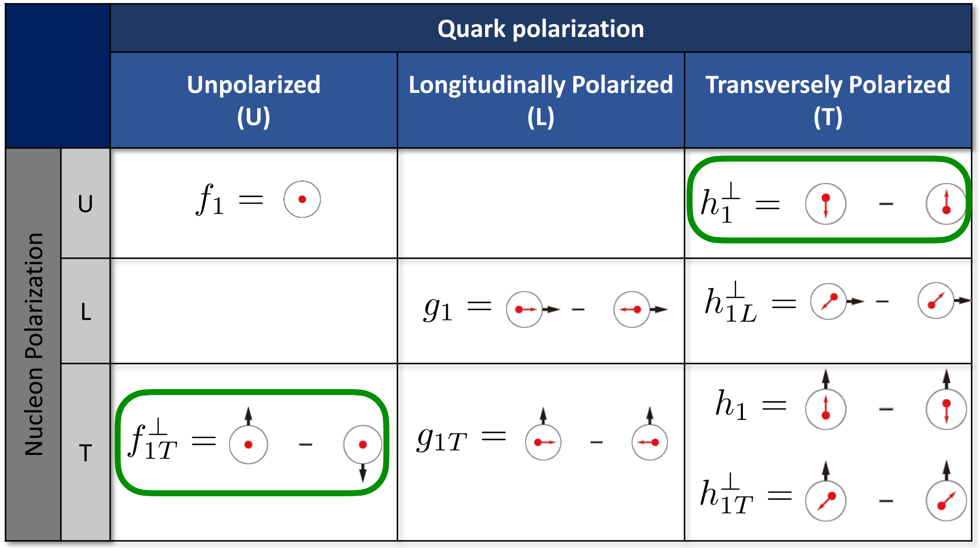
polarizations

quark



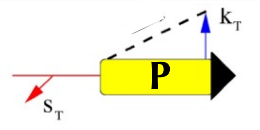
nucleon



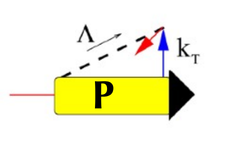


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^\perp =$ ⊙ - ⊙
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \to f_1 - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot\hat{\mathbf{P}}}{M}$$



$\mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$



$\mathbf{s}_T \cdot \mathbf{k}_\perp \times \mathbf{P}$

$$\frac{1}{2}\mathrm{Tr}[\Phi\, i\sigma_{+i}\gamma_5] \to \ldots + h_1^\perp \frac{(\mathbf{k}_\perp \times \mathbf{s}_T)\cdot\hat{\mathbf{P}}}{M}$$

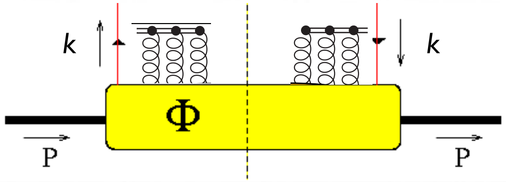
T-odd TMD PDFs

Sivers and Boer-Mulders TMD PDFs vanish without gauge link *U*

$$\Phi(x, \mathbf{k}_\perp, S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3} \, e^{-ik\cdot\xi} \, \langle P, S \,|\, \bar{\psi}(\xi) \, U_{[\xi,0]} \, \psi(0) \,|\, P, S \rangle_{\xi_+=0}$$

$$U_{[a,b]} = \mathscr{P} \exp\Big[-ig \int_a^b d\eta_\mu \, A^\mu(\eta) \Big]$$

They are generated by interference of different channels.
(for example, $f_{1T}^{\perp}$ can be reproduced by interference of model LC wave functions with different orbital angular momentum)



Gauge link *U* represents the residual color interactions that generate the necessary phase difference for the interference. As such, time reversal puts no constraints on these structures.

Sivers and Boer-Mulders TMD PDFs are conventionally named "T-odd" TMD PDFs

Boer & Mulders, P.R. D**57** (98)

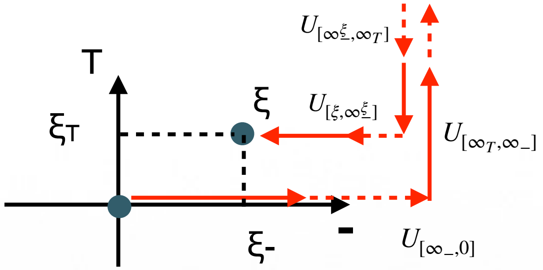
The gauge link

$$\Phi(x,\mathbf{k}_\perp,S) = \int \frac{d\xi_- d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}(\xi)\, U_{[\xi,0]}\, \psi(0)\,|\,P,S\rangle_{\xi_+=0}$$

TMD factorisation for SIDIS process suggests a trick similar to collinear framework case:

$$\langle P,S\,|\,\bar{\psi}(\xi)\, U_{[\xi,0]}\, \psi(0)\,|\,P,S\rangle = \langle P,S\,|\,\bar{\psi}(\xi)\, U_{[\xi,\infty^{\xi}_{-}]}\, U_{[\infty^{\xi}_{-},\infty_T]}\, U_{[\infty_T,\infty_-]}\, U_{[\infty_-,0]}\, \psi(0)\,|\,P,S\rangle$$

$$= \langle P,S\,|\,\{\bar{\psi}(\xi)\}\,\{\psi(0)\}\,|\,P,S\rangle$$



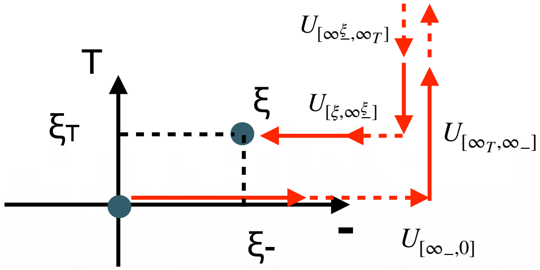
The gauge link

$$\Phi(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar{\psi}(\xi)\, U_{[\xi,0]}\, \psi(0)\,|\,P,S\rangle_{\xi_+=0}$$

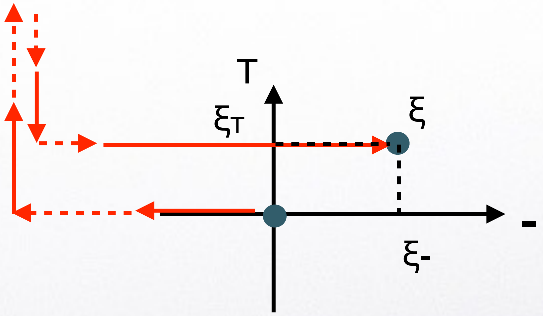
TMD factorisation for SIDIS process suggests a trick similar to collinear framework case:

$$\langle P,S\,|\,\bar{\psi}(\xi)\,U_{[\xi,0]}\,\psi(0)\,|\,P,S\rangle = \langle P,S\,|\,\bar{\psi}(\xi)\,U_{[\xi,\infty^{\xi}_{-}]}\,U_{[\infty^{\xi}_{-},\infty_T]}\,U_{[\infty_T,\infty_-]}\,U_{[\infty_-,0]}\,\psi(0)\,|\,P,S\rangle$$

$$= \langle P,S\,|\,\{\bar{\psi}(\xi)\}\,\{\psi(0)\}\,|\,P,S\rangle$$



In Drell-Yan process, TMD factorisation gives the following path for gauge link:



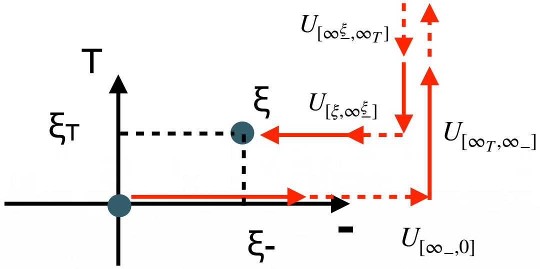
The gauge link

$$\Phi(x,\mathbf{k}_\perp,S) = \int \frac{d\xi\_d^2\xi_T}{(2\pi)^3}\, e^{-ik\cdot\xi}\, \langle P,S\,|\,\bar\psi(\xi)\, U_{[\xi,0]}\, \psi(0)\,|\,P,S\rangle_{\xi_+=0}$$

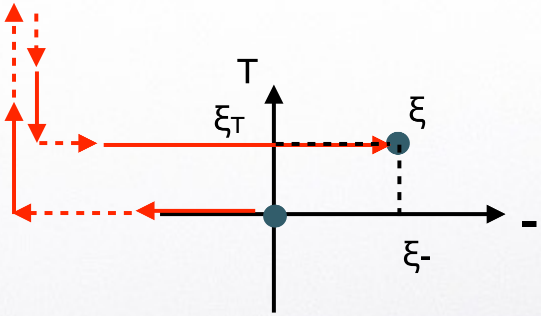
TMD factorisation for SIDIS process suggests a trick similar to collinear framework case:

$$\langle P,S\,|\,\bar\psi(\xi)\,U_{[\xi,0]}\,\psi(0)\,|\,P,S\rangle = \langle P,S\,|\,\bar\psi(\xi)\,U_{[\xi,\infty^{\xi}_{-}]}\,U_{[\infty^{\xi}_{-},\infty_T]}\,U_{[\infty_T,\infty_-]}\,U_{[\infty_-,0]}\,\psi(0)\,|\,P,S\rangle$$

$$= \langle P,S\,|\,\{\bar\psi(\xi)\}\,\{\psi(0)\}\,|\,P,S\rangle$$



In Drell-Yan process, TMD factorisation gives the following path for gauge link:



Notations: gauge link $U_{[+]}$ for SIDIS; $U_{[-]}$ for Drell-Yan

Important result: T-even TMD $\text{PDF}_{[+]}$ = TMD $\text{PDF}_{[-]}$
T-odd TMD $\text{PDF}_{[+]}$ = - TMD $\text{PDF}_{[-]}$

← breaking universality!
(but in a calculable way)

Process dependence

Sivers $f_{1T}^{\perp\,[+]} = -\, f_{1T}^{\perp\,[-]}$

Boer-Mulders $h_1^{\perp\,[+]} = -\, h_1^{\perp\,[-]}$

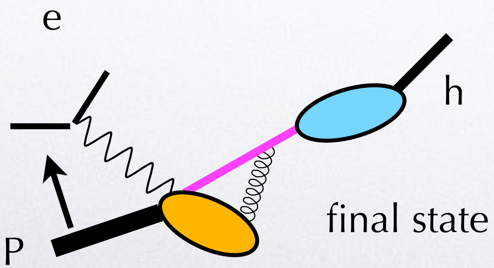
SIDIS Drell-Yan

Prediction of QCD based on interplay between time-reversal and (color) gauge symmetry

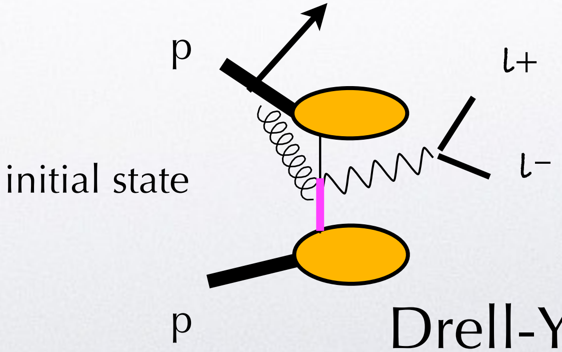
Intense experimental work to test this prediction

Intuition: in SIDIS, gauge link $U_{[+]}$ describes color final-state interactions between struck parton and spectators

in Drell-Yan, gauge link $U_{[-]}$ describes color initial-state interactions between struck parton and spectators



SIDIS



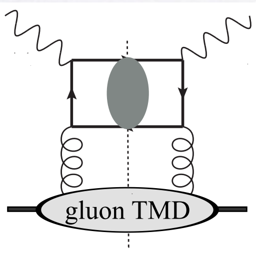
Drell-Yan

gluon TMDs

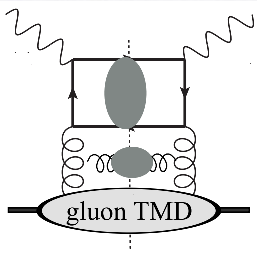
Gluon TMDs are phenomenologically unknown.
Why ?

- gluons carry no electric charge → in SIDIS they appear only at higher orders
- gluons carries "two color charges" → in general, difficult to neutralise them all
- in hadronic collisions, gluons appear at tree level, but :
  - factorisation theorem available only for Drell-Yan processes
  - for H_1+H_2 → h+X no factor. th. but also no counterexample disproving it
- useful processes under study:

$e\,p^{\uparrow} \to e + J/\psi + X$

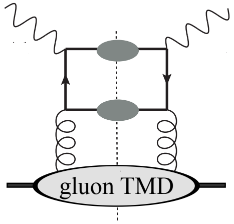


$e\,p^{\uparrow} \to e + J/\psi + \text{jet} + X$

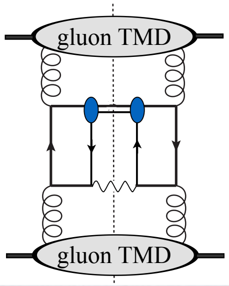


$e\,p \to e\ \text{jet jet}\ X$

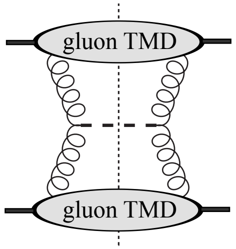
$e\,p^{\uparrow} \to e + h_1 + h_2 + X$



$p\,p \to J/\psi\,\gamma\,X$



$p\,p \to \eta_c\,X$



Boer et al., P.R.L. ***108*** *(12) 032002*
den Dunnen et al., P.R.L. ***112*** *(14) 212001*
Mukherjee & Rajesh, arXiv:1609.05596
Boer et al., arXiv:1605.07934
Godbole et al., arXiv:1703.01991
D'Alesio et al., arXiv:1705.04169
Rajesh et al., arXiv:1802.10359
Zheng et al., arXiv:1805.05290
Bacchetta et al., arXiv:1809.02056
D'Alesio et al., arXiv:1908.00446
D'Alesio et al., arXiv:1910.09640
….

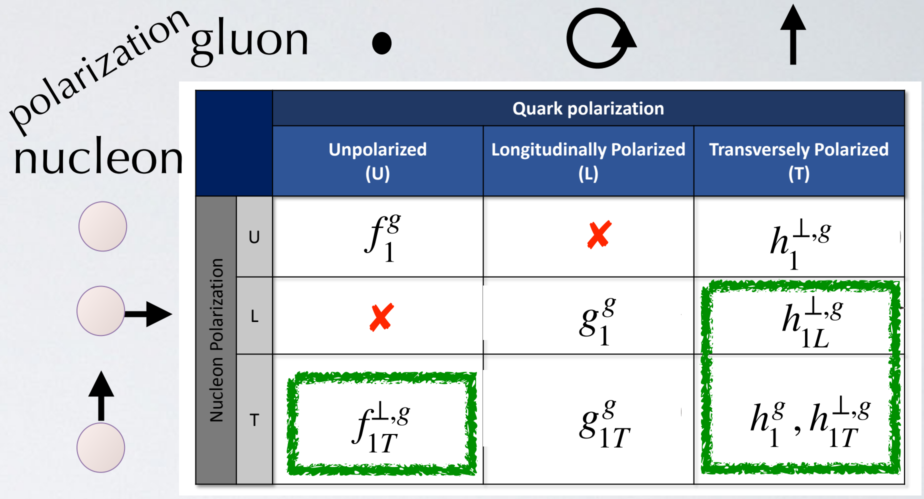
gluon TMDs

- First classification given in

  Mulders & Rodrigues,
  *P.R. D**63** (01) 094021, arXiv:hep-ph/0009343*

- Factorization, evolution & universality studied in

  *Ji et al., JHEP **07** (05) 020, arXiv:hep-ph/0503015*
  *Buffing et al., P.R. D**88** (13) 054027, arXiv:1306.5897*
  *Boer & Van Dunnen, N.P. **B886** (14) 421, arXiv:1404.6753*
  *Echevarria et al., JHEP **07** (15) 158 [E: **05** (17) 073], arXiv:1502.05354*



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1^g	✗	$h_1^{\perp,g}$
	L	✗	g_1^g	$h_{1L}^{\perp,g}$
	T	$f_{1T}^{\perp,g}$	g_{1T}^g	$h_1^g\,, h_{1T}^{\perp,g}$

T-odd TMDs

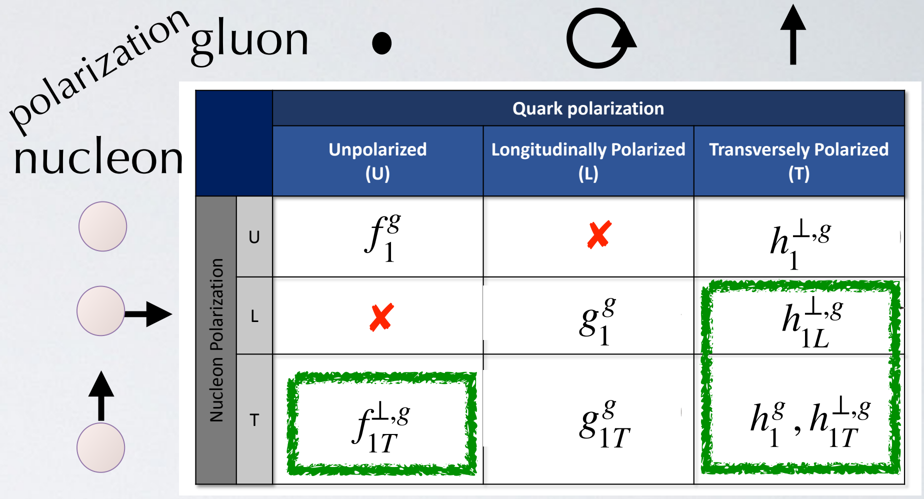
gluon TMDs

- First classification given in

  Mulders & Rodrigues,
  *P.R. D**63** (01) 094021, arXiv:hep-ph/0009343*

- Factorization, evolution & universality studied in

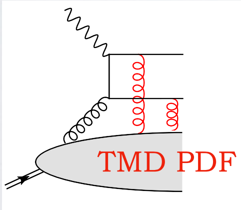
  *Ji et al., JHEP **07** (05) 020, arXiv:hep-ph/0503015*
  *Buffing et al., P.R. D**88** (13) 054027, arXiv:1306.5897*
  *Boer & Van Dunnen, N.P. **B886** (14) 421, arXiv:1404.6753*
  *Echevarria et al., JHEP **07** (15) 158 [E: **05** (17) 073], arXiv:1502.05354*



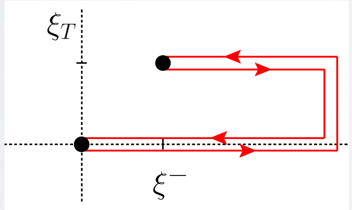
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1^g	✗	$h_1^{\perp,g}$
	L	✗	g_1^g	$h_{1L}^{\perp,g}$
	T	$f_{1T}^{\perp,g}$	g_{1T}^g	$h_1^g\,,h_{1T}^{\perp,g}$

T-odd TMDs

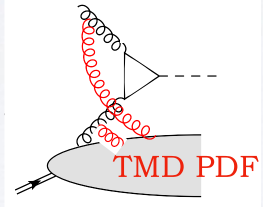
gluons carry "two color charges" → intricate non-universality



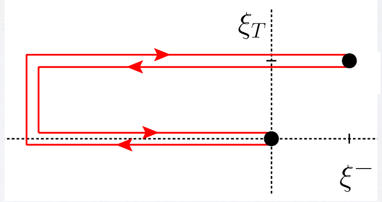
two-jets SIDIS



color structure [+,+]



gluon fusion to Higgs



[-,-]

T-even: $f_1^{[+,+]} = f_1^{[-,-]}$

T-odd: $f_{1T}^{\perp\,[+,+]} = -f_{1T}^{\perp\,[-,-]}$

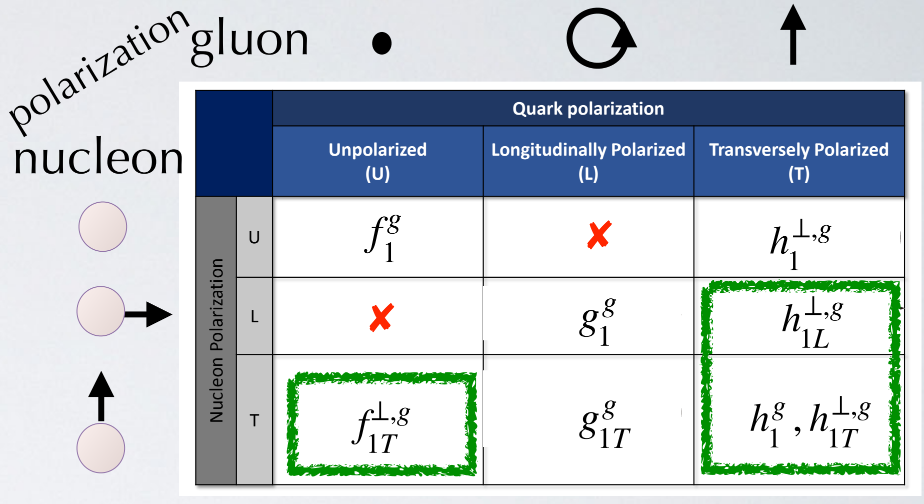
gluon TMDs

- First classification given in

  Mulders & Rodrigues,
  *P.R. D**63** (01) 094021, arXiv:hep-ph/0009343*

- Factorization, evolution & universality studied in

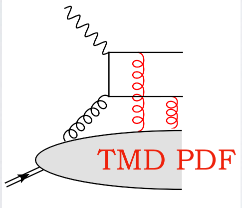
  *Ji et al., JHEP **07** (05) 020, arXiv:hep-ph/0503015*
  *Buffing et al., P.R. D**88** (13) 054027, arXiv:1306.5897*
  *Boer & Van Dunnen, N.P. **B886** (14) 421, arXiv:1404.6753*
  *Echevarria et al., JHEP **07** (15) 158 [E: **05** (17) 073], arXiv:1502.05354*



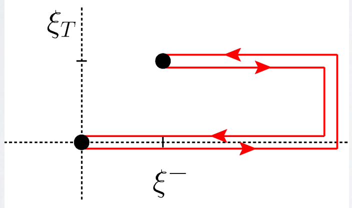
Nucleon Polarization	Quark polarization Unpolarized (U)	Quark polarization Longitudinally Polarized (L)	Quark polarization Transversely Polarized (T)
U	f_1^g	✘	$h_1^{\perp,g}$
L	✘	g_1^g	$h_{1L}^{\perp,g}$
T	$f_{1T}^{\perp,g}$	g_{1T}^g	$h_1^g\, ,\, h_{1T}^{\perp,g}$

T-odd TMDs

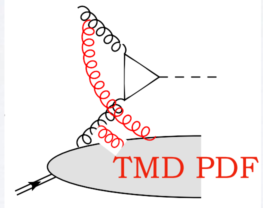
gluons carry "two color charges" → intricate non-universality



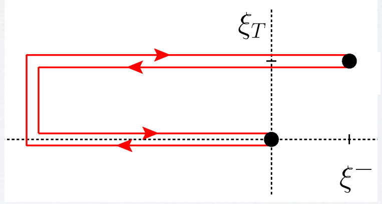
two-jets SIDIS



color structure [+,+]



gluon fusion to Higgs



[-,-]

T-even: $f_1^{[+,+]} = f_1^{[-,-]}$

T-odd: $f_{1T}^{\perp\,[+,+]} = -\, f_{1T}^{\perp\,[-,-]}$

WW-type TMDs

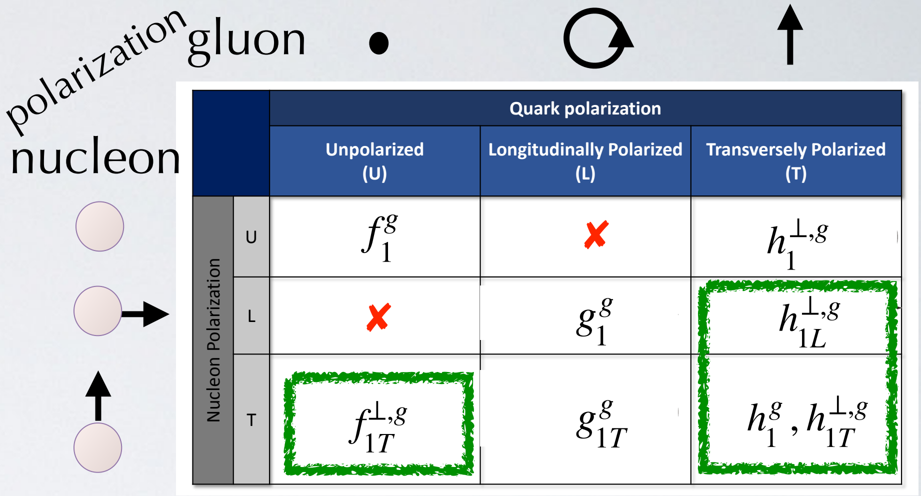
gluon TMDs

- First classification given in

  Mulders & Rodrigues,
  *P.R. D**63** (01) 094021, arXiv:hep-ph/0009343*

- Factorization, evolution & universality studied in

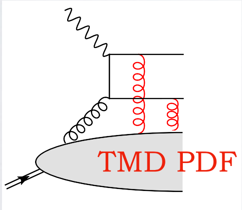
  *Ji et al., JHEP **07** (05) 020, arXiv:hep-ph/0503015*
  *Buffing et al., P.R. D**88** (13) 054027, arXiv:1306.5897*
  *Boer & Van Dunnen, N.P. **B886** (14) 421, arXiv:1404.6753*
  *Echevarria et al., JHEP **07** (15) 158 [E: **05** (17) 073], arXiv:1502.05354*



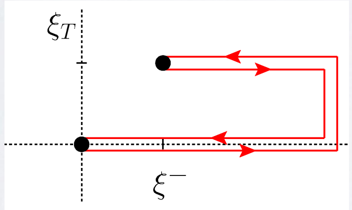
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
gluon		●	↻	↑
Nucleon Polarization	U	f_1^g	✘	$h_1^{\perp,g}$
	L	✘	g_1^g	$h_{1L}^{\perp,g}$
	T	$f_{1T}^{\perp,g}$	g_{1T}^g	$h_1^g\,,\,h_{1T}^{\perp,g}$

T-odd TMDs

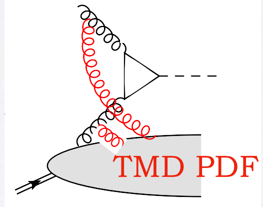
gluons carry “two color charges” → intricate non-universality



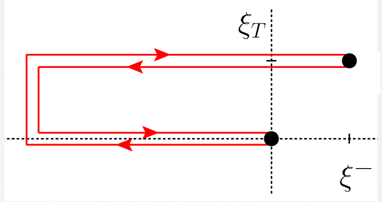
two-jets SIDIS



color structure [+,+]



gluon fusion to Higgs

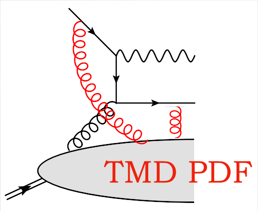


[-,-]

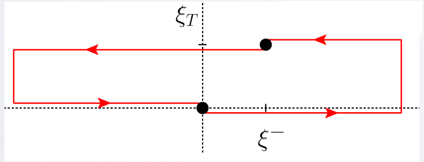
T-even: $f_1^{[+,+]} = f_1^{[-,-]}$

T-odd: $f_{1T}^{\perp\,[+,+]} = -f_{1T}^{\perp\,[-,-]}$

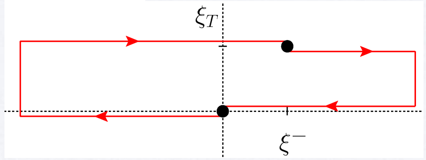
WW-type TMDs



p p → γ^*+jet



[+,-]



[-,+]

T-even: $f_1^{[+,-]} = f_1^{[-,+]}$

T-odd: $f_{1T}^{\perp\,[+,-]} = -f_{1T}^{\perp\,[-,+]}$

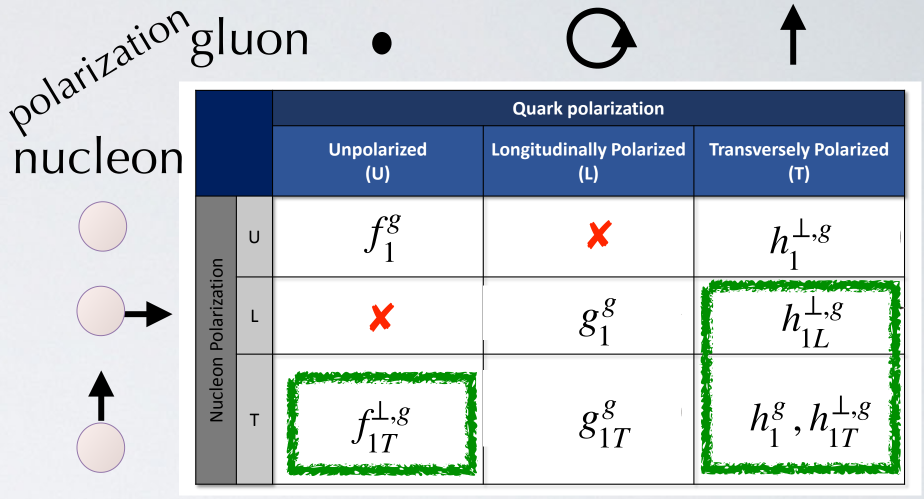
gluon TMDs

- First classification given in

  Mulders & Rodrigues,
  *P.R. D**63** (01) 094021, arXiv:hep-ph/0009343*

- Factorization, evolution & universality studied in

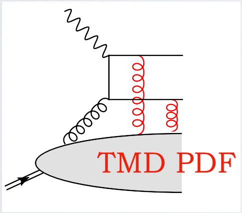
  *Ji et al., JHEP **07** (05) 020, arXiv:hep-ph/0503015*
  *Buffing et al., P.R. D**88** (13) 054027, arXiv:1306.5897*
  *Boer & Van Dunnen, N.P. **B886** (14) 421, arXiv:1404.6753*
  *Echevarria et al., JHEP **07** (15) 158 [E: **05** (17) 073], arXiv:1502.05354*



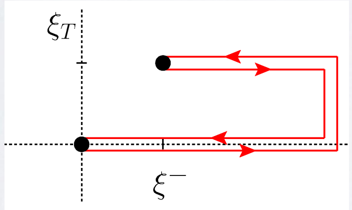
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1^g	✗	$h_1^{\perp,g}$
	L	✗	g_1^g	$h_{1L}^{\perp,g}$
	T	$f_{1T}^{\perp,g}$	g_{1T}^g	$h_1^g\,,h_{1T}^{\perp,g}$

T-odd TMDs

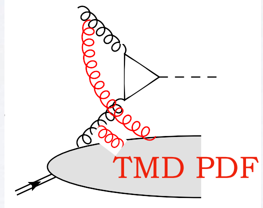
gluons carry “two color charges” → intricate non-universality



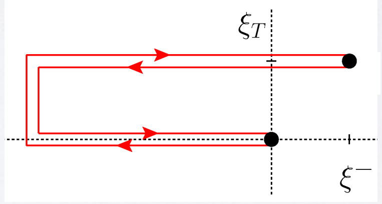
two-jets SIDIS



color structure [+,+]



gluon fusion to Higgs

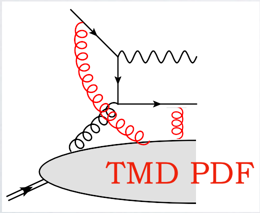


[-,-]

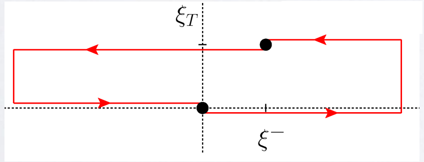
T-even: $f_1^{[+,+]} = f_1^{[-,-]}$

T-odd: $f_{1T}^{\perp\,[+,+]} = -f_{1T}^{\perp\,[-,-]}$

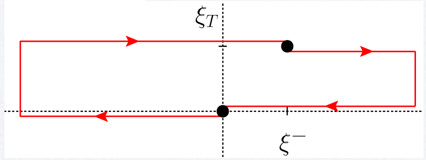
WW-type TMDs



p p → γ^*+jet



[+,-]



[-,+]

T-even: $f_1^{[+,-]} = f_1^{[-,+]}$

T-odd: $f_{1T}^{\perp\,[+,-]} = -f_{1T}^{\perp\,[-,+]}$

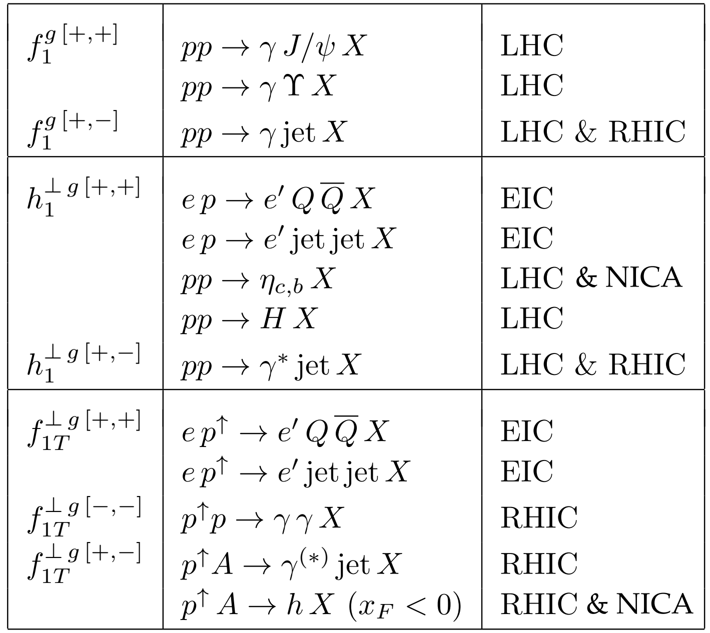
≠

dipole-type TMDs

gluon TMDs

many papers exploring useful channels at colliders to extract WW and dipole gluon TMDs. Handy pocket list:

Boer, talk at IWHSS 2020



$f_1^{g\,[+,+]}$	$pp \to \gamma\, J/\psi\, X$	LHC
	$pp \to \gamma\, \Upsilon\, X$	LHC
$f_1^{g\,[+,-]}$	$pp \to \gamma\, \text{jet}\, X$	LHC & RHIC
$h_1^{\perp\, g\,[+,+]}$	$e\, p \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p \to e'\, \text{jet}\, \text{jet}\, X$	EIC
	$pp \to \eta_{c,b}\, X$	LHC & NICA
	$pp \to H\, X$	LHC
$h_1^{\perp\, g\,[+,-]}$	$pp \to \gamma^*\, \text{jet}\, X$	LHC & RHIC
$f_{1T}^{\perp\, g\,[+,+]}$	$e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p^\uparrow \to e'\, \text{jet}\, \text{jet}\, X$	EIC
$f_{1T}^{\perp\, g\,[-,-]}$	$p^\uparrow p \to \gamma\, \gamma\, X$	RHIC
$f_{1T}^{\perp\, g\,[+,-]}$	$p^\uparrow A \to \gamma^{(*)}\, \text{jet}\, X$	RHIC
	$p^\uparrow\, A \to h\, X\ (x_F < 0)$	RHIC & NICA

(see also recent review on quarkonium physics)

Boer et al., arXiv:2409.03691

gluon TMDs

many papers exploring useful channels at colliders to extract WW and dipole gluon TMDs. Handy pocket list:

Boer, talk at IWHSS 2020

$f_1^{g\,[+,+]}$	$pp \to \gamma\, J/\psi\, X$	LHC
	$pp \to \gamma\, \Upsilon\, X$	LHC
$f_1^{g\,[+,-]}$	$pp \to \gamma\, \text{jet}\, X$	LHC & RHIC
$h_1^{\perp\, g\,[+,+]}$	$e\,p \to e'\, Q\,\overline{Q}\, X$	EIC
	$e\,p \to e'\, \text{jet jet}\, X$	EIC
	$pp \to \eta_{c,b}\, X$	LHC & NICA
	$pp \to H\, X$	LHC
$h_1^{\perp\, g\,[+,-]}$	$pp \to \gamma^*\, \text{jet}\, X$	LHC & RHIC
$f_{1T}^{\perp\, g\,[+,+]}$	$e\,p^\uparrow \to e'\, Q\,\overline{Q}\, X$	EIC
	$e\,p^\uparrow \to e'\, \text{jet jet}\, X$	EIC
$f_{1T}^{\perp\, g\,[-,-]}$	$p^\uparrow p \to \gamma\,\gamma\, X$	RHIC
$f_{1T}^{\perp\, g\,[+,-]}$	$p^\uparrow A \to \gamma^{(*)}\, \text{jet}\, X$	RHIC
	$p^\uparrow\, A \to h\, X \;(x_F < 0)$	RHIC & NICA

(see also recent review on quarkonium physics)

Boer et al., arXiv:2409.03691

- TMD factorization $\xrightarrow{\text{small x}}$ UGD k_t factorization

WW $f_1^{g\,[+,+]}$ $\longrightarrow$ # density of gluons in CGC

dipole $f_1^{g\,[+,-]}$ $\longrightarrow$ Fourier Transform of color-dipole cross section in CGC

*Dominguez et al., P.R.L. **106** (11) 022301, arXiv:1009.2141*
*Dominguez et al., P.R. D**83** (11) 105005, arXiv:1101.0715*

gluon TMDs

many papers exploring useful channels at colliders to extract WW and dipole gluon TMDs. Handy pocket list:

Boer, talk at IWHSS 2020

$f_1^{g\,[+,+]}$	$pp \to \gamma\, J/\psi\, X$	LHC
	$pp \to \gamma\, \Upsilon\, X$	LHC
$f_1^{g\,[+,-]}$	$pp \to \gamma\, \text{jet}\, X$	LHC & RHIC
$h_1^{\perp\, g\,[+,+]}$	$e\, p \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p \to e'\, \text{jet jet}\, X$	EIC
	$pp \to \eta_{c,b}\, X$	LHC & NICA
	$pp \to H\, X$	LHC
$h_1^{\perp\, g\,[+,-]}$	$pp \to \gamma^*\, \text{jet}\, X$	LHC & RHIC
$f_{1T}^{\perp\, g\,[+,+]}$	$e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X$	EIC
	$e\, p^\uparrow \to e'\, \text{jet jet}\, X$	EIC
$f_{1T}^{\perp\, g\,[-,-]}$	$p^\uparrow p \to \gamma\, \gamma\, X$	RHIC
$f_{1T}^{\perp\, g\,[+,-]}$	$p^\uparrow A \to \gamma^{(*)}\, \text{jet}\, X$	RHIC
	$p^\uparrow\, A \to h\, X\ (x_F < 0)$	RHIC & NICA

(see also recent review on quarkonium physics)

Boer et al., arXiv:2409.03691

- TMD factorization $\xrightarrow{\text{small x}}$ UGD k_t factorization

WW $f_1^{g\,[+,+]}$ $\longrightarrow$ # density of gluons in CGC

dipole $f_1^{g\,[+,-]}$ $\longrightarrow$ Fourier Transform of color-dipole cross section in CGC

*Dominguez et al., P.R.L. **106** (11) 022301, arXiv:1009.2141*
*Dominguez et al., P.R. D**83** (11) 105005, arXiv:1101.0715*

- small-x limit of T-odd gluon TMDs:

spin-dependent T-odd part of dipole amplitude describes the colorless C-odd t-channel 3-gluon exchange

WW $\quad f_{1T}^{\perp}, h_1, h_{1T}^{\perp} \to 0$

dipole $\quad xf_{1T}^{\perp} = xh_1 = xh_{1T}^{\perp} \to -\dfrac{k_T^2 N_c}{4\pi\alpha_s}\, O_{1T}^{\perp}(x, k_T^2)$ spin Odderon

*Boer et al., P.R.L. **116** (16) 122001, arXiv:1511.03485*

gluon TMDs : only models

- Available experimental information on gluon TMDs is scarce.
- Very few attempts of phenomenological studies:

*Lansberg et al., P.L. **B784** (18) 217 [E: P.L. **B791** (19) 420], arXiv:1710.01684*
*D'Alesio et al., P.R. D**96** (17) 036011, arXiv:1705.04169*
*D'Alesio et al., P.R. D**99** (19) 036013, arXiv:1811.02970*
*D'Alesio et al., P.R. D**102** (20) 094011, arXiv:2007.03353*

- Many models on the market (list of references too long).

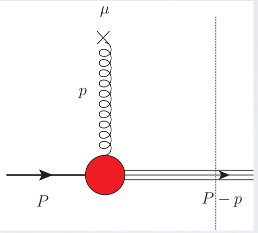
Let me advertise our one, for first time providing systematically
all T-even and T-odd gluon TMDs at leading twist:

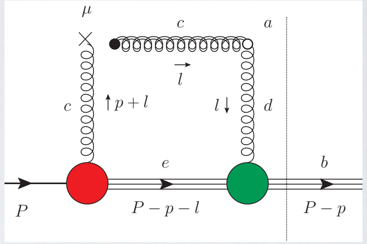
*Bacchetta et al., E.P.J.C **80** (20) 733, arXiv:2005.02288* T-even

*Bacchetta et al., E.P.J.C **84** (24) 576, arXiv:2402.17556* T-odd

spectator model of gluon TMDs

- Nucleon = gluon + spectator on-shell
  spin-1/2 particle





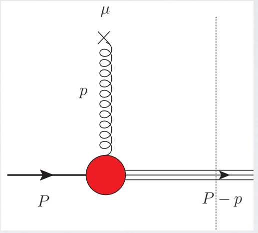
- T-odd generated by gluon-spectator FSI via 1 gluon-exchange
- Spectator mass takes continuous range of values through a parametric spectral function
- Parameters fixed by reproducing collinear gluon PDFs f_1 and g_1 from NNPDF3.0

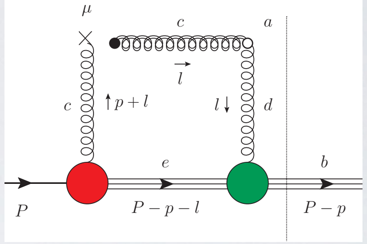
Bacchetta et al., E.P.J.C ***80*** *(20) 733, arXiv:2005.02288*
Bacchetta et al., E.P.J.C ***84*** *(24) 576, arXiv:2402.17556*

S_L $-S_L$

spectator model of gluon TMDs

- Nucleon = gluon + spectator on-shell spin-1/2 particle

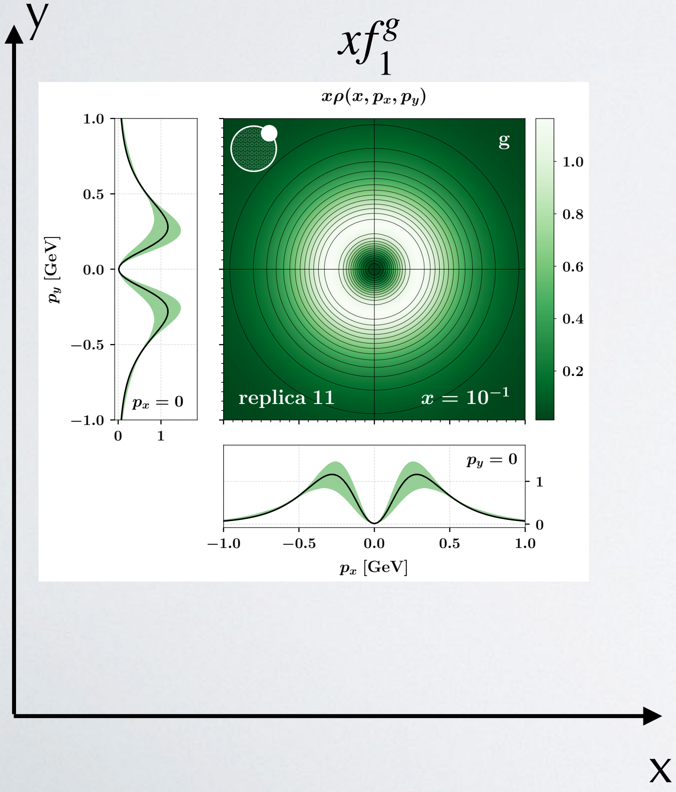




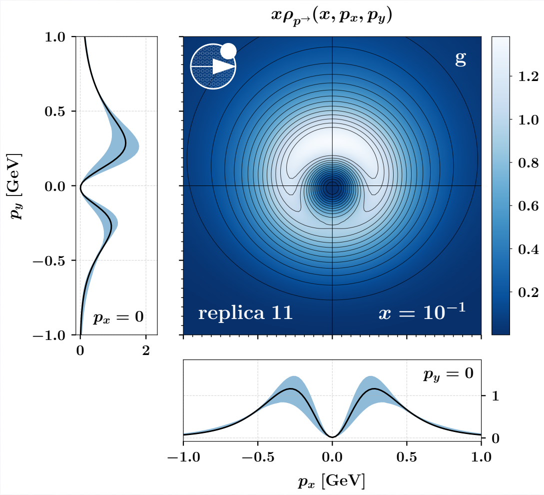
- T-odd generated by gluon-spectator FSI via 1 gluon-exchange
- Spectator mass takes continuous range of values through a parametric spectral function
- Parameters fixed by reproducing collinear gluon PDFs f₁ and g₁ from NNPDF3.0

Bacchetta et al., E.P.J.C ***80*** *(20) 733, arXiv:2005.02288*
Bacchetta et al., E.P.J.C ***84*** *(24) 576, arXiv:2402.17556*

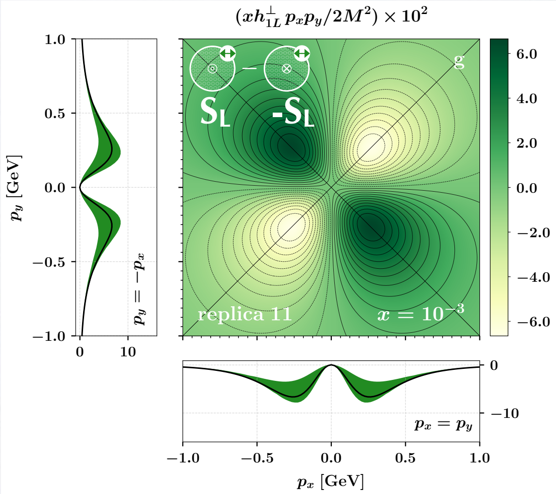
xf_1^g



$xf_1^g + (p_y/M)\, xf_{1T}^{\perp g}$



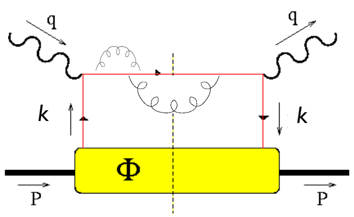
"propeller" $h_{1L}^{\perp g}$

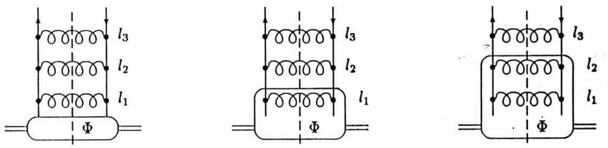


Outline

- Pause

More on factorisation → evolution





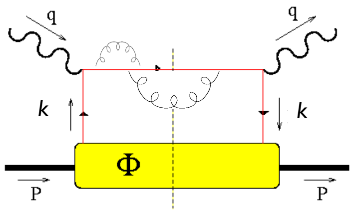
inclusive DIS: QCD corrections generate soft and collinear divergences

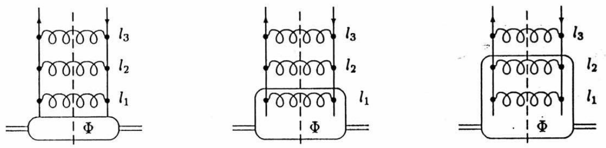
sum of real and virtual diagrams cancel soft divergences

collinear divergences reabsorbed in collinear PDFs

factorisation scale μ determines what is perturbative (calculable) from what is non perturbative (inside PDFs)
→ scale dependence given by DGLAP evolution eq.'s

More on factorisation → evolution



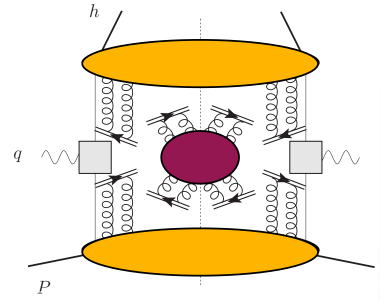


inclusive DIS: QCD corrections generate soft and collinear divergences

sum of real and virtual diagrams cancel soft divergences

collinear divergences reabsorbed in collinear PDFs

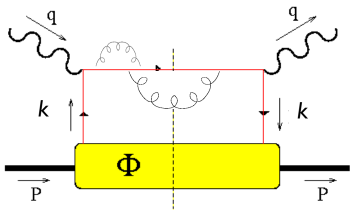
factorisation scale μ determines what is perturbative (calculable) from what is non perturbative (inside PDFs)
→ scale dependence given by DGLAP evolution eq.'s

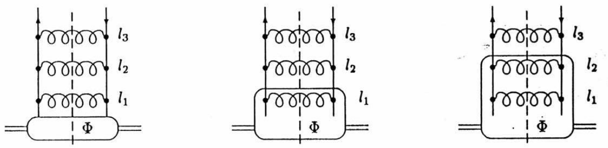


SIDIS: soft divergences do not cancel anymore
new class of light-cone (rapidity) divergences

need to introduce a **soft factor** convoluted with TMD PDFs and FFs

More on factorisation → evolution



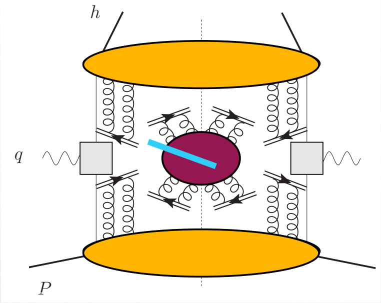


inclusive DIS: QCD corrections generate soft and collinear divergences

sum of real and virtual diagrams cancel soft divergences

collinear divergences reabsorbed in collinear PDFs

factorisation scale μ determines what is perturbative (calculable) from what is non perturbative (inside PDFs) → scale dependence given by DGLAP evolution eq.'s



SIDIS: soft divergences do not cancel anymore
new class of light-cone (rapidity) divergences

need to introduce a **soft factor** convoluted with TMD PDFs and FFs

need to introduce a new **“rapidity scale” ζ** that regulates the rapidity divergences and splits the soft factor content between TMD PDFs and FFs → new scale dependence

DGLAP eq.'s $\frac{d\log \text{TMD}}{d\log\mu} = \gamma_D(\mu,\zeta)$ CSS eq.'s $\frac{d\log \text{TMD}}{d\log\sqrt{\zeta}} = K(\mu)$

TMDs in position space

TMD evolution from initial (μ_0,ζ_0) scales is better studied in position space b_T (↔ P_{hT})

In fact, by Fourier transforming the complicate convolution between internal transverse momenta gets broken

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{C}\Big[\mathrm{Tr}\big[\Phi(x_B,\mathbf{k}_\perp,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{P}_\perp)\,\gamma^\nu\big]\Big]$$

$$\mathcal{C}\big[\ldots\big] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\big[\ldots\big]$$

$$\int d^2\mathbf{b}_T\,e^{-i\mathbf{b}_T\cdot\mathbf{P}_{hT}}\quad\ldots\quad\int d\mathbf{P}_\perp\,e^{i\mathbf{b}_T\cdot\mathbf{P}_\perp}\quad\ldots\quad\int d\mathbf{k}_\perp\,e^{i\mathbf{b}_T\cdot\mathbf{k}_\perp}\ldots$$

TMDs in position space

TMD evolution from initial (μ_0,ζ_0) scales is better studied in position space b_T ($\leftrightarrow$ P_{hT})

In fact, by Fourier transforming the complicate convolution between internal transverse momenta gets broken

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathscr{C}\Big[\mathrm{Tr}\big[\Phi(x_B,\mathbf{k}_\perp,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{P}_\perp)\,\gamma^\nu\big]\Big]$$

$$\mathscr{C}\big[\dots\big] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\big[\dots\big]$$

$$\int d^2\mathbf{b}_T\,e^{-i\mathbf{b}_T\cdot\mathbf{P}_{hT}}\ \dots\quad \int d\mathbf{P}_\perp\,e^{i\mathbf{b}_T\cdot\mathbf{P}_\perp}\ \dots\quad \int d\mathbf{k}_\perp\,e^{i\mathbf{b}_T\cdot\mathbf{k}_\perp}\dots$$

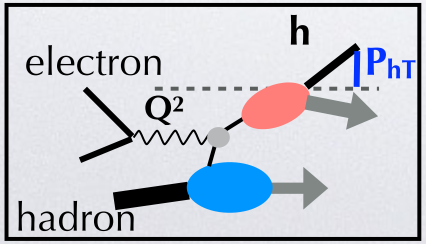
$$\frac{d\sigma}{dxdzdq_TdQ} \sim \mathscr{H}^{\mathrm{SIDIS}}(Q^2)\,\frac{1}{2\pi}\int_0^\infty db_T\,b_T\,J_0(b_T,q_T)\ \ \tilde{f}_1^q(x,b_T^2;Q^2)\ \ \tilde{D}_1^{q\to h}(z,b_T^2;Q^2)$$

hard part

TMDPDF

TMDFF

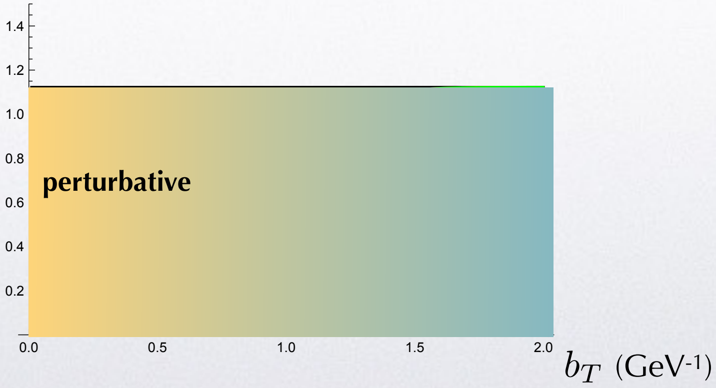
$$q_T^2 = \frac{P_{hT}^2}{z^2} \ll Q^2$$



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$



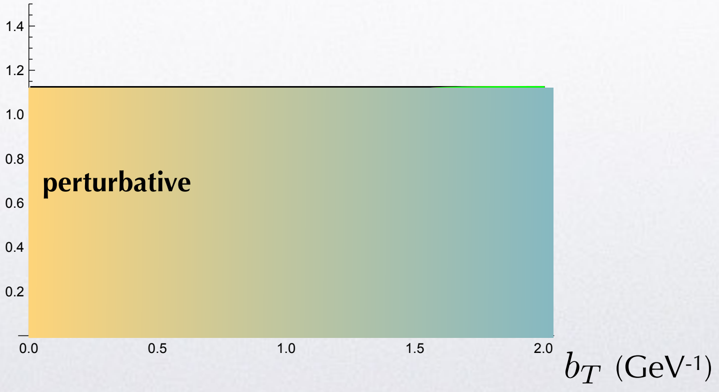
More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

DGLAP+CSS eqs. ↓

$$\exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu,\zeta) + K(\mu_0)\log\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

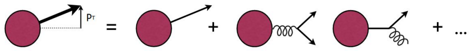
DGLAP+CSS eqs. ↓

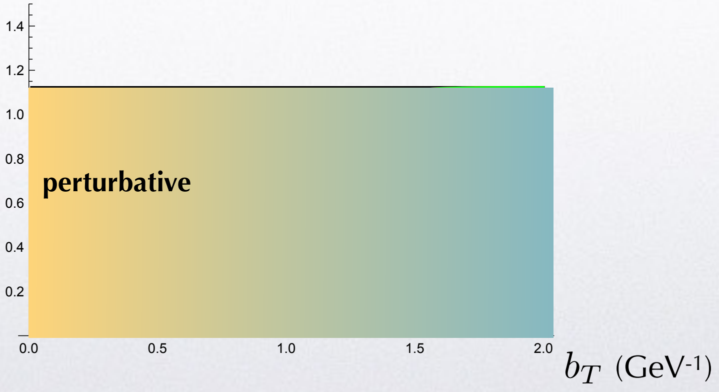
$$\exp\Big[\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$

OPE on PDFs

$$= \sum_i \big[C_{q \to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\big]$$

small b_T (large k_T) from perturbative splitting





More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

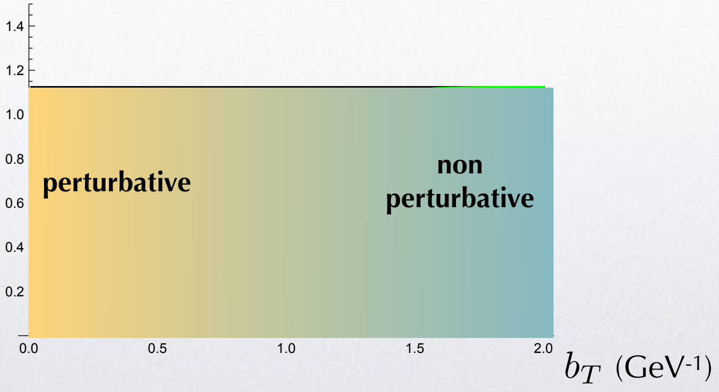
DGLAP+CSS eqs. ↓

$$\exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$

↘ OPE on PDFs

$$= \sum_i \left[C_{q\to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\right]$$

For large b_T perturbation theory breaks down; need to find a suitable function that smoothly connects the two regions



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

DGLAP+CSS eqs. ↓ ↘ OPE on PDFs

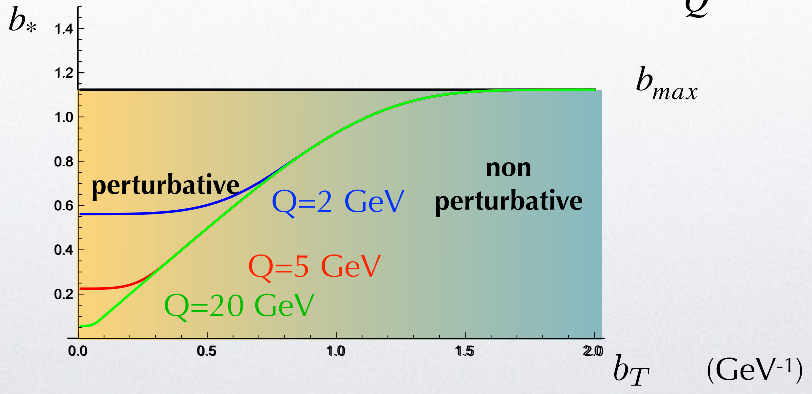
$$\exp\Big[\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big] \qquad = \sum_i \Big[C_{q\to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\Big]$$

For large b_T perturbation theory breaks down; need to find a suitable function that smoothly connects the two regions

$\overline{\text{MS}}$ factorization scheme suggests the following scale:

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

$$b_{min} = \frac{2e^{-\gamma_E}}{Q} \leq b^*(b_T) \leq \quad b_{max} = 2e^{-\gamma_E}$$



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b_T^2; \mu, \zeta) = \quad \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)\Big] \quad f_1^q(x, b_T^2; \mu_0, \zeta_0)$$

DGLAP+CSS eqs. ↓

$$\exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_0) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$

OPE on PDFs ↘

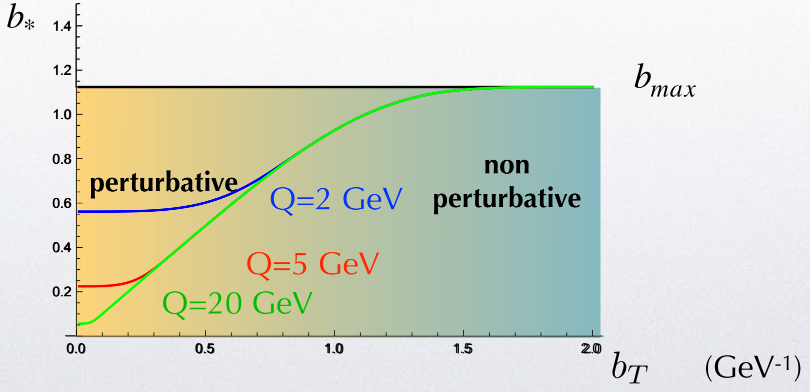
$$= \sum_i \left[C_{q\to i}(x, b_T^2; \mu_0, \zeta_0) \otimes f_1^i(x, \mu_0)\right]$$

➡

$$f_1^q(x, b_T^2; \mu, \zeta) = \boxed{\frac{f_1^q(x, b_T^2; \mu, \zeta)}{f_1^q(x, b^*(b_T); \mu, \zeta)}} \quad f_1^q(x, b^*(b_T); \mu, \zeta)$$

$\tilde{f}_{NP}(x, b_T^2; Q_0^2)$

Q_0 = scale at which the nonperturbative term is parametrised



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b^*(b_T); \mu, \zeta) = \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_b, \zeta_0)\Big] \qquad f_1^q(x, b^*(b_T); \mu_b, \zeta_0) \qquad \times \qquad F_{NP}(b_T; Q_0^2)$$

DGLAP+CSS eqs. ↓

$$\exp\Big[\Big[\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_b) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$

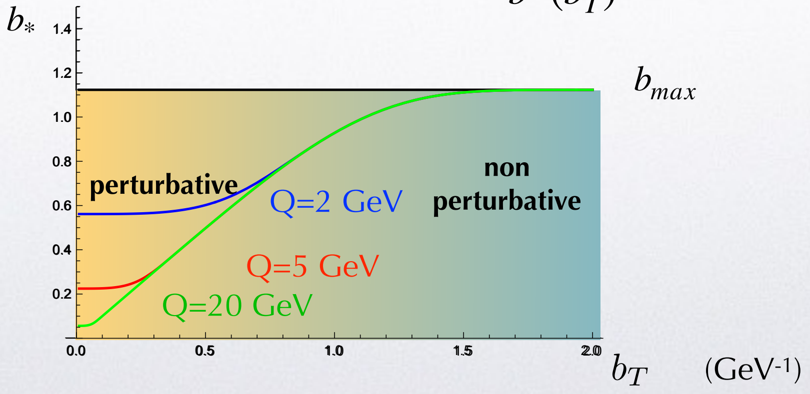
OPE on PDFs

$$= \sum_i \left[C_{q\to i}(x, b^*(b_T); \mu_b, \zeta_0) \otimes f_1^i(x, \mu_b)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$$

$$K \to K + g_{NP}(b_T)$$

conventional choice: $\mu = \sqrt{\zeta} = Q \qquad \mu_0 = \sqrt{\zeta_0} = \mu_b = \dfrac{2e^{-\gamma_E}}{b^*(b_T)}$



More on factorisation → evolution

For $b_T \ll 1/\Lambda_{QCD}$ perturbation theory is valid

$$f_1^q(x, b^*(b_T); \mu, \zeta) = \text{Evo}\Big[(\mu, \zeta) \leftarrow (\mu_b, \zeta_0)\Big] \quad f_1^q(x, b^*(b_T); \mu_b, \zeta_0) \quad \times \quad F_{NP}(b_T; Q_0^2)$$

DGLAP+CSS eqs. ↓

$$\exp\Big[\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_D(\mu, \zeta) + K(\mu_b) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\Big]$$

↓

$$K \to K + g_{NP}(b_T)$$

OPE on PDFs ↓

$$= \sum_i \left[C_{q\to i}(x, b^*(b_T); \mu_b, \zeta_0) \otimes f_1^i(x, \mu_b)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

Final formula

$$f_1^q(x, b^*; Q^2) = \exp\Big[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \gamma_D(Q) + K(\mu_b) \log\left(\frac{Q}{\mu_b}\right) + g_{NP}(b_T) \log\left(\frac{Q}{Q_0}\right)\Big] \sum_i \left[C_{q\to i} \otimes f_1^i\right](x, b^*, \mu_b) \, F_{NP}(b_T, Q_0^2)$$

Collins, Soper, Sterman, N.P. ***B250*** *(85)*
Collins, "Foundations of Perturbative QCD" (2011)
*Rogers and Aybat, P.R. D**83** (11)*

More on factorisation → evolution

others schemes possible:

Laenen, Sterman Vogelsang, P.R.L. ***84*** *(00)*
Bozzi et al., N.P. ***B737*** *(06)*
Echevarria et al., E.P.J. ***C73*** *(13) …*

CSS evolution formula for TMD

$$f_1^q(x, b^*; Q^2) = \exp\Big[\Big[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\, \gamma_D(Q) + K(\mu_b)\, \log\left(\frac{Q}{\mu_b}\right) + g_{NP}(b_T)\, \log\left(\frac{Q}{Q_0}\right)\Big]\ \sum_i\, [C_{q\to i} \otimes f_1^i](x, b^*, \mu_b)\ F_{NP}(b_T, Q_0^2)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

More on factorisation → evolution

others schemes possible:

Laenen, Sterman Vogelsang, P.R.L. ***84*** *(00)*
Bozzi et al., N.P. ***B737*** *(06)*
Echevarria et al., E.P.J. ***C73*** *(13) …*

CSS evolution formula for TMD

$$f_1^q(x, b^*; Q^2) = \exp\Big[\int_{\enclose{circle}{\mu_b}}^{Q} \frac{d\mu'}{\mu'}\, \gamma_D(Q) + K(\enclose{circle}{\mu_b}) \log\left(\frac{Q}{\enclose{circle}{\mu_b}}\right) + \enclose{circle}{g_{NP}}(b_T) \log\left(\frac{Q}{Q_0}\right)\Big] \sum_i \left[C_{q\to i} \otimes f_1^i\right](x, b^*, \enclose{circle}{\mu_b})\, \enclose{circle}{F_{NP}}(b_T, Q_0^2)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

arbitrariness of nonperturbative components

- choice of $b_*(b_T)$ functional form
- choice of $g_{NP}(b_T)$ functional form
- choice of $F_{NP}(b_T,Q_0)$ functional form

each one affects evolution: how k⊥-distribution changes with scale

→ source of theoretical bias/uncertainty

need to be constrained by experimental data with large lever arm in Q^2

EIC is the suitable machine for that

Quality parameters of TMD extraction

$$\frac{d\sigma}{dxdzdq_TdQ} \sim \mathcal{H}^{\mathrm{SIDIS}}(Q^2)\ \frac{1}{2\pi}\int_0^\infty db_T\, b_T\ J_0(b_T, q_T)\ \tilde{f}_1^q(x, b^*(b_T); Q^2)\ \ \tilde{D}_1^{q\to h}(z, b^*(b_T); Q^2)$$

$$f_1^q(x, b^*; Q^2) = \exp\left[\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\,\gamma_D(Q) + K(\mu_b)\ \log\left(\frac{Q}{\mu_b}\right) + g_{NP}(b_T)\ \log\left(\frac{Q}{Q_0}\right)\right]\ \sum_i\ [C_{q\to i}\otimes f_1^i](x, b^*, \mu_b)\ F_{NP}(b_T, Q_0^2)$$

$$\searrow\ \gamma_D = \gamma_F - \gamma_K\ \log(\sqrt{\zeta}/\mu) \qquad \frac{dK}{d\log\mu} = -\gamma_K$$

cusp anomalous dimension

Quality parameters of TMD extraction

$$\frac{d\sigma}{dx\,dz\,dq_T\,dQ} \sim \mathscr{H}^{\rm SIDIS}(Q^2)\,\frac{1}{2\pi}\int_0^\infty db_T\, b_T\, J_0(b_T,q_T)\;\tilde f_1^q(x,b^*(b_T);Q^2)\;\tilde D_1^{q\to h}(z,b^*(b_T);Q^2)$$

$$f_1^q(x,b^*;Q^2) = \exp\left[\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\,\gamma_D(Q) + K(\mu_b)\,\log\left(\frac{Q}{\mu_b}\right) + g_{NP}(b_T)\,\log\left(\frac{Q}{Q_0}\right)\right]\sum_i\,[C_{q\to i}\otimes f_1^i](x,b^*,\mu_b)\;F_{NP}(b_T,Q_0^2)$$

$$\gamma_D = \gamma_F - \gamma_K\,\log(\sqrt{\zeta}/\mu) \qquad \frac{dK}{d\log\mu} = -\,\gamma_K$$

cusp anomalous dimension

α_S^n

perturbative accuracy	$\mathscr{H}$ and C	K and γ_F	γ_K	PDF and α_S evol.	FF
LL	0	-	1	-	-
NLL	0	1	2	LO	LO
NLL'	1	1	2	NLO	NLO
NNLL	1	2	3	NLO	NLO
NNLL'	2	2	3	NNLO	NNLO
N^{3}LL(-)	2	3	4	NNLO	NLO
N^{3}LL	2	3	4	NNLO	NNLO

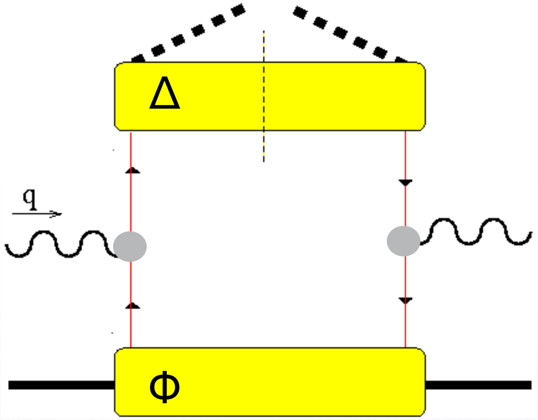
nonperturbative accuracy: quality of the fit from χ^2 value e number of data points

Outline

- Where to find TMDs

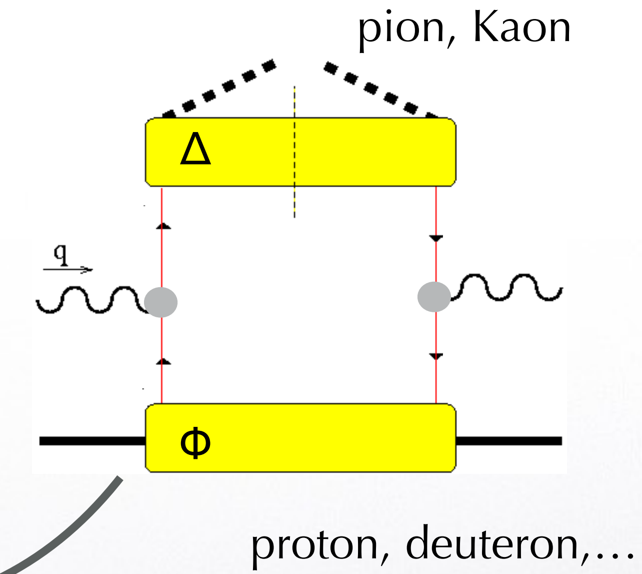
link TMD ↔ structure functions

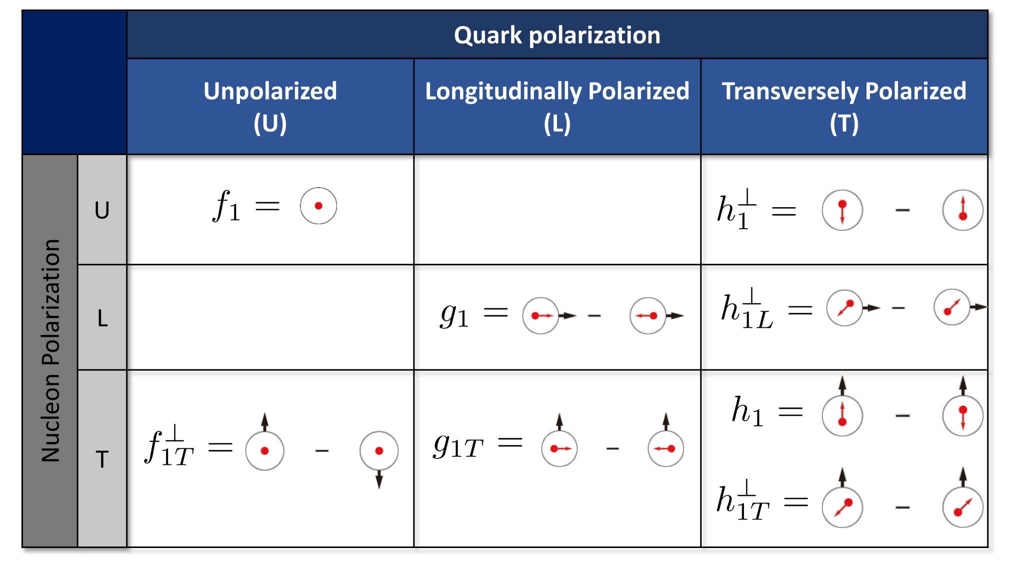
pion, Kaon



proton, deuteron,…

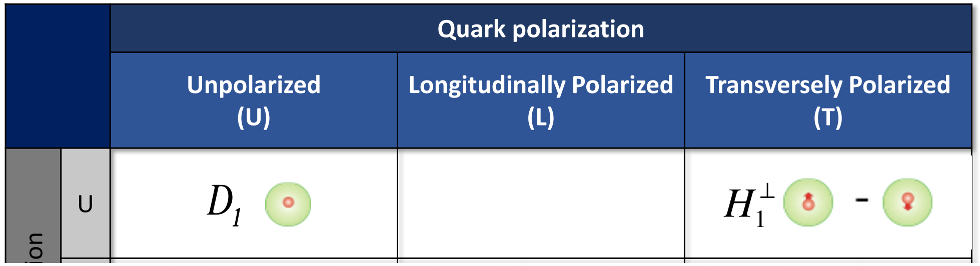
link TMD ↔ structure functions



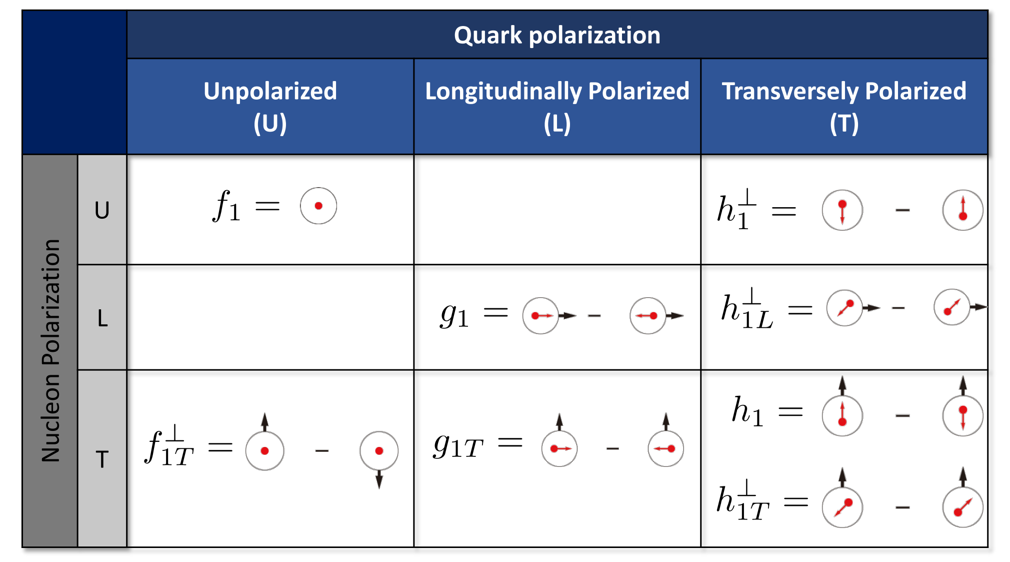


		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

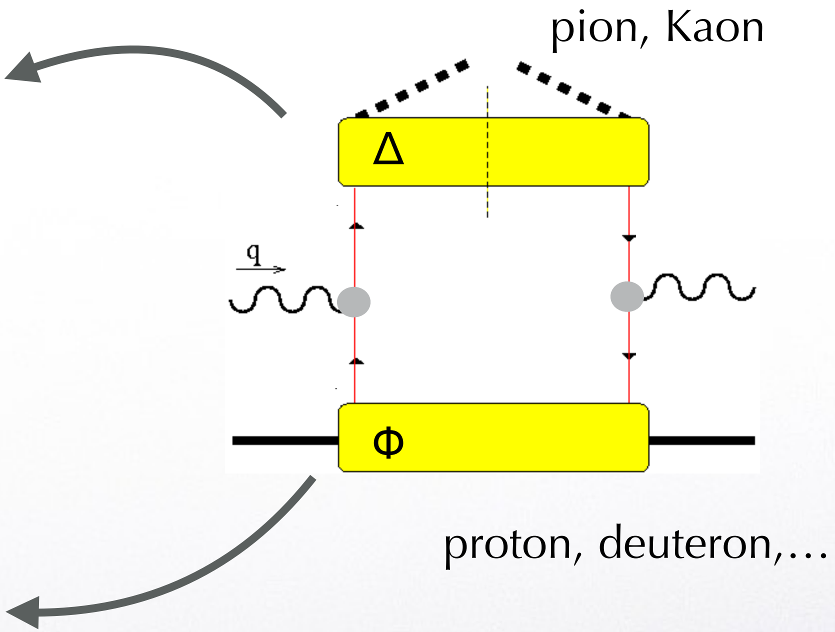
link TMD ↔ structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1		$H_1^{\perp}$ -



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ -
	L		$g_1 =$ -	$h_{1L}^{\perp} =$ -
	T	$f_{1T}^{\perp} =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^{\perp} =$ -

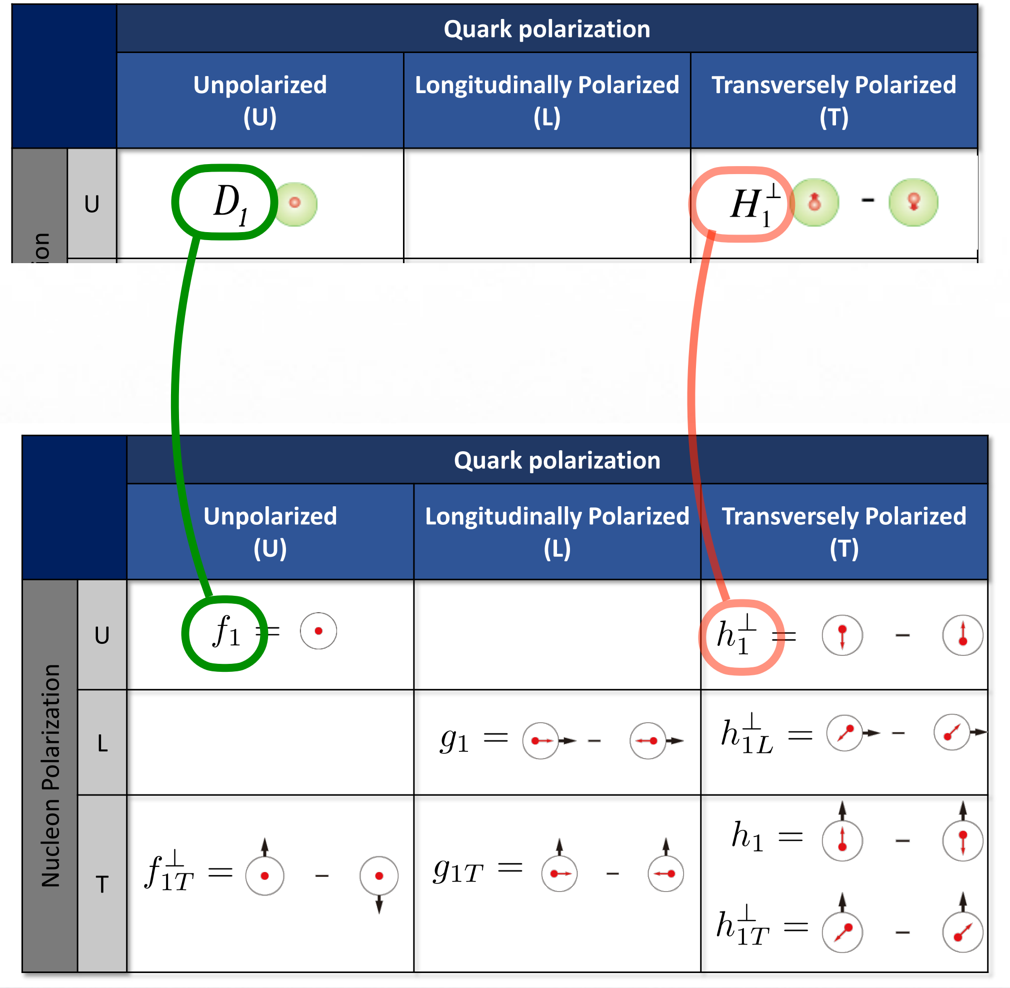


each structure function ~

$$F \sim d\hat{\sigma}(Q^2)\,\mathscr{C}\left[\mathrm{TMDPDF}(x,\mathbf{k}_\perp^2)\,,\mathrm{TMDFF}(z,\mathbf{P}_\perp^2)\right]$$

$$\mathscr{C}[\ldots] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\left[\ldots\right]$$

link TMD ↔ structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1 ⊙		$H_1^\perp$ ⊙ - ⊙

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^\perp =$ ⊙ - ⊙
	L		$g_1 =$ ⊙ - ⊙	$h_{1L}^\perp =$ ⊙ - ⊙
	T	$f_{1T}^\perp =$ ⊙ - ⊙	$g_{1T} =$ ⊙ - ⊙	$h_1 =$ ⊙ - ⊙ $h_{1T}^\perp =$ ⊙ - ⊙

target polariz. $\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$

○ $A(y)\,F_U \quad + \quad B(y)\,\cos 2\phi_h\,F_U^{\cos 2\phi_h}$

$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$

$+\;A(y)\,\sin(\phi_h - \phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$

$+\;B(y)\,\sin(\phi_h + \phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$

$+\;B(y)\,\sin(3\phi_h - \phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$

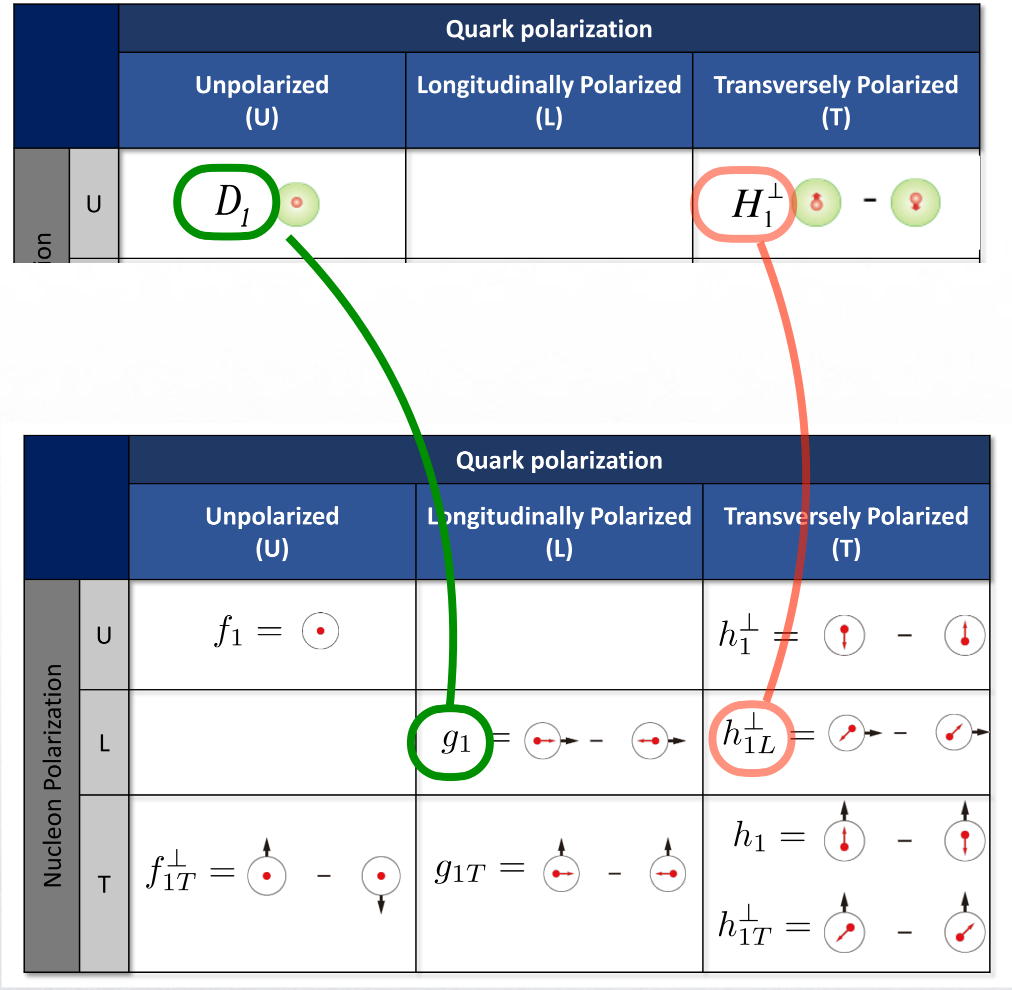
$+\;C(y)\,\cos(\phi_h - \phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$

each structure function ~

$$F \sim d\hat{\sigma}(Q^2)\,\mathscr{C}\left[\mathrm{TMDPDF}(x,\mathbf{k}_\perp^2)\,,\mathrm{TMDFF}(z,\mathbf{P}_\perp^2)\right]$$

$$\mathscr{C}[\ldots] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\left[\ldots\right]$$

link TMD ↔ structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
ion	U	D_1		$H_1^\perp$ -

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

target
polariz.

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$$

$$A(y)\,F_U \quad + \quad B(y)\,\cos 2\phi_h\,F_U^{\cos 2\phi_h}$$

$$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$$

$$+ \;\; A(y)\,\sin(\phi_h-\phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$$

$$+ \;\; B(y)\,\sin(\phi_h+\phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$$

$$+ \;\; B(y)\,\sin(3\phi_h-\phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$$

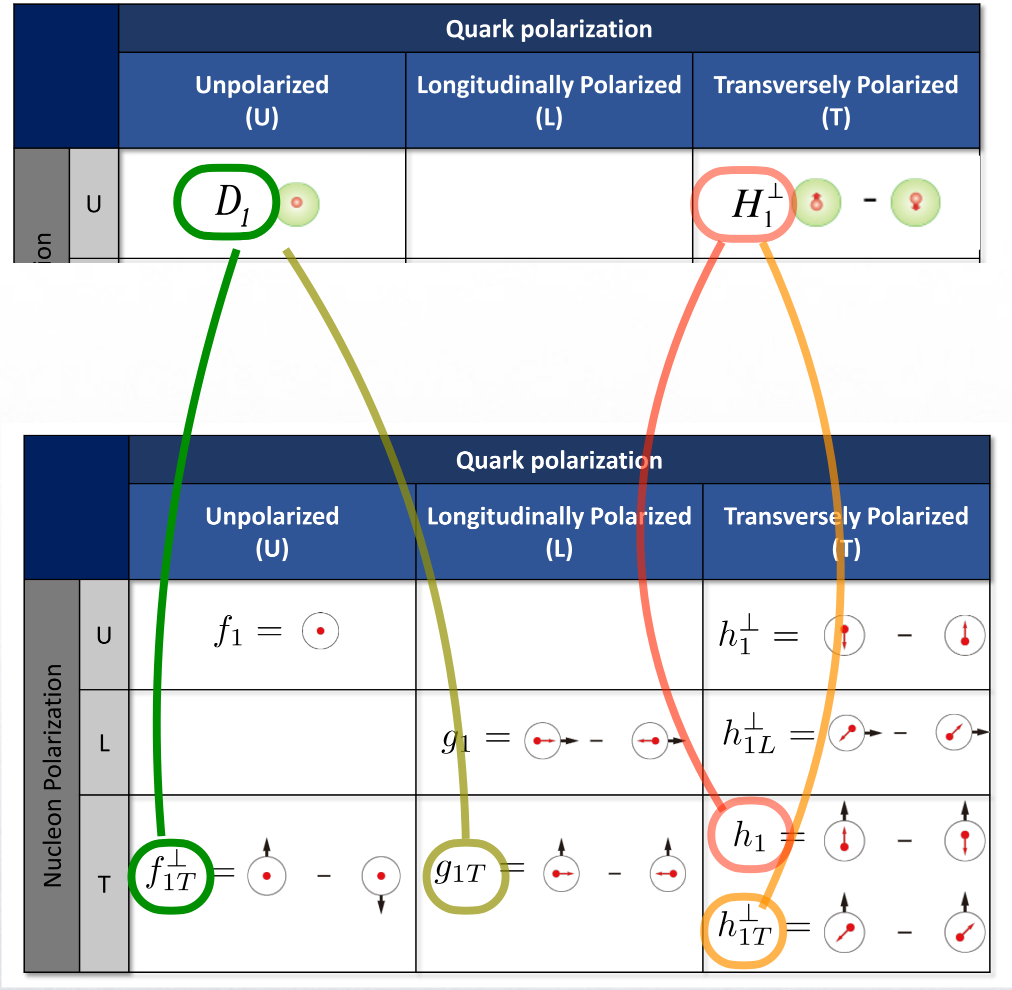
$$+ \;\; C(y)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

each structure function ~

$$F \sim d\hat{\sigma}(Q^2)\,\mathcal{C}\left[\mathrm{TMDPDF}(x,\mathbf{k}_\perp^2)\,,\mathrm{TMDFF}(z,\mathbf{P}_\perp^2)\right]$$

$$\mathcal{C}[\ldots] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\left[\ldots\right]$$

link TMD ↔ structure functions



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
	U	D_1		$H_1^\perp$ -

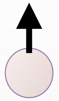
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

target
polariz.

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{hT}^2} \sim$$

$$A(y)\,F_U \quad + \quad B(y)\,\cos 2\phi_h\,F_U^{\cos 2\phi_h}$$

$$+\,C(y)\,F_{LL} \quad + \quad B(y)\,\sin 2\phi_h\,F_L^{\sin 2\phi_h}$$



$$+\;A(y)\,\sin(\phi_h-\phi_S)\,F_T^{\sin(\phi_h-\phi_S)}$$

$$+\;B(y)\,\sin(\phi_h+\phi_S)\,F_T^{\sin(\phi_h+\phi_S)}$$

$$+\;B(y)\,\sin(3\phi_h-\phi_S)\,F_T^{\sin(3\phi_h-\phi_S)}$$

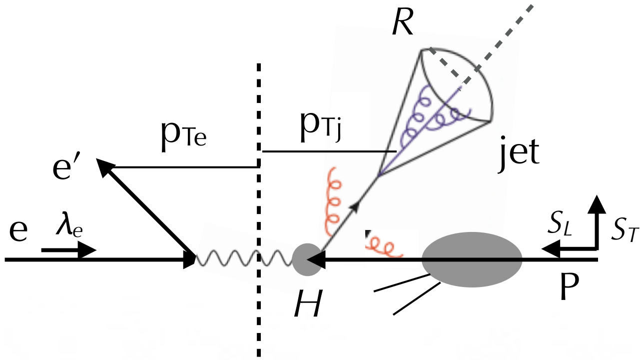
$$+\;C(y)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

each structure function ~

$$F \sim d\hat{\sigma}(Q^2)\,\mathcal{C}\left[\text{TMDPDF}(x,\mathbf{k}_\perp^2)\,,\,\text{TMDFF}(z,\mathbf{P}_\perp^2)\right]$$

$$\mathcal{C}[\ldots] = \int d\mathbf{P}_\perp d\mathbf{k}_\perp\,\delta^{(2)}(z\mathbf{k}_\perp + \mathbf{P}_\perp - \mathbf{P}_{hT})\,[\ldots]$$

TMDs with jets: SIDIS



$$\mathbf{q}_T = \mathbf{p}_{Te} + \mathbf{p}_{Tj} \ll \mathbf{p}_T = \frac{\mathbf{p}_{Te} - \mathbf{p}_{Tj}}{2}$$

"familiar" expression

$$\frac{d\sigma}{dy_j d\mathbf{p}_T d\mathbf{q}_T} = \overset{f_1}{F_{UU}} + \lambda_e S_L \overset{g_{1L}}{F_{LL}}$$

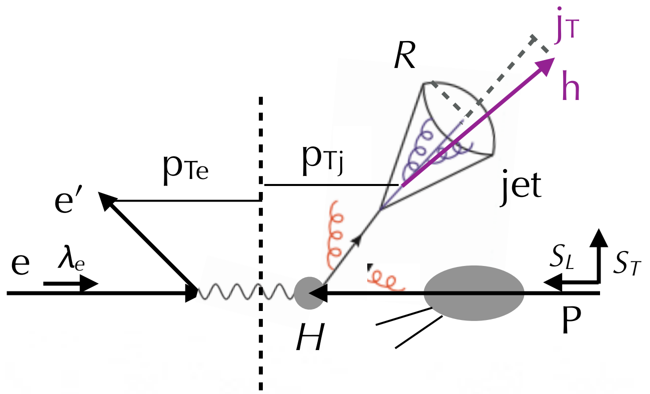
$$+ S_T \sin(\phi_j - \phi_S) \overset{f_{1T}^\perp}{F_{UT}^{\sin(\phi_j - \phi_S)}} + \lambda_e S_T \cos(\phi_j - \phi_S) \overset{g_{1T}}{F_{LT}^{\cos(\phi_j - \phi_S)}}$$

$$F_{UU} \sim \underbrace{H(Q)}_{\text{hard}} \, \underbrace{J(p_T R, Q)}_{\text{jet}} \, \underbrace{\{f_1(x, q_T, Q)\}}_{\text{"dressed" TMD}}$$

similarly for other *F*..

Kang et al., arXiv:2106.15624

TMDs with jets: SIDIS



$$\mathbf{q}_T = \mathbf{p}_{Te} + \mathbf{p}_{Tj} \ll \mathbf{p}_T = \frac{\mathbf{p}_{Te} - \mathbf{p}_{Tj}}{2}$$

"familiar" expression

$$\frac{d\sigma}{dy_j d\mathbf{p}_T d\mathbf{q}_T} = \overset{f_1\,\mathscr{D}_1}{F_{UU}} + \cos(\phi_j - \phi_h)\, \overset{h_1^\perp\,\mathscr{H}_1^\perp}{F_{UU}^{\cos(\phi_j-\phi_h)}} + \lambda_e S_L\, \overset{g_{1L}\,\mathscr{D}_1}{F_{LL}}$$

$$+ S_L\, \sin(\phi_j - \phi_h)\, \overset{h_{1L}^\perp\,\mathscr{H}_1^\perp}{F_{UL}^{\sin(\phi_j-\phi_h)}}$$

$$+ S_T\, \sin(\phi_j - \phi_S)\, \overset{f_{1T}^\perp\,\mathscr{D}_1}{F_{UT}^{\sin(\phi_j-\phi_S)}} + \lambda_e S_T\, \cos(\phi_j - \phi_S)\, \overset{g_{1T}\,\mathscr{D}_1}{F_{LT}^{\cos(\phi_j-\phi_S)}}$$

$$+ S_T\, \sin(\phi_h - \phi_S)\, \overset{h_1\,\mathscr{H}_1^\perp}{F_{UT}^{\sin(\phi_h-\phi_S)}} + S_T\, \sin(2\phi_j - \phi_h - \phi_S)\, \overset{h_{1T}^\perp\,\mathscr{H}_1^\perp}{F_{UT}^{\sin(2\phi_j-\phi_h-\phi_S)}}$$

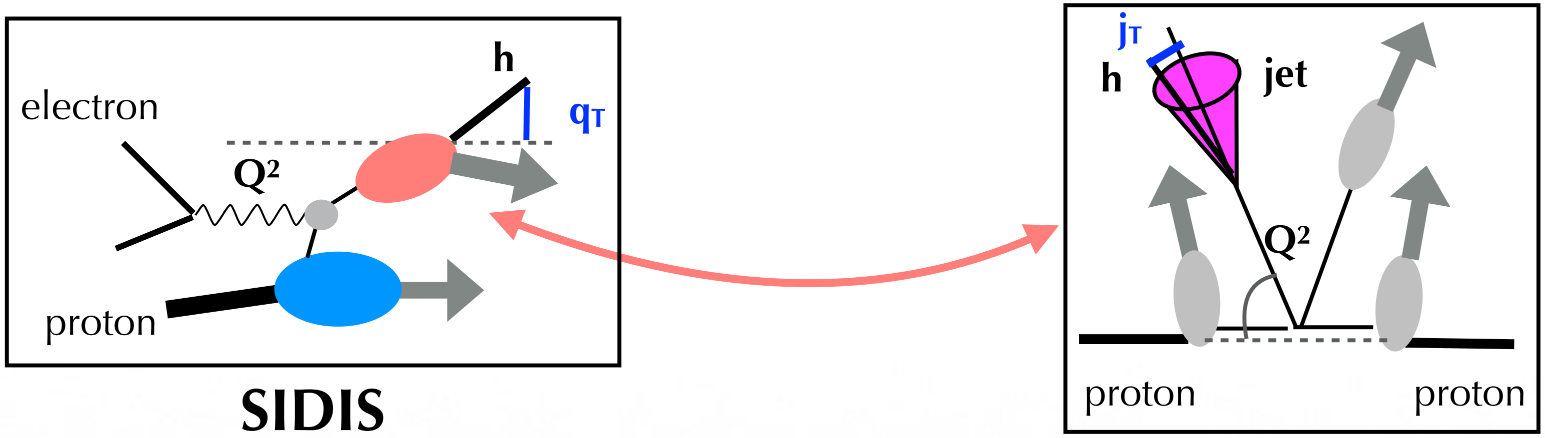
$$F_{UU} \sim H(Q)\, \mathrm{TMDJFF}(z_h, p_T R, Q)\, \{f_1(x, q_T, Q)\}$$

hard jet "dressed" TMD

similarly for other *F*..

Kang et al., arXiv:2106.15624

TMDs with jets: hybrid factorisation



hybrid scheme:
- TMD framework for TMD **fragmentation**
- collinear framework for PDF

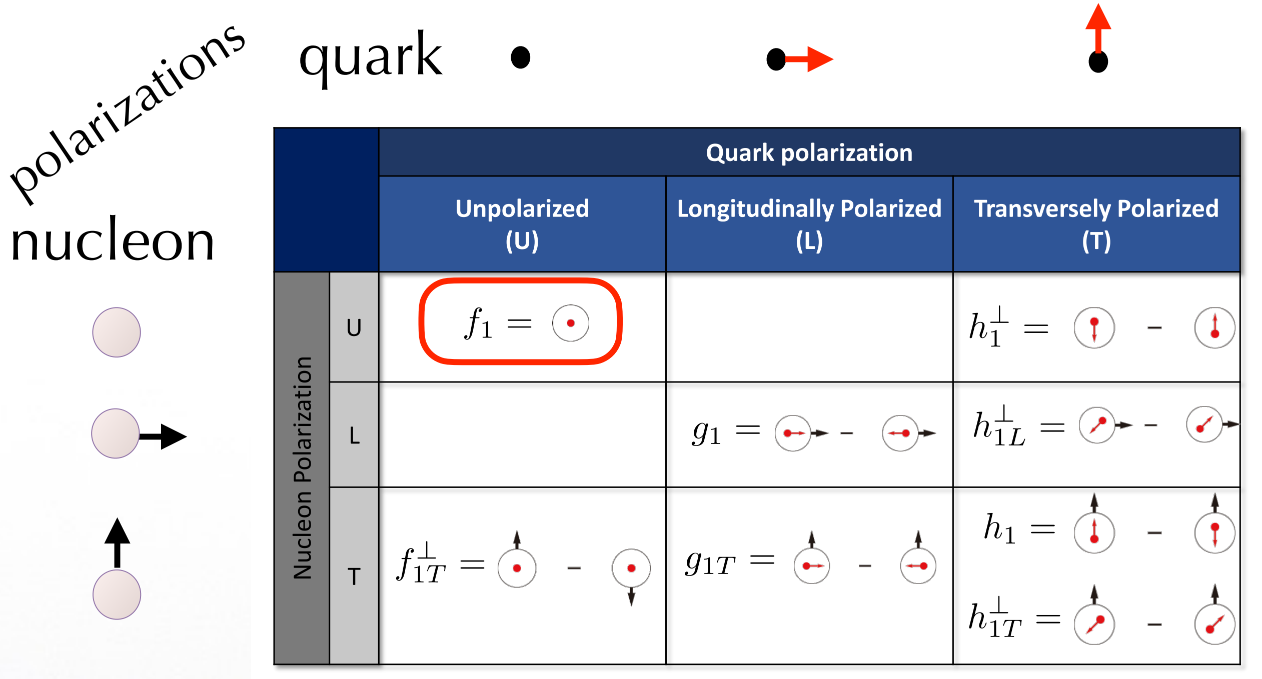
Factorization theorem for **$j_T \ll Q$**
universality for TMD **fragmentation**

*Kang, Liu, Ringer, Xing, JHEP **1711** (17), arXiv:1705.08443*
*Kang, Prokudin, Ringer, Yuan, P.L. **B774** (17), arXiv:1707.00913*

Outline

- Phenomenology of TMDs

The unpolarized TMD PDF



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp = \quad - $
	L		$g_1 = \quad - $	$h_{1L}^\perp = \quad - $
	T	$f_{1T}^\perp = \quad - $	$g_{1T} = \quad - $	$h_1 = \quad - $ $h_{1T}^\perp = \quad - $

$f_1^q(x, \mathbf{k}_\perp^2)$ probability density of finding a quark q with "longitudinal" (along "+" LC direction) fraction x of nucleon momentum, and transverse momentum $\mathbf{k}_\perp$

Most recent extractions of unpolarized TMD f_1

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY	W / Z production	N of points	χ^2/N_{points}
PV 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.5
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL’	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	N^{3}LL(-)	✔	✔	✔	✔	1039	1.06
PV 2019 arXiv:1912.07550	N^{3}LL	✘	✘	✔	✔	353	1.07
SV19 + flavor dep. arXiv:2201.07114	N^{3}LL	✘	✘	✔	✔	309	<1.08>
MAPTMD 2022 arXiv:2206.07598	N^{3}LL(-)	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^{4}LL	✘	✘	✔	✔	627	0.96
MAPTMD 2024 arXiv:2405.13833	N^{3}LL	✔	✔	✔	✔	2031	1.08
MAPNN 2025 arXiv:2502.04166	N^{3}LL	✘	✘	✔	✔	482	0.97

first use of Neural Networks

Most recent extractions of unpolarized TMD f_1

		SIDIS					
	Accuracy	HERMES	COMPASS	DY	W / Z production	N of points	χ^2/N_{points}
PV 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.5
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL’	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	$N^3LL(-)$	✔	✔	✔	✔	1039	1.06
PV 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.07
SV19 + flavor dep. arXiv:2201.07114	N^3LL	✘	✘	✔	✔	309	<1.08>
MAPTMD 2022 arXiv:2206.07598	$N^3LL(-)$	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^4LL	✘	✘	✔	✔	627	0.96
MAPTMD 2024 arXiv:2405.13833	N^3LL	✔	✔	✔	✔	2031	1.08
MAPNN 2025 arXiv:2502.04166	N^3LL	✘	✘	✔	✔	482	0.97

increasing accuracy & precision

Most recent extractions of unpolarized TMD f_1

		SIDIS					
	Accuracy	HERMES	COMPASS	DY	W / Z production	N of points	χ^2/N_{points}
PV 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.5
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL’	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	N^{3}LL(-)	✔	✔	✔	✔	1039	1.06
PV 2019 arXiv:1912.07550	N^{3}LL	✘	✘	✔	✔	353	1.07
SV19 + flavor dep. arXiv:2201.07114	N^{3}LL	✘	✘	✔	✔	309	<1.08>
MAPTMD 2022 arXiv:2206.07598	N^{3}LL(-)	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^{4}LL	✘	✘	✔	✔	627	0.96
MAPTMD 2024 arXiv:2405.13833	N^{3}LL	✔	✔	✔	✔	2031	1.08
MAPNN 2025 arXiv:2502.04166	N^{3}LL	✘	✘	✔	✔	482	0.97

only four global fits

increasing accuracy & precision

Most recent extractions of unpolarized TMD f_1

		SIDIS					
	Accuracy	HERMES	COMPASS	DY	W / Z production	N of points	χ^2/N_{points}
PV 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.5
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL’	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	$N^3LL(-)$	✔	✔	✔	✔	1039	1.06
PV 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.07
SV19 + flavor dep. arXiv:2201.07114	N^3LL	✘	✘	✔	✔	309	<1.08>
MAPTMD 2022 arXiv:2206.07598	$N^3LL(-)$	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^4LL	✘	✘	✔	✔	627	0.96
MAPTMD 2024 arXiv:2405.13833	N^3LL	✔	✔	✔	✔	2031	1.08
MAPNN 2025 arXiv:2502.04166	N^3LL	✘	✘	✔	✔	482	0.97

MAPTMD24 : introduce **flavor sensitivity of k_T-dependence**

The MAPTMD24 data sets

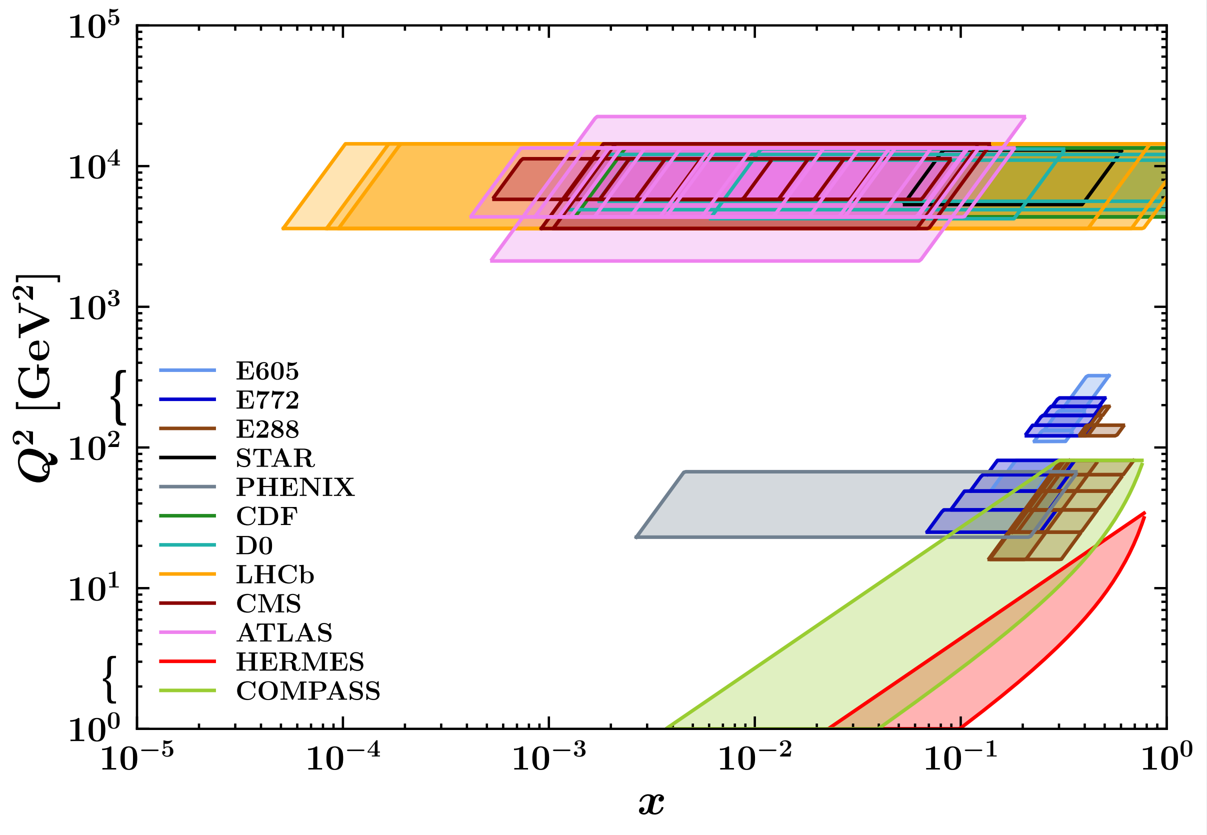
N_{data} after cuts

Drell Yan { 233 fixed target

251 collider

1547 SIDIS

2031 data points



kinematic cuts

$\langle Q \rangle > 1.4$ GeV

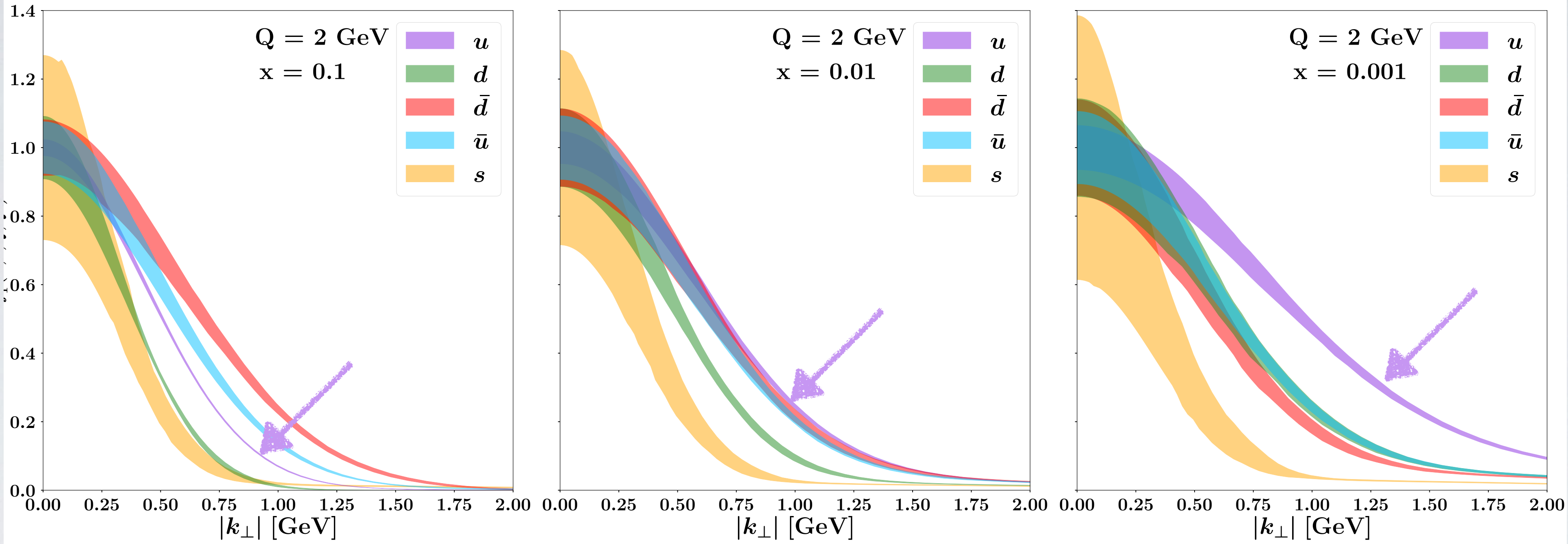
$0.2 < z < 0.7$

Drell-Yan $q_T < 0.2\,Q$

SIDIS $P_{hT} < \min\Big[\min\big[0.2\,Q,\, 0.5\,Qz\big] + 0.3\,\mathrm{GeV},\, zQ\Big]$

"Normalized" MAPTMD24 TMD PDF

$$\frac{f_1(x, k_T; Q)}{f_1(x, 0; Q)}$$

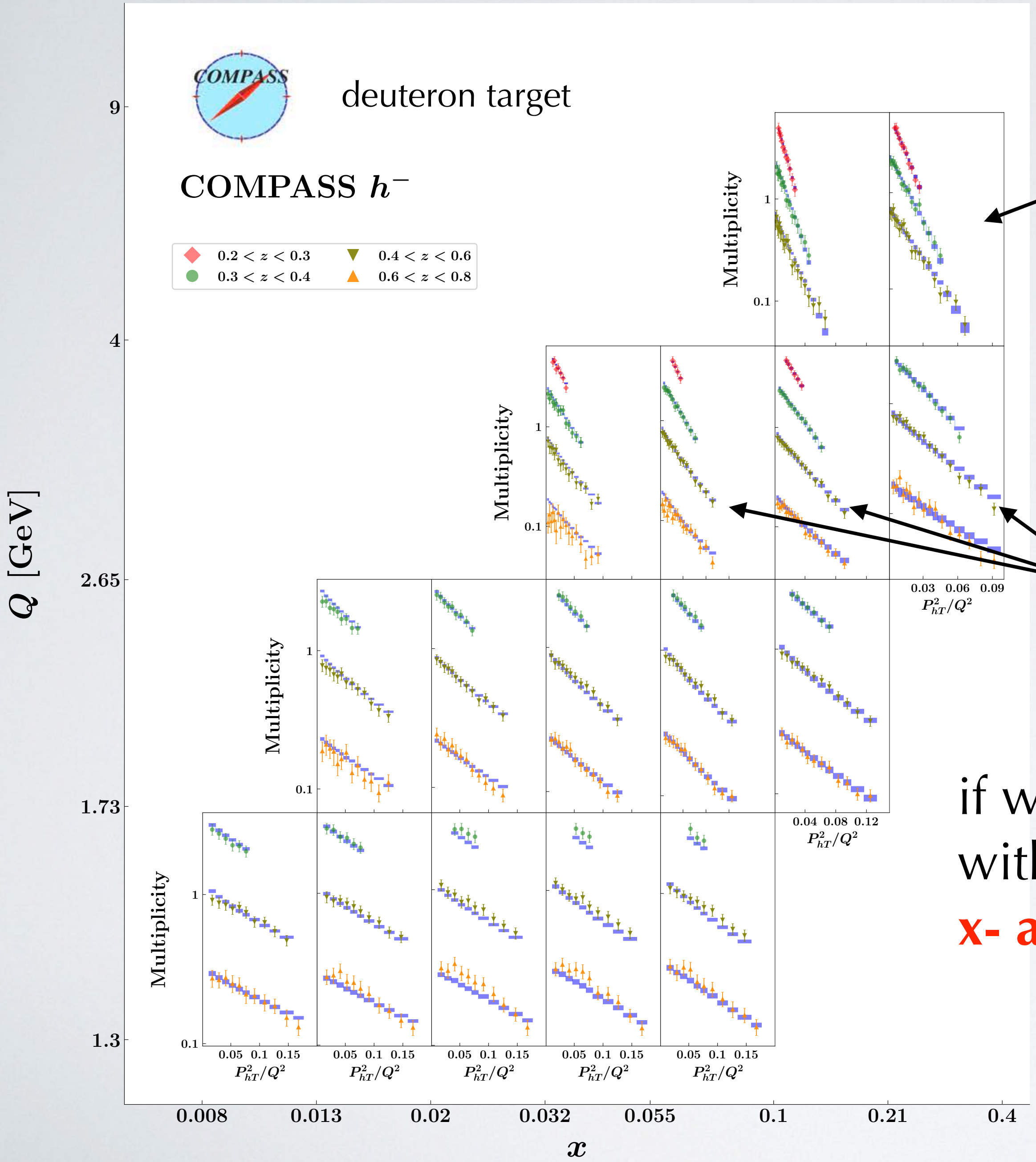


th. error band =
68% of all replicas

- very different k_T behavior
- it changes with x
- potential impact on the extraction of W mass parameter from collider data

Bacchetta et al., P.L. **B788** (19) 542, arXiv:1807.02101
Bozzi & Signori, Adv.HighEn.Phys. **2019** (19) 2526897, arXiv:1901.01162

Data-driven nonperturbative TMD



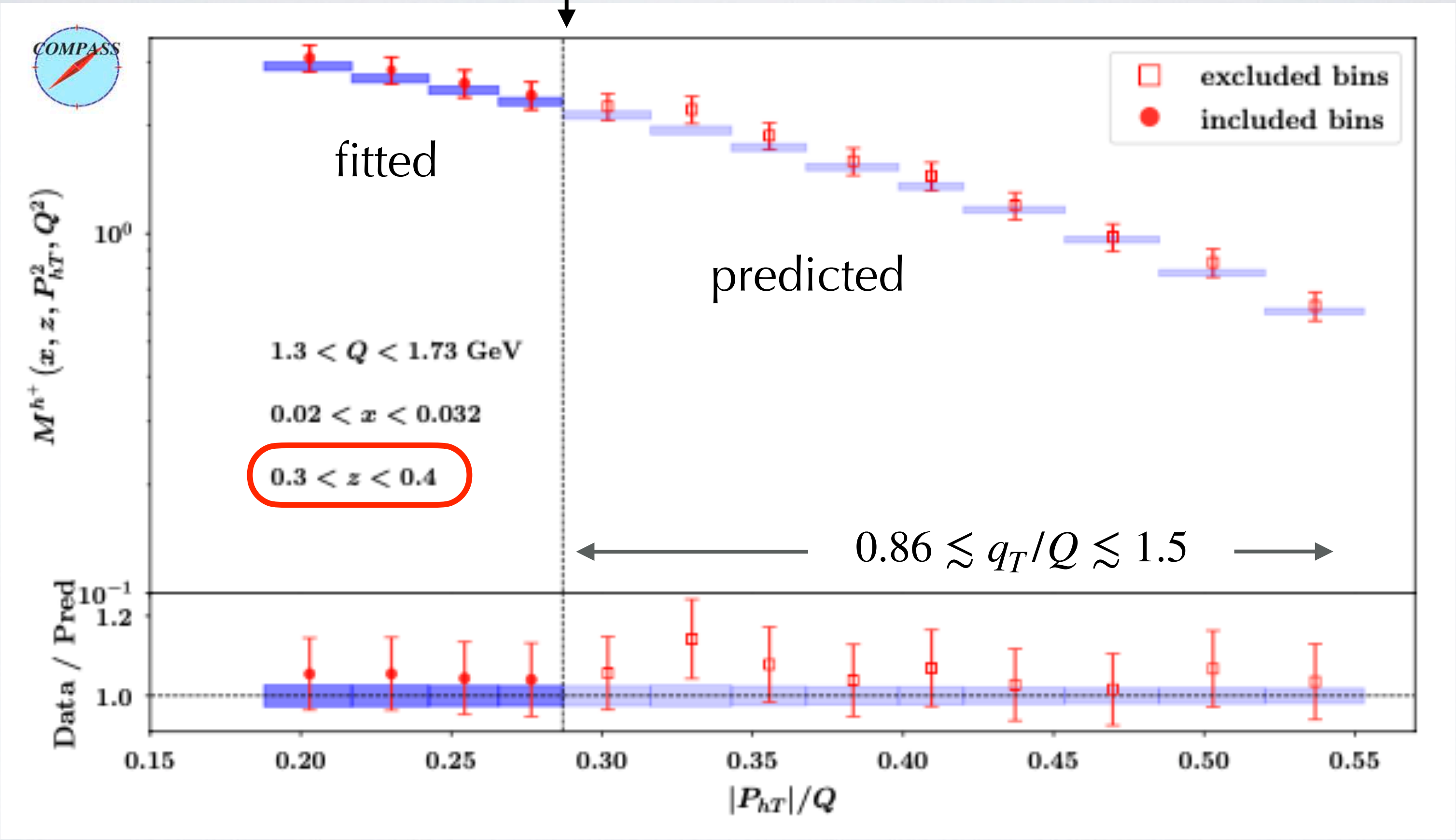
at given (x,Q^2),
different slopes for different z

at given z,
different slopes for different x

if we model nonperturbative TMDs with Gaussians, we need
x- and z-dependent widths!

MAPTMD22: validity of TMD region?

cut of baseline fit



fitted

predicted

$0.86 \lesssim q_T/Q \lesssim 1.5$

validity of TMD factorization seems to extend well beyond $P_{hT}/z << Q$!

Collins-Soper evolution kernel

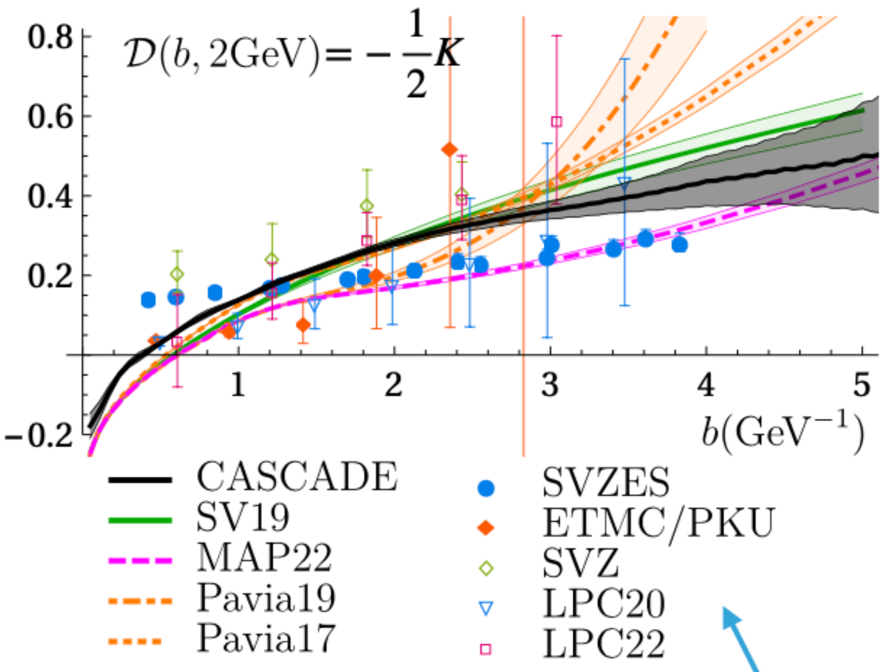
universal flavor-independent $K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$

drives evolution in rapidity ζ

perturbative (computed)

non-perturbative (fitted)

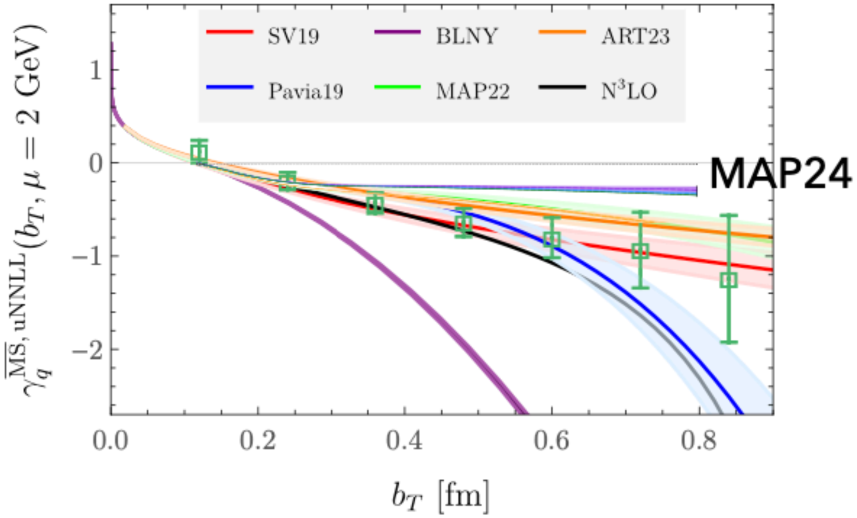
Bermudez Martinez, Vladimirov, arXiv:2206.01105



TMD phenomenology

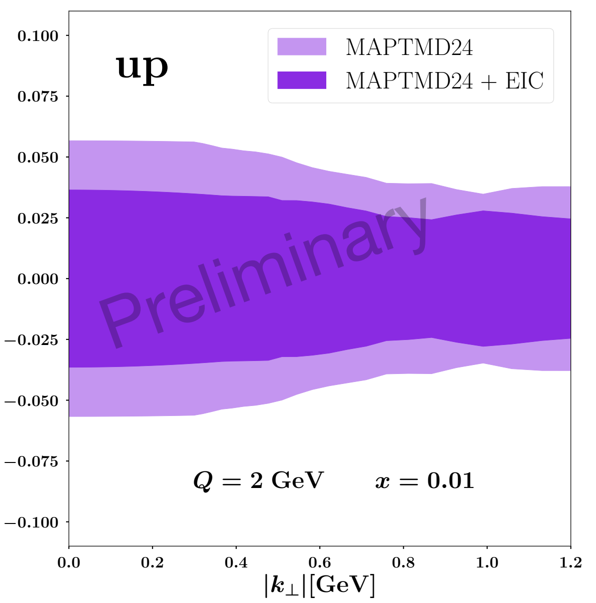
Lattice QCD

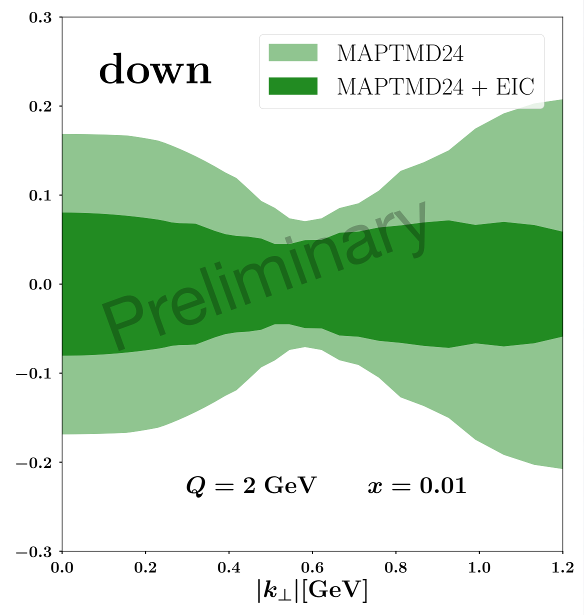
Avkhadiev, Shanahan, Wagman, Zhao, arXiv:2307.12359



Bacchetta, ePIC 2025 general meeting

The EIC impact at x=0.01



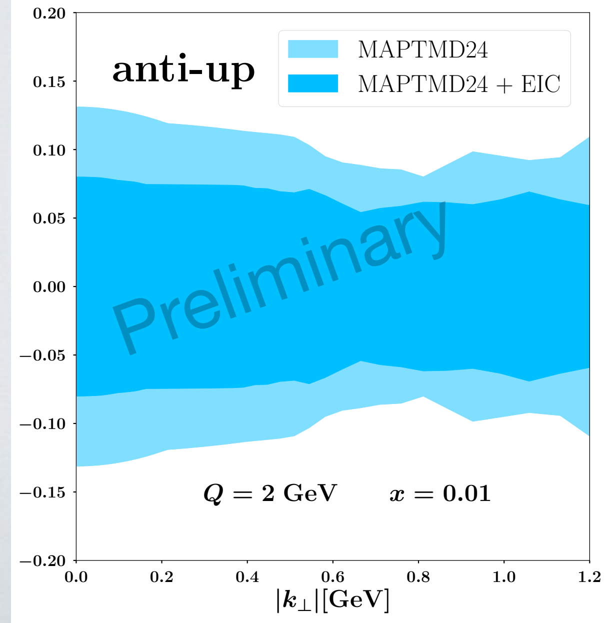


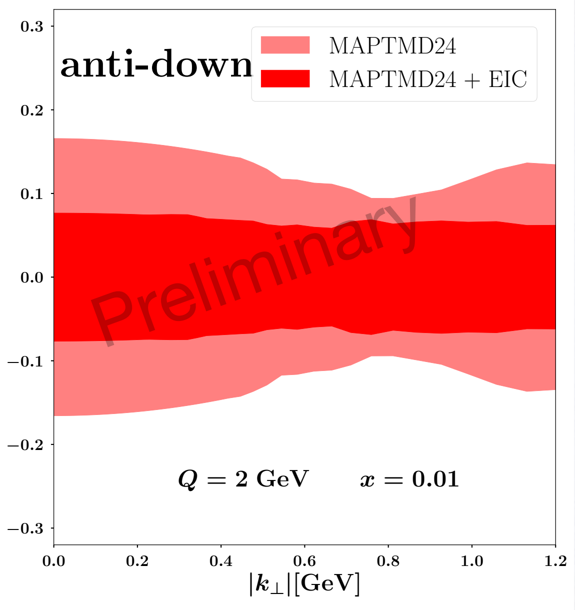
$$\frac{\text{TMD}^q - \langle\text{TMD}^q\rangle}{\langle\text{TMD}^q\rangle}$$

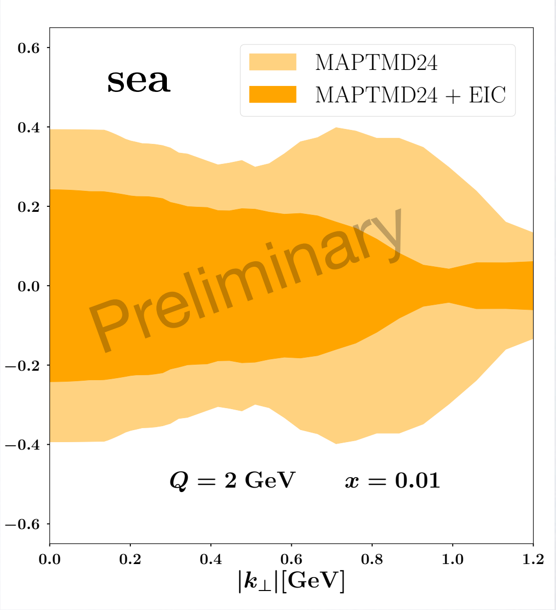
x=0.01

MAPTMD24 2031

EIC	**# pts.**	**lumi [fb^{-1}]**
5x41	1273	2.85
10x100	1611	51.3
18x275	1648	10

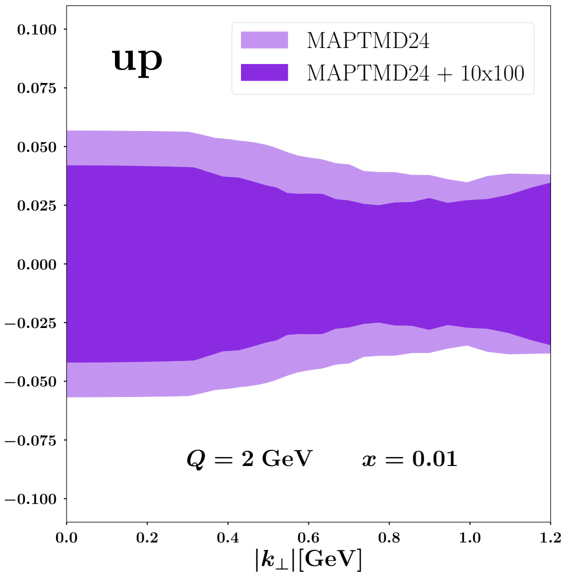


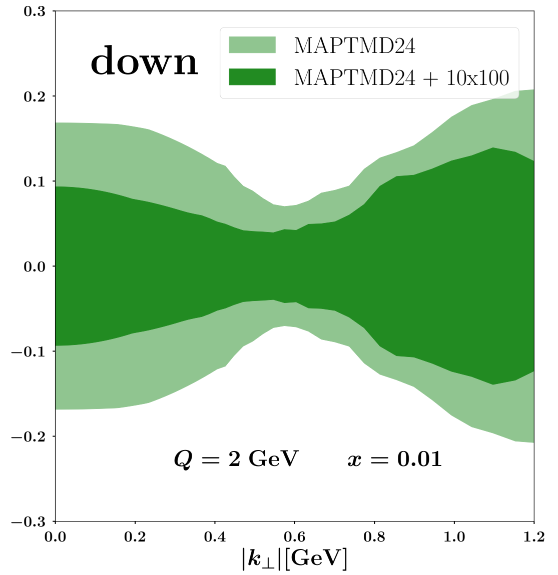




L. Rossi, Ph.D. Thesis

The EIC impact with 10x100 at x=0.01



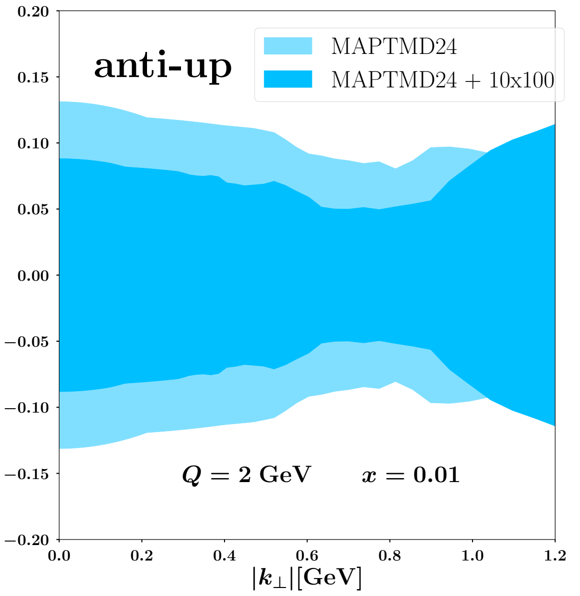


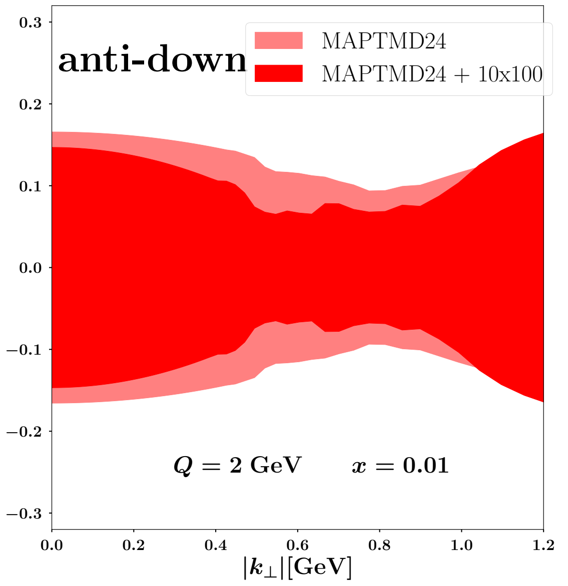
$$\frac{\mathrm{TMD}^q - \langle \mathrm{TMD}^q \rangle}{\langle \mathrm{TMD}^q \rangle} \qquad \mathbf{x=0.01}$$

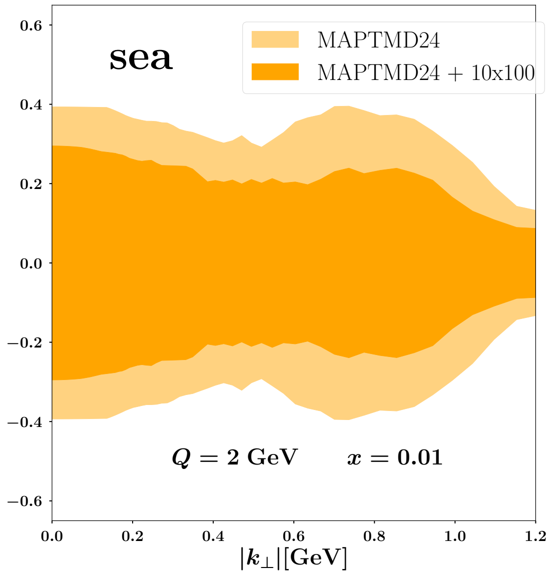
MAPTMD24 2031

EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	51.3

(simulation campaign of May 2024)







courtesy L. Rossi

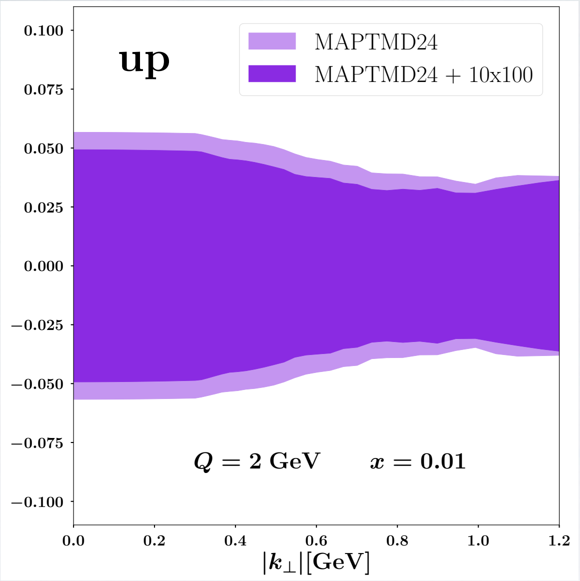
The EIC impact with 10x100 at x=0.01

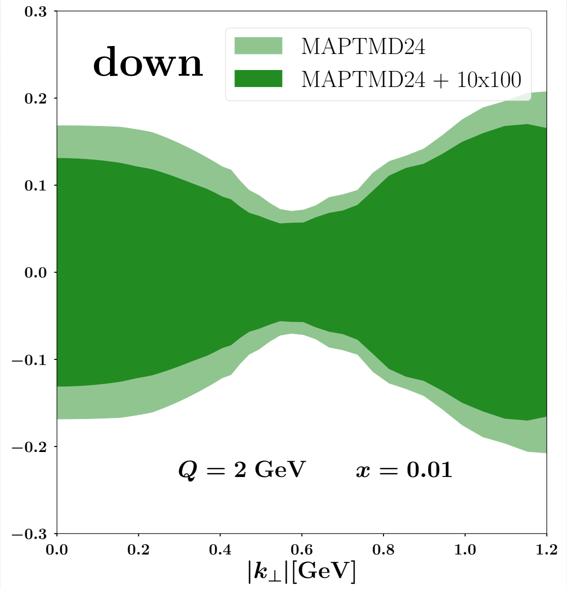
$$\frac{\mathrm{TMD}^q - <\mathrm{TMD}^q>}{<\mathrm{TMD}^q>}$$

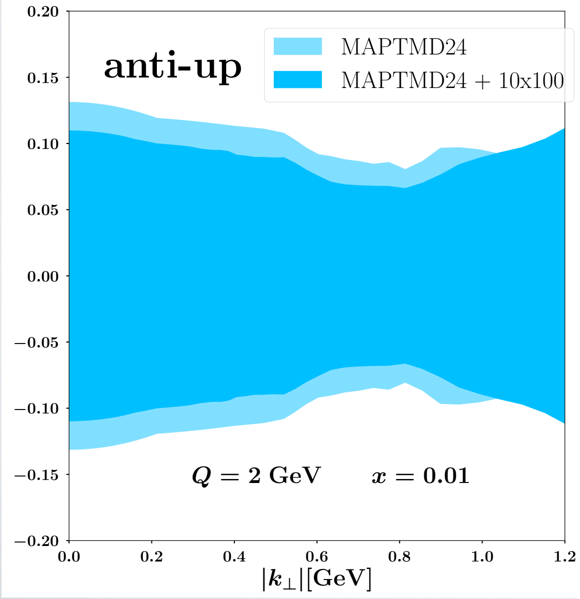
x=0.01

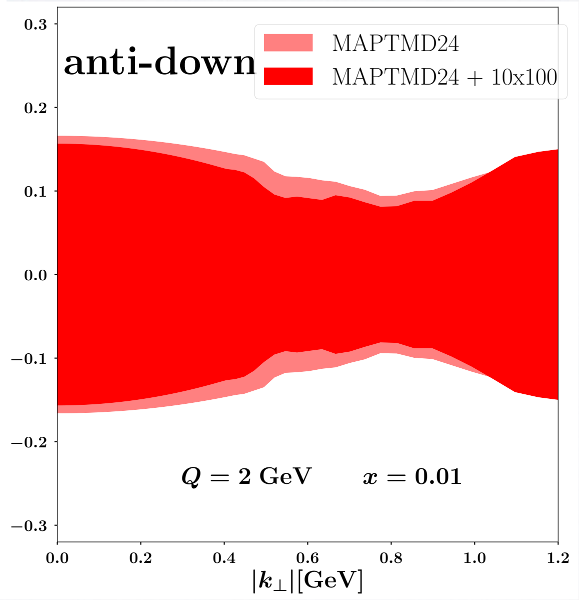
MAPTMD24	2031	
EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	5

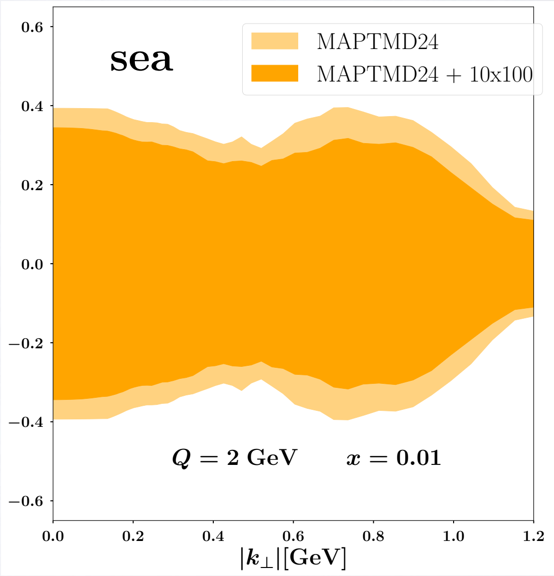
(early Science conditions)





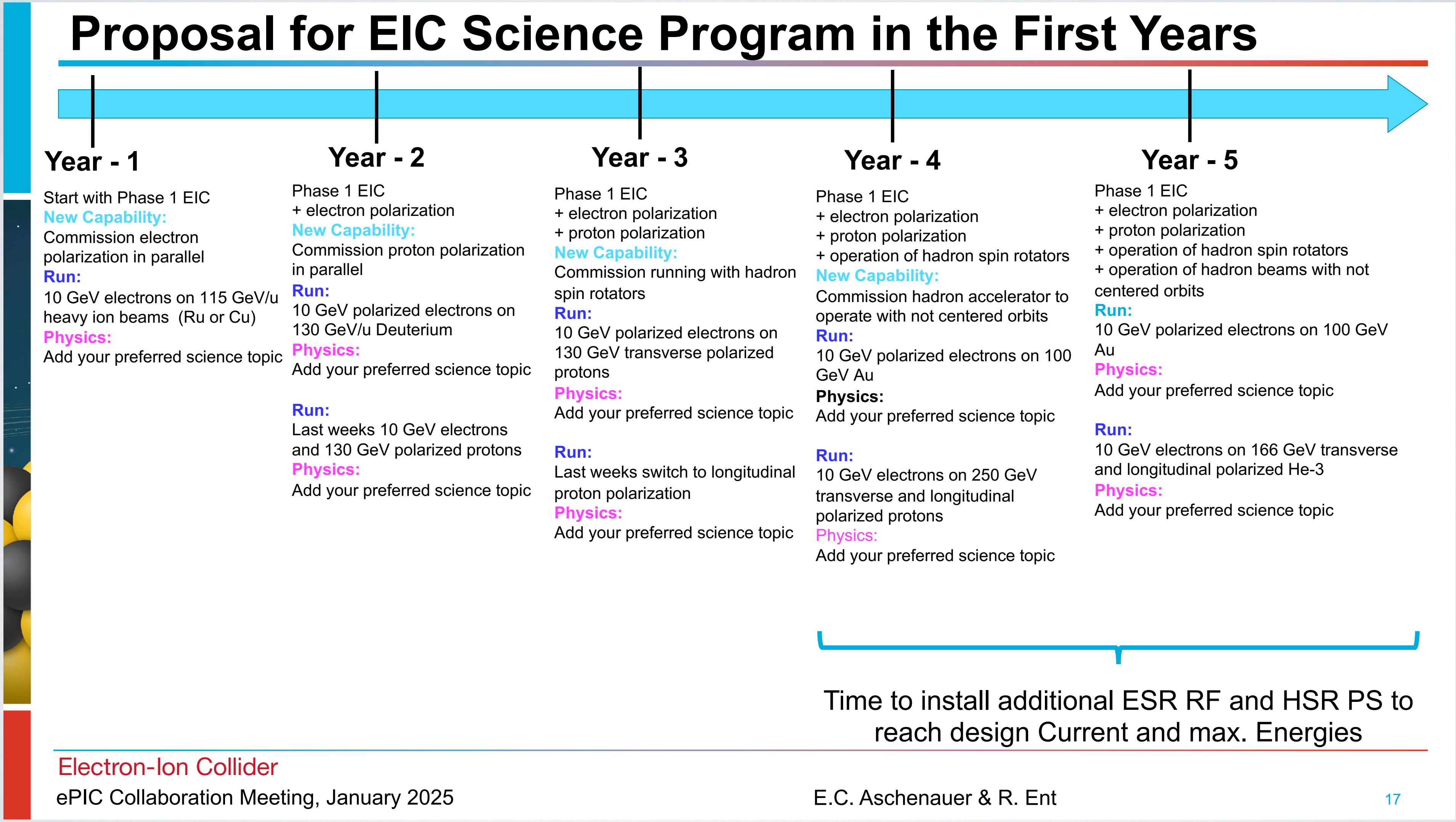






courtesy L. Rossi

Early Science Conditions



Proposal for EIC Science Program in the First Years

Year - 1

Start with Phase 1 EIC
New Capability:
Commission electron polarization in parallel
Run:
10 GeV electrons on 115 GeV/u heavy ion beams (Ru or Cu)
Physics:
Add your preferred science topic

Year - 2

Phase 1 EIC
+ electron polarization
New Capability:
Commission proton polarization in parallel
Run:
10 GeV polarized electrons on 130 GeV/u Deuterium
Physics:
Add your preferred science topic

Run:
Last weeks 10 GeV electrons and 130 GeV polarized protons
Physics:
Add your preferred science topic

Year - 3

Phase 1 EIC
+ electron polarization
+ proton polarization
New Capability:
Commission running with hadron spin rotators
Run:
10 GeV polarized electrons on 130 GeV transverse polarized protons
Physics:
Add your preferred science topic

Run:
Last weeks switch to longitudinal proton polarization
Physics:
Add your preferred science topic

Year - 4

Phase 1 EIC
+ electron polarization
+ proton polarization
+ operation of hadron spin rotators
New Capability:
Commission hadron accelerator to operate with not centered orbits
Run:
10 GeV polarized electrons on 100 GeV Au
Physics:
Add your preferred science topic

Run:
10 GeV electrons on 250 GeV transverse and longitudinal polarized protons
Physics:
Add your preferred science topic

Year - 5

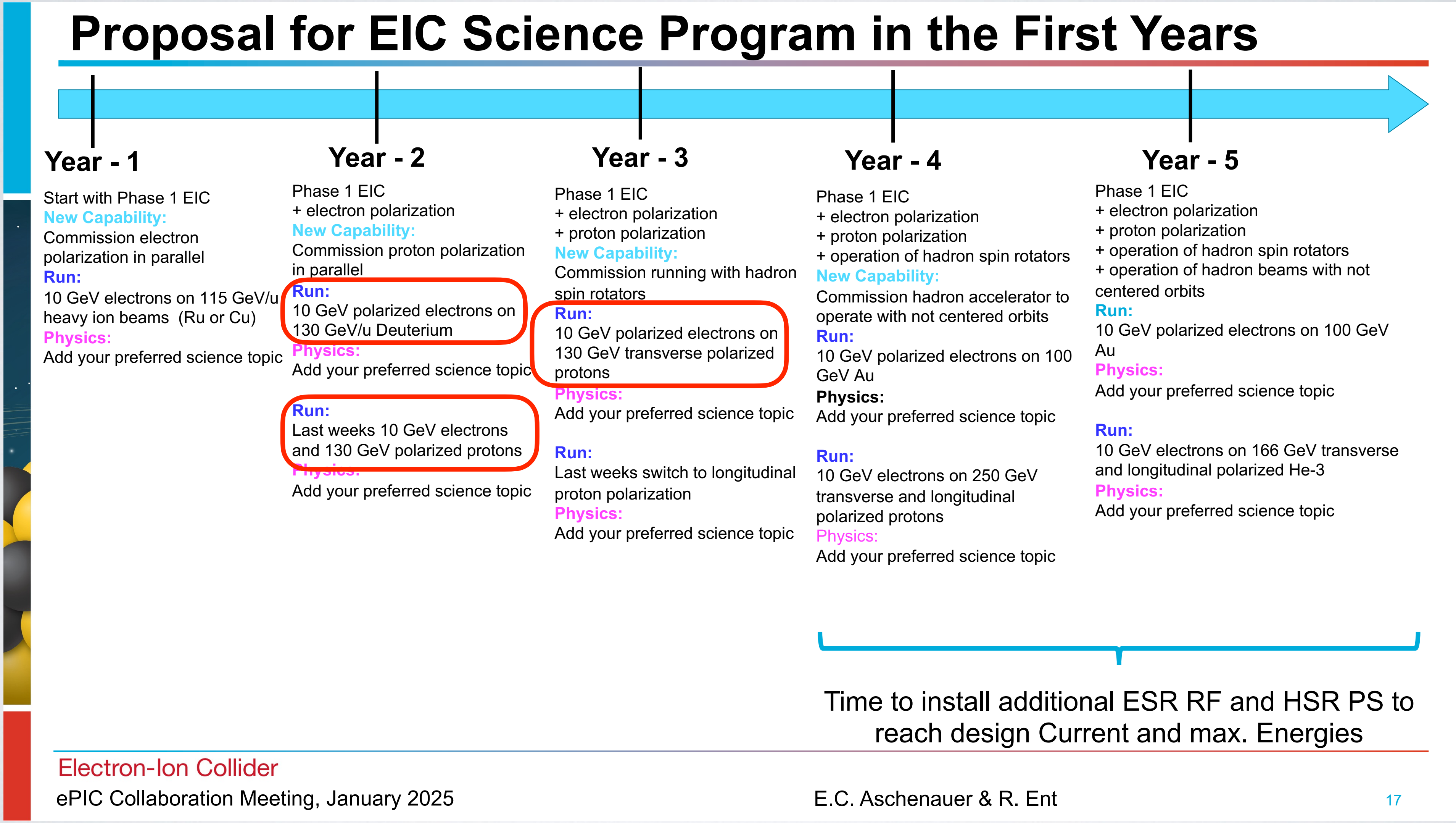
Phase 1 EIC
+ electron polarization
+ proton polarization
+ operation of hadron spin rotators
+ operation of hadron beams with not centered orbits
Run:
10 GeV polarized electrons on 100 GeV Au
Physics:
Add your preferred science topic

Run:
10 GeV electrons on 166 GeV transverse and longitudinal polarized He-3
Physics:
Add your preferred science topic

Time to install additional ESR RF and HSR PS to reach design Current and max. Energies

Electron-Ion Collider
ePIC Collaboration Meeting, January 2025
E.C. Aschenauer & R. Ent
17

Early Science Conditions



Proposal for EIC Science Program in the First Years

Year - 1

Start with Phase 1 EIC
New Capability:
Commission electron polarization in parallel
Run:
10 GeV electrons on 115 GeV/u heavy ion beams (Ru or Cu)
Physics:
Add your preferred science topic

Year - 2

Phase 1 EIC
+ electron polarization
New Capability:
Commission proton polarization in parallel
Run:
10 GeV polarized electrons on 130 GeV/u Deuterium
Physics:
Add your preferred science topic

Run:
Last weeks 10 GeV electrons and 130 GeV polarized protons
Physics:
Add your preferred science topic

Year - 3

Phase 1 EIC
+ electron polarization
+ proton polarization
New Capability:
Commission running with hadron spin rotators
Run:
10 GeV polarized electrons on 130 GeV transverse polarized protons
Physics:
Add your preferred science topic

Run:
Last weeks switch to longitudinal proton polarization
Physics:
Add your preferred science topic

Year - 4

Phase 1 EIC
+ electron polarization
+ proton polarization
+ operation of hadron spin rotators
New Capability:
Commission hadron accelerator to operate with not centered orbits
Run:
10 GeV polarized electrons on 100 GeV Au
Physics:
Add your preferred science topic

Run:
10 GeV electrons on 250 GeV transverse and longitudinal polarized protons
Physics:
Add your preferred science topic

Year - 5

Phase 1 EIC
+ electron polarization
+ proton polarization
+ operation of hadron spin rotators
+ operation of hadron beams with not centered orbits
Run:
10 GeV polarized electrons on 100 GeV Au
Physics:
Add your preferred science topic

Run:
10 GeV electrons on 166 GeV transverse and longitudinal polarized He-3
Physics:
Add your preferred science topic

Time to install additional ESR RF and HSR PS to reach design Current and max. Energies

Electron-Ion Collider
ePIC Collaboration Meeting, January 2025

E.C. Aschenauer & R. Ent

17

Early Science Conditions

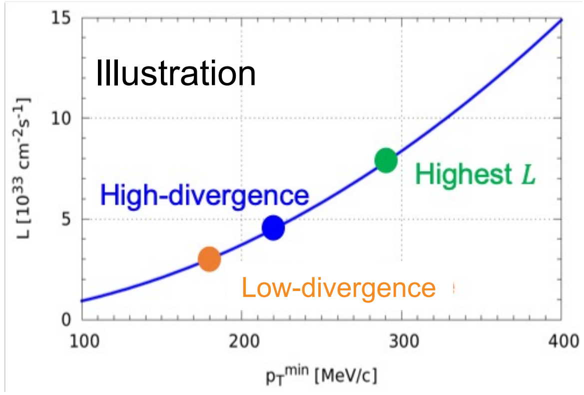
ep Luminosity for Phase-1

High Divergence	Lumi per Fill (5 h)	Lumi per Year
5 GeV e x 250 GeV p	9.26 pb^{-1}	6.48 fb^{-1}
10 GeV e x 250 GeV p	13.12 pb^{-1}	9.18 fb^{-1}
5 GeV e x 130 GeV p	6.3 pb^{-1}	4.36 fb-1
10 GeV e x 130 GeV p	7.6 pb^{-1}	5.33 fb^{-1}

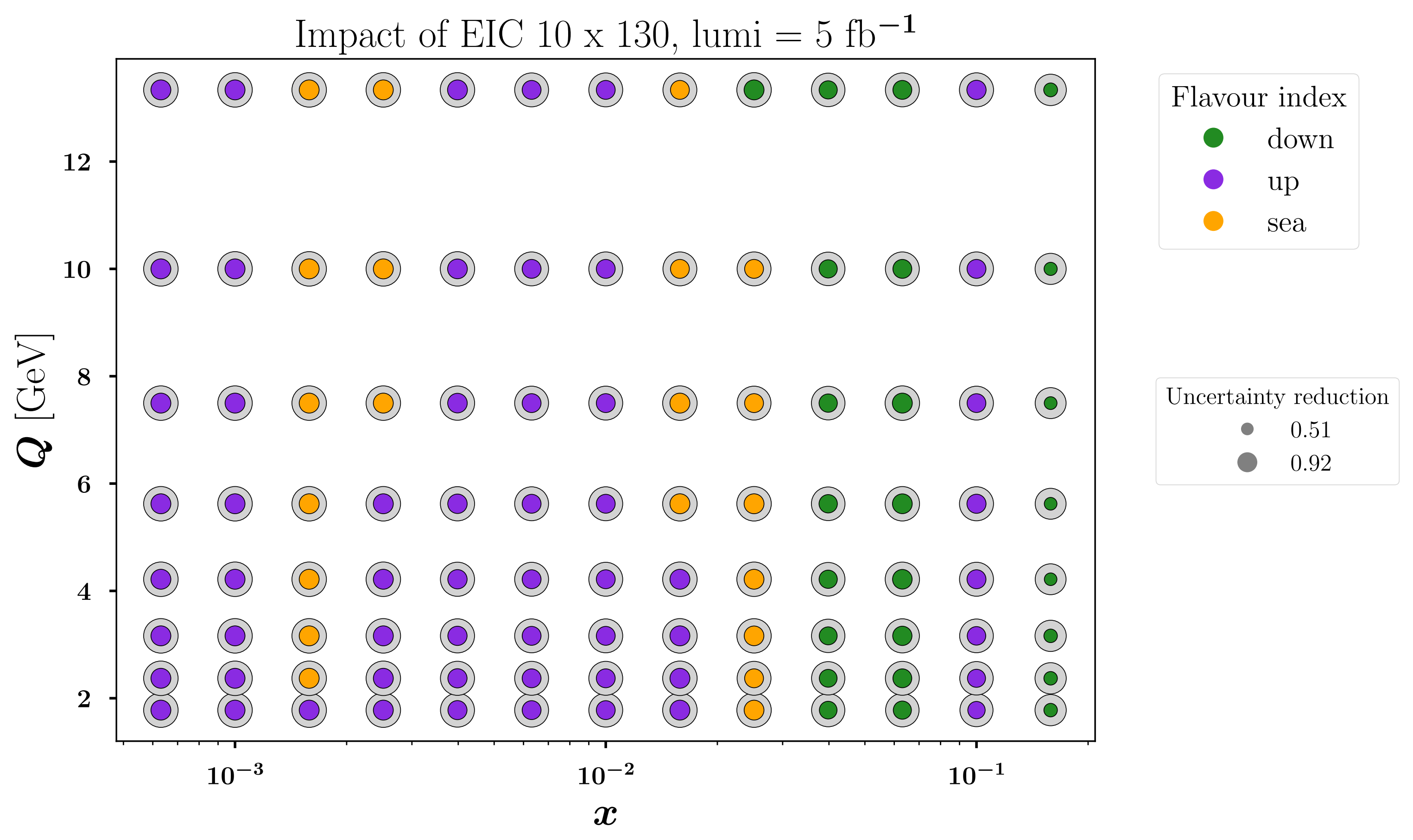
Low Divergence	Lumi per Fill (5 h)	Lumi per Year
5 GeV e x 250 GeV p	6.81 pb^{-1}	4.78 fb^{-1}
10 GeV e x 250 GeV p	8.8 pb^{-1}	6.19 fb^{-1}
5 GeV e x 130 GeV p	5.8 pb^{-1}	4.1 fb^{-1}
10 GeV e x 130 GeV p	7.1 pb^{-1}	4.95 fb^{-1}

Compare to HERA integrated luminosity 1992 – 2007: 0.6 fb^{-1}

Remember:
high divergence: higher lumi, but reduced acceptance for low forward particle p_T^{min}
low divergence: lower lumi, but increased acceptance for low forward particle p_T^{min}
→ important for exclusive processes



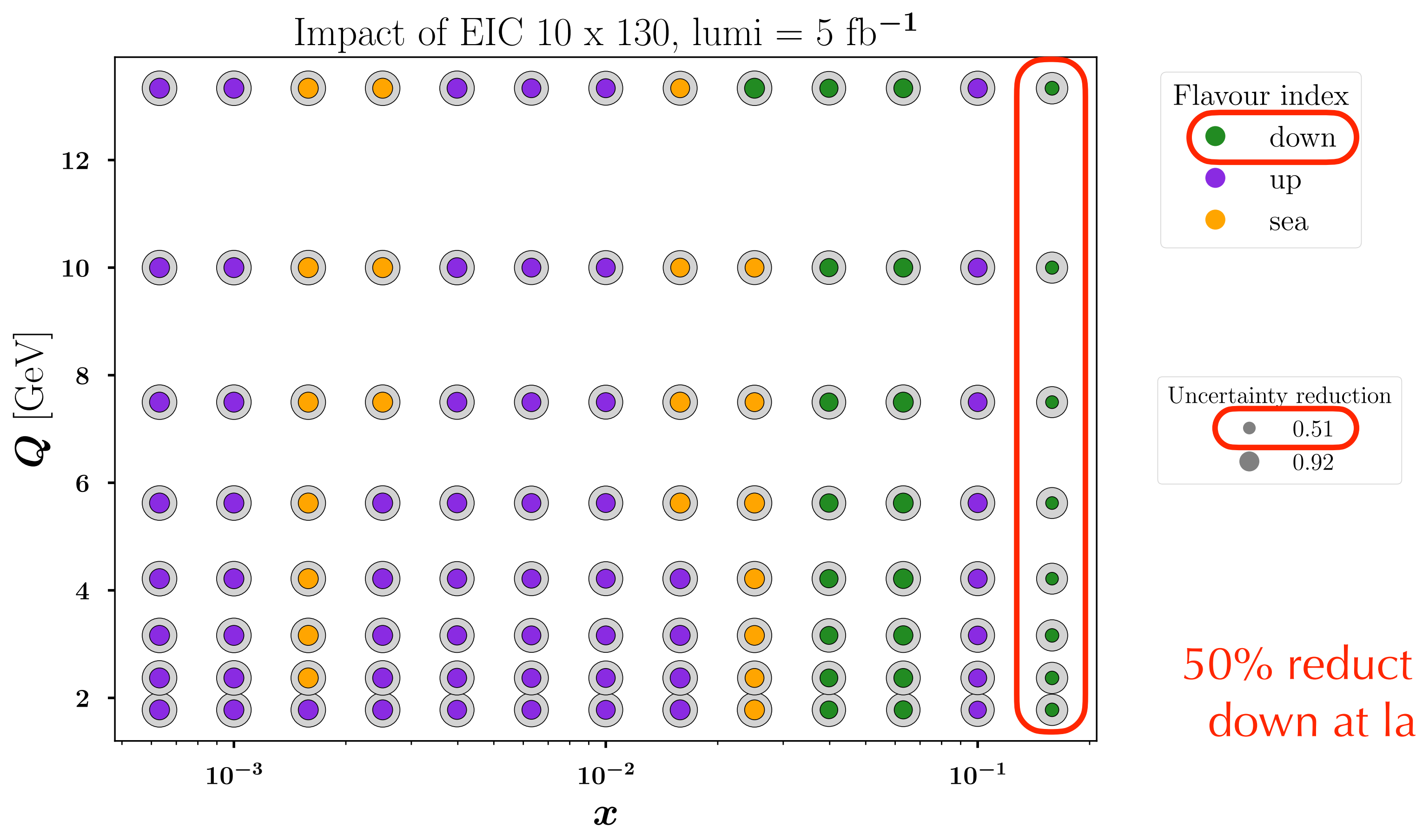
EIC impact in Early Science Conditions



For each (x,Q^2) bin:

- ○ from MAPTMD24, max. uncertainty of $f_1^q(x,k_T;Q)$ over all k_T and all flavors q
- ◉ including EIC pseudodata, color code indicates the flavor with max. reduction in uncertainty over all k_T

EIC impact in Early Science Conditions



50% reduction on down at large x

For each (x,Q^2) bin:

- ○ from MAPTMD24, max. uncertainty of $f_1^q(x,k_T;Q)$ over all k_T and all flavors q
- ◉ including EIC pseudodata, color code indicates the flavor with max. reduction in uncertainty over all k_T

The EIC impact with 10x130 at x=0.16

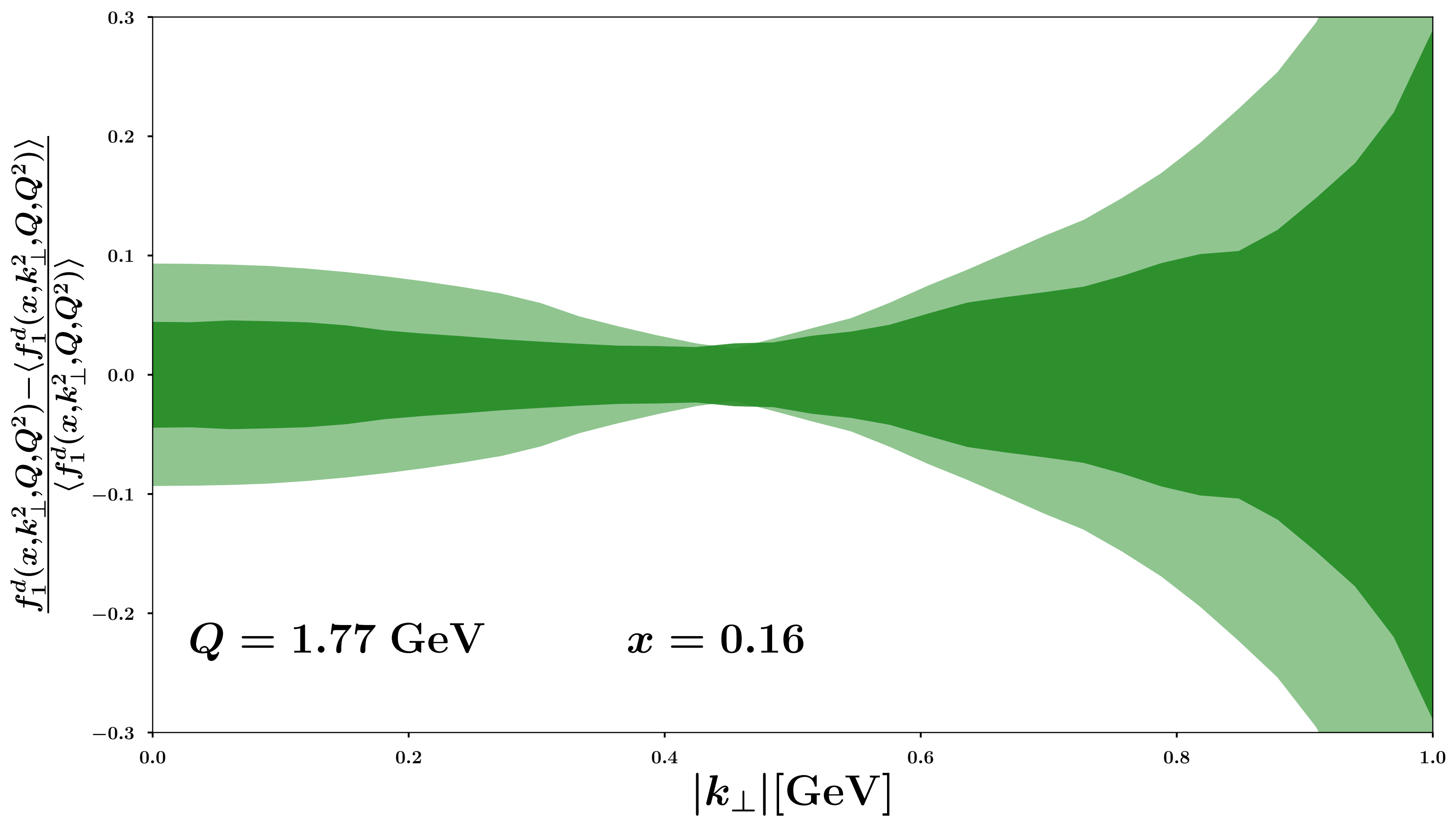
MAPTMD24 2031

EIC	**# pts.**	**lumi [fb^{-1}]**
10x130	~1620	5

(early Science conditions)

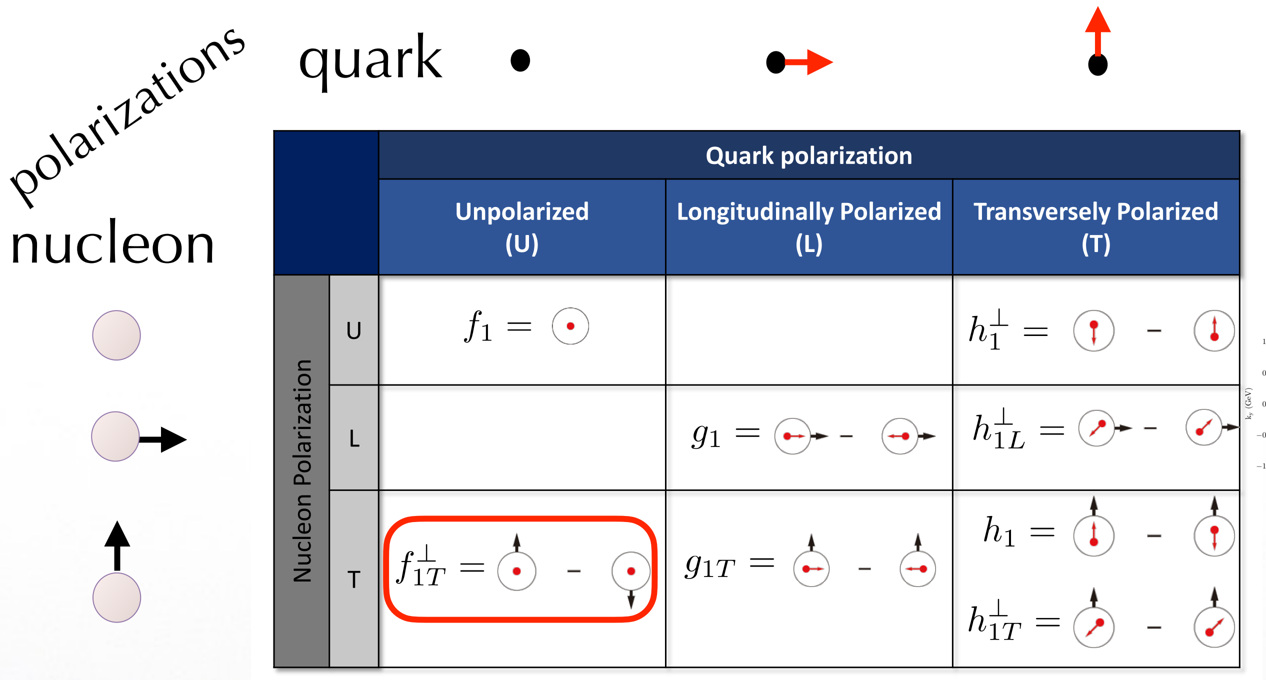
$$\frac{\mathrm{TMD}^q - \langle \mathrm{TMD}^q \rangle}{\langle \mathrm{TMD}^q \rangle}$$

x=0.16, Q=1.77 GeV



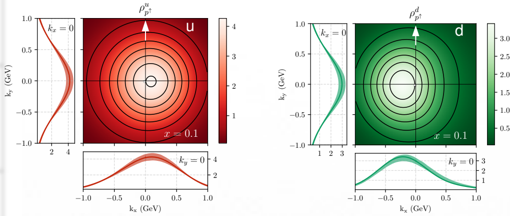
courtesy L. Rossi

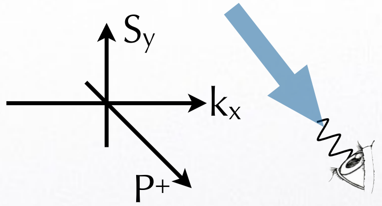
The Sivers TMD PDF



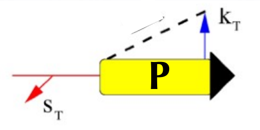
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ −
	L		$g_1 =$ −	$h_{1L}^\perp =$ −
	T	$f_{1T}^\perp =$ −	$g_{1T} =$ −	$h_1 =$ − $h_{1T}^\perp =$ −

Bacchetta et al.,
P.L. ***B827*** *(22) 136961,*
arXiv:2004.14278





$$\frac{1}{2}\mathrm{Tr}[\Phi\,\gamma_+] \to f_1 - f_{1T}^\perp \frac{(\mathbf{k}_\perp \times \mathbf{S}_T)\cdot \hat{\mathbf{P}}}{M}$$



$$\mathbf{S_T} \cdot \mathbf{k}_\perp \times \mathbf{P}$$

Sivers effect: how the momentum distribution of quarks is distorted by the transverse polarization of parent nucleon ("spin-orbit" correlation)

Sivers $f_{1T}^\perp$ → indirect access to quark orbital angular momentum

Burkardt, P.R. D**66** (2002) 114005;
N.P. **A735** (2004) 185
Bacchetta & Radici, P.R.L. **107** (2011) 212001
Ji et al., N.P. **B652** (2003) 383

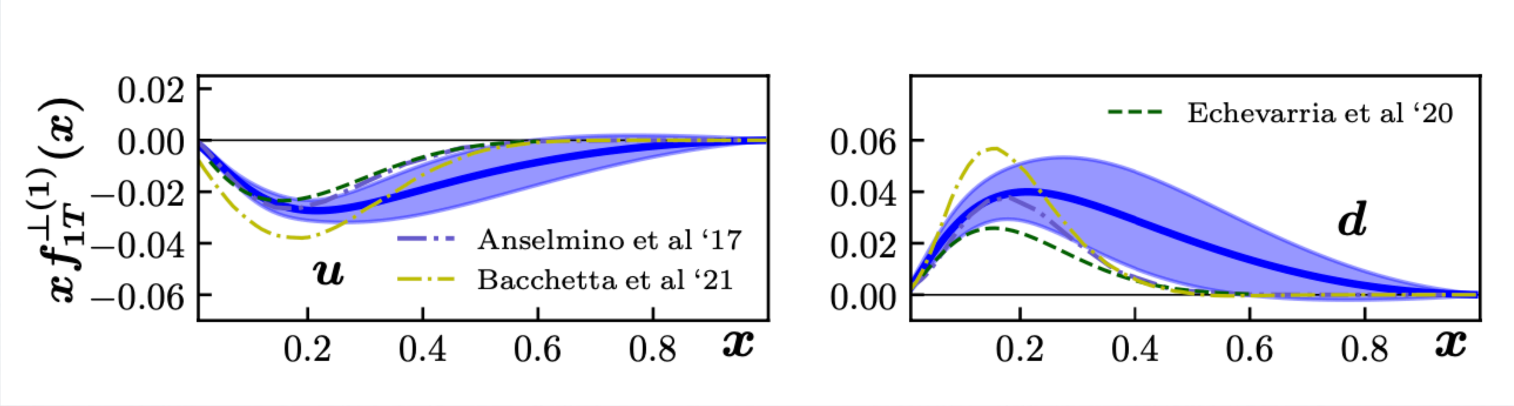
Most recent Sivers extractions

	Framework	SIDIS	DY	W/Z production	forward EM jet	e+e-	N. of points	χ^2/N	
JAM 2020 arXiv:2002.08384	generalized parton model	✔	✔	✔	✘	✔	517	1.04	
PV 2020 arXiv:2004.14278	LO+NLL	✔	✔	✔	✘	✘	125	1.08	
EKT 2020 arXiv:2009.10710	NLO+N^{2}LL	✔	✔	✔	✘	✘	226/452	0.99 /1.45	SIDIS / +STAR
BPV 2020 arXiv:2012.05135 arXiv:2103.03270	ζ prescription	✔	✔	✔	✘	✘	76	0.88	
TO-CA 2021 arXiv:2101.03955	generalized parton model	✔	✘	✘	✔	✘	238	$1.05^{+0.03}_{-0.01}$	SIDIS + reweighting
JAM 2022 arXiv:2205.00999	generalized parton model	✔	✔	✔	✘	✘	255	1.10	+ A_N^π data
Fernando-Keller arXiv:2304.14328	generalized parton model	✔	✔	✘	✘	✘	732	1.66	first using Neural Networks

lower accuracy and less data w.r.t. unpolarized TMD

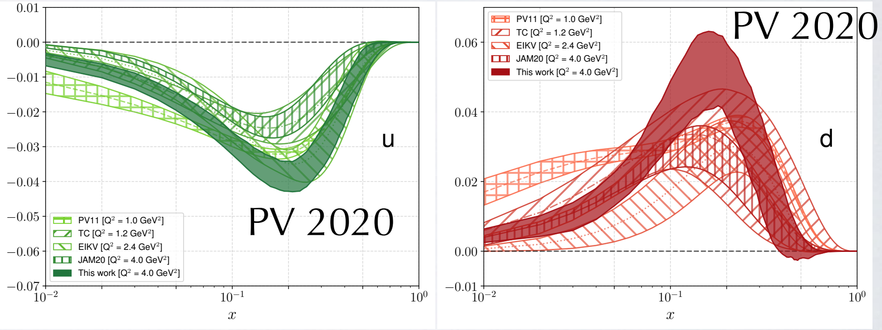
Most recent Sivers extractions

first k_T-moment $f_{1T}^{\perp(1)}(x)$



all parametrizations are in fair agreement for x-dependence of valence flavors

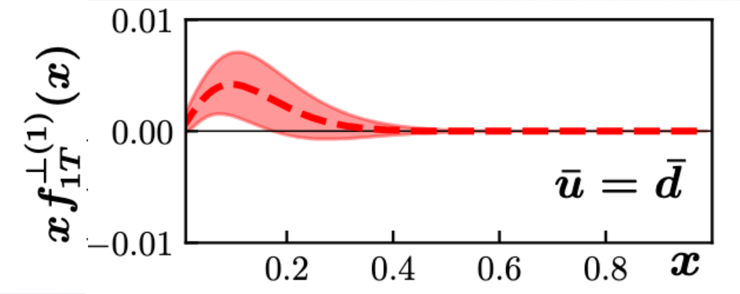
k_T-dependence is still much unconstrained



Bacchetta et al., P.L. ***B827*** *(22) 136961 arXiv:2004.14278*

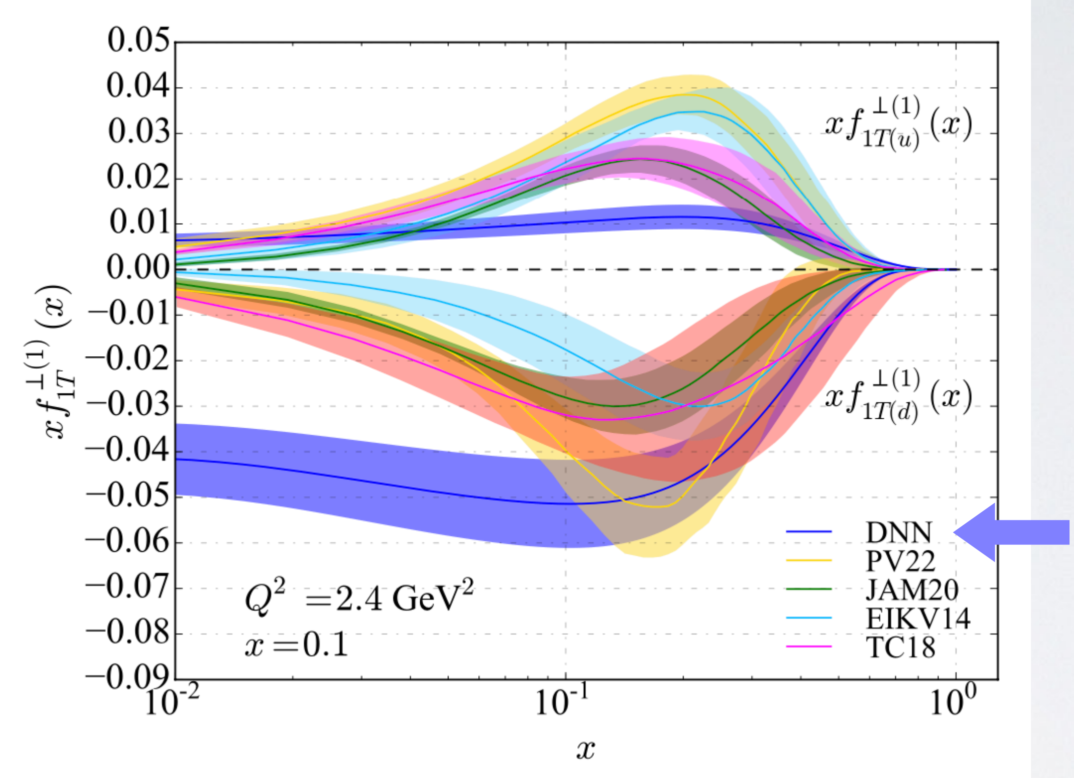
sea-quarks ~ $O(10^{-3})$ smaller, large errors

=> impact of EIC



Sivers extraction using Neural Networks

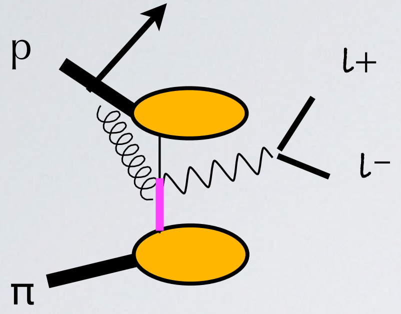
first k$_T$-moment $f_{1T}^{\perp(1)}(x)$



*Fernando & Keller, P.R.D**108** (23) 054007 arXiv:2304.14328*

but limited analysis: - parton model => no TMD evolution
- no consistent knowledge of unpolarized TMD in denominator of spin asymmetry

Sign change puzzle

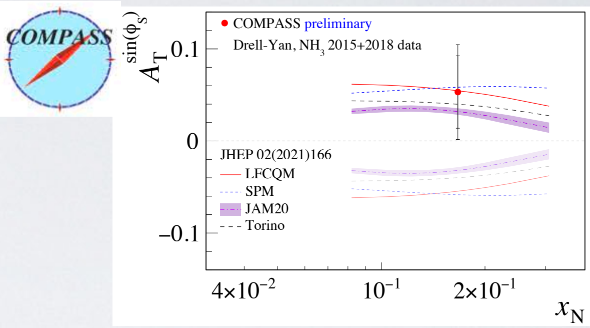


π-p↑ Drell-Yan

spin asymmetry

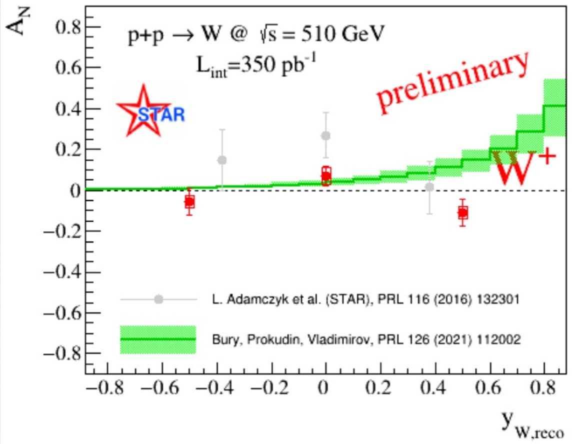
$A_T \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$

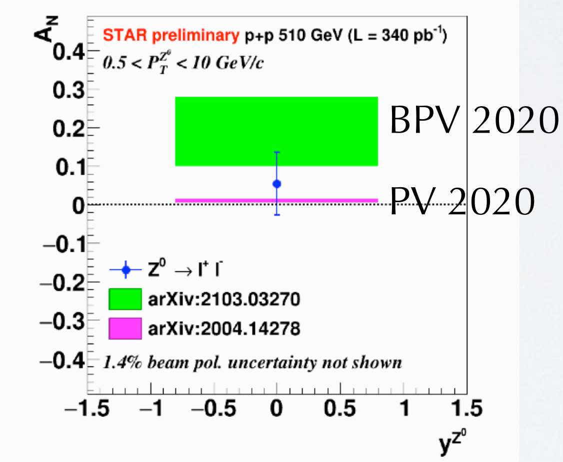
Aghasyan et al., P.R.L. ***119*** *(17) 112002*



sign change

no sign change



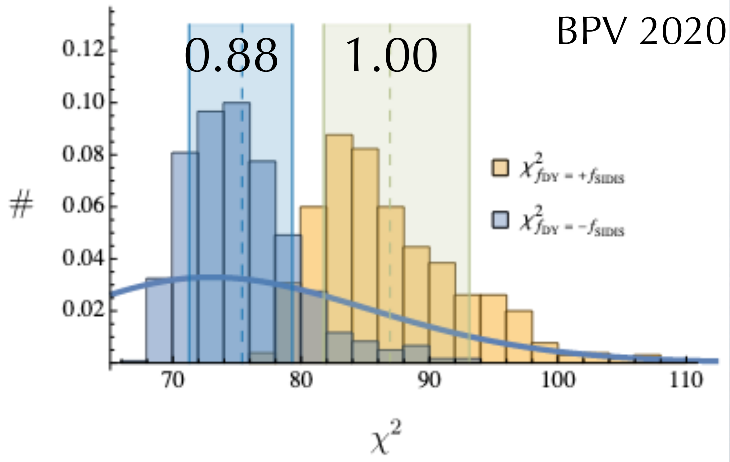


p-p↑ → W+X

p-p↑ → Z⁰+X

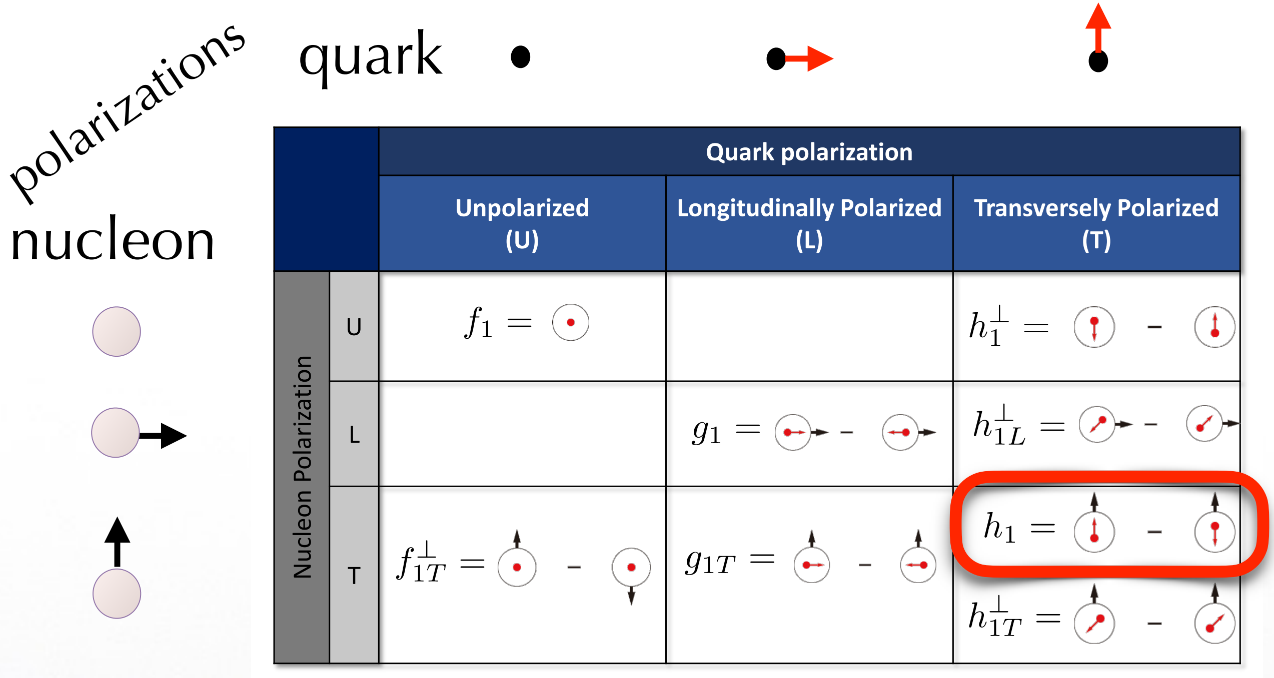
$A_N \sim f_{1,p} \otimes f_{1T,p}^{\perp}$

Adamczyk et al., P.R.L. ***116*** *(16) 132301*



still not enough to confirm sign change ?

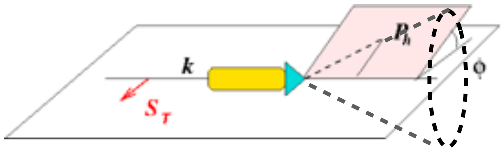
Transversity



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ -
	L		$g_1 =$ -	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ -	$g_{1T} =$ -	$h_1 =$ - $h_{1T}^\perp =$ -

- transversity is the prototype of chiral-odd structures
- the only chiral-odd structure that survives in collinear kinematics
- only way to determine the tensor charge $\delta^q(Q^2) = \int_0^1 dx\, h_1^{q-\bar{q}}(x, Q^2)$

Analyzers of transversity at leading twist



Collins effect

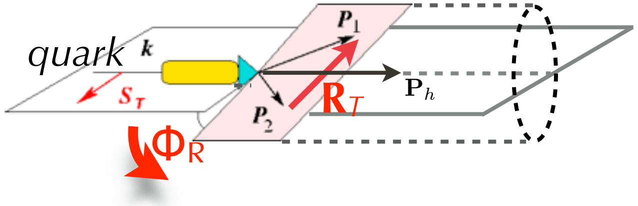
$S_T\cdot\mathbf{k}\times\mathbf{P}_{hT}$

Collins, N.P. **B396** (93) 161

$\propto\ h_1(x,k_\perp)\otimes\underline{H_1^\perp(z,P_\perp)}$

TMD framework

SIDIS



di-hadron mechanism

$S_T\cdot\mathbf{P}_2\times\mathbf{P}_1=S_T\cdot\mathbf{P}_h\times\mathbf{R}_T$

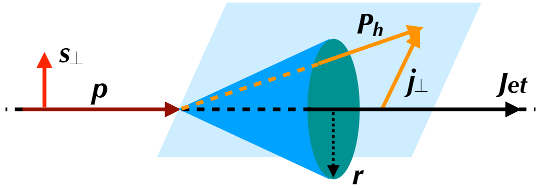
Collins et al., N.P. **B420** (94)

$\propto\ h_1(x)\,\underline{H_1^{\sphericalangle}(z,R_T^2\propto M_{h_1h_2}^2)}$

collinear framework

SIDIS

p p↑



hadron-in-jet Collins effect

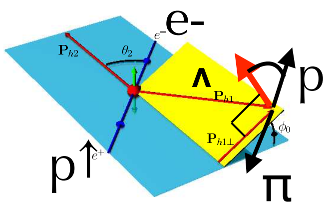
$j_T^2\ll Q^2=(P_T^{jet})^2$

Yuan, P.R.L. **100** (08)

$\propto\ h_1(x)\,[C(z,\mu)\otimes\underline{H_1^\perp(z_h,j_T,P_T^{\rm jet}r)}]$

hybrid framework

SIDIS p p↑



Λ spin transfer

Jaffe, P.R. D**54** (96)

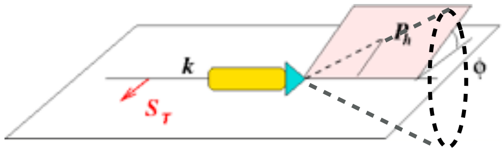
$\propto\ h_1(x)\,\underline{H_1(z)}$

collinear framework

SIDIS

p p↑

Analyzers of transversity at leading twist



Collins effect

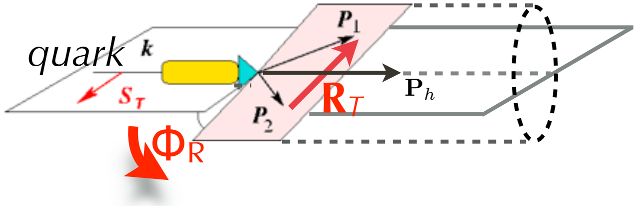
$\mathbf{S}_T \cdot \mathbf{k} \times \mathbf{P}_{hT}$

Collins, N.P. **B396** (93) 161

$\propto\ h_1(x, k_\perp) \otimes \underline{H_1^\perp(z, P_\perp)}$

TMD framework

SIDIS



di-hadron mechanism

$\mathbf{S}_T \cdot \mathbf{P}_2 \times \mathbf{P}_1 = \mathbf{S}_T \cdot \mathbf{P}_h \times \mathbf{R}_T$

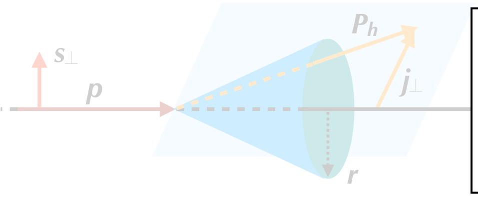
Collins et al., N.P. **B420** (94)

$\propto\ h_1(x)\, \underline{H_1^{\sphericalangle}(z, R_T^2 \propto M_{h_1 h_2}^2)}$

collinear framework

SIDIS

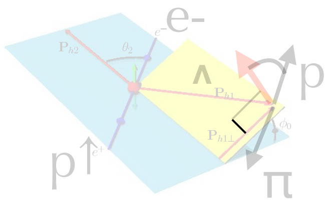
p p↑



mechanisms used so far
to extract transversity from data

$\underline{H_1^\perp(z_h, j_T, P_T^{\mathrm{jet}} r)]}$

SIDIS p p↑



Λ spin transfer

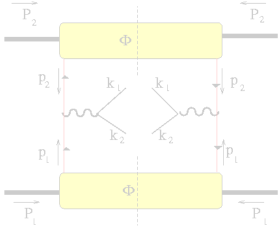
Jaffe, P.R. D**54** (96)

$\propto\ h_1(x)\, \underline{H_1(z)}$

collinear framework

SIDIS

p p↑



single-polarised Drell-Yan

Boer, P.R. D**60** (99)

$\propto\ \underline{h_1^\perp(x_1, k_{\perp 1})} \otimes h_1(x_2, k_{\perp 2})$

TMD framework

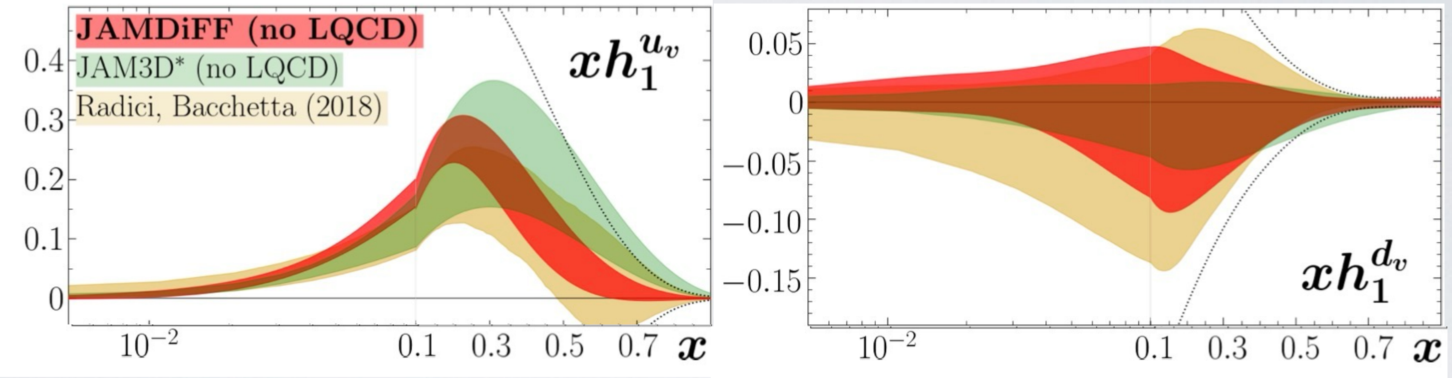
π p↑

Most recent extractions

Collins effect	Framework	e+e-	SIDIS	Drell-Yan A_N	Lattice
Anselmino 2015 P.R. **D92** (15) 114023	parton model	✔	✔	✘	✘
Kang et al. 2016 P.R. **D93** (16) 014009	TMD / CSS	✔	✔	✘	✘
Lin et al. 2018 P.R.L. **120** (18) 152502	parton model	✘	✔	✘	✔ g_T
D'Alesio et al. 2020 (CA) P.L. **B803** (20) 135347	parton model	✔	✔	✘	✘
JAM3D-20 P.R. **D102** (20) 054002	parton model	✔	✔	✔	✘
JAM3D-22 P.R. **D106** (22) 034014	parton model	✔	✔	✔	✔ g_T
Boglione et al. 2024 (TO) P.L. **B854** (24) 138712	parton model	✔	✔	✔ reweighting	✘

Dihadron mechanism	e+e- unpol. $d\sigma^0$	e+e- asymmetry	SIDIS	p-p collisions	Lattice
Radici & Bacchetta 2018 P.R.L. **120** (18) 192001	PYTHIA (separately)	✔ (separately)	✔	✔	✘
Benel et al. 2020 E.P.J. **C80** (20) 5	PYTHIA (separately)	✔ (separately)	✔	✘	✘
JAMDIFF 2024 P.R.L. **132** (24) 091901	✔	✔	✔	✔	✔ δu, δd

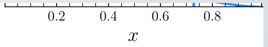
Transversity



D. Pitonyak, QCD Evolution 24

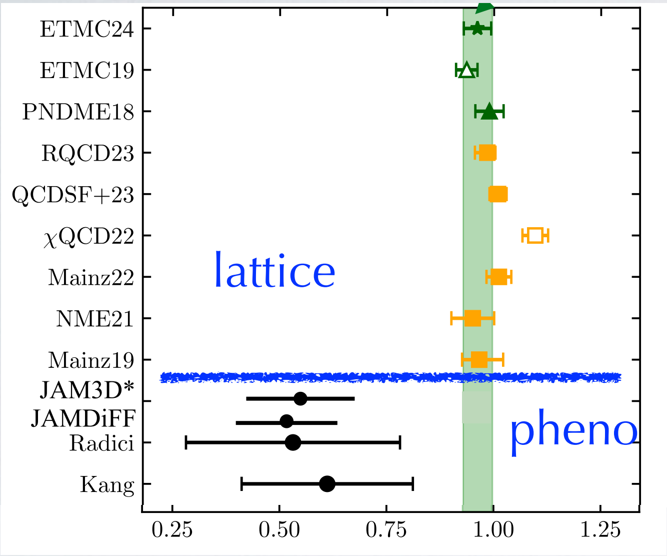
* JAM3D includes $\bar{u} = -\bar{d}$ w.r.t. JAM22

consistency of phenomenological extractions from a variety of exp. data with different approaches
(provided that no LQCD points are included in the fit)

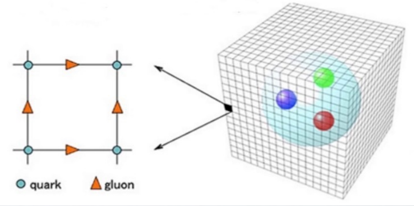


Pheno - lattice : tensor charge

$g_T = \delta u - \delta d$



adapted from C. Alexandrou, QCD Evolution 24

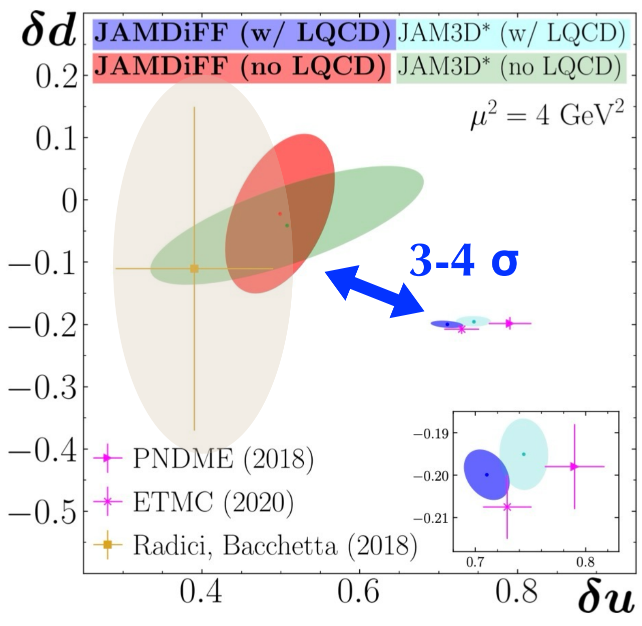


green N_f=2+1+1

open symbols = no continuum extrapolation

yellow N_f=2+1

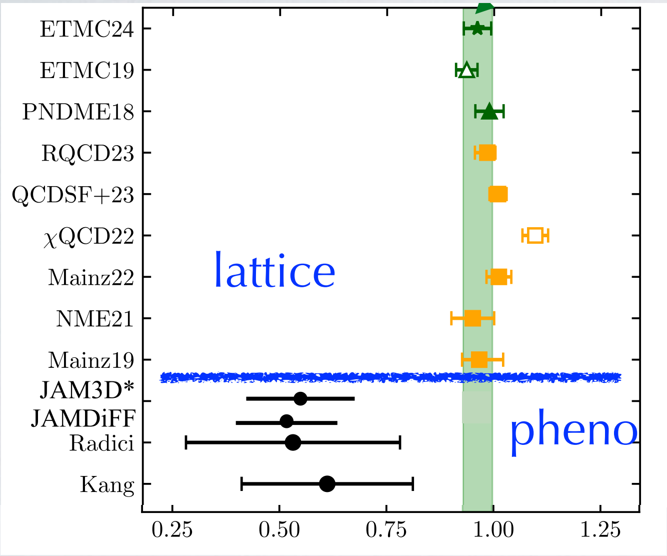
tension between pheno and lattice ?



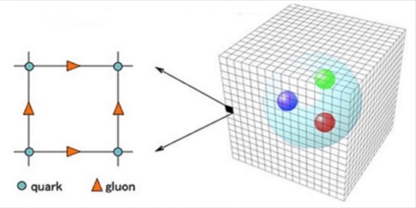
adapted from D. Pitonyak, QCD Evolution 24

Pheno - lattice : tensor charge

$g_T = \delta u - \delta d$



adapted from C. Alexandrou, QCD Evolution 24



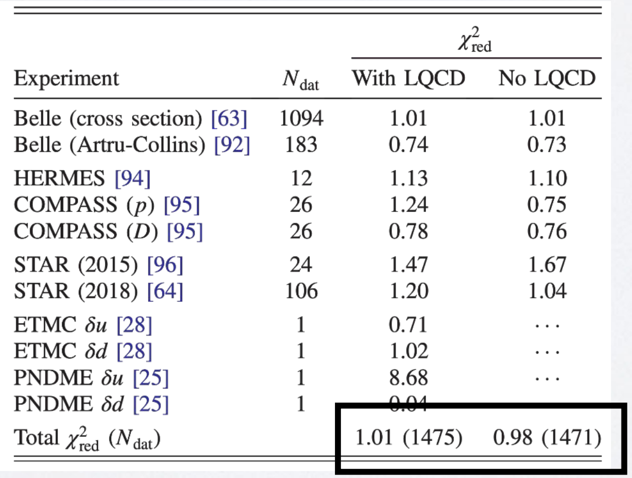
green N_f=2+1+1

open symbols = no continuum extrapolation

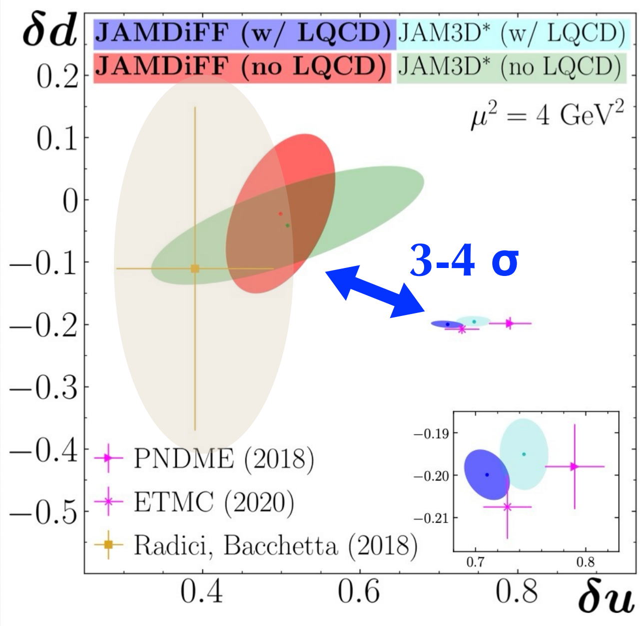
yellow N_f=2+1

tension between pheno and lattice ?

Including lattice data,
JAM finds **compatibility,**
still under discussion…



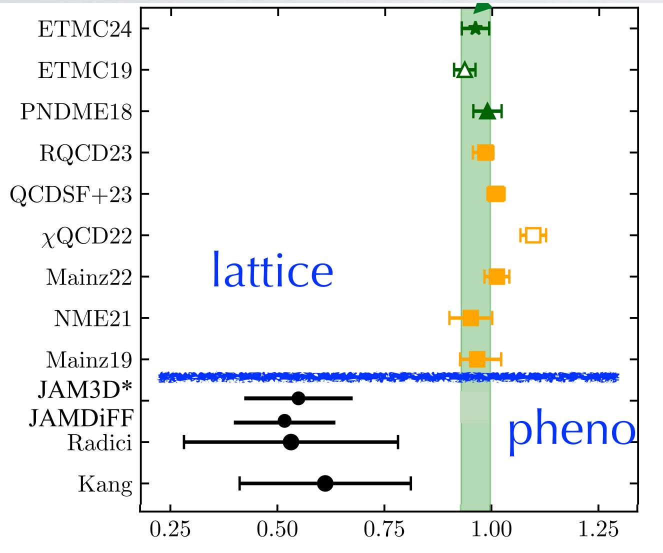
Experiment	N_{dat}	χ^2_{red}	
		With LQCD	No LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [94]	12	1.13	1.10
COMPASS (p) [95]	26	1.24	0.75
COMPASS (D) [95]	26	0.78	0.76
STAR (2015) [96]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	…
ETMC δd [28]	1	1.02	…
PNDME δu [25]	1	8.68	…
PNDME δd [25]	1	0.04	…
Total χ^2_{red} (N_{dat})		1.01 (1475)	0.98 (1471)



adapted from D. Pitonyak, QCD Evolution 24

Pheno - lattice : tensor charge

$g_T = \delta u - \delta d$

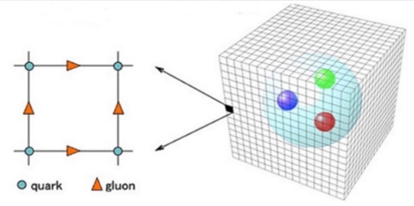


adapted from C. Alexandrou, QCD Evolution 24

green N_f=2+1+1

open symbols = no continuum extrapolation

yellow N_f=2+1

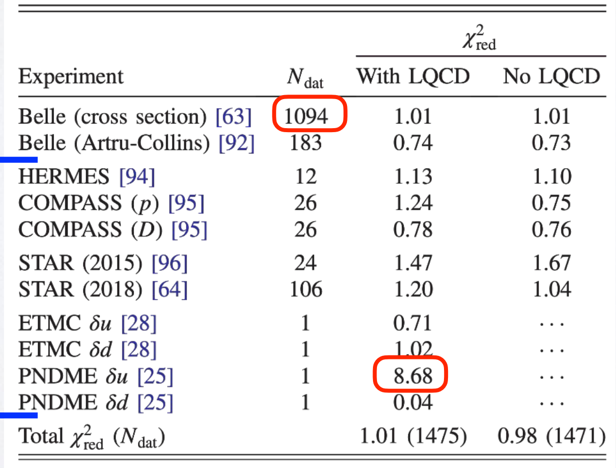


tension between pheno and lattice ?

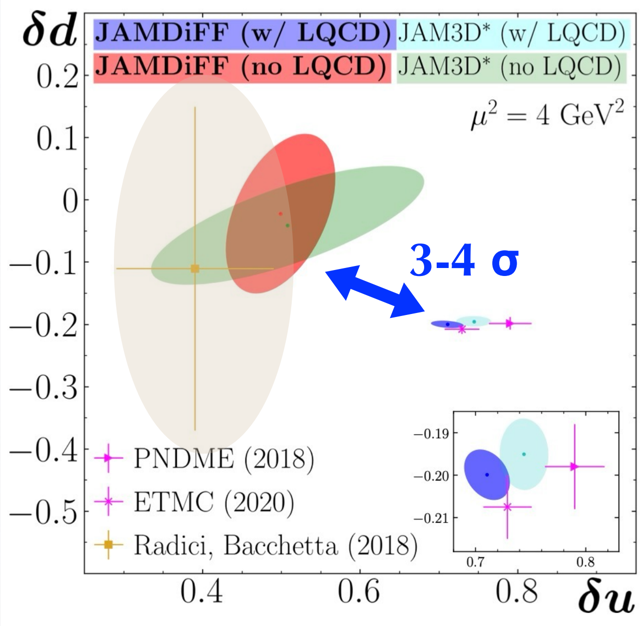
Including lattice data,
JAM finds **compatibility,**
still under discussion…

But most data
insensitive to tensor charge

For data sensitive to δu, δd
$\chi^2 = 203 \rightarrow 239$
$\chi^2/N_{dat} = 1.02 \rightarrow 1.21$



Experiment	N_{dat}	χ^2_{red}	
		With LQCD	No LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [94]	12	1.13	1.10
COMPASS (p) [95]	26	1.24	0.75
COMPASS (D) [95]	26	0.78	0.76
STAR (2015) [96]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	…
ETMC δd [28]	1	1.02	…
PNDME δu [25]	1	8.68	…
PNDME δd [25]	1	0.04	…
Total χ^2_{red} (N_{dat})		1.01 (1475)	0.98 (1471)



adapted from D. Pitonyak, QCD Evolution 24

Future

New data already available:

- Compass SIDIS spin asymmetry on deuteron target with Collins effect & di-hadron mechanism S. Asatryan, DIS 2024

COMPASS Alexeev *et al.*, arXiv:2401.00309

- updated Hermes SIDIS spin asymmetry Airapetian *et al.*, JHEP **12** (20) 010

$$p^{\uparrow} + p \to \Lambda^{\uparrow} + X$$

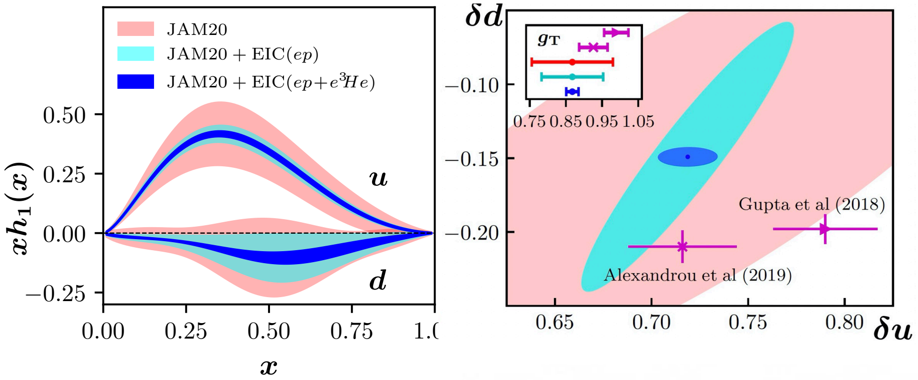
- Compass π-p$^{\uparrow}$ Drell-Yan A$_T$ asymmetry Alexeev *et al.*, arXiv:2312.17379
- STAR asymmetry in $p^{\uparrow} + p \to \text{jet} + \pi^{\pm} + X$ hadron-in-jet Collins effect

X. Chu, DIS 2024

- STAR asymmetry in $p^{\uparrow} + p \to \Lambda^{\uparrow} + X$ Λ spin transfer STAR, P.R. D**109** (24) 012004
- STAR asymmetry in $p^{\uparrow} + p \to \pi^{+}\pi^{-} + X$ di-hadron mechanism

B. Surrow, DIS 2024

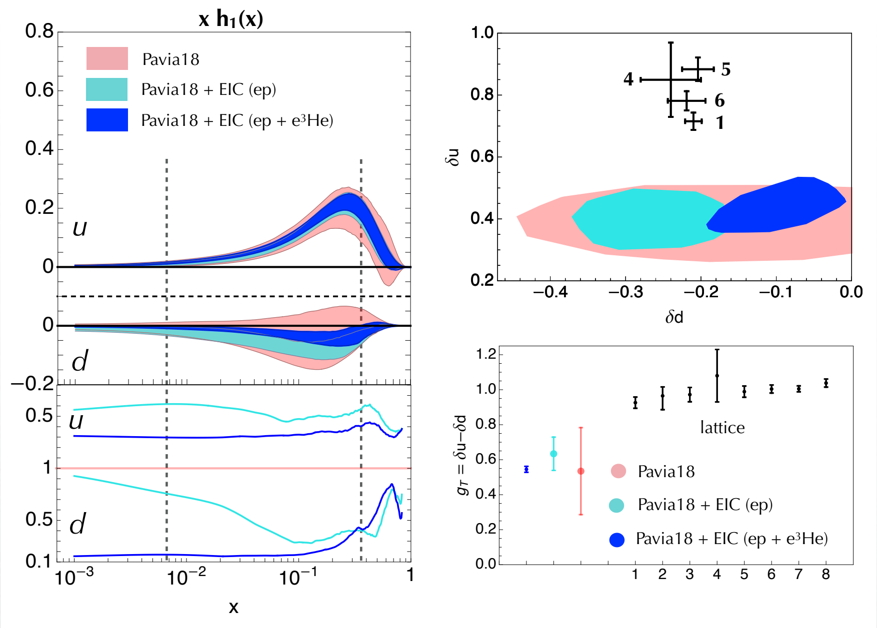
EIC impact on tensor charge



Collins effect

🟦 proton 🟦 + ^{3}He

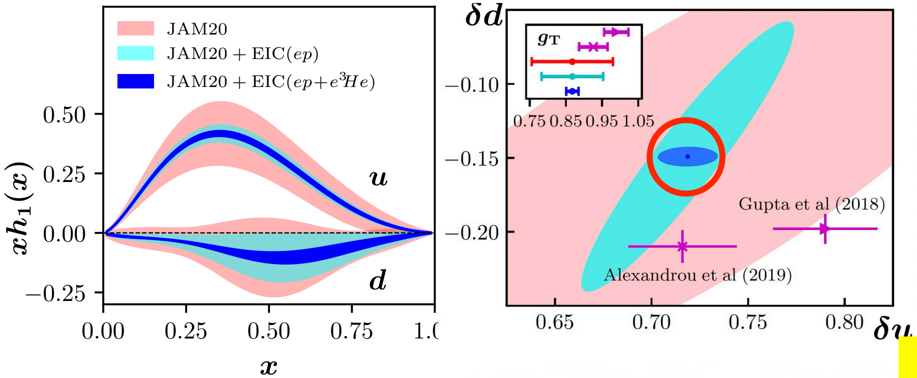
Abdul-Khalek *et al.*
(EIC Yellow Report),
N.P. **A1026** (22) 122447



di-hadron mechanism

1)	ETMC '19	*Alexandrou et al., arXiv:1909.00485*
2)	Mainz '19	*Harris et al., P.R. D**100** (19) 034513*
3)	LHPC '19	*Hasan et al., P.R. D**99** (19) 114505*
4)	JLQCD '18	*Yamanaka et al., P.R. D**98** (18) 054516*
5)	PNDME '18	*Gupta et al., P.R. D**98** (18) 034503*
6)	ETMC '17	*Alexandrou et al., P.R. D**95** (17) 114514; (E) P.R. D**96** (17) 099906*
7)	RQCD '14	*Bali et al., P.R. D**91** (15) 054501*
8)	LHPC '12	*Green et al., P.R. D**86** (12) 114509*

EIC impact on tensor charge

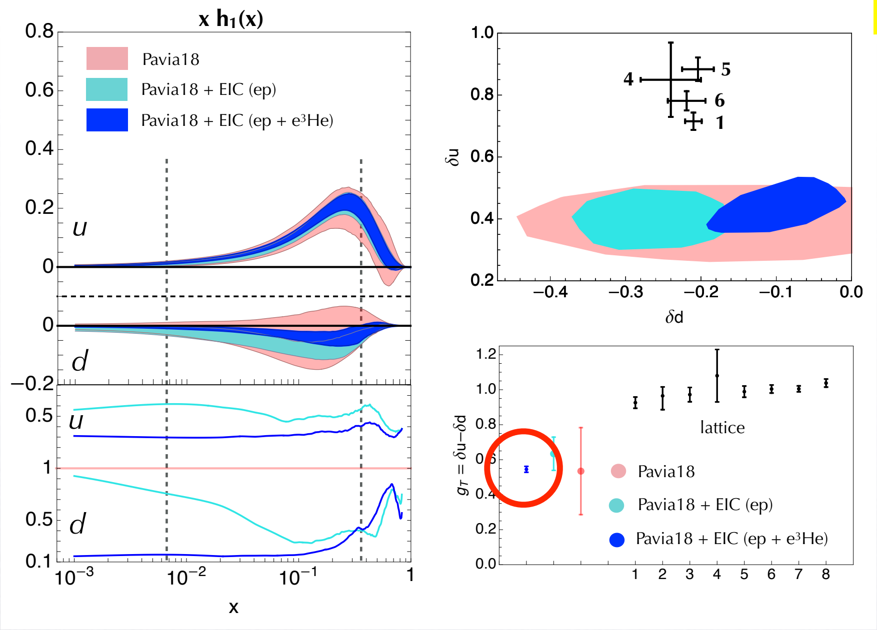


Collins effect

🟦 proton   🟦 + ³He

Abdul-Khalek *et al.*
(EIC Yellow Report),
N.P. **A1026** (22) 122447

expected precision close to (or higher than) lattice



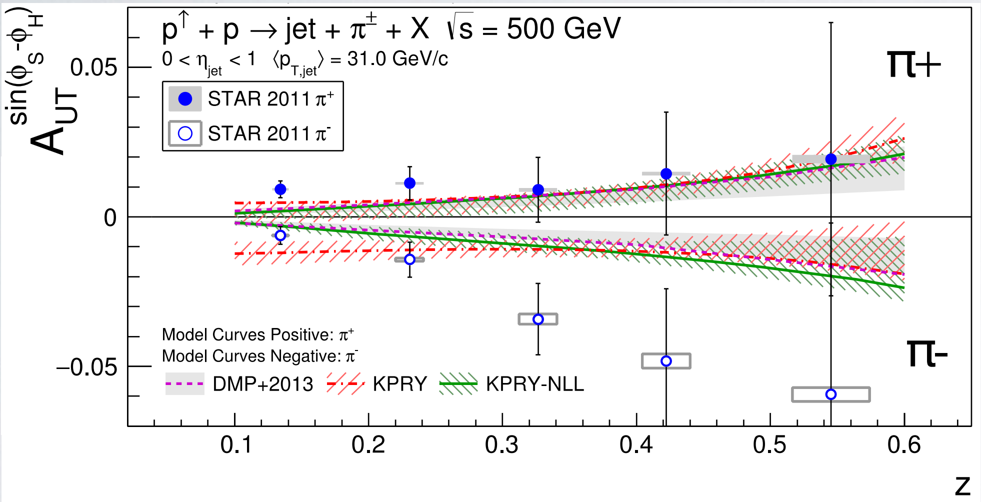
di-hadron mechanism

1)	ETMC '19	*Alexandrou et al., arXiv:1909.00485*
2)	Mainz '19	*Harris et al., P.R. D**100** (19) 034513*
3)	LHPC '19	*Hasan et al., P.R. D**99** (19) 114505*
4)	JLQCD '18	*Yamanaka et al., P.R. D**98** (18) 054516*
5)	PNDME '18	*Gupta et al., P.R. D**98** (18) 034503*
6)	ETMC '17	*Alexandrou et al., P.R. D**95** (17) 114514; (E) P.R. D**96** (17) 099906*
7)	RQCD '14	*Bali et al., P.R. D**91** (15) 054501*
8)	LHPC '12	*Green et al., P.R. D**86** (12) 114509*

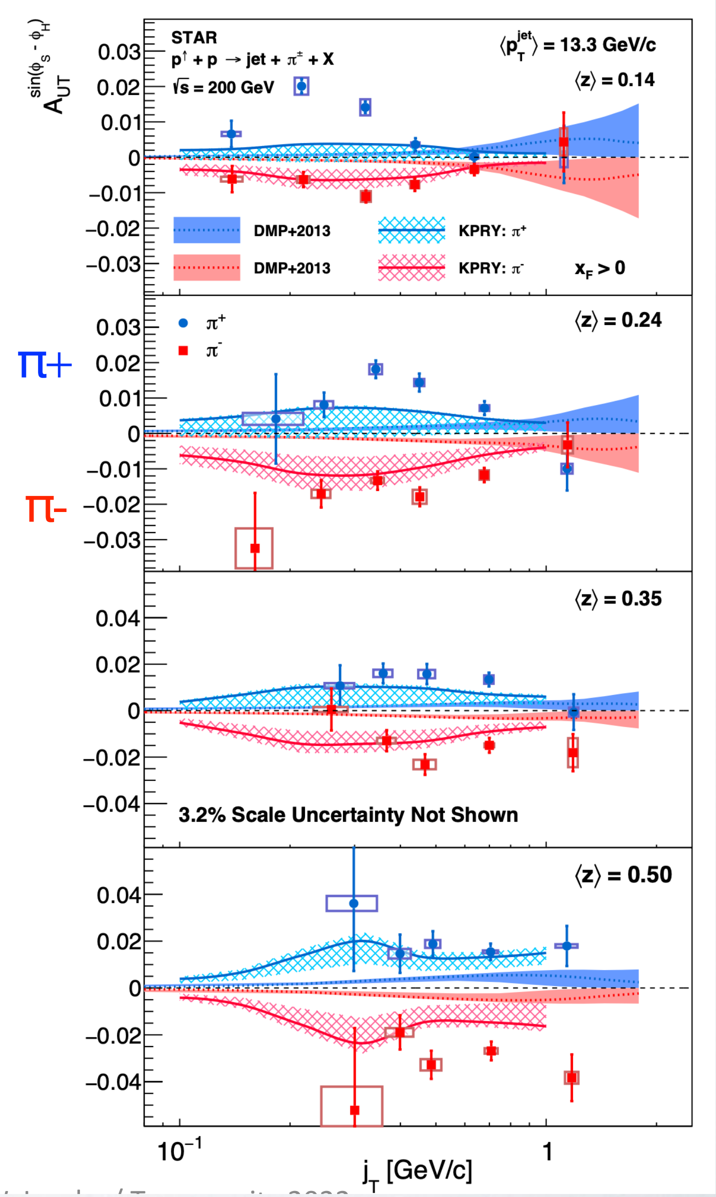
Hadron-in-jet Collins effect

$$p^{\uparrow} + p \rightarrow \text{jet} + \pi^{\pm} + X$$

M. Grosse-Perdekamp, Transversity 2022



STAR 2010-11 $\sqrt{s} = 500$ GeV



▨ D'Alesio et al., P.L. **B773** (17) 300 🟦 π+ 🟥 π-

no evolution ▨

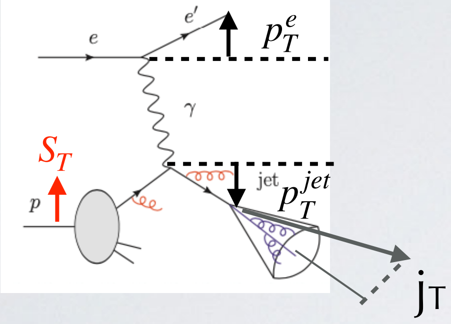
TMD evolution ▨

Kang et al., P.L. **B774** (17) 635 ▨ π+ ▨ π-

STAR 2012-15 $\sqrt{s} = 200$ GeV

Future: EIC impact

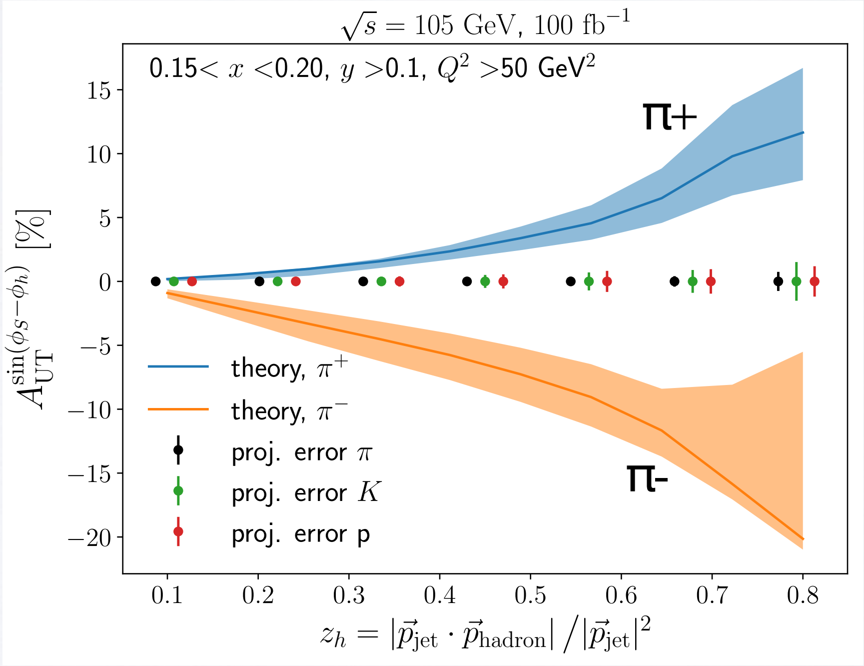
hadron-in-jet Collins effect



electron - hadron-in-jet azimuthal correlations

$|p_T^e + p_T^{\text{jet}}| \ll |p_T^e - p_T^{\text{jet}}|/2 \Rightarrow$ factorization theorem

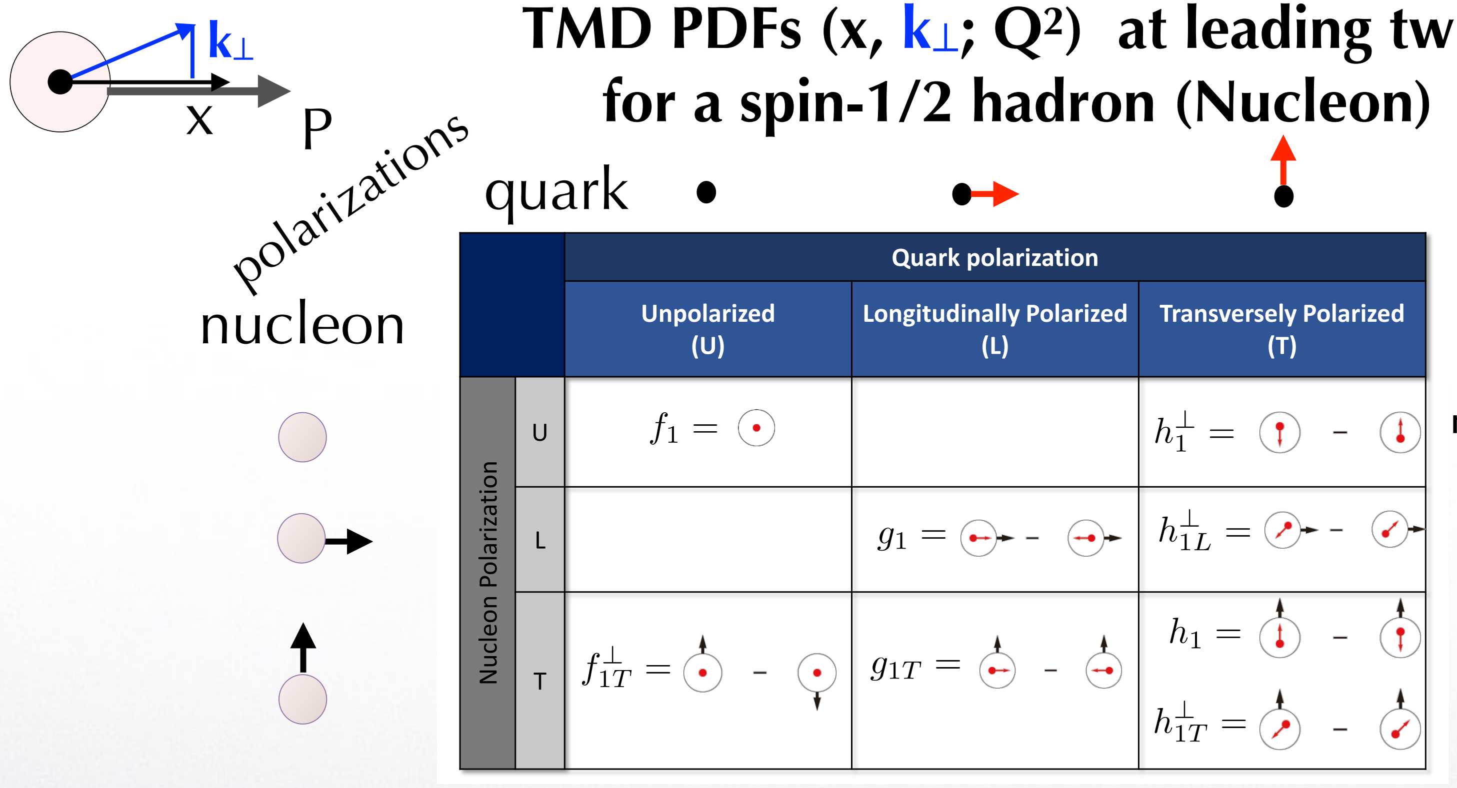
theory uncertainty
bands from



Arratia *et al.*, P.R. D**102** (20) 074015

Summary

TMD PDFs (x, k⊥; Q²) at leading twist for a spin-1/2 hadron (Nucleon)



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ ⊙		$h_1^{\perp} = \quad - $
	L		$g_1 = \quad - $	$h_{1L}^{\perp} = \quad - $
	T	$f_{1T}^{\perp} = \quad - $	$g_{1T} = \quad - $	$h_1 = \quad - $ $h_{1T}^{\perp} = \quad - $

nomenclature

no-name Boer-Mulders

helicity Kotzinian-Mulders

transversity

pretzelocity

Sivers worm gear

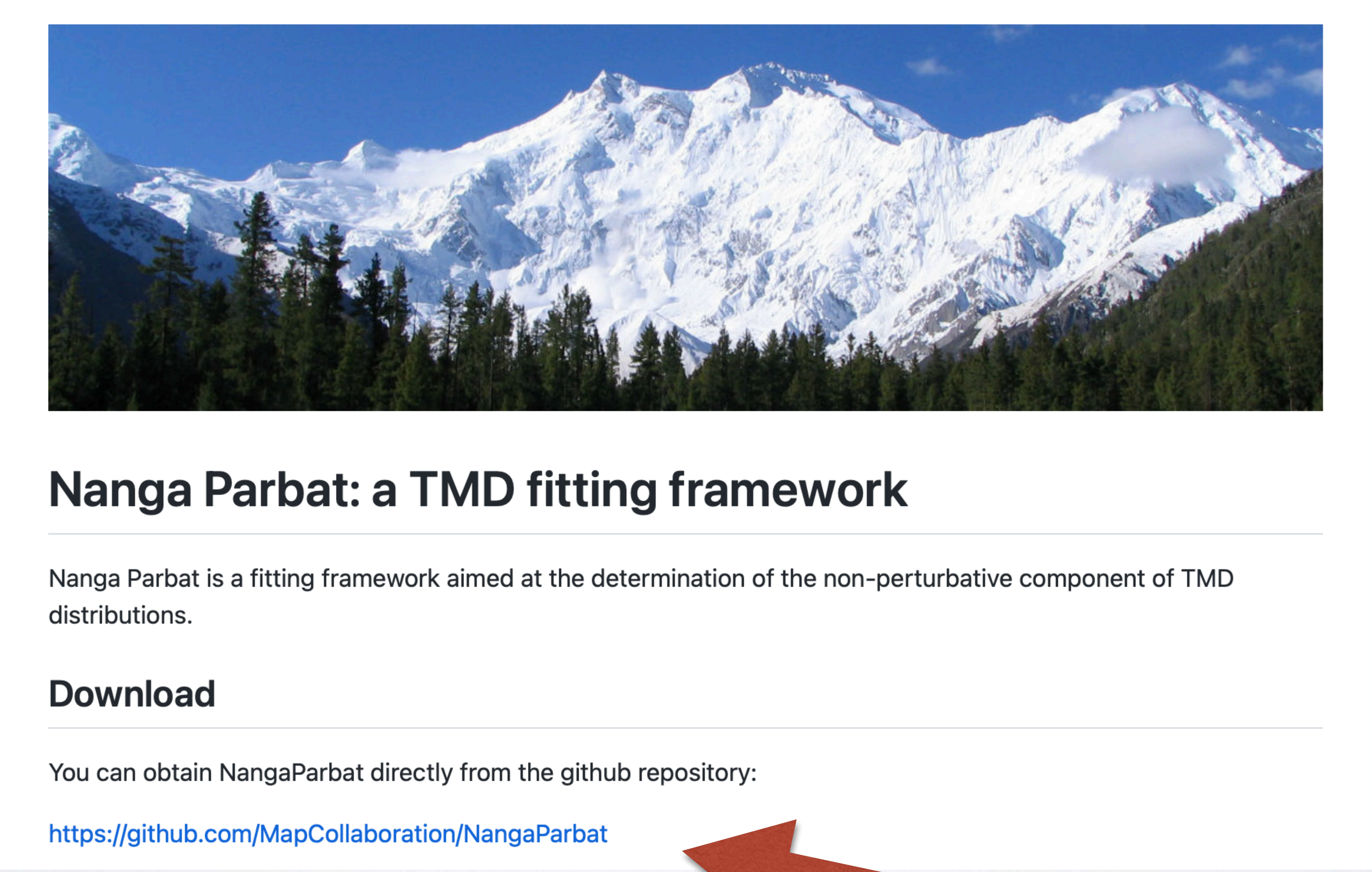
- very good knowledge of x-dependence of f_1 and g_1
- good knowledge of k$_T$-dependence of f_1
- fair knowledge of x-dependence of h_1 and k$_T$-moments of $f_{1T}^{\perp}$
- some hints about all others

List of latest extractions

Unpol. TMD	MAP 22 arXiv:2206.07598, ART23 2305.07473, MAP24 arXiv:2405.13833
Helicity	arXiv:2409.08110, MAP24, arXiv:2409.18078
Transversity	arXiv:1505.05589, arXiv:1612.06413, arXiv:2205.00999
Sivers	MAP20 arXiv:2004.14278, arXiv:2009.10710, arXiv:2103.03270, arXiv:2205.00999, arXiv:2304.14328
Boer-Mulders	arXiv:2004.02117, arXiv:2407.06277
Worm-gear g1T	arXiv:2110.10253, arXiv:2210.07268
Worm-gear h1L	
Pretzelosity	arXiv:1411.0580

not mentioned pion TMDs, TMD fragmentation functions, nuclear TMDs

The Nanga Parbat fitting framework



Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

The Artemide fitting framework

https://teorica.fis.ucm.es/artemide/

arTeMiDe

News

12 Dec 2019: Version 2.02 released (+manual update).

23 Feb 2019: Version 1.4 released (+manual update).

21 Jan 2019: Artemide now has a repository.

Archive of older links/news.

Articles, presentations & supplementary materials

Extra pictures for the paper arXiv:1902.08474

Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.

Link to the text in Inspire.

Archive of older links/news.

Download

Recent version/release can be found in repository

About us & Contacts

If you have found mistakes, or have suggestions/questions, please, contact us.

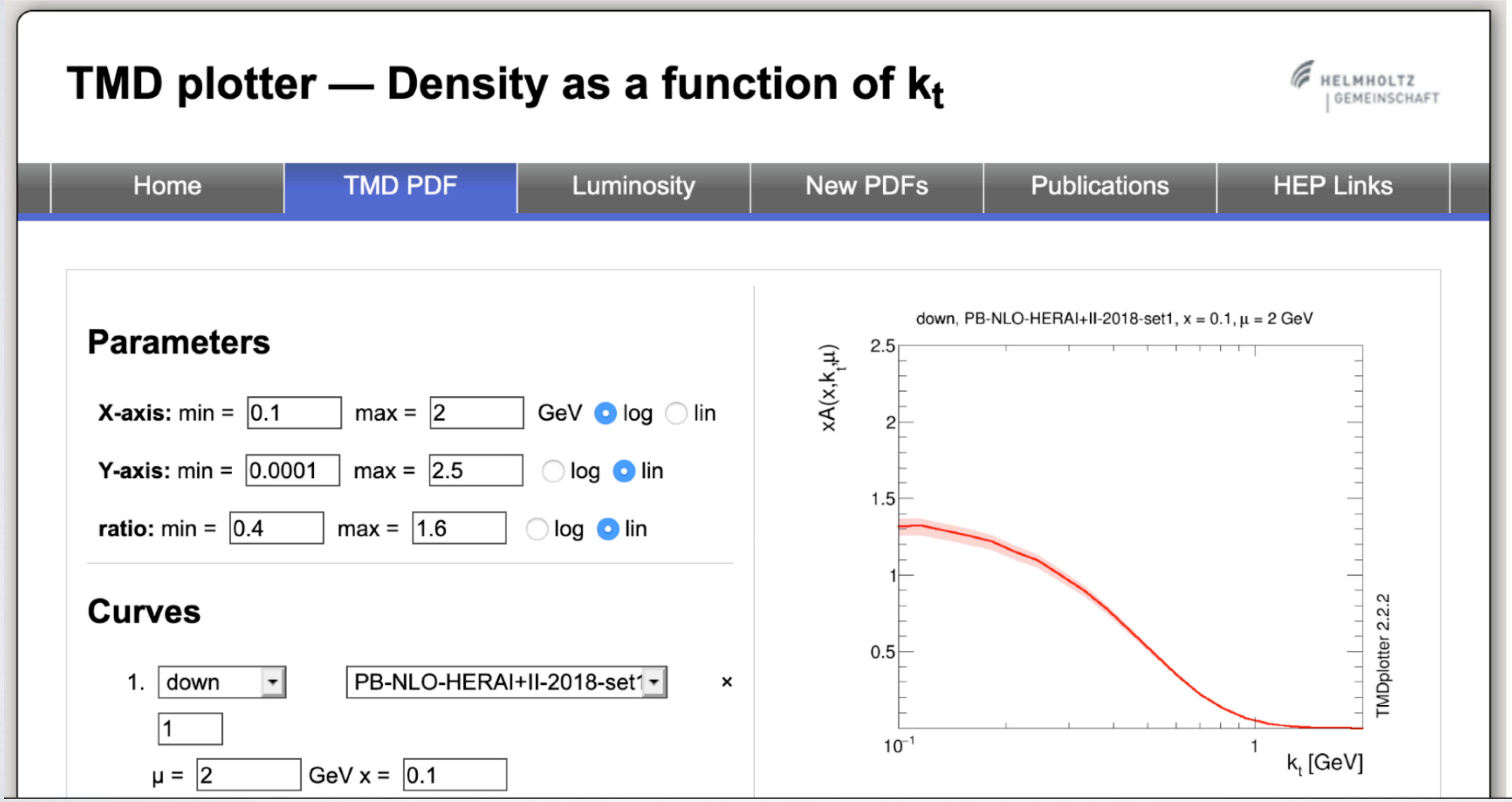
Some extra materials can be found on Alexey's web-page

Alexey Vladimirov Alexey.Vladimirov@physik.uni-regensburg.de

Ignazio Scimemi ignazios@fis.ucm.es

The TMDLib and TMDPlotter tools

https://tmdlib.hepforge.org/



TMD plotter — Density as a function of k_t

Parameters

X-axis: min = 0.1 max = 2 GeV ● log ○ lin

Y-axis: min = 0.0001 max = 2.5 ○ log ● lin

ratio: min = 0.4 max = 1.6 ○ log ● lin

Curves

1. down PB-NLO-HERAI+II-2018-set1 ×

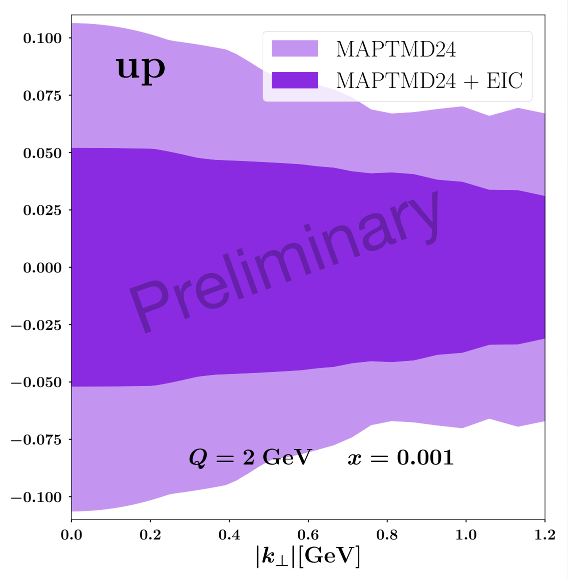
1

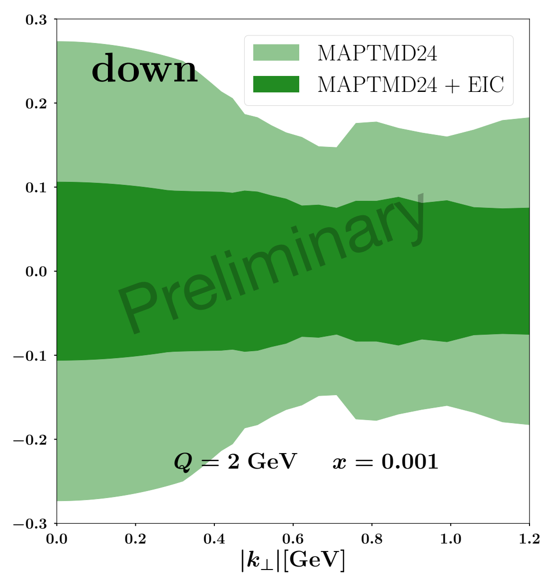
μ = 2 GeV x = 0.1

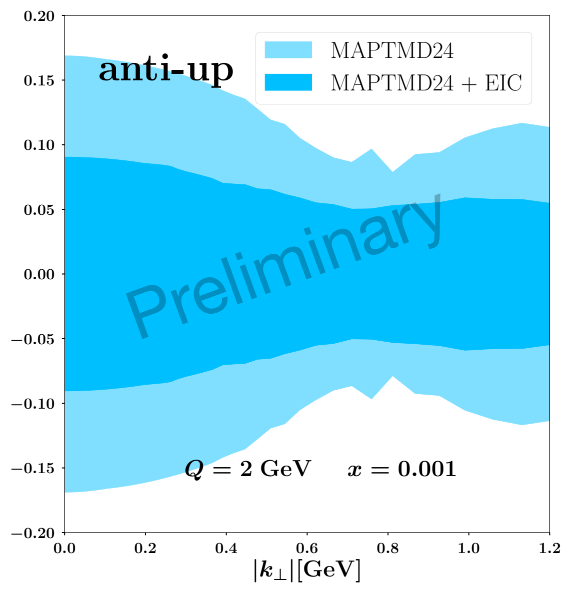
Outline

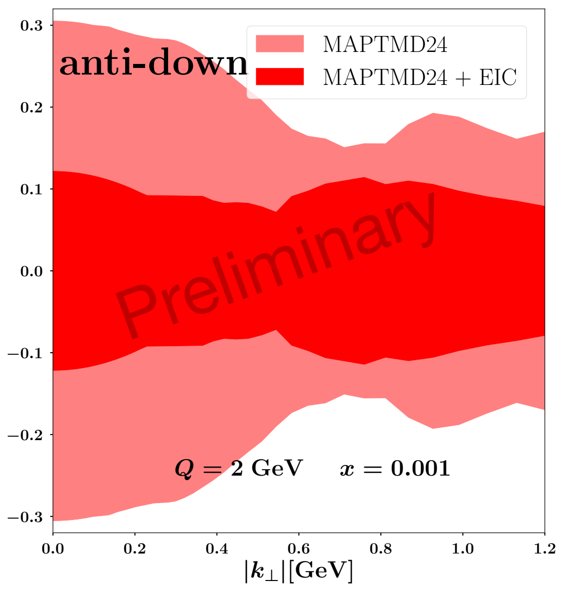
- Backup

The EIC impact at x=0.001









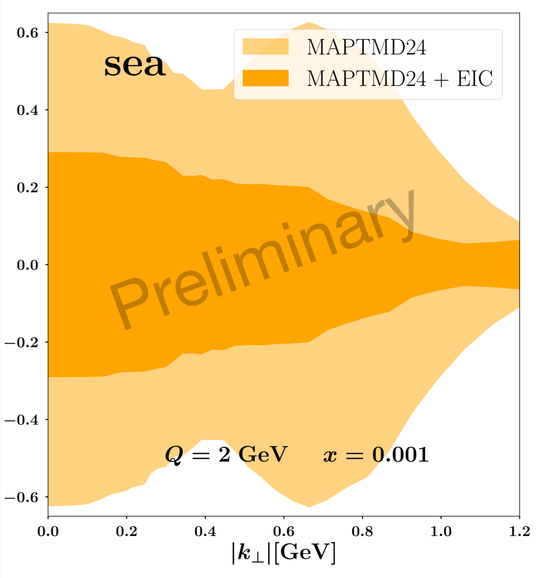
$$\frac{\mathrm{TMD}^q - \langle\mathrm{TMD}^q\rangle}{\langle\mathrm{TMD}^q\rangle}$$

x=0.001

MAPTMD24 2031

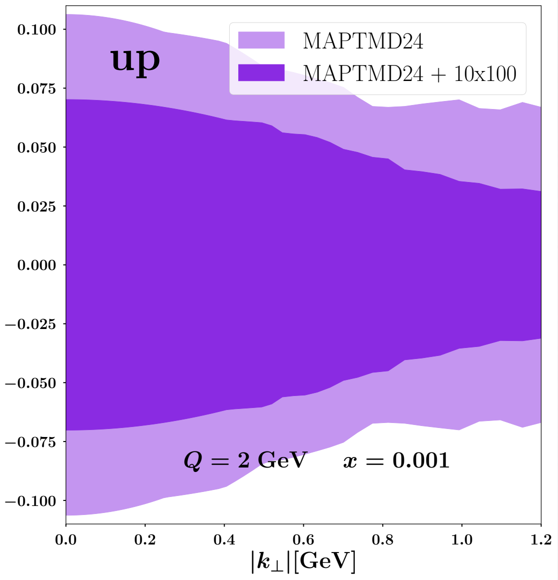
EIC	**# pts.**	**lumi [fb^{-1}]**
5x41	1273	2.85
10x100	1611	51.3
18x275	1648	10

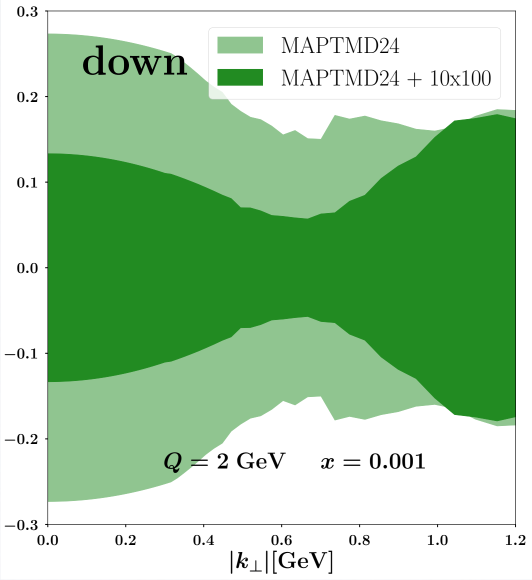
(simulation campaign of May 2024)



L. Rossi, Ph.D. Thesis

The EIC impact with 10x100 at x=0.001



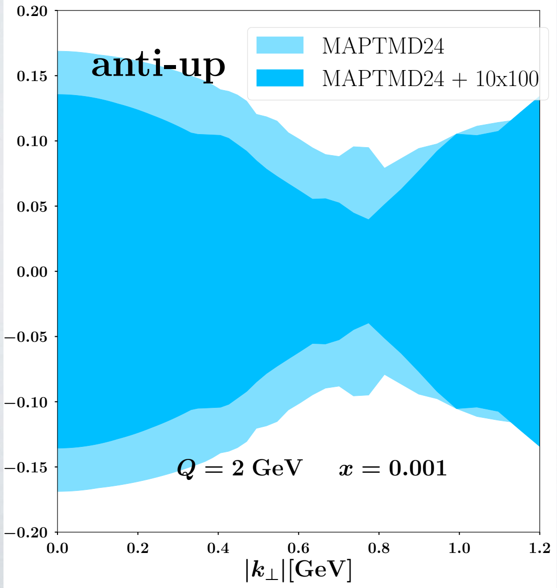


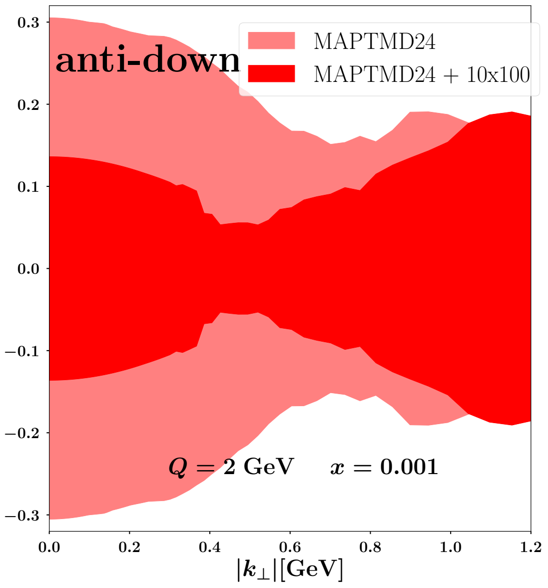
$$\frac{\text{TMD}^q - \langle\text{TMD}^q\rangle}{\langle\text{TMD}^q\rangle}$$

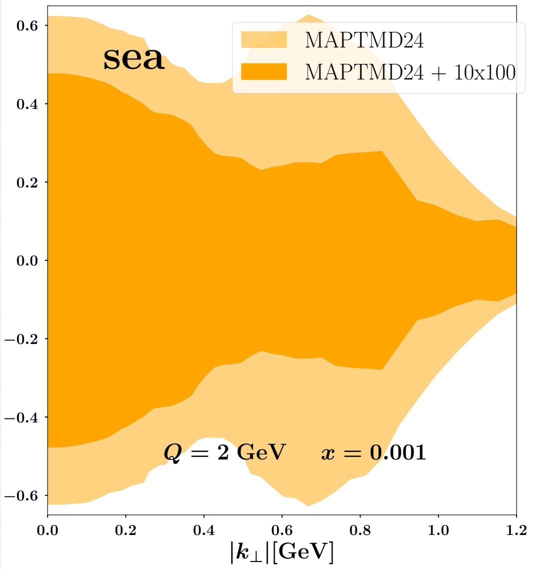
x=0.001

MAPTMD24	2031	
EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	51.3

(simulation campaign of May 2024)

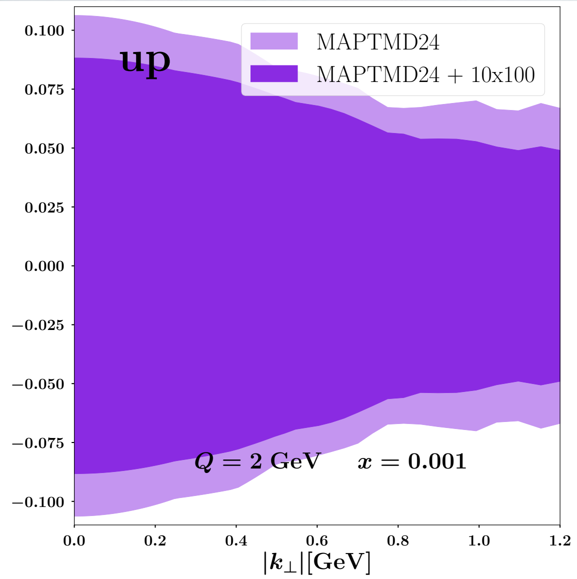


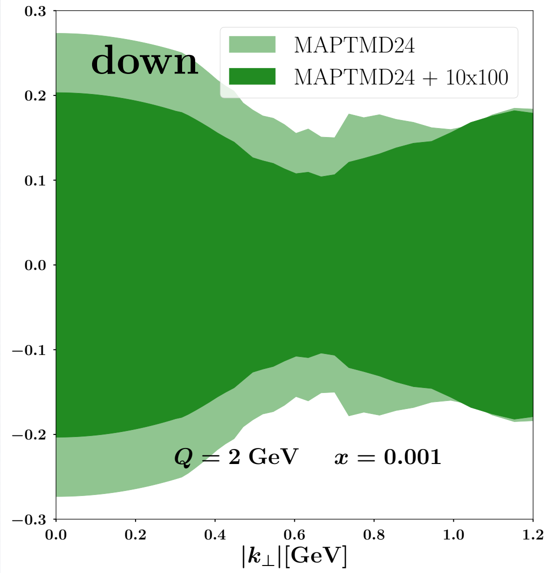




L. Rossi, Ph.D. Thesis

The EIC impact with 10x100 at x=0.001



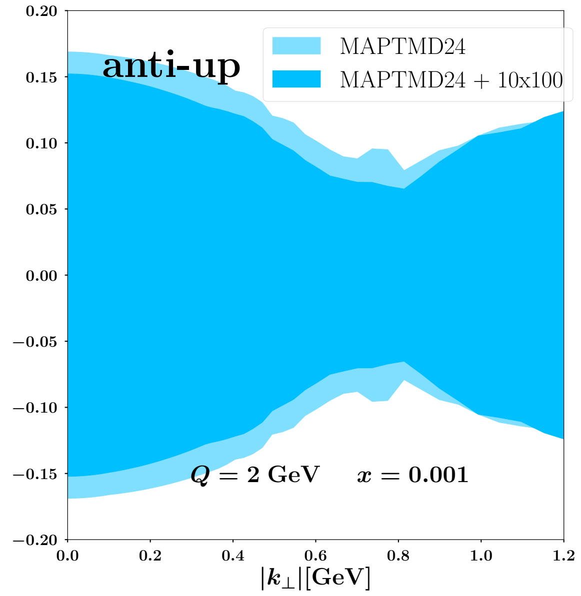


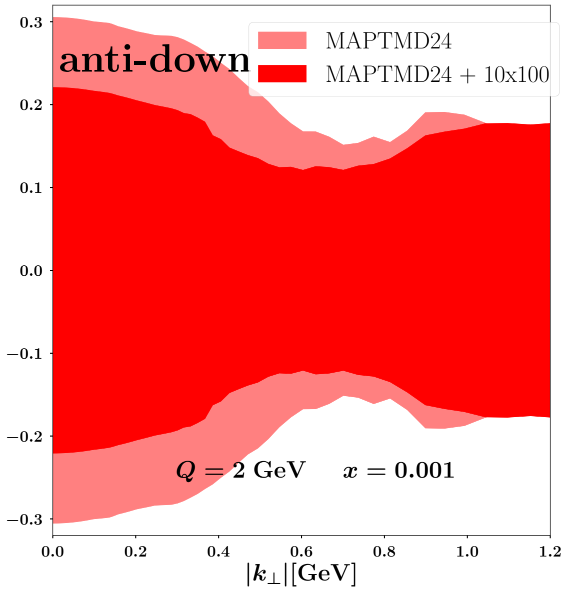
$$\frac{\mathrm{TMD}^q - \langle \mathrm{TMD}^q \rangle}{\langle \mathrm{TMD}^q \rangle}$$

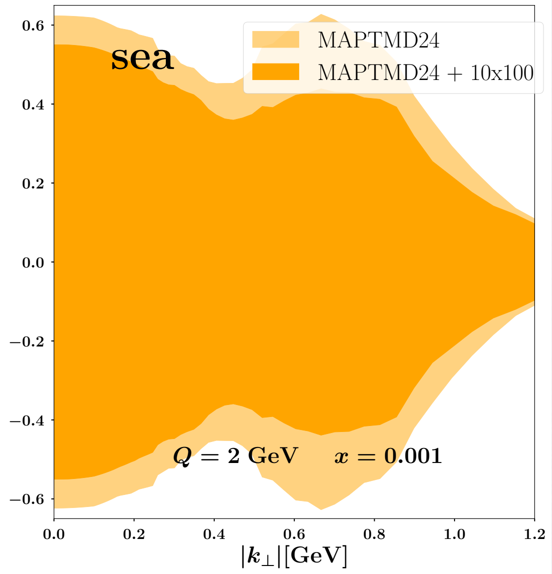
x=0.001

MAPTMD24	2031	
EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	5

(early Science conditions)

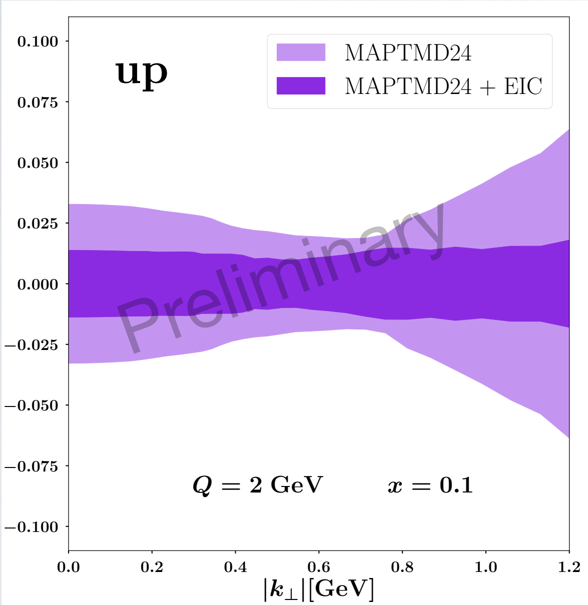


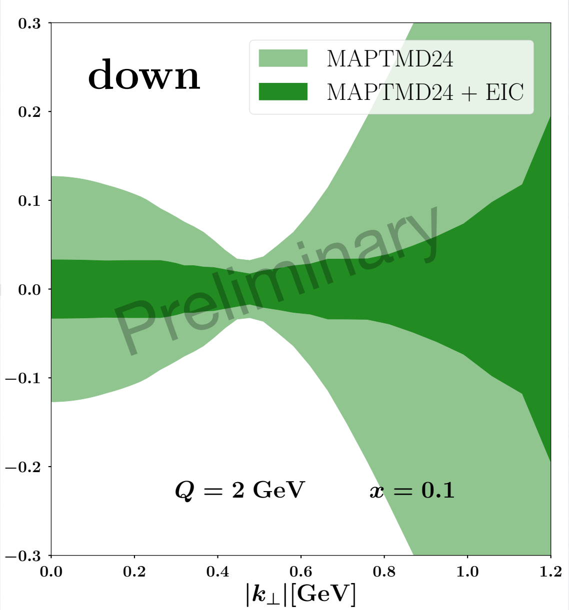


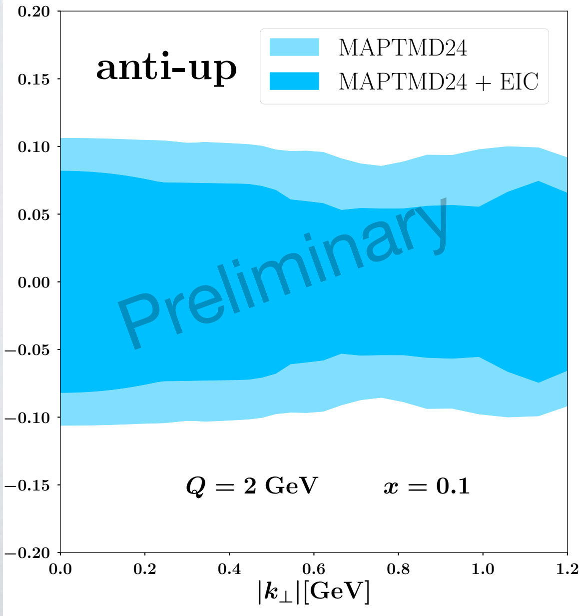


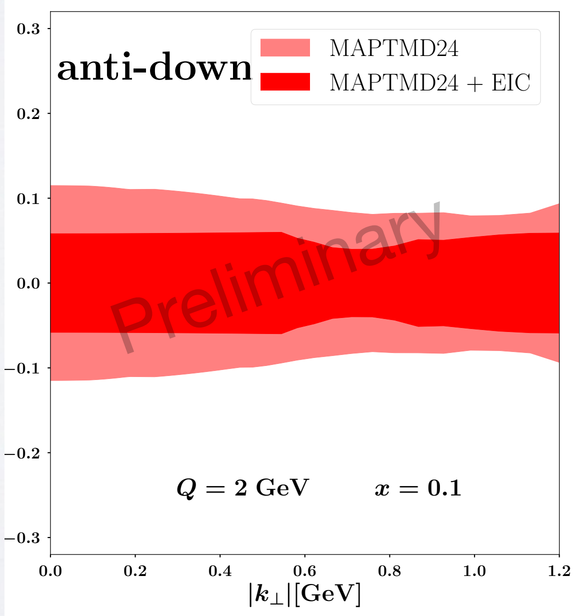
courtesy L. Rossi

The EIC impact at x=0.1







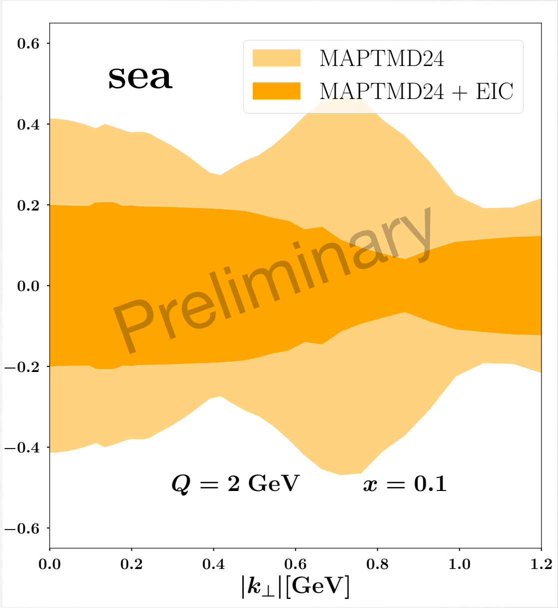


$$\frac{\text{TMD}^q - \langle\text{TMD}^q\rangle}{\langle\text{TMD}^q\rangle}$$

x=0.1

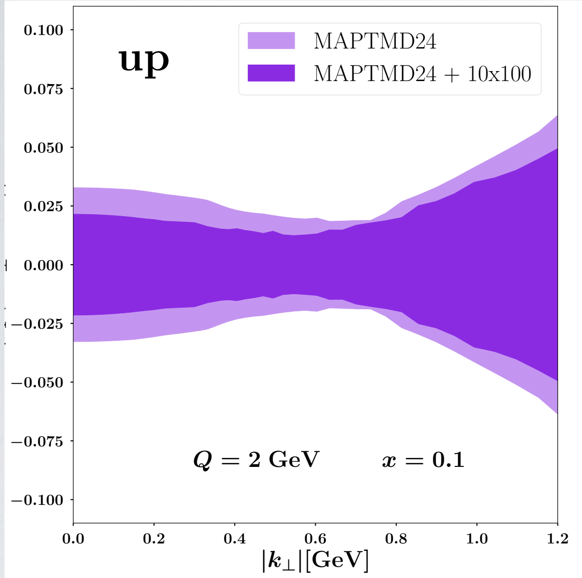
MAPTMD24 2031

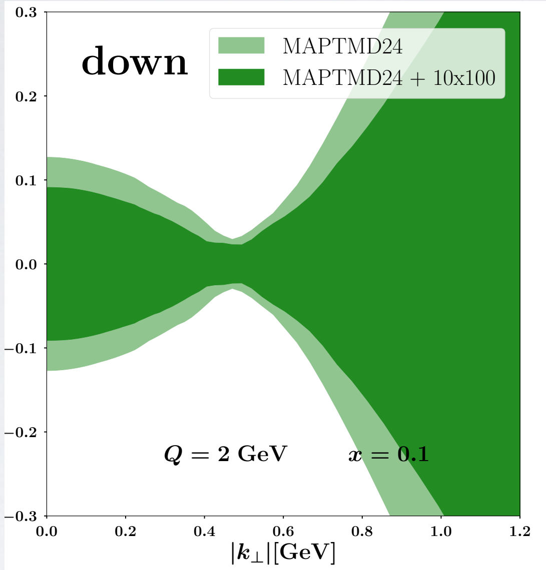
EIC	**# pts.**	**lumi [fb^{-1}]**
5x41	1273	2.85
10x100	1611	51.3
18x275	1648	10

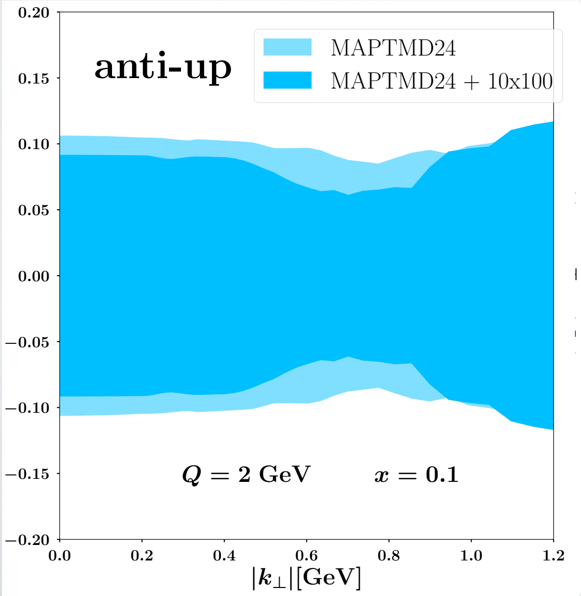


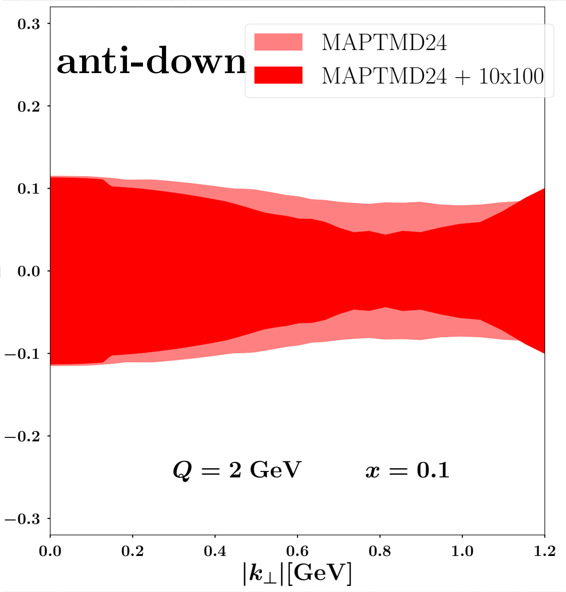
L. Rossi, Ph.D. Thesis

The EIC impact with 10x100 at x=0.1







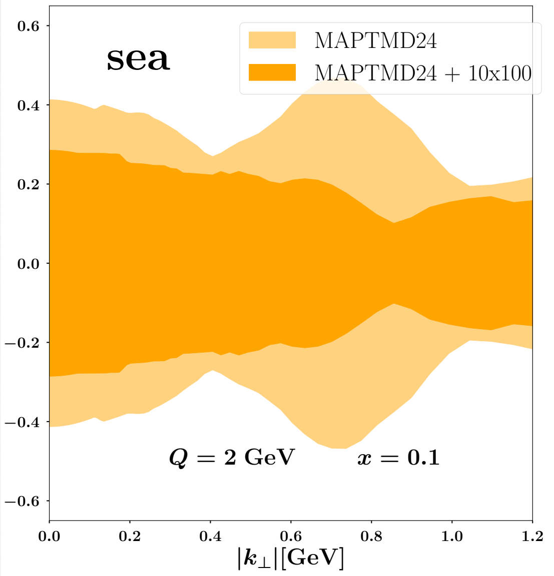


$$\frac{\mathrm{TMD}^q - \langle \mathrm{TMD}^q \rangle}{\langle \mathrm{TMD}^q \rangle}$$

x=0.1

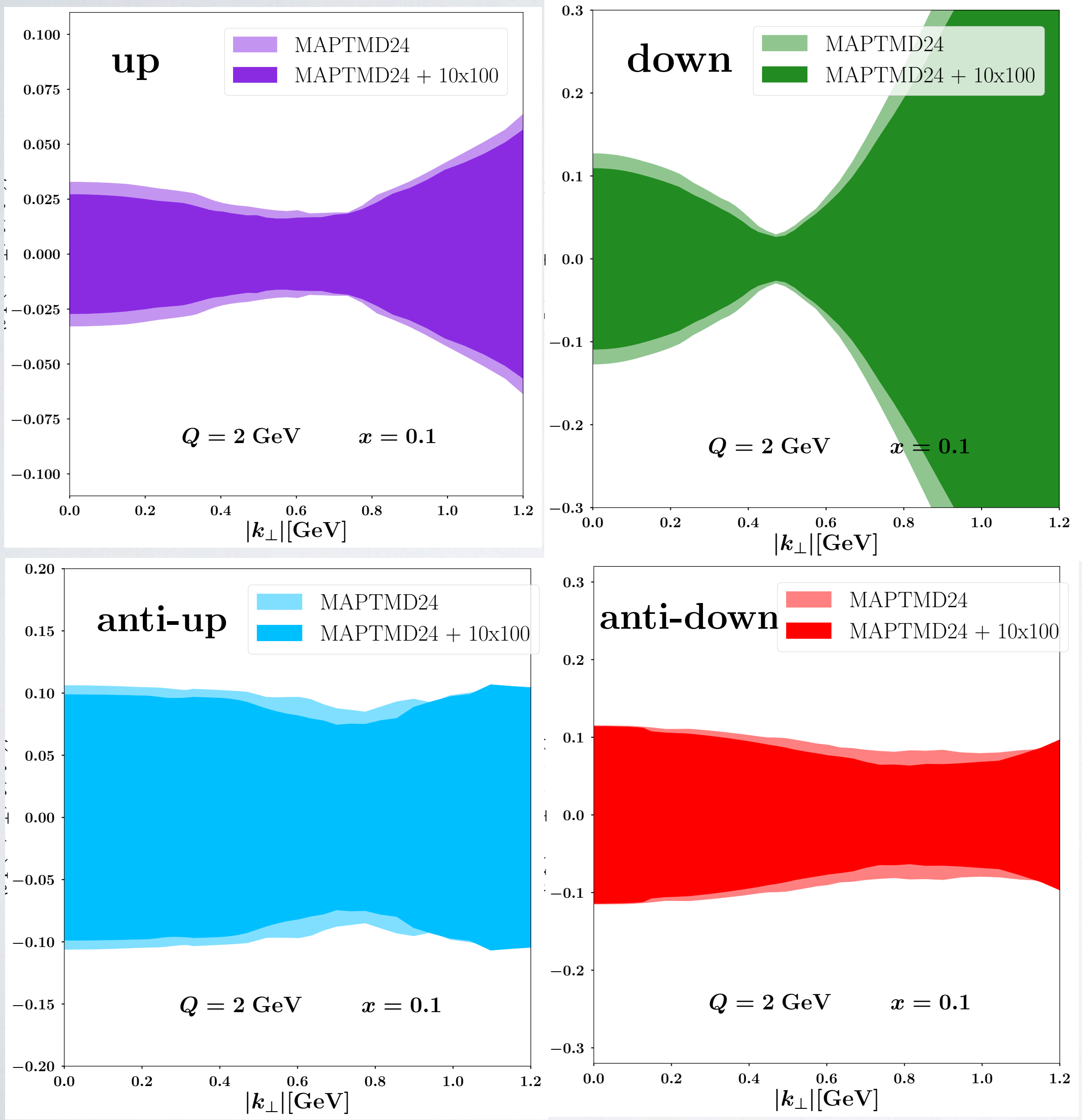
MAPTMD24	2031	
EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	51.3

(simulation campaign of May 2024)



L. Rossi, Ph.D. Thesis

The EIC impact with 10x100 at x=0.1

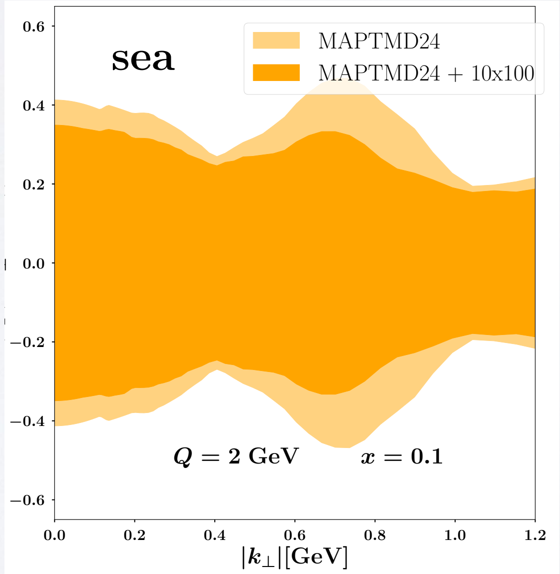


$$\frac{\mathrm{TMD}^q - \langle \mathrm{TMD}^q \rangle}{\langle \mathrm{TMD}^q \rangle}$$

x=0.1

MAPTMD24	2031	
EIC	**# pts.**	**lumi [fb^{-1}]**
10x100	1611	5

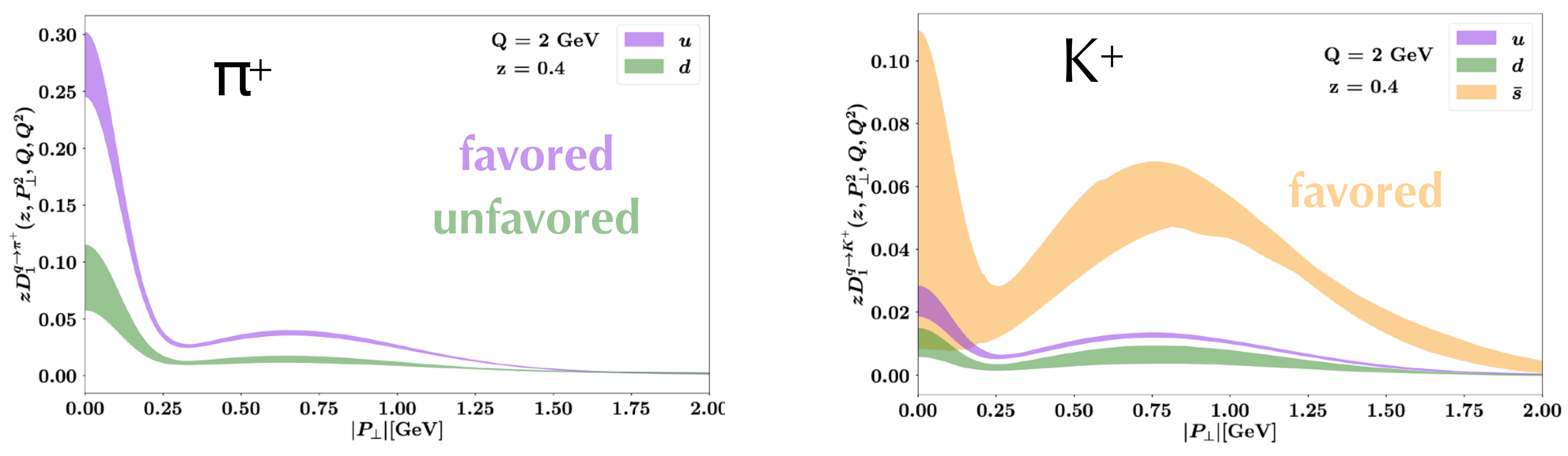
(early Science conditions)



courtesy L. Rossi

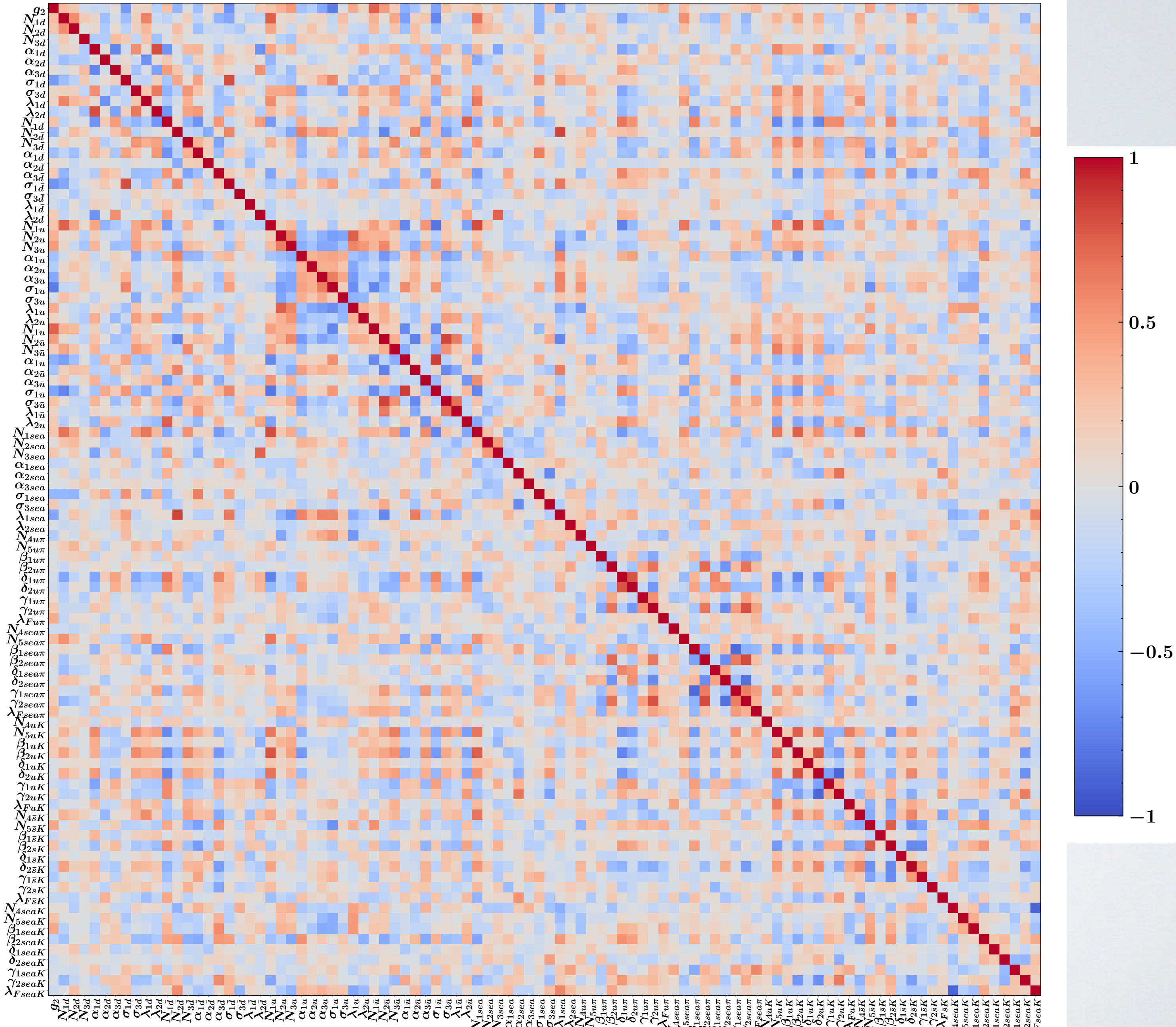
“Normalized” MAPTMD24 TMD FF

$$\frac{D_1(z,P_T;Q)}{D_1(z,0;Q)}$$

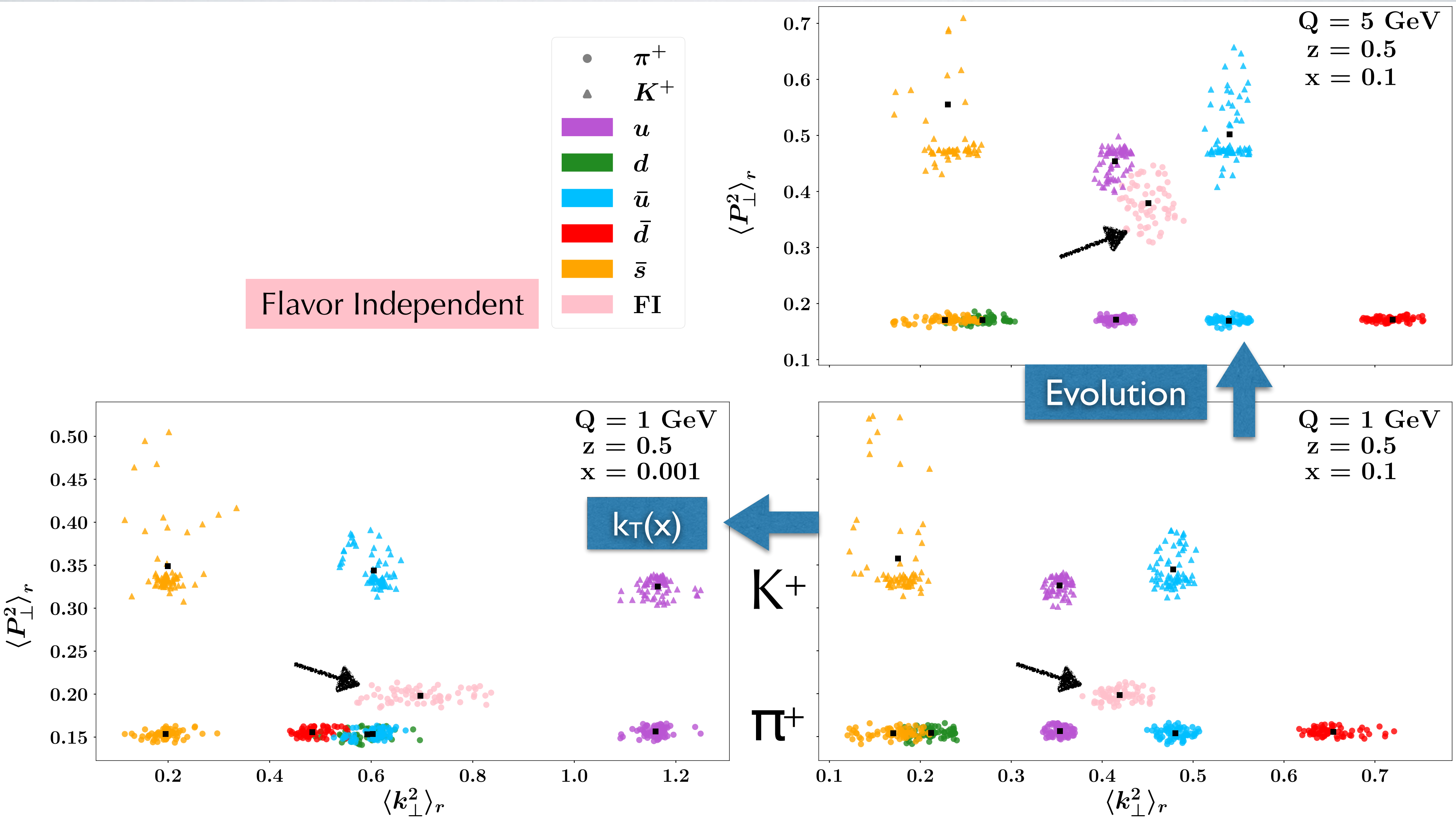


MAPTMD 2024
arXiv:2405.13833

Correlation matrix



Average transverse momenta



Flavor Independent

Evolution

$k_T(x)$

K^+

π^+

clusters = 68% of all replicas