Amplitudes for Black Holes and Hawking Radiation

Matteo Sergola
matteo.sergola@ipht.fr

Institut de Physique Théorique/CEA, Université Paris-Saclay

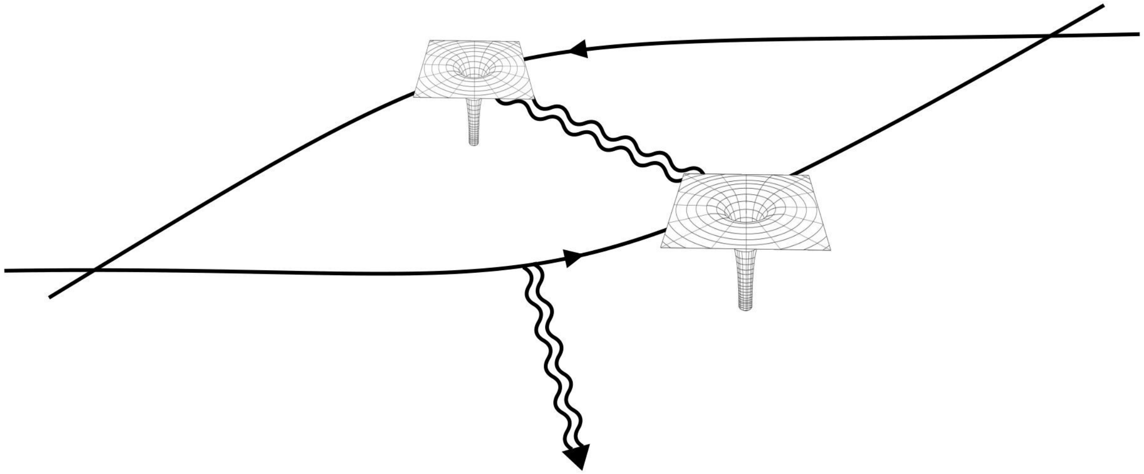
Based (not only) on [2303.06211], [2412.05267].
With *Donal O'Connell, Ingrid A. Vazquez-Holm, Asaad Elkhidir, Rafael Aoude*.

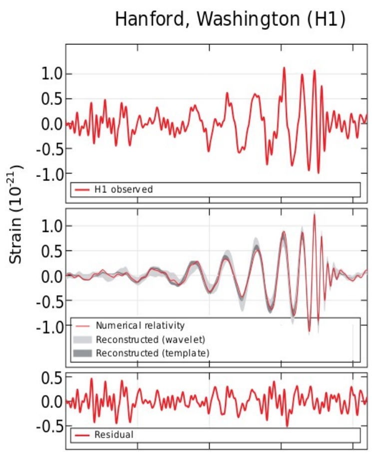
String Theory as a Bridge between Gauge Theory and Quantum Gravity,
Università di Roma - Tor Vergata, 08/05/2025

Outline

- Motivation
- On-Shell observables and KMOC (with examples)
- One for all: the final state ansatz
- Hawking radiation from amplitudes

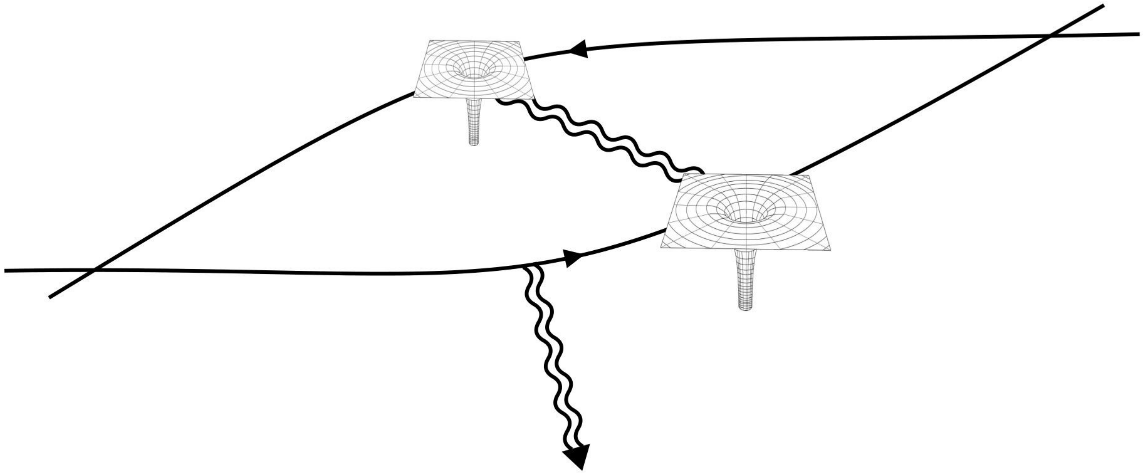
Background and motivation

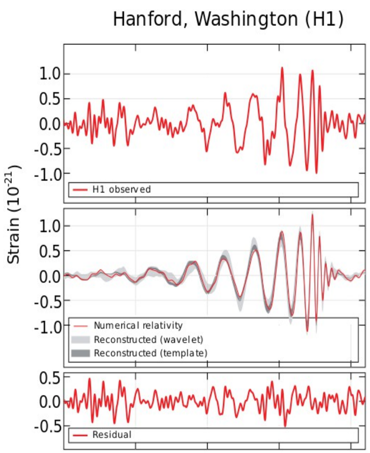




LIGO collaboration, 2016

Background and motivation





LIGO collaboration, 2016

$$\Rightarrow \langle R_{\mu\nu\rho\sigma}(x) \rangle = \int_k \mathcal{A}(k)\, e^{-ik\cdot x}$$

Kosower, O'Connell, Gonzo & Cristofoli, 2019
Snowmass White Paper: Gravitational Waves and Scattering Amplitudes 2022

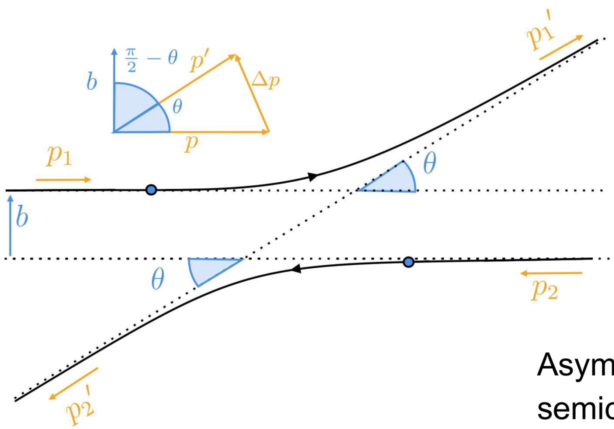
Classical observables from amplitudes: KMOC

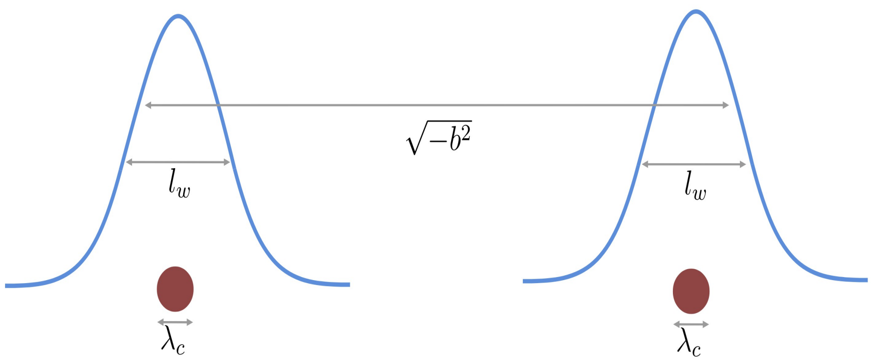
- Classical gravity/EM from amplitudes: $\Delta p^\mu, \ \Delta s^\mu, \ F^{\mu\nu}_{\text{rad}}, \ \Psi^{\alpha\beta\gamma\delta}_{\text{rad}}$

$$S = 1 + iT$$

$$\begin{aligned}\Delta p_1^\mu &= \langle\psi|S^\dagger \hat{P}_1^\mu S|\psi\rangle - p_1^\mu \\ &= \langle\psi|[\hat{P}_1^\mu, iT]|\psi\rangle + \langle\psi|T^\dagger[\hat{P}_1^\mu, T]|\psi\rangle\end{aligned}$$

$$|\psi\rangle \sim \int \varphi(p_1, p_2)|p_1, p_2\rangle$$





$$\lambda_c \ll l_w \ll \sqrt{-b^2}$$

Asymptotic states with semiclassical features

Kosower, Maybee & O'Connell, 2018.
Vines, Maybee & O'Connell, 2019.
Kosower, Cristofoli, Gonzo & O'Connell, 2021.

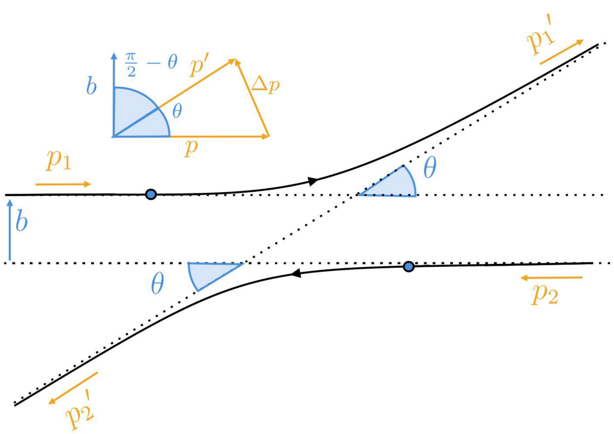
Classical observables from amplitudes: KMOC

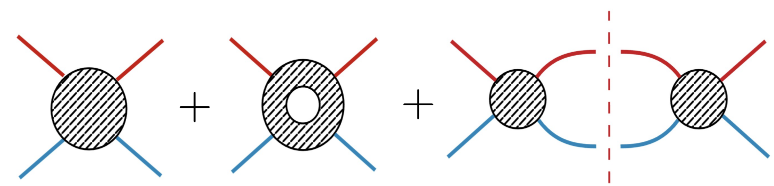
- Classical gravity/EM from amplitudes: $\Delta p^\mu, \;\; \Delta s^\mu, \;\; F^{\mu\nu}_{\rm rad}, \;\; \Psi^{\alpha\beta\gamma\delta}_{\rm rad}$

$$\Delta p_1^\mu = \int \hat{d}^4 q \, \hat{\delta}(p_1 \cdot q)\hat{\delta}(p_2 \cdot q) e^{-ib\cdot q}\Big[q^\mu i\mathcal{A}(p_1 \to p_1 + q) + \int \hat{d}^4 l \, \hat{\delta}(p_1 \cdot l)\hat{\delta}(p_2 \cdot l) l^\mu \mathcal{A}^*(p_1 + q_1 \to p_1 + l)\mathcal{A}(p_1 \to p_1 + l)\Big]$$

classical limit:

$$p_i^\mu = m_i u_i^\mu$$
$$q^\mu = \hbar \bar{q}^\mu$$
$$p_i \gg q$$



$\Delta p_1^\mu =$  $+ \cdots$

tree $\sim \mathcal{O}(G)$ 1-loop $\sim \mathcal{O}(G^2)$

Kosower, Maybee & O'Connell, 2018.
Vines, Maybee & O'Connell, 2019.
Kosower, Cristofoli, Gonzo & O'Connell, 2021.

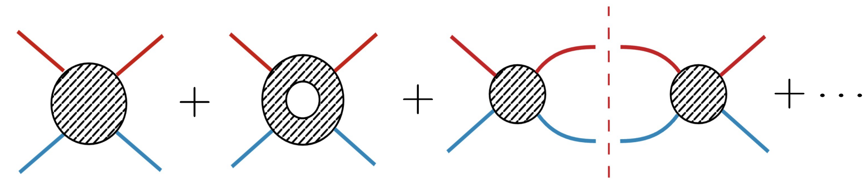
Classical observables from amplitudes: KMOC

- Classical gravity/EM from amplitudes: $\Delta p^\mu, \ \Delta s^\mu, \ F^{\mu\nu}_{\text{rad}}, \ \Psi^{\alpha\beta\gamma\delta}_{\text{rad}}$

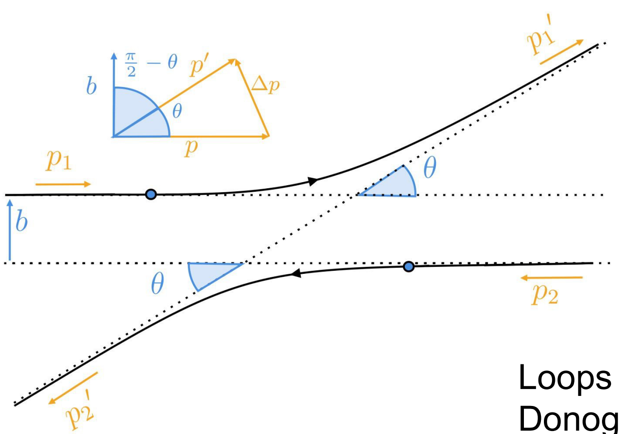
$$\Delta p_1^\mu = \int \hat{d}^4 q\, \hat{\delta}(p_1 \cdot q)\hat{\delta}(p_2 \cdot q) e^{-ib\cdot q}\Big[q^\mu i\mathcal{A}(p_1 \to p_1 + q) + \int \hat{d}^4 l\, \hat{\delta}(p_1 \cdot l)\hat{\delta}(p_2 \cdot l) l^\mu \mathcal{A}^*(p_1 + q_1 \to p_1 + l)\mathcal{A}(p_1 \to p_1 + l)\Big]$$

classical limit:

$$p_i^\mu = m_i u_i^\mu$$
$$q^\mu = \hbar \bar{q}^\mu$$
$$p_i \gg q$$

$\Delta p_1^\mu =$ 

tree $\sim \mathcal{O}(G)$ 1-loop $\sim \mathcal{O}(G^2)$



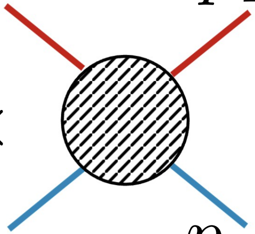
Loops can be classical!
Donoghue, Holstein, 2004

Kosower, Maybee & O'Connell, 2018.
Vines, Maybee & O'Connell, 2019.
Kosower, Cristofoli, Gonzo & O'Connell, 2021.

Momentum kick from the double copy and generalized unitarity

Kosower, Bern, Dixon, Dunbar, 1994

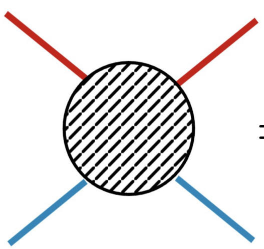
$$\Delta p_1^\mu = \int \hat{d}^4 q\, \hat{\delta}(2p_1 \cdot q)\, \hat{\delta}(2p_2 \cdot q)\, e^{-ib\cdot q} q^\mu \times$$



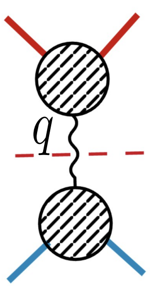
Minimally-coupled scalar → $2p_1 \cdot \epsilon_h(q)$

$$\sum_h \epsilon^\mu_{-h} \epsilon^\nu_h = P^{\mu\nu} = -\eta^{\mu\nu} + \frac{q^{(\mu} v^{\nu)}}{q \cdot v} - v^2 \frac{v^\mu v^\nu}{(q\cdot v)^2}$$

$\Rightarrow$



$$= \frac{1}{q^2} \sum_h$$



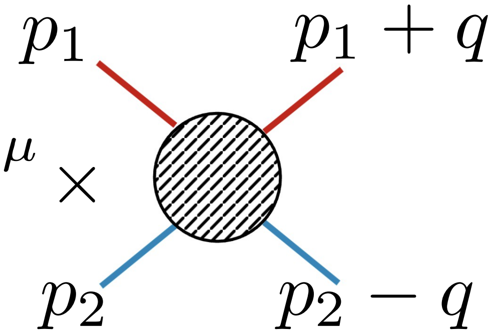
$$\epsilon_h^2 = 0 = \epsilon_h \cdot v = \epsilon_h \cdot q$$

$2p_2 \cdot \epsilon_{-h}(q)$

Momentum kick from the double copy and generalized unitarity

Kosower, Bern, Dixon, Dunbar, 1994

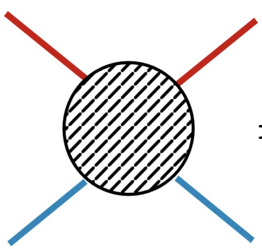
$$\Delta p_1^\mu = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu \times$$



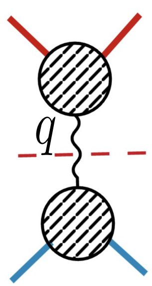
Minimally-coupled scalar → $2p_1 \cdot \epsilon_h(q)$

$$\sum_h \epsilon^\mu_{-h}\epsilon^\nu_h = P^{\mu\nu} = -\eta^{\mu\nu} + \frac{q^{(\mu}v^{\nu)}}{q\cdot v} - v^2 \frac{v^\mu v^\nu}{(q\cdot v)^2}$$

$\Rightarrow$



$$= \frac{1}{q^2}\sum_h$$



Impact parameter

$$\Rightarrow \Delta p_1^\mu \sim Q_1 Q_2 \frac{b^\mu}{b^2}\gamma$$

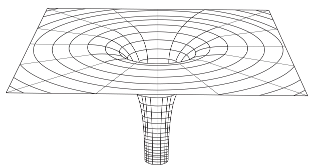
$$\epsilon_h^2 = 0 = \epsilon_h \cdot v = \epsilon_h \cdot q$$

$$2p_2 \cdot \epsilon_{-h}(q)$$

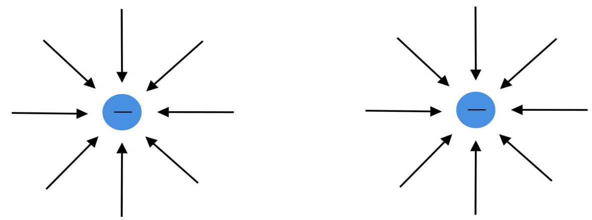
where $\gamma = \dfrac{p_1 \cdot p_2}{m_1 m_2}$

Momentum kick from the double copy and generalized unitarity

Kosower, Bern, Dixon, Dunbar, 1994



$=$

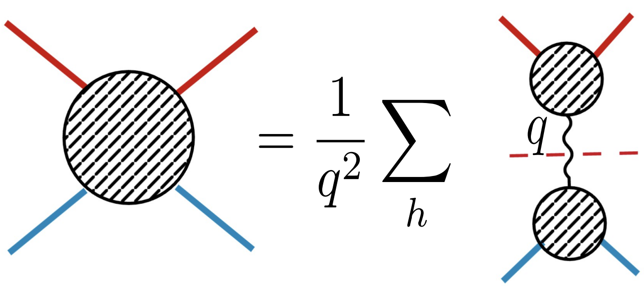


Double copy!

$$(2p_1 \cdot \epsilon_h(q))^2$$

$$\sum_h \epsilon^{\mu}_{-h}\epsilon^{\nu}_{h}\epsilon^{\rho}_{-h}\epsilon^{\sigma}_{h} = P^{\mu(\rho}P^{\nu)\sigma} - P^{\mu\nu}P^{\rho\sigma}$$

$\Rightarrow$



$$\Rightarrow \Delta p_1^{\mu} \sim G m_1 m_2 \frac{b^{\mu}}{b^2}(2\gamma^2 - 1)$$

$$\epsilon_h^2 = 0 = \epsilon_h \cdot v = \epsilon_h \cdot q$$

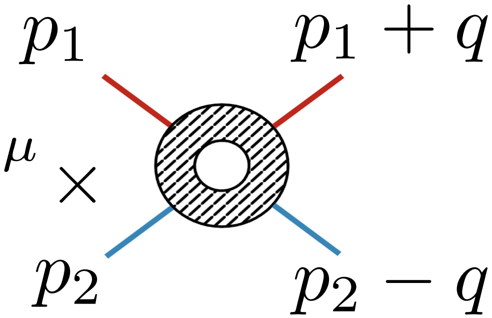
$$(2p_2 \cdot \epsilon_{-h}(q))^2$$

Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansson, 2008
Monteiro, O'Connell & White, 2014

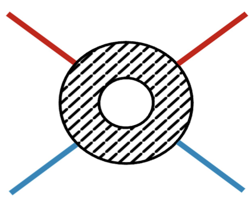
Momentum deflection – one loop

Kosower, Bern, Dixon, Dunbar, 1994

$$\Delta p_1^\mu = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu \times$$



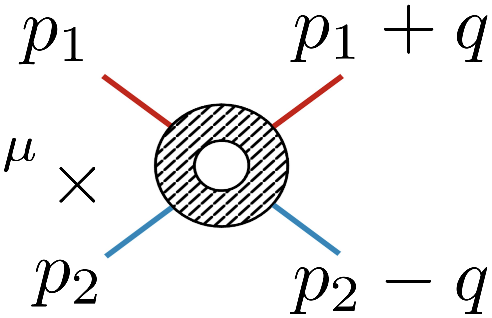
$\Rightarrow$



Momentum deflection – one loop

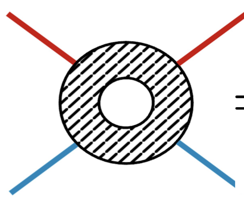
Kosower, Bern, Dixon, Dunbar, 1994

$$\Delta p_1^\mu = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu \times$$



Two sums over helicities now, to be dealt with just like before!

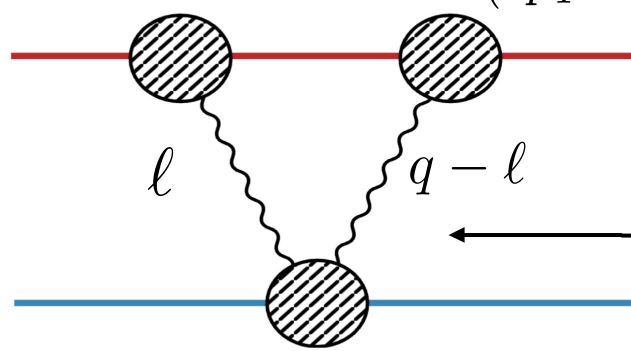
$$\Rightarrow$$



$$= \frac{1}{p_1 \cdot \ell \, \ell^2 (q-\ell)^2} \sum_{h,h'}$$

$(2p_1 \cdot \epsilon_h(\ell))^2$

$(2p_1 \cdot \epsilon_h(q-\ell))^2$



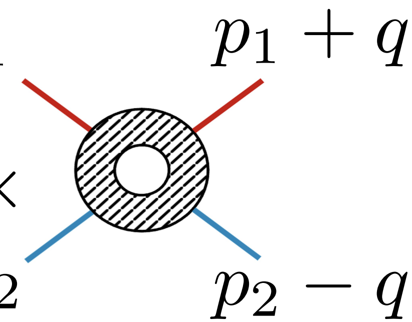
$(\epsilon_h(\ell) \cdot \epsilon_{-h}(q-\ell))^2$

Internal lines are placed on-shell to find the integrand

Momentum deflection – one loop

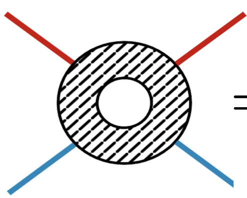
Kosower, Bern, Dixon, Dunbar, 1994

$$\Delta p_1^\mu = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu \times$$

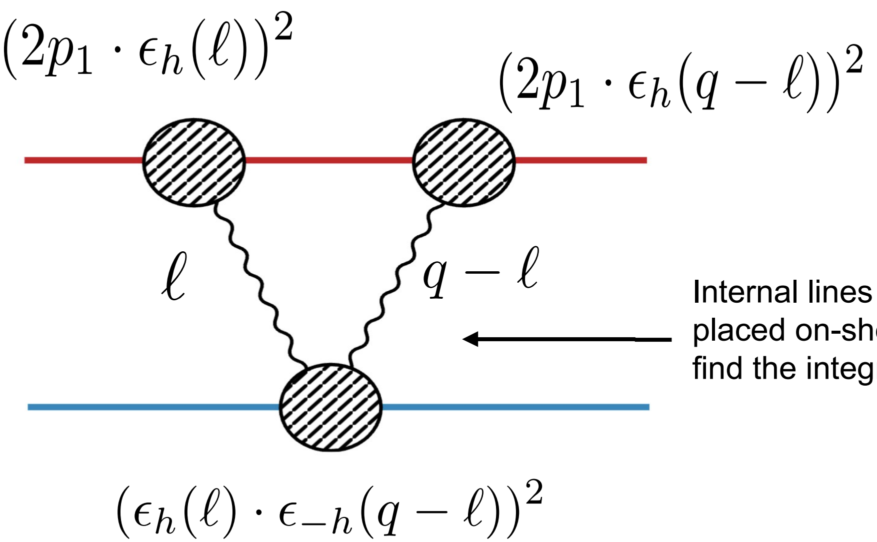


Two sums over helicities now,
to be dealt with just like before!

$$\Rightarrow$$



$$= \frac{1}{p_1 \cdot \ell\, \ell^2 (q-\ell)^2} \sum_{h,h'}$$



Internal lines are placed on-shell to find the integrand

$$\Rightarrow \Delta p_1^\mu \sim \cdots + G^2 m_1 m_2 \frac{b^\mu}{b^3}(5\gamma^2 - 1) + \cdots + \mathcal{O}(G^5)$$

See review:snowmass White Paper:
Gravitational Waves and Scattering Amplitudes 2022

Plefka, Mogull, Jakobsen, Nega, Klemm 2024

Waveforms at one loop: 5-point amplitudes

Riemann as a Second Quantization field operator: $\mathbb{R} \sim \int_k (a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ik\cdot x})$

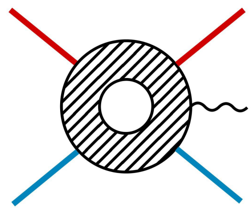
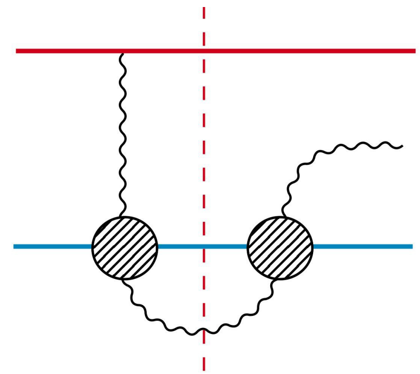
$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle = -\sum_\eta \int \mathrm{d}\Phi(k)\left(k_{[\mu}\varepsilon^\eta_{\nu]}k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k)e^{-ik\cdot x} + \text{h.c.}\right)$$

Vazquez-Holm, Elkhidir, O'Connell, **Sergola** 2023
Heissenberg, Georgoudis,2023
Russo, De Angelis, Travaglini,
Brandhuber, Brown, Gowdy,Chen 2023
Herdeschee, Roiban, Teng, 2023

Waveforms at one loop:
5-point amplitudes

Riemann as a Second Quantization field operator: $\mathbb{R} \sim \int_k (a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ik\cdot x})$

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x) S|\psi\rangle = -\sum_\eta \int \mathrm{d}\Phi(k)\left(k_{[\mu}\varepsilon^\eta_{\nu]}k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k)e^{-ik\cdot x} + \text{h.c.}\right)$$

$\alpha_\eta(k) =$  $+\ i$ 

$\mathcal{A}_{(1)}(p_1, p_2 \to p_1', p_2', k)$

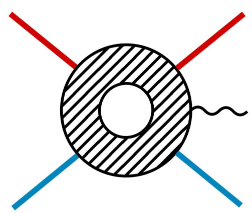
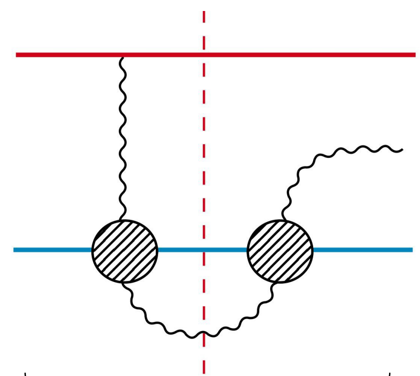
$\text{Cut}_{(1)}(p_1, p_2 \to p_1', p_2', k)$
$\sim \mathcal{A}_{(0)} \times \mathcal{A}_{(0)}$

Vazquez-Holm, Elkhidir, O'Connell, **Sergola** 2023
Heissenberg, Georgoudis,2023
Russo, De Angelis, Travaglini,
Brandhuber, Brown, Gowdy,Chen 2023
Herdeschee, Roiban, Teng, 2023

Waveforms at one loop: 5-point amplitudes

Riemann as a Second Quantization field operator: $\mathbb{R} \sim \int_k (a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ik\cdot x})$

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle = -\sum_\eta \int \mathrm{d}\Phi(k)\left(k_{[\mu}\varepsilon^\eta_{\nu]}k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k)e^{-ik\cdot x} + \mathrm{h.c.}\right)$$

$\alpha_\eta(k) =$  $+ \; i$ 

Radiation Kernel
Generated by
geodesic forces

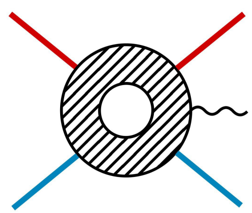
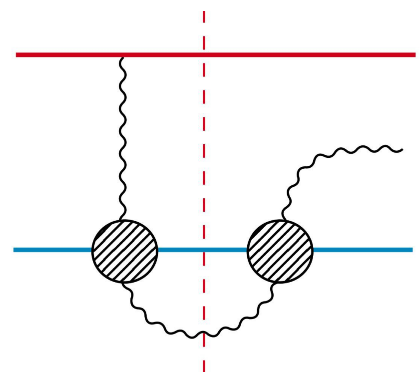
Radiation Kernel
Generated by the black hole's own field: radiation reaction.
Computed by ALD force in the QED case.

Vazquez-Holm, Elkhidir, O'Connell, **Sergola** 2023
Heissenberg, Georgoudis, 2023
Russo, De Angelis, Travaglini,
Brandhuber, Brown, Gowdy, Chen 2023
Herdeschee, Roiban, Teng, 2023

Waveforms at one loop: 5-point amplitudes

Riemann as a Second Quantization field operator: $\mathbb{R} \sim \int_k (a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ik\cdot x})$

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x) S|\psi\rangle = -\sum_\eta \int \mathrm{d}\Phi(k)\left(k_{[\mu}\varepsilon^\eta_{\nu]}k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k)e^{-ik\cdot x} + \text{h.c.}\right)$$

$\alpha_\eta(k) =$  $+\ i$ 

Radiation Kernel Generated by geodesic forces

Radiation Kernel Generated by the black hole's own field: radiation reaction. Computed by ALD force in the QED case.

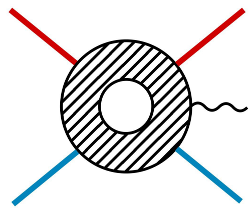
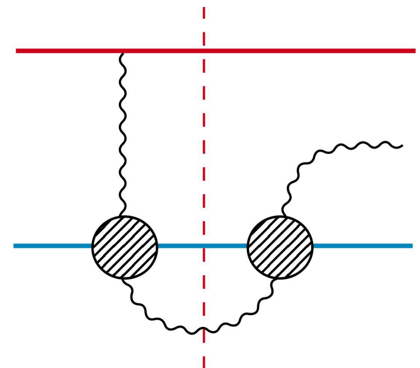
Vazquez-Holm, Elkhidir, O'Connell, **Sergola** 2023
Heissenberg, Georgoudis,2023
Russo, De Angelis, Travaglini,
Brandhuber, Brown, Gowdy,Chen 2023
Herdeschee, Roiban, Teng, 2023

→ **Also see Gabriele's talk!**

Waveforms at one loop: 5-point amplitudes

What can be said about the out state?

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle = -\sum_\eta \int \mathrm{d}\Phi(k)\left(k_{[\mu}\varepsilon^\eta_{\nu]}k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k)e^{-ik\cdot x} + \mathrm{h.c.}\right)$$

$\alpha_\eta(k) =$  $+\ i$ 

Radiation Kernel Generated by geodesic forces

Radiation Kernel Generated by the black hole's own field: radiation reaction. Computed by ALD force in the QED case.

Vazquez-Holm, Elkhidir, O'Connell, **Sergola** 2023
Heissenberg, Georgoudis,2023
Russo, De Angelis, Travaglini,
Brandhuber, Brown, Gowdy,Chen 2023
Herdeschee, Roiban, Teng, 2023

Uncertainty relations and classical physics

- We demand that expectation values factorise:

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle \sim \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle S^\dagger|\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle$$

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S\mathbb{P}^\lambda|\psi\rangle \sim \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|\mathbb{P}^\lambda|\psi\rangle$$

Uncertainty relations and classical physics

- We demand that expectation values factorise:

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle \sim \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle S^\dagger|\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle$$

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S\mathbb{P}^\lambda|\psi\rangle \sim \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|\mathbb{P}^\lambda|\psi\rangle$$

- Then, we derive relations for classical amplitudes:

$$\mathcal{A}^{tree}_{(6)} \sim \hbar$$

$$\mathcal{A}^{1loop}_{(4)} \sim \mathcal{A}^{tree}_{(4)} \otimes \mathcal{A}^{tree}_{(4)}$$

$$\mathcal{A}^{1loop}_{(5)} \sim \mathcal{A}^{tree}_{(5)} \otimes \mathcal{A}^{tree}_{(4)}$$

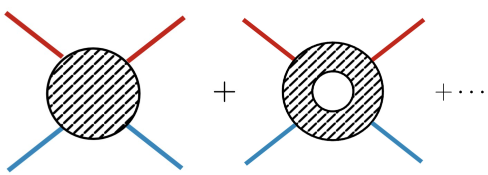
$$\vdots$$

The final state ansatz

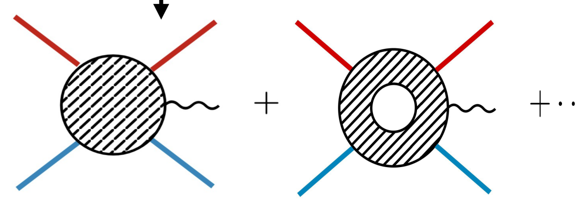
- We find that the most general semiclassical outgoing state is of the following form:

$$\Rightarrow S|\psi\rangle = \int d\Phi(p)\varphi(p)e^{i\chi(b)}\exp\left[\int d\Phi(k)\alpha_\eta(k)a^\dagger_\eta(k)\right]|p\rangle$$

$i\chi(b) \sim$



$\alpha_\eta(k) \sim$



Accetulli-Huber, De Angelis, Brandhuber, Travaglini, 2020
Sergola, Monteiro, O'Connell, Peinador-Vega, 2021
Damgaard, Plante, Vanhove, 2021
Cristofoli, Gonzo, Moynihan, O'Connell, Ross, White, **Sergola**, 2021
Di Vecchia, Heissenberg, Russo, Veneziano, 2022

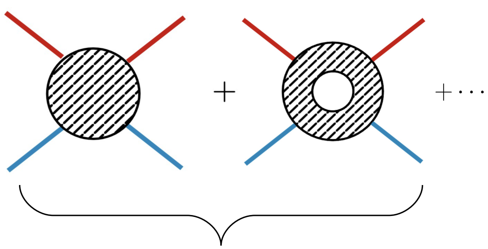
Confirmed by Daamgard, Vanhove, Hansen & Planté (3-loop impulse) and Georgoudis, Heissenberg, Holm (1-loop radiation)

The final state ansatz

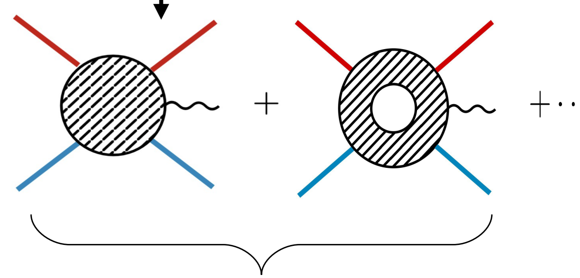
- We find that the most general semiclassical outgoing state is of the following form:

$$\Rightarrow S|\psi\rangle = \int d\Phi(p)\varphi(p)e^{i\chi(b)}\exp\left[\int d\Phi(k)\alpha_\eta(k)a^\dagger_\eta(k)\right]|p\rangle$$

$i\chi(b) \sim$



$\alpha_\eta(k) \sim$



Recovers conservative Scattering via saddle point: $\Delta p^\mu = \partial_b^\mu \chi(b)$

Recovers waveform via: $a(k)S|\psi\rangle = \alpha(k)S|\psi\rangle$ and explains why we only need single graviton emission

Accetulli-Huber, De Angelis, Brandhuber, Travaglini, 2020
Sergola, Monteiro, O'Connell, Peinador-Vega, 2021
Damgaard, Plante, Vanhove, 2021
Cristofoli, Gonzo, Moynihan, O'Connell, Ross, White, **Sergola**, 2021
Di Vecchia, Heissenberg, Russo, Veneziano, 2022

Confirmed by Daamgard, Vanhove, Hansen & Planté (3-loop impulse) and Georgoudis, Heissenberg, Holm (1-loop radiation)

In a good place to tackle Hawking radiation!

$\Rightarrow$ Set up a scattering problem for it

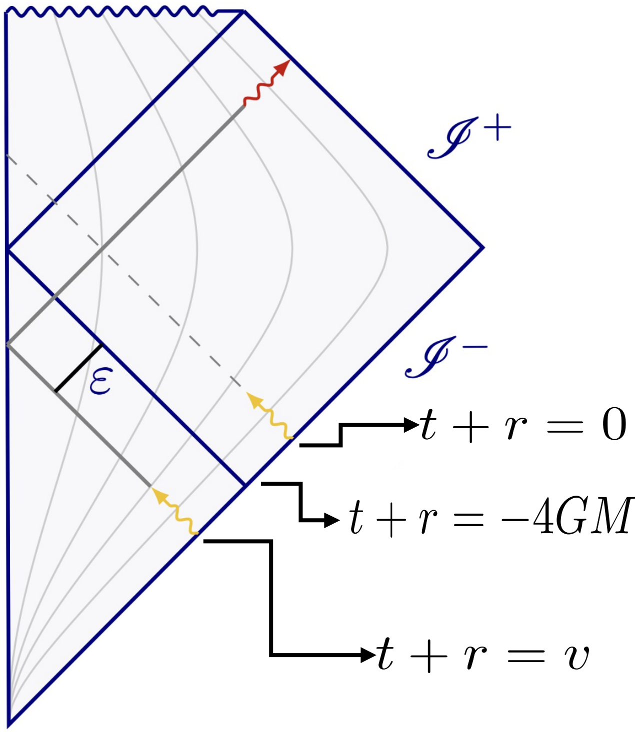
Hawking scattering:
The Vaidya background

In-falling shell of
Classical radiation

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu$$

$$k^\mu(x) = (1, \hat{r})$$

Kerr-Schild null vector field



Vaidya, 1966

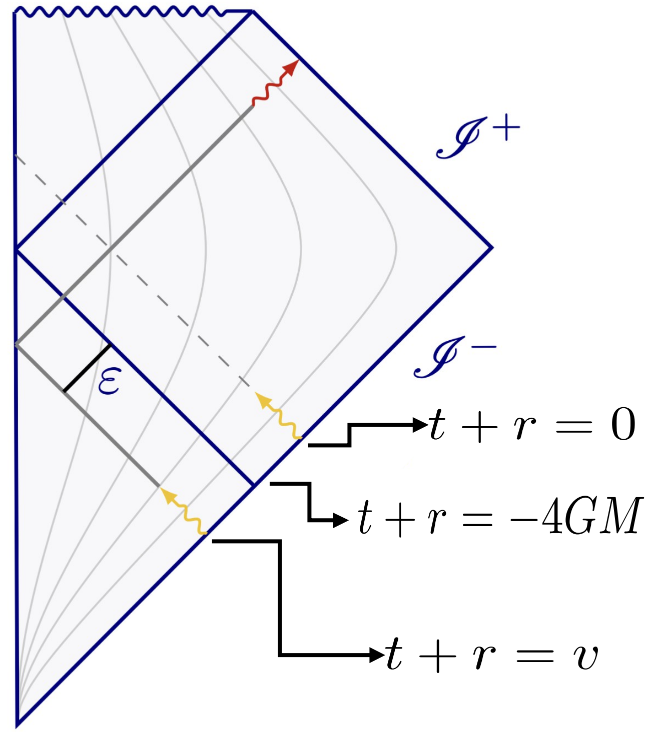
Hawking scattering: The Vaidya background

In-falling shell of
Classical radiation

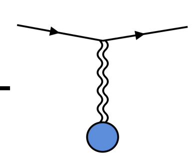
$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{2GM\Theta(t+r)}{r} k_\mu k_\nu$$

$$k^\mu(x) = (1, \hat{r})$$

Kerr-Schild null vector field



$$\mathcal{L} = \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$
$$= \mathcal{L}_{free} + \mathcal{L}_{cubic}$$



Vaidya, 1966

The final state in a time-varying background: Hawking radiation

- Consider the LO eikonal for a spherically symmetric massless scalar on a time-varying background:

$$|\psi\rangle = \int d\Phi(p)\varphi(p)|p\rangle = \int d\Phi(p) \int dv\, \varphi(v)e^{iEv}|p\rangle$$

$$= \int d\Phi(p)|p\rangle \int dv\, \varphi(v)e^{ib\cdot p}$$

On-shell Lorentz invariant phase space of a massless quantum scalar

$$b^\mu = (v, 0, 0, 0)$$

Sergola, O'Connell, Aoude, 2024

The final state in a time-varying background: Hawking radiation

- Consider the LO eikonal for a spherically symmetric massless scalar on a time-varying background:

$$|\psi\rangle = \int d\Phi(p)\varphi(p)|p\rangle = \int d\Phi(p) \int dv\, \varphi(v) e^{iEv}|p\rangle$$

$$= \int d\Phi(p)|p\rangle \int dv\, \varphi(v) e^{ib\cdot p}$$

On-shell Lorentz invariant phase space of a massless quantum scalar

$$b^\mu = (v, 0, 0, 0)$$

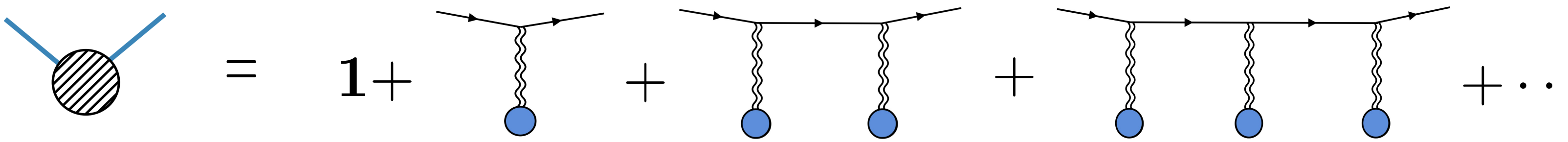
- The final state has the form:

$$S|\psi\rangle = \int d\Phi(p)|p\rangle \int dv\, \varphi(v) e^{iEv} e^{i\chi(v)}$$

$\Rightarrow$ *Now* $p^2 = 0$ *but still* $p \gg q$ *: Geometric-optics approximation!*

Sergola, O'Connell, Aoude, 2024

LO Eikonal exponentiation



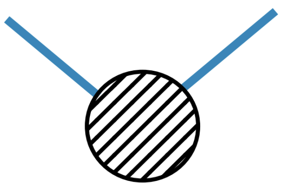
$$\Rightarrow \langle p|S|\psi\rangle = \int dv\,\varphi(v) e^{ib\cdot p} e^{-4iGME\log(-v/\mu)}$$

Higher loops resum into an exponential in the leading eikonal/geometric-optics approximation $p \gg q$

Sergola, O'Connell, Aoude, 2024

Computing the resummed amplitude

We choose the spherical initial state $\varphi(v) = e^{-iE_0 v}$



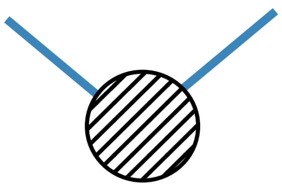
$$= \int_{-\infty}^{\infty} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

$$\Rightarrow \int_{-\infty}^{0} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

Cannot integrate over all v
Because of the logarithm cut:
not an inclusive result!

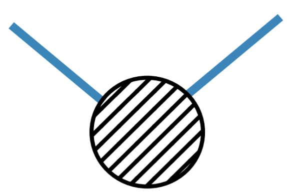
Computing the resummed amplitude

We choose the spherical initial state $\varphi(v) = e^{-iE_0 v}$



$$= \int_{-\infty}^{\infty} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

$$\Rightarrow \int_{-\infty}^{0} dv\, e^{iv(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

$\Rightarrow \Big|$  $\Big|^2 = N \dfrac{e^{8\pi GME}}{e^{8\pi GME}-1}$

Looks familiar..

Bogoliubov and amplitudes

Our EOM $\partial^2\phi = -\partial_\mu h^{\mu\nu}(x)\partial_\nu\phi$ can be resolved

- Onto a past basis: $\phi(x) = \int d\Phi(p)\,(a(p)P(x,p) + h.c.)$

  $P(x,p) \xrightarrow{t \to -\infty} e^{-ip\cdot x}$

- Or a future one: $\phi(x) = \int d\Phi(p)\,(b(p)F(x,p) + h.c.)$

  $F(x,p) \xrightarrow{t \to \infty} e^{-ip\cdot x}$

Bogoliubov and amplitudes

Our EOM $\partial^2\phi = -\partial_\mu h^{\mu\nu}(x)\partial_\nu\phi$ can be resolved

- Onto a past basis: $\phi(x) = \int d\Phi(p)\,(a(p)P(x,p) + h.c.)$

$$P(x,p) \xrightarrow[t\to-\infty]{} e^{-ip\cdot x}$$

$$||$$

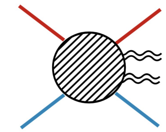
- Or a future one: $\phi(x) = \int d\Phi(p)\,(b(p)F(x,p) + h.c.)$

$$F(x,p) \xrightarrow[t\to\infty]{} e^{-ip\cdot x}$$

$$\Rightarrow b(p) = S^\dagger a(p) S = \int d\Phi(k)\left(A(p,k)a(k) + B(p,k)a^\dagger(k)\right)$$

Bogoliubov and amplitudes

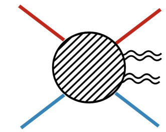
$$S = \exp\left[\int d\Phi(p,k)\xi(p,k)a^\dagger(p)a^\dagger(k)\right] = \exp\left[\quad\right]$$



$$\Rightarrow b(p) = S^\dagger a(p) S = \int d\Phi(k)\left(A(p,k)a(k) + B(p,k)a^\dagger(k)\right)$$

Bogoliubov and amplitudes

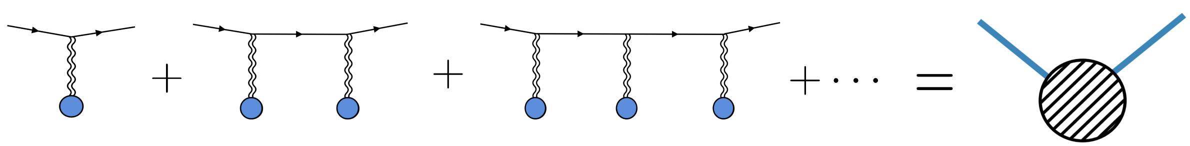
$$S = \exp\left[\int d\Phi(p,k)\xi(p,k)a^\dagger(p)a^\dagger(k)\right] = \exp\left[\quad\right]$$



$$\Rightarrow b(p) = S^\dagger a(p) S = \int d\Phi(k)\left(A(p,k)a(k) + B(p,k)a^\dagger(k)\right)$$

- $A(p,k) = \langle\Omega|S^\dagger a(p) S a^\dagger(k)|\Omega\rangle = \int dv r^2 F(v, E_p) i\partial_v P(v, -E_k)$

$= \mathbf{1} +$

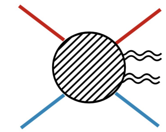


Solve the EOM with Born iterations

Caron-Huot, Giroux, Hannesdottir & Mizera, 2023

Bogoliubov and amplitudes

$$S = \exp\left[\int d\Phi(p,k)\xi(p,k)a^\dagger(p)a^\dagger(k)\right] = \exp\left[\quad\right]$$

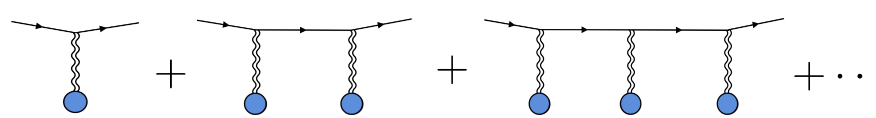


$$\Rightarrow b(p) = S^\dagger a(p) S = \int d\Phi(k)\left(A(p,k)a(k) + B(p,k)a^\dagger(k)\right)$$

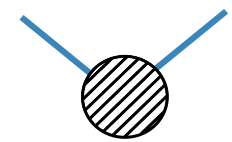
- $A(p,k) = \langle\Omega|S^\dagger a(p) S a^\dagger(k)|\Omega\rangle = \int dv r^2 F(v,E_p) i\partial_v P(v,-E_k)$

Solve the EOM with Born iterations

$= \mathbf{1} +$

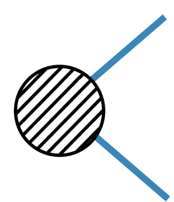


$+ \cdots =$



- $B(p,k) = \langle\Omega|a(k) S^\dagger a(p) S|\Omega\rangle = \int dv r^2 F(v,E_p) i\partial_v P(v,E_k)$

$=$

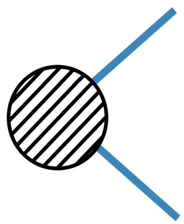


Caron-Huot, Giroux, Hannesdottir & Mizera, 2023

Crossing and spectrum

- The number of particles in the future is

$$\langle \Omega | S^\dagger a^\dagger(p) a(p) S | \Omega \rangle = \int d\Phi(k) |B(p,k)|^2$$

$B(p,k) =$  $= \int_{-\infty}^{0} dv\, e^{iv(E+E_0)} e^{-4iGM \log(-v/\mu)}$

$$|B(p,k)|^2 = N \frac{1}{e^{8\pi GME} - 1}$$

Hawking spectrum with $T = \dfrac{1}{8\pi GM}$

Hawking, 1975
Aoude, O'Connell, **Sergola** 2024

The spectrum is tree-exact

We also computed sub leading
corrections to the Hawking phase in

$$B(E, E_0) = \int dv\, e^{iv(E+E_0)} e^{i(\chi_{(0)}+\chi_{(1)}+\cdots)}$$

We find are computing the eikonal $\chi(b)$..
This is non trivially related to the amplitude through

$$e^{i\chi(b)}(1+\Delta(b)) = 1 + i\mathcal{A}(b)$$

Di Vecchia, Heissenberg,
Russo & Veneziano, 2023

The spectrum is tree-exact

We also computed sub leading corrections to the Hawking phase in

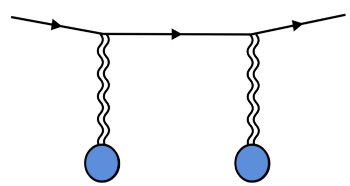
$$B(E, E_0) = \int dv\, e^{iv(E+E_0)} e^{i(\chi_{(0)}+\chi_{(1)}+\cdots)}$$

We find are computing the eikonal $\chi(b)$..
This is non trivially related to the amplitude through

$$e^{i\chi(b)}(1+\Delta(b)) = 1 + i\mathcal{A}(b)$$

$$\chi_{(0)}(b) = \mathcal{A}^0_{(0)}(b)$$

$$\chi_{(1)}(b) = \mathcal{A}^0_{(1)}(b) - i\mathcal{A}^0_{(0)}(b)\Delta_{(0)}(b)$$



$$\Delta_{(0)}(b) = \mathcal{A}^1_{(0)}(b)$$

Di Vecchia, Heissenberg, Russo & Veneziano, 2023

The spectrum is tree-exact

We also computed sub leading corrections to the Hawking phase in

$$B(E, E_0) = \int dv\, e^{iv(E+E_0)} e^{i(\chi_{(0)}+\chi_{(1)}+\cdots)}$$

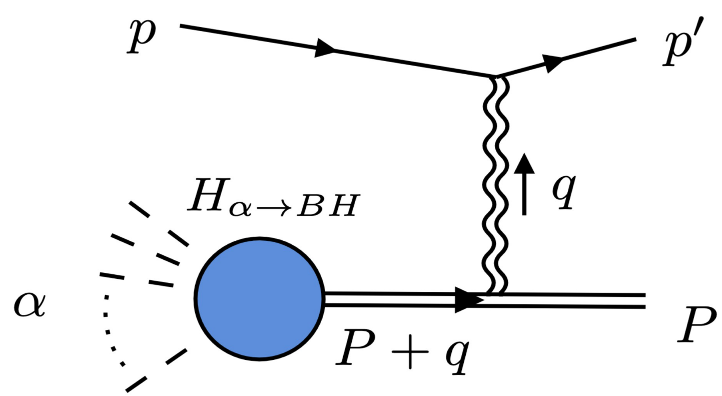
We find $\chi_{(0)} = -4GM\log(-v/\mu), \quad \chi_{(1)} = -\frac{16G^2M^2E}{v}$

$$e^{i(\chi_{(0)}+\chi_{(1)}+\cdots)} \approx e^{-4iGME\log(-(v+4GM)/\mu)}$$

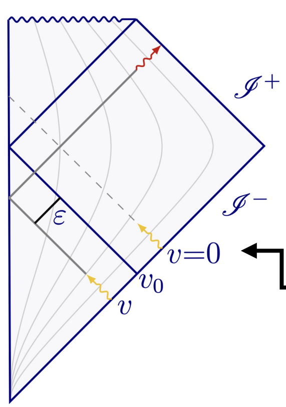
Simple translation which results in a pure phase
For the Bogoliubov coefficient

Some work in progress..

So far we have rephrased Hawking's derivation using amplitudes, but we think we can do more!



$$\sim \frac{GM\Theta(t+r)}{4\pi r}$$



Coherent state which describes the in-falling classical radiation shell at $t{+}r{=}v{=}0$

Conclusions

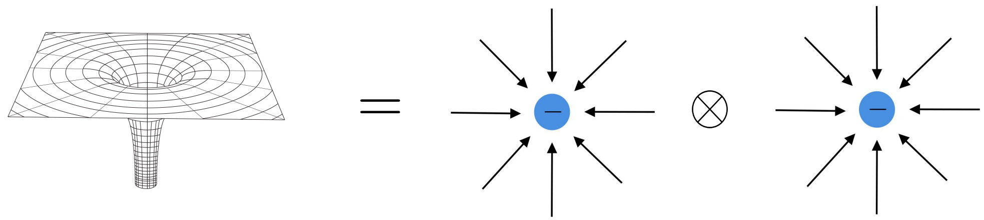
- On-shell methods, QFT technology and the double copy can be powerful tools for computations of classical GW observables
- Amplitudes can describe Hawking radiation too!

Next:

- What's the EM-analogue of this: “single copy” of Hawking!
- More complicate Hawking temperatures (Kerr, RN)
- Back reaction on the metric? Page curve?

Thank you!

The classical double copy



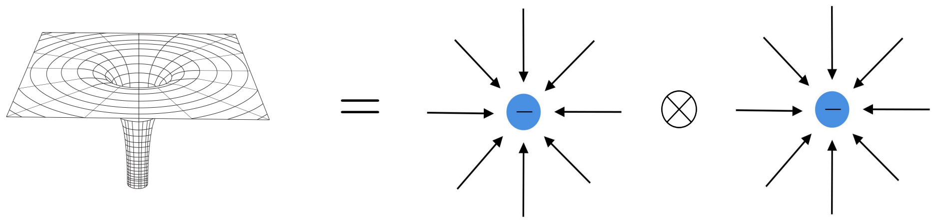
$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \phi(x) K_\mu(x) K_\nu(x)$$

$$K^\nu \partial_\nu K^\mu = 0, \quad K^2 = 0$$

$$\phi(x) = \frac{2GM}{r}, \quad K^\mu = \left(1, \frac{\vec{x}}{r}\right) \Rightarrow ds^2 = g^{Sch.}_{\mu\nu}(x) dx^\mu dx^\nu$$

Monteiro, O’Connell & White, 2014
Luna, Monteiro, Nicholson, White & O'Connell, 2016
Sergola, Peinador Veiga, Monteiro & O'Connell 2021

The classical double copy



$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \phi(x) K_\mu(x) K_\nu(x)$$

$$K^\nu \partial_\nu K^\mu = 0, \quad K^2 = 0$$

$$A^\mu(x) = \phi(x) K^\mu(x)$$

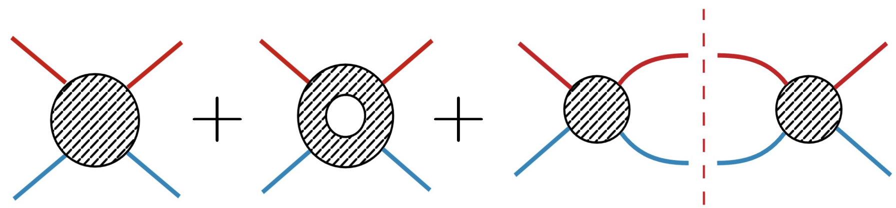
$$R^{\mu\nu}(x) = 0 \Rightarrow \underbrace{\partial_\mu\left(\partial^\mu(\phi K^\nu) - \partial^\nu(\phi K^\mu)\right)}_{\partial_\mu F^{\mu\nu}(x) = 0} = 0$$

Coulomb ⇔ Schwarzschild

Monteiro, O'Connell & White, 2014
Luna, Monteiro, Nicholson, White & O'Connell, 2016
Sergola, Peinador Veiga, Monteiro & O'Connell 2021

Adding spin: a^μ

$$\Delta p^\mu_{Kerr} = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu$$

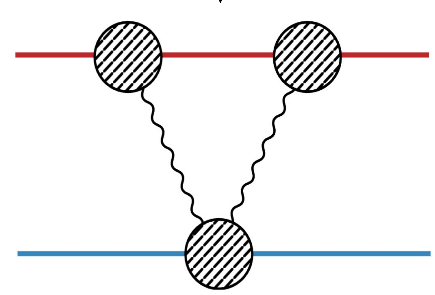
$\times$ 

$$\mathcal{A}_a = \mathcal{A}_{a=0} e^{iq\cdot(a_1+a_2)}$$

$$\Delta p^\mu_{Kerr} = G m_1 m_2 \mathrm{Re}\left[\frac{\cosh 2\eta b^\mu + 2i\cosh\eta \epsilon^{\mu\nu\rho\sigma} b_{\perp,\nu} u_\rho u_\sigma}{b_\perp^2}\right]$$

$$b^\mu_\perp = b^\mu + i a^\mu_1 + i a^\mu_2$$

Now massive and spinning particles



Compton sub-amplitude with spurious poles for $a^{n>4}$

Vines 2018
Arkani-Hamed, Huang & O Connell 2019
Vines, Maybee & O'Connell, 2019
Vines, Ochirov & Guevara 2019
Cangemi, Pichini, Johannson, Bohnenblust 2024
Sergola, Bautista, Vines,Kavanagh, Khalil 2024
Bjerrum-Bohr, Chen, Su, Wang 2025
Luna, Vazquez-Holm 2025