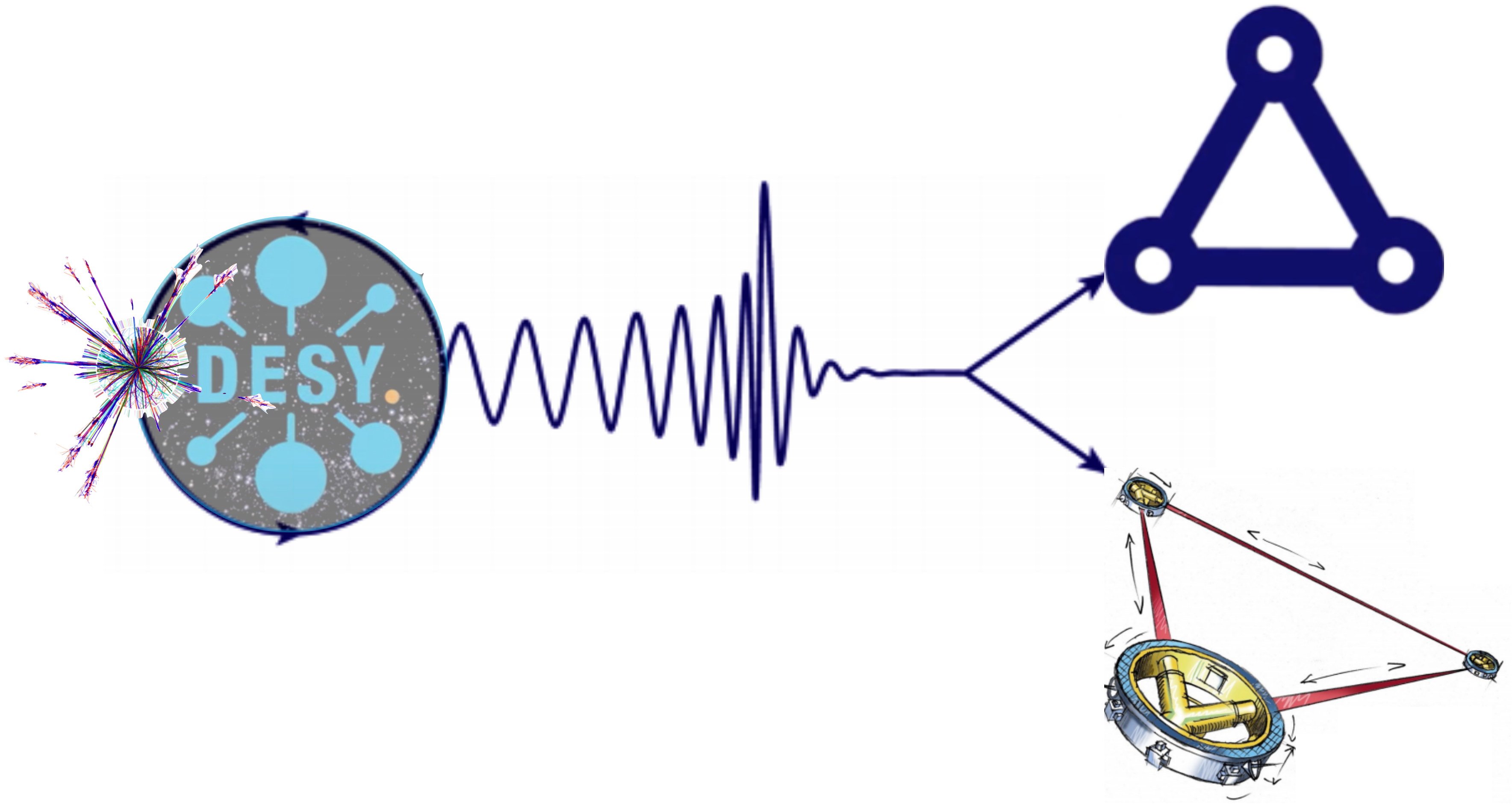
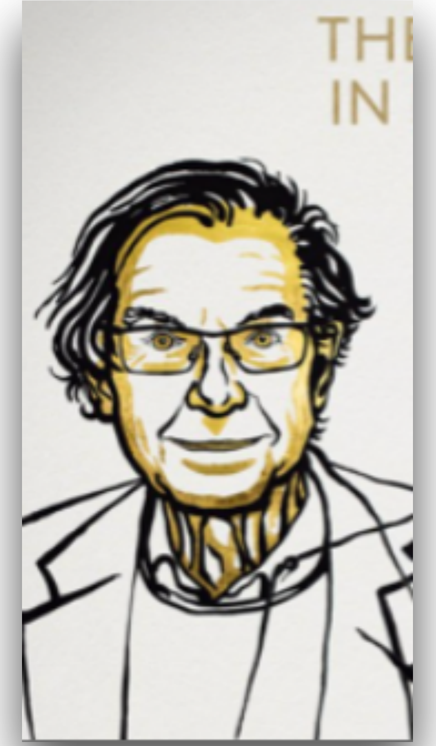
Bootstrapping the relativistic 2-body problem

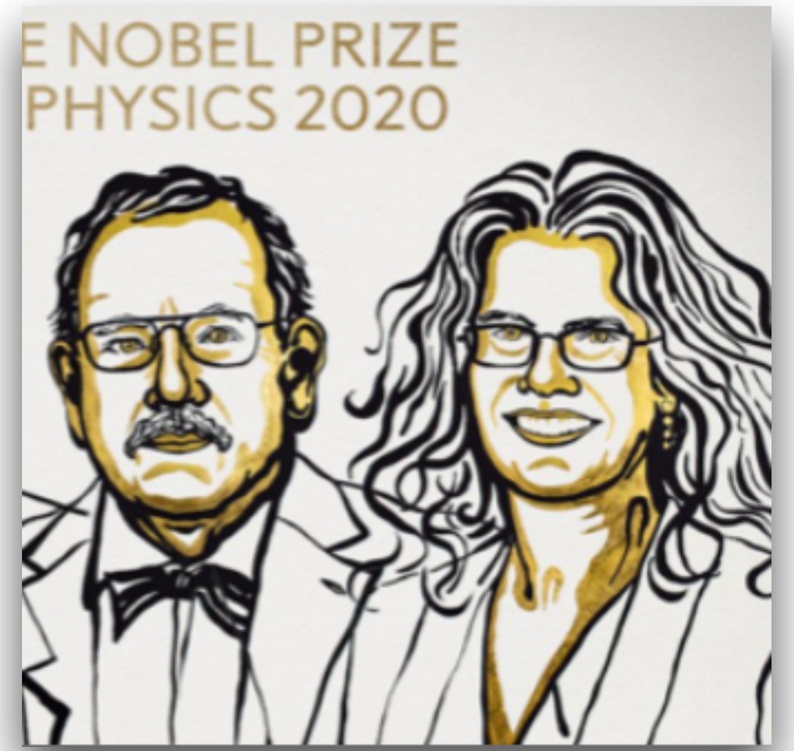


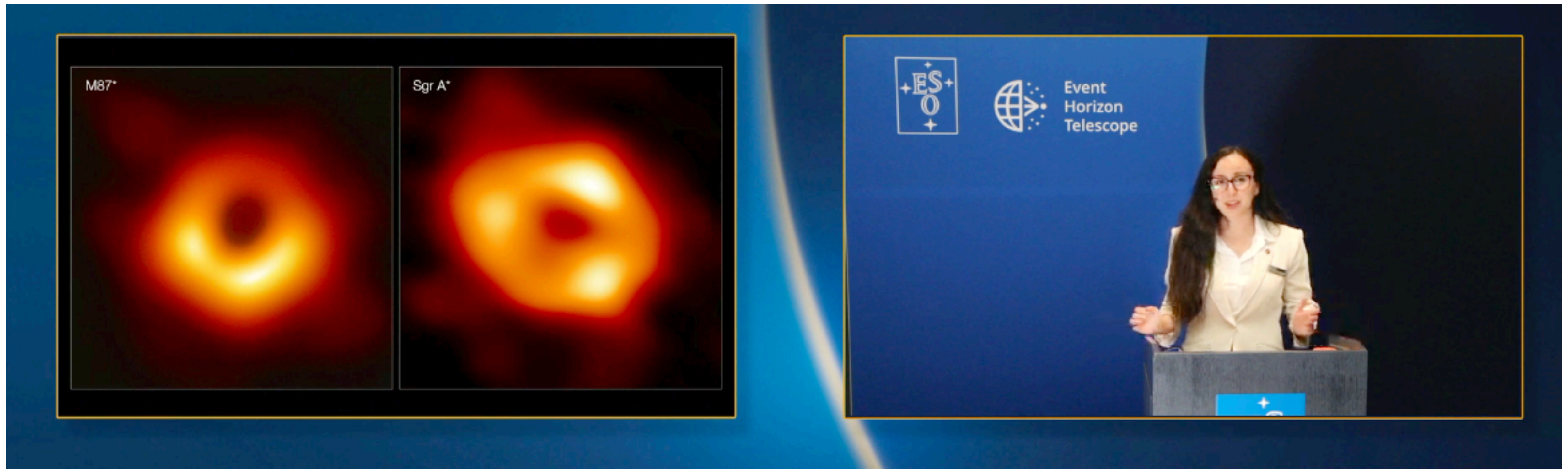
Rafael A. Porto

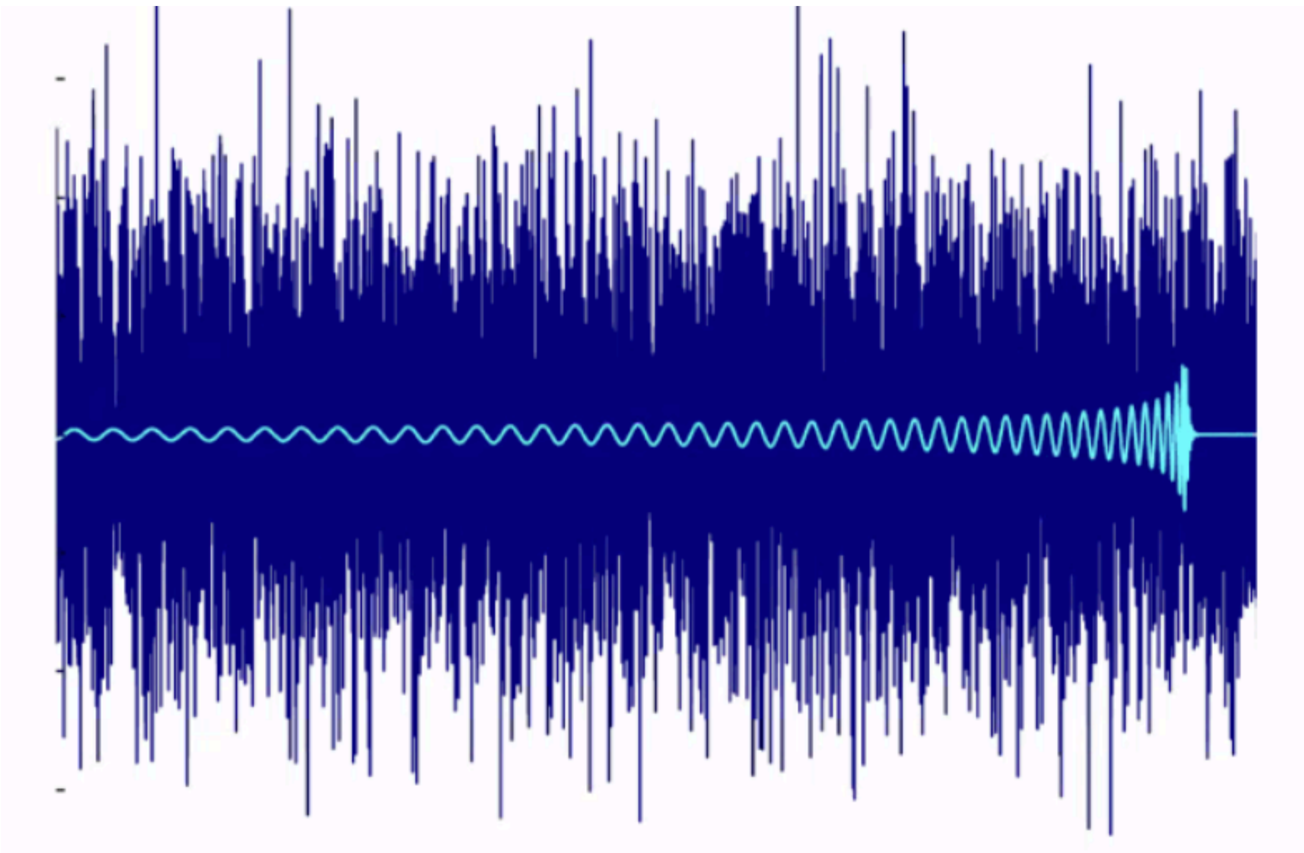


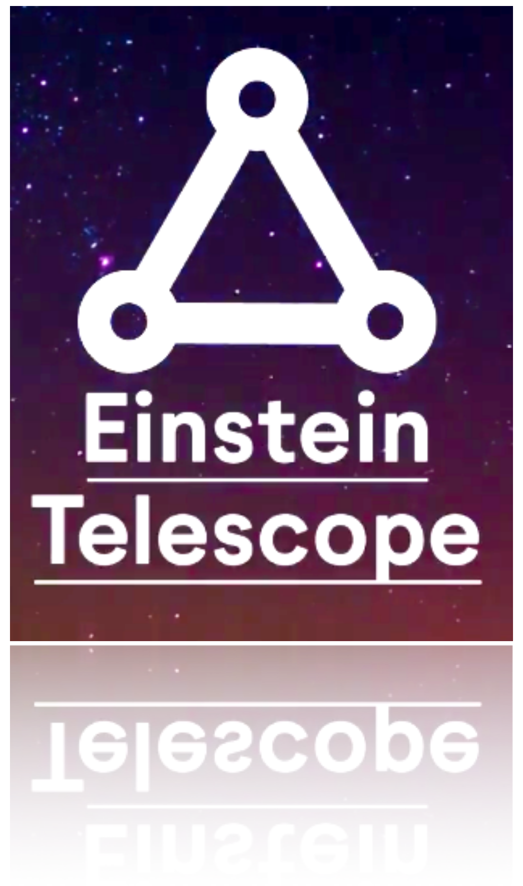
"for the discovery that black hole formation is a robust prediction of the general theory of relativity"

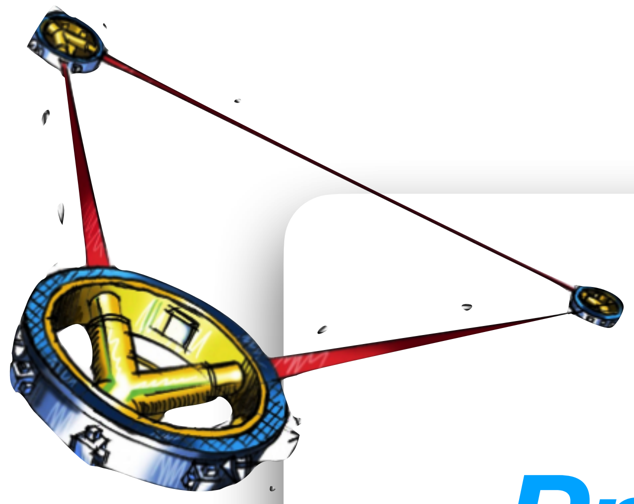
"for the discovery of a supermassive compact object at the centre of our galaxy"









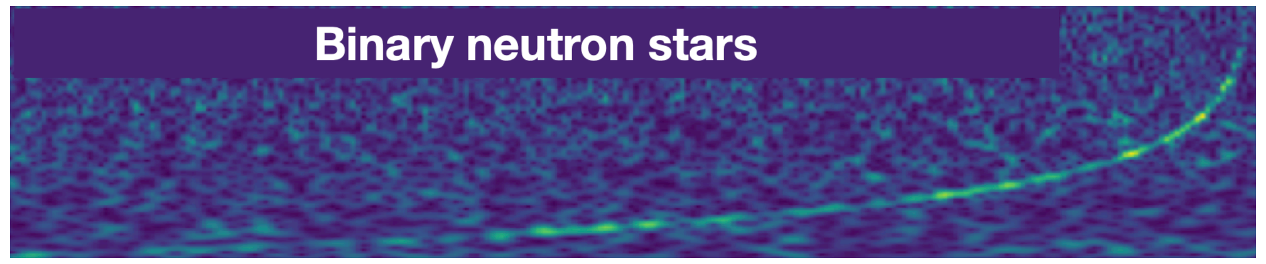


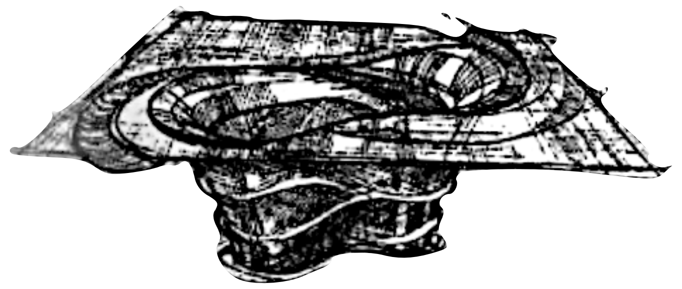
Discovery Potential =

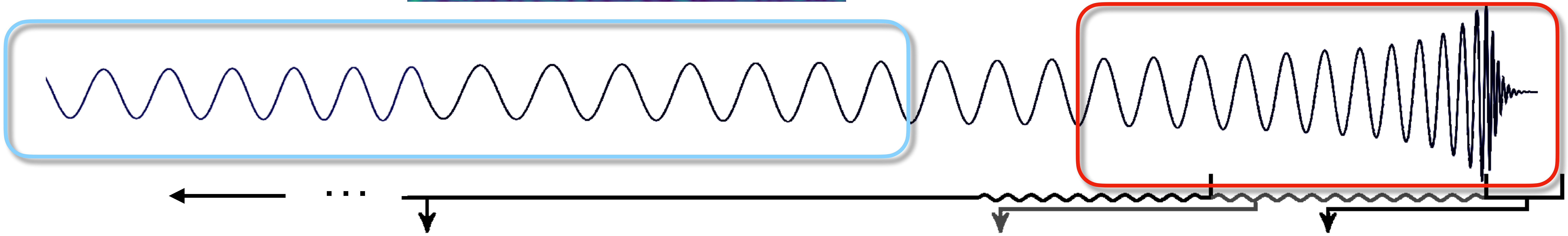
Precise Theoretical Predictions

Challenge in GW Science

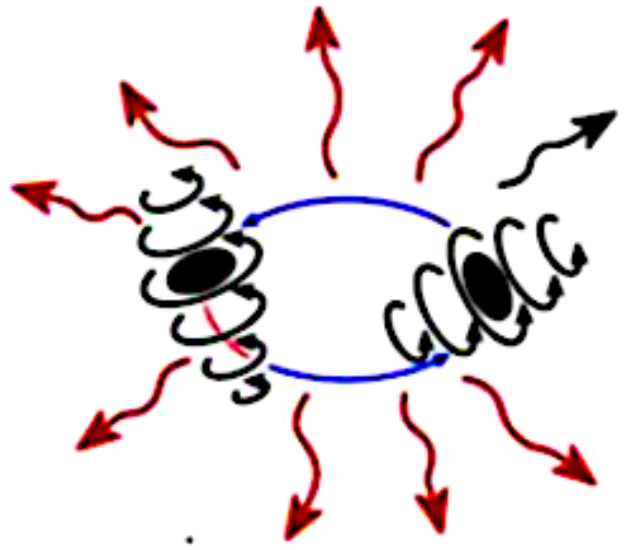
$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^{l}_{i\rho}\Gamma^{\rho}_{ml} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right)$$







Inspiral

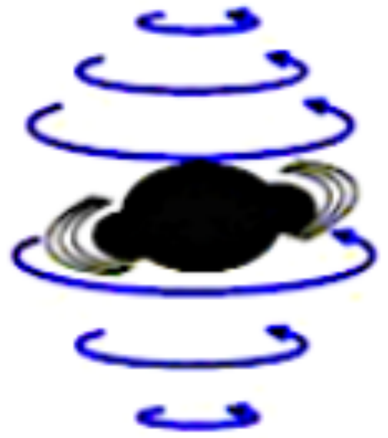


Analytic
(Approx. but fast)

Merger

Numerical
(exact but slow)

Ringing



Analytic/
Perturbative

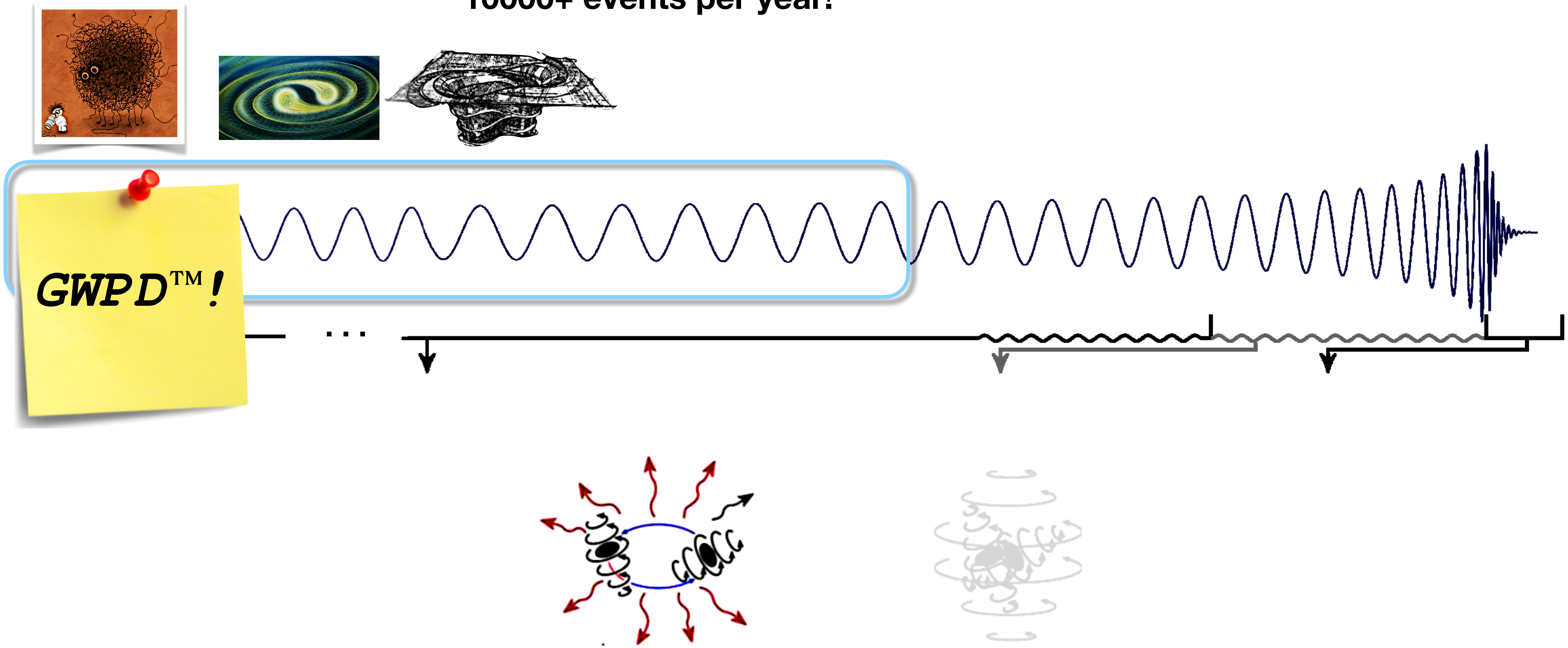
"It's one thing in physics to write down the equations…
but then you have to solve them!…
That sometimes is easier said than done"

Ed Witten

‘GW Precision Data’

10000+ cycles in band @ Design-Sensitivity

10000+ events per year!



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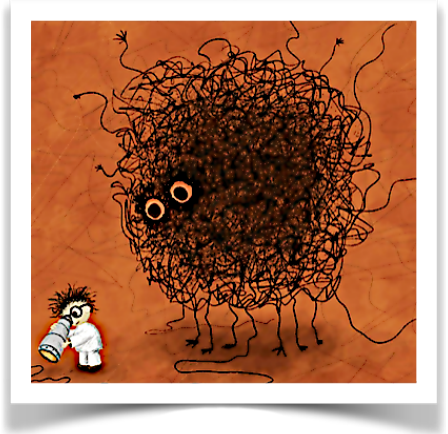
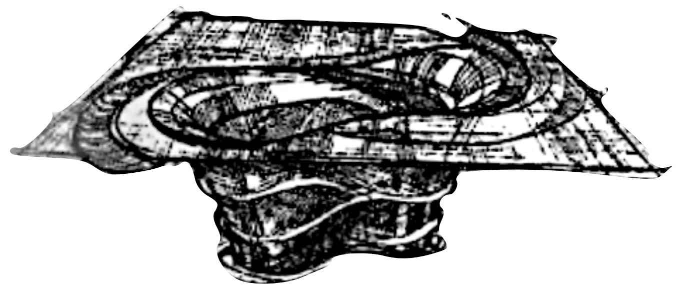
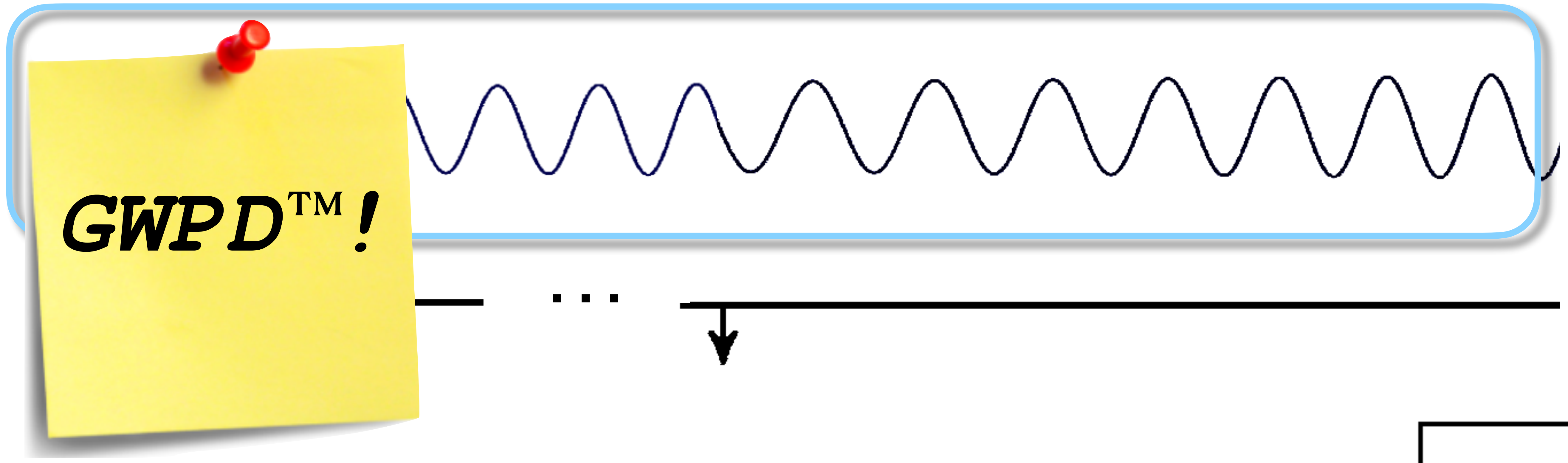




state of the art

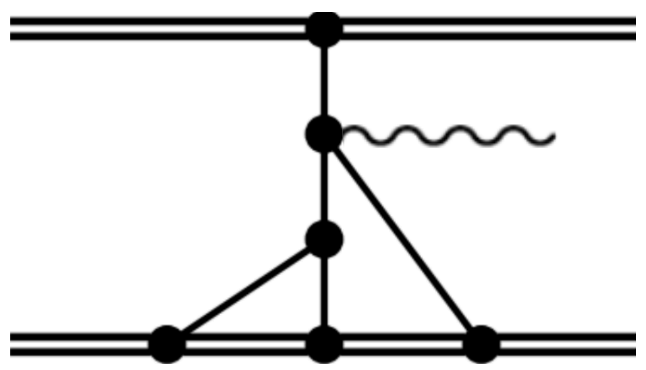


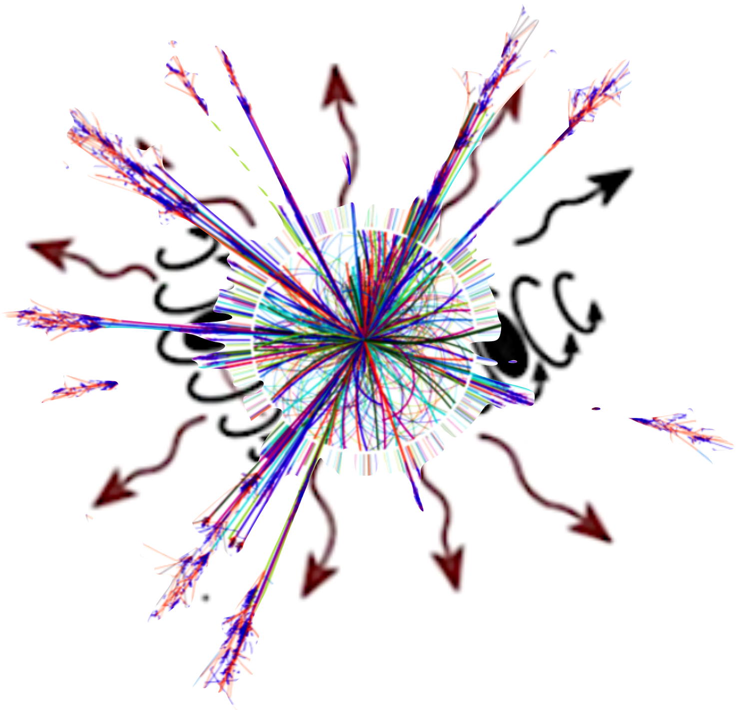
Fourth-order PN accuracy

$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^4\right\}$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$





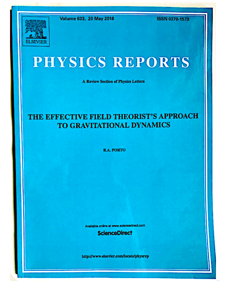
$$4\pi R^2 \mathcal{T} = \frac{\kappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right]$$

The effective field theorist's approach to gravitational dynamics

Rafael A. Porto

Physics Reports

Volume 633, 20 May 2016, Pages 1-104



‘GW Precision Data’

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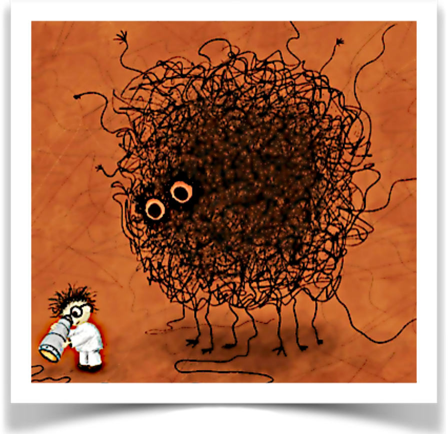
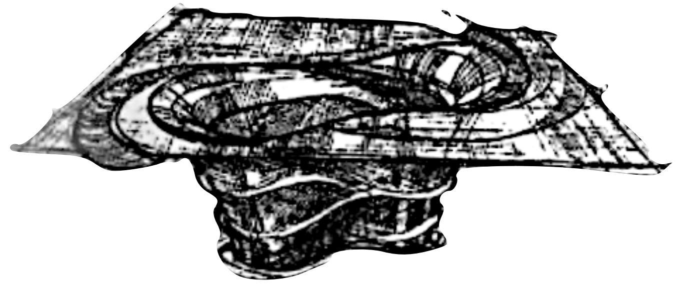
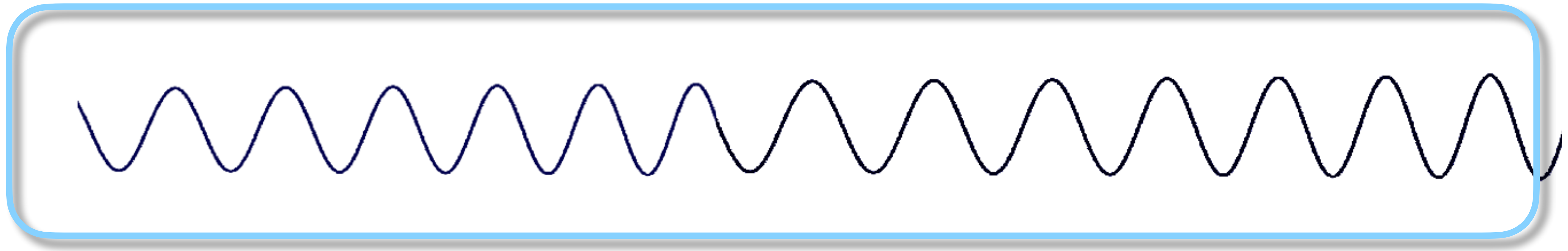
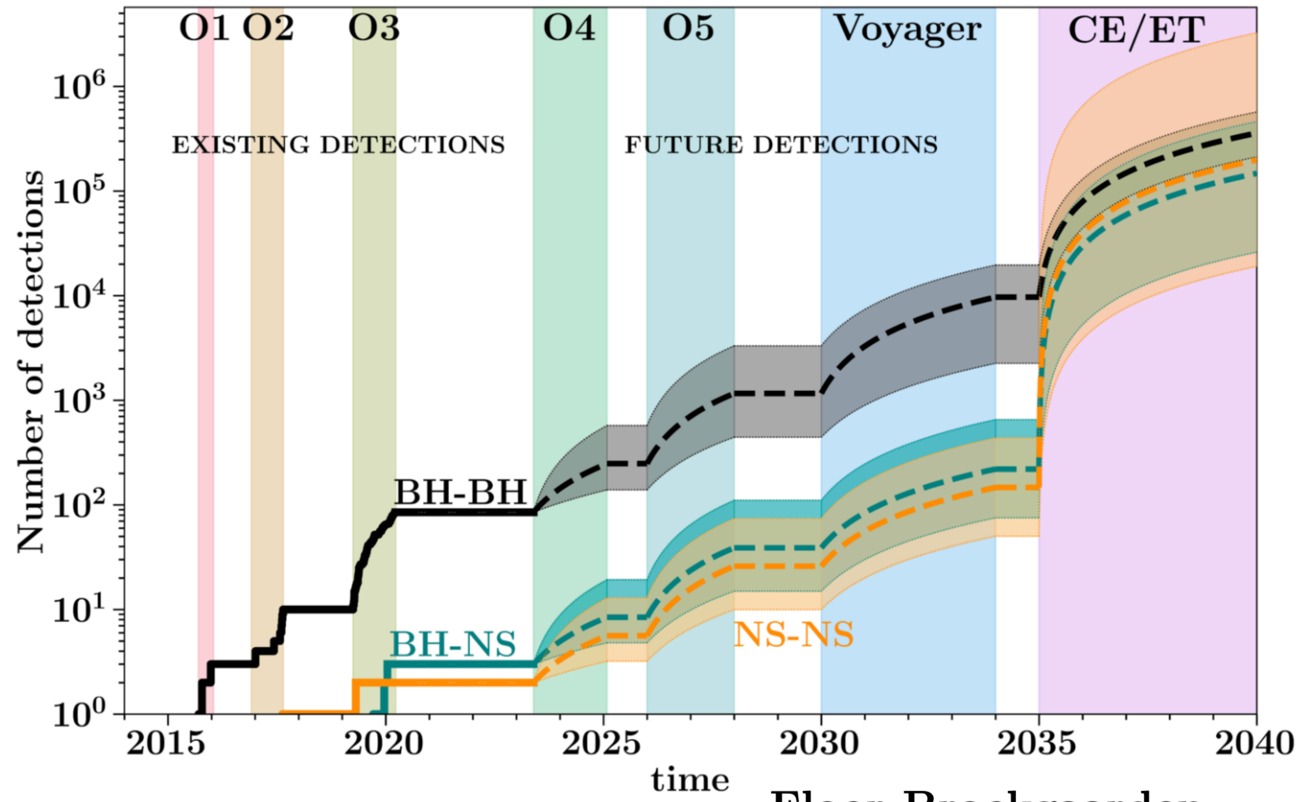
10000+ events per year!







Are we ready for the future?



Floor Broekgaarden

LIGO VIRGO KAGRA

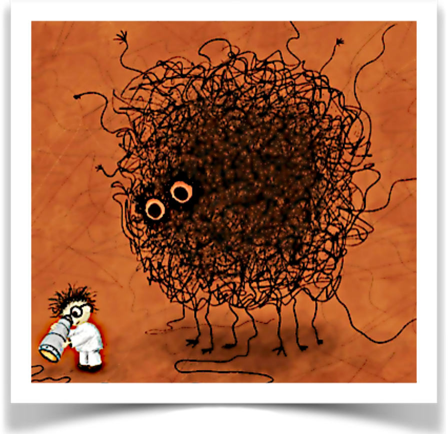
Einstein Telescope

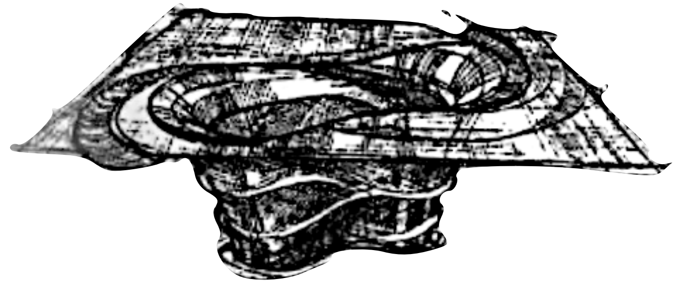
‘Ligo/Virgo’ **‘LISA/ET’** (+20)

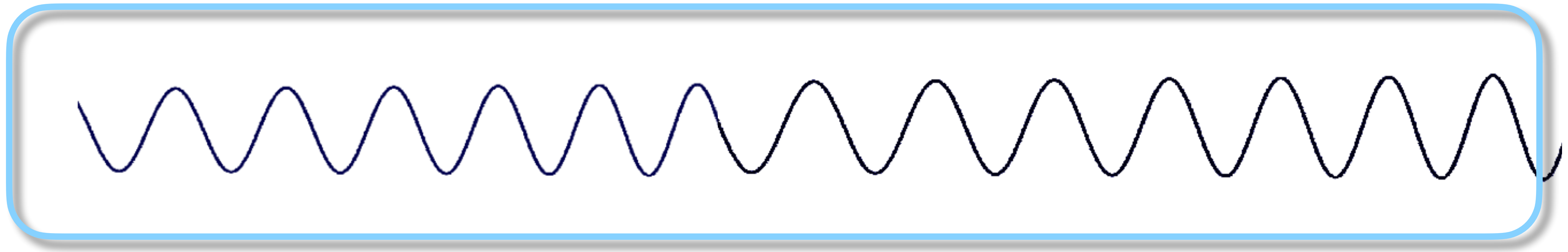
Cobe (92) Planck (13)

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \overbrace{\cdots + [\cdots]x^4}\right\}$$

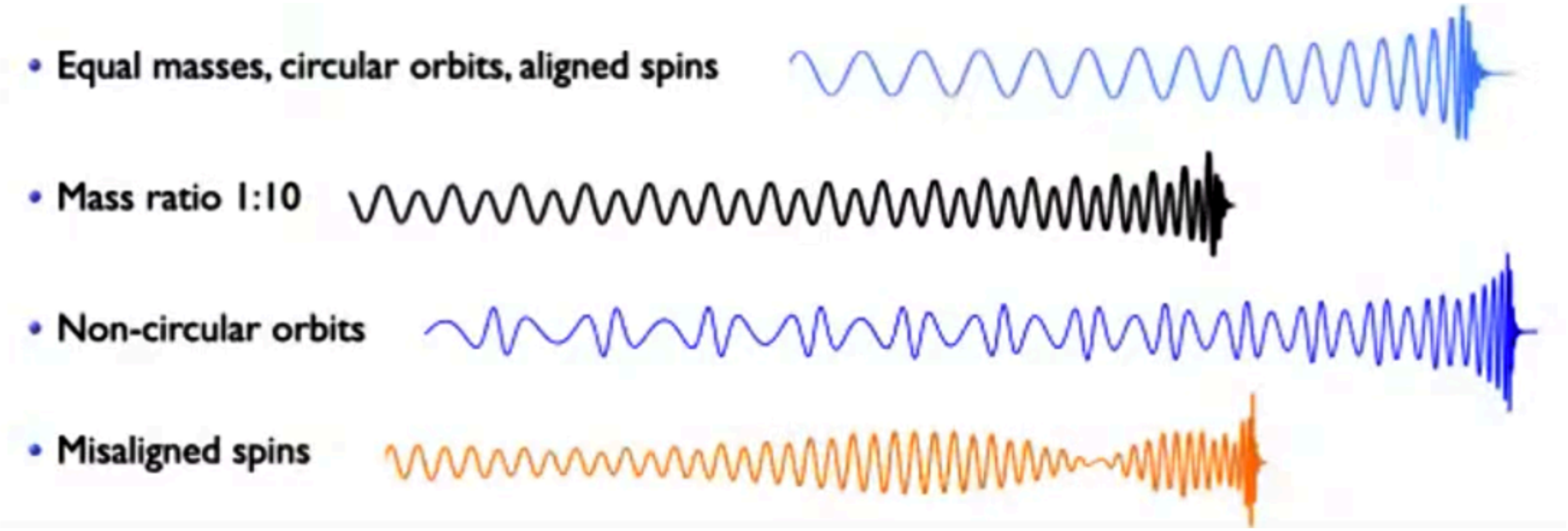
Theoretical uncertainties dominate over planned empirical reach







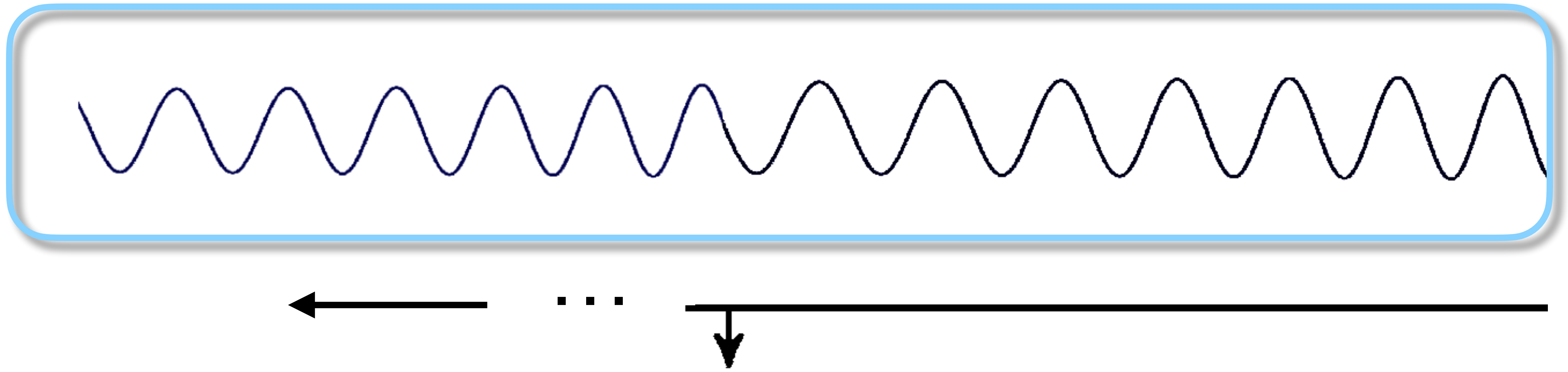
...

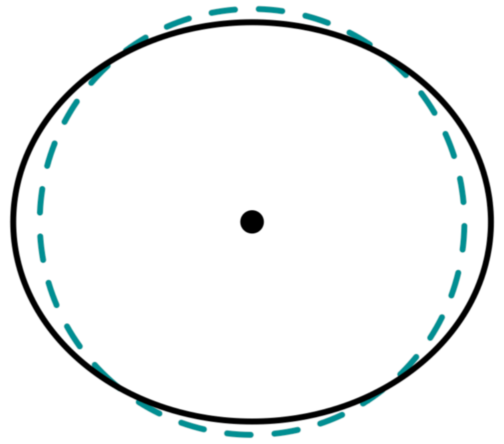


Not GOOD ENOUGH

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{\overbrace{1 + \cdots + [\cdots] x^4}\right\}$$

We haven't reached the analytic precision to distinguish between compact bodies!

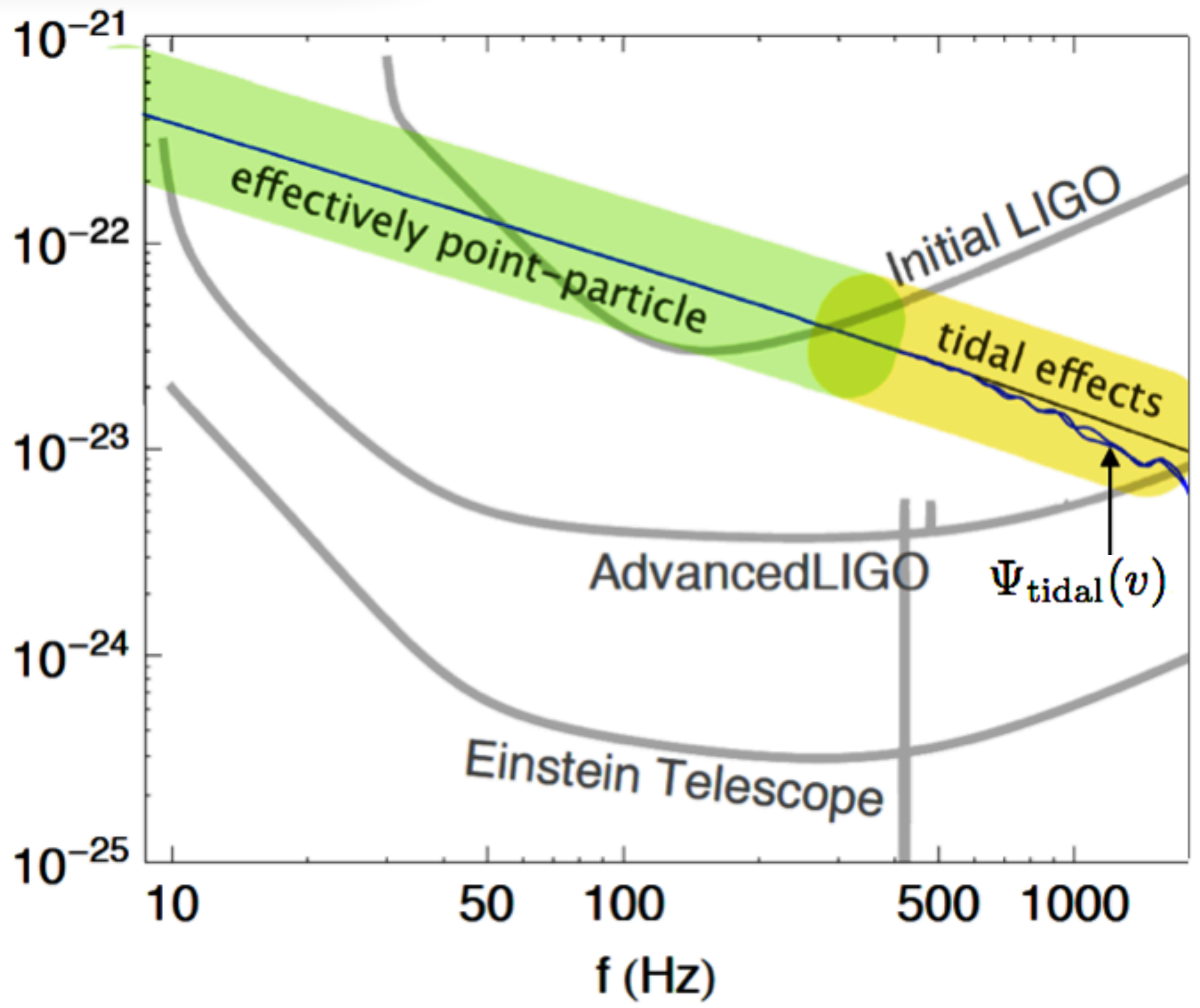




("susceptibility")

$$Q_{ij} = c_E E_{ij}$$

tidal effects



$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^4 + \mathcal{O}(x^5)\right\}$$

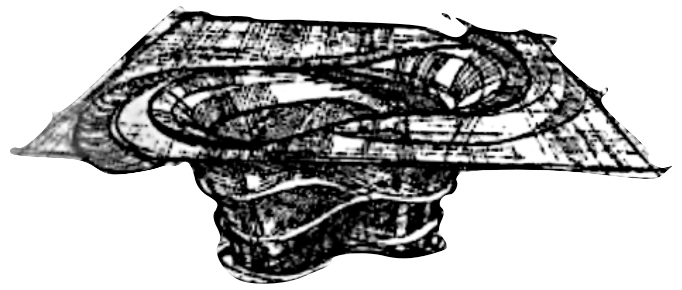
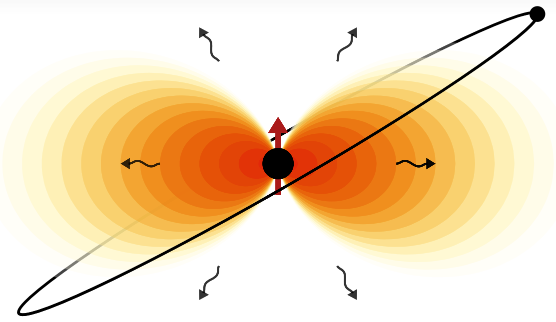
N^5LO

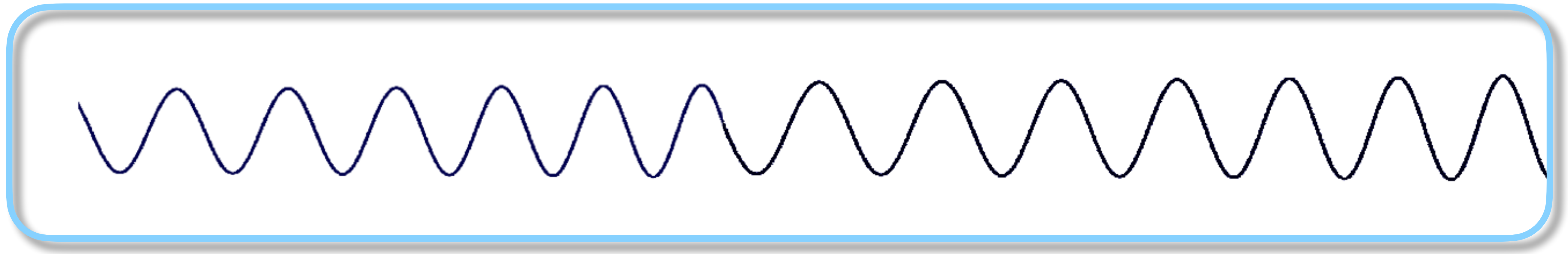
$5PN$

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

e.g. Equation of State of Neutron Stars

We haven't reached the analytic precision to distinguish between compact bodies!



···

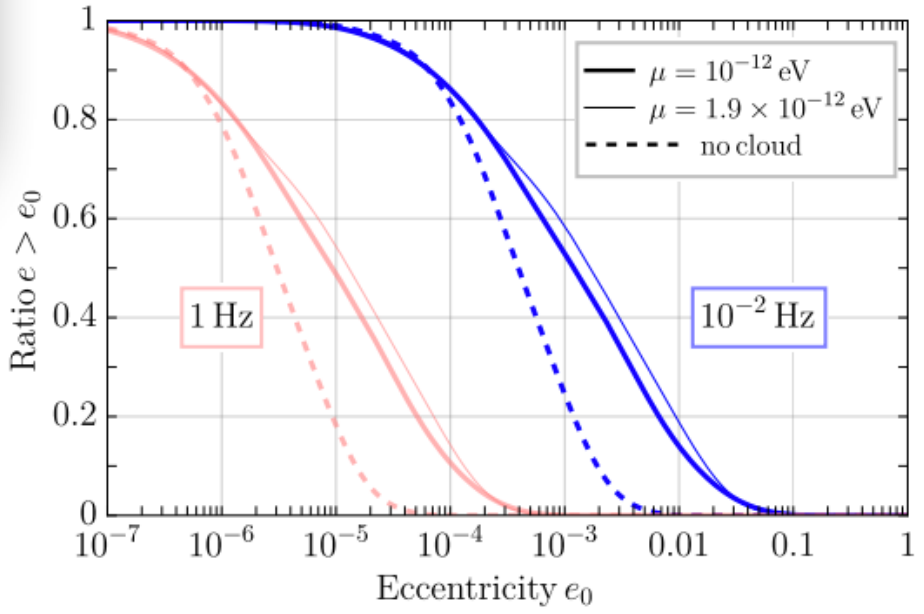
'New Physics' <u>Threshold</u>

Probing ultralight bosons with binary black holes

Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto

Phys. Rev. D 99, 044001 (2019)

Published February 4, 2019



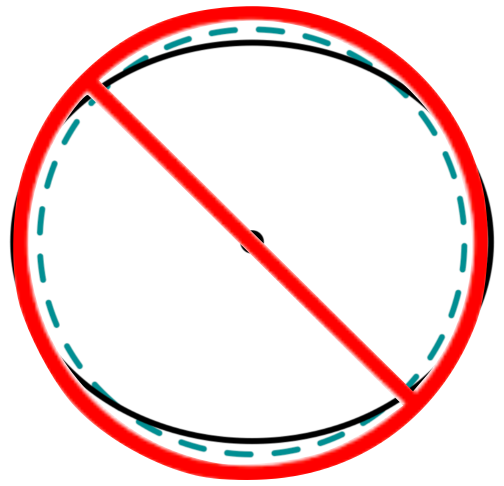
Signatures of Ultralight Bosons in the Orbital Eccentricity of Binary Black Holes

Mateja Bošković, Matthias Koschnitzke, and Rafael A. Porto

Phys. Rev. Lett. 133, 121401 (2024)

Published September 16, 2024

("susceptibility")



vanishes for black holes in Einstein's gravity (4d)

$$Q_{ij} = c_E E_{ij}$$

$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^4 + \mathcal{O}(x^5)\right\}$$

N^5LO 5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

NO 'Standard Model' Background!

Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

The tune of love and the *nature(ness)* of spacetime

*Rafael A. Porto**

Outline of the (remaining) talk…

Discovery Potential =

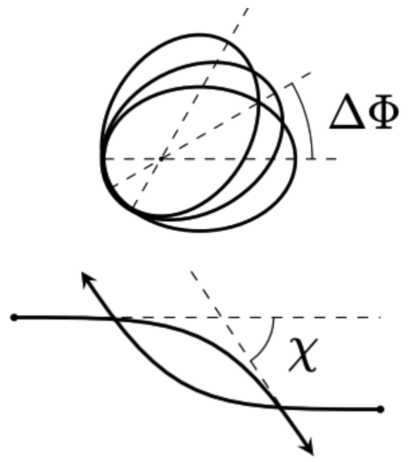
Precise Theoretical Predictions

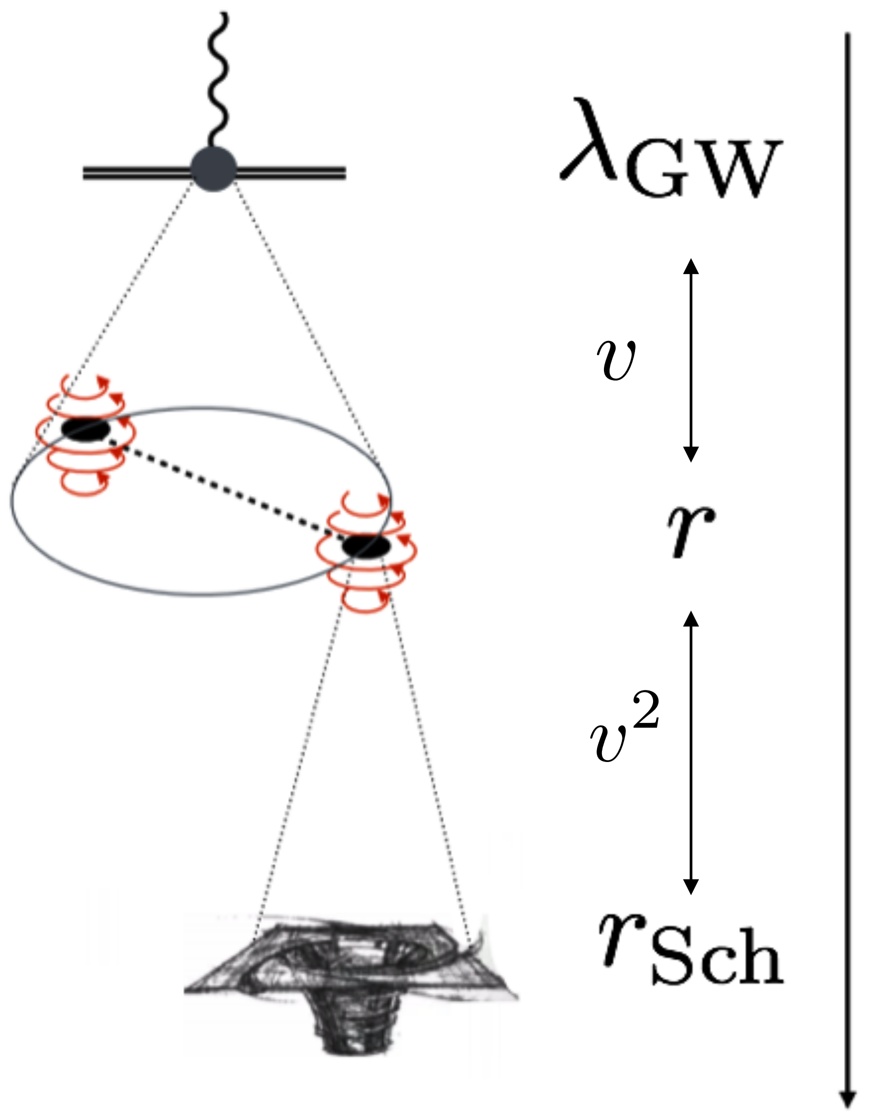
- **Part I: EFT for Bound (PN)/Unbound(PM) states**
- **Part II:** **B**oundary**2B**ound correspondence

Goldberger Rothstein (NRGR 2006)
RAP (2006)
Goldberger Ross (2009)
Foffa Sturani (2011)

Kalin RAP (2020)

EFT approach to GW physics PN PM

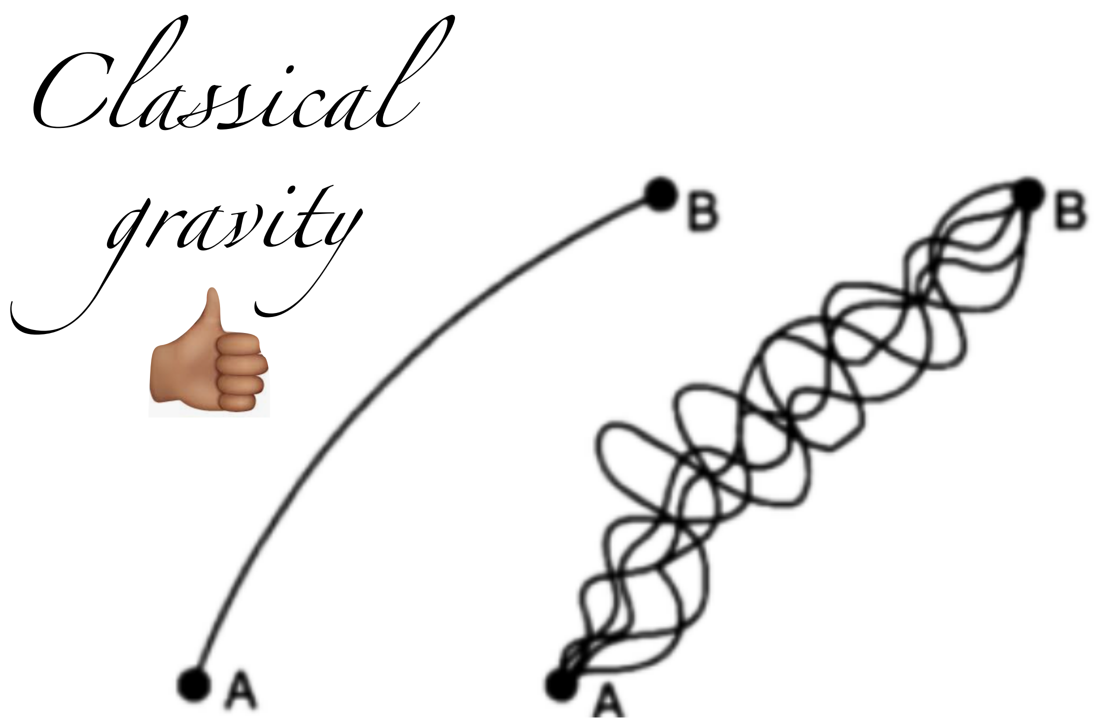




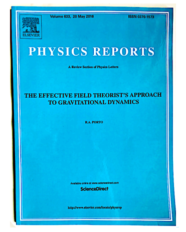
- **Separation of Scales for PN sources:**

$$r_{\rm Sch} \ll r \ll \lambda_{\rm GW}$$

- **Effective Field Theory:**
  Classical effective action (saddle point) *one scale at a time* (method of regions)



$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$


The effective field theorist's approach to gravitational dynamics
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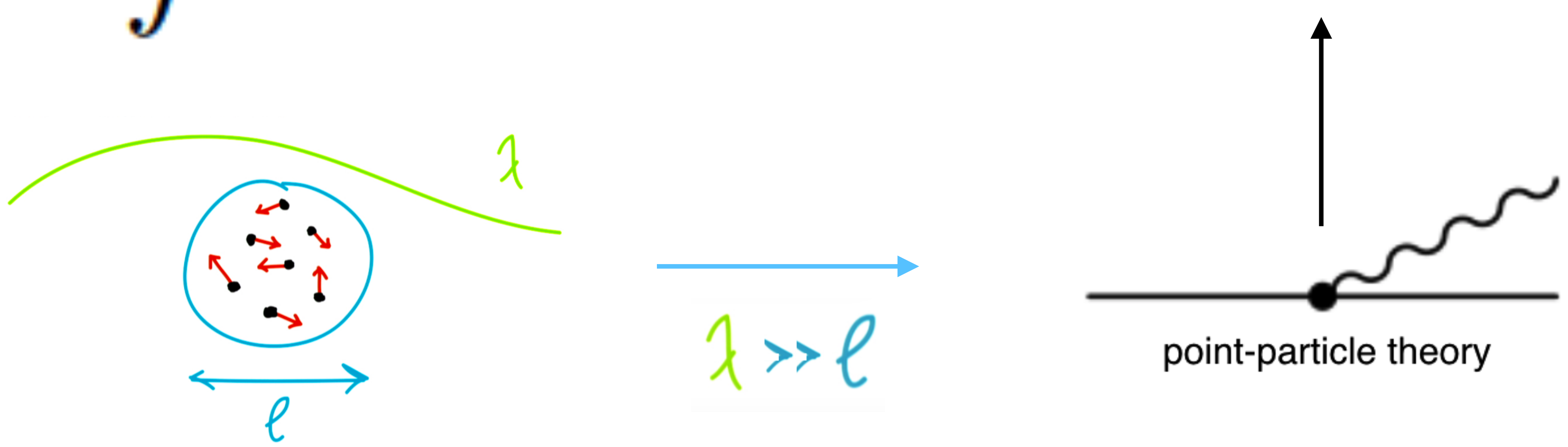
Rafael A. Porto
Volume 633, 20 May 2016, Pages 1-104



$$\ldots = \frac{1}{2} Q_{ij} E^{ij} + \ldots$$

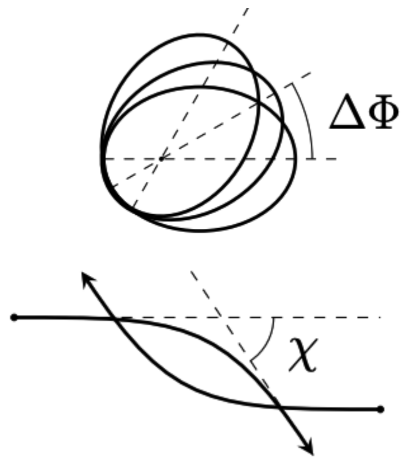
$$S_{\rm pp} = -\sum \frac{m_a}{2} \int d\tau_a\, g_{\mu\nu}(x_a(\tau_a)) v_a^\mu(\tau_a) v_a^\nu(\tau_a) + \ldots$$

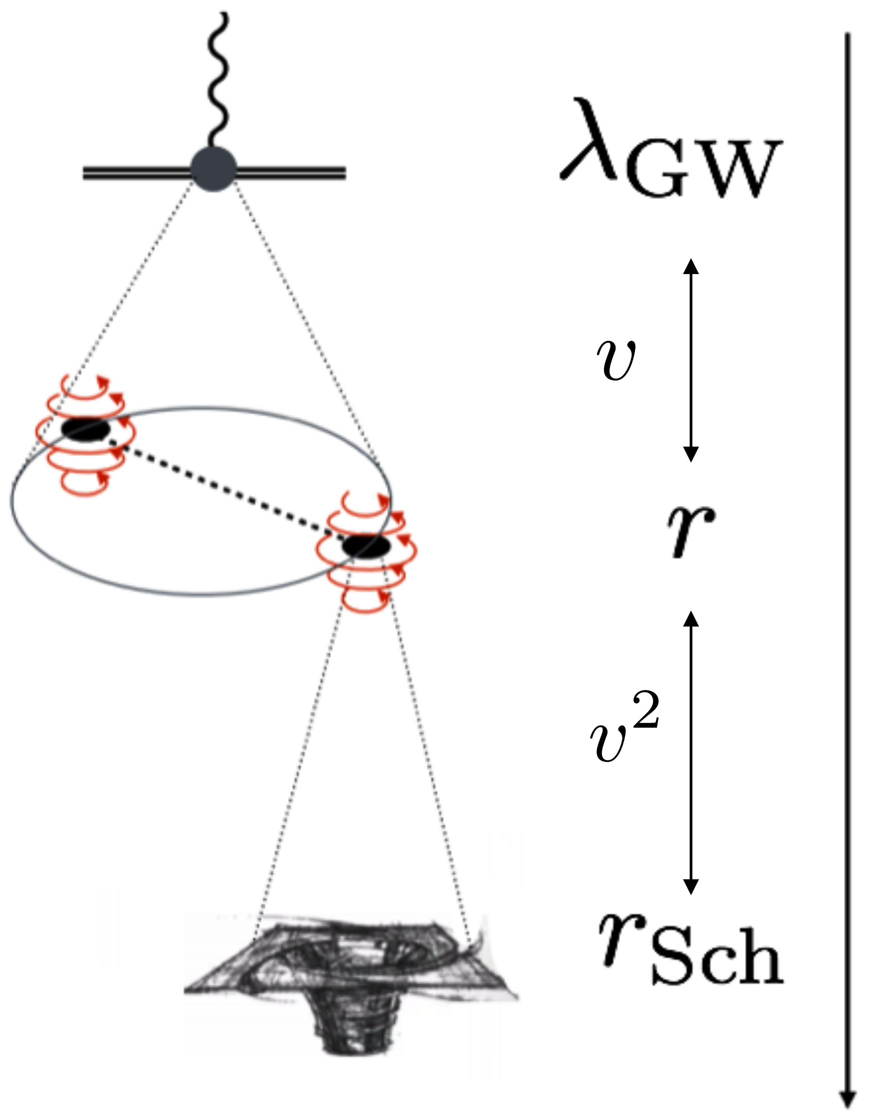
$$\frac{1}{2} \omega_\mu^{ab} S_{ab} u^\mu$$

Goldberger Rothstein (NRGR 2006)
RAP (2006)
Goldberger Ross (2009)
Foffa Sturani (2011)

Kalin **RAP (2020)**

EFT approach to GW physics PN PM





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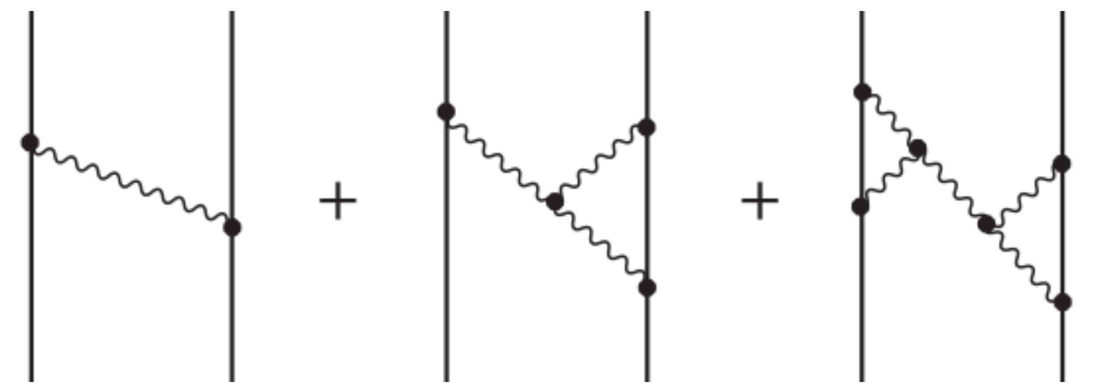
  Classical effective action (saddle point) *one scale at a time* (method of regions)

Classical Electrodynamics in Terms of Direct Interparticle Action[1]

John Archibald Wheeler and Richard Phillips Feynman[2]
Princeton University, Princeton, New Jersey

$$J = -\sum_a m_a c \int (-da_\mu da^\mu)^{\frac{1}{2}} + \sum_{a<b} (e_a e_b / c)$$
$$\times \int\int \delta(ab_\mu ab^\mu)(da_\nu db^\nu) = \text{extremum.} \quad (1)$$



$$S_{\rm red}(T) = \frac{1}{2} T G T + V_3(GT, GT, GT) + \ldots$$

e.g. Duff (70's);
Damour et al. (90's)

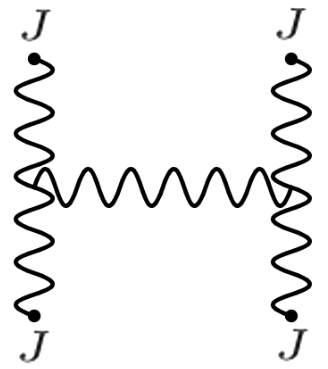
$$\underbrace{{\rm Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{{\rm Im}\, W[x_a]}_{\text{radiation}}$$

$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, e^{iS_{\rm pp}}$$

Radiation modes

Potential modes

$$\log \langle 0|0\rangle^J =$$



UV Divergences: localized sources

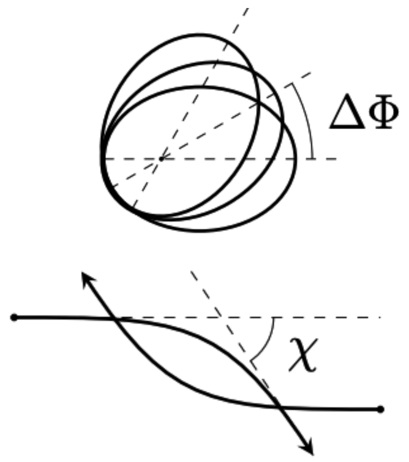
Classical 'loop' = iterated Green's function + point-like sources

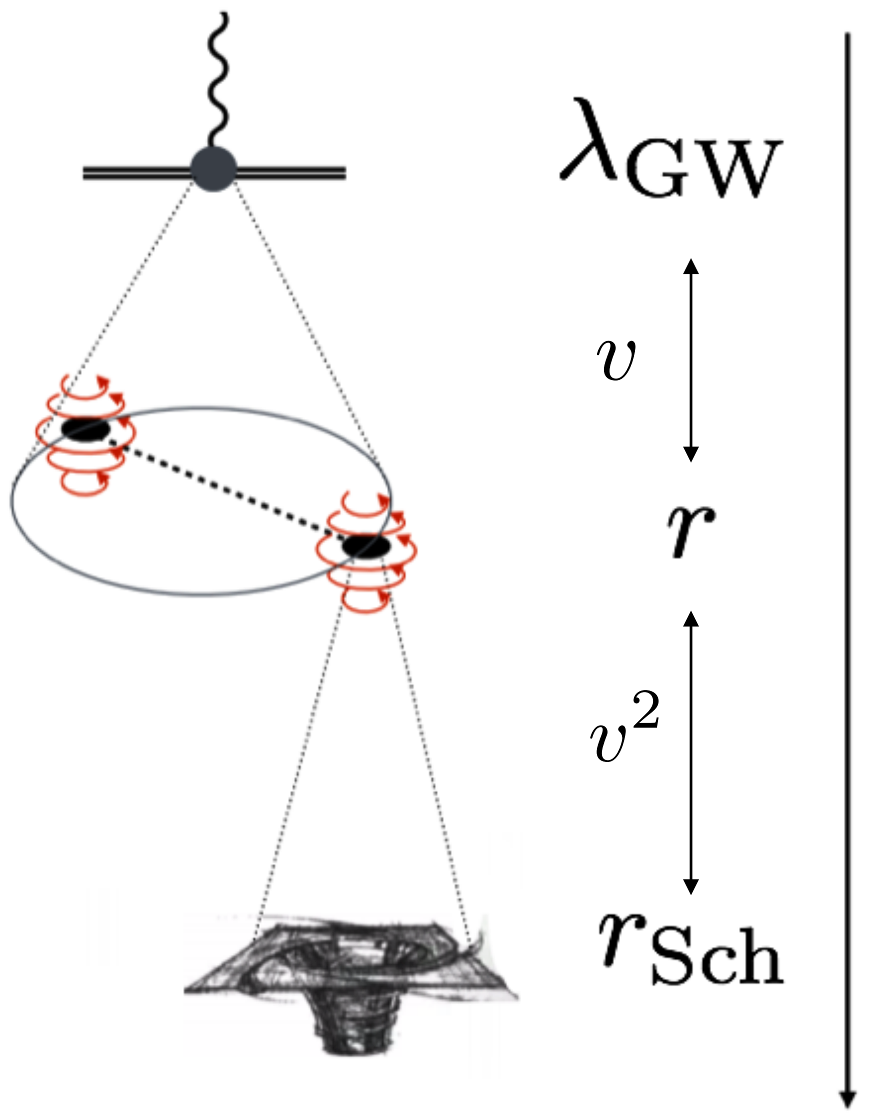
$$\frac{1}{p_0^2 - p^2} = \frac{1}{p^2}\left(-1 + p_0^2/p^2 + \cdots\right)$$

Goldberger Rothstein (NRGR 2006)
RAP (2006)
Goldberger Ross (2009)
Foffa Sturani (2011)

Kalin **RAP (2020)**

EFT approach to GW physics PN PM





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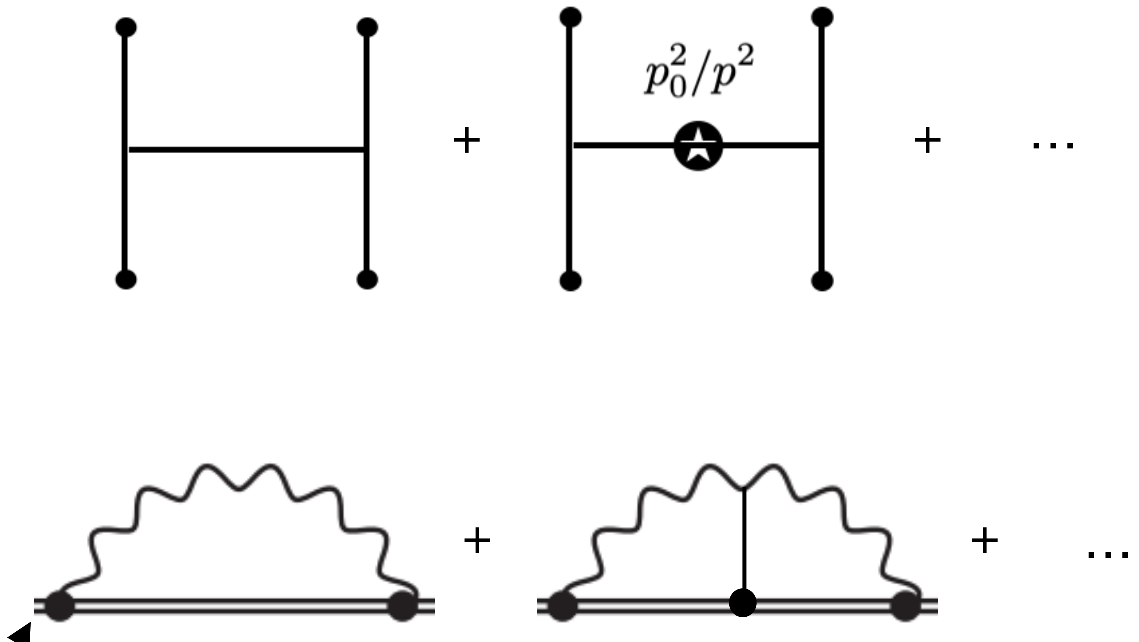


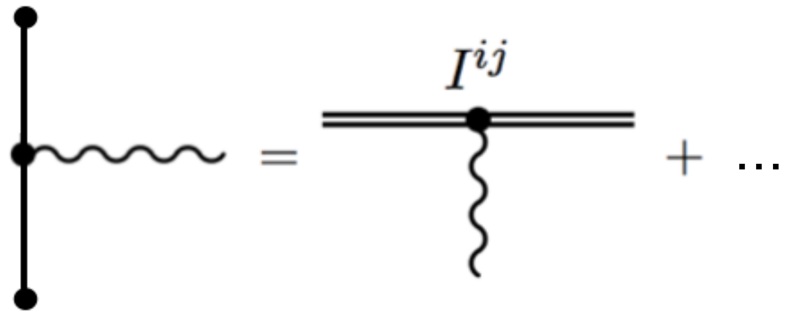
$$\underbrace{{\rm Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{{\rm Im}\, W[x_a]}_{\text{radiation}}$$

$$e^{iW} = \int D[\lambda^{-1}_{\rm rad}]\, D[r^{-1}]\, e^{iS_{\rm pp}}$$

Radiation modes

Potential modes

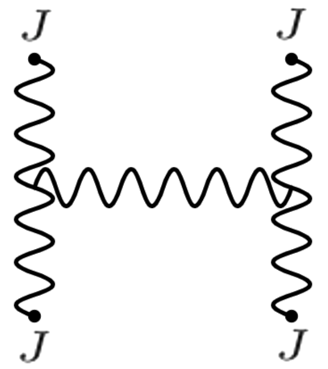
Matching to (source) multipoles (known to NNNLO)



$$\frac{1}{2} I_{ij} E^{ij}$$

binary

$$\log \langle 0|0\rangle^J =$$



UV Divergences: localized sources

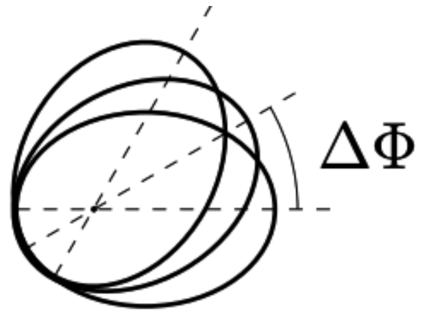
PHYSICAL REVIEW D 110, 044046 (2024)

Gravitational radiation from inspiralling compact binaries to N^{3}LO in the effective field theory approach

Loris Amalberti,[1,2,*] Zixin Yang,[1,†] and Rafael A. Porto[1,‡]

* *Conservative non-spinning*

EFT approach to GW physics PN



$\mathcal{E} < 0$

Halley Hooke **Newton** (16XX)	Droste EIH (1917)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa RAP et al.** (2015-19')	Bluemlein et al Foffa Sturani et al. **RAP Riva Yang** (2021-2024)	**'New Physics Threshold'**
0PN	1PN	2PN	3PN	4PN	5PN	6PN

$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right)$$

$$G^2\left(1+v^2+v^4+v^6+v^8\quad+\cdots\right)$$

$$G^3\left(1+v^2+v^4+v^6\quad+\cdots\right)$$

$$G^4\left(1+v^2+v^4\quad+\cdots\right)$$

$$G^5\left(1+v^2\quad+\cdots\right)$$

$$\vdots \qquad\qquad \vdots$$

LIGO VIRGO KAGRA

'Ligo/Virgo' **'LISA/ET'** (+20)

Cobe (92) Planck (13)

$$\begin{aligned}
\phi_{\rm spin} = -\frac{x^{-5/2}}{32\,\nu}\Bigg[& x^{3/2}\left(\frac{235}{6}\,\hat S_\ell+\frac{125}{8}\,\delta\,\hat\Sigma_\ell\right)+x^2\Big\{\left(-50-25\,\kappa_+\right)\hat S_\ell^2\\
&+\Big[25\,\kappa_-+\delta\left(-50-25\,\kappa_+\right)\Big]\hat S_\ell\,\hat\Sigma_\ell+\Big[-\frac{5}{16}+50\,\nu+\frac{25}{2}\,\delta\,\kappa_-+\left(-\frac{25}{2}+25\,\nu\right)\kappa_+\Big]\hat\Sigma_\ell^2\Big\}\\
&+x^{5/2}\log x\left[\left(-\frac{554345}{2016}-\frac{55}{8}\,\nu\right)\hat S_\ell+\left(-\frac{41745}{448}+\frac{15}{8}\,\nu\right)\delta\,\hat\Sigma_\ell\right]\\
&+x^3\Bigg(\frac{940}{3}\,\pi\,\hat S_\ell+\frac{745}{6}\,\pi\,\delta\,\hat\Sigma_\ell+\Big[-\frac{31075}{126}+60\,\nu+\frac{2215}{48}\,\delta\,\kappa_-+\left(\frac{15635}{84}+30\,\nu\right)\kappa_+\Big]\hat S_\ell^2\\
&+\Big\{\left(-\frac{47035}{336}-\frac{2575}{12}\,\nu\right)\kappa_-+\delta\Big[-\frac{9775}{42}+60\,\nu+\left(\frac{47035}{336}+30\,\nu\right)\kappa_+\Big]\Big\}\hat S_\ell\,\hat\Sigma_\ell\\
&+\Big[-\frac{410825}{2688}+\frac{23535}{112}\,\nu-60\,\nu^2+\left(-\frac{47035}{672}-\frac{2935}{48}\,\nu\right)\delta\,\kappa_-+\left(\frac{47035}{672}-\frac{4415}{56}\,\nu-30\,\nu^2\right)\kappa_+\Big]\hat\Sigma_\ell^2\Bigg)\\
&+x^{7/2}\Big\{\left(-\frac{8980424995}{6096384}+\frac{6586595}{6048}\,\nu-\frac{305}{288}\,\nu^2\right)\hat S_\ell+\left(-\frac{170978035}{387072}+\frac{2876425}{5376}\,\nu+\frac{4735}{1152}\,\nu^2\right)\delta\,\hat\Sigma_\ell\\
&+\left(-100\,\pi-50\,\pi\,\kappa_+\right)\hat S_\ell^2+\Big[50\,\pi\,\kappa_-+\delta\left(-100\,\pi-50\,\pi\,\kappa_+\right)\Big]\hat S_\ell\,\hat\Sigma_\ell+\Big[-\frac{15}{16}\,\pi+100\,\nu\,\pi+25\,\pi\,\delta\,\kappa_-+\left(-25\,\pi+50\,\nu\,\pi\right)\kappa_+\Big]\hat\Sigma_\ell^2\Big\}\\
&+x^4\Bigg(\left(\frac{2388425\,\pi}{3024}-\frac{9925\,\pi}{36}\,\nu\right)\hat S_\ell+\delta\left(\frac{3237995\,\pi}{12096}-\frac{258245\,\pi}{2016}\,\nu\right)\hat\Sigma_\ell+\Big[-\frac{83427805}{72576}-\frac{19720}{63}\,\nu+\frac{475}{24}\,\nu^2+\left(\frac{3284125}{24192}+\frac{1115}{72}\,\nu\right)\delta\,\kappa_-\\
&+\left(\frac{55124675}{145152}-\frac{32825}{756}\,\nu+\frac{475}{48}\,\nu^2\right)\kappa_+\Big]\hat S_\ell^2+\Big\{\left(-\frac{35419925}{145152}-\frac{975955}{2016}\,\nu-\frac{10345}{144}\,\nu^2\right)\kappa_-\\
&+\delta\Big[-\frac{66536845}{72576}-\frac{109535}{378}\,\nu+\frac{475}{24}\,\nu^2+\left(\frac{35419925}{145152}-\frac{89065}{1512}\,\nu+\frac{475}{48}\,\nu^2\right)\kappa_+\Big]\Big\}\hat S_\ell\,\hat\Sigma_\ell\\
&+\Big[-\frac{17815050265}{48771072}+\frac{26426305}{41472}\,\nu+\frac{12570535}{48384}\,\nu^2-\frac{475}{24}\,\nu^3+\left(-\frac{35419925}{290304}-\frac{2571605}{24192}\,\nu-\frac{5885}{288}\,\nu^2\right)\delta\,\kappa_-\\
&+\left(\frac{35419925}{290304}-\frac{19990295}{145152}\,\nu+\frac{479845}{6048}\,\nu^2-\frac{475}{48}\,\nu^3\right)\kappa_+\Big]\hat\Sigma_\ell^2\Bigg)\Bigg],
\end{aligned}$$

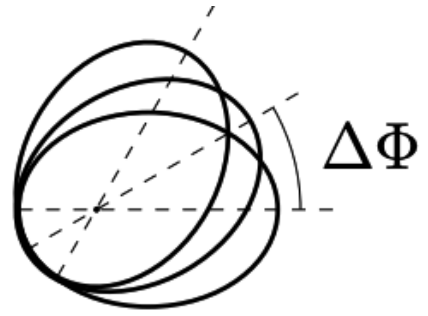
4PN spin!
Cho **RAP** Yang
(2022)

Galley Leibovich **RAP** Ross 1511.07379

Foffa **RAP** Rothstein Sturani 1903.05118

RAP Rothstein 1703.06433

EFT approach to GW physics *PN*



$\mathcal{E} < 0$

Halley Hooke **Newton** (16XX)	Droste EIH (1916)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa RAP et al.** (2015-19')		
0PN	**1PN**	**2PN**	**3PN**	**4PN**	**5PN**	**6PN**

$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right)$$

$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right)$$

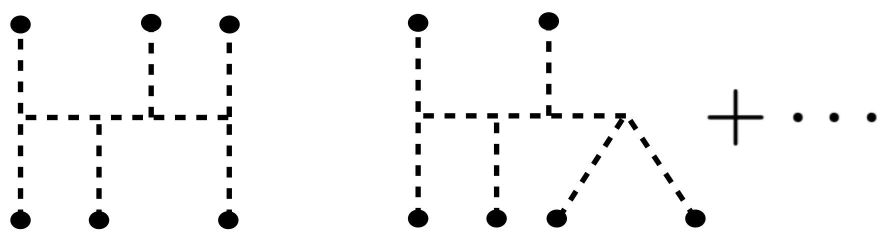
$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right)$$

$$G^4\left(1 + v^2 + v^4 + \cdots\right) \leftarrow \log v$$

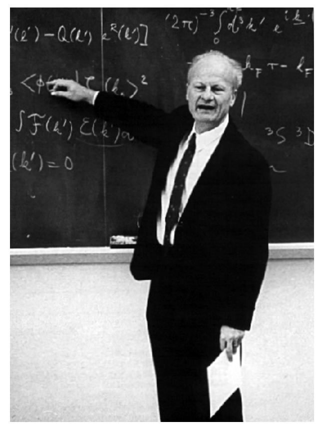
$$G^5\left(1 \right.$$

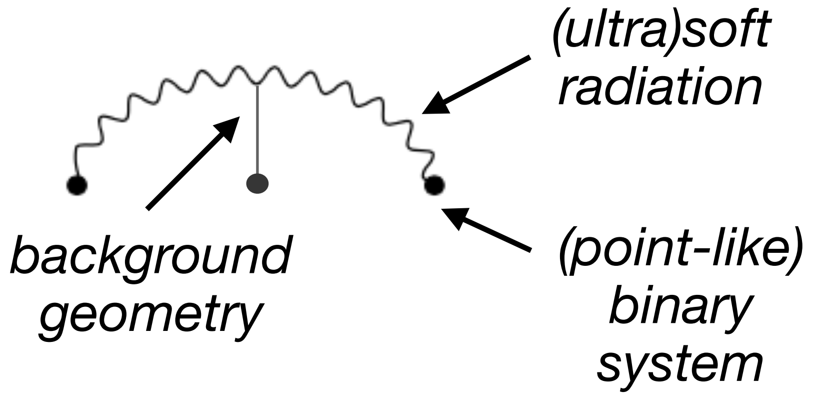
"Tail effect" (scattering off of the geometry sourced by the binary)

Lamb shift and the gravitational binding energy for binary black holes
R. A. Porto, Phys. Rev. D **96**, 024063 (2017).

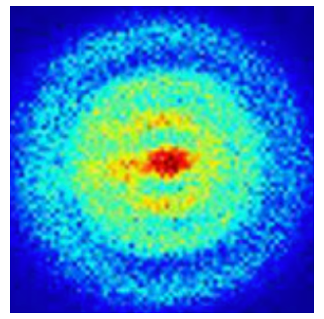


$$\equiv \int_{k_1,k_2,k_3,k_4} \frac{N_{49}}{k_1^2\, p_2^2\, k_3^2\, p_4^2\, k_{12}^2\, k_{13}^2\, k_{23}^2\, k_{24}^2\, k_{34}^2},$$





EFT approach to Atomic physics



$$(e^2/4\pi)\left(\frac{1}{2m}(\boldsymbol{qa}-\boldsymbol{aq})+\frac{4q^2}{3m^2}\boldsymbol{a}\left(\ln\frac{m}{\lambda_{\min}}-\frac{3}{8}\right)\right), \quad (24)$$

which shows the change in magnetic moment and the Lamb shift as interpreted in more detail in B.[13]

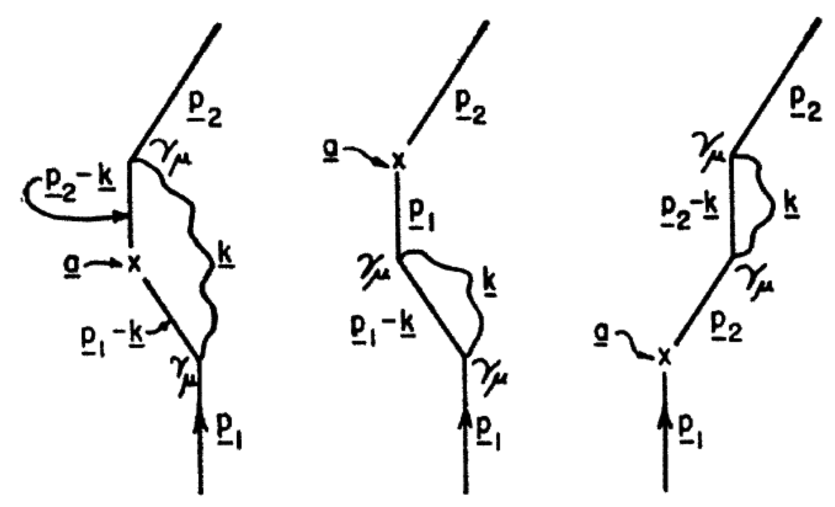
[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\max}-1=\ln\lambda_{\min}$ used by the author should have been $\ln 2k_{\max}-5/6=\ln\lambda_{\min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,

. The author feels unhappily responsible for the very considerable delay in the publication of French's result occasioned by this error. This footnote is appropriately numbered.

Space-Time Approach to Quantum Electrodynamics

R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)



a. Eq.12 b. Eq.13 c. Eq.14

H. A. Bethe, The electromagnetic shift of energy levels, Phys. Rev. **72**, 339 (1947).
F. J. Dyson, The electromagnetic shift of energy levels, Phys. Rev. **73**, 617 (1948).
J. B. French and V. F. Weisskopf, The electromagnetic shift of energy levels, Phys. Rev. **75**, 1240 (1949).
N. M. Kroll and W. E. Lamb, On the self-energy of a bound electron, Phys. Rev. **75**, 388 (1949).

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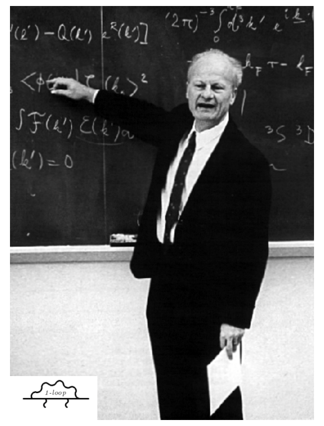
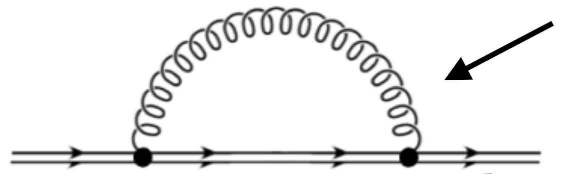
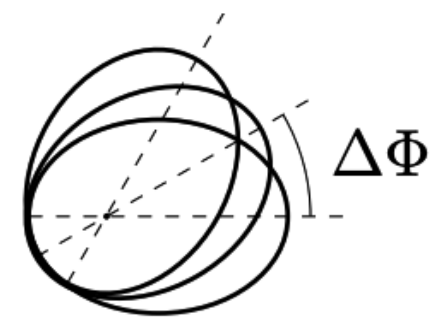




EFT approach to GW physics *PN*



$\mathcal{E} < 0$

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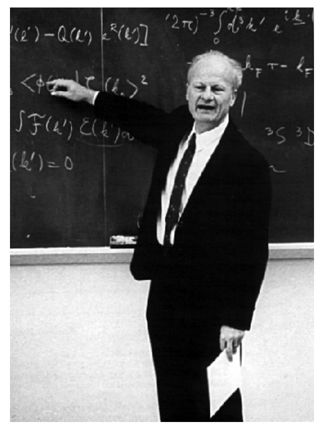
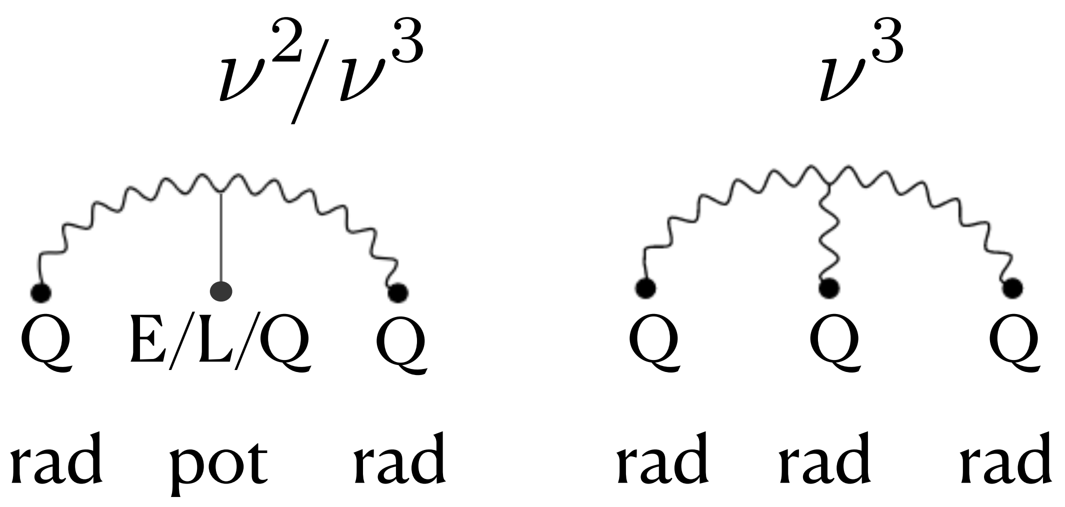
$$1$$

Novel nonlocal-in-time memory effects!

Conservative-like effects from radiation-reaction square!

(apparent) discrepancy with PM EFT results resolved!





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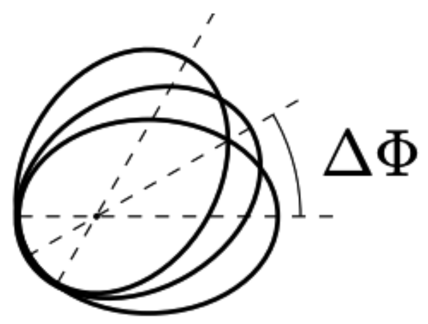
EFT approach to GW physics *PN*



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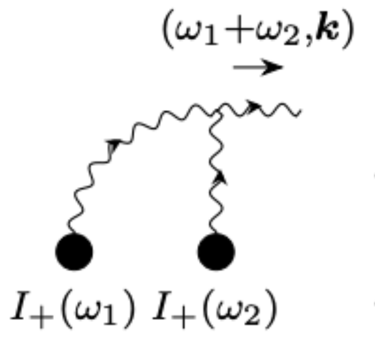
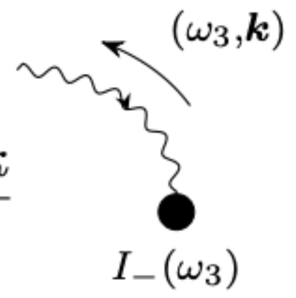
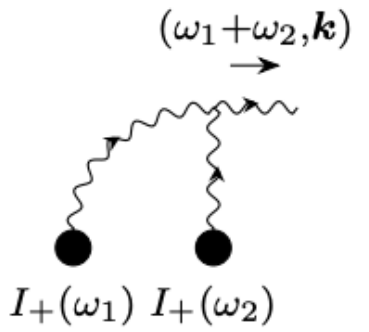
$$G^5\left(1+v^2+\cdots\right)$$

$$1$$

Novel ν^3 nonlocal-in-time memory effects!

$$\int_{\boldsymbol{k}}\left[\frac{-i\kappa}{2}\right.$$  $$\left.\right]^{\mu\nu}\frac{iP_{\mu\nu\rho\sigma}}{(\omega_1+\omega_2+i0^+)^2-\boldsymbol{k}^2}\left[\frac{-i\kappa}{2}\right.$$  $$\left.\right]^{\rho\sigma}$$

$$\left[\lim_{\omega_1+\omega_2\to 0}\right.$$  $$\left.\right]^{\mu\nu}=\frac{1}{\omega_1+\omega_2+i0^+}T^{\mu\nu}_{\rm SFL(M)}(\omega_1+\omega_2,\boldsymbol{k})\,.$$

$$S_{\rm SFL(M)}=-\frac{2G^2}{35}\int {\rm d}t\left[I_+^{(6)ij}(t)I_+^{jk}(t)I_-^{(2)ki}(t)-\frac{\rm d}{{\rm d}t}\left\{I_-^{(2)ij}(t)\int {\rm d}\tau\,\vartheta(t-\tau)I_+^{(6)jk}(\tau)I_+^{ki}(\tau)\right\}\right]$$

↑

Boundary term responsible for flux of CANONICAL angular momentum

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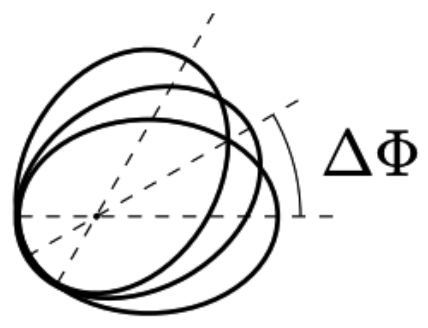
EFT approach to GW physics *PN*



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$$1$$

Novel ν^3 nonlocal-in-time memory effects!

Also in the conservative sector:

$$i\Delta_F(\tau, \boldsymbol{x}=0) = -\int \frac{d\omega}{2\pi}\int_{\boldsymbol{k}} \frac{e^{-i\omega\tau}}{\omega^2 - \boldsymbol{k}^2 + i0} = \frac{i}{4\pi}\int \frac{d\omega}{2\pi}|\omega| e^{-i\omega\tau}$$

$$S^{\text{cons}}_{(\text{M})} = \frac{8G^2\pi^2}{35}\int \frac{d\omega d\omega_1}{(2\pi)^2}\int_{\boldsymbol{q},\boldsymbol{k}} \frac{f(\omega,\omega_1)}{[\omega^2 - \boldsymbol{k}^2 + i0^+][\omega_1^2 - \boldsymbol{q}^2 + i0^+]} I^{ij}(\omega_1) I^{jk}(\omega-\omega_1) I^{ki}(-\omega)$$
$$= -\frac{G^2}{70}\int \frac{d\omega d\omega_1}{(2\pi)^2}|\omega||\omega_1| f(\omega,\omega_1) I^{ij}(\omega_1) I^{jk}(\omega-\omega_1) I^{ki}(-\omega)\,,$$

Cannot keep mass polynomiality AND locality at the same time! ONLY AT G4:

$$S^{\text{cons}}_{G^4(\text{M})} = -\frac{G^2}{70}\int \frac{d\omega d\omega_1}{(2\pi)^2}\omega^2\omega_1^2\left(2\omega^4 - 6\omega^3\omega_1 + 15\omega^2\omega_1^2 - 6\omega\omega_1^3 + 2\omega_1^4\right) I^{ij}_{(1)}(\omega_1) I^{jk}_{(0)}(\omega-\omega_1) I^{ki}_{(1)}(-\omega)$$
$$= \frac{G^2}{5}\int dt\left(\frac{1}{7} I^{(2)ij} I^{(3)jk} I^{(3)ik} - \frac{1}{2} I^{ij} I^{(4)jk} I^{(4)ik}\right)_{G^2},$$

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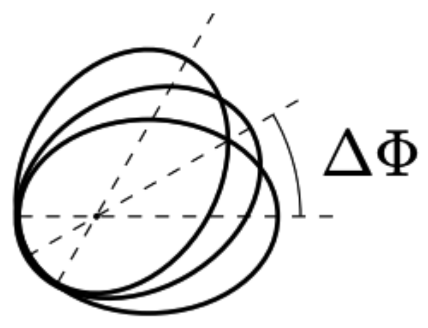

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$$G^4\left(1 + v^2 + v^4 + \cdots\right)$$

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$$1$$

Conservative-like effects from radiation-reaction square!

$$S^{\rm cons}_{({\rm RR}^2)} = \frac{i}{10\pi}\int {\rm d}t_1{\rm d}t_2 I^{ij}(t_1)\frac{\mathscr{P}}{t_1 - t_2} I^{(5)ij}_{({\rm RR})}(t_2)\,,$$

↓

$$\begin{aligned} S^{\rm cons}_{({\rm RR}^2)} = -\frac{2G^2}{25\pi^2}\int {\rm d}t_1{\rm d}t_2 \frac{\mathscr{P}}{t_1 - t_2} I_{ij}(t_1)\Big\{ & L_{k\langle i}(t_2)\int {\rm d}t_3 \frac{\mathscr{P}}{t_2 - t_3} I^{(7)}_{j\rangle k}(t_3) \\ & + 4Q^{(1)}_{k\langle i}(t_2)\int {\rm d}t_3\frac{\mathscr{P}}{t_2 - t_3} I^{(7)}_{j\rangle k}(t_3) + 5Q^{(2)}_{k\langle i}(t_2)\int {\rm d}t_3\frac{\mathscr{P}}{t_2 - t_3} I^{(6)}_{j\rangle k}(t_3) \\ & + 2Q^{(3)}_{k\langle i}(t_2)\int {\rm d}t_3\frac{\mathscr{P}}{t_2 - t_3} I^{(5)}_{j\rangle k}(t_3) + Q_{k\langle i}(t_2)\int {\rm d}t_3\frac{\mathscr{P}}{t_2 - t_3} I^{(8)}_{j\rangle k}(t_3)\Big\}\,. \end{aligned}$$

↑

Cannot keep mass polynomiality AND locality at the same time! ONLY AT G4:

$$\begin{aligned} S^{\rm cons}_{G^4({\rm RR}^2)} &= \frac{2G^2}{25}\int\frac{{\rm d}\omega{\rm d}q_0}{(2\pi)^2}(-i\omega^7)L^{ki}(q_0)I^{kj}(-\omega)I^{ij}(\omega - q_0) \\ &+ \frac{2G^2}{25}\int\frac{{\rm d}\omega{\rm d}\omega_1}{(2\pi)^2}\left\{\left(2\omega_1^3\omega^5 - \omega_1^2\omega^6\right)I^{ij}(\omega_1)Q^{ki}_{(0)}(\omega - \omega_1)I^{jk}(-\omega)\right\} \\ &= \frac{2G^2}{25}\int {\rm d}t\left(-L_{ki}I^{(4)}_{kj}I^{(3)}_{ji} + Q_{ki}I^{(4)}_{kj}I^{(4)}_{ij} + Q^{(2)}_{ki}I^{(3)}_{kj}I^{(3)}_{ij}\right)_{G^2}\,. \end{aligned}$$

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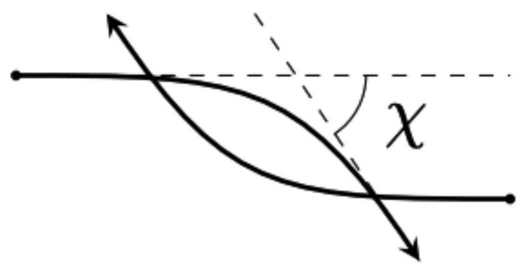

$\mathcal{E} > 0$

$$\tilde{\chi}^{(n,\nu^2)}_{j,\text{rel}} \equiv \Gamma^{n-1} \chi^{(n,\nu^2)}_{j,\text{rel}}$$

(apparent) discrepancy with PM EFT results resolved!

$$\tilde{\chi}^{(4,\nu^2)\text{cons}}_{j(\text{pot+T})} = -\frac{8919}{1400}\pi\nu^2 v_\infty^6$$

$$\tilde{\chi}^{(4,\nu^2)}_{j(\text{FT})} = \frac{23}{16}\pi\nu^2 v_\infty^6\,,$$

$$\tilde{\chi}^{(4,\nu^2)}_{j(\text{M})} = -\frac{529}{280}\pi\nu^2 v_\infty^6\,,$$

$$\tilde{\chi}^{(4,\nu^2)}_{j(\text{RR}^2)} = \frac{211}{20}\pi\nu^2 v_\infty^6\,,$$

$$\tilde{\chi}^{(4,\nu^2)\text{tot}}_{j(\text{even})} = \frac{1491}{400}\pi\nu^2 v_\infty^6$$

Also in the conservative sector:

$$\tilde{\chi}^{(4,\nu^2)\text{cons}}_{j(\text{pot+T+FT+M+RR}^2)} = \left(-\frac{8919}{1400} + \frac{69}{80} - \frac{477}{560} + \frac{159}{25}\right)\pi\nu^2 v_\infty^6 = 0$$

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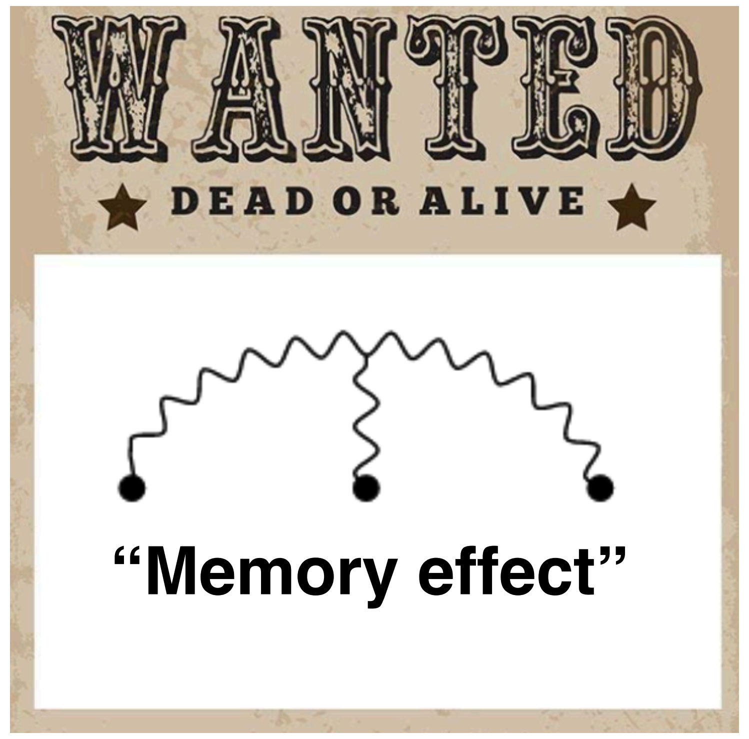
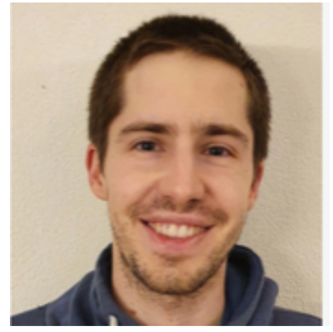
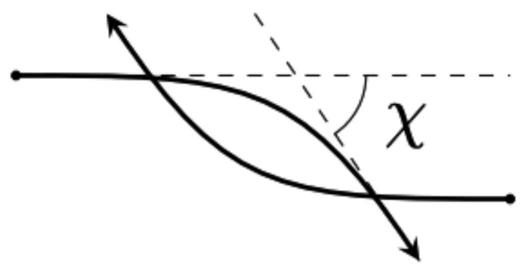
$$S^{\text{cons}}_{\text{5PN/4PM}} = S_{(\text{pot})} + S^{\text{cons}}_{(\text{T})} + \frac{G^2}{5}\int \mathrm{d}t\Big\{ -\frac{1}{6}L^{kl} I^{(4)ki} I^{(3)li} + \frac{1}{7} I^{(2)ij} I^{(3)jk} I^{(3)ik} - \frac{1}{2} I^{ij} I^{(4)jk} I^{(4)ik} + \frac{2}{5}\left(-L^{ki} I^{(4)kj} I^{(3)ij} + Q^{ki} I^{kj(4)} I^{(4)ij} + Q^{(2)ki} I^{(3)kj} I^{(3)ij}\right)\Big\},$$



EFT approach to GW physics *PM*



Kalin **RAP**
(2020)



$\mathcal{E} > 0$

Post-Minkowskian expansion

$$\frac{Gm}{b} \ll 1$$

* *Conservative non-spinning*

$$\int \underbrace{D[\lambda_{\rm rad}^{-1}]D[r^{-1}]D[r_s^{-1}]}_{\text{classical 'soft' region}} e^{iS_{\rm full}}$$

Linear WEFT coupling

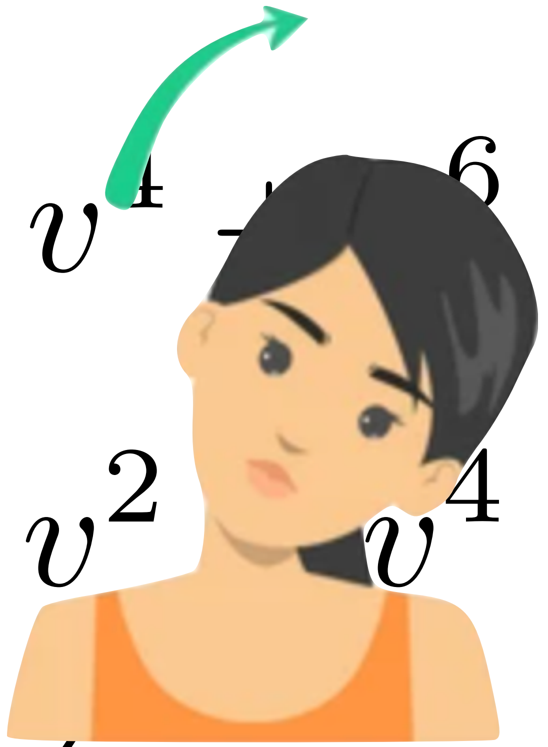
$$S_{\rm pp} = -\sum \frac{m_a}{2}\int {\rm d}\tau_a\, g_{\mu\nu}(x_a(\tau_a))v_a^\mu(\tau_a)v_a^\nu(\tau_a) + \dots$$

Compute total impulse and scattering angle

$$\Delta p_a^\mu = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} {\rm d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a))$$

Fully (special) relativistic integration problem!

$$\int \left(\prod_{i=1}^{n} \frac{d^D\ell_i\, e^{\gamma_E \epsilon}}{\pi^{(D-1)/2}} \frac{\delta(\ell_i\cdot u_{a_i})}{(\pm \ell_i\cdot u_{\phi_i} - i0)^{\alpha_i}}\right)\frac{1}{D_1^{\nu_1}D_2^{\nu_2}\cdots D_N^{\nu_N}},$$



$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) \quad \textbf{1PM}$$

$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) \quad \textbf{2PM}$$

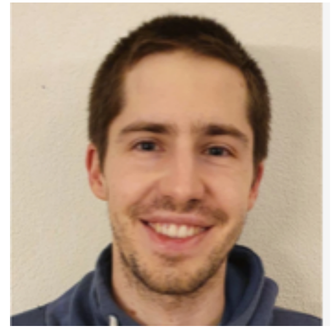
$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right) \quad \textbf{3PM}$$

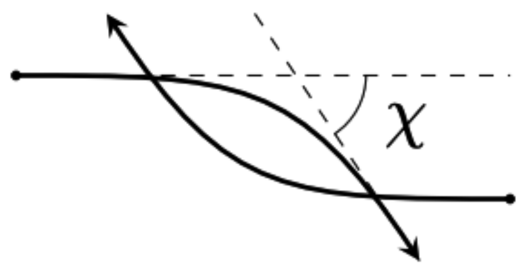
$$G^4\left(1 + v^2 + v^4 + \cdots\right) \quad \textbf{4PM}$$

$$G^5\left(1 + v^2 + \cdots\right) \quad \textbf{5PM}$$

$$\vdots$$

EFT approach to GW physics *PM*





$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)}\,,$$

* *Conservative non-spinning*

Westphal (1985)

Cheung et al (2019)
Kalin **RAP** (2020)

$$\frac{\chi_b^{(1)}}{\Gamma} = \frac{2\gamma^2-1}{\gamma^2-1}\,,$$

$$\frac{\chi_b^{(2)}}{\Gamma} = \frac{3\pi}{8}\frac{5\gamma^2-1}{\gamma^2-1}\,,$$

$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right) \quad \text{1PM}$$

$$G^2\left(1+v^2+v^4+v^6+v^8\ +\cdots\right) \quad \text{2PM}$$

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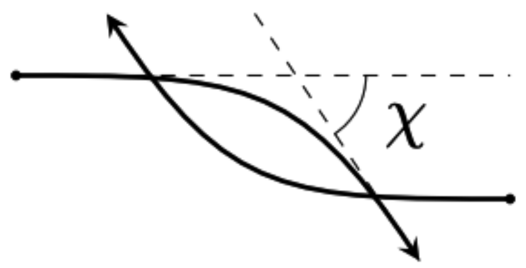
EFT approach to GW physics *PM*



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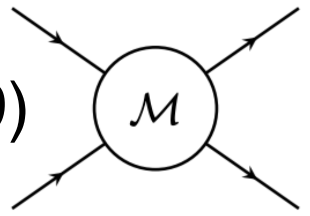
Kalin Liu **RAP** (2020)

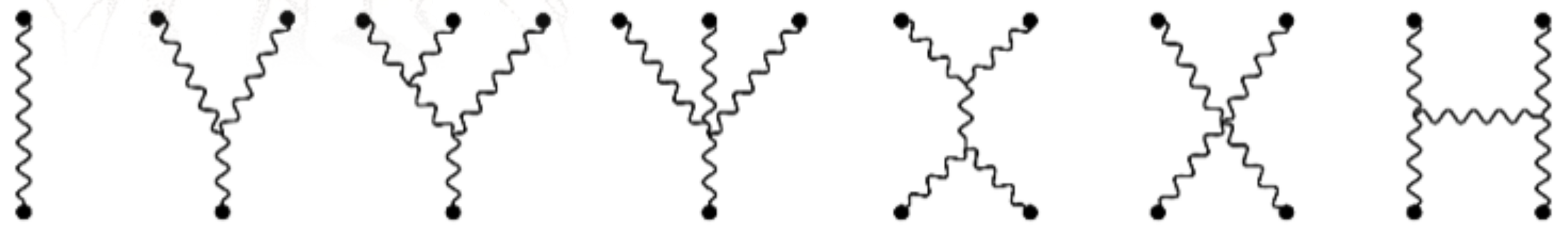
$$G^3\left(1+v^2+v^4+v^6+\cdots\right) \quad \text{3PM}$$

Bern et al (2019)



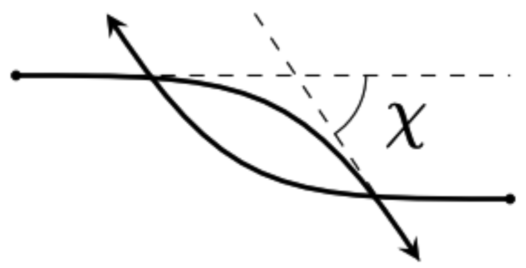
$$G^4\left(1+v^2+v^4+\cdots\right) \quad \text{4PM}$$

$$G^5\left(1+v^2+\cdots\right) \quad \text{5PM}$$



$$\frac{\chi_b^{(3)}}{\Gamma} = \frac{1}{(\gamma^2-1)^{3/2}}\Bigg[-\frac{4\nu}{3}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) + \frac{(64\gamma^6-120\gamma^4+60\gamma^2-5)(1+2\nu(\gamma-1))}{3(\gamma^2-1)^{3/2}} - 8\nu(4\gamma^4-12\gamma^2-3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}\Bigg],$$

EFT approach to GW physics PM



'PM-bootstrapping two-body problem' =

*D*ifferential Equations + boundary conditions from **PN!**

$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)}\,,$$

* *Conservative non-spinning*

canonical

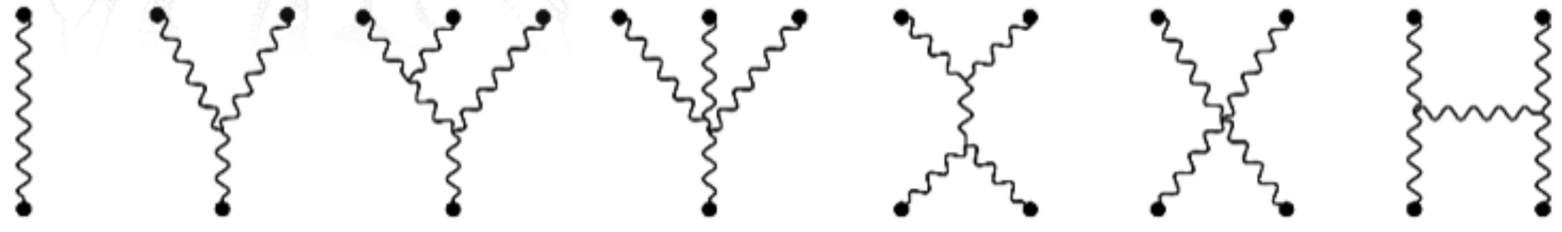
to N2LO!

polylogarithms

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$

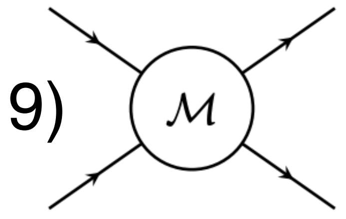
$$\gamma = \frac{1+x^2}{2x}$$

Kalin Liu **RAP** (2020)



$$G^3\left(1+v^2+v^4+v^6 \;+\cdots\right) \quad \textbf{3PM}$$

Bern et al (2019)



$$G^4\left(1+v^2+v^4 \;+\cdots\right) \quad \textbf{4PM}$$

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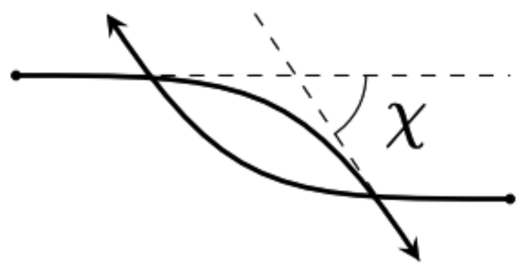
$$\frac{\chi_b^{(3)}}{\Gamma} = \frac{1}{(\gamma^2-1)^{3/2}}\Bigg[-\frac{4\nu}{3}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) + \frac{(64\gamma^6-120\gamma^4+60\gamma^2-5)(1+2\nu(\gamma-1))}{3(\gamma^2-1)^{3/2}} - 8\nu(4\gamma^4-12\gamma^2-3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}\Bigg], \quad \log x$$

EFT approach to GW physics PM



'PM-bootstrapping two-body problem' =

*D*ifferential Equations + boundary conditions from **PN!**

* *Conservative non-spinning*

<u>**Not canonical**</u>

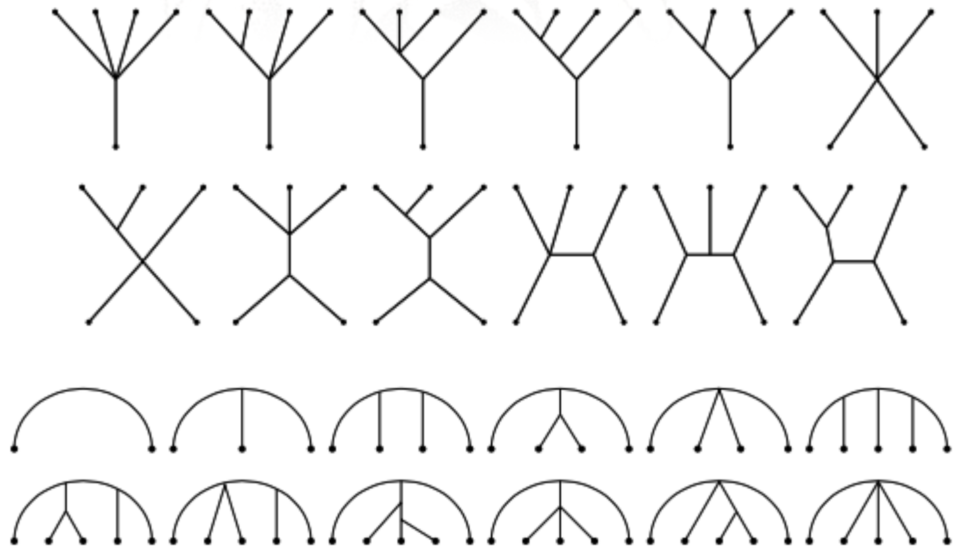
at N3LO

elliptic integrals!

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\, \vec{h}(x,\epsilon)$$

$$\mathrm{K}(z) \equiv \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-zt^2)}}\,,$$

$$\mathrm{E}(z) \equiv \int_0^1 dt\, \frac{\sqrt{1-zt^2}}{\sqrt{1-t^2}}\,,$$



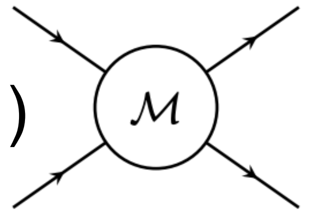
Kalin Liu **RAP** (2020)

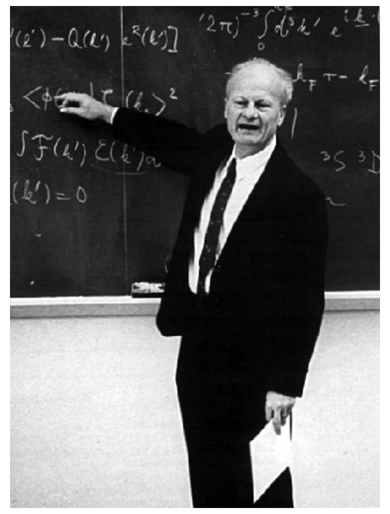
$$G^3\left(1 + v^2 + v^4 + v^6 \;+ \cdots\right) \quad \textbf{3PM}$$

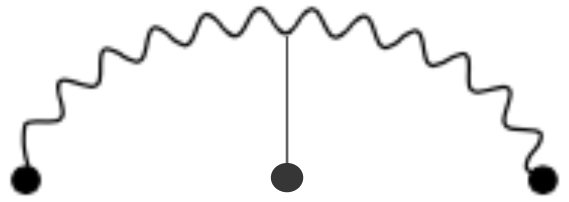
Dlapa Kalin Liu **RAP** (2021)

$$G^4\left(1 + v^2 + v^4 \;+ \cdots\right) \quad \textbf{4PM}$$

Bern et al (2021)







"Tail effect"

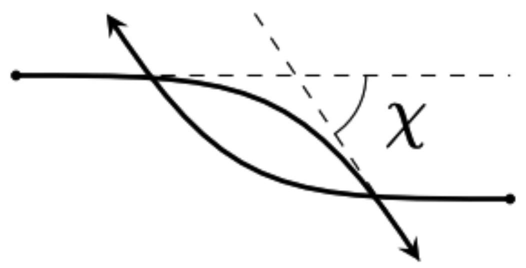
$$\frac{\chi^{(4)}_{b\,(\mathrm{comb})}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + 2\chi_{2\epsilon}(x)\log(1-x)\Big),$$

"Bethe logarithm" $\log v$

various aspects confirmed in a number of studies

Bini et al. (2021)
Blumlein et al. (2021)
Foffa et al. (2021)

EFT approach to GW physics *PM*

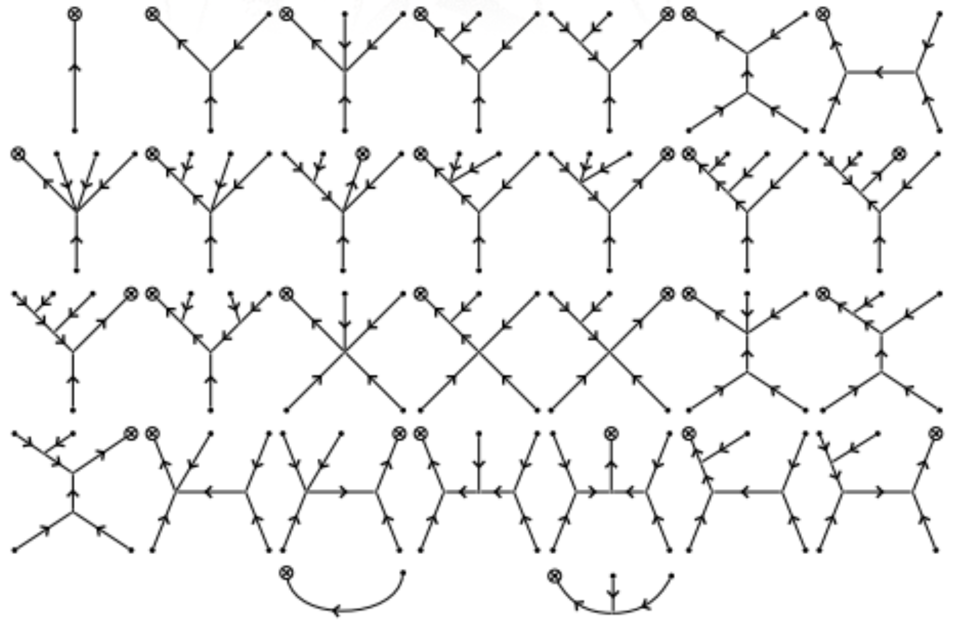


'PM-bootstrapping two-body problem' =

*D*ifferential Equations + boundary conditions from **PN!**

* *Conservative* + *dissipative* = ***TOTAL!***

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$



In-In boundary conditions (causality preserving)

Kalin Liu **RAP** (2020)

$$G^3\left(1+v^2+v^4+v^6\ +\cdots\right) \quad \textbf{3PM}$$

Dlapa Kalin Liu **RAP** (2021)
Dlapa Kalin Liu Neef **RAP (2022)**

$$G^4\left(1+v^2+v^4\ +\cdots\right) \quad \textbf{4PM}$$

$$e^{iW^{(+,-)}} = \int D[\lambda_{\rm rad}^{-1}]\,D[r^{-1}]\,D[r_s^{-1}]\,e^{iS^{C}_{\rm full}}$$

$$\begin{pmatrix} 0 & -\Delta_{\rm adv}(x-y) \\ -\Delta_{\rm ret}(x-y) & \frac{1}{2}\Delta_H(x-y) \end{pmatrix}$$

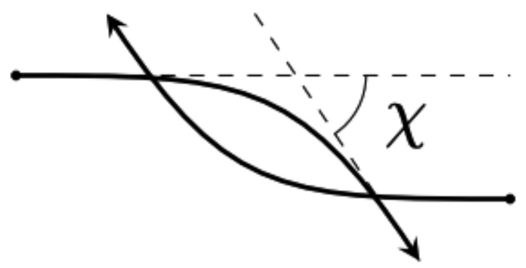
$$\Delta p^\mu_{\rm tot} = \Delta p^\mu_{\rm cons} + \Delta p^\mu_{\rm diss}\,,$$

$$\Delta p^\mu_{\rm cons} \equiv \mathbb{R}\Delta p^\mu_{\rm F}$$

various aspects confirmed in a number of studies

Bini et al. (2022)
Manohar et al. (2022)
Damgaard et al. (2023)
Jakobsen et al. (2023)

EFT approach to GW physics *PM*



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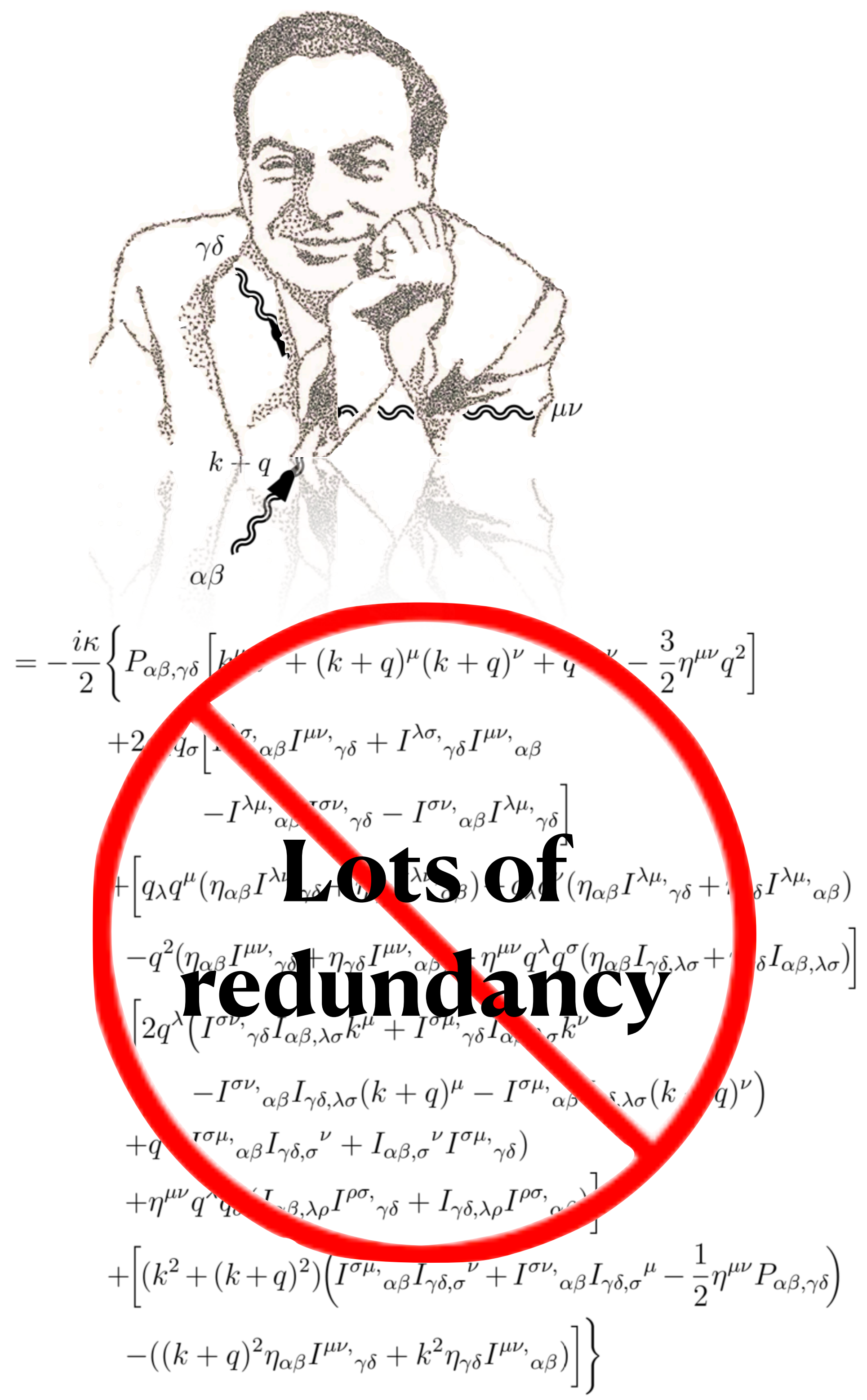
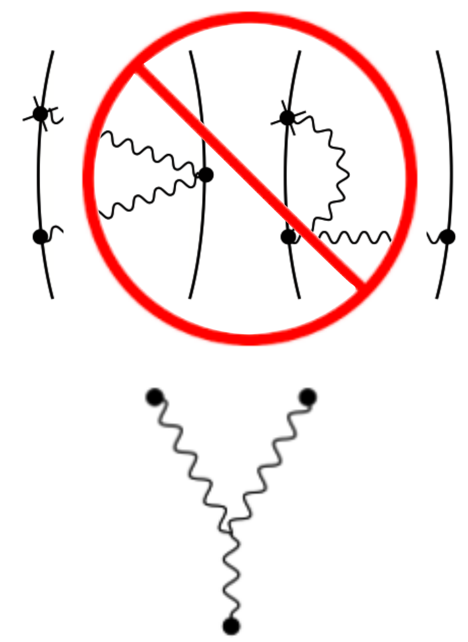


Kalin Liu **RAP** (2020) $G^3\left(1+v^2+v^4+v^6\ +\cdots\right)$ **3PM**

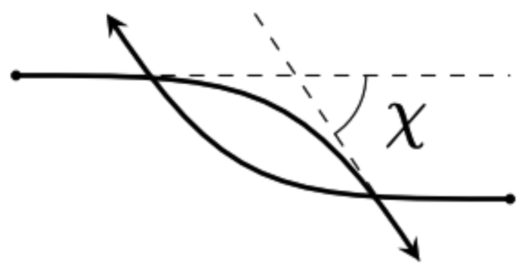
Dlapa Kalin Liu **RAP** (2021)
Dlapa Kalin Liu Neef **RAP (2022)** $G^4\left(1+v^2+v^4\ +\cdots\right)$ **4PM**

Driesse et al. (2024) $G^5\left(1+v^2\ +\cdots\right)$ **5PM (1SF)**



⋮

EFT approach to GW physics *PM*



Kalin **RAP** (2020)
Kalin Liu **RAP** (2020)
Liu **RAP** Yang (2021)
Kalin Neef **RAP** (2022)
Dlapa Kalin Liu **RAP** (2021)
Dlapa Kalin Liu Neef **RAP** (2022)

* *Conservative* + *dissipative* = ***TOTAL!***

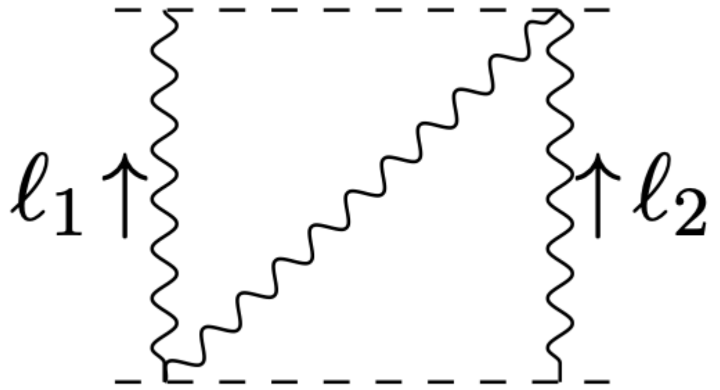
Schwinger-Keldysh Formalism

$$e^{iW} \overset{1,2}{=} \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS^{\rm C}_{\rm full}}$$

Family of integrals

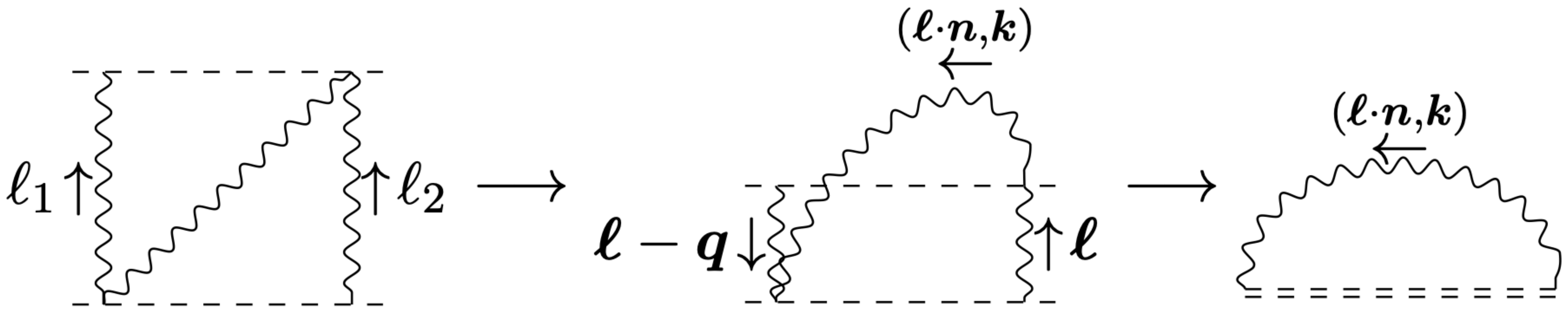
$$\int \prod_{i=1}^{3} \frac{{\rm d}^d \ell_i}{\pi^{d/2}} \frac{\delta(\ell_i \cdot u_{a_i})}{(\pm \ell_i \cdot u_{\phi_i} - i0)^{n_i}} \prod_{k=1}^{9} \frac{1}{D_k^{\nu_k}}\,, \quad \leftarrow \{(\ell^0 \pm i0)^2 - \boldsymbol{\ell}^2, \dots\}$$

Boundary conditions: Method of regions



$$= \int_{\ell_1 \ell_2} \frac{\delta(\ell_1 \cdot u_1)\, \delta(\ell_2 \cdot u_2)}{\ell_1^2\, \ell_2^2\, (\ell_1 + \ell_2 - q)^2}$$

Reversed NRGR
(do the rad integral first
then the left-over potential)



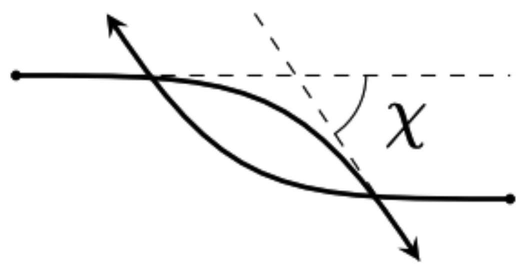
$$I_1^{\rm pot} = -\int_{\boldsymbol{\ell}\boldsymbol{k}} \frac{1}{[(\boldsymbol{k} - \boldsymbol{\ell} + \boldsymbol{q})^2]\,[\boldsymbol{\ell}^2]\,[\boldsymbol{k}^2]} + \mathcal{O}(v_\infty^2)\,,$$

$$I_1^{\rm rad} = -\int_{\boldsymbol{\ell}} \frac{1}{[(\boldsymbol{\ell} - \boldsymbol{q})^2]\,[\boldsymbol{\ell}^2]} \int_{\tilde{\boldsymbol{k}}} \frac{v_\infty^{d-2}}{[\tilde{\boldsymbol{k}}^2 - (\ell^z)^2]} + \mathcal{O}(v_\infty^d)\,,$$

$v_\infty^{-2\epsilon}$

Kalin **RAP** (2020)
Kalin Liu **RAP** (2020)
Liu **RAP** Yang (2021)
Kalin Neef **RAP** (2022)
Dlapa Kalin Liu **RAP** (2021)
Dlapa Kalin Liu Neef **RAP** (2022)

EFT approach to GW physics *PM*



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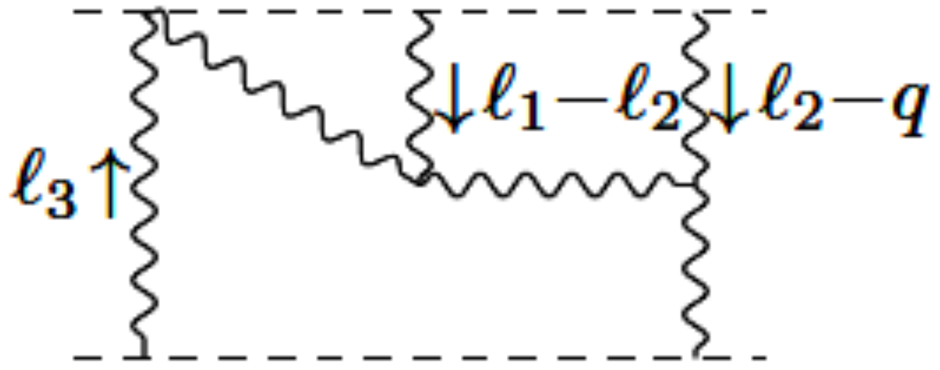
$$\int \prod_{i=1}^{3} \frac{\mathrm{d}^d \ell_i}{\pi^{d/2}} \frac{\delta(\ell_i \cdot u_{a_i})}{(\pm \ell_i \cdot u_{\phi_i} - i0)^{n_i}} \prod_{k=1}^{9} \frac{1}{D_k^{\nu_k}}\,,$$

← $\{(\ell^0 \pm i0)^2 - \boldsymbol{\ell}^2, \dots\}$

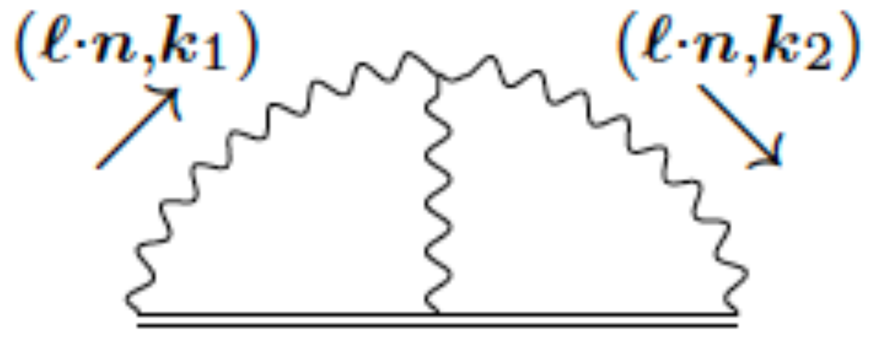
Boundary conditions: Method of regions



$$= \int_{\ell_1,\ell_2,\ell_3} \frac{\delta(\ell_1\cdot u_1)\delta(\ell_2\cdot u_1)\delta(\ell_3\cdot u_2)}{\ell_3^2(\ell_2-q)^2(\ell_3-q)^2(\ell_1-\ell_2)^2(\ell_2-\ell_3)^2(\ell_3-\ell_1)^2}$$

$$\begin{aligned}
\text{pot}: &\quad k_1 \sim (v_\infty, 1)\,, \quad k_2 \sim (v_\infty, 1)\,, \quad \ell \sim (v_\infty, 1)\,,\\
\text{1rad}: &\quad k_1 \sim (v_\infty, v_\infty)\,, \quad k_2 \sim (v_\infty, 1)\,, \quad \ell \sim (v_\infty, 1)\,,\\
\text{2rad}: &\quad k_1 \sim (v_\infty, v_\infty)\,, \quad k_2 \sim (v_\infty, v_\infty)\,, \quad \ell \sim (v_\infty, 1)\,,
\end{aligned}$$

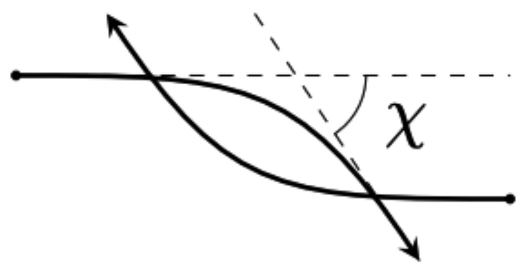
→



“Bethe logarithm” $\log v$ ← FROM pole in tail region (aka Lamb shift)

$v_\infty^{-2\epsilon}$

EFT approach to GW physics *4PM*



PHYSICAL REVIEW LETTERS **130**, 101401 (2023)

Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[2,1] Jakob Neef,[3,4] and Rafael A. Porto[1]

$$\Delta^{(n)} p_1^\mu = c_{1b}^{(n)} \frac{\hat{b}^\mu}{b^n} + \frac{1}{b^n} \sum_a c_{1\check{u}_a}^{(n)} \check{u}_a^\mu ,$$

* *Conservative + dissipative = TOTAL!*

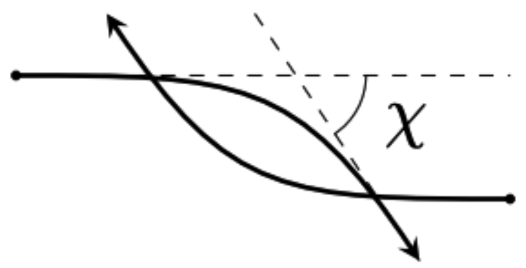
Perfect agreement with PN (Bini-Damour-Geralico)

$$\begin{aligned}
\frac{c_{1b}^{(4)\text{tot}}}{\pi} =& -\frac{3h_1 m_1 m_2 (m_1^3+m_2^3)}{64(\gamma^2-1)^{5/2}} + m_1^2 m_2^2 (m_1+m_2)\Bigg[\frac{21 h_2 \mathrm{E}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{32(\gamma-1)\sqrt{\gamma^2-1}} + \frac{3h_3 \mathrm{K}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^{3/2}} - \frac{3h_4 \mathrm{E}\left(\frac{\gamma-1}{\gamma+1}\right)\mathrm{K}\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^{3/2}} + \frac{\pi^2 h_5}{8\sqrt{\gamma^2-1}} + \frac{h_6 \log\left(\frac{\gamma-1}{2}\right)}{16(\gamma^2-1)^{3/2}} \\
&+ \frac{3h_7 \mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{(\gamma-1)(\gamma+1)^2} - \frac{3h_7 \mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{4(\gamma-1)(\gamma+1)^2}\Bigg] + m_1^3 m_2^2 \Bigg[\frac{h_8}{48(\gamma^2-1)^3} + \frac{\sqrt{\gamma^2-1}\, h_9}{768(\gamma-1)^3\gamma^9(\gamma+1)^4} + \frac{h_{10}\log\left(\frac{\gamma+1}{2}\right)}{8(\gamma^2-1)^2} - \frac{h_{11}\log\left(\frac{\gamma+1}{2}\right)}{32(\gamma^2-1)^{5/2}} + \frac{h_{12}\log(\gamma)}{16(\gamma^2-1)^{5/2}} \\
&- \frac{h_{13}\,\mathrm{arccosh}(\gamma)}{8(\gamma-1)(\gamma+1)^4} + \frac{h_{14}\,\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{15}\log\left(\frac{\gamma+1}{2}\right)\log\left(\frac{\gamma-1}{\gamma+1}\right)}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\,\mathrm{arccosh}(\gamma)\log\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^2} - \frac{3h_{17}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{64\sqrt{\gamma^2-1}} - \frac{3}{32}\sqrt{\gamma^2-1}\,h_{18}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)\Bigg] \\
&+ m_1^2 m_2^3\Bigg[\frac{3h_{15}\log\left(\frac{2}{\gamma-1}\right)\log\left(\frac{\gamma+1}{2}\right)}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\log\left(\frac{\gamma-1}{2}\right)\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{19}}{48(\gamma^2-1)^3} + \frac{h_{20}}{192\gamma^7(\gamma^2-1)^{5/2}} + \frac{h_{21}\log\left(\frac{\gamma+1}{2}\right)}{8(\gamma^2-1)^2} + \frac{h_{22}\log\left(\frac{\gamma+1}{2}\right)}{16(\gamma^2-1)^{3/2}} + \frac{h_{23}\log(\gamma)}{2(\gamma^2-1)^{3/2}} \\
&- \frac{h_{24}\,\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^3} + \frac{h_{25}\,\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{26}\,\mathrm{arccosh}^2(\gamma)}{32(\gamma^2-1)^{7/2}} + \frac{3h_{27}\log^2\left(\frac{\gamma+1}{2}\right)}{2\sqrt{\gamma^2-1}} + \frac{3h_{28}\log\left(\frac{\gamma+1}{2}\right)\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{29}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)}{4\sqrt{\gamma^2-1}} + \frac{3h_{30}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{8\sqrt{\gamma^2-1}}\Bigg],
\end{aligned}$$

$$c_{1\check{u}_1}^{(4)\text{tot}} = \frac{9\pi^2 h_{31} m_1 m_2^2 (m_1+m_2)^2}{32(\gamma^2-1)} + \frac{2h_{32} m_1 m_2^2 (m_1^2+m_2^2)}{(\gamma^2-1)^3} + m_1^2 m_2^3\Bigg[\frac{4h_{33}}{3(\gamma^2-1)^3} - \frac{8h_{34}}{3(\gamma^2-1)^{5/2}} + \frac{8h_{35}\,\mathrm{arccosh}(\gamma)}{(\gamma^2-1)^3} - \frac{16h_{36}\,\mathrm{arccosh}(\gamma)}{(\gamma^2-1)^{3/2}}\Bigg],$$

$$\begin{aligned}
c_{1\check{u}_2}^{(4)\text{tot}} =& -m_1^4 m_2\left(\frac{9\pi^2 h_{31}}{32(\gamma^2-1)} + \frac{2h_{32}}{(\gamma^2-1)^3}\right) + m_1^3 m_2^2\Bigg[-\frac{4h_{37}}{3(\gamma^2-1)^3} + \frac{h_{38}}{705600\gamma^8(\gamma^2-1)^{5/2}} + \frac{\pi^2 h_{39}}{192(\gamma^2-1)^2} + \frac{h_{40}\,\mathrm{arccosh}(\gamma)}{6720\gamma^9(\gamma^2-1)^3} + \frac{32h_{41}\,\mathrm{arccosh}(\gamma)}{3(\gamma^2-1)^{3/2}} \\
&- \frac{8h_{42}\,\mathrm{arccosh}^2(\gamma)}{(\gamma^2-1)^2} + \frac{32h_{43}\,\mathrm{arccosh}^2(\gamma)}{(\gamma^2-1)^{7/2}} + \frac{h_{44}\log(2)\,\mathrm{arccosh}(\gamma)}{8(\gamma^2-1)^2} + \frac{3h_{45}\left(\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right) - 4\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)\right)}{16(\gamma^2-1)^2} \\
&+ \frac{3h_{46}\left(\log\left(\frac{\gamma+1}{2}\right)\mathrm{arccosh}(\gamma) - 2\mathrm{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)\right)}{8(\gamma^2-1)^2} - \frac{h_{47}\left(\mathrm{Li}_2\left(-\left(\gamma-\sqrt{\gamma^2-1}\right)^2\right) - 2\log(\gamma)\,\mathrm{arccosh}(\gamma)\right)}{16(\gamma^2-1)^2}\Bigg] + m_1^2 m_2^3\Bigg[-\frac{2h_{48}}{45(\gamma^2-1)^3} \\
&+ \frac{h_{49}}{1440\gamma^7(\gamma^2-1)^{5/2}} + \frac{\pi^2 h_{50}}{48(\gamma^2-1)^2} + \frac{h_{51}\,\mathrm{arccosh}(\gamma)}{480\gamma^8(\gamma^2-1)^3} - \frac{16h_{52}\,\mathrm{arccosh}(\gamma)}{5(\gamma^2-1)^{3/2}} - \frac{16h_{53}\,\mathrm{arccosh}^2(\gamma)}{(\gamma^2-1)^2} - \frac{32h_{54}\,\mathrm{arccosh}^2(\gamma)}{(\gamma^2-1)^{7/2}} - \frac{h_{55}\log(2)\,\mathrm{arccosh}(\gamma)}{4(\gamma^2-1)^2} \\
&+ \frac{h_{56}\left(\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right) - 4\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)\right)}{32(\gamma^2-1)^2} + \frac{h_{57}\left(\log\left(\frac{2}{\gamma+1}\right)\mathrm{arccosh}(\gamma) + 2\mathrm{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)\right)}{4(\gamma^2-1)^2} + \frac{h_{58}\left(\mathrm{Li}_2\left(-\left(\gamma-\sqrt{\gamma^2-1}\right)^2\right) - 2\log(\gamma)\,\mathrm{arccosh}(\gamma)\right)}{8(\gamma^2-1)^2}\Bigg].
\end{aligned}$$

EFT approach to GW physics *4PM*



PHYSICAL REVIEW LETTERS **130,** 101401 (2023)

Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[2,1] Jakob Neef,[3,4] and Rafael A. Porto[1]

* *Conservative* + *dissipative* = ***TOTAL!***

Perfect agreement with PN (Bini-Damour -Geralico)

NLO radiation (G^4) beyond Kovacs and Thorne (G^3)

$$
\begin{aligned}
\Delta E^{4\mathrm{PM}}_{\mathrm{hyp}} = -\frac{G^4M^5\nu^2}{b^4\Gamma}\Bigg\{ & \frac{15\pi^2(\gamma^2-1)\left(27(\gamma^2-1)h_{31}+2h_{50}\right)+64(45h_{32}-h_{48})}{1440(\gamma^2-1)^3} + \frac{h_{49}}{1440\gamma^7(\gamma^2-1)^{5/2}} - \operatorname{arccosh}^2(\gamma)\left(\frac{16h_{53}}{(\gamma^2-1)^2}+\frac{32h_{54}}{(\gamma^2-1)^{7/2}}\right) \\
& -\frac{h_{55}\log(2)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} + \frac{h_{57}\log\left(\frac{2}{\gamma+1}\right)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} - \frac{h_{58}\log(\gamma)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} + \operatorname{arccosh}(\gamma)\left(\frac{h_{51}}{480\gamma^8(\gamma^2-1)^3}-\frac{16h_{52}}{5(\gamma^2-1)^{3/2}}\right) \\
& -\frac{h_{56}\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{8(\gamma^2-1)^2} + \frac{h_{56}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{32(\gamma^2-1)^2} + \frac{h_{57}\mathrm{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)}{2(\gamma^2-1)^2} + \frac{h_{58}\mathrm{Li}_2\left(-\left(\gamma-\sqrt{\gamma^2-1}\right)^2\right)}{8(\gamma^2-1)^2} \\
& +\nu\Bigg[\frac{4(-45h_{32}+30h_{33}-30h_{37}+h_{48})}{45(\gamma^2-1)^3} + \frac{\pi^2\left(54(\gamma^2-1)h_{31}+h_{39}-4h_{50}\right)}{96(\gamma^2-1)^2} - \operatorname{arccosh}^2(\gamma)\left(\frac{16(h_{42}-2h_{53})}{(\gamma^2-1)^2}-\frac{64(h_{43}+h_{54})}{(\gamma^2-1)^{7/2}}\right) \\
& +\frac{h_{38}-490\gamma(3840\gamma^7h_{34}+h_{49})}{352800\gamma^8(\gamma^2-1)^{5/2}} + \frac{(3h_{46}+2h_{57})\log\left(\frac{\gamma+1}{2}\right)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} + \frac{(h_{44}+2h_{55})\log(2)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} + \frac{(h_{47}+2h_{58})\log(\gamma)\operatorname{arccosh}(\gamma)}{4(\gamma^2-1)^2} \\
& +\operatorname{arccosh}(\gamma)\left(\frac{53760\gamma^9h_{35}-14\gamma h_{51}+h_{40}}{3360\gamma^9(\gamma^2-1)^3}-\frac{32(15h_{36}-10h_{41}-3h_{52})}{15(\gamma^2-1)^{3/2}}\right) + \frac{(h_{56}-6h_{45})\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{4(\gamma^2-1)^2} - \frac{(h_{56}-6h_{45})\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^2} \\
& -\frac{(3h_{46}+2h_{57})\mathrm{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)}{2(\gamma^2-1)^2} - \frac{(h_{47}+2h_{58})\mathrm{Li}_2\left(-\left(\gamma-\sqrt{\gamma^2-1}\right)^2\right)}{8(\gamma^2-1)^2}\Bigg]\Bigg\}.
\end{aligned}
$$

g) *The Feynman-Diagram Approach*

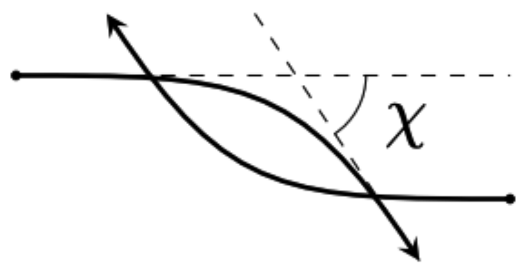
Any classical problem can be solved quantum-mechanically; and sometimes the quantum solution is easier than the classical. There is an extensive literature on the Feynman-diagram, quantum-mechanical treatment of gravitational bremsstrahlung radiation (e.g., Feynman 1961, 1963; Barker, Gupta, and Kaskas 1969; Barker and Gupta 1974). Unfortunately, the regime of validity of these quantum calculations does not overlap the classical regime.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

SÁNDOR J. KOVÁCS, JR.
AND
KIP S. THORNE

1977!

EFT approach to GW physics 4PM



PHYSICAL REVIEW LETTERS **130,** 101401 (2023)

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PHYSICAL REVIEW LETTERS **128,** 161104 (2022)

Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-Eccentricity Expansion

Christoph Dlapa, Gregor Kälin, Zhengwen Liu, and Rafael A. Porto
Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

PHYSICAL REVIEW LETTERS **128,** 161103 (2022)

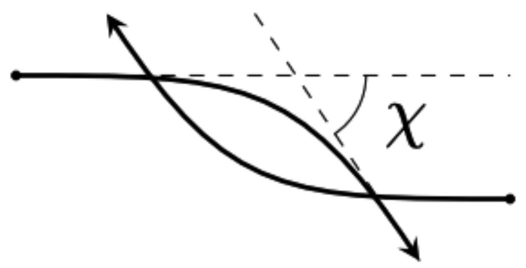
Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

Zvi Bern,[1] Julio Parra-Martinez,[2] Radu Roiban,[3] Michael S. Ruf,[1] Chia-Hsien Shen,[4] Mikhail P. Solon,[1] and Mao Zeng[5]

$$\begin{aligned}
\frac{\chi^{(4)\text{cons}}_{b,\text{rel}}(\gamma)}{\pi\Gamma} = {} & \frac{3h_{61}}{128(\gamma^2-1)^3} + \nu\Bigg[-\frac{3h_3 \text{K}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{32(\gamma^2-1)^2} + \frac{3h_4 \text{E}\left(\frac{\gamma-1}{\gamma+1}\right)\text{K}\left(\frac{\gamma-1}{\gamma+1}\right)}{32(\gamma^2-1)^2} + \frac{\pi^2 h_5}{16(1-\gamma^2)} + \frac{3h_{27}\log^2\left(\frac{\gamma+1}{2}\right)}{4(1-\gamma^2)} - \frac{h_6\log\left(\frac{\gamma-1}{2}\right)}{32(\gamma^2-1)^2} + \frac{3h_{15}\log\left(\frac{\gamma-1}{2}\right)\log\left(\frac{\gamma+1}{2}\right)}{16(\gamma^2-1)} \\
& - \frac{h_{22}\log\left(\frac{\gamma+1}{2}\right)}{32(\gamma^2-1)^2} - \frac{h_{23}\log(\gamma)}{4(\gamma^2-1)^2} + \frac{3h_{26}\,\text{arccosh}^2(\gamma)}{64(\gamma^2-1)^4} + \frac{h_{24}\,\text{arccosh}(\gamma)}{32(\gamma^2-1)^{7/2}} - \frac{3h_{16}\log\left(\frac{\gamma-1}{2}\right)\text{arccosh}(\gamma)}{32(\gamma^2-1)^{5/2}} - \frac{3h_{28}\log\left(\frac{\gamma+1}{2}\right)\text{arccosh}(\gamma)}{32(\gamma^2-1)^{5/2}} \\
& - \frac{h_{62}}{384\gamma^7(\gamma^2-1)^3} - \frac{21h_2\text{E}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{64(\gamma-1)^2(\gamma+1)} - \frac{3\sqrt{\gamma^2-1}\,h_7\text{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{2(\gamma-1)^2(\gamma+1)^3} + \frac{h_{29}\text{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)}{8(1-\gamma^2)} + \left(\frac{3\sqrt{\gamma^2-1}\,h_7}{8(\gamma-1)^2(\gamma+1)^3} + \frac{3h_{30}}{16-16\gamma^2}\right)\text{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)\Bigg],
\end{aligned}$$

$$\begin{aligned}
\frac{\Gamma\chi^{(4)1\text{rad}}_{b,\text{rel}}(\gamma)}{\pi\nu} = {} & \frac{h_{64}}{96(\gamma^2-1)^{7/2}} + \frac{h_{65}\log\left(\frac{\gamma+1}{2}\right)}{16(\gamma^2-1)^{5/2}} + \frac{h_{63}\,\text{arcsinh}\left(\frac{\sqrt{\gamma-1}}{\sqrt{2}}\right)}{8(\gamma^2-1)^4} - \frac{h_{25}\,\text{arccosh}(\gamma)}{32(\gamma^2-1)^4} \\
& + \nu\left[\frac{h_{67}}{96(\gamma^2-1)^{7/2}} + \frac{h_{68}\log\left(\frac{\gamma+1}{2}\right)}{16(\gamma^2-1)^{5/2}} - \frac{\text{arccosh}(\gamma)\left((\gamma+1)h_{14}+(\gamma-3)h_{25}\right)}{32(\gamma^2-1)^4} + \frac{h_{66}\,\text{arcsinh}\left(\frac{\sqrt{\gamma-1}}{\sqrt{2}}\right)}{8(\gamma-1)^2(\gamma+1)^4}\right], \\
\frac{\Gamma\chi^{(4)2\text{rad}}_{b,\text{rel}}(\gamma)}{\pi\nu^2} = {} & \frac{\log\left(\frac{\gamma+1}{2}\right)\left(2(\gamma^2-1)h_{22}+h_{11}\right)}{64(\gamma-1)^3(\gamma+1)^2} - \frac{\log(\gamma)\left(h_{12}-8(\gamma^2-1)h_{23}\right)}{32(\gamma-1)^3(\gamma+1)^2} + \frac{\text{arccosh}(\gamma)\left(2(\gamma-1)^2h_{13}-(\gamma+1)h_{24}\right)}{32(\gamma^2-1)^{7/2}} \\
& + \frac{3\sqrt{\gamma^2-1}\,(h_{16}+h_{28})\log\left(\frac{\gamma+1}{2}\right)\text{arccosh}(\gamma)}{32(\gamma-1)^3(\gamma+1)^2} - \frac{h_9-4\gamma^2(\gamma+1)h_{20}}{1536\gamma^9(\gamma^2-1)^3} - \frac{3(h_{15}-4h_{27})\log^2\left(\frac{\gamma+1}{2}\right)}{16(\gamma-1)} \\
& - \frac{3h_{26}\,\text{arccosh}^2(\gamma)}{64(\gamma-1)^4(\gamma+1)^3} + \left(\frac{3}{64}(\gamma+1)h_{18} + \frac{h_{29}}{8(\gamma-1)}\right)\text{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right) + \frac{3(h_{17}+8h_{30})\text{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{128(\gamma-1)},
\end{aligned}$$

EFT approach to GW physics *4PM*



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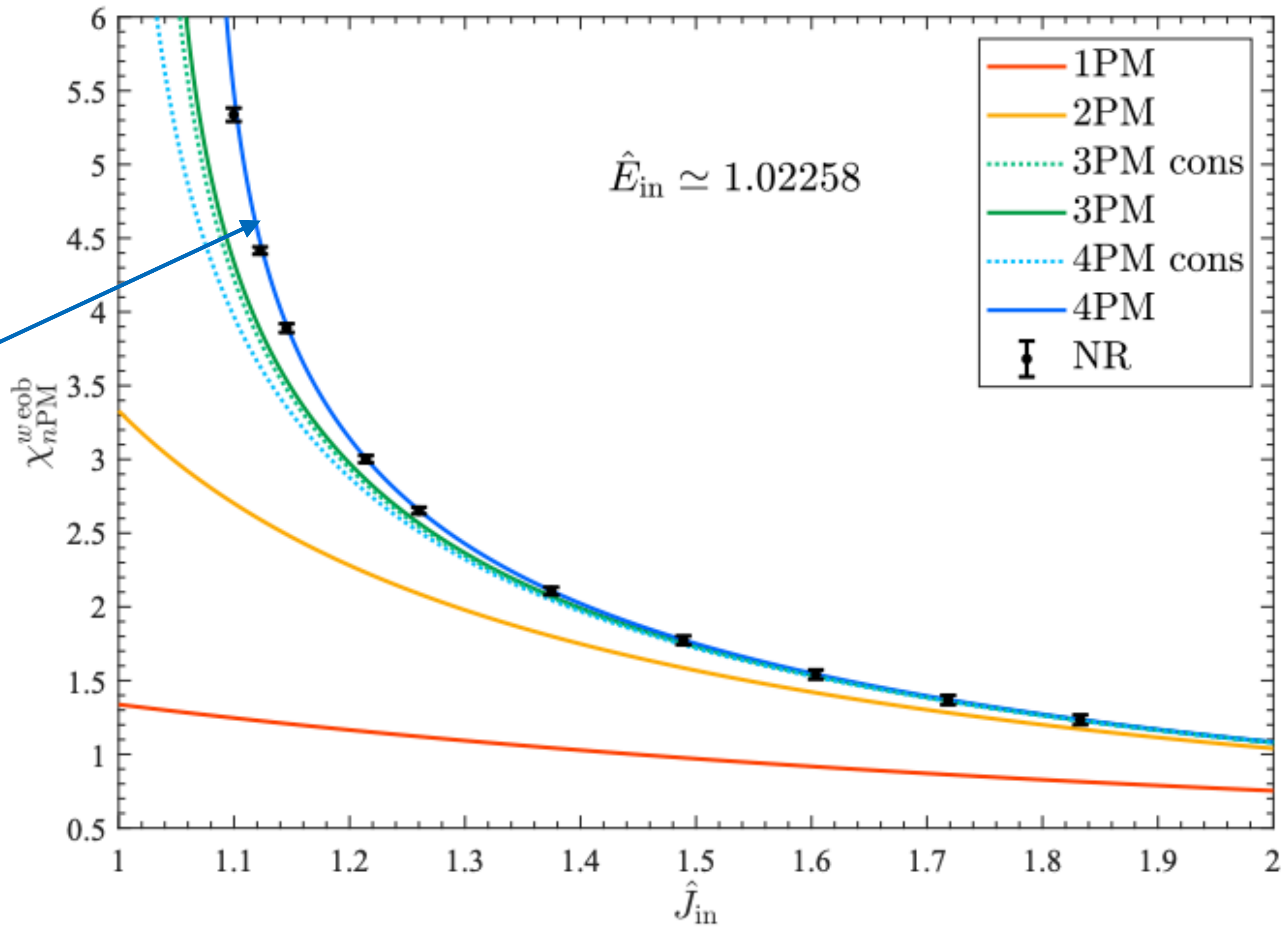
$$\frac{\chi^{(4)\rm cons}_{b,\rm rel}(\gamma)}{\pi\Gamma} = \frac{3h_{61}}{128\left(\gamma^2-1\right)^3} + \nu\Bigg[-\frac{3h_3{\rm K}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{32\left(\gamma^2-1\right)^2} + \frac{3h_4{\rm E}\left(\frac{\gamma-1}{\gamma+1}\right){\rm K}\left(\frac{\gamma-1}{\gamma+1}\right)}{32\left(\gamma^2-1\right)^2} + \frac{\pi^2 h_5}{16(1-\gamma^2)} + \frac{3h_{27}\log^2\left(\frac{\gamma+1}{2}\right)}{4(1-\gamma^2)} - \frac{h_6\log\left(\frac{\gamma-1}{2}\right)}{32\left(\gamma^2-1\right)^2} + \frac{3h_{15}\log\left(\frac{\gamma-1}{2}\right)\log\left(\frac{\gamma+1}{2}\right)}{16\left(\gamma^2-1\right)}$$
$$-\frac{h_{22}\log\left(\frac{\gamma+1}{2}\right)}{32\left(\gamma^2-1\right)^2} - \frac{h_{23}\log(\gamma)}{4\left(\gamma^2-1\right)^2} + \frac{3h_{26}\,{\rm arccosh}^2(\gamma)}{64\left(\gamma^2-1\right)^4} + \frac{h_{24}\,{\rm arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{7/2}} - \frac{3h_{16}\log\left(\frac{\gamma-1}{2}\right){\rm arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{5/2}} - \frac{3h_{28}\log\left(\frac{\gamma+1}{2}\right){\rm arccosh}(\gamma)}{32\left(\gamma^2-1\right)^{5/2}}$$
$$-\frac{h_{62}}{384\gamma^7\left(\gamma^2-1\right)^3} - \frac{21h_2{\rm E}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{64(\gamma-1)^2(\gamma+1)} - \frac{3\sqrt{\gamma^2-1}\,h_7{\rm Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{2(\gamma-1)^2(\gamma+1)^3} + \frac{h_{29}{\rm Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)}{8(1-\gamma^2)} + \left(\frac{3\sqrt{\gamma^2-1}\,h_7}{8(\gamma-1)^2(\gamma+1)^3} + \frac{3h_{30}}{16-16\gamma^2}\right){\rm Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)\Bigg],$$

$$\frac{\Gamma\chi^{(4)\rm 1rad}_{b,\rm rel}(\gamma)}{\pi\nu} = \frac{h_{64}}{96\left(\gamma^2-1\right)^{7/2}} + \frac{h_{65}\log\left(\frac{\gamma+1}{2}\right)}{16\left(\gamma^2-1\right)^{5/2}} + \frac{h_{63}\,{\rm arcsinh}\left(\frac{\sqrt{\gamma-1}}{\sqrt{2}}\right)}{8\left(\gamma^2-1\right)^4} - \frac{h_{25}\,{\rm arccosh}(\gamma)}{32\left(\gamma^2-1\right)^4}$$
$$+\nu\left[\frac{h_{67}}{96\left(\gamma^2-1\right)^{7/2}} + \frac{h_{68}\log\left(\frac{\gamma+1}{2}\right)}{16\left(\gamma^2-1\right)^{5/2}} - \frac{{\rm arccosh}(\gamma)\left((\gamma+1)h_{14}+(\gamma-3)h_{25}\right)}{32\left(\gamma^2-1\right)^4} + \frac{h_{66}\,{\rm arcsinh}\left(\frac{\sqrt{\gamma-1}}{\sqrt{2}}\right)}{8(\gamma-1)^2(\gamma+1)^4}\right],$$
$$\frac{\Gamma\chi^{(4)\rm 2rad}_{b,\rm rel}(\gamma)}{\pi\nu^2} = \frac{\log\left(\frac{\gamma+1}{2}\right)\left(2\left(\gamma^2-1\right)h_{22}+h_{11}\right)}{64(\gamma-1)^3(\gamma+1)^2} - \frac{\log(\gamma)\left(h_{12}-8\left(\gamma^2-1\right)h_{23}\right)}{32(\gamma-1)^3(\gamma+1)^2} + \frac{{\rm arccosh}(\gamma)\left(2(\gamma-1)^2h_{13}-(\gamma+1)h_{24}\right)}{32\left(\gamma^2-1\right)^{7/2}}$$
$$+\frac{3\sqrt{\gamma^2-1}\left(h_{16}+h_{28}\right)\log\left(\frac{\gamma+1}{2}\right){\rm arccosh}(\gamma)}{32(\gamma-1)^3(\gamma+1)^2} - \frac{h_9-4\gamma^2(\gamma+1)h_{20}}{1536\gamma^9\left(\gamma^2-1\right)^3} - \frac{3\left(h_{15}-4h_{27}\right)\log^2\left(\frac{\gamma+1}{2}\right)}{16(\gamma-1)}$$
$$-\frac{3h_{26}\,{\rm arccosh}^2(\gamma)}{64(\gamma-1)^4(\gamma+1)^3} + \left(\frac{3}{64}(\gamma+1)h_{18} + \frac{h_{29}}{8(\gamma-1)}\right){\rm Li}_2\left(\frac{1-\gamma}{\gamma+1}\right) + \frac{3\left(h_{17}+8h_{30}\right){\rm Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{128(\gamma-1)},$$

PHYSICAL REVIEW D **107,** 064051 (2023)

Strong-field scattering of two black holes: Numerical relativity meets post-Minkowskian gravity

Thibault Damour[1] and Piero Rettegno[2,3]



Resumed ala Firsov/Impetus
(B2B Kalin RAP)

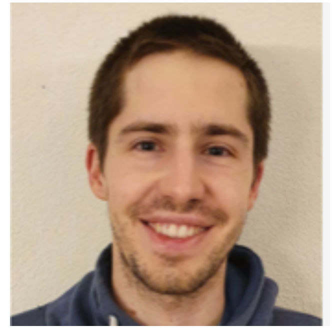
$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i = p_\infty^2(E)\left(1+\sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

Discovery Potential =

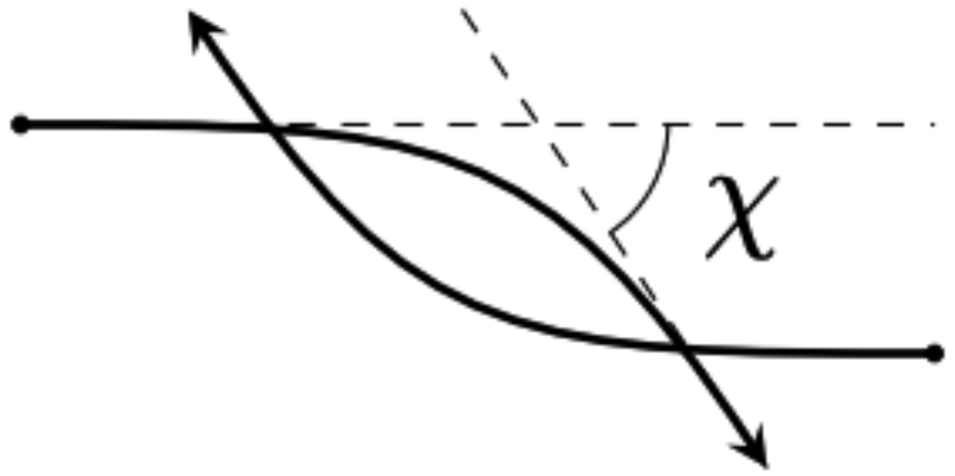
Precise Theoretical Predictions

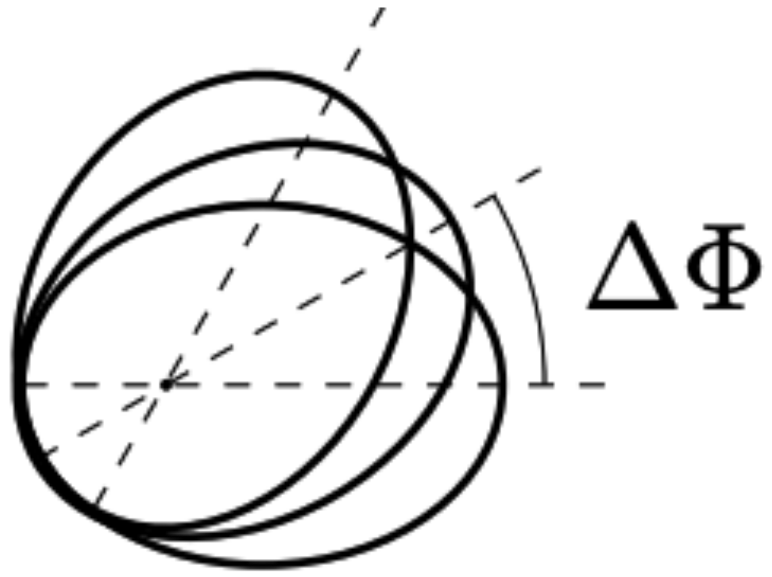
- Part I: EFT for Bound (PN)/Unbound(PM) states
- **Part II:** Boundary2Bound correspondence

Kalin **RAP**
1910.03008
1911.09130



How do we compute *bound* observables from *boundary* data?

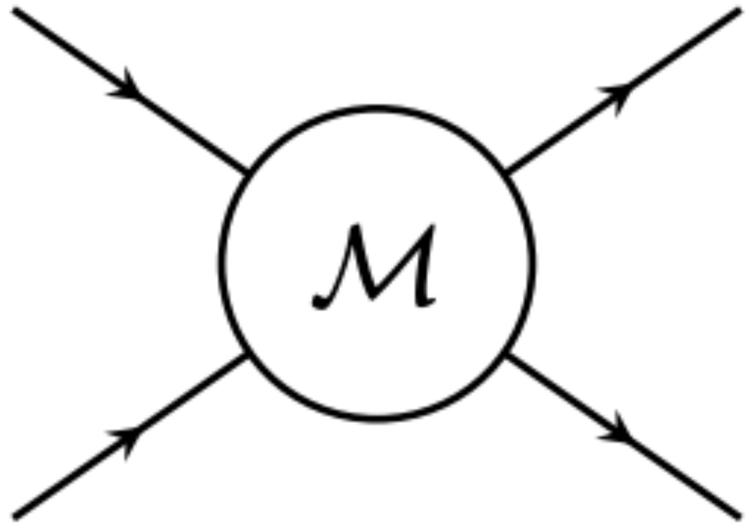


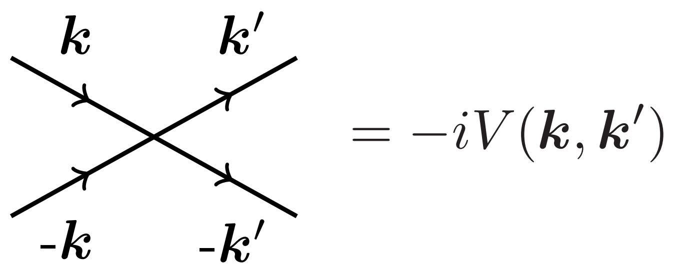


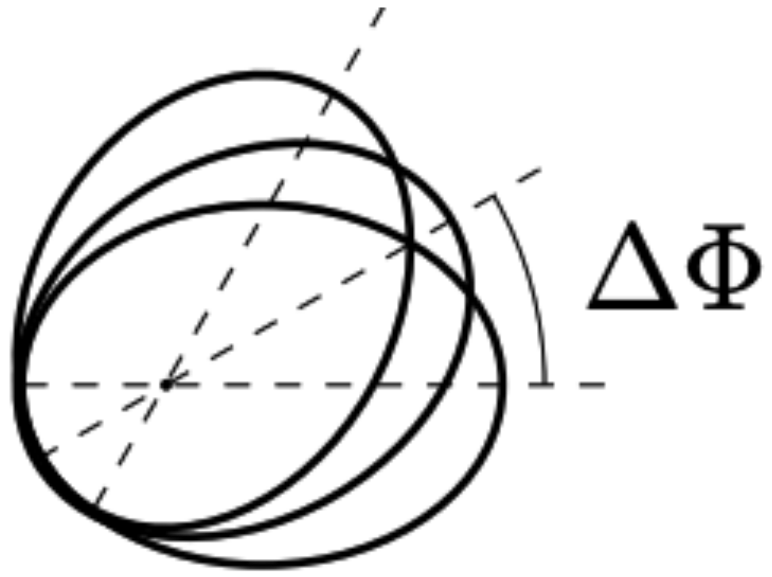
How do we compute *bound* observables from *boundary* data?

Cheung et al 1808.02489

Bern et al. 1908.01493







Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian:
$$H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r})$$
$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

Newton in here

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} - \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{2\nu^3(3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6\xi^4}\right],$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ v^{14}$	$+ \cdots) G$
2PM		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ \cdots) G^2$
3PM			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ \cdots) G^3$
4PM				$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ \cdots) G^4$
5PM					$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ \cdots) G^5$

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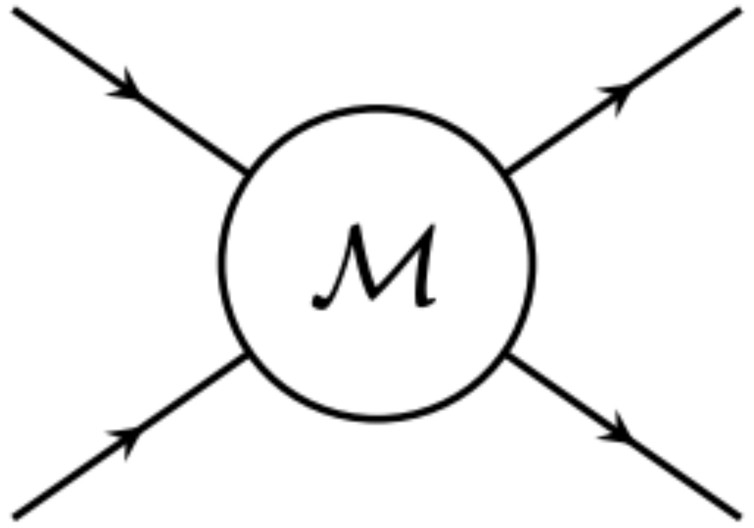
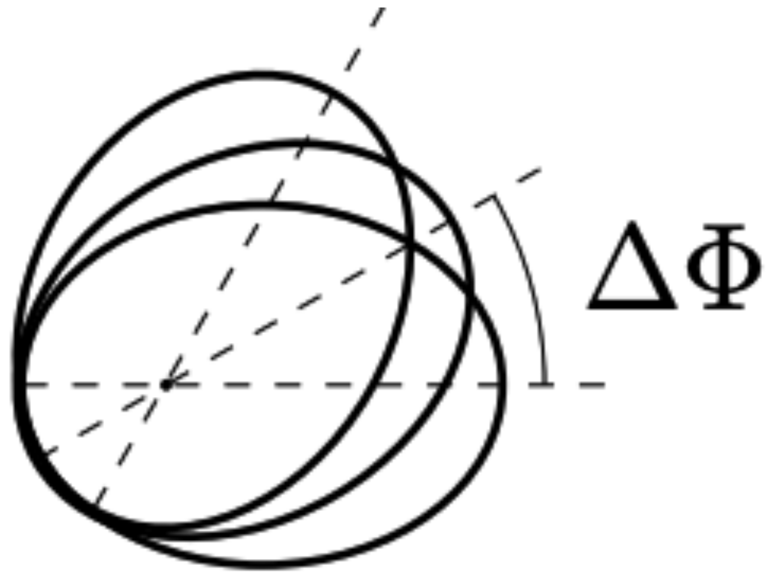
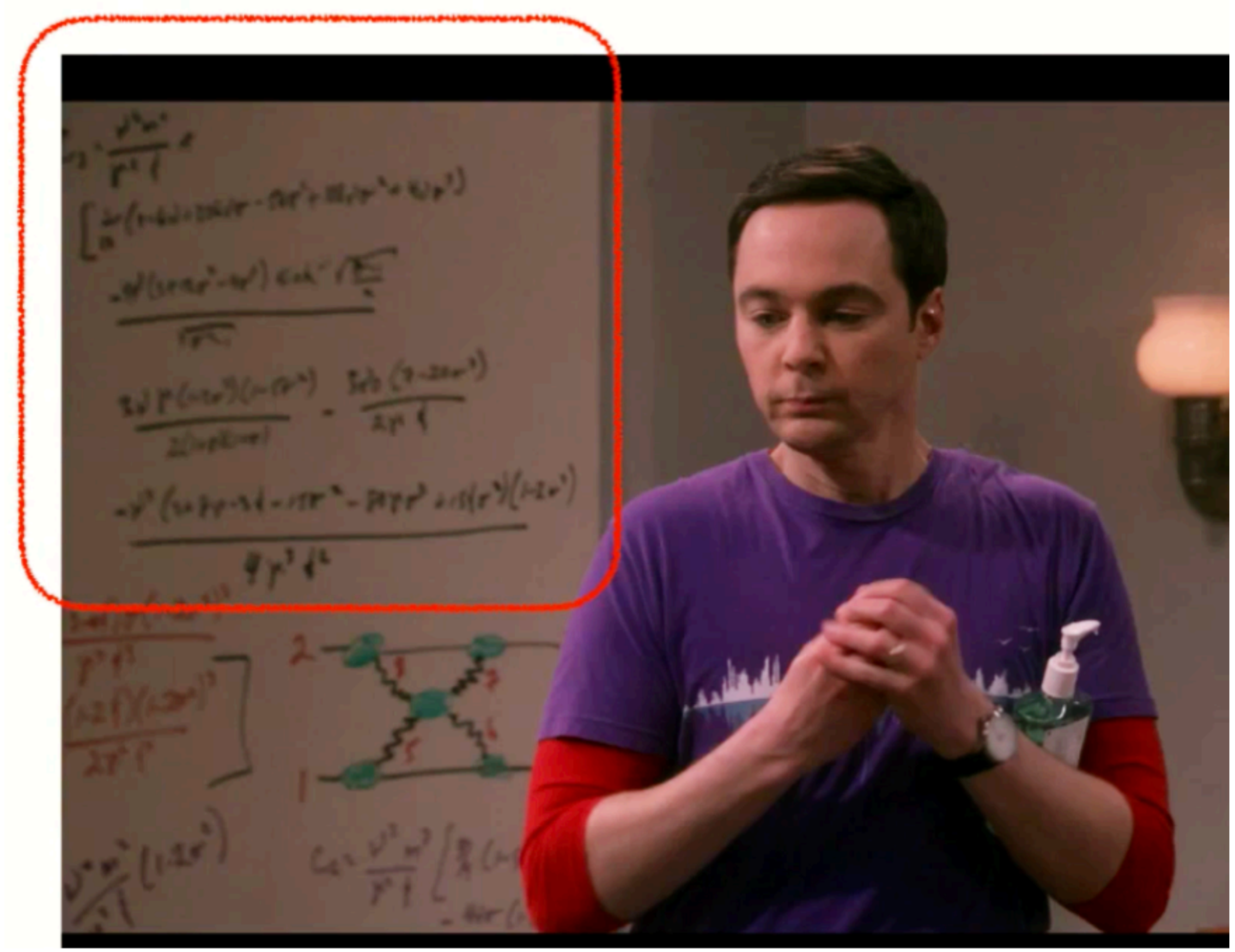






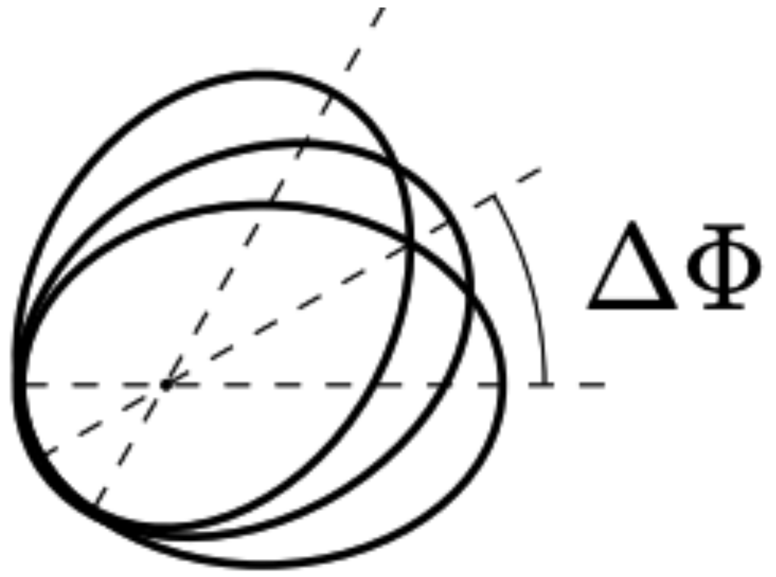
	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)$	G
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)$	G^2
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)$	G^3
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)$	G^4
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)$	G^5

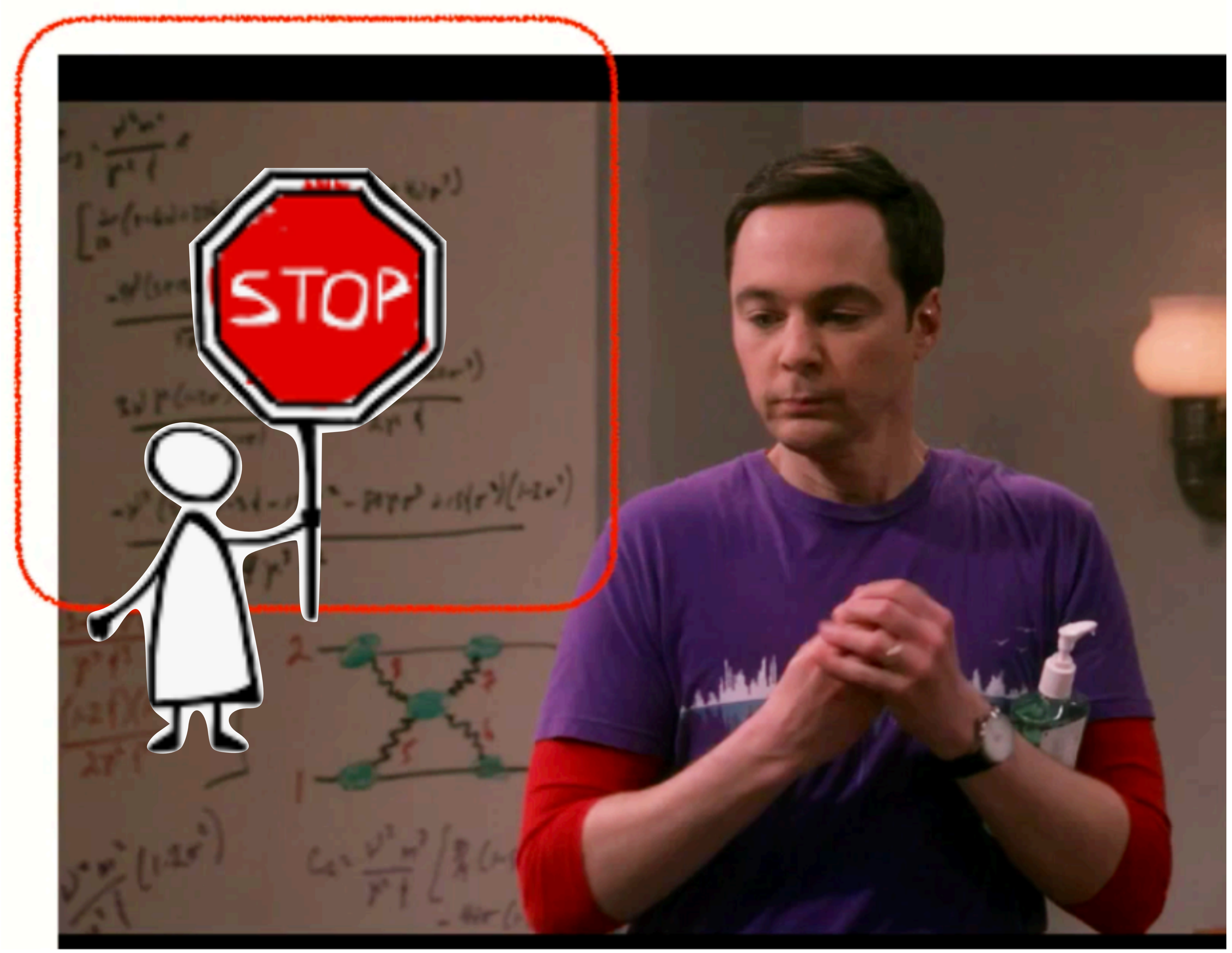
Watch for typos! (Radu R.)

How do we compute *bound* observables from *boundary* data?

IN THE ON-SHELL SPIRIT…

Do we really need the cumbersome & gauge-dependent Hamiltonian?

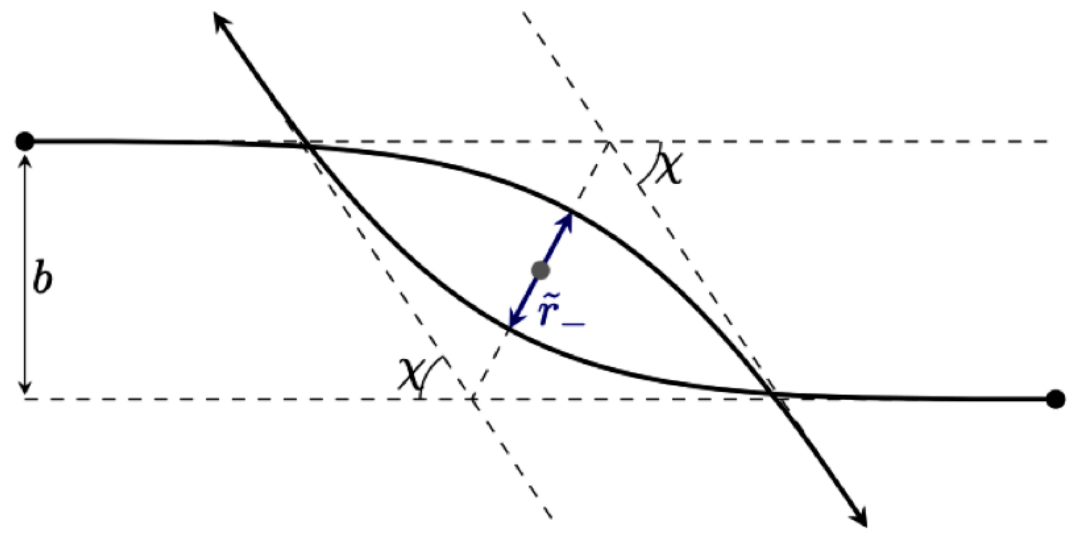




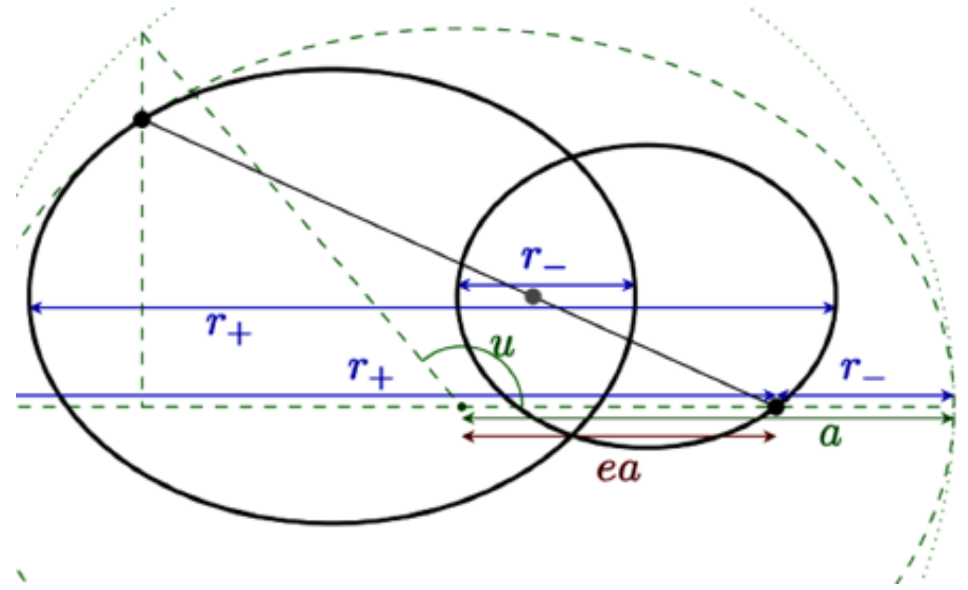
* Sheldon had wrong sign of QCD beta function!

Kalin **RAP**
1910.03008
1911.09130

B2B correspondence



Conservative effects



$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \underbrace{\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}}_{\mathcal{E}>0}\mathrm{d}r$$

Scattering angle

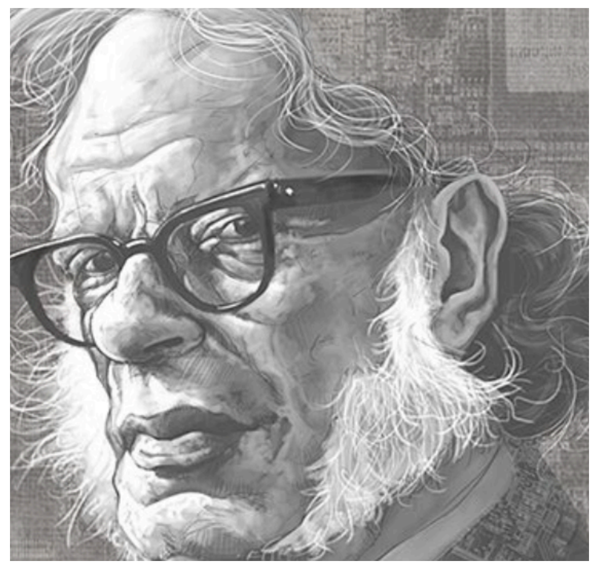
$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \underbrace{\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}}_{\mathcal{E}<0}\mathrm{d}r$$

Periastron advance

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

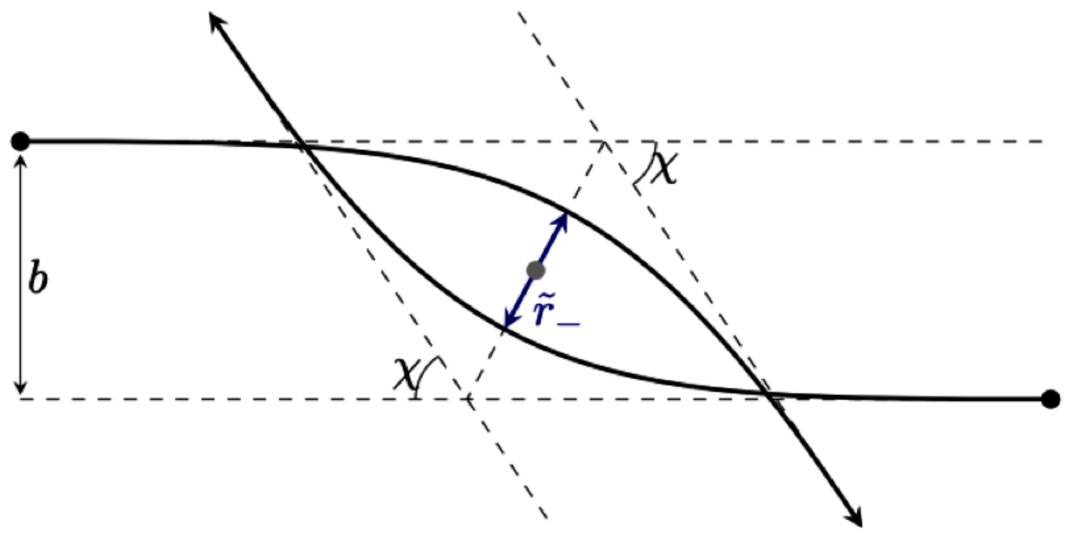
endpoints related by analytic continuation

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."
—Isaac Asimov

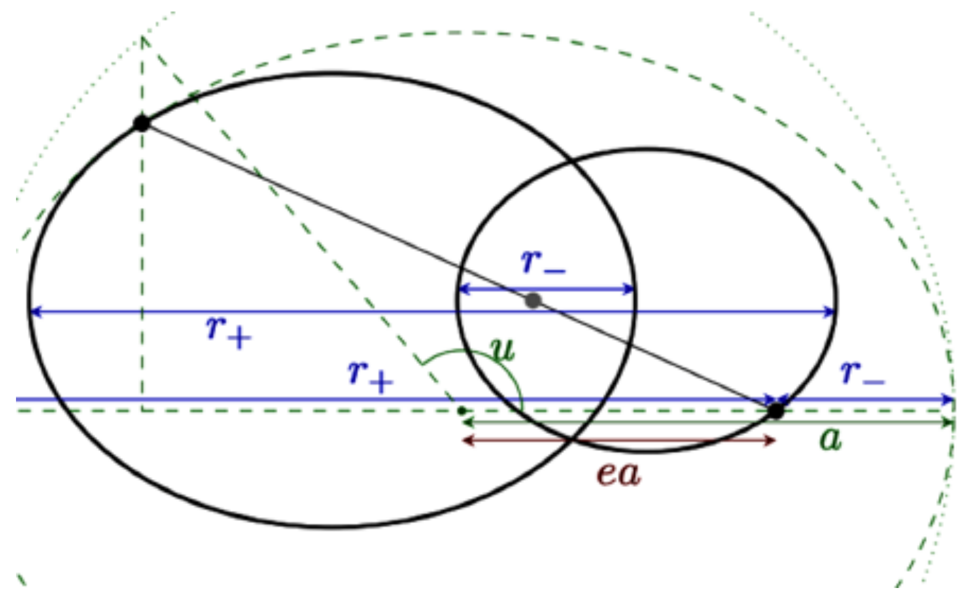


Kalin **RAP**
1910.03008
1911.09130

B2B correspondence



Conservative effects

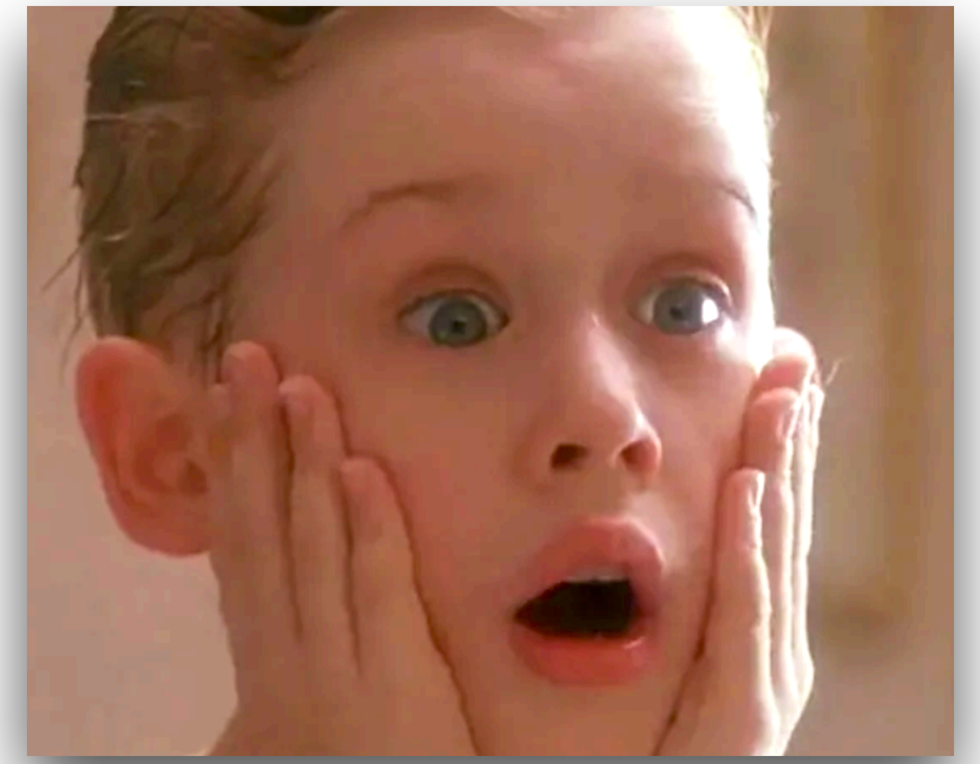


$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \underbrace{\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}}_{\mathcal{E}>0}\mathrm{d}r$$

Scattering angle

$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \underbrace{\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}}_{\mathcal{E}<0}\mathrm{d}r$$

Periastron advance

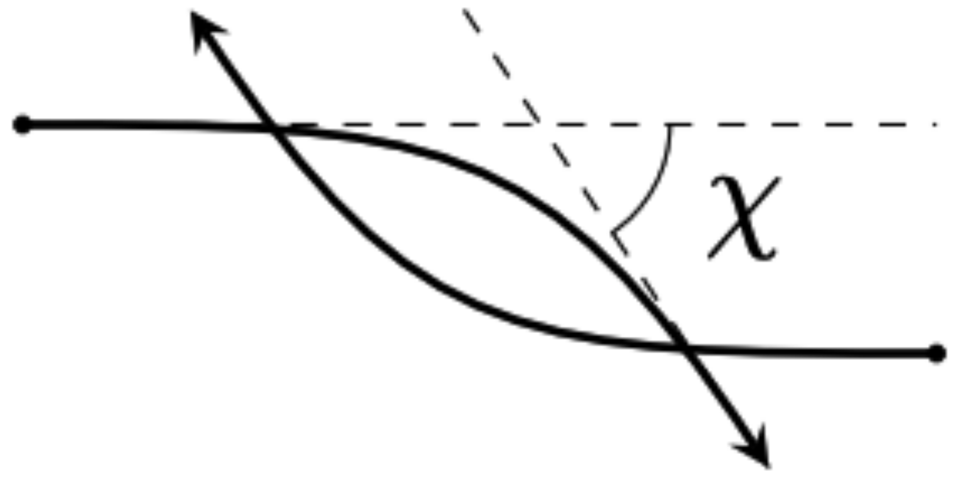


LOOP AROUND INFINITY!

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

Kalin **RAP**
1910.03008
1911.09130

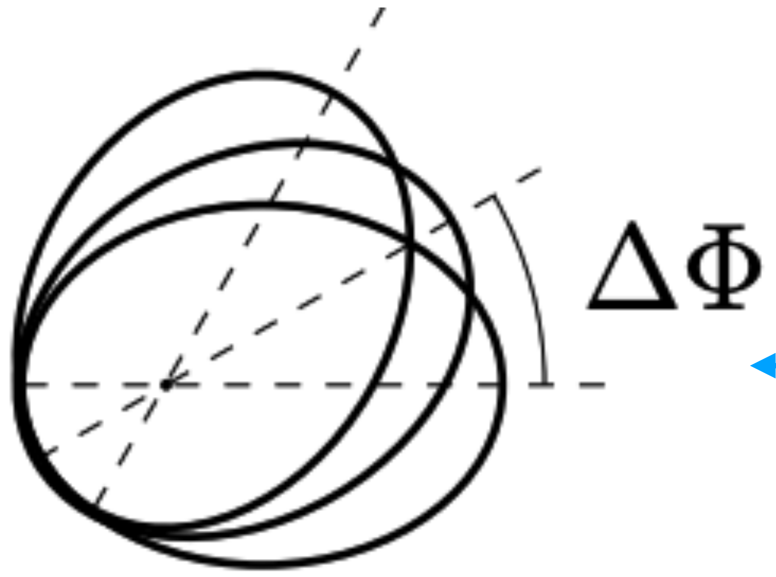
B2B correspondence



$\chi/2\pi = -\partial_j i_r^{(\mathrm{unbound})}$ $\quad j = \frac{J}{GM^2\nu}$

Conservative effects

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



Analytic continuation

At the level of the radial action:

$$i_r^{(\mathrm{bound})}(\mathcal{E}<0,\, J) = i_r^{(\mathrm{unbound})}(\mathcal{E}<0,\, J) - i_r^{(\mathrm{unbound})}(\mathcal{E}<0, -J)$$

Central object for the **bound** problem:

$$(GM^2\nu\,\times)\;\; \delta i_r^{(\mathrm{bound})}(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

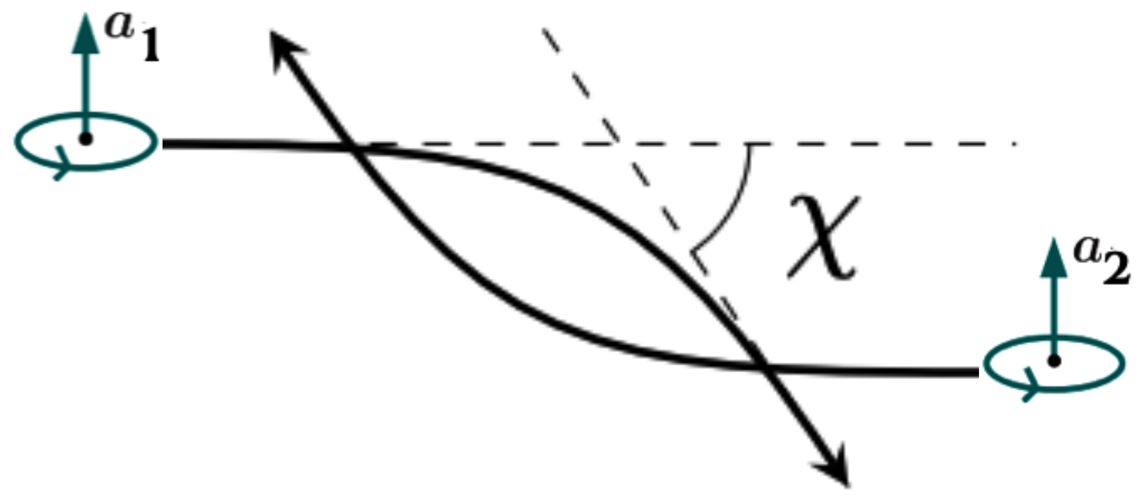
ALL the (conservative) observables!

Kalin **RAP** 1910.03008 1911.09130

Liu **RAP** Yang 2102.10059

B2B correspondence

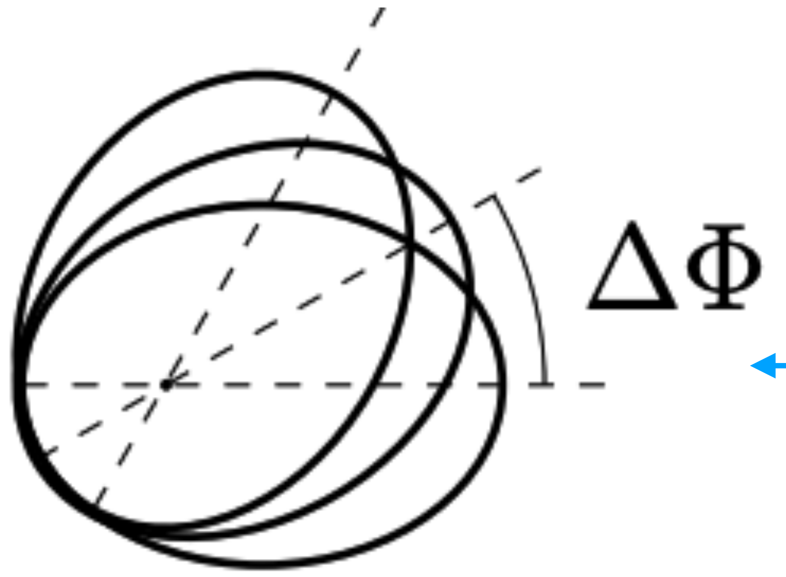
valid for (planar) spin dynamics



J total (canonical) angular momentum

Conservative effects

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



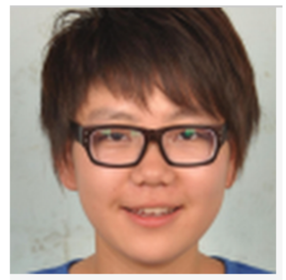
Analytic continuation

At the level of the radial action:

$$i_r^{(\mathrm{bound})}(\mathcal{E}<0,\ell,\tilde{a}_\pm) = i_r^{(\mathrm{unbound})}(\mathcal{E}<0,\ell,\tilde{a}_\pm) - i_r^{(\mathrm{unbound})}(\mathcal{E}<0,-\ell,-\tilde{a}_\pm)\,,$$

Conservative bound radial action to 2PM order:

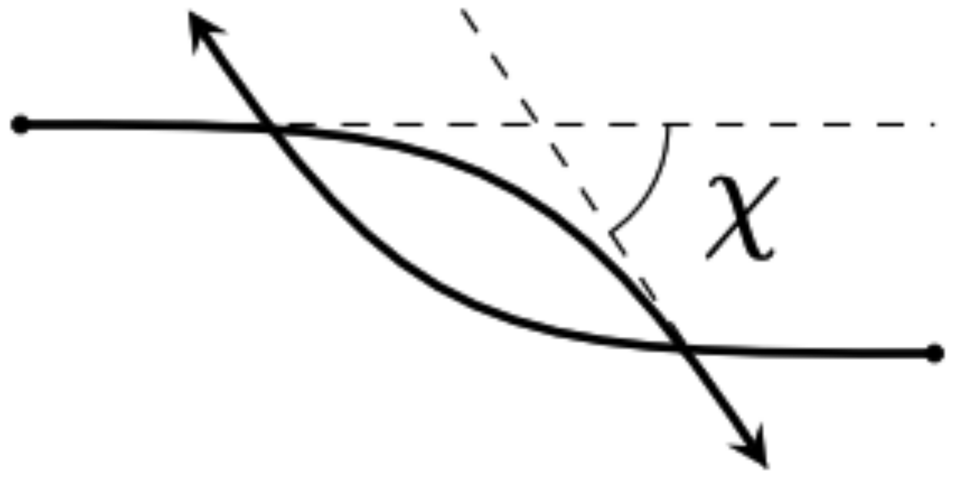
$$i_r^{2\mathrm{PM}}(\mathcal{E},\ell,\tilde{a}_\pm) = -\ell + \frac{2\gamma^2-1}{\sqrt{1-\gamma^2}} + \frac{3}{4\ell}\frac{5\gamma^2-1}{\Gamma} + \frac{1}{\pi}\sum_{A=\pm}\chi_A^{(3)}(\gamma)\frac{\tilde{a}_A}{\ell^2} + \frac{2}{3\pi}\sum_{\{A,B\}=\pm}\chi_{AB}^{(4)}(\gamma)\frac{\tilde{a}_A\tilde{a}_B}{\ell^3}\,,$$



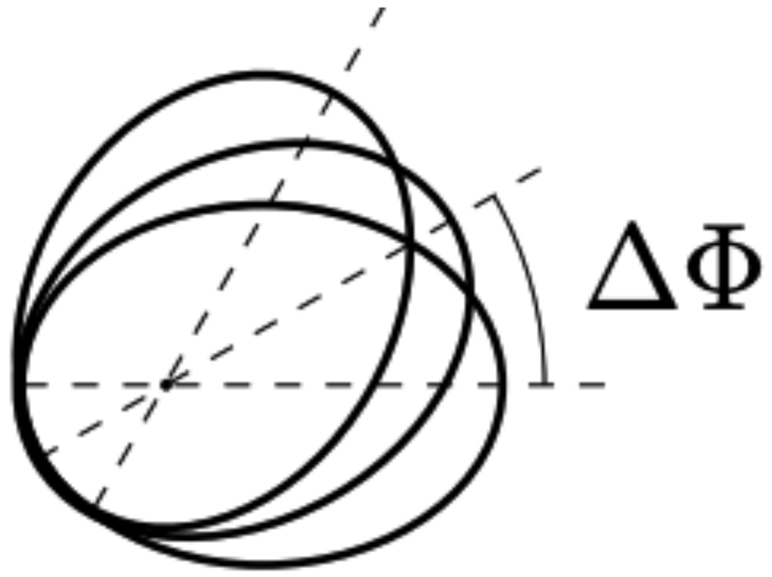
$\tilde{a}_\pm \equiv a_\pm/(GM)$, $\ell \equiv L/GM\mu$

Kalin RAP
1910.03008
1911.09130

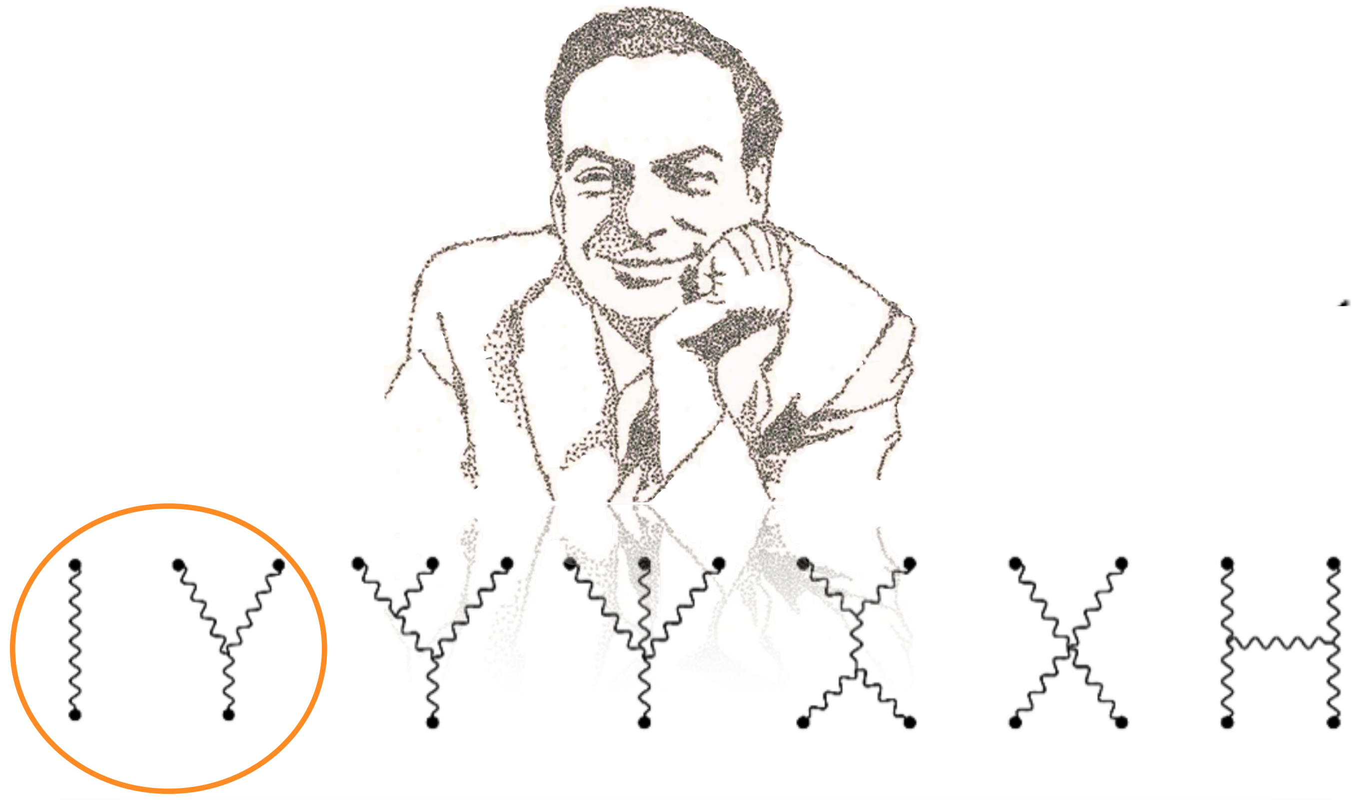
B2B correspondence



Conservative effects



$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



ONE-LOOP EXACT!

$$\frac{\chi_b^{(2)}}{\Gamma} = \frac{3\pi}{8}\frac{5\gamma^2-1}{\gamma^2-1}$$

$$i_r^{2\mathrm{PM}}(\mathcal{E},\ell,\tilde{a}_\pm) = -\ell + \frac{2\gamma^2-1}{\sqrt{1-\gamma^2}} + \frac{3}{4\ell}\frac{5\gamma^2-1}{\Gamma}$$

1PN, 2PN, 2PN, 3PN, 4PN, 5PN

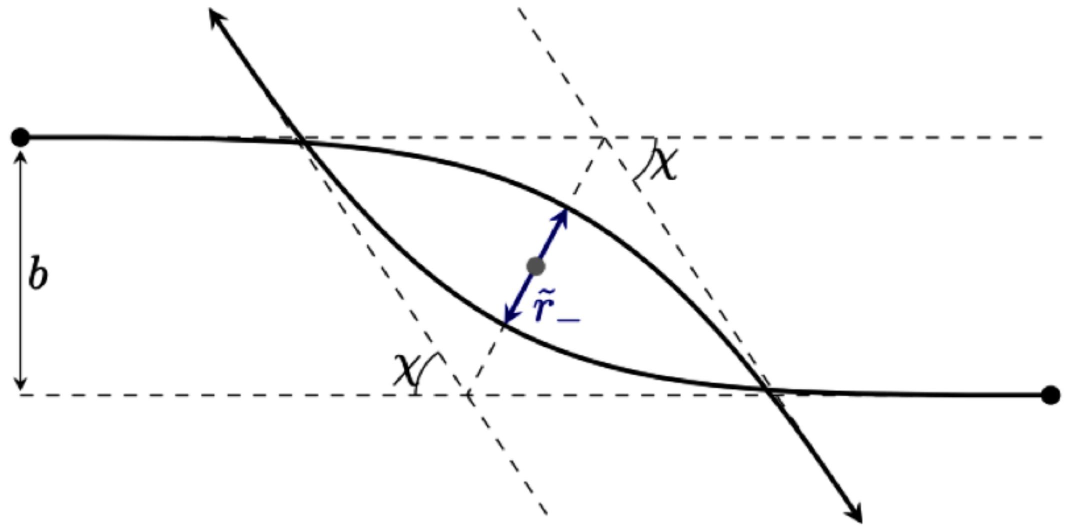
$$\begin{aligned}\frac{\Delta\Phi}{2\pi} &= \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10-4\nu + \frac{194-184\nu+23\nu^2}{j^2}\right)\mathcal{E} \\ &+ \frac{3}{4j^2}\left(5-5\nu+4\nu^2 + \frac{3535-6911\nu+3060\nu^2-375\nu^3}{10j^2}\right)\mathcal{E}^2 \\ &+ \frac{3}{4j^2}\left((5-4\nu)\nu^2 + \frac{35910-126347\nu+125559\nu^2-59920\nu^3+7385\nu^4}{140j^2}\right)\mathcal{E}^3 \\ &+ \frac{3}{4j^2}\left((5-20\nu+16\nu^2)\frac{\nu^2}{4}\right)\mathcal{E}^4 + \cdots,\end{aligned}$$

$$\gamma \equiv \frac{1}{2}\frac{E^2-m_1^2-m_2^2}{m_1 m_2} = 1+\mathcal{E}+\frac{1}{2}\nu\mathcal{E}^2,$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)} = 1+\nu\mathcal{E}.$$

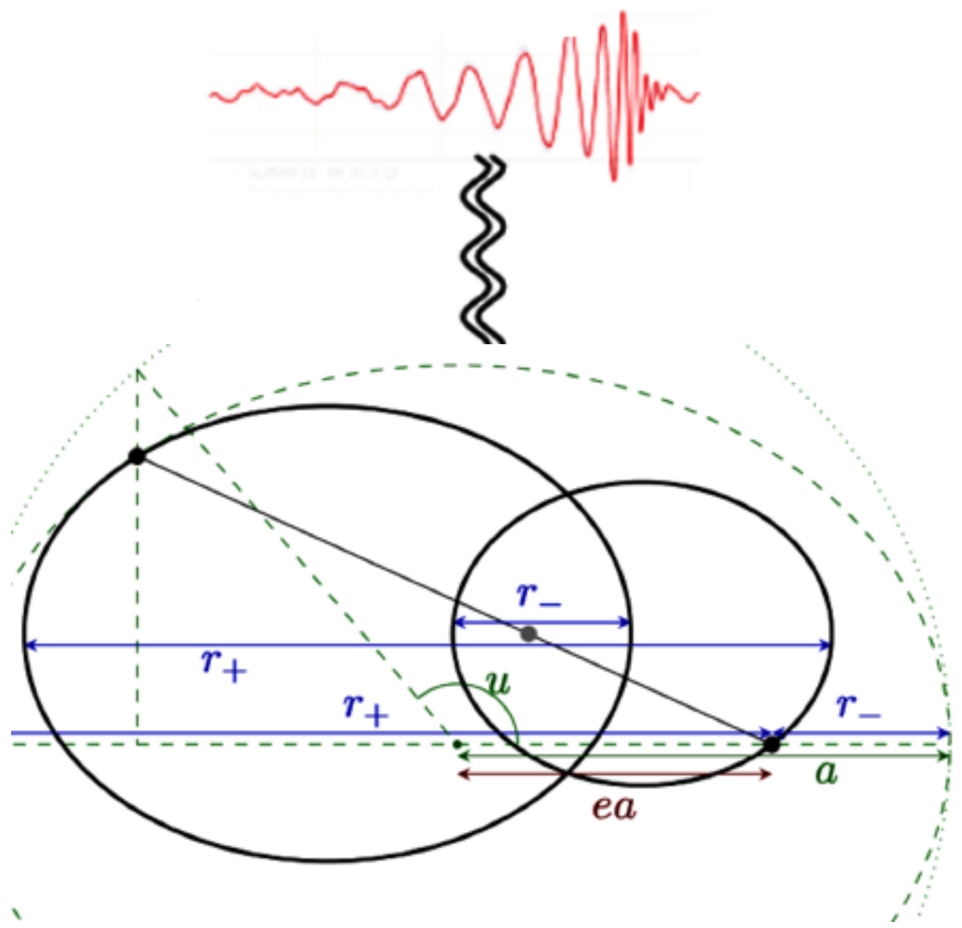
Cho Kalin **RAP**
2112.03976

B2B correspondence



Radiative effects?!

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



$$\Delta E_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt\,\frac{dE}{dt} \qquad\qquad \Delta E_{\rm ell}(J,\mathcal{E}) = \oint dt\,\frac{dE}{dt}$$

$$\frac{dE}{dt}(r,J,\mathcal{E}) = \frac{dE}{dt}(r,-J,\mathcal{E})$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot r}\frac{dE}{dt}(r,J,\mathcal{E}) \qquad\qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot r}\frac{dE}{dt}(r,J,\mathcal{E})$$

Similar to radial action: **Loop-around!**

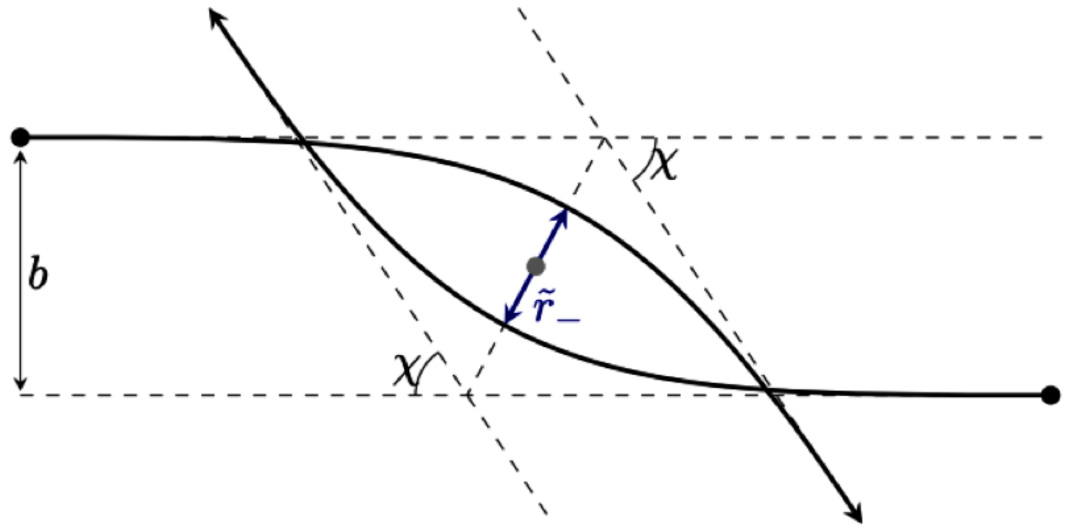
$$\boxed{\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})} \qquad \mathcal{E}<0$$

Aligned-spin configurations

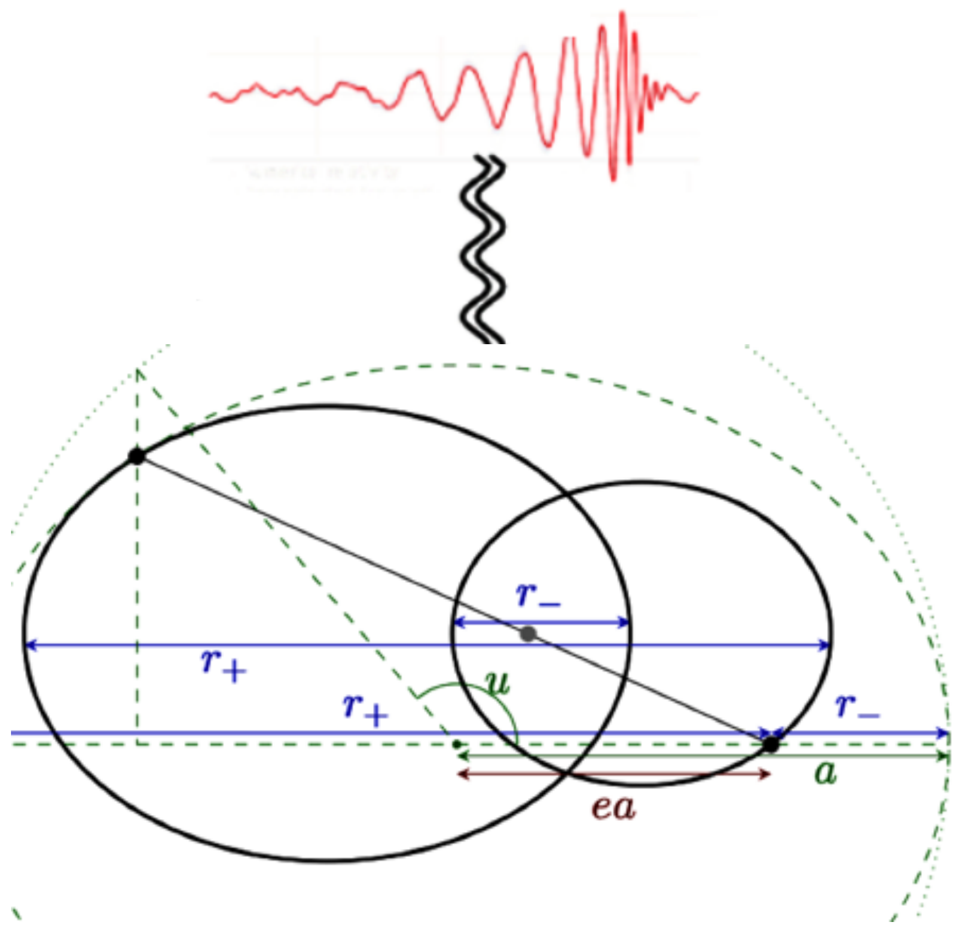
Adiabatic Approx.

Cho Kalin **RAP**
2112.03976

B2B correspondence



Radiative effects?!



$$r_-(J,\mathcal{E}) = \tilde r_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde r_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

$$\Delta J_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dJ}{dt} \qquad \longleftrightarrow \qquad \Delta J_{\rm ell}(J,\mathcal{E}) = \oint dt \frac{dJ}{dt}$$

$$\frac{dJ}{dt}(r,J,\mathcal{E}) = -\frac{dJ}{dt}(r,-J,\mathcal{E})$$

$$2\int_{\tilde r_-}^{+\infty} \frac{dr}{\dot r}\frac{dJ}{dt}(r,J,\mathcal{E}) \qquad\qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot r}\frac{dJ}{dt}(r,J,\mathcal{E})$$

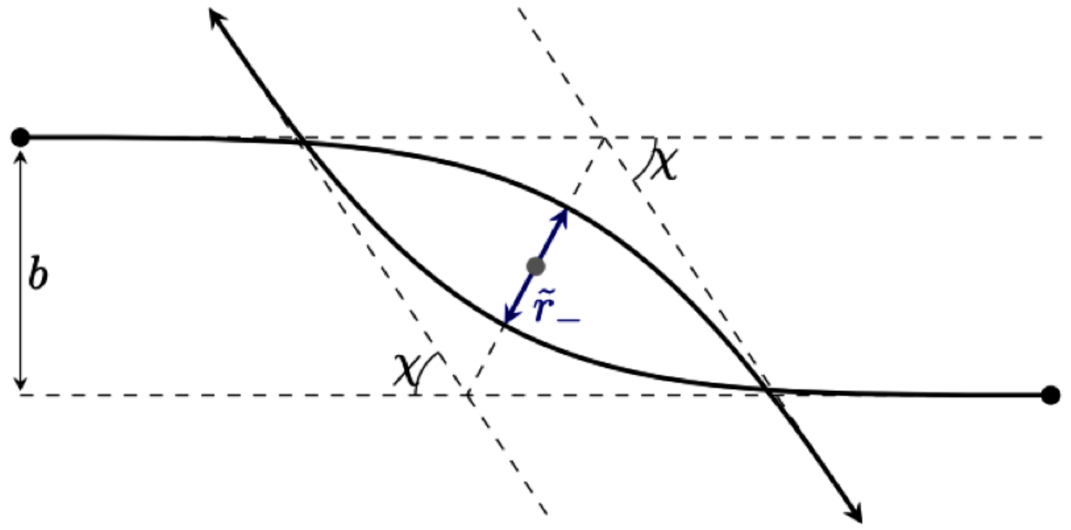
Similar to radial action: **Loop-around!**

$$\Delta J_{\rm ell}(J,\mathcal{E}) = \Delta J_{\rm hyp}(J,\mathcal{E}) + \Delta J_{\rm hyp}(-J,\mathcal{E}) \qquad \mathcal{E}<0$$

Sign flips
Similar to periastron to angle

Cho Kalin **RAP**
2112.03976

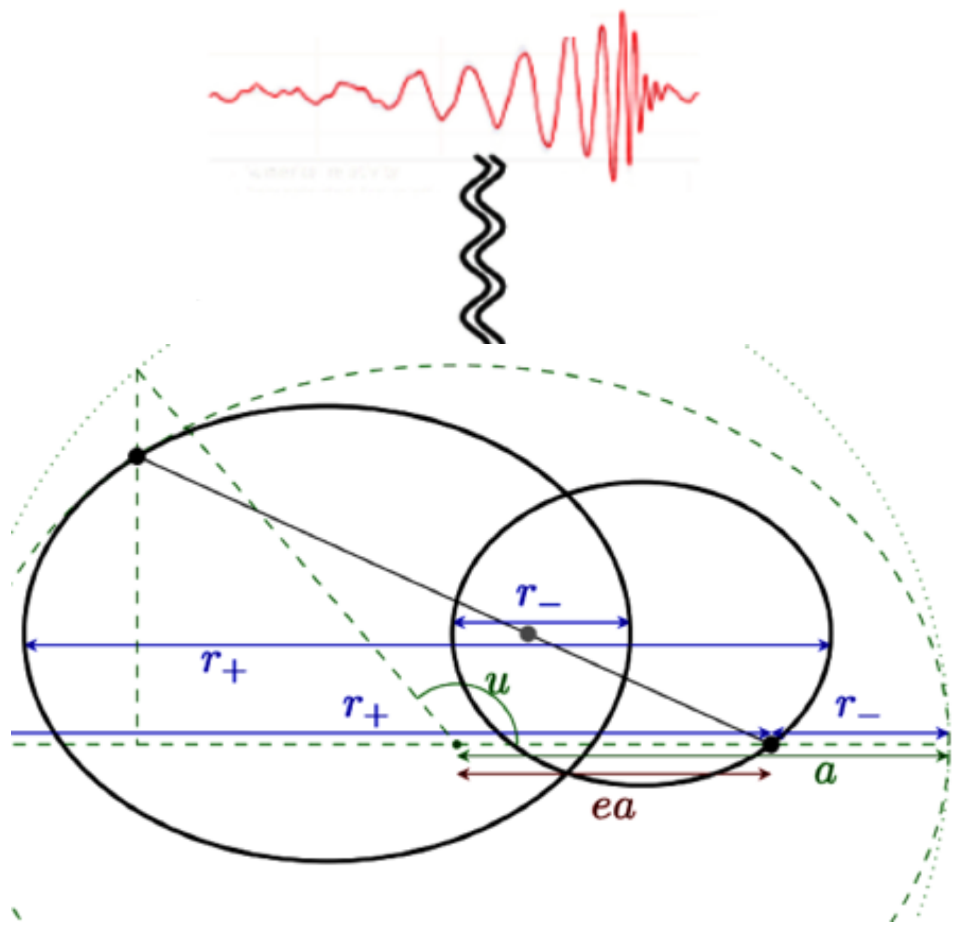
B2B correspondence



Radiative effects?!

analytic continuation:

$$\cos^{-1}\left(\frac{1}{e}\right) + \cos^{-1}\left(-\frac{1}{e}\right) = \pi$$



NLO PN result:

$$\Delta E_{\rm hyp}(j,\mathcal{E}) = \frac{M\,\nu^2}{15}\Bigg[\frac{850\sqrt{2}\sqrt{\mathcal{E}}}{j^6} + \frac{2692\sqrt{2}\mathcal{E}^{3/2}}{3j^4} + \left(\frac{850}{j^7} + \frac{1464\mathcal{E}}{j^5} + \frac{296\mathcal{E}^2}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)$$
$$+ \frac{\sqrt{2}\mathcal{E}^{5/2}(2506431 - 3009160\nu)}{105(1+2\mathcal{E}j^2)j^4} + \frac{\mathcal{E}^{3/2}(182337 - 140480\nu)}{3\sqrt{2}(1+2\mathcal{E}j^2)j^6} - \frac{7\sqrt{\mathcal{E}}(-5763+3220\nu)}{2\sqrt{2}(1+2\mathcal{E}j^2)j^8}$$
$$- \frac{2\sqrt{2}\mathcal{E}^{7/2}(-89907+156380\nu)}{35(1+2\mathcal{E}j^2)j^2} + \left(\frac{\mathcal{E}\left(\frac{33885}{2} - 15900\nu\right)}{j^7} + \frac{\mathcal{E}^2\left(\frac{46617}{7} - 10464\nu\right)}{j^5}\right.$$
$$\left. + \frac{\frac{40341}{4} - 5635\nu}{j^9} + \frac{\mathcal{E}^3\left(\frac{4786}{7} - 888\nu\right)}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)$$

$$\Delta E_{\rm ell}(j,\mathcal{E}) = \frac{M\,\nu^2}{15}\Bigg[\frac{850\pi}{j^7} + \frac{1464\mathcal{E}\pi}{j^5} + \frac{296\mathcal{E}^2\pi}{j^3} + \frac{\mathcal{E}^2\pi}{j^5}\left(\frac{46617}{7} - 10464\nu\right)$$
$$+ \frac{7\pi(5763 - 3220\nu)}{4j^9} + \frac{15\mathcal{E}\pi(2259 - 2120\nu)}{2j^7} + \frac{\mathcal{E}^3\pi}{j^3}\left(\frac{4786}{7} - 888\nu\right)$$

$$\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})$$

only odd terms survive

From losses we can reconstruct fluxes and/or F_rr

Nonlocality in time

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

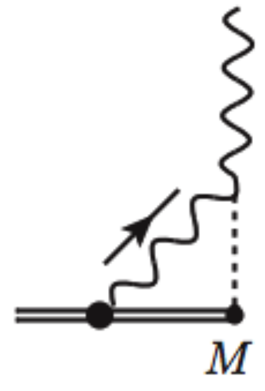
Fails for nonlocal Interactions

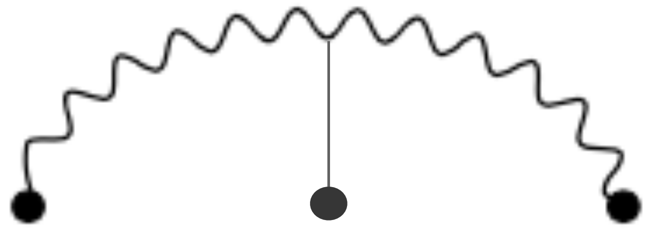
Halley Hooke **Newton** (16XX)	Droste EIH (1916)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa et al.** (2015-19')		
0PN	1PN	2PN	3PN	4PN	5PN	6PN

$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right)$$
$$G^2\left(1+v^2+v^4+v^6+v^8+\cdots\right)$$
$$G^3\left(1+v^2+v^4+v^6+\cdots\right)$$
$$G^4\left(1+v^2+v^4+\cdots\right) \longleftarrow \log v$$
$$G^5\left(1 \cdots\right.$$

Hyperbolic does not fully describe elliptic!

$$\bar{E}^{\rm iso,hyp}_{\rm 4PN(4PM)} - \bar{E}^{\rm iso,ell}_{\rm 4PN(4PM)} = \nu x^5\left[\frac{37933}{45}+\frac{1036\gamma_E}{45}-\frac{113847608\ln 2}{45} - \frac{1472499\ln 3}{20}+\frac{13671875\ln 5}{12}\right] \simeq 14.94\nu x^5\,,$$





"Tail effect"
(scattering off of the geometry sourced by the binary)

$$S_r^{(\rm nloc)} = -\frac{GE}{2\pi}\int_\omega \frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$

UNIVERSAL FORM (IR/UV MIXING + OPTICAL THM)

Energetics and scattering of gravitational two-body systems at fourth post-Minkowskian order

Mohammed Khalil,[1,2,*] Alessandra Buonanno,[1,2,†] Jan Steinhoff,[1,‡] and Justin Vines[1,§]

MOST ACCURATE DESCRIPTION TO DATE DERIVED FROM SCATTERING DATA!

PHYSICAL REVIEW LETTERS 132, 221401 (2024)

Local in Time Conservative Binary Dynamics at Fourth Post-Minkowskian Order

Christoph Dlapa[1,*] Gregor Kälin[1,†] Zhengwen Liu[2,3,‡] and Rafael A. Porto[1,§]

$$\mathcal{S}_r^{(\text{nloc})} = -\frac{GE}{2\pi}\int_\omega \frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$

$$\uparrow$$

$$(2e^{\gamma_E}k\cdot u_{\text{com}})^{2\tilde{\epsilon}}$$

$$u_{\text{com}} \equiv \frac{(m_1u_1+m_2u_2)}{E}$$

Integrand read off from rad1 (on shell k) region of 3PM

$$\frac{1}{\pi\Gamma}\chi_{b(\text{nloc})}^{(4)(n\text{SF})} = \frac{\nu}{(\gamma^2-1)^2}\left\{h_1 + \frac{\pi^2 h_2}{\sqrt{\gamma^2-1}} + h_3\log\left(\frac{\gamma+1}{2}\right) + \frac{h_4\text{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_5\log\left(\frac{\gamma-1}{8}\right) + h_6\log^2\left(\frac{\gamma+1}{2}\right)\right.$$
$$+ h_7\text{arccosh}(\gamma)^2 + \frac{h_8\log(2)\text{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_9\log\left(\frac{\gamma-1}{8}\right)\log\left(\frac{\gamma+1}{2}\right)$$
$$\left. + \frac{h_{10}\log(\frac{\gamma^2-1}{16})\text{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_{11}\text{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right) + \frac{h_{12}\left[\text{arccosh}(\gamma)^2 + 4\text{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)\right]}{\sqrt{\gamma^2-1}}\right\}.$$

$\uparrow$ nSF order (expanded in Mass ratio)

$$\frac{1}{\pi\Gamma}\chi_{b(\text{nloc})}^{(4)\log} = -2\nu\chi_{2\epsilon}(\gamma)$$
$$= \frac{-2\nu}{(\gamma^2-1)^2}\left(h_5 + h_9\log\left(\frac{\gamma+1}{2}\right) + \frac{h_{10}\text{arccosh}(\gamma)}{\sqrt{\gamma^2-1}}\right),$$

$$h_i = h_i^{(0)}(\gamma) + \sqrt{1-4\nu}\,h_i^{(1)}(\gamma) + \Delta h_i(\gamma,\nu),$$

The $\Delta h_i(\gamma,\nu)$ vanish except when $i = 1,3,4$, for which they become polynomials both in γ and ν, up to $\mathcal{O}(\nu^n)$.

Integration problem depends on two scales (velocity and mass ratio)! EXACT SOLUTION ITERATED ELLPTICS

Apply B2B!

$$\chi_{b(\text{loc})}^{(4)} = \chi_{b(\text{tot})}^{(4)\text{cons}} - \chi_{b(\text{nloc})}^{(4)}$$

$$\chi_{b(\text{loc})}^{(4)\log} = -\chi_{b(\text{nloc})}^{(4)\log}$$

$$i_{r(\text{loc})}^{4\text{PM}} = \frac{2v_\infty^4}{3(\Gamma j)^3}\left(\frac{\chi_{b(\text{loc})}^{(4)}}{\pi\Gamma} + \frac{\chi_{b(\text{loc})}^{(4)\log}}{2\pi\Gamma}\log\frac{j^2}{v_\infty^2}\right)$$

$$i_{r(\log)}^{4\text{PM}} = -\frac{E}{(2\pi)M^2\nu}\Delta E_{\text{ell}}(j)\log(-\mathcal{E})$$
$$= \frac{2\nu}{3}\frac{(1-\gamma^2)^2}{(\Gamma j)^3}\chi_{2\epsilon}(\gamma)\log(-\mathcal{E}) + \cdots,$$

LOGS ARE UNIVERSAL!

MOST ACCURATE DESCRIPTION TO DATE DERIVED FROM SCATTERING DATA!

Local-in-Time Conservative Binary Dynamics at Fifth Post-Minkowskian and First Self-Force Orders

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[2] and Rafael A. Porto[1]

$$\mathcal{S}_r^{(\text{nloc})} = -\frac{GE}{2\pi}\int_\omega \frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$

$$\uparrow$$

$$(2e^{\gamma_E}k\cdot u_{\text{com}})^{2\tilde{\epsilon}}$$

$$u_{\text{com}} \equiv \frac{(m_1u_1+m_2u_2)}{E}$$

Integrand read off from rad1 (on shell k) regions of 4PM

$$\frac{1}{\pi\Gamma}\chi_{b(\text{nloc})}^{(5)(1\text{SF})} = \text{COMING SOON}$$

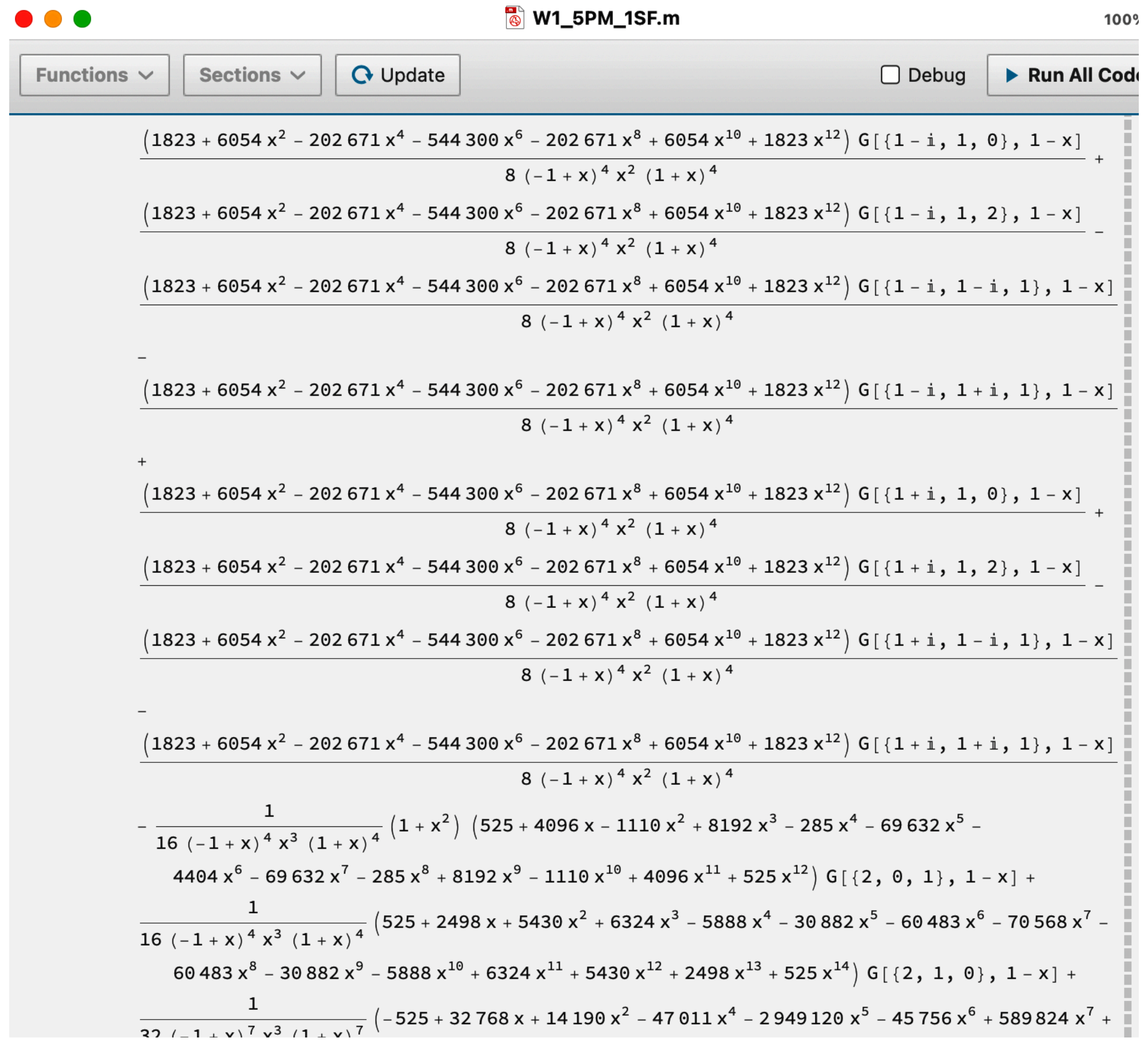
$$\frac{1}{\pi\Gamma}\chi_{b(\text{nloc})}^{(5)\,\log} = \text{COMING SOON}$$

Answer depends on MPLs up to weight 3 with y=1-x and 5 letters (0,1,2,1+i,1-i)

$$G(a_1,\ldots,a_n;y) = \int_0^y \frac{dt}{t-a_1}G(a_2,\ldots,a_n;y)\,,\quad G(0,\ldots,0;y) = \frac{1}{n!}\log^n y$$

Derive local-in-time counterpart

$$\chi_{b(\text{loc})}^{(5)(1\text{SF})} = \chi_{b(\text{even})}^{(5)(1\text{SF})} - \chi_{b(\text{nloc})}^{(5)}\,,\qquad \chi_{b(\text{loc})}^{(5)\,\log} = -\chi_{b(\text{nloc})}^{(5)\,\log}\,,$$



MOST ACCURATE DESCRIPTION TO DATE DERIVED FROM SCATTERING DATA!

PHYSICAL REVIEW LETTERS 132, 221401 (2024)

Local in Time Conservative Binary Dynamics at Fourth Post-Minkowskian Order

Christoph Dlapa[1,*] Gregor Kälin[1,†] Zhengwen Liu[2,3,‡] and Rafael A. Porto[1,§]

All orders in velocity!

Known to 6PN order

$$\hat{H}^{\rm ell}_{\rm 4PM} = \sum_{i=1}^{i=4} \frac{\hat{c}_{i({\rm loc})}}{\hat{r}^i} + \sum_{i=1}^{i=4} \frac{\hat{c}_{i({\rm nloc})}}{\hat{r}^i} + \frac{4\nu^2}{3\hat{r}^4}\frac{(\gamma^2-1)}{\Gamma^2\xi}\chi_{2\epsilon}\log\left(\frac{\hat{r}}{e^{2\gamma_E}}\right)$$

Logs are universal (PN-exact) fixed by radiated flux

5PM/1SF results on the way!

Energetics and scattering of gravitational two-body systems at fourth post-Minkowskian order

Mohammed Khalil[1,2,*] Alessandra Buonanno,[1,2,†] Jan Steinhoff[1,‡] and Justin Vines[1,§]

$$\begin{aligned}
\hat{H}^{\rm ell,iso}_{\rm 6PN(4PM)} &= \hat{H}^{\rm ell,iso}_{\rm 5PN(4PM)} + \left[\frac{33}{2048} + \frac{429\nu^6}{2048} - \frac{3003\nu^5}{2048} + \frac{3003\nu^4}{1024} - \frac{1287\nu^3}{512} + \frac{2145\nu^2}{2048} - \frac{429\nu}{2048}\right]p^{14} \\
&+ \frac{Gp^{12}}{r}\left[\frac{273}{1024} - \nu^6 - \frac{3\nu^5}{2} + \frac{75\nu^4}{4} + \nu^3\left(-\frac{218307}{140} + \frac{10834496\ln 2}{3} + \frac{19775583\ln 3}{140} - \frac{138671875\ln 5}{84}\right)\right. \\
&+ \nu^2\left(\frac{10614711}{22400} - \frac{5417248}{5}\ln 2 + \frac{27734375\ln 5}{56} - \frac{59326749\ln 3}{1400}\right) \\
&\left.+ \nu\left(-\frac{1860381}{44800} + \frac{1354312\ln 2}{15} + \frac{19775583\ln 3}{5600} - \frac{27734375\ln 5}{672}\right)\right] \\
&+ \frac{G^2p^{10}}{r^2}\left[\frac{441}{256} + \frac{693\nu^6}{512} + \frac{17175\nu^5}{512} - \frac{2505\nu^4}{256}\right. \\
&+ \nu^3\left(\frac{1752882443}{134400} - \frac{50772177511\ln 2}{3780} + \frac{15140243287719\ln 3}{2867200} + \frac{15746212109375\ln 5}{3096576} - \frac{1065779114477\ln 7}{442368}\right) \\
&+ \nu^2\left(\frac{930216823}{107520} - \frac{188966394467\ln 2}{7560} + \frac{147239183828125\ln 5}{12386304} + \frac{484445052035\ln 7}{589824} - \frac{7125985899279\ln 3}{2293760}\right) \\
&\left.+ \nu\left(-\frac{29016839}{35840} + \frac{154094423\ln 2}{72} + \frac{527065116993\ln 3}{2293760} - \frac{96889010407\ln 7}{1769472} - \frac{12533579921875\ln 5}{12386304}\right)\right] \\
&+ \frac{G^3p^8}{r^3}\left[\frac{2805}{512} - \frac{19425\nu^5}{256} - \frac{168131\nu^4}{512} + \nu^3\left(-\frac{5539742599}{120960} + \frac{38790406370519\ln 2}{1786050} + \frac{1009279694921875\ln 5}{877879296}\right.\right. \\
&\left.+ \frac{453841966033589\ln 7}{89579520} - \frac{244047465883413\ln 3}{10035200}\right) + \nu^2\left(-\frac{180308862367}{2822400} + \frac{116606471572979\ln 2}{1071630}\right. \\
&\left.+ \frac{368097237751268 9\ln 7}{358318080} - \frac{448065058976289\ln 3}{40140800} - \frac{181279182489765625\ln 5}{3511517184}\right) + \nu\left(-\frac{3456473588783}{304819200}\right. \\
&\left.\left.+ \frac{369057536315537\ln 2}{9185400} + \frac{607401830370627\ln 3}{80281600} - \frac{1267373911442149\ln 7}{429981696} - \frac{44240036362654375\ln 5}{2341011456}\right)\right] \\
&+ \frac{G^4p^6}{r^4}\left[\frac{2275}{256} - \frac{105\nu^6}{128} + \frac{1855\nu^5}{32} + \left(\frac{146987}{192} - \frac{41\pi^2}{64}\right)\nu^4 + \nu^3\left(-\frac{74\ln r}{5} - \frac{25729\pi^2}{4096} + \frac{5104603957}{60480} + \frac{148\gamma_E}{5}\right.\right. \\
&\left.- \frac{2348423027149\ln 2}{51030} + \frac{8674336284777\ln 3}{286720} + \frac{250707235071713\ln 7}{17915904} - \frac{2232609748046875\ln 5}{125411328}\right) \\
&+ \nu^2\left(\frac{197\ln r}{140} - \frac{197\gamma_E}{70} - \frac{104939\pi^2}{16384} + \frac{2714234991803}{16934400} - \frac{126132398166437\ln 2}{1071630} + \frac{763693932388383\ln 3}{8028160}\right. \\
&\left.+ \frac{204623745011171875\ln 5}{3511517184} - \frac{4304025048065071\ln 7}{71663616}\right) + \nu\left(-\frac{5827\ln r}{1008} - \frac{2337139\pi^2}{25165824} + \frac{3571766093993}{76204800}\right. \\
&+ \frac{5827\gamma_E}{504} - \frac{616925145960877\ln 2}{3214890} + \frac{52541416380715625\ln 5}{585252864} + \frac{1554400159532395\ln 7}{107495424} \\
&\left.\left.- \frac{144912376553769\ln 3}{4014080}\right)\right]. \qquad \text{(A6)}
\end{aligned}$$

Perfect agreement with state of the art in PN!

Implemented also in the "EOB gauge"

Fourth post-Minkowskian local-in-time conservative dynamics of binary systems

Donato Bini and Thibault Damour

Phys. Rev. D 110, 064005 – Published 3 September 2024

effective one-body Hamiltonian (in energy gauge). Our computation capitalizes on the tutti frutti approach [D. Bini *et al.*, Novel approach to binary dynamics: Application to the fifth post-Newtonian level, Phys. Rev. Lett. **123**, 231104 (2019)] and on recent post-Minkowskian advances [Z. Bern *et al.*, Scattering amplitudes, the tail effect, and conservative binary dynamics at O(G4), Phys. Rev. Lett. **128**, 161103 (2022); C. Dlapa *et al.*, Conservative dynamics of binary systems at fourth post-Minkowskian order in the large-eccentricity expansion, Phys. Rev. Lett. **128**, 161104 (2022); C. Dlapa *et al.*, Local in time conservative binary dynamics at fourth post-Minkowskian order, **132**, 221401 (2024)].

PHYSICAL REVIEW LETTERS 128, 161104 (2022)

Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-Eccentricity Expansion

Christoph Dlapa, Gregor Kälin, Zhengwen Liu, and Rafael A. Porto
Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

PHYSICAL REVIEW LETTERS 128, 161103 (2022)

Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

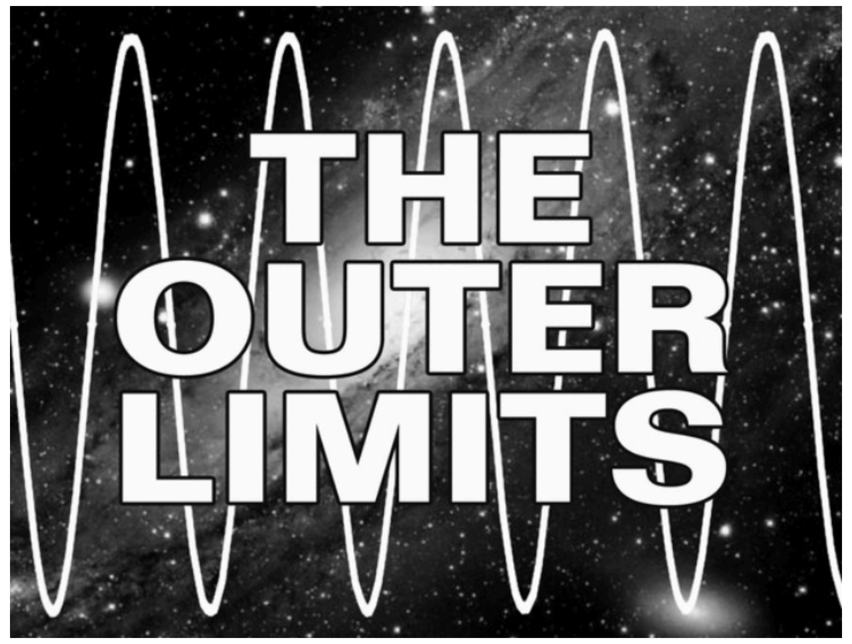
Zvi Bern,[1] Julio Parra-Martinez,[2] Radu Roiban,[3] Michael S. Ruf,[1] Chia-Hsien Shen,[4] Mikhail P. Solon,[1] and Mao Zeng[5]

We provide a resummed (gauge-invariant) version of local+W2 instead

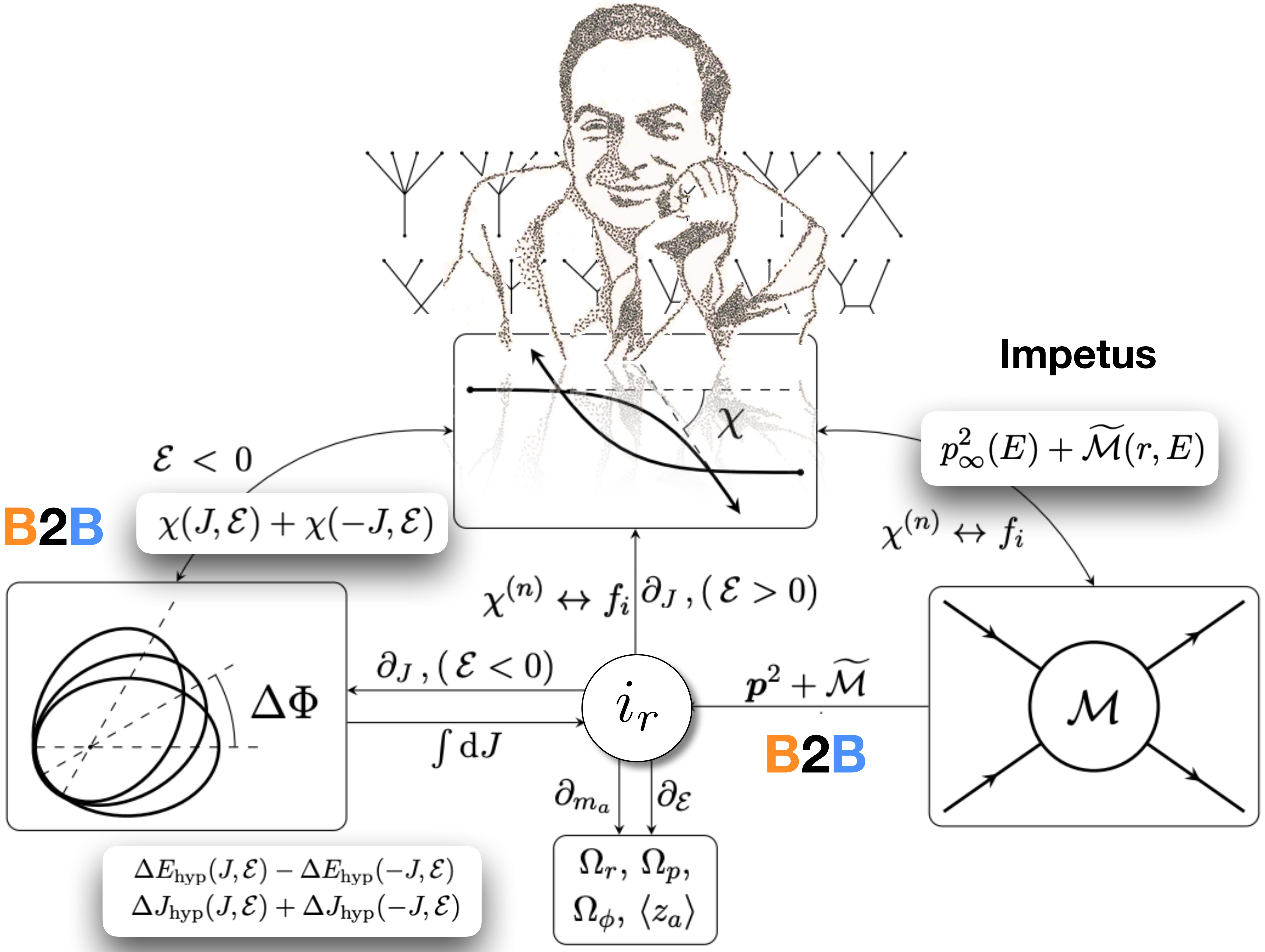
Worldline EFT approach state-of-the-art in PN/PM

$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right)$$
$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right)$$
$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right)$$
$$G^4\left(1 + v^2 + v^4 + \cdots\right)$$
$$G^5\left(1 + v^2 + \cdots\right)$$

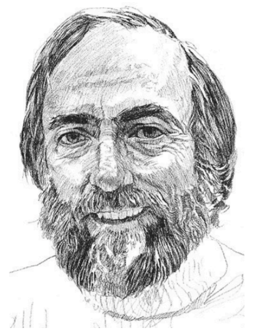
1



"[...] We control the vertical...
We control the horizontal"

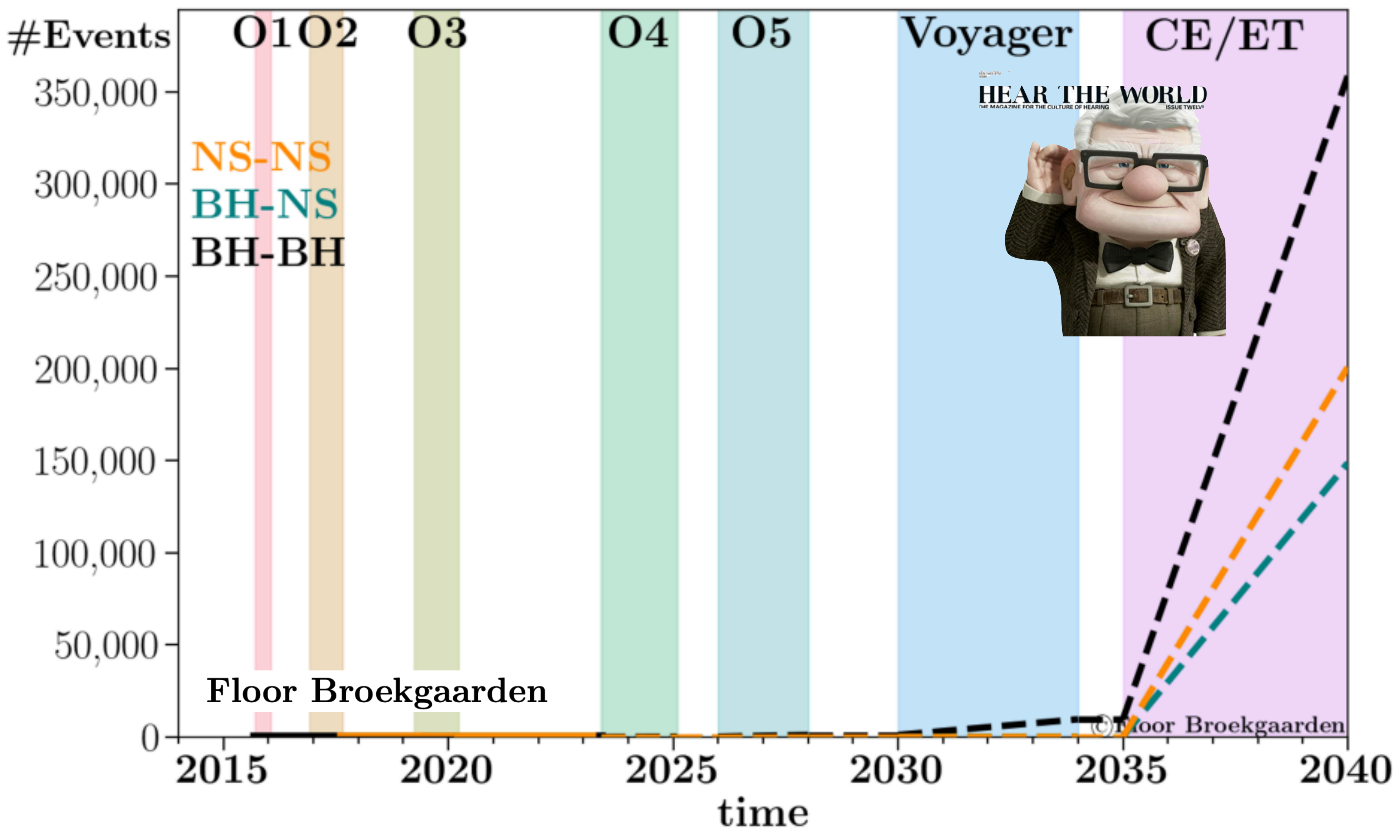


$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$



"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW's information"

Kip Thorne
(Last 3 mins) ~~1993~~ 2024

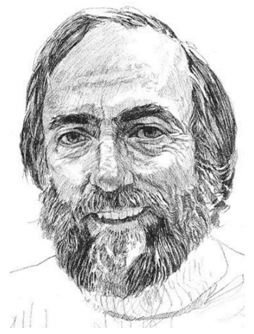


German Center for Astrophysics to Lausitz!

DZA

COMING SOON

Are we ready for the future?



"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW's information"

Kip Thorne
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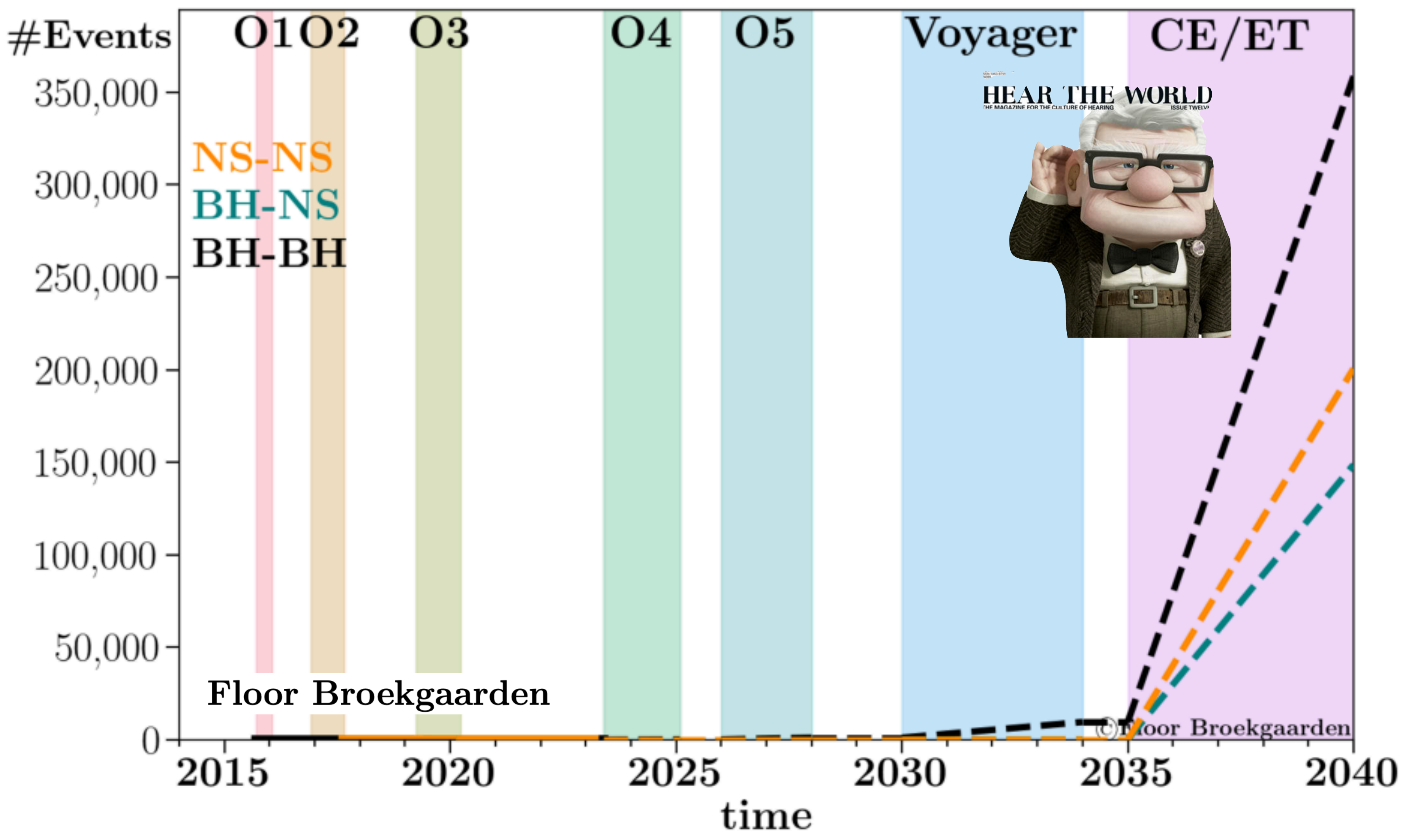
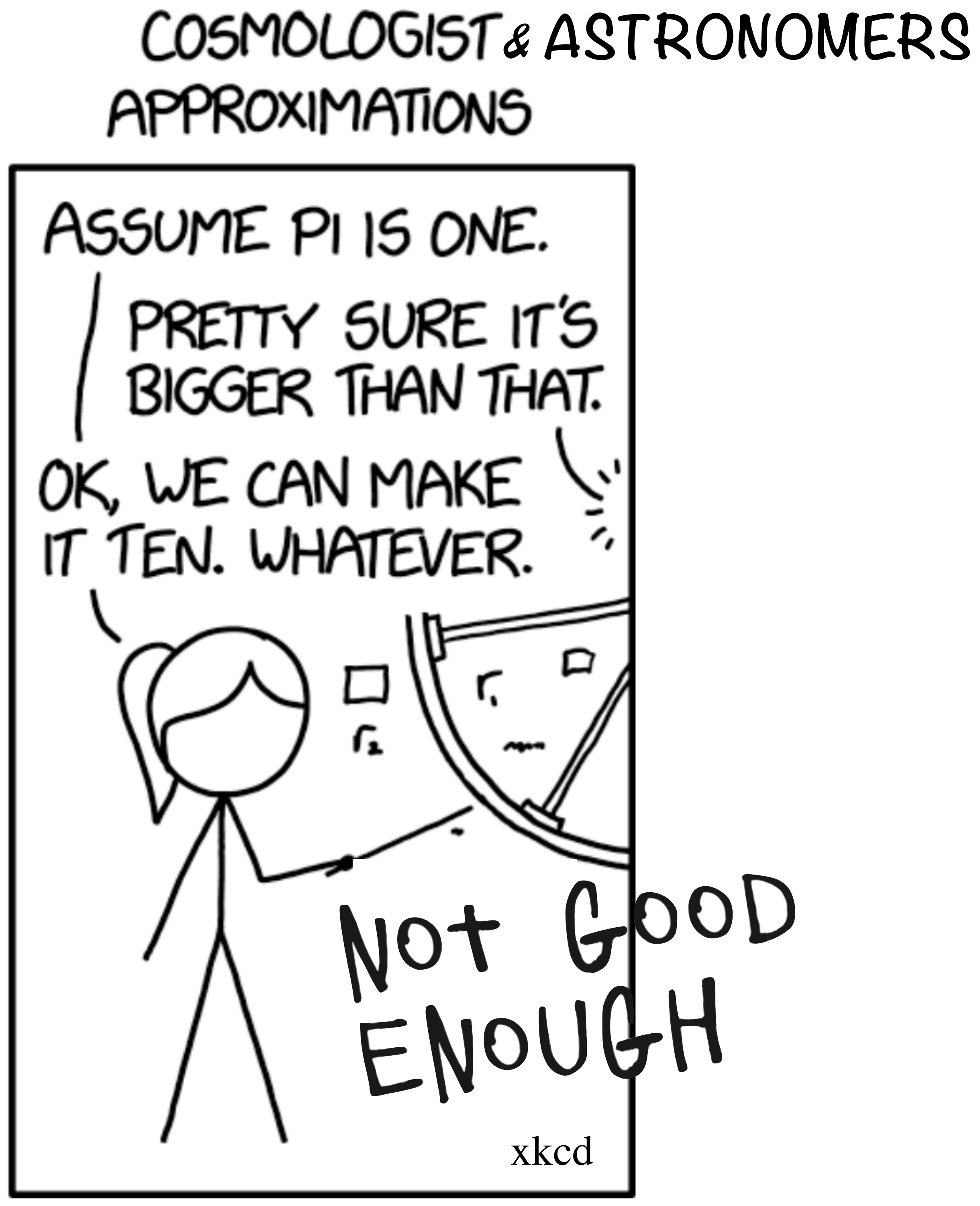
~~1993~~ 2024

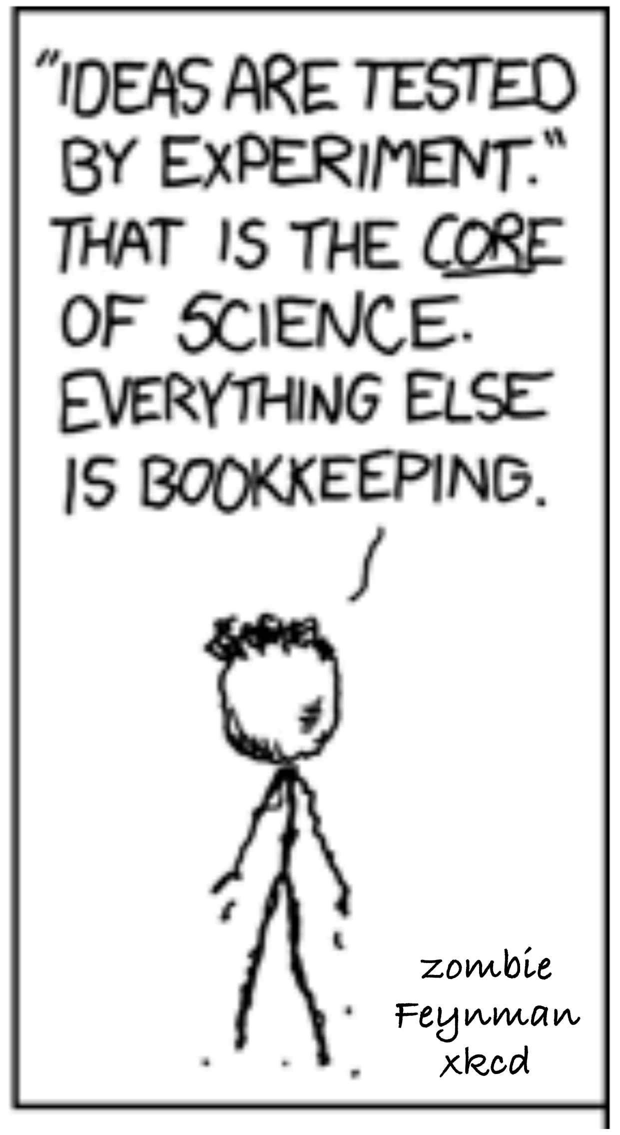




German Center for Astrophysics to Lausitz!

NYT 1991


Experts Clash Over Project To Detect Gravity Wave



Die Zeit

no.203.078

01.01.203X

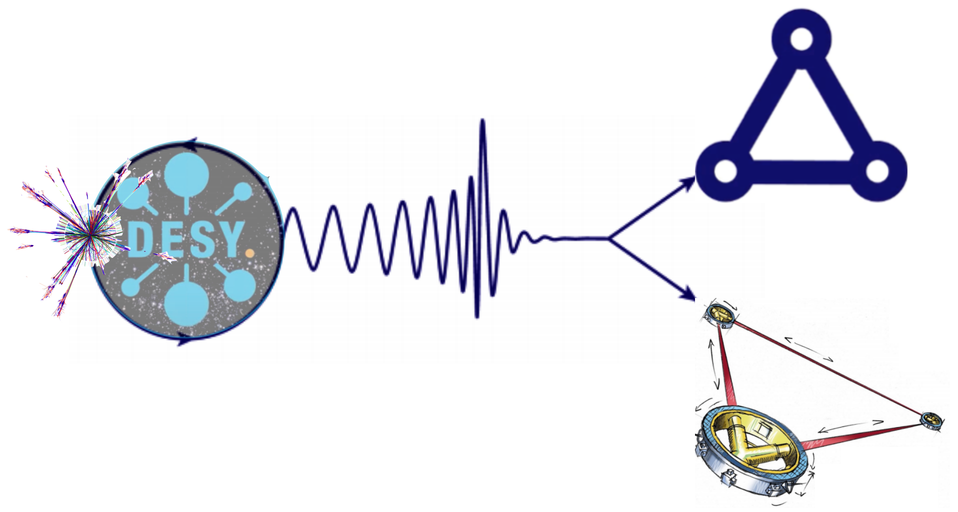
EinsTein reloaded!

New era of investigations through GW precision data!

New particles discovered!

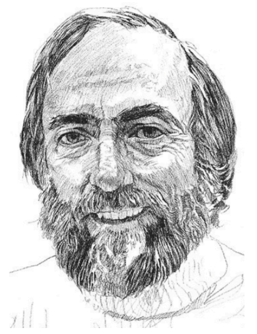
Black holes unveiled!

Einstein was right?!



Thank you!

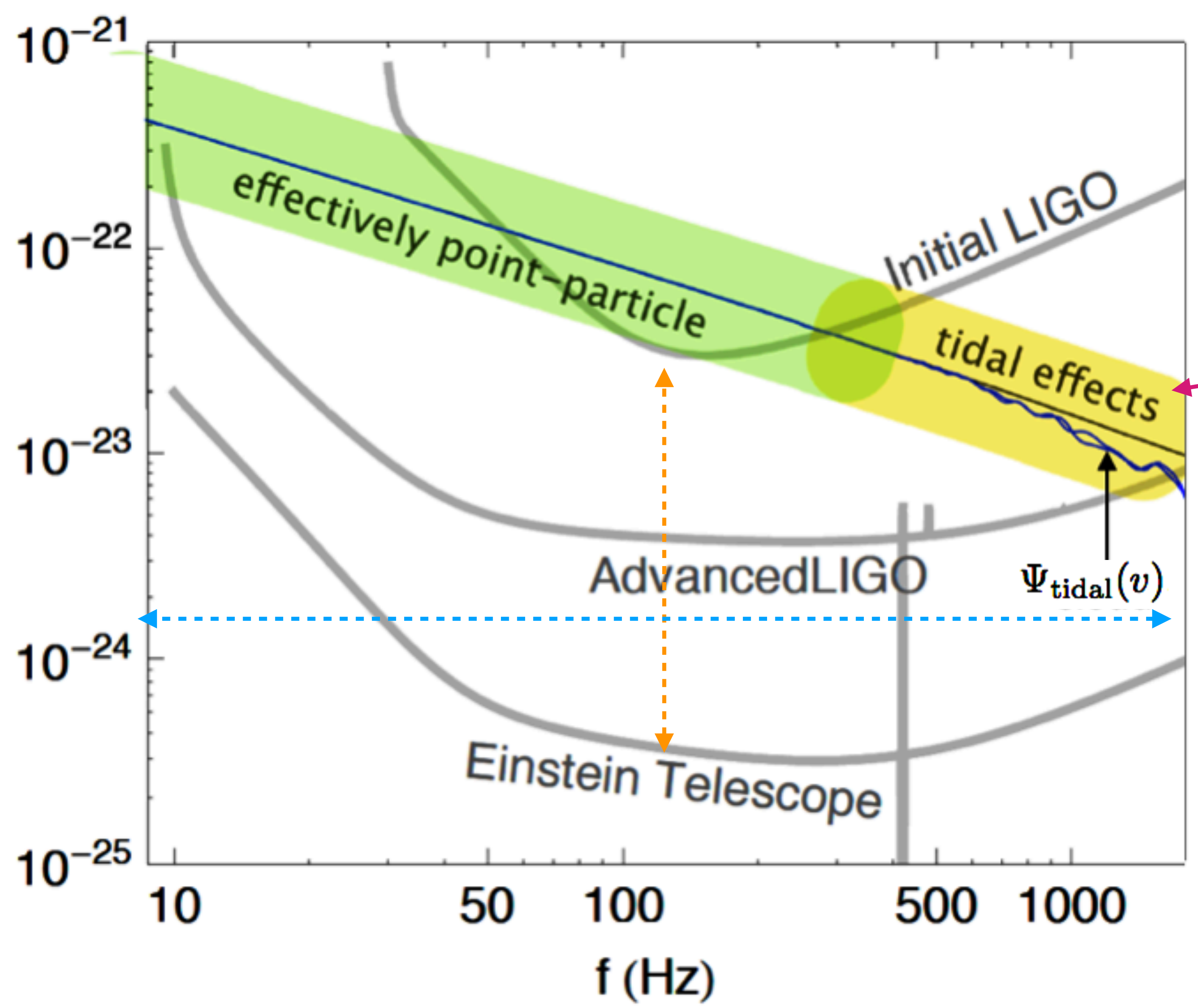
*This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 817791).

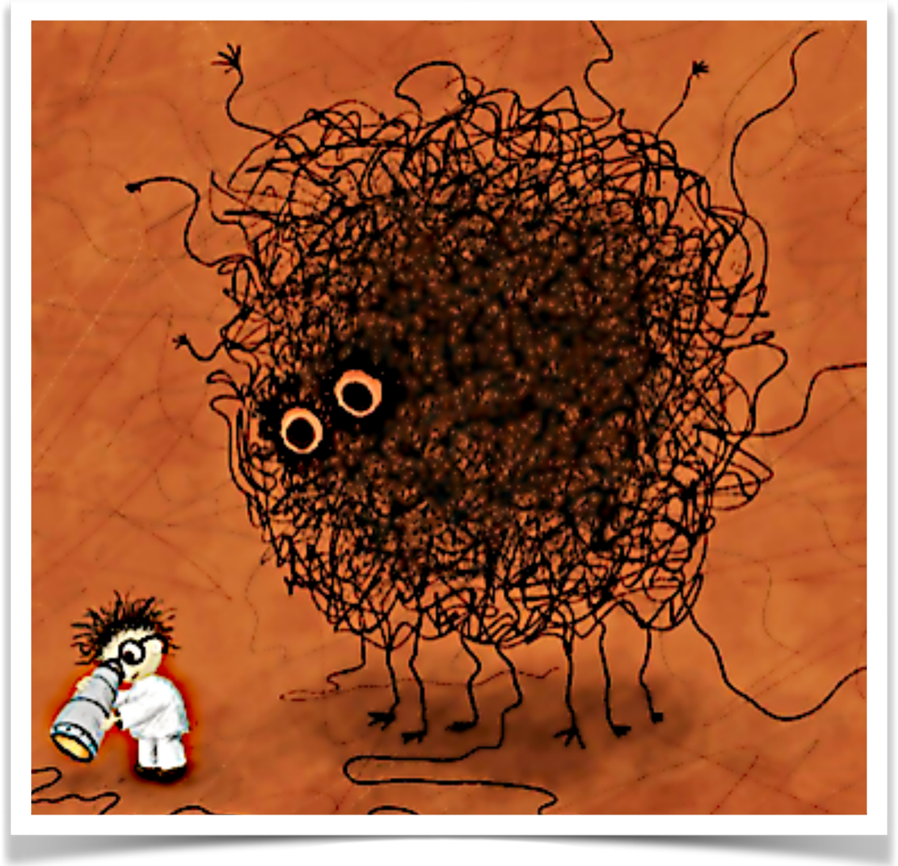


“Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW’s information”

Kip Thorne ~~1993~~ 2024

- **Luminosity Frontier**
- **Energy/Frequency Frontier**



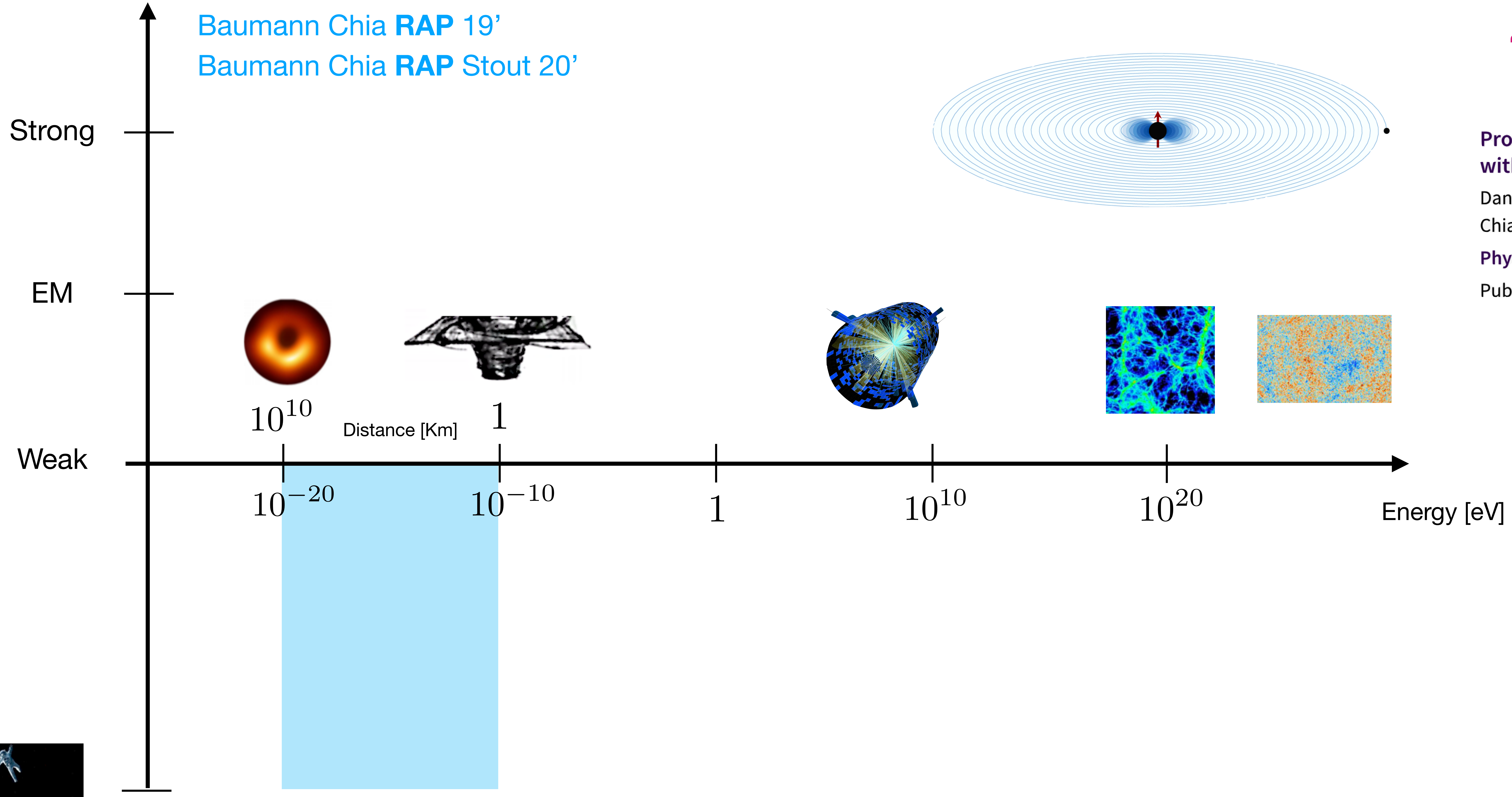


‘New Physics Threshold’

COSMOLOGIST & ASTRONOMERS APPROXIMATIONS

NEW frontier in particle physics

Gravitational Collider Physics



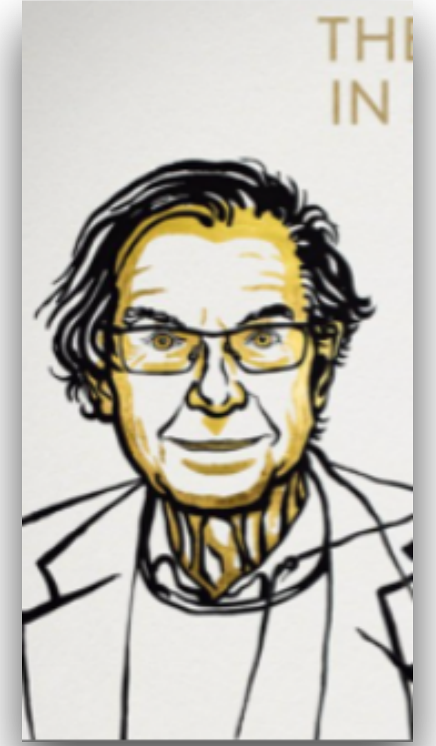
'New Physics'
Threshold

Probing ultralight bosons with binary black holes

Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto

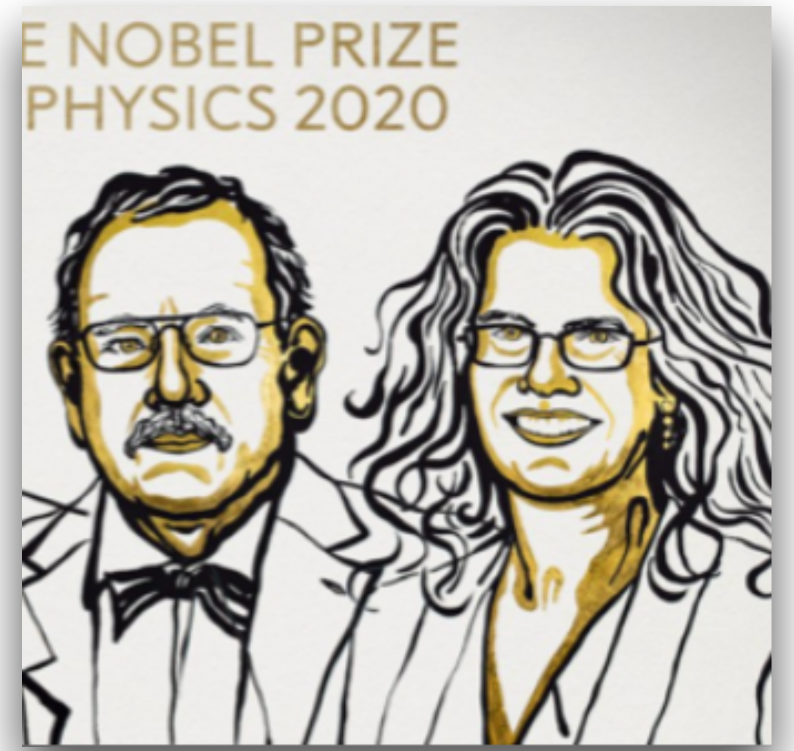
Phys. Rev. D 99, 044001 (2019)

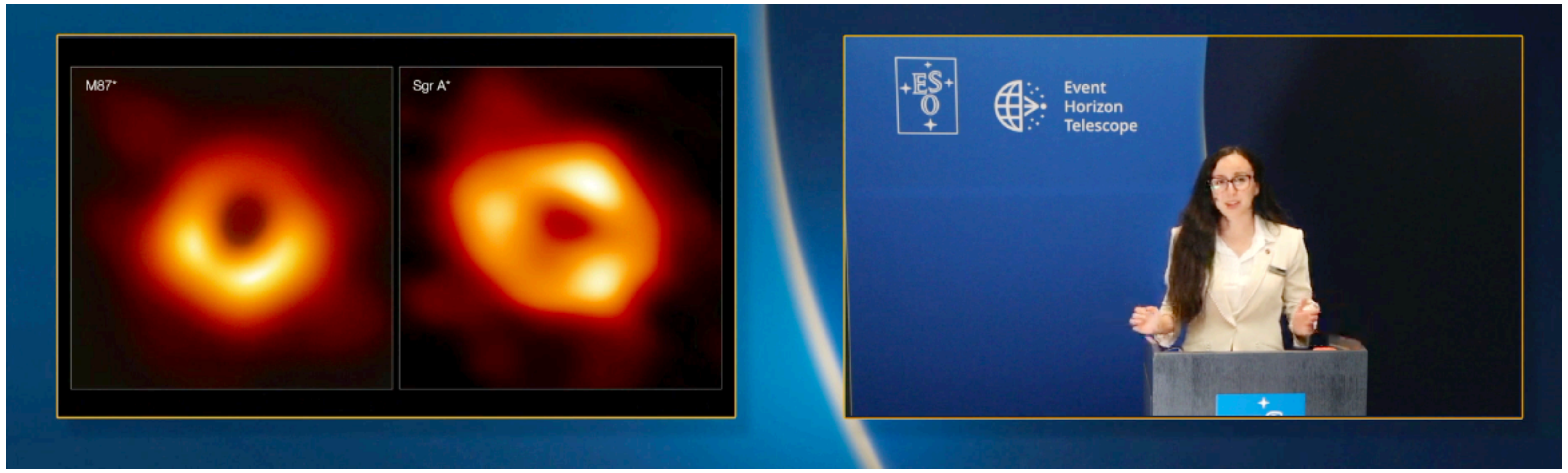
Published February 4, 2019

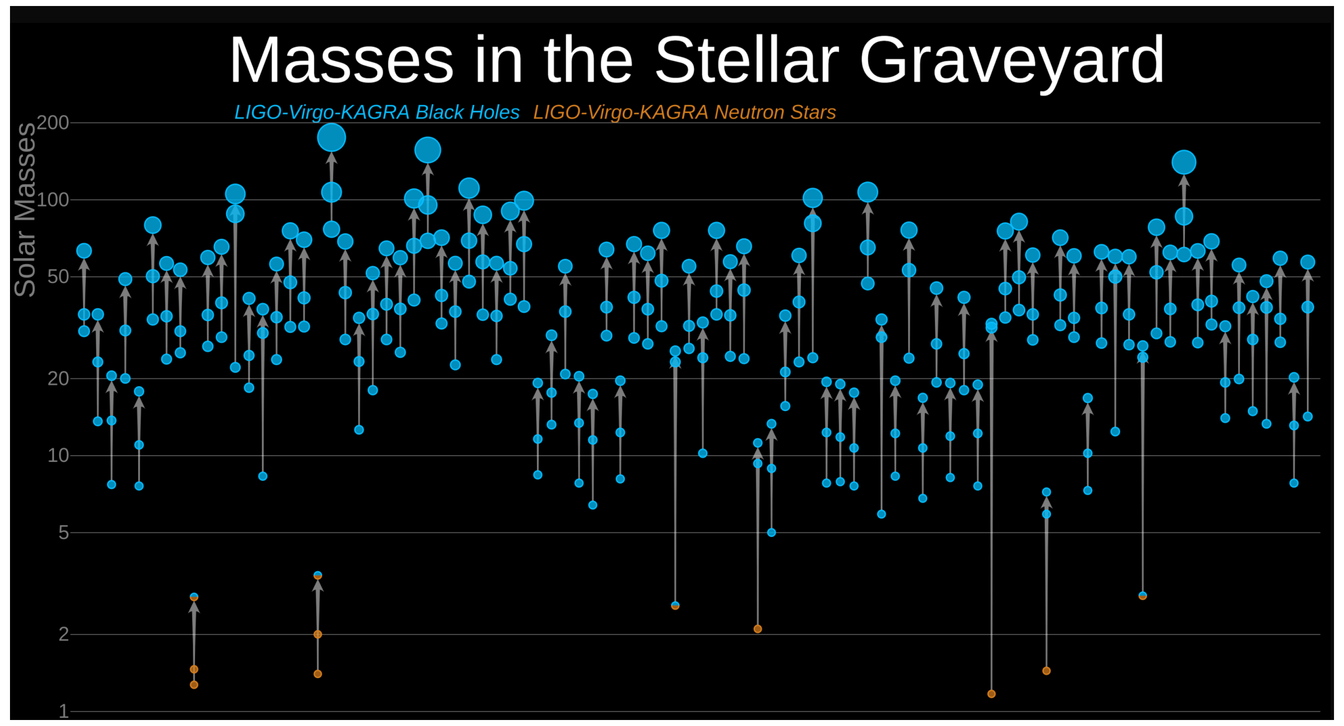


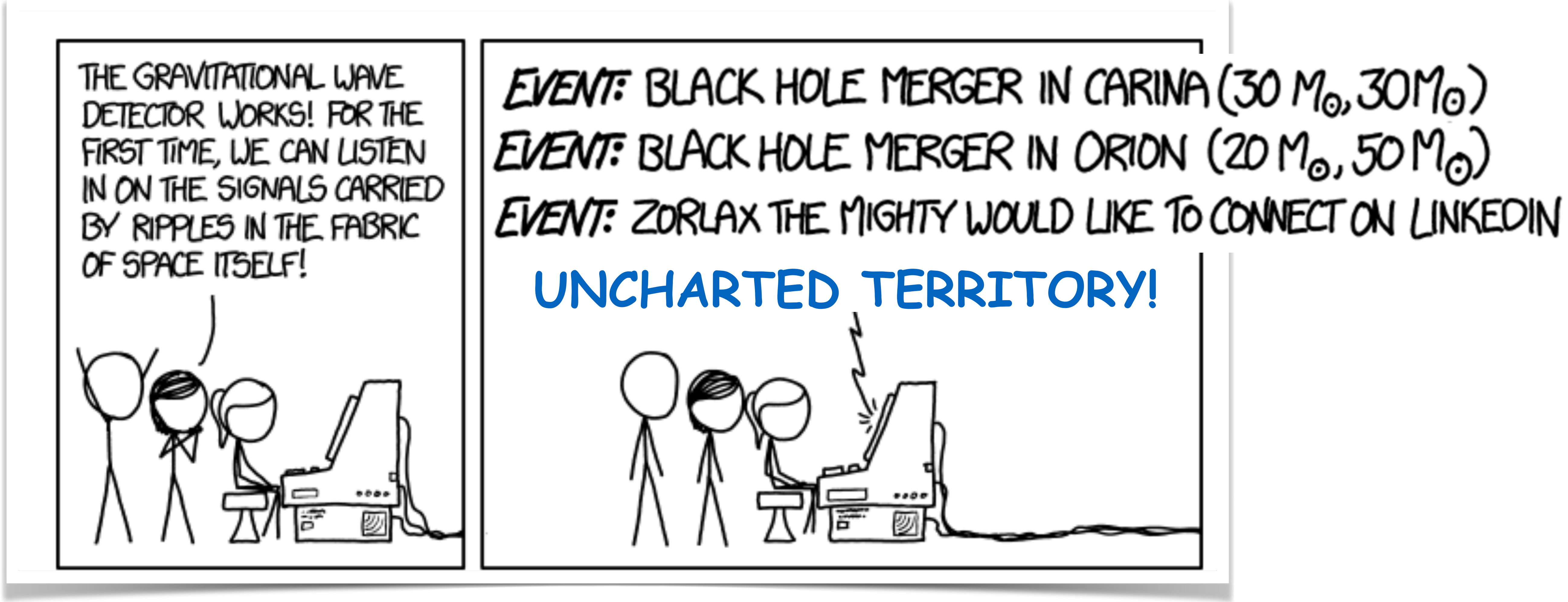
"for the discovery that black hole formation is a robust prediction of the general theory of relativity"

"for the discovery of a supermassive compact object at the centre of our galaxy"



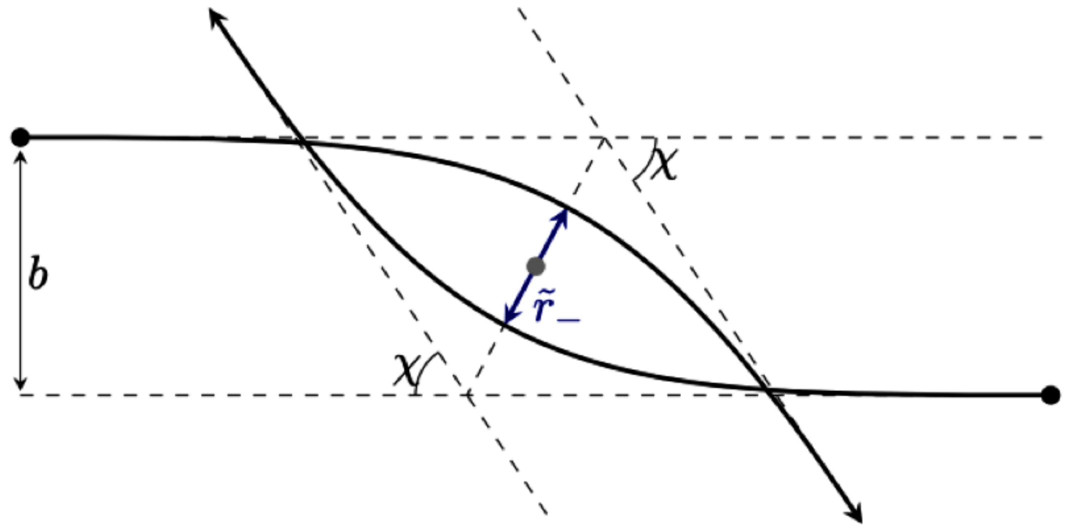




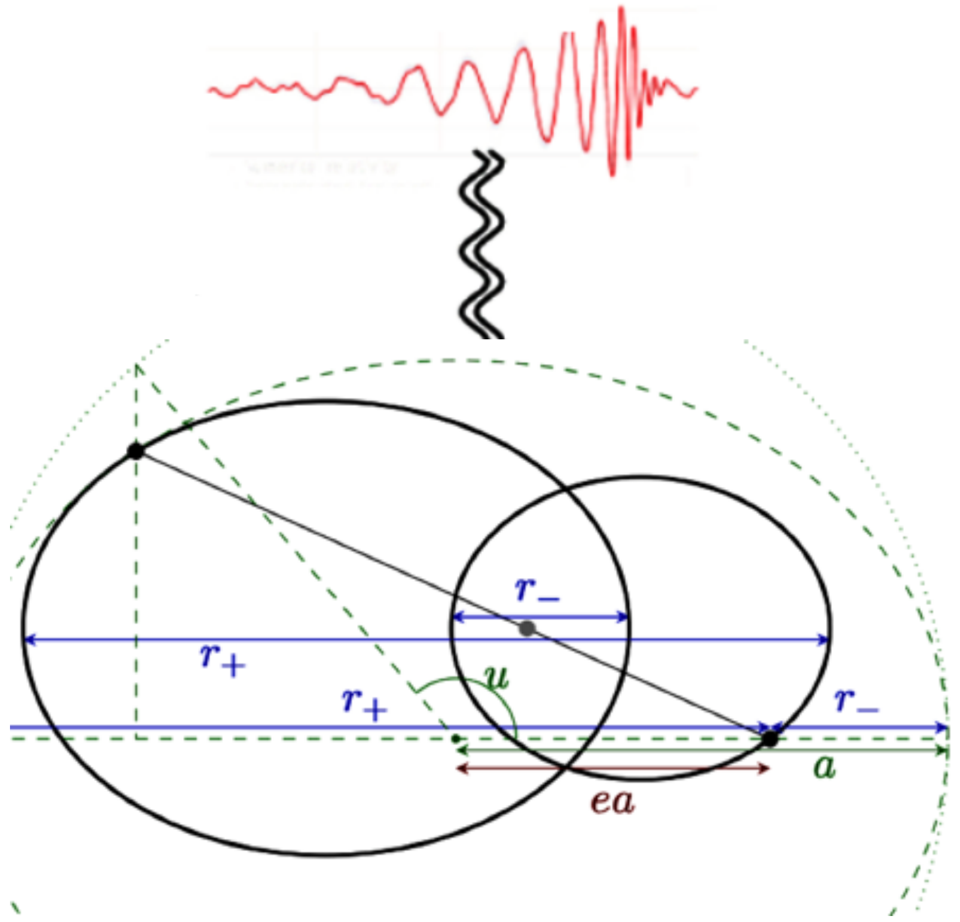


To appear

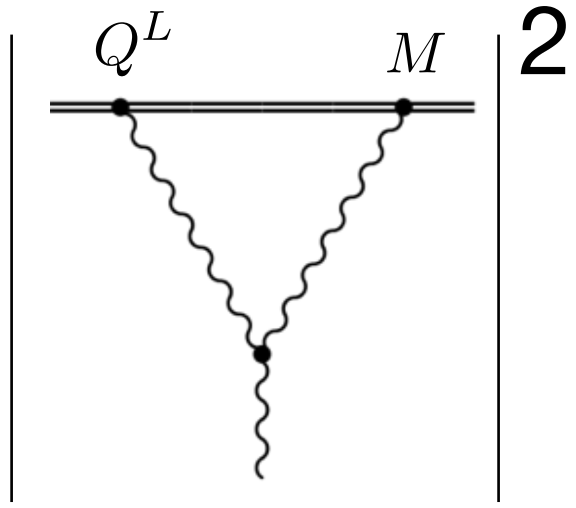
B2B correspondence



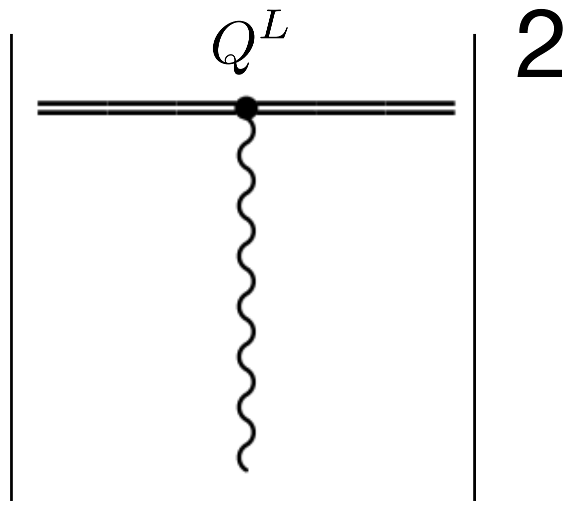
Conservative!
Radiative effects



Goldberger Ross
0912.4254

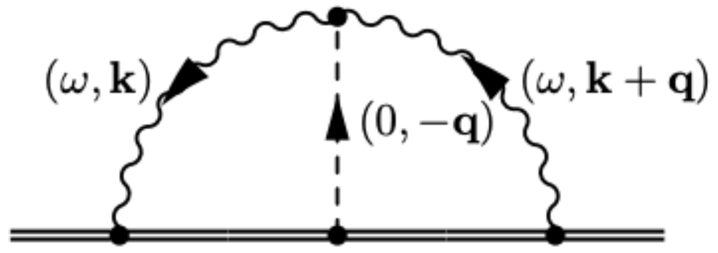


= $(1 + 2\pi G M \omega)$



$\omega \frac{d\Gamma}{d\omega}$

OPTICAL THEOREM



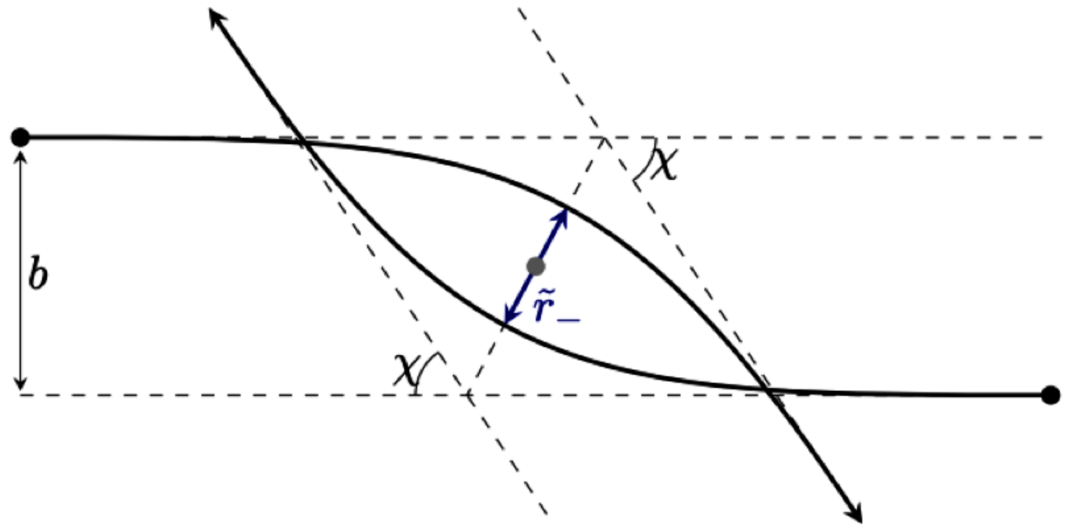
(Feynman b.c.)

= $\int d\omega \left(i\pi G M \frac{dE}{d\omega} + \cdots \right)$

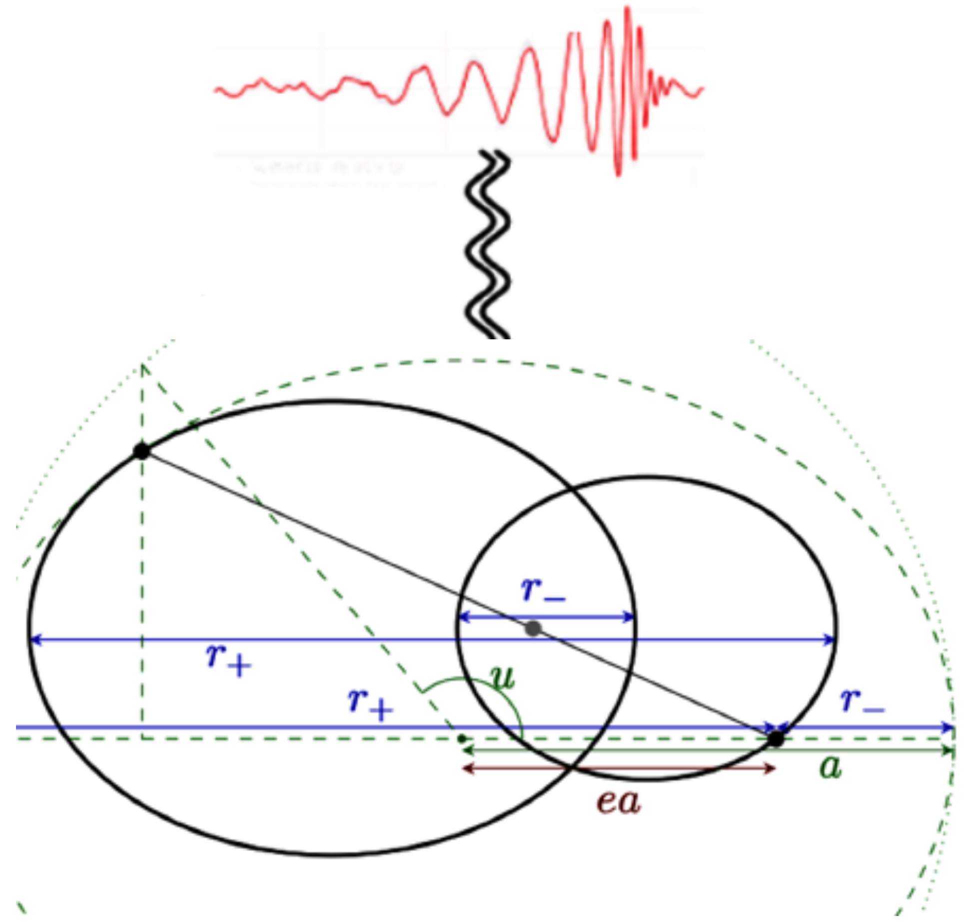
Energy spectrum!

To appear

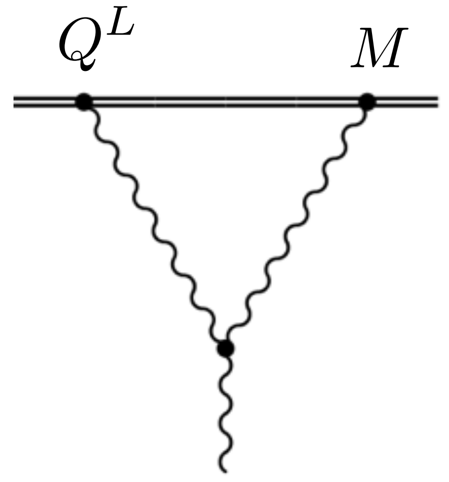
B2B correspondence

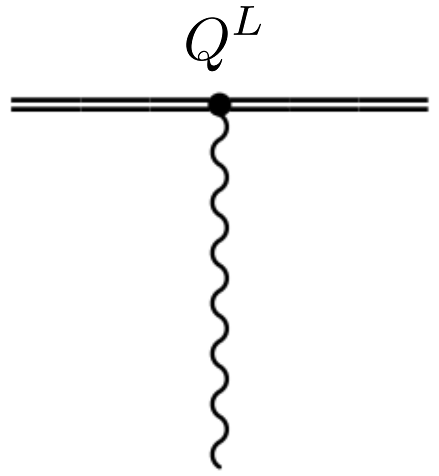


Conservative!
Radiative effects



Goldberger Ross
0912.4254

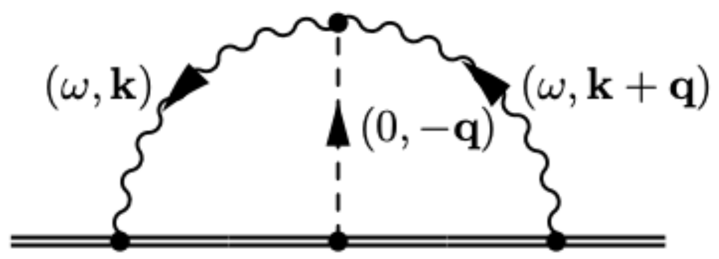
$\left|\right.$ 

$\left.\right|^2 = (1 + 2\pi G M \omega) \left|\right.$ 

$\left.\right|^2$

See also
Foffa Sturani
2103.03190

OPTICAL THEOREM



(Feynman b.c.)

$$= \int d\omega \left(i\pi G M \frac{dE}{d\omega} + \cdots \right)$$

pole+log coincide
conservative tail

Galley Leibovich
RAP Ross
1511.07379

in-in b.c. $= GM \int d\omega \frac{dE}{d\omega} \left(-\frac{1}{d-4} - \log(\omega^2/\mu^2) + i\pi \mathrm{sign}(\omega) \cdots \right)$

Ret $\frac{1}{d-4}[-(\omega + i0^+)^2]^{d-4}$

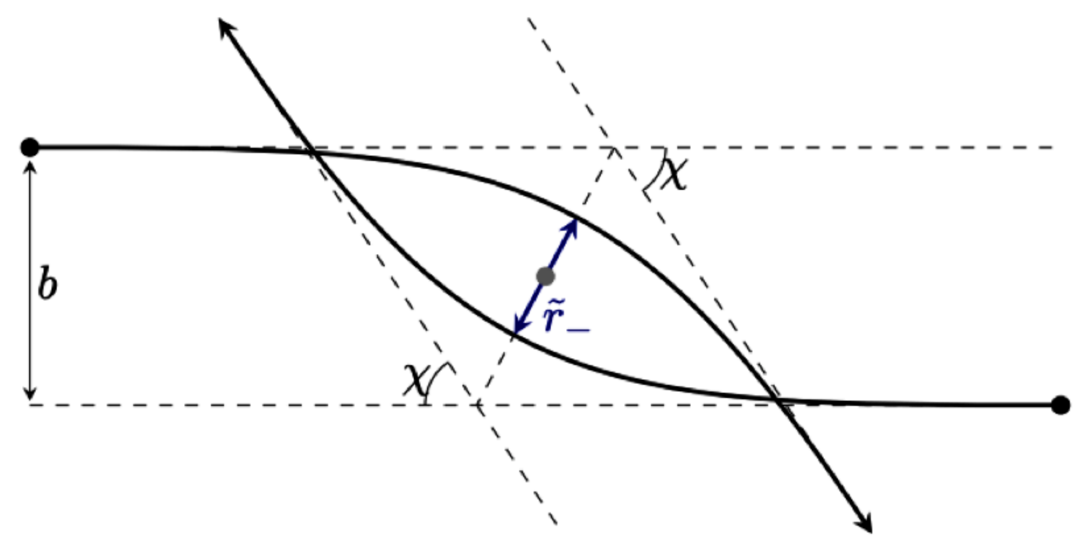
Feyn $\frac{1}{d-4}[-\omega^2 - i0^+]^{d-4}$

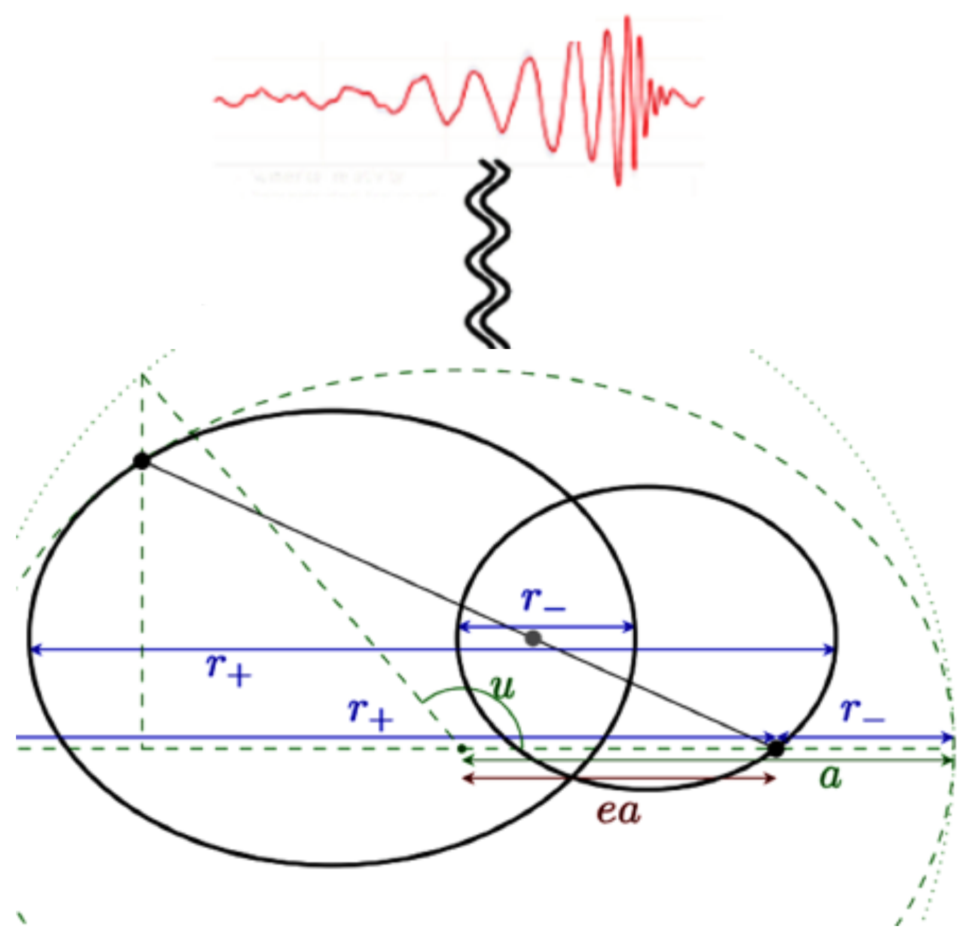
Dissipative
term

$$S_{\rm eff} = -\int H_{\rm tail} dt \longrightarrow H^{\rm loc}_{\rm tail} \propto \frac{dE}{dt}$$

To appear

B2B correspondence





Local Conservative
Radiative effects

$$\Delta E_{\text{ell}}(J,\mathcal{E}) = \Delta E_{\text{hyp}}(J,\mathcal{E}) - \Delta E_{\text{hyp}}(-J,\mathcal{E})$$

See also
Bini Damour
2007.11239

$$\delta\mathcal{S}_r^{bound} = -\frac{1}{2\pi}\oint H_{\text{tail}}dt \qquad \delta\mathcal{S}_r^{unbound} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} H_{\text{tail}}dt$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$ **(local)**

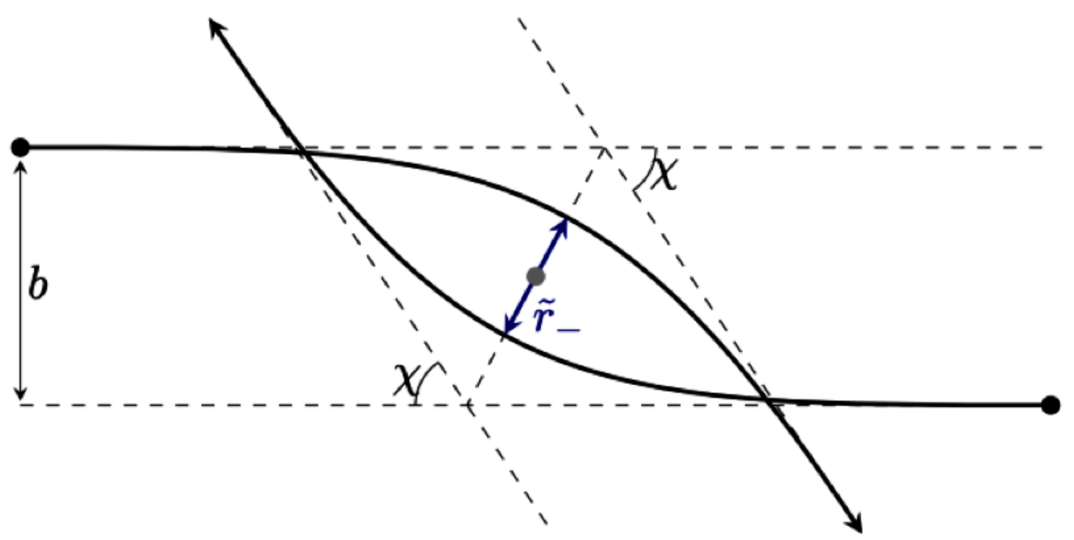
The *local* conservative B2B map remains the same! (runs both ways)

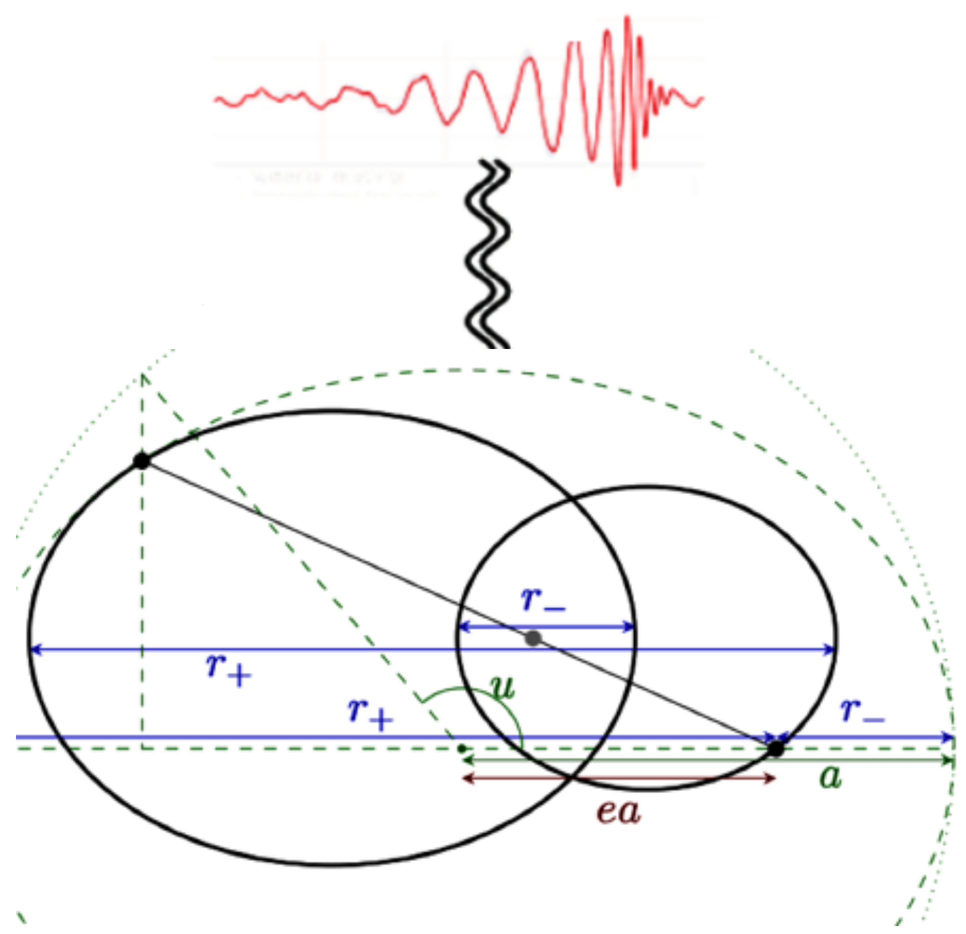
$$i_r(j,\mathcal{E}) \equiv \frac{\mathcal{S}_r}{GM\mu} = \text{sg}(\hat{p}_\infty)\chi_j^{(1)}(\mathcal{E}) - j\left(1 + \frac{2}{\pi}\sum_{n=1}\frac{\chi_j^{(2n)}(\mathcal{E})}{(1-2n)j^{2n}}\right)$$ **(local)**

$$S_{\text{eff}} = -\int H_{\text{tail}}dt \longrightarrow H_{\text{tail}}^{\text{loc}} \propto \frac{dE}{dt}$$ ⟵ Same map for orbital elements and Firsov

To appear

B2B correspondence





Non-local Conservative Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

$$\delta\mathcal{S}_r^{bound} = -\frac{1}{2\pi}\oint H_{\rm tail}dt \qquad \delta\mathcal{S}_r^{unbound} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} H_{\rm tail}dt$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Total (L+NL) Conserv.

What about the non-local part? **Loop around again!**

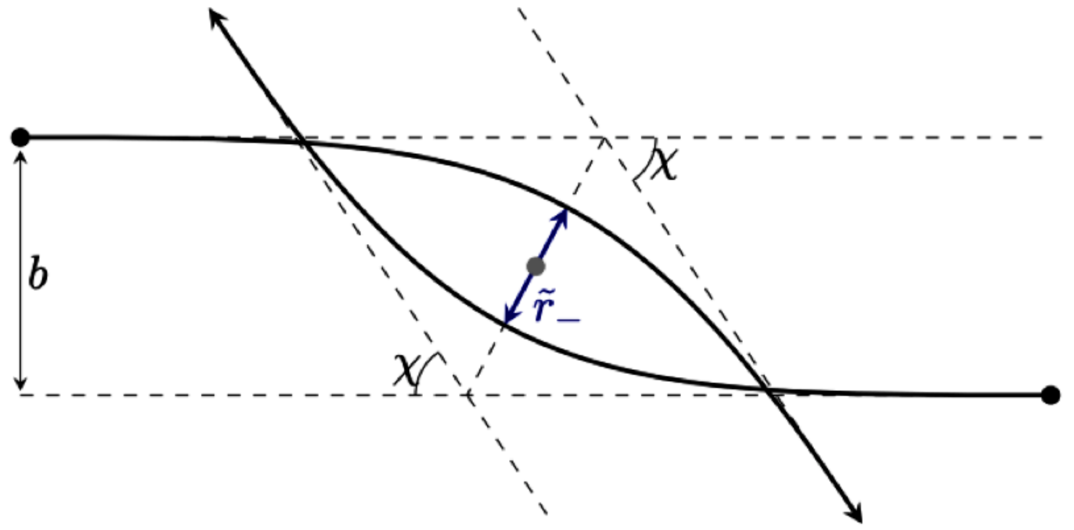
Valid in the "large-j" limit ONLY

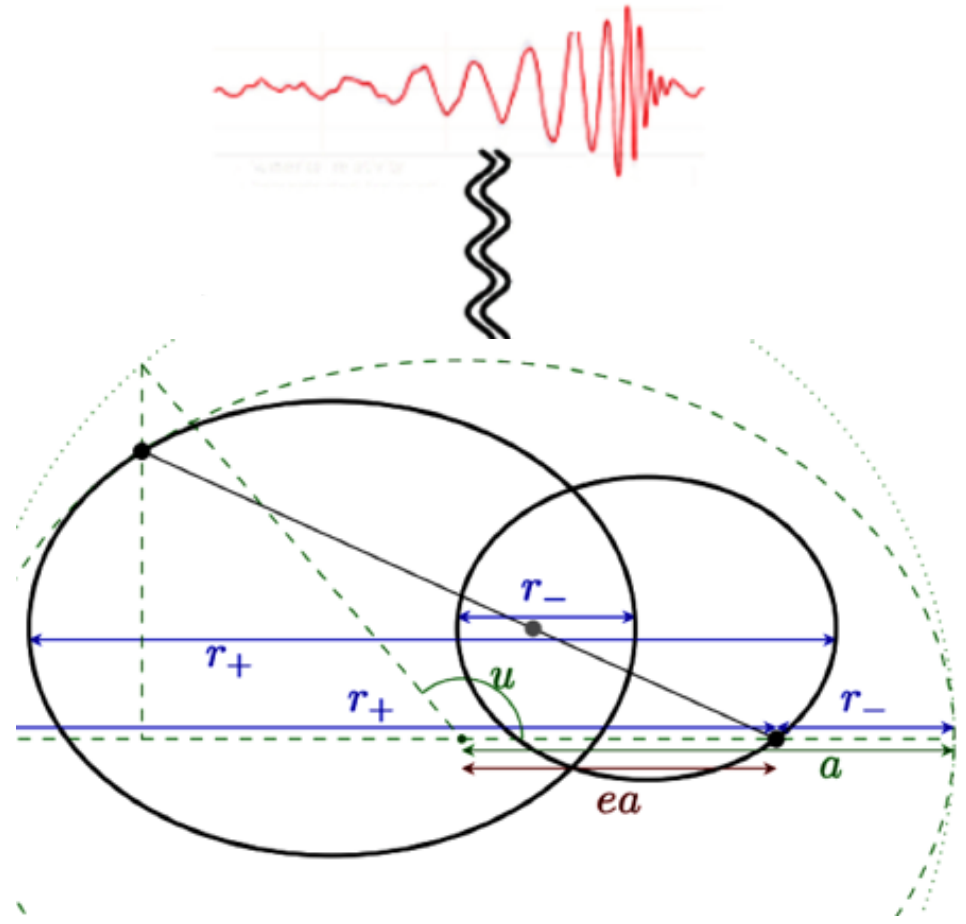
$$\int_{\tilde{r}_-}^{\infty}\frac{dr}{p_r}H_{\rm tail} \qquad H_{\rm tail}(r,\mathcal{E},j) = H_{\rm tail}(r,\mathcal{E},-j) \qquad \int_{r_-}^{r_+}\frac{dr}{p_r}H_{\rm tail}$$

Unlike the local ***(and logarithms!!!!)*** this Hamiltonian does not interpolate from large to small eccentricity unscathed

To appear

B2B correspondence





Non-local Conservative Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,.$$
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$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Galley Leibovich
RAP Ross
1511.07379

Jakobsen et al.
2101.12688

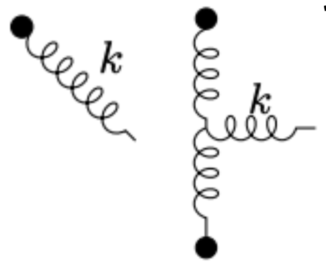
Mougiakakos
Riva Vernizzi
2102.08339

BUT we already know the ***universal*** structure of the tail term

dissipative

$$GM\int d\omega \frac{dE}{d\omega}\left(-\frac{1}{d-4} - \log(\omega^2/\mu^2) + i\pi\text{sign}(\omega)\cdots\right)$$

Known at G^3!



Universal UV pole cancels potential IR

Universal non-local part

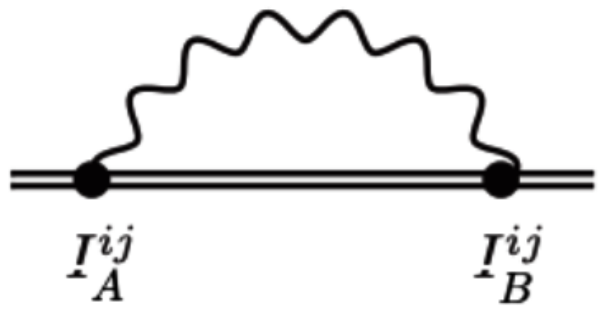
Local terms
Must compute in PM!

EFT approach to GW physics PM

Galley Leibovich
RAP Ross
1511.07379

Schwinger-Keldysh

$$\left[\frac{\delta W}{\delta \boldsymbol{x}^i_{a-}(t)}\right]_{\rm PL} = 0 \quad \Longrightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{x}^i_a} - \frac{\partial L}{\partial \boldsymbol{x}^i_a} = \left[\frac{\partial R}{\partial \boldsymbol{x}^i_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}^i_{a-}}\right]_{\rm PL}, \qquad \dot{M} = \sum_a \boldsymbol{v}_a \cdot \left[\frac{\partial R}{\partial \boldsymbol{x}_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}_{a-}}\right]_{\rm PL}.$$



$$W[\boldsymbol{x}^\pm_a] = -\frac{G_N}{5}\int dt\, I^{ij}_-(t) I^{(5)}_{+ij}(t) \equiv \int dt\, R_{\rm rad}[\boldsymbol{x}^\pm_a],$$

$$\boldsymbol{a}_{\rm RR} = \frac{M^2\nu}{15 r^4}\dot{r}\left(\frac{136M}{r} + 72\boldsymbol{v}^2\right)\boldsymbol{r} - \frac{8M^2\nu}{5r^3}\left(\frac{3M}{r} + \boldsymbol{v}^2\right)\boldsymbol{v}.$$

G^2

$$\dot{M} = -\frac{G_N}{5} I^{(1)ij}(t) I^{(5)ij}(t)\,,$$

$$\dot{\mathcal{E}}_N = \frac{8}{15}\frac{G^3 m^2 \mu^2}{c^5 r^4}\left\{12v^2 - 11\dot{r}^2\right\}$$

G^3

$$\dot{E} = -\frac{G_N}{5} I^{(3)ij}(t) I^{(3)ij}(t)\,,$$

EFT approach to GW physics PM

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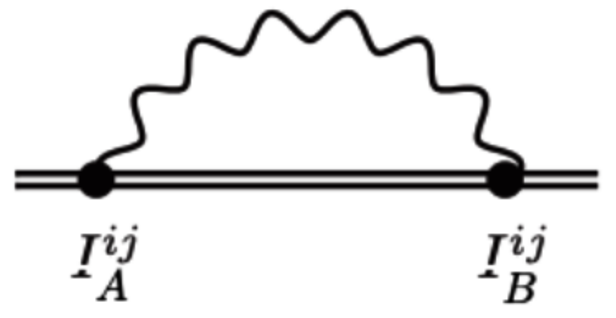
1511.07379

Schwinger-Keldysh

$$\left[\frac{\delta W}{\delta \boldsymbol{x}^i_{a-}(t)}\right]_{\rm PL} = 0 \quad \Longrightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{x}^i_a} - \frac{\partial L}{\partial \boldsymbol{x}^i_a} = \left[\frac{\partial R}{\partial \boldsymbol{x}^i_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}^i_{a-}}\right]_{\rm PL},$$

$$\dot M = \sum_a \boldsymbol{v}_a \cdot \left[\frac{\partial R}{\partial \boldsymbol{x}_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}_{a-}}\right]_{\rm PL}.$$



$$W[\boldsymbol{x}^\pm_a] = -\frac{G_N}{5}\int dt\, I^{ij}_-(t) I^{(5)}_{+ij}(t) \equiv \int dt\, R_{\rm rad}[\boldsymbol{x}^\pm_a],$$

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G^2

$$\dot M = -\frac{G_N}{5} I^{(1)ij}(t) I^{(5)ij}(t),$$

$$\dot{\mathcal{E}}_N = \frac{8}{15}\frac{G^3 m^2\mu^2}{c^5 r^4}\left\{12v^2 - 11\dot r^2\right\}$$

$$\sqrt{\Delta p^2} \to G\int dt\, I^{(5)ij}x^i x^j \sim G\int dt I^{(5)ij} I^{ij}$$

$$\mathcal{J}_N = \frac{8}{5}\frac{G^2 m\,\mu^2}{c^5 r^3}\tilde{\mathbf{L}}_{\rm N}\left\{2v^2 - 3\dot r^2 + 2\frac{Gm}{r}\right\}$$

$$\sqrt{\Delta p^2} \sim G\int dt I^{(3)ij} I^{(2)ij} \sim G\int dt\frac{dL}{dt} \sim G\Delta L \sim G^3$$

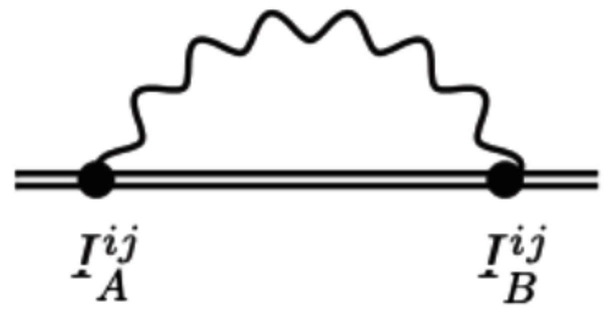
$$L^{ij} \equiv -\int d^3\mathbf{x}\,(T^{0i}x^j - T^{0j}x^i)$$

EFT approach to GW physics PM

Schwinger-Keldysh

$$\left[\frac{\delta W}{\delta \boldsymbol{x}^i_{a-}(t)}\right]_{\rm PL} = 0 \qquad \Longrightarrow \qquad \frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{x}^i_a} - \frac{\partial L}{\partial \boldsymbol{x}^i_a} = \left[\frac{\partial R}{\partial \boldsymbol{x}^i_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}^i_{a-}}\right]_{\rm PL}, \qquad \dot{M} = \sum_a \boldsymbol{v}_a \cdot \left[\frac{\partial R}{\partial \boldsymbol{x}_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}_{a-}}\right]_{\rm PL}.$$

$$W[\boldsymbol{x}^\pm_a] = \int dt \left(L[\boldsymbol{x}^{(1)}_a] - L[\boldsymbol{x}^{(2)}_a] + R[\boldsymbol{x}^{(1)}_a, \boldsymbol{x}^{(2)}_a]\right), \qquad R[\boldsymbol{x}^{(1)}_a, \boldsymbol{x}^{(2)}_a] \supset \underbrace{F[\boldsymbol{x}^{(1)}_a] - F[\boldsymbol{x}^{(2)}_a]}_{\text{conservative}},$$



2.5PN

$$W[\boldsymbol{x}^\pm_a] = -\frac{G_N}{5}\int dt\, I^{ij}_-(t) I^{(5)}_{+ij}(t) \equiv \int dt\, R_{\rm rad}[\boldsymbol{x}^\pm_a],$$

$$(\boldsymbol{a}^i_a)_{\rm rr}(t) = -\frac{2G_N}{5} I^{(5)ij}(t)\boldsymbol{x}^j_a(t).$$

$$\dot{M} = -\frac{G_N}{5} I^{(1)ij}(t) I^{(5)ij}(t), \quad \dot{E} = -\frac{G_N}{5} I^{(3)ij}(t) I^{(3)ij}(t),$$

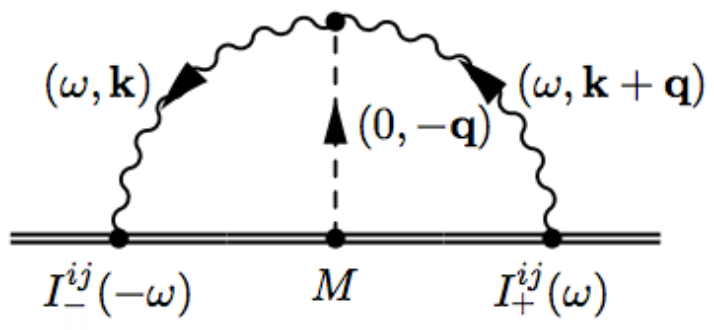
$$E = M + \underbrace{\frac{G_N}{5} I^{(1)ij}(t) I^{(4)ij}(t) - \frac{G_N}{5} I^{(2)ij}(t) I^{(3)ij}(t)}_{\textbf{Schott terms}}.$$

Galley Leibovich
RAP Ross
1511.07379

EFT approach to GW physics PM

Schwinger-Keldysh

$$\left[\frac{\delta W}{\delta \boldsymbol{x}^i_{a-}(t)}\right]_{\rm PL} = 0 \quad \Longrightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{x}^i_a} - \frac{\partial L}{\partial \boldsymbol{x}^i_a} = \left[\frac{\partial R}{\partial \boldsymbol{x}^i_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}^i_{a-}}\right]_{\rm PL}, \qquad \dot{M} = \sum_a \boldsymbol{v}_a \cdot \left[\frac{\partial R}{\partial \boldsymbol{x}_{a-}} - \frac{d}{dt}\frac{\partial R}{\partial \boldsymbol{v}_{a-}}\right]_{\rm PL}.$$



4PN

$$W_{\rm tail}[\boldsymbol{x}^\pm_a] = \frac{2G_N^2 M}{5}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^6\, I^{ij}_-(-\omega) I^{ij}_+(\omega)\left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega)\right].$$

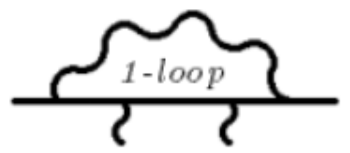
$$\left(\boldsymbol{a}^j_a\right)_{\rm cons}(t,\mu) = -\frac{4G_N^2 M}{5}\boldsymbol{x}^i_a(t)\left(I^{ij(6)}(t)\log\mu^2 + {\rm PV}\int_{-\infty}^{\infty} dt'\, I^{ij(6)}(t')\left[\frac{1}{|t-t'|}\right]\right)$$

$$\left(\boldsymbol{a}^j_a\right)_{\rm diss}(t) = -\frac{4G_N^2 M}{5}\boldsymbol{x}^i_a(t)\,{\rm PV}\int_{-\infty}^{\infty} dt'\, I^{ij(6)}(t')\left[\frac{1}{t-t'}\right].$$

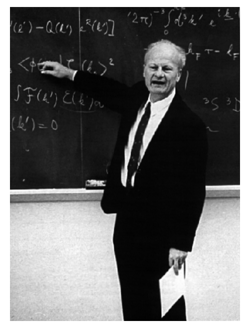
$$R[\boldsymbol{x}^{(1)}_a, \boldsymbol{x}^{(2)}_a] \supset F[\boldsymbol{x}^{(1)}_a] - F[\boldsymbol{x}^{(2)}_a],$$

conservative

Balance equation:



$$\underbrace{\langle \dot{E}(t)\rangle}_{\text{source + cons-RR}} = -P_{\rm local} - \frac{2G_N^2 M}{5}\left\langle \underbrace{I^{ij(1)}(t)\,{\rm PV}\int_{-\infty}^{\infty} dt'\, I^{ij(6)}(t')\left[\frac{1}{t-t'}\right]}_{\text{dissipative}}\right\rangle$$



Galley Leibovich
RAP Ross
1511.07379