First-order phase transitions in radiative simmetry breaking models

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Mainly based on:
JCAP **04** (2023), 051 arXiv:2302.10212 [hep-ph], A. Salvio
JCAP **12** (2023), 046 arXiv:2307.04694 [hep-ph], A. Salvio
arXiv:2412.06889 [hep-ph] I.K. Banerjee, F.Rescigno, A.Salvio

ENP meeting 2025

Roadmap

Roadmap

Radiative Symmetry Breaking (RSB)

- We start from the most general no-scale Lagrangian:

$$\mathcal{L}_{\rm ns} = -\frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A + \frac{1}{2}D_\mu\phi_a D^\mu\phi_a + \bar{\psi}_j i\not{D}\psi_j - \frac{1}{2}(Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

$$D_\mu\phi_a = \partial_\mu\phi_a + i\theta^A_{ab}V^A_\mu\phi_b, \qquad D_\mu\psi_j = \partial_\mu\psi_j + it^A_{jk}V^A_\mu\psi_k,$$

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d.$$

Radiative Symmetry Breaking (RSB)

- At quantum level the couplings depend on the RG energy μ, there may be some specific value $\mu = \tilde{\mu}$ at which the potential $V_{\rm ns}(\phi)$ develops a flat direction parametrized as $\phi_a = \chi \nu_a$

$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad \lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d,$$

such that $\lambda_\chi(\tilde{\mu}) = 0$, $\lambda_{ijkl}(\tilde{\mu})\nu_i\nu_j\nu_k = 0$ (flat direction).

- The one-loop effective potential at zero temperature is

$$V_q(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4 + \frac{\beta_{\lambda_\chi}}{4}\left(\log\frac{\chi}{\mu} - \frac{1}{4}\right)\chi^4, \qquad \beta_{\lambda_\chi} \equiv \mu\frac{d\lambda_\chi}{d\mu}.$$

Hierarchy of Scales

The RG running is **logarithmic**: starting with order-one value of $\lambda_\chi(\mu)$ at some high-energy scale μ, the value $\tilde{\mu}$ at which $\lambda_\chi(\tilde{\mu}) = 0$ is typically exponentially large, generating **exponentially large hierarchies** in a natural way.

Radiative Symmetry Breaking (RSB)

- Setting $\mu = \tilde{\mu}$ we obtain $\lambda_\chi = 0$, i.e.

$$V_q(\chi) = \frac{\bar{\beta}}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4,$$

$$\bar{\beta} \equiv [\beta_{\lambda_\chi}]_{\mu=\tilde{\mu}}, \qquad \chi_0 \equiv \frac{\tilde{\mu}}{e^{1/4+a_s}}.$$

- When the conditions,

$$\begin{cases} \lambda_\chi(\tilde{\mu}) = 0 & \text{(flat direction)} \\ \beta_{\lambda_\chi}(\tilde{\mu}) > 0 & \text{(minimum condition)} \end{cases}$$

  are fulfilled χ_0 is the zero temperature vacuum-expectation value of χ and is the new absolute minima of $V(\chi)$

- the fluctuation around χ_0 have mass $m_\chi^2 = \bar{\beta}\chi_0^2$

Radiative Symmetry Breaking and EW scale generation

- The non trivial minimum can generically break global and/or local symmetries and thus generate the particle masses, where χ_0 plays the role of symmetry breaking scale

$$\mathcal{L}_{\chi h} \equiv \frac{1}{2}\lambda_{ab}\phi_a\phi_b|\mathcal{H}|^2, \qquad \lambda_{\chi h}(\mu) \equiv \lambda_{ab}(\mu)\nu_a\nu_b.$$

- RG improving and setting $\mu = \tilde{\mu}$

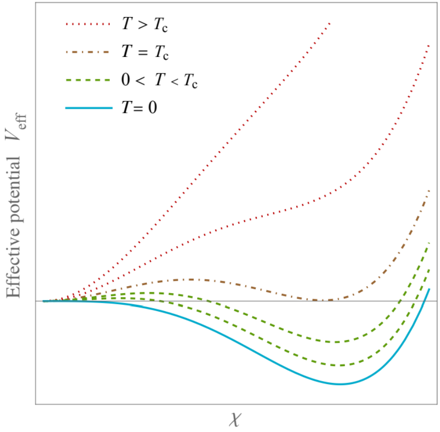
$$\mathcal{L}_{\chi\mathcal{H}} = \frac{1}{2}\lambda_{\chi\mathcal{H}}(\tilde{\mu})\chi^2|\mathcal{H}|^2, \qquad \lambda_{\chi\mathcal{H}}(\tilde{\mu}) > 0,$$

evaluated at the minimum

$$\mu_h^2 = \frac{1}{2}\lambda_{\chi h}(\tilde{\mu})\chi_0^2.$$

Roadmap

- FOPT can occur in the early Universe when the temperature T drops below a critical value T_c
- The finite temperature effective potential $V(\chi, T)$ develops a new stable minima (**true vacuum**), while the old minima becomes metastable (**false vacuum**)
- The false vacuum eventually **decay in the true vacuum**
- The false vacuum decay manifest as the **nucleation of true vacuum bubbles** in a background of false vacuum



Effective thermal potential

Effective Thermal Potential

$$V_{\text{eff}}(\chi, T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0,$$

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

$$J_F(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 + e^{-\sqrt{p^2+x}}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\log\left(\frac{x}{a_F}\right) + O(x^3),$$

$$a_B = 16\pi^2 \exp(3/2 - 2\gamma_E), \qquad a_F = \pi^2 \exp(3/2 - 2\gamma_E).$$

Decay rate of the false vacuum per unit of volume (time independent bounce)

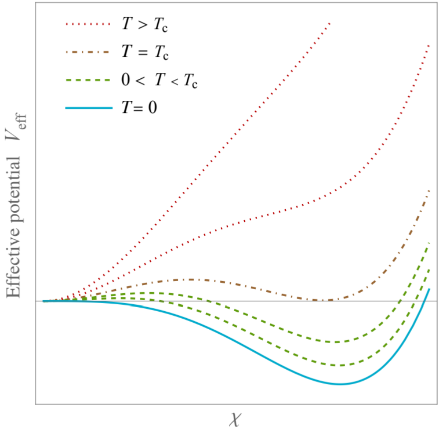
$$\Gamma \approx T^4 \mathsf{exp}(-S_3/T),$$

$$S_3 \equiv 4\pi \int_0^\infty dr\, r^2 \left(\frac{1}{2}\chi'^2 + \bar{V}_{\rm eff}(\chi, T) \right) = -8\pi \int_0^\infty dr\, r^2 \bar{V}_{\rm eff}(\chi, T),$$

$$\chi'' + \frac{2}{r}\chi' = \frac{d\bar{V}_{\rm eff}}{d\chi}, \qquad \chi'(0) = 0, \qquad \lim_{r\to\infty} \chi(r) = 0, \qquad \bar{V}_{\rm eff}(\chi, T) = V_{\rm eff}(\chi, T) - V_{\rm eff}(0, T).$$

Properties of Phase Transition in RSB models

- Phase Transitions in RSB models are **always FOPT**
- FOPTs in RSB feature always a **supercooling phase**, i.e. the temperature drops much below T_c before the bubble nucleation became effective.
  This ensures that thermal effect do **not spoil perturbation theory**
- Supercooling implies a new stage of **inflation**
- FOPT in RSB models are always **strong**, the PT release a large amount of energy



Supercool Expansion (LO)

If supercooling is strong enough in a generic theory of the form $\mathcal{L}^{\rm ns}$, to good accuracy, the full effective action for relevant values of χ can be described by **three and only three parameters**: χ_0, $\bar{\beta}$ and g defined as

$$g^2\chi^2 \equiv \sum_b n_b m_b^2(\chi) + \sum_f m_f^2(\chi).$$

The conditions for supercool expansion is verified if

$$\epsilon \equiv \frac{g^4}{6\bar{\beta}\log\frac{\chi_0}{T}} \quad \text{is } \textbf{small}.$$

Supercool Expansion

- The LO of this expansion corresponds to approximate the effective potential as

$$J_B(x) \approx J_B(0) + \frac{\pi^2}{12}x, \qquad J_F(x) \approx J_F(0) - \frac{\pi^2}{24}x,$$

Supercool Expansion (LO)

$$\bar{V}_{\text{eff}}(\chi, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{\lambda(T)}{4}\chi^4, \qquad S_3 \approx c_3\frac{m}{\lambda},$$

where

$$m^2(T) \equiv \frac{g^2T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta}\log\frac{\chi_0}{T}, \qquad c_3 = 18.8973\ldots$$

Supercool Expansion

- It is possible to calculate several FOTP parameters:
  - **Nucleation temperature**: Defined as the temperature T_n for which

$$\Gamma(T_n) \approx H(T_n)^4 \equiv H_n^4.$$

  - **Inverse duration**: Defined as

$$\beta \equiv \left[\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right]_{t_n}.$$

Supercool Expansion

- The NLO expansion includes higher order terms in the expansion of the thermal function

$$J_B(x) \approx J_B(0) + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2}, \qquad J_F(x) \approx J_F(0) - \frac{\pi^2}{24}x.$$

- The NLO effective potential includes a perturbative cubic term

$$\bar{V}_{\text{eff}}(\chi, T) \approx \frac{m^2(\chi)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4$$

$$k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}, \qquad \tilde{g}^3\chi^3 \equiv \sum_b n_b m_b^3(\chi), \qquad \tilde{g} \leq g.$$

Improved Supercool Expansion

- In general, when $\epsilon \sim 1$, in the effective potential **the cubic term cannot be treated as a perturbation** (i.e. the term of order $x^{3/2}$ is not perturbative in the expansion of the thermal functions)
- However we can treat the **terms beyond the quartic as perturbation**
- We can rewrite the effective potential and the 3d euclidean action at the LO of the **Improved Supercool Expansion** as

Improved Supercool Expansion (LO)

$$\bar{V}_{\rm eff}(\chi, T) \approx \frac{m^2(\chi)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4,$$

$$m^2(T) \equiv \frac{g^2 T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta} \log \frac{\chi_0}{T}, \qquad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}$$

(Improved) Supercool Expansion: Results

Inverse duration β/H_n, nucleation temperature

$$\frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4, \qquad \textbf{(Supercool expansion LO)}$$

$$T_n \approx \chi_0 \exp\left(\frac{\sqrt{c^2 - 16a} - c}{8}\right), \qquad \textbf{(Supercool expansion LO)}$$

$$\frac{\beta}{H_n} \approx \frac{\pi^3 g^5}{6\sqrt{3}\tilde{g}^8} \frac{(4\pi)^2 \bar{\beta}}{\tilde{g}^4} (-F'(\tilde{\lambda}_n)) - 4, \qquad \textbf{(Improved supercool expansion LO)}$$

$$T_n \approx \chi_0 \exp\left(-\frac{12\tilde{g}^6/g^2}{(4\pi)^2\bar{\beta}} \tilde{\lambda}_n\right), \qquad \textbf{(Improved supercool expansion LO)}$$

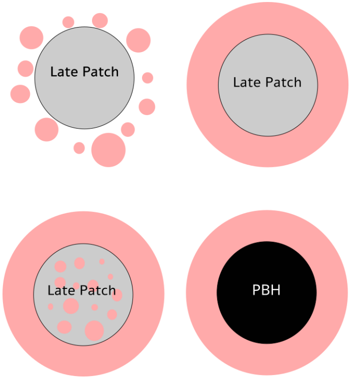
$$a \equiv \frac{c_3 g}{\sqrt{12\bar{\beta}}}, \qquad c \equiv \log \frac{4\sqrt{3}\bar{M}_P}{\sqrt{\bar{\beta}}\chi_0} + \frac{3}{2}\log\frac{a}{2\pi},$$

$\tilde{\lambda}_n$ needs numerical calculation.

Roadmap

Phys.Rev.D 105 (2022) 2, L021303, J. Liu, L. Bian, R. G. Cai, Z. K. Guo and S. J. Wang

- Vacuum decay is a **probabilistic process**
- There can be some regions that **persist in the false vacuum** for a longer time than the background
- These regions can eventually **collapse into Primordial Black Holes (PBH)** if the mass excess reaches the critical value δ_c



Late Blooming Mechanism

Late Blooming Mechanism

Expanding the vacuum decay rate as

$$\Gamma(t) = \Gamma(t_n)\text{exp}(\beta(t - t_n) + \beta_2(t - t_n)^2 + ...) \approx H_n^4 e^{\beta(t-t_n)},$$

one can characterize the late blooming mechanism in supercooled 1OPT:
Phys.Rev.D 110 (2024) 4, 043514, arXiv:2305.04942 [hep-ph], Y. Gouttenoire, T. Volansky

Collapse Probability, PBH Fraction and PBH Mass

$$\mathcal{P}_{\rm coll} \approx \text{exp}\left[-a_{\mathcal{P}}\left(\frac{\beta}{H_n}\right)^{b_{\mathcal{P}}}(1+\delta_c)^{c_{\mathcal{P}}\frac{\beta}{H_n}}\right], \qquad \text{for } \alpha = \left(\frac{T_{\rm eq}}{T_n}\right)^4 \gg 1, \qquad \delta_c \simeq 0.45,$$

$$a_{\mathcal{P}} \approx 0.5646, \qquad b_{\mathcal{P}} \approx 1.266, \qquad c_{\mathcal{P}} \approx 0.6639,$$

$$f_{\rm PBH} \approx \frac{\mathcal{P}_{\rm coll}}{6.0 \times 10^{-12}}\frac{T_{\rm eq}}{500\text{GeV}}, \qquad M_{\rm PBH} \approx M_\odot \left(\frac{20}{g_*(T_{\rm eq})}\right)^{1/2}\left(\frac{140\text{MeV}}{T_{\rm eq}}\right)^2.$$

It is also possible calculate the RMS Kerr Parameter of the PBHs
arXiv:2409.06494 [gr-qc], I. K. Banerjee, T. Harada

PBH Initial Spin

$$\sqrt{\langle a_*^2 \rangle} \approx \frac{2.1 \times 10^{-3}}{23.484 - 1.25 \log_{10}(f_{\rm PBH}) - 1.25 \log_{10}\left(\frac{\Omega_{\rm CDM}}{0.26}\right) - 0.625 \log_{10}\left(\frac{M_{\rm PBH}}{10^{15}{\rm g}}\right)}.$$

$$a_K \equiv \frac{J}{Mc}, \qquad a_* \equiv \frac{a_K c^2}{G_N M}$$

We can express the physical parameters of the Late Blooming Mechanism in terms of $(\chi_0, \bar{\beta}, g, \tilde{g})$

Late Blooming & Radiative Symmetry Breaking Models

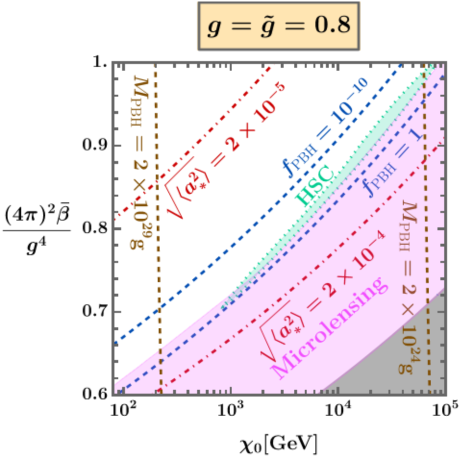
$$T_{\rm eq}^4 \approx \frac{15\bar{\beta}\chi_0^4}{8\pi^2 g_*(T_{\rm eq})}, \qquad M_{\rm PBH} \approx M_\odot \left(\frac{2\pi^2}{3\bar{\beta}}\right)^{1/2} \left(\frac{280\text{MeV}}{\chi_0}\right)^2, \qquad H_n \sim H_I \approx \frac{\sqrt{\bar{\beta}}\chi_0^2}{4\sqrt{3}\bar{M}_P},$$

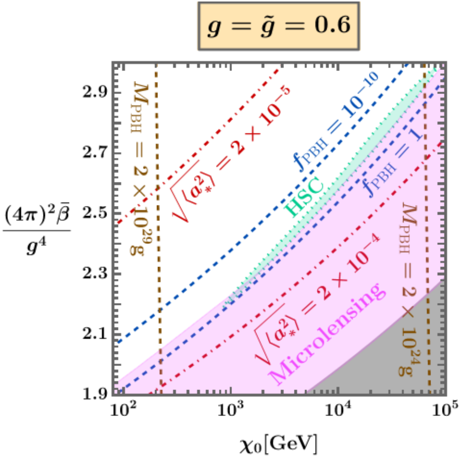
$$\frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4 \qquad \textbf{(Supercool expansion LO)},$$

$$\frac{\beta}{H_n} \approx \frac{\pi^3 g^5}{6\sqrt{3}\tilde{g}^8}\frac{(4\pi)^2\bar{\beta}}{\tilde{g}^4}(-F'(\tilde{\lambda}_n)) - 4 \qquad \textbf{(Improved supercool expansion LO)}.$$

Then we can calculate:

$$f_{\rm PBH}(\chi_0, \bar{\beta}, g, \tilde{g}), \qquad \sqrt{\langle a_*^2\rangle}(f_{\rm PBH}(\chi_0, \bar{\beta}, g, \tilde{g}), M_{\rm PBH}(T_{\rm eq}(\chi_0, \bar{\beta}))).$$





Improved supercool expansion at LO

Roadmap

FOPT in the B-L Model

- $U(1)_{B-L}$ model phenomenology can be described with Improved Supercool Expansion

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{\text{ns}} + D_\mu A^\dagger D^\mu A + \bar{N}_j i\gamma_\mu D^\mu N_j - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu}$$
$$+ \left(Y_{ij}L_i\mathcal{H}N_j + \frac{1}{2}y_{ij}AN_iN_j + \text{h.c.}\right)$$
$$- \lambda_a|A|^4 + \lambda_{ah}|A|^2|\mathcal{H}|^2$$

- The gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

FOPT in the B-L Model

- To generate the EW scale, the flat direction should be a mixing of $|\mathcal{H}|$ and $|A|$ with a small mixing angle (χ is mostly $|A|$): we can then approximate the flat direction condition with $\lambda_a(\tilde{\mu}) = 0$
- the one-loop RG equation for the quartic coupling λ_a

$$(4\pi)^2 \mu \frac{d}{d\mu} \lambda_a = 96 g_1'^4 - 48 \lambda_a g_1'^2 + 20 \lambda_a^2 + 2 \lambda_{ah}^2 + 2 \lambda_a \mathrm{Tr}(y y^\dagger) - \mathrm{Tr}(y y^\dagger y y^\dagger)$$

- Neglecting λ_{ah} and y (right-handed neutrino Majorana masses are taken below EW scale)

$$\bar{\beta} = \frac{96 g_1'^4}{(4\pi)^2}$$

- The background dependent Z' mass is

$$M_{Z'}(\chi) = 2 |g_1'| \chi$$

FOPT in the B-L Model

- The collective couplings are:

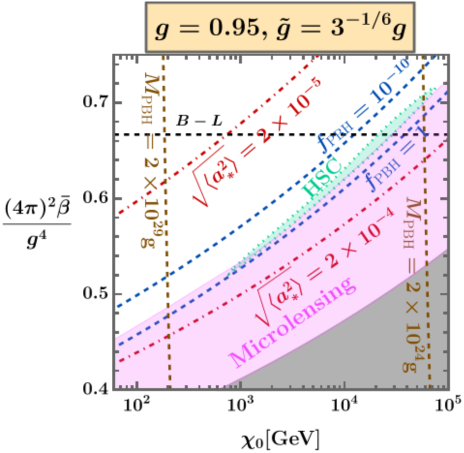
$$g = 2\sqrt{3}|g_1'|, \qquad \tilde{g} = 2\sqrt[3]{3}|g_1'| = \frac{g}{\sqrt[6]{3}},$$

we get

$$\bar{\beta} = \frac{2g^4}{3(4\pi)^2}$$

and

$$m_\chi = \sqrt{\frac{2}{3}\frac{g^2}{4\pi}}\chi_0, \qquad M_h = \sqrt{\lambda_{ah}}\chi_0.$$



Improved Supercool at LO

Conclusions

- The (improved) supercool expansion is a **powerful tool** to study the **phenomenology of FOPT** when there is **enough supercooling**
- The FOPT phenomenology related to a **general RSB model** can be described by using **just few parameters** $(\chi_0, \bar{\beta}, g, \tilde{g})$
- We described using (improved) supercool expansion the production of PBH via late blooming mechanism, and provided a model that can account for an appreciable fraction of dark matter in the form of PBH

Future perspectives

- The supercooling leads to a period of **inflation** in the universe, after which **reheating** is needed to heat the universe: can we describe it in a general way?
- FOPT are related to several sources of **particle production**: bubble collision, reheating, preheating. How can we describe these production mechanism in RSB models?
- Is it possible to produce dark matter?

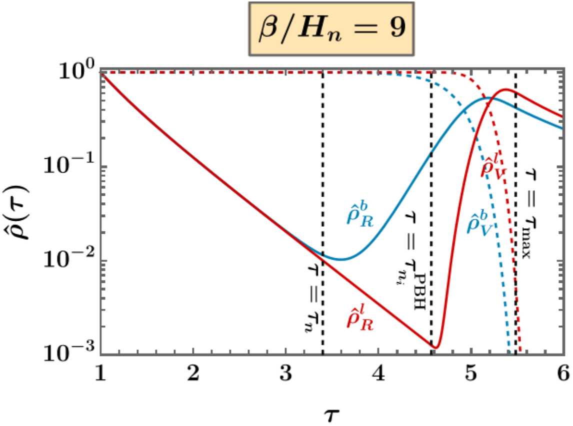


Figure 1: Energy Densities Late blooming

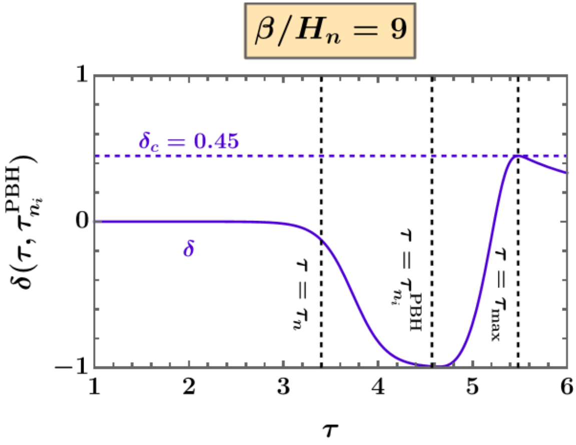


Figure 2: Density Contrast

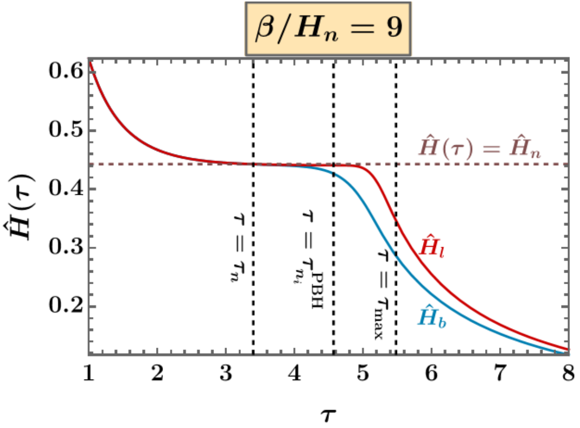


Figure 3: Hubble rate Late Blooming Mechanism

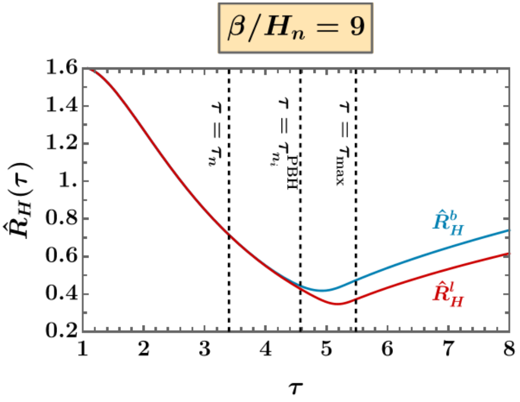


Figure 4: Hubble Radius late blooming mechanism

Supercool Expansion

- The nucleation temperature T_n is obtained solving the equation

$$\Gamma(T_n) \approx H_n^4 \approx H_I^4 \qquad \longrightarrow \qquad cX - 4X^2 - a \approx 0$$

where $H_I = \frac{\sqrt{\bar{\beta}}\chi_0^2}{4\sqrt{3}\bar{M}_P}$ is the Hubble rate when the vacuum is still dominant respect the true vacuum, and

$$X \equiv \log\frac{\chi_0}{T_n}, \qquad a \equiv \frac{c_3 g}{\sqrt{12\bar{\beta}}}, \qquad c \equiv \log\frac{4\sqrt{3}\bar{M}_P}{\sqrt{\bar{\beta}}\chi_0} + \frac{3}{2}\log\frac{a}{2\pi}.$$

- Solving the equation we obtain

$$T_n \approx \chi_0 \exp\left(\frac{\sqrt{c^2 - 16a} - c}{8}\right).$$

Extending the validity of the supercool expansion

- The supercool expansion is still valid for $\epsilon \sim 1$ only if the number of degrees of freedom N with dominant coupling to the field χ is large

$$g \sim \sqrt{N}\tau, \qquad \tilde{g} \lesssim \sqrt[3]{N}\tau \quad \longrightarrow \quad \tilde{g}^3/g^3 \lesssim 1/\sqrt{N}$$

- The cubic term in the effective potential gets suppressed by a factor $\lesssim 1/\sqrt{N}$
- $1/X = 6\bar{\beta}/\epsilon g^4$ is still small for $\epsilon \sim 1$
- Truncating the small-x expansion of the thermal function up to order $x^{3/2}$ because the higher-order terms involve smaller and smaller coefficients

- The inverse duration can be written as

$$\frac{\beta}{H_n} \approx \left[T \frac{d}{dT}(S_3/T) - 4 - \frac{3}{2} T \frac{d}{dT} \log(S_3/T) \right]_{T=T_n}$$
$$\approx \frac{a}{\log^2(\chi_0/T_n)} - 4 - \frac{3}{2} \frac{1}{\log(\chi_0/T_n)},$$

neglecting the last term

$$\frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4.$$

Improved Supercool Expansion

- In general, when $\epsilon \sim 1$, in the effective potential **the cubic term cannot be treated as a perturbation** (i.e. the term of order $x^{3/2}$ is not perturbative in the expansion of the thermal functions)
- However we can **treat the cubic term as LO** and the other higher order term as a perturbation
- We can rewrite the effective potential and the 3d euclidean action at the LO of the **Improved Supercool Expansion** as

$$\tilde{V}_{\text{eff}}(\phi, T) = \frac{1}{2}\phi^2 - \frac{1}{3}\phi^3 - \frac{\tilde{\lambda}}{4}\phi^4, \qquad S_3 = -\frac{8\pi m^3}{k^2}\int_0^\infty d\rho\, \rho^2 \tilde{V}_{\text{eff}}(\phi, T),$$

$$\tilde{\lambda}(T) \equiv \frac{\lambda m^2}{k^2} = \frac{(4\pi)^2\bar{\beta}}{12\,\tilde{g}^6/g^2}\log(\chi_0/T) \geq \frac{2\pi^2}{9\epsilon}.$$

Improved Supercool Expansion

- We are interested for values of $\tilde{\lambda} \sim 1$: for such values we can write S_3 as
JHEP **02** (2023), 125, arXiv:2212.08085 [hep-ph], N. Levi, T. Opferkuch, D. Redigolo

$$S_3 = \frac{27\pi m^3}{2k^2} \frac{1 + \exp(-1/\sqrt{\tilde{\lambda}})}{1 + \frac{9}{2}\tilde{\lambda}}.$$

- This expression reproduces the numerical calculation at the $\sim 1\%$ level for the values of $\tilde{\lambda}$ we are interested in

- The validity of this expression of S_3 has been established in a **model independent way within the improved supercool expansion**

Improved Supercool Expansion

- Solving the equation $\Gamma(T_n) \approx H_n^4$

$$a_1 - a_2\tilde{\lambda} = F(\tilde{\lambda}) \equiv \frac{1 + \exp(-1/\sqrt{\tilde{\lambda}})}{2/9 + \tilde{\lambda}},$$

$$a_1 \equiv \frac{c\, c_3 k^3}{3\pi a \bar{\beta} m^2}, \qquad a_2 \equiv \frac{4 c_3 k^4}{3\pi a \bar{\beta}^2 m^4}.$$

- We are interested in the smallest real and positive solution $\tilde{\lambda}_n(T) \equiv \tilde{\lambda}(T_n)$ for which the straight line $a_1 - a_2\tilde{\lambda}$ reaches $F(\tilde{\lambda})$ from below in increasing $\tilde{\lambda}$ (if it exist)

- Following the same steps as before we can calculate β/H_n

$$\frac{\beta}{H_n} \approx \frac{\pi^3 g^5}{6\sqrt{3}\tilde{g}^8} \frac{(4\pi)^2 \bar{\beta}}{\tilde{g}^4} (-F'(\tilde{\lambda_n})) - 4.$$

Late Blooming & Supercool Expansion

The expansion

$$\Gamma(t) = \Gamma(t_n)\text{exp}(\beta(t-t_n) + \beta_2(t-t_n)^2 + ...) \approx H_n^4 e^{\beta(t-t_n)},$$

can be justified on the ground of the supercool expansion. Using:

$$dt = -dT/(TH) \approx -dT/(TH_I) \longrightarrow T(t) \approx T_n e^{-H_I(t-t_n)},$$

for the supercool expansion at LO

$$\frac{S_3}{T} \approx c_3\frac{m}{T\lambda} = \frac{c_3 g}{\sqrt{12\bar{\beta}}\text{log}\frac{\chi_0}{T}} \equiv \frac{a}{X + \text{log}\frac{T_n}{T}} \approx \frac{a}{X + H_I(t-t_n)},$$

$$X \equiv \text{log}\frac{\chi_0}{T_n},$$

Late Blooming & Supercool Expansion

we get

$$\Gamma(t) \approx T^4(t)\mathsf{exp}(-S_3/T(t)) \approx T_n^4\mathsf{exp}\left(-\frac{a}{X + H_I(t-t_n)} - 4H_I(t-t_n)\right).$$

Expanding for t around t_n

$$\frac{1}{1+\frac{H_I(t-t_n)}{X}} = 1 - \frac{H_I(t-t_n)}{X} + \left(\frac{H_I(t-t_n)}{X}\right)^2 + \cdots + (-1)^k\left(\frac{H_I(t-t_n)}{X}\right)^k + \ldots$$

noting that $H_I = H_{\rm eq}/\sqrt{2} = \gamma/(\sqrt{3}t_{\rm eq})$

$$\frac{H_I(t-t_n)}{X} = \frac{\gamma(\tau-\tau_n)}{\sqrt{3}X}, \qquad \gamma = 0.76329\ldots, \qquad \tau \equiv \frac{t}{t_{\rm eq}},$$

Late Blooming & Supercool Expansion

- As long as $\tau - \tau_n$ is small respect to $\sqrt{3}X/\gamma$, $\Gamma(t) \approx H_n^4 e^{\beta(t-t_n)}$ is a **good approximation**.
- This holds also for the improved supercool expansion ($\epsilon \sim 1$ at $T = T_n$)

$$\bar{\beta} \sim \frac{g^4}{(4\pi)^2} \quad (\textbf{loop suppressed}) \quad \longrightarrow \quad X \sim 26 \quad \longrightarrow \quad \frac{\gamma}{\sqrt{3}X} \sim 10^{-2}$$

Late Blooming & Supercool Expansion

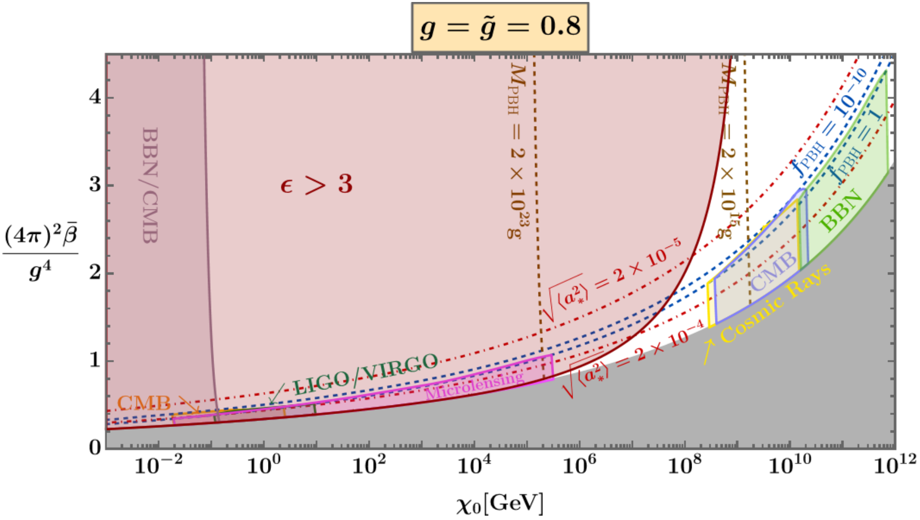


Figure 5: Improved Supercool at LO

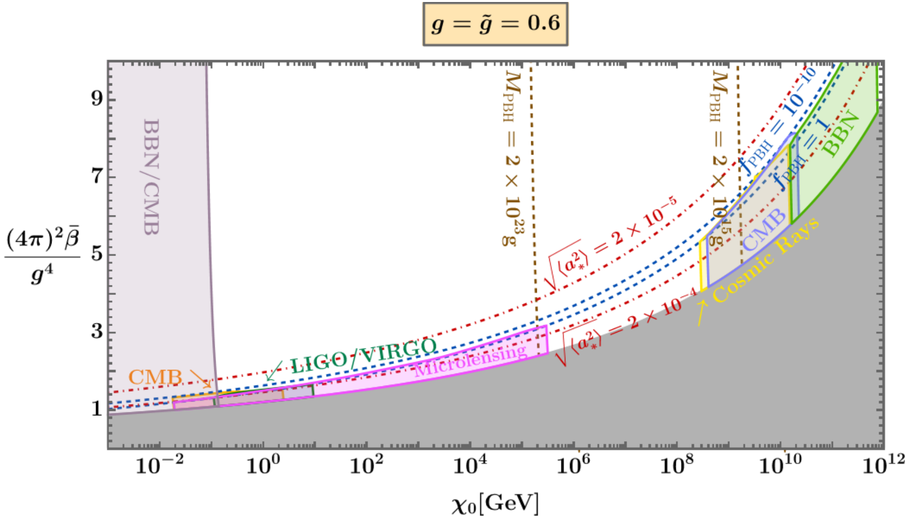


Figure 6: Improved Supercool at LO

Supercool expansion details

- The dominant contributions to the bounce action S are those from field values around the barrier.
- Therefore, we first need to estimate the barrier size, which we can define as the field value

$$\bar{V}_{\rm eff}(\chi_b, T) = 0$$

- The log term inside V_q taking χ around χ_b

$$\log \frac{\chi_b}{\chi_0} - \frac{1}{4} = log \frac{\chi_b}{T} - \frac{1}{4} + \log \frac{T}{\chi_0} \approx \log \frac{T}{\chi_0},$$

- One then can show that for enough supercooling

$$\frac{\chi_b^2}{T^2} \approx \frac{g^2}{6\bar{\beta} \log \frac{\chi_0}{T}}$$