Unitarization procedures in VBS processes at LHC

Costanza Carrivale
(INFN Perugia, Università degli studi di Perugia)

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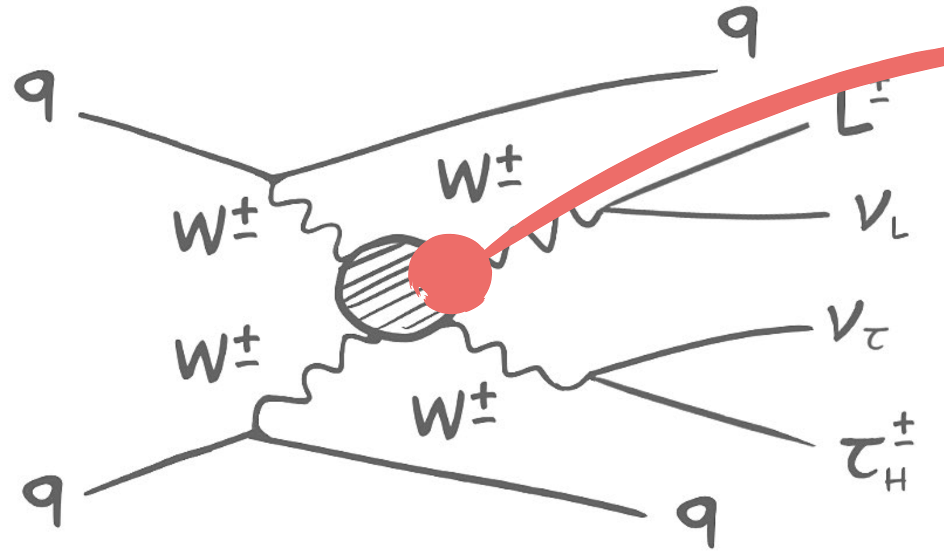
Outline

- An overview on Effective Field Theories
- Unitarity violation in EFT
- Why multiboson processes?
- Impact of unitarity bounds for EFT@dim6 operators in VBS ssWW processes
- Unitarization procedures in ATLAS and CMS
- Unitarization in multiboson processes
- Ongoing work

Effective Field Theories

Effective Field Theories are an indirect approach for New Physics searches. SM EFT is based on few basic assumptions:

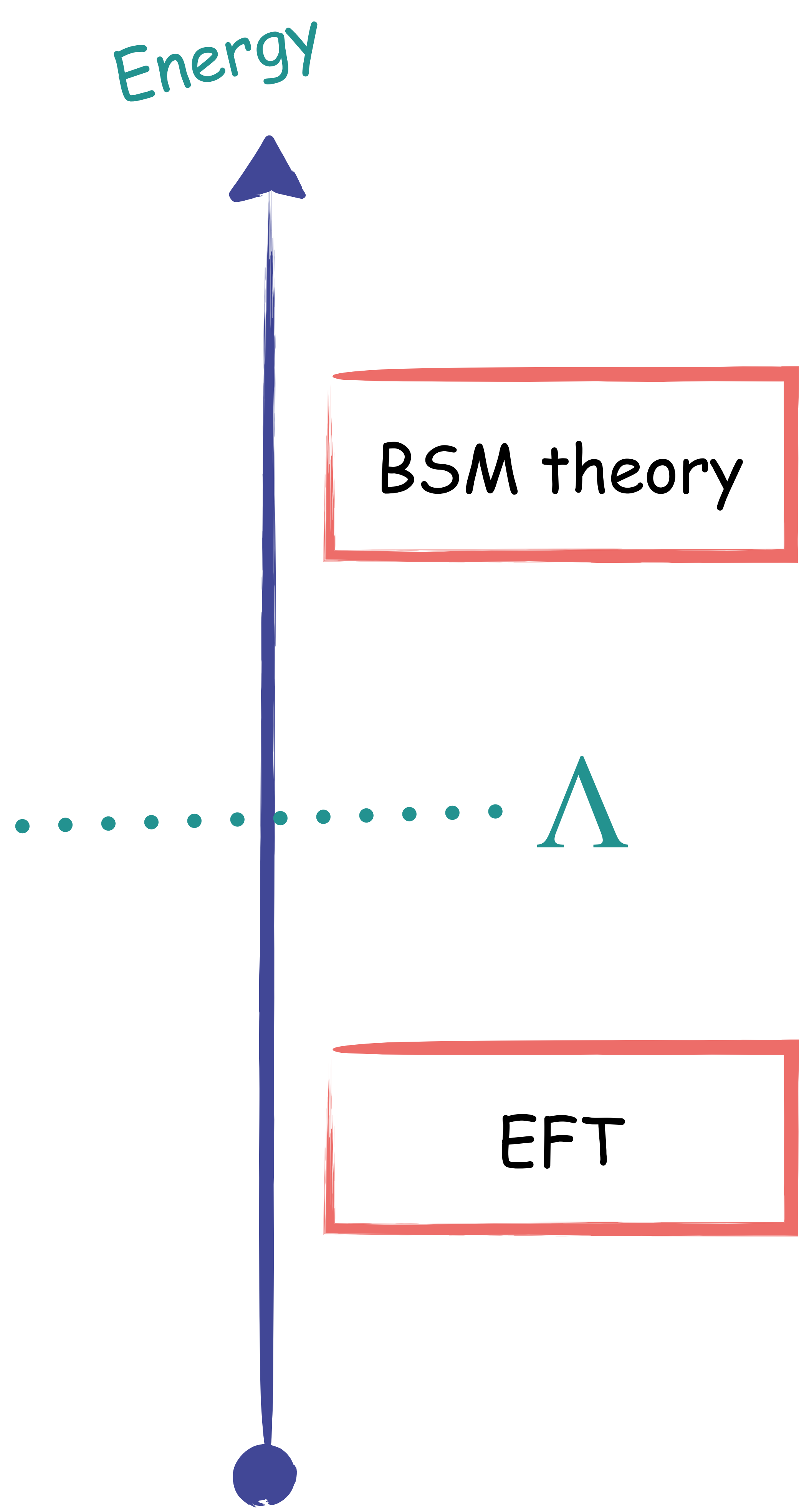
- High-energy theory with **same field content** of SM and same symmetries;
- Dynamics at high energy cannot be directly proben;
- Low-energy effects are parametrized by a **Taylor expansion** of SM Lagrangian:



$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{f_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)} + \ldots$$

* Wilson coefficients

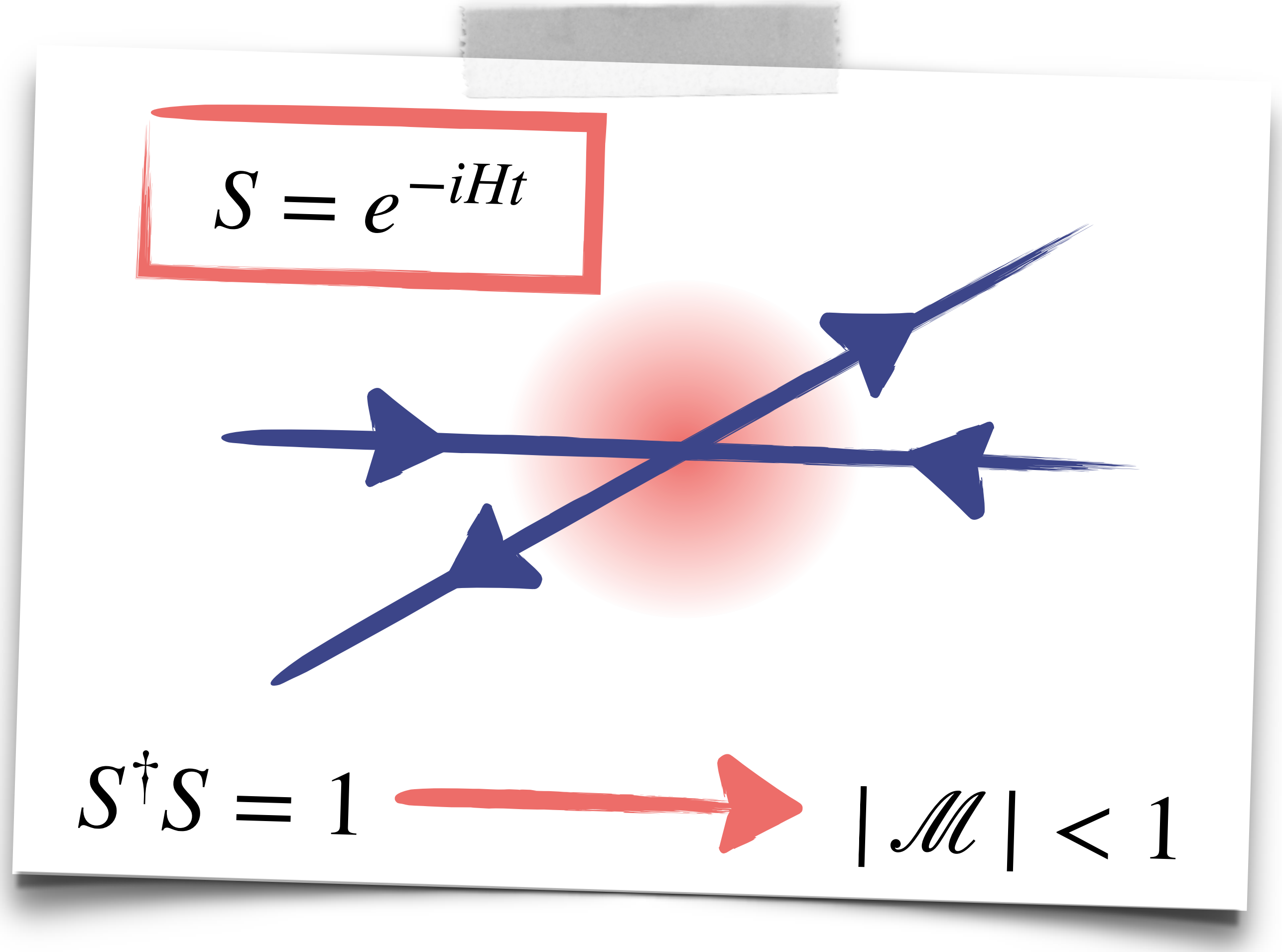
** New Physics scale



Optical theorem

Scattering matrix S defines the kinematics of the process and unitarity of S means conservation of probability.

This translates in constraints on scattering amplitude of the process and, though, on the coefficients of related partial wave expansion:



- $\sigma_{\text{tot}}(2 \to 2) = \dfrac{1}{32\pi E_{\text{CM}}^2} \int |\mathscr{M}(\theta)|^2 d\cos\theta$
- $\mathscr{M}(\theta) = 16\pi \sum_{j=0}^{\infty} a^j (2j+1) \mathscr{P}_j(\cos(\theta))$

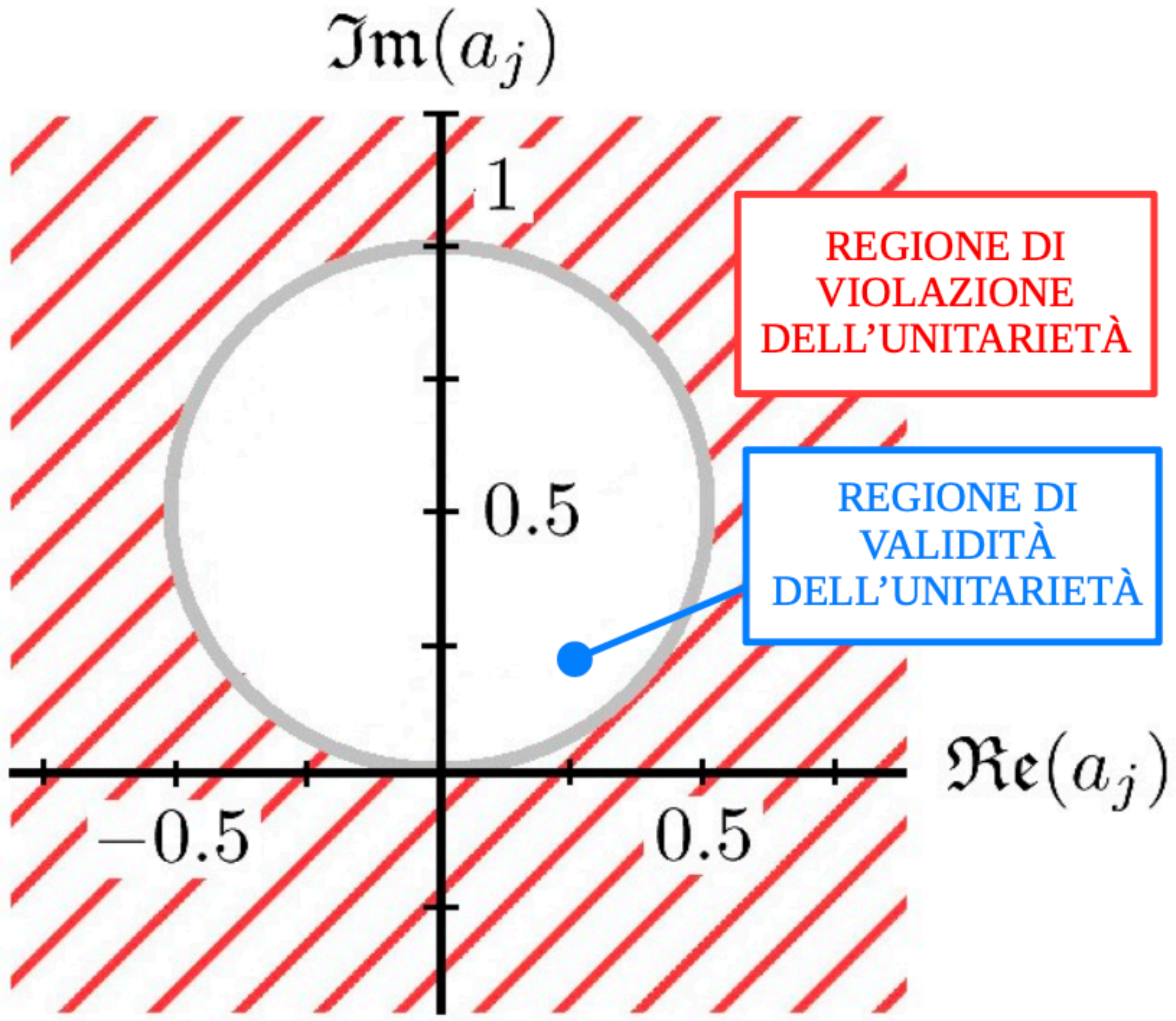
$$\boxed{\sigma_{\text{tot}}(2 \to 2) = \frac{16\pi}{E_{CM}^2} \sum_{j=0}^{\infty} (2j+1)|a^j|^2}$$

$$\mathfrak{Im}[\mathscr{M}(\theta = 0)] = 2E_{CM}|\vec{p_i}| \sum_X \sigma(2 \to X) \geq 2E_{CM}|\vec{p_i}|\sigma_{tot}(2 \to 2)$$

Unitarity violation in EFT

- $0 \leq \mathfrak{Im}(a^j) \leq 1$
- $|\mathfrak{Re}(a^j)| \leq \frac{1}{2}$

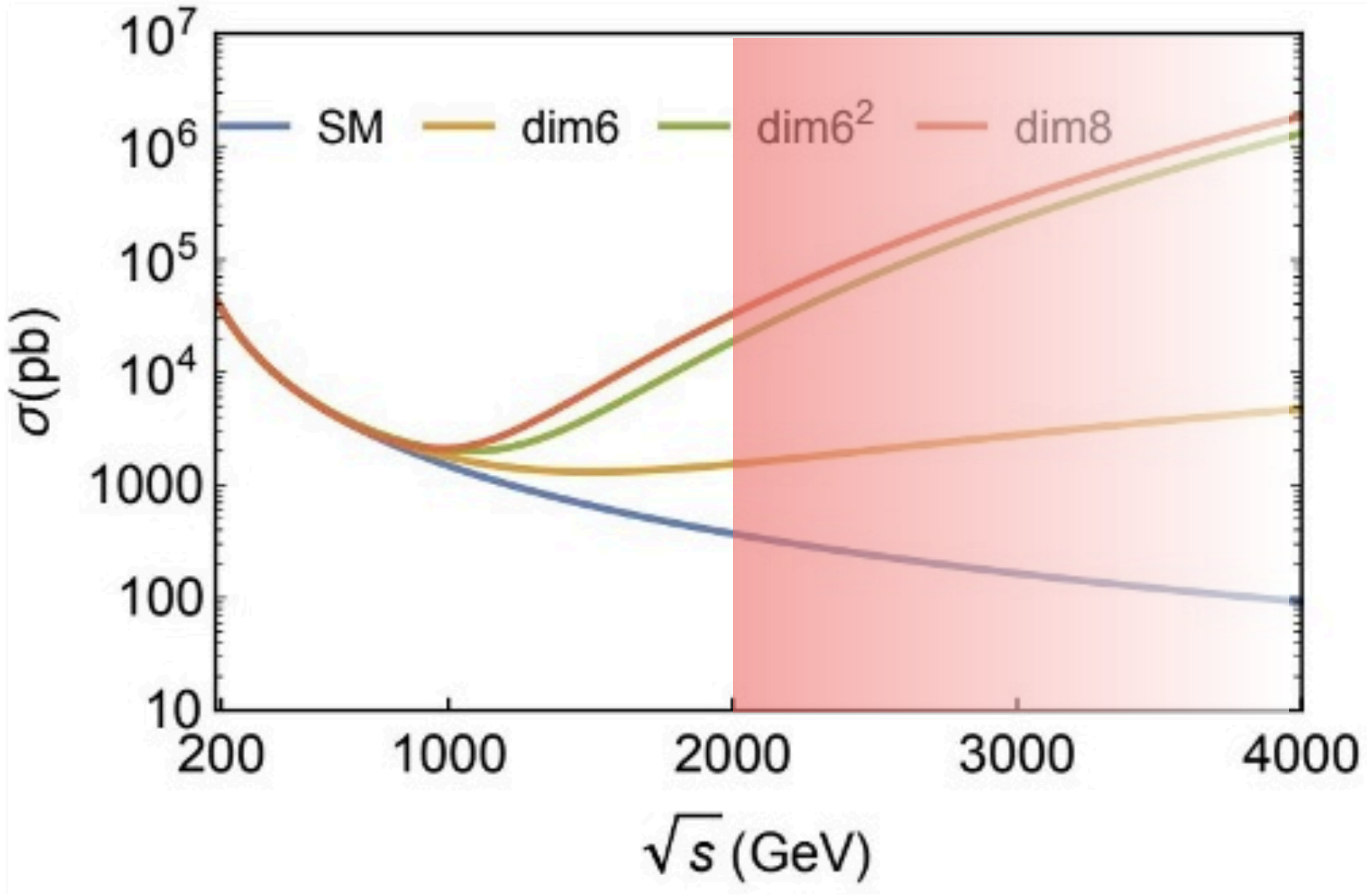
$$\boxed{|a^j| \leq 1}$$



EFT amplitude exhibits terms with different behaviour:

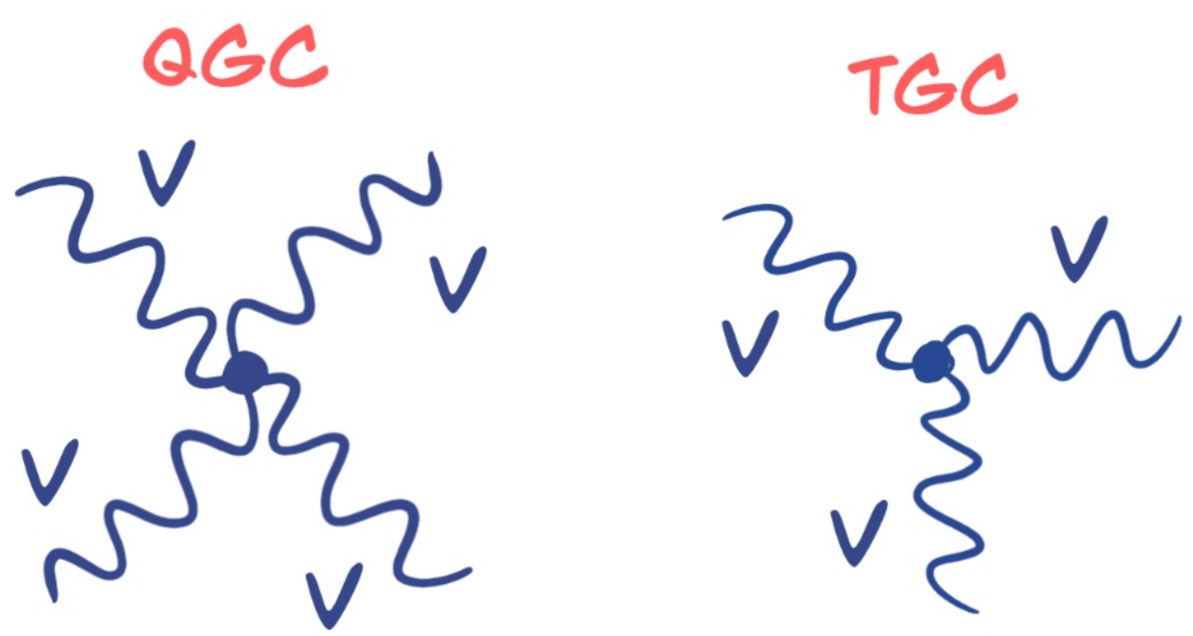
$$\mathcal{M}_{\text{eft}} = \mathcal{M}_{\text{sm}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{M}_6^i + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{M}_8^i + \dots$$

- In **SM** unitarity is never violated
- **EFT terms** can grow with center of mass energy ($\mathcal{M}_{\text{eft}} \sim \mathcal{O}(\hat{s})$)



Vector Boson Scattering in EFT

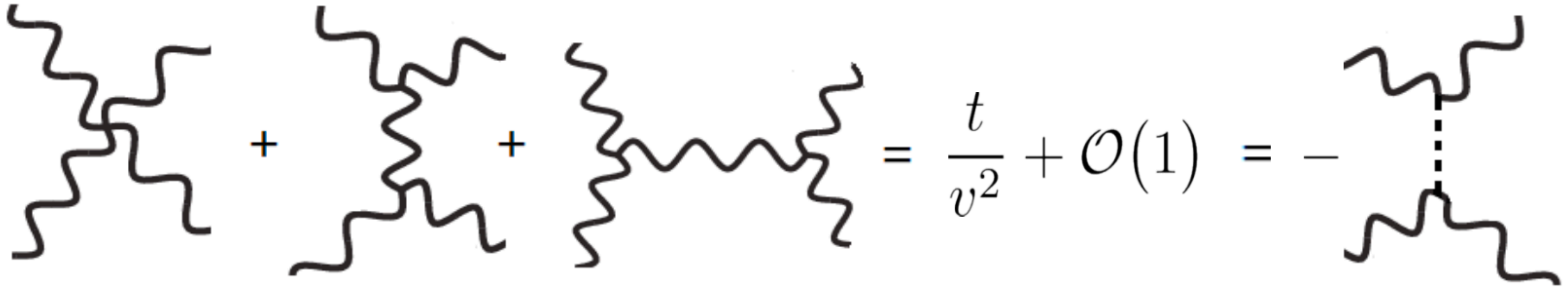
- EW sector of the Standard Model is based on $SU(2)_L \otimes SU(1)_Y$ symmetry group. The non-abelian nature of the group results in self-interaction of the gauge bosons (triple and quartic gauge couplings, **TGCs and QGCs**). VBS processes exhibit both types of interaction.



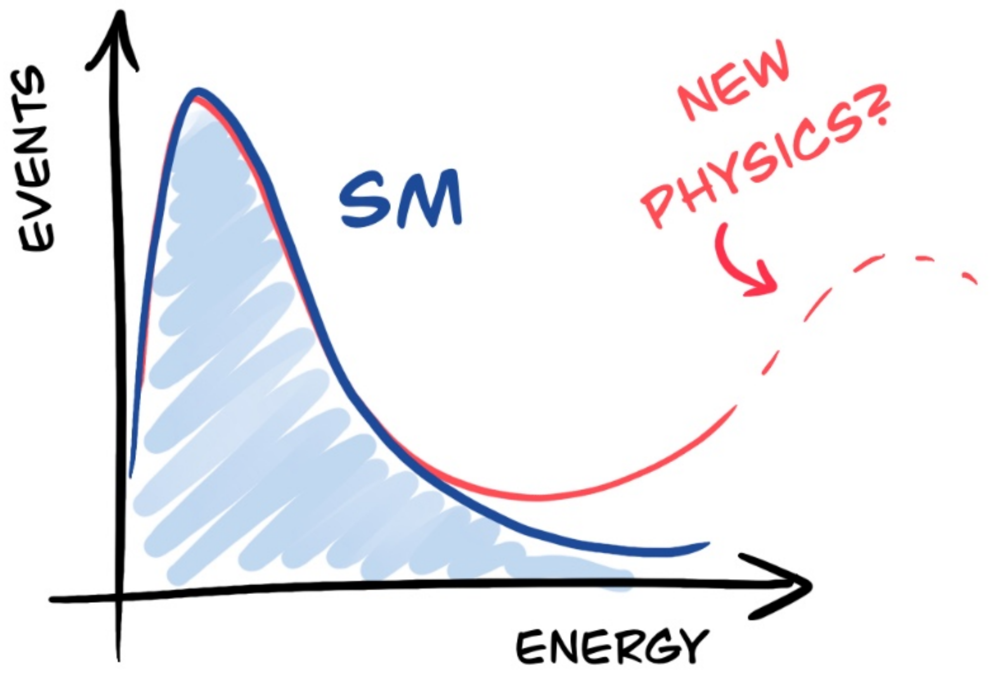
- VBS processes are strictly related to **unitarity violation** in SM and precise measurements of VBS can probe the nature of the Higgs sector

$$m_h \leq \sqrt{\frac{16\pi}{3}\frac{1}{v}} \simeq 1\ \text{TeV}$$

Lee-Quigg-Thacker bound



- Powerful instrument to search for **effects beyond the SM** using model-independent approaches



Study of the impact of unitarity bounds for dim6 operators in VBS same-sign WW

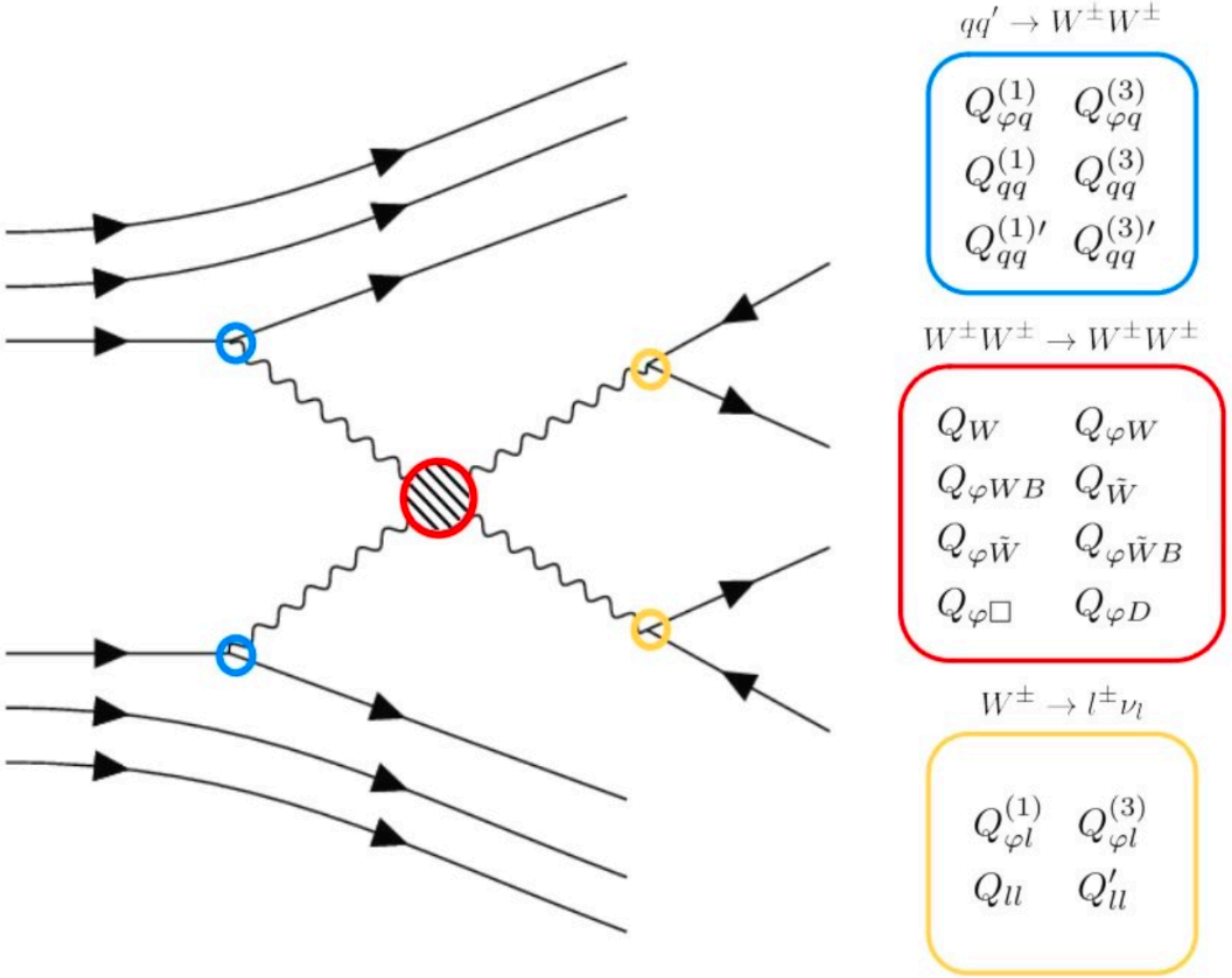
10.1393/ncc/i2024-24280-y

$$W^{\pm}(p_1,\lambda_1)W^{\pm}(p_2,\lambda_2) \to W^{\pm}(p_3,\lambda_3)W^{\pm}(p_4,\lambda_4)$$

VBS same-sign WW in the fully-leptonic final state is considered the golden channel for VBS studies.

Generator level analysis:

- Sample generation using **MadGraph5_aMC@NLO v2.7.3** (13TeV);
- Reweighting procedure to extract single operator contributions;
- Derivation of unitarity bounds using partial waves expansion of scattering amplitudes;
- Determination of confidence intervals of Wilson coefficients on samples w/ and w/o unitarity bounds.



Helicity amplitude computation

- Tree-level computation performed using **Mathematica v_12.1** and specific packages for HEP computations (FeynArtsv_3.11 and FormCalc v_9.9)
- 5 CP-conserving bosonic operators from Warsaw basis are included using **SMEFTsim_U35_MwScheme** model
- One EFT contribution at a time in only one vertex, i.e. $\mathcal{O}(c_i/\Lambda^2)$ orders

EFT operator	$\mathcal{M}_{\mp\mp\mp\pm}$	$\mathcal{M}_{\pm\pm\mp\mp}$	$\mathcal{M}_{0\pm0\mp}$	$\mathcal{M}_{0\pm\mp0}$	$\mathcal{M}_{0000}$
Q_W	$-6g\frac{c_W}{\Lambda^2}s$	$-12g\frac{c_W}{\Lambda^2}s$	$-\frac{3}{4}g\frac{c_W}{\Lambda^2}s(3+\cos\theta)$	$\frac{3}{4}g\frac{c_W}{\Lambda^2}s(3-\cos\theta)$	0
$Q_{\varphi W}$	0	0	$g\frac{c_{\varphi W}}{\Lambda^2}s(1-\cos\theta)$	$-g\frac{c_{\varphi W}}{\Lambda^2}s(1+\cos\theta)$	0
$Q_{\varphi WB}$	0	0	0	0	0
$Q_{\varphi DD}$	0	0	0	0	$2\frac{c_{\varphi DD}}{\Lambda^2}s$
$Q_{\varphi\Box}$	0	0	0	0	$2\frac{c_{\varphi\Box}}{\Lambda^2}s$

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$Q_{\varphi W}$	0	0	[illegible] $g\frac{c_{\varphi W}}{\ldots}s(1-\cos\theta)$	[illegible] $g\frac{c_{\varphi W}}{\ldots}s(1+\cos\theta)$	0
$Q_{\varphi WB}$	0	0	[illegible]	[illegible]	[illegible]
$Q_{\varphi DD}$	0	0	[illegible]	[illegible]	[illegible]
$Q_{\varphi\Box}$	0	0	[illegible]	[illegible]	[illegible]

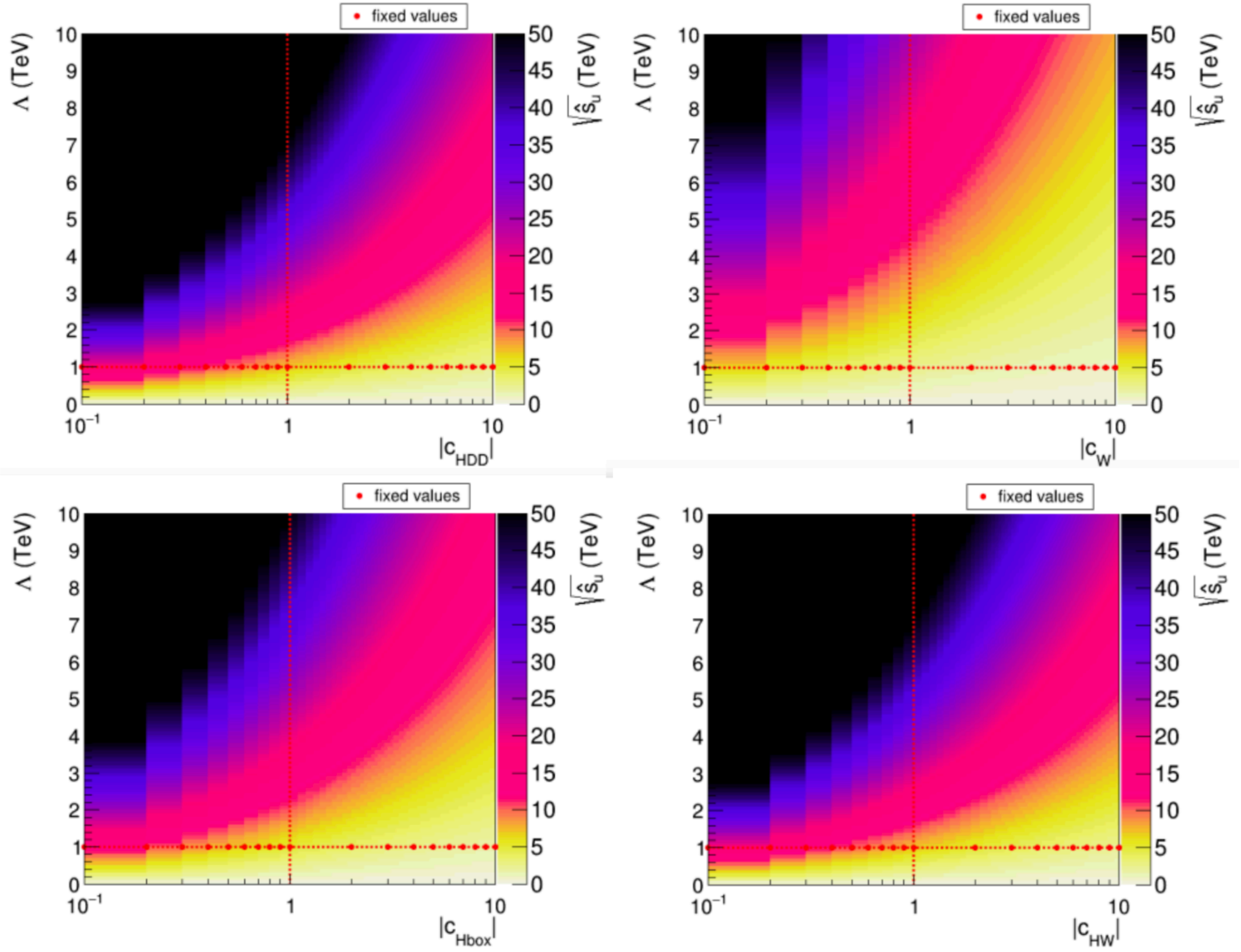
For each EFT contribution, optical theorem condition has been applied to corresponding partial amplitudes and most stringent bound has been identified as unitarity bound:

$$|a^j_{\lambda_1\lambda_2\lambda_3\lambda_4}(s;\Lambda,c_i)| \leq 1 \longrightarrow \hat{s} \leq \hat{s}_u(\Lambda, c_i)$$

Perturbative unitarity bounds

Unitarity violation threshold has been derived as a function of EFT parameters c_i and Λ. Chosen working points are $c_i = 1$ and $\Lambda = 1$ TeV.

EFT operator	Unitarity bound
Q_W	$2\left(\frac{\Lambda^2\pi}{3g}\right)^{1/2}\frac{1}{\lvert c_W\rvert^{1/2}} \leq 2.3$ TeV
$Q_{\varphi W}$	$4\left(\Lambda^2\pi\right)^{1/2}\frac{1}{\lvert c_{\varphi W}\rvert^{1/2}} \leq 7.1$ TeV
$Q_{\varphi WB}$	–
$Q_{\varphi DD}$	$4\left(\Lambda^2\pi\right)^{1/2}\frac{1}{\lvert c_{\varphi W}\rvert^{1/2}} \leq 7.1$ TeV
$Q_{\varphi\Box}$	$2\left(2\Lambda^2\pi\right)^{1/2}\frac{1}{\lvert c_{\varphi\Box}\rvert^{1/2}} \leq 5.0$ TeV



Likelihood scan extraction

1D Likelihood scan technique for the extraction of 68%CL and 95%CL limits on Wilson coefficients using CMS tools for statistical analysis (Combine).

$$k = \frac{c_i}{\Lambda^2}$$

$$\mathcal{F}(\xi) = f_{sm}(\xi) + k \cdot f_{int}(\xi) + k^2 \cdot f_{bsm}(\xi)$$

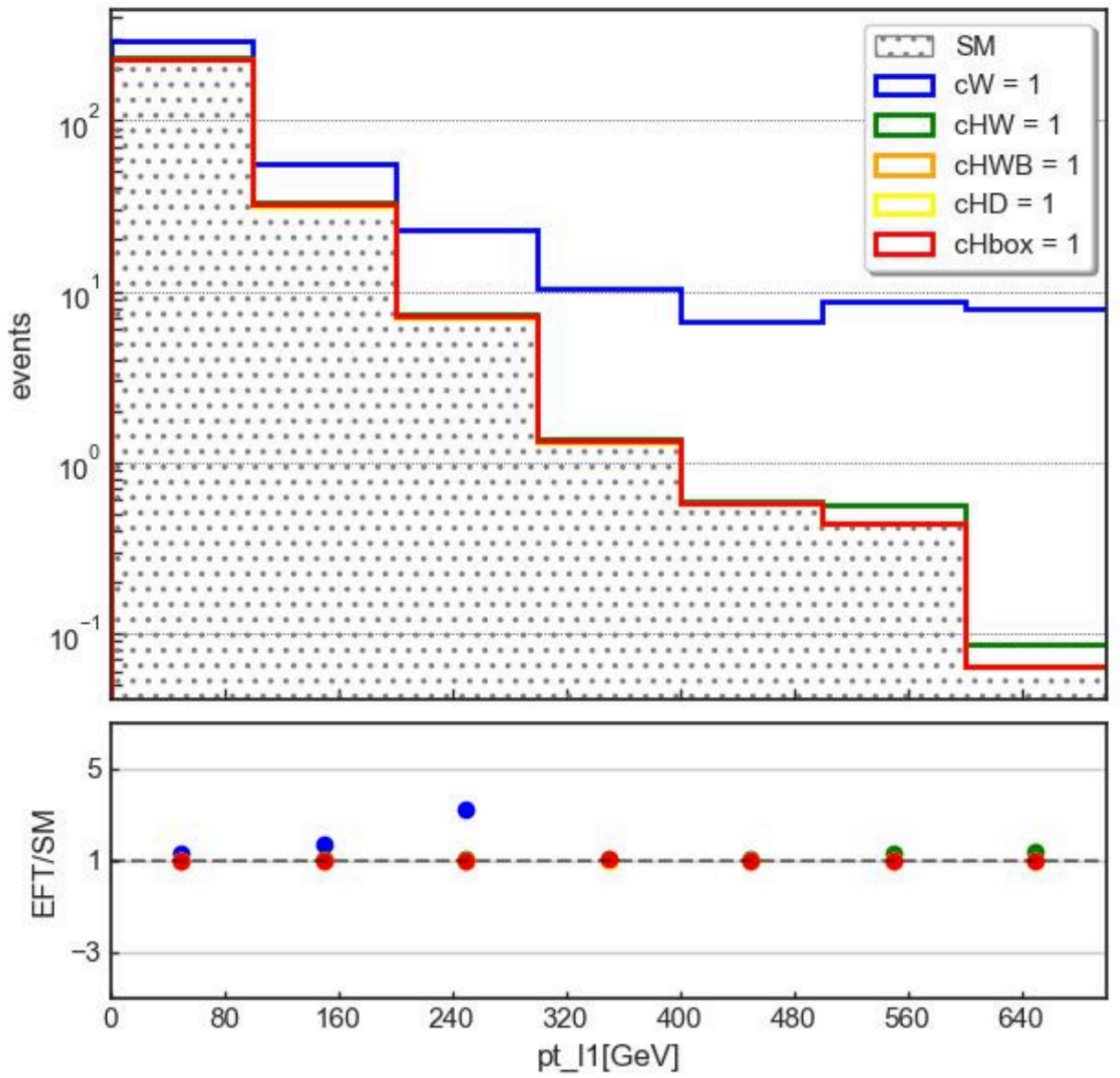
$$\mathcal{L}(\mathrm{Asimov}\,|\,\mathrm{MC(k)}) = \prod_{i=1}^{M} \mathcal{P}(n_i; \lambda_i(k))$$

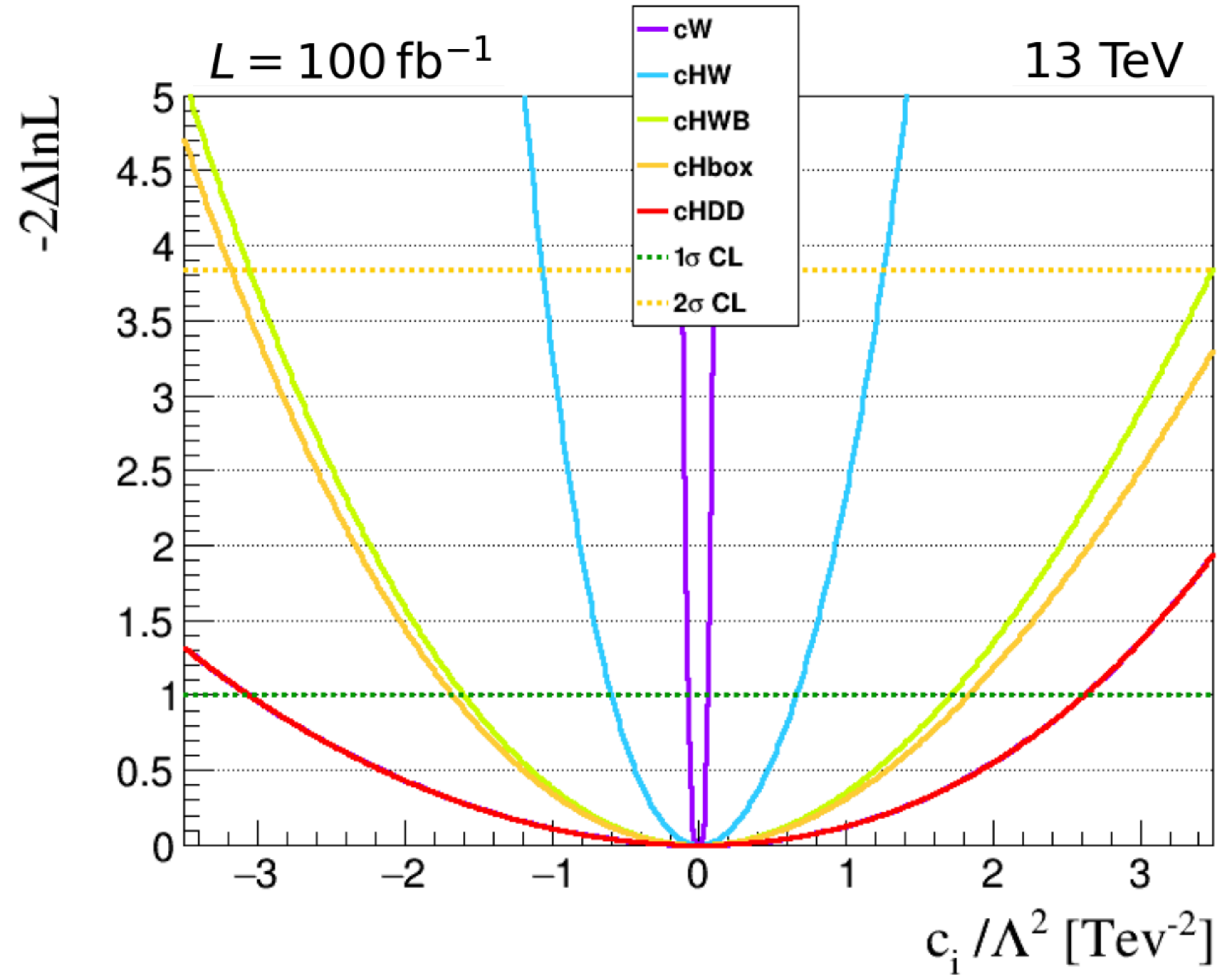
$$\Lambda(k) = \frac{\mathcal{L}(\mathrm{Asimov}\,|\,H_1(k))}{\mathcal{L}(\mathrm{Asimov}\,|\,H_0)}$$

Likelihood Ratio

$$\boxed{q(k) = -2\log\Lambda(k) = -2\Delta\log\mathcal{L}(k)}$$

Ho = SM only
H1 = SM + EFT





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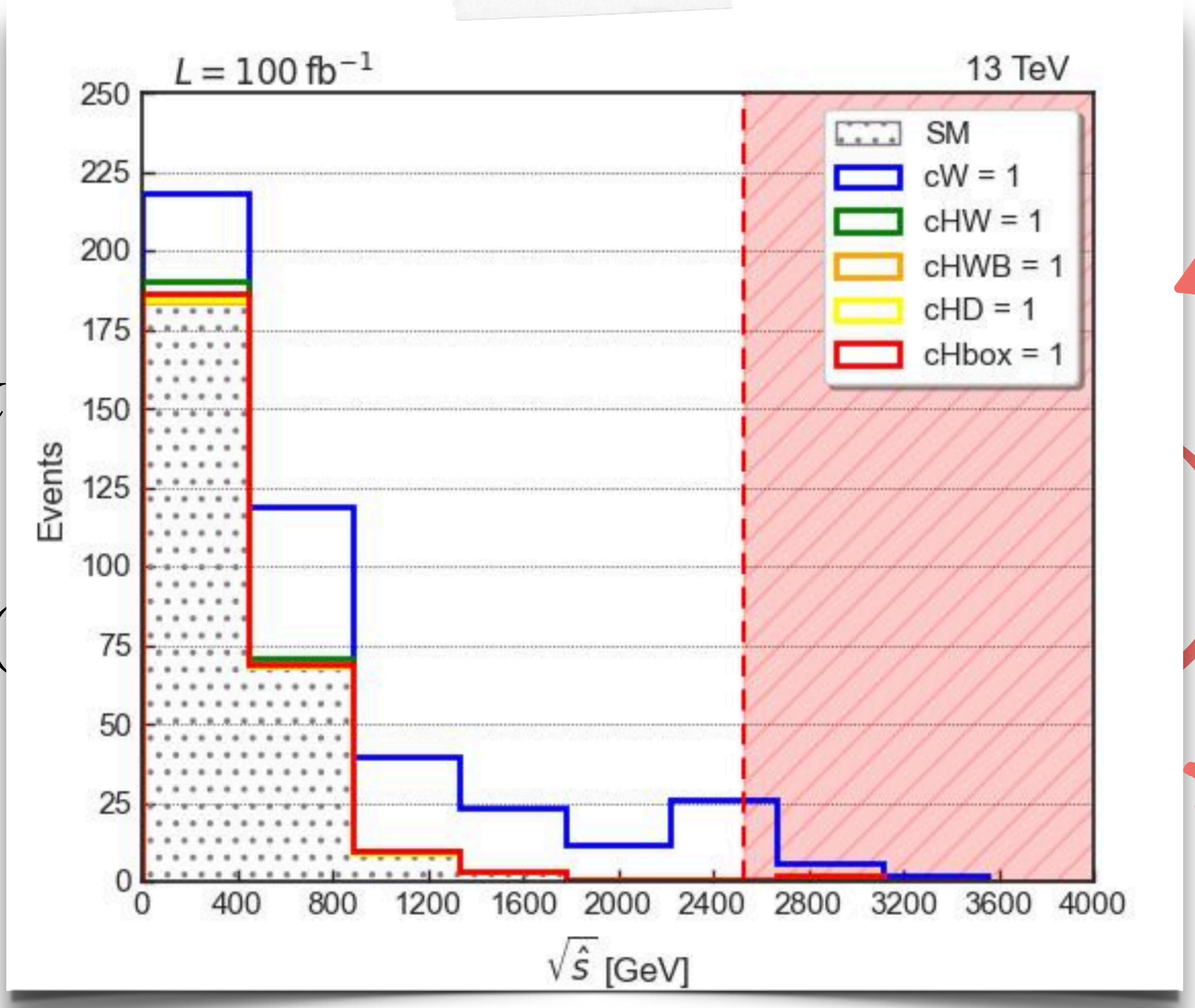
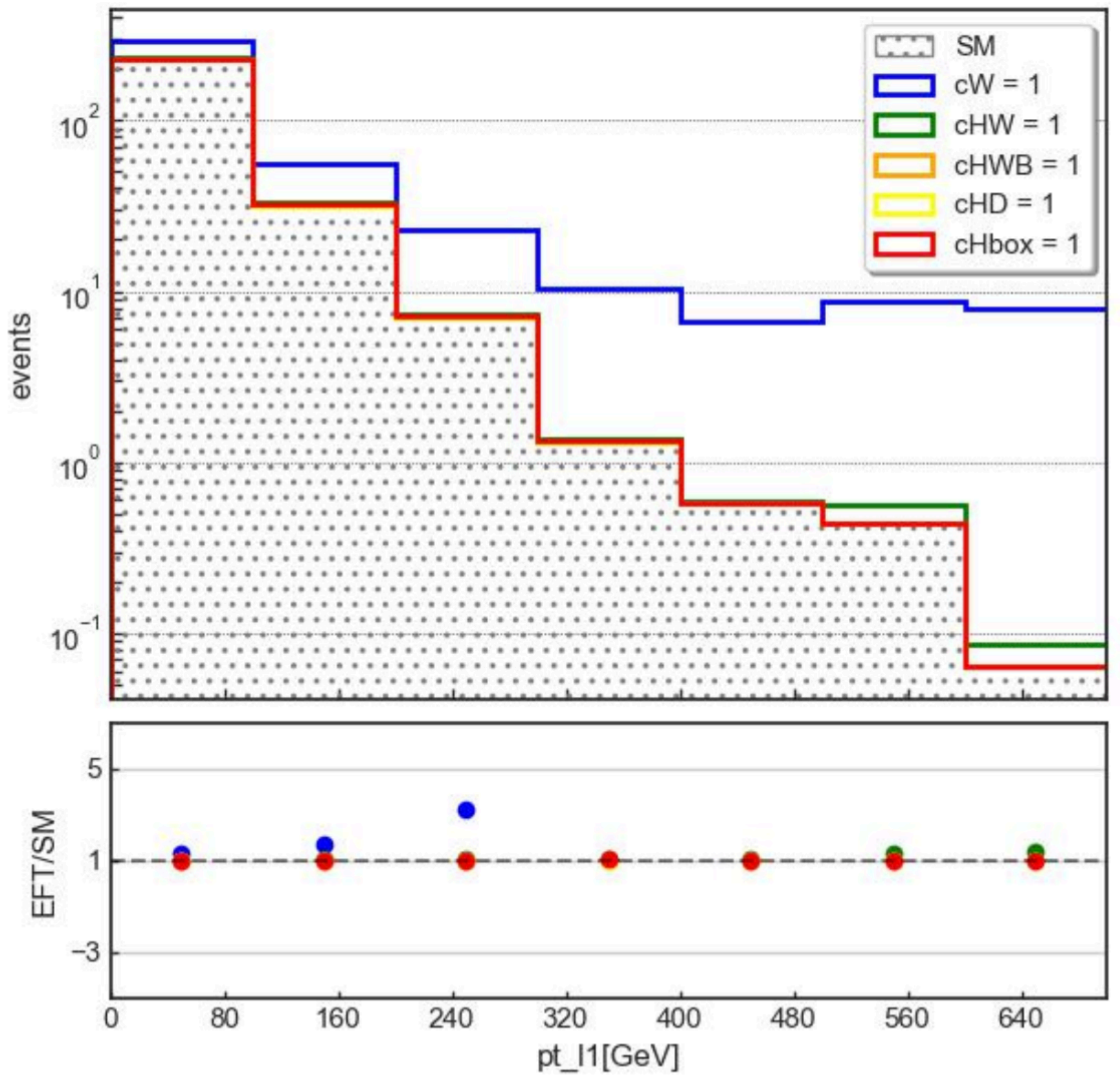
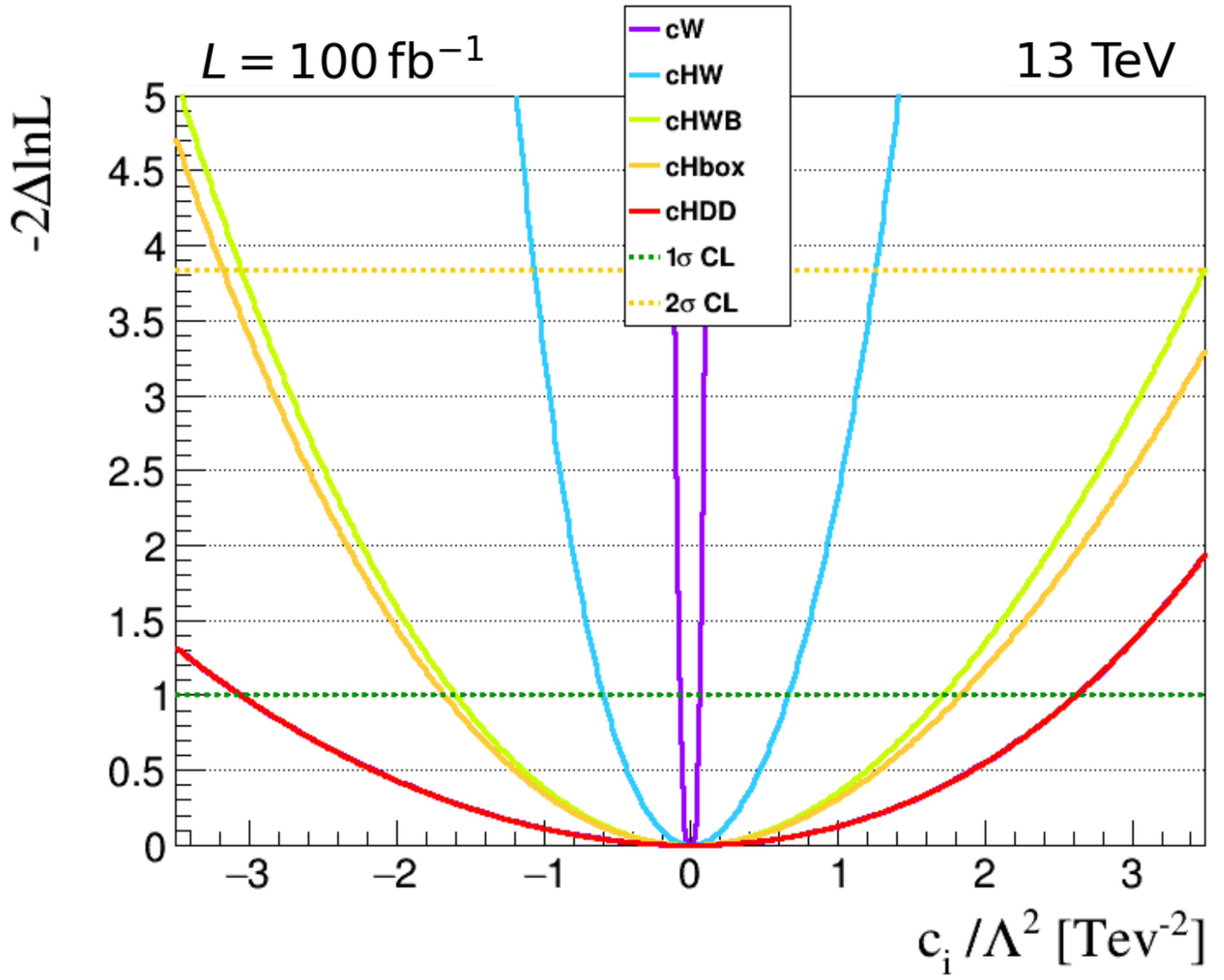
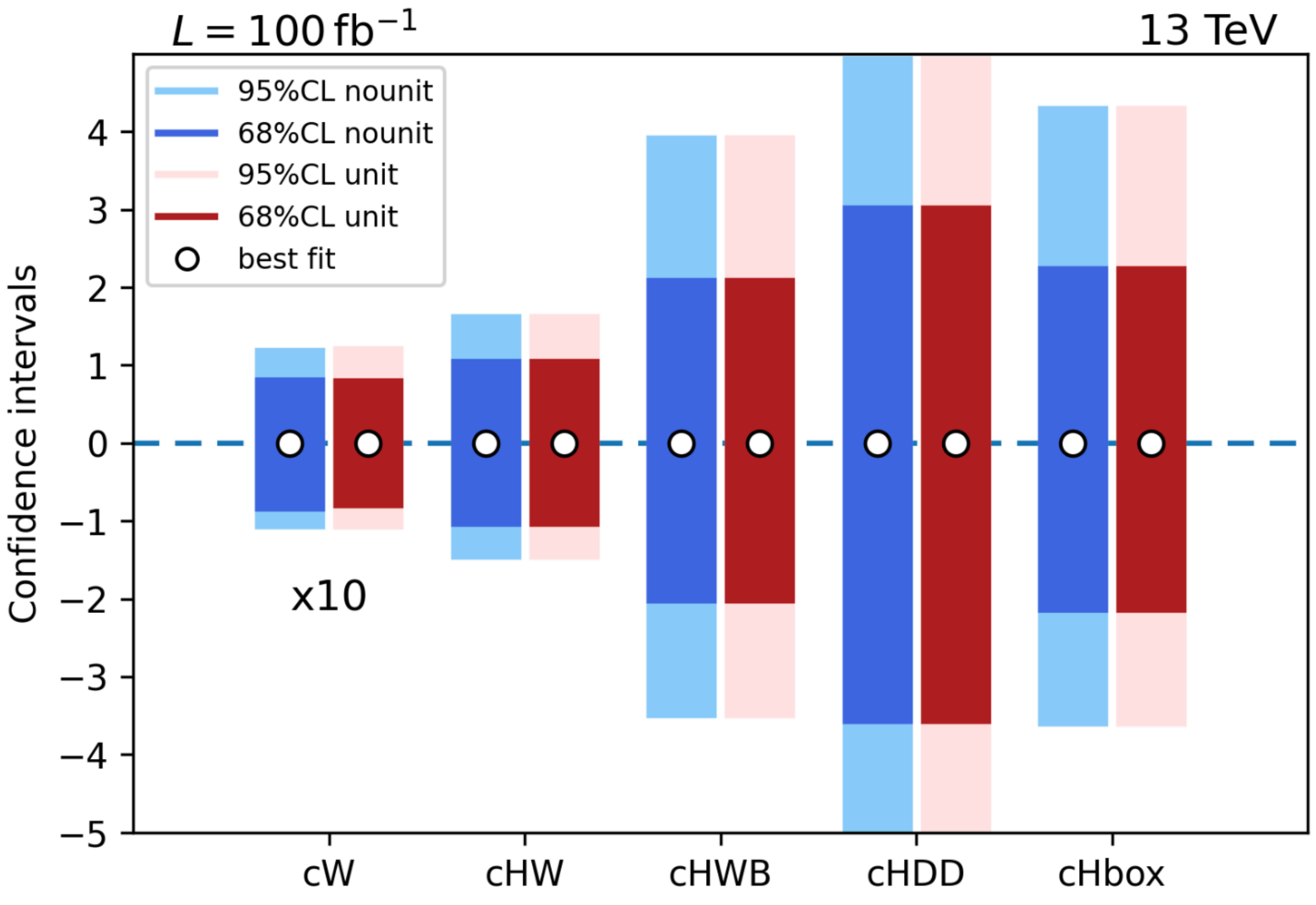






Results - Impact of unitarity bounds

After unitarity bounds implementation, CIs for dim6 Wilson coefficients remain unchanged.



WC	Bound	68%CL	95%CL	Impact (%)
cW	**Yes**	[-0.042 ; 0.040]	[-0.065 ; 0.078]	**1.4**
	No	[-0.038 ; 0.039]	[-0.065 ; 0.080]	
cHW	**Yes**	[-0.62 ; 0.64]	[-1.04 ; 1.21]	**0.0**
	No	[-0.62 ; 0.64]	[-1.04 ; 1.21]	
cHWB	**Yes**	[-1.60 ; 1.68]	[-3.08 ; 3.50]	**0.0**
	No	[-1.60 ; 1.68]	[-3.08 ; 3.50]	
cHDD	**Yes**	[-1.72 ; 1.83]	[-3.18 ; 3.88]	**0.0**
	No	[-1.72 ; 1.83]	[-3.18 ; 3.88]	
cHbox	**Yes**	[-3.15 ; 2.61]	[-5.73 ; 4.52]	**0.0**
	No	[-3.15 ; 2.61]	[-5.73 ; 4.52]	

Clipping method in LHC analyses

Theoretical unitarity bounds (arXiv:2004.05174) for a given limit value are calculated from partial-wave unitarity constraints.

- **T_0 and T_1 amplitude matrices** in particle and parameter space as a function of the WCs for dim8 operators;
- Matrices formed with the s-divergent parts of the amplitudes corresponding to all combinations of *Z*, *W* and *H* pairs with a given total charge Q = 2, 1, 0 with possible projections on a given partial wave J.

Example for $(Q, J) = (2{,}0)$, corresponding to $(W_+^+W_+^+, W_0^+W_0^+, W_-^+W_-^+)$ basis

$$\frac{1}{96\pi}\frac{s^2}{\Lambda^4}\begin{pmatrix} 6f_{T_1}+3f_{T_2}+3f_{T_3} & 0 & 4f_{T_0}+8f_{T_1}+f_{T_2}+3f_{T_3} \\ 0 & 3f_{S_0}+f_{S_1}+f_{S_2} & 0 \\ 4f_{T_0}+8f_{T_1}+f_{T_2}+3f_{T_3} & 0 & 6f_{T_1}+3f_{T_2}+3f_{T_3} \end{pmatrix}$$

$$\left|\frac{2f_{T_0}+7f_{T_1}+2f_{T_2}+3f_{T_3}}{\Lambda^4}s^2\right| \leq 48\pi$$

$$\left|\frac{2f_{T_0}+f_{T_1}-f_{T_2}}{\Lambda^4}s^2\right| \leq 48\pi$$

$$\left|\frac{3f_{S_0}+f_{S_1}+f_{S_2}}{\Lambda^4}s^2\right| \leq 96\pi$$

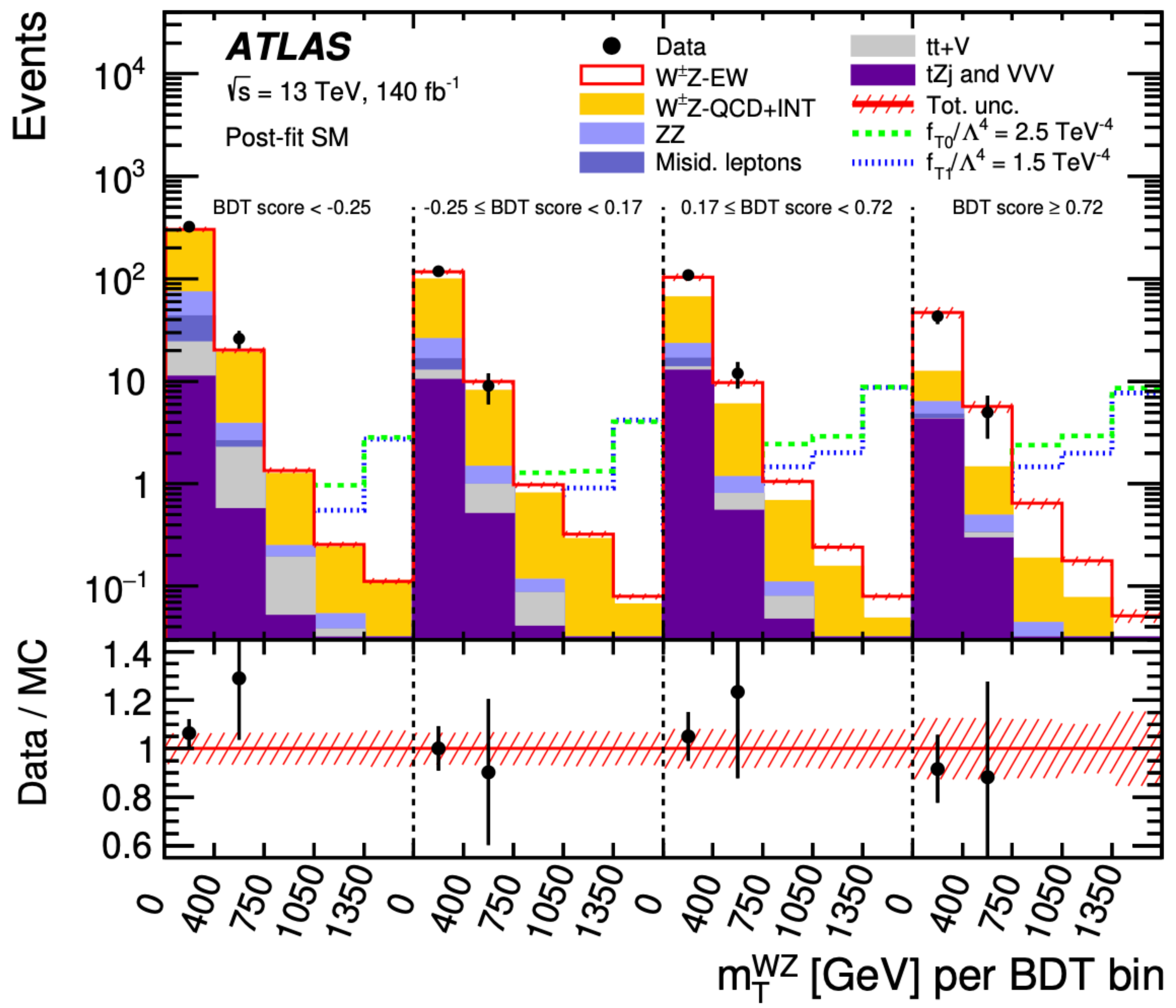
Strongest unitarity bounds come from the eigenvalues of the matrix

Unitarity constraints for S-type dim8 operators

Wilson Coefficient	Bound: 1 operator		Bound: all 3 operators	
		For $\sqrt{s} < 1.5\,(3)$ TeV		For $\sqrt{s} < 1.5\,(3)$ TeV
$\left\|\frac{f_{S,0}}{\Lambda^4}\right\|$	$32\,\pi\,s^{-2}$	$20\ (1.2)\ \text{TeV}^{-4}$	$48\,\pi\,s^{-2}$	$30\ (1.9)\ \text{TeV}^{-4}$
$\left\|\frac{f_{S,1}}{\Lambda^4}\right\|$	$\frac{96}{7}\,\pi\,s^{-2}$	$8.5\ (0.53)\ \text{TeV}^{-4}$	$\frac{288\,\pi}{5}\,s^{-2}$	$35\ (2.2)\ \text{TeV}^{-4}$
$\left\|\frac{f_{S,2}}{\Lambda^4}\right\|$	$\frac{96}{5}\,\pi\,s^{-2}$	$8.5\ (0.53)\ \text{TeV}^{-4}$	$\frac{288\,\pi}{5}\,s^{-2}$	$35\ (2.2)\ \text{TeV}^{-4}$

ATLAS side: a VBS WZ analysis

arXiv:2403.15296 [hep-ex]



Clipping technique:

Introduced to preserve unitarity at high energy by setting the EFT prediction to zero when the energy of the process is above a given clipping energy (E_C). EFT limits are derived for a scan of different clipping energies.

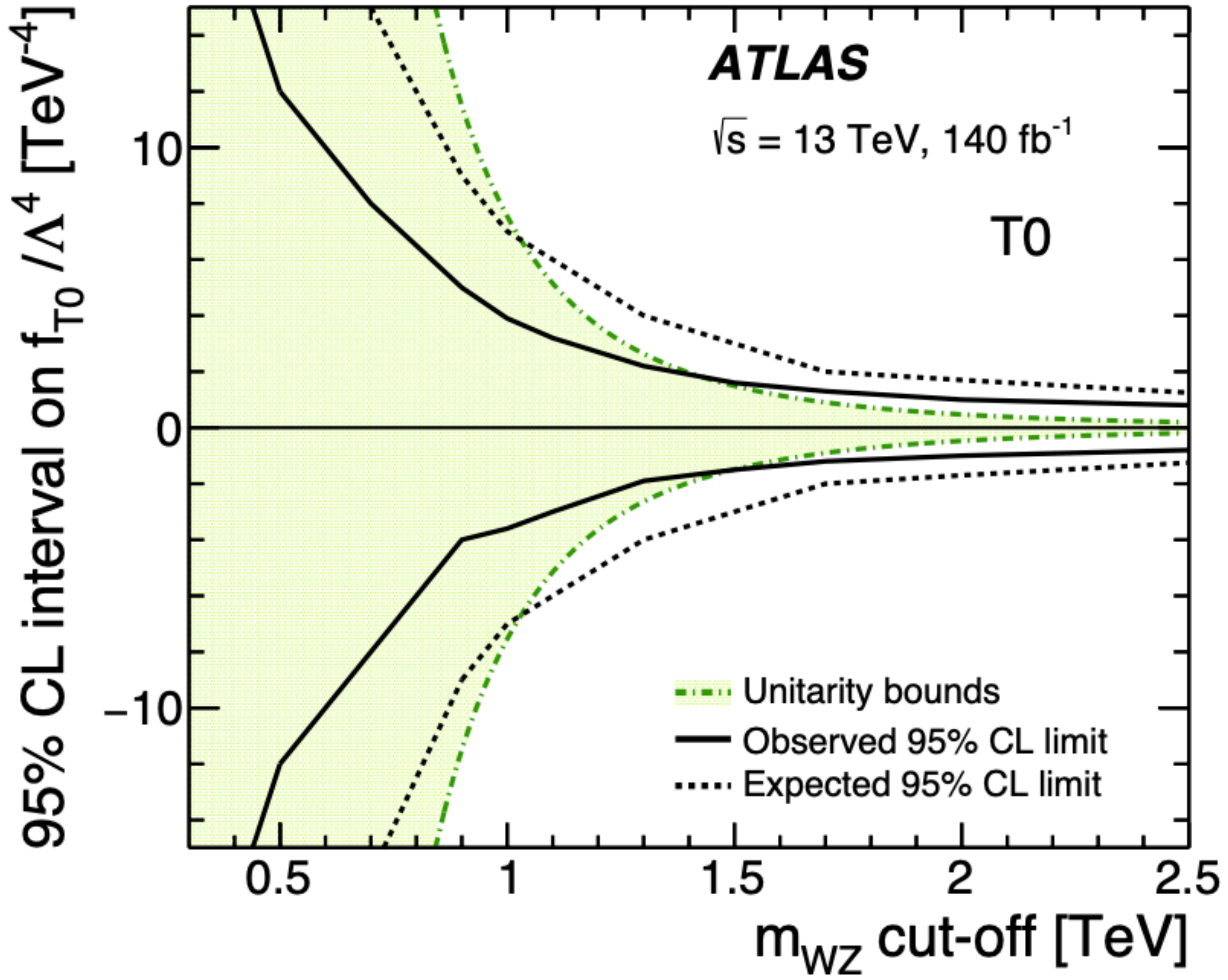
The unitarized limit is chosen given the intersection of the clipping scan and the unitarity bounds.

Before

	Expected [TeV $^{-4}$]	Observed [TeV $^{-4}$]
f_{T0}/Λ^4	[-0.80, 0.80]	[-0.57, 0.56]
f_{T1}/Λ^4	[-0.52, 0.49]	[-0.39, 0.35]
f_{T2}/Λ^4	[-1.6, 1.4]	[-1.2, 1.0]
f_{M0}/Λ^4	[-8.3, 8.3]	[-5.8, 5.6]
f_{M1}/Λ^4	[-12.3, 12.2]	[-8.6, 8.5]
f_{M7}/Λ^4	[-16.2, 16.2]	[-11.3, 11.3]
f_{S02}/Λ^4	[-14.2, 14.2]	[-10.4, 10.4]
f_{S1}/Λ^4	[-42, 41]	[-30, 30]

After

	Expected [TeV $^{-4}$]	Observed [TeV $^{-4}$]
f_{T0}/Λ^4	[-7.0, 7.0]	[-1.5, 1.6]
f_{T1}/Λ^4	[-1.1, 1.0]	[-0.7, 0.6]
f_{T2}/Λ^4	[-12, 6]	[-2.4, 1.8]
f_{M0}/Λ^4	[-60, 60]	[-12, 12]
f_{M1}/Λ^4	[-32, 32]	[-15, 15]
f_{M7}/Λ^4	[-30, 30]	[-15, 15]
f_{S02}/Λ^4	[-41, 41]	[-18, 18]
f_{S1}/Λ^4	—	—

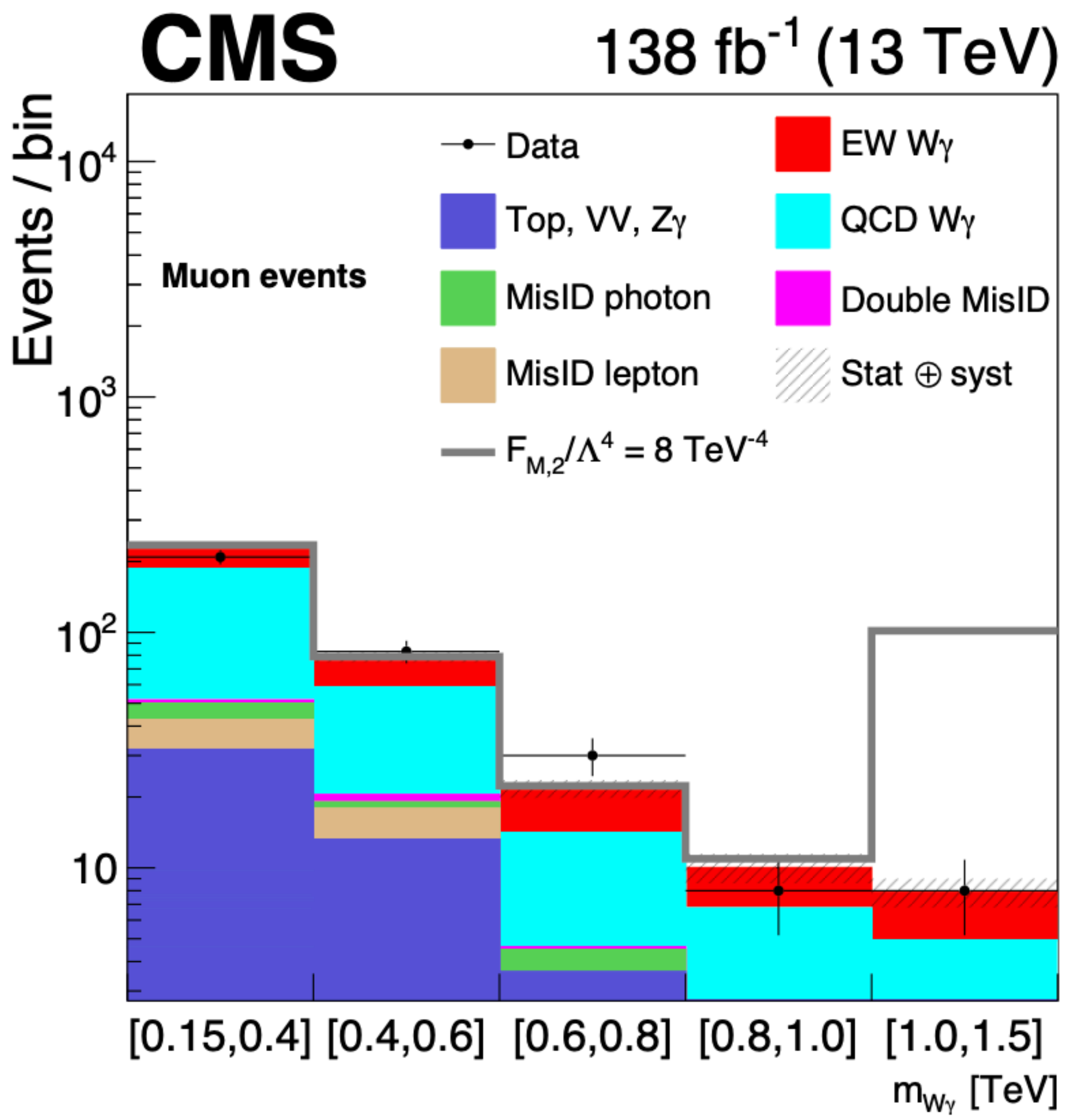


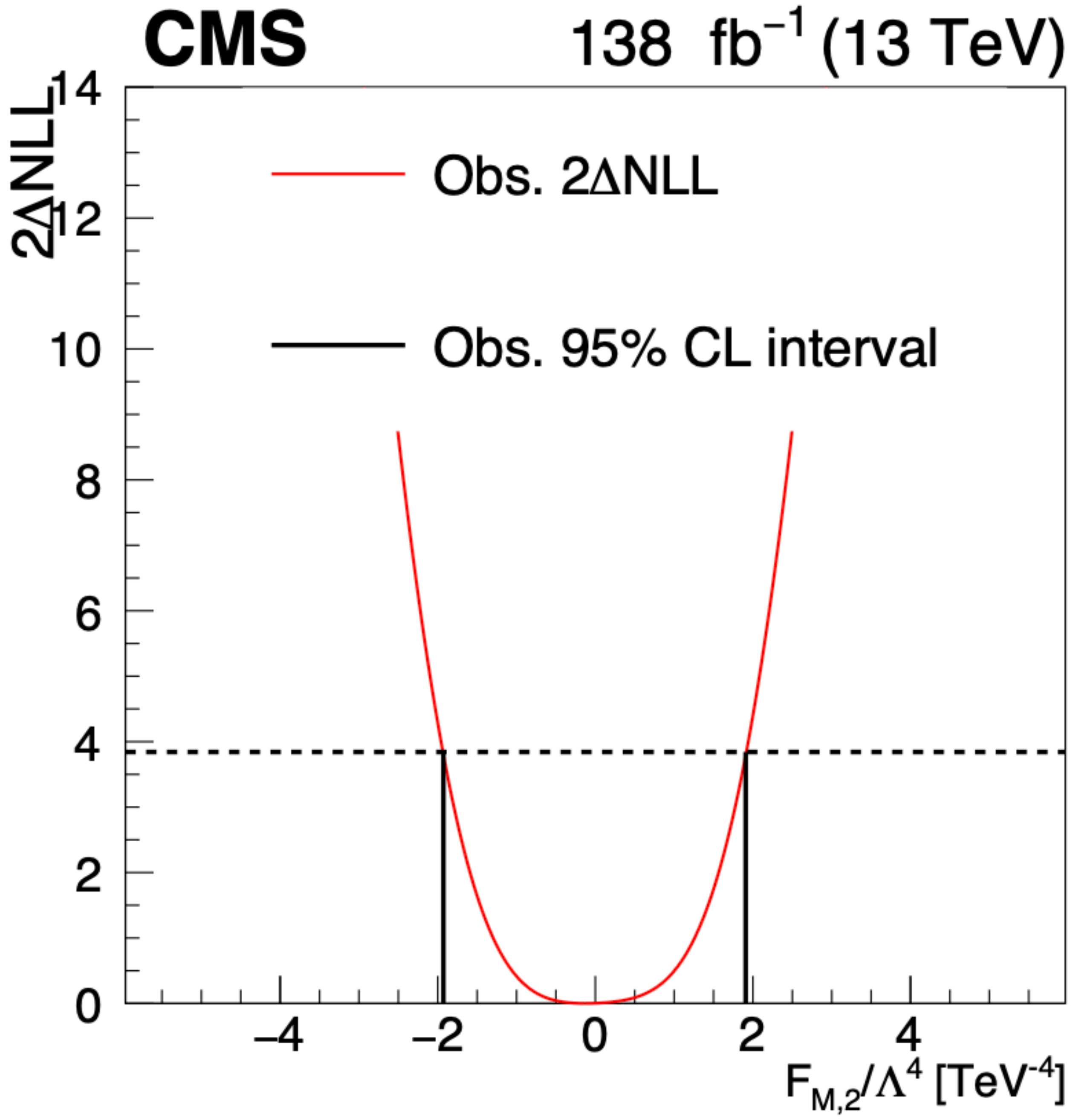
CMS side: a VBS $W\gamma$ analysis

arXiv:2212.12592 [hep-ex]

The unitarity bound (U_{bound}) is defined as the scattering energy at which the aQGC coupling strength, when set equal to the observed limit, would result in a scattering amplitude that violates unitarity.

Expected limit	Observed limit	U_{bound}
$-5.1 < f_{M,0}/\Lambda^4 < 5.1$	$-5.6 < f_{M,0}/\Lambda^4 < 5.5$	1.7
$-7.1 < f_{M,1}/\Lambda^4 < 7.4$	$-7.8 < f_{M,1}/\Lambda^4 < 8.1$	2.1
$-1.8 < f_{M,2}/\Lambda^4 < 1.8$	$-1.9 < f_{M,2}/\Lambda^4 < 1.9$	2.0
$-2.5 < f_{M,3}/\Lambda^4 < 2.5$	$-2.7 < f_{M,3}/\Lambda^4 < 2.7$	2.7
$-3.3 < f_{M,4}/\Lambda^4 < 3.3$	$-3.7 < f_{M,4}/\Lambda^4 < 3.6$	2.3
$-3.4 < f_{M,5}/\Lambda^4 < 3.6$	$-3.9 < f_{M,5}/\Lambda^4 < 3.9$	2.7
$-13 < f_{M,7}/\Lambda^4 < 13$	$-14 < f_{M7}/\Lambda^4 < 14$	2.2
$-0.43 < f_{T,0}/\Lambda^4 < 0.51$	$-0.47 < f_{T,0}/\Lambda^4 < 0.51$	1.9
$-0.27 < f_{T,1}/\Lambda^4 < 0.31$	$-0.31 < f_{T,1}/\Lambda^4 < 0.34$	2.5
$-0.72 < f_{T,2}/\Lambda^4 < 0.92$	$-0.85 < f_{T,2}/\Lambda^4 < 1.0$	2.3
$-0.29 < f_{T,5}/\Lambda^4 < 0.31$	$-0.31 < f_{T,5}/\Lambda^4 < 0.33$	2.6
$-0.23 < f_{T,6}/\Lambda^4 < 0.25$	$-0.25 < f_{T,6}/\Lambda^4 < 0.27$	2.9
$-0.60 < f_{T,7}/\Lambda^4 < 0.68$	$-0.67 < f_{T,7}/\Lambda^4 < 0.73$	3.1

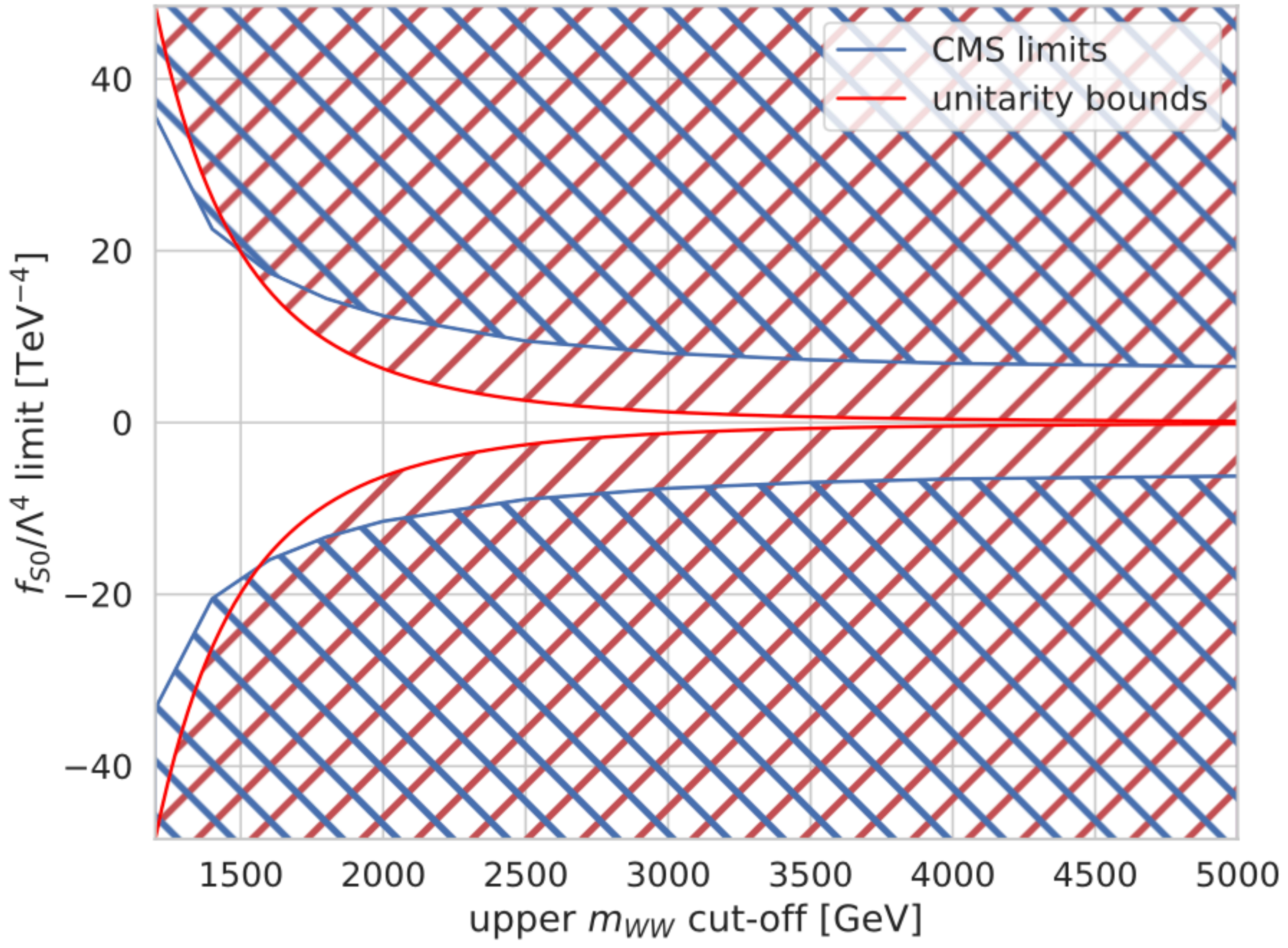


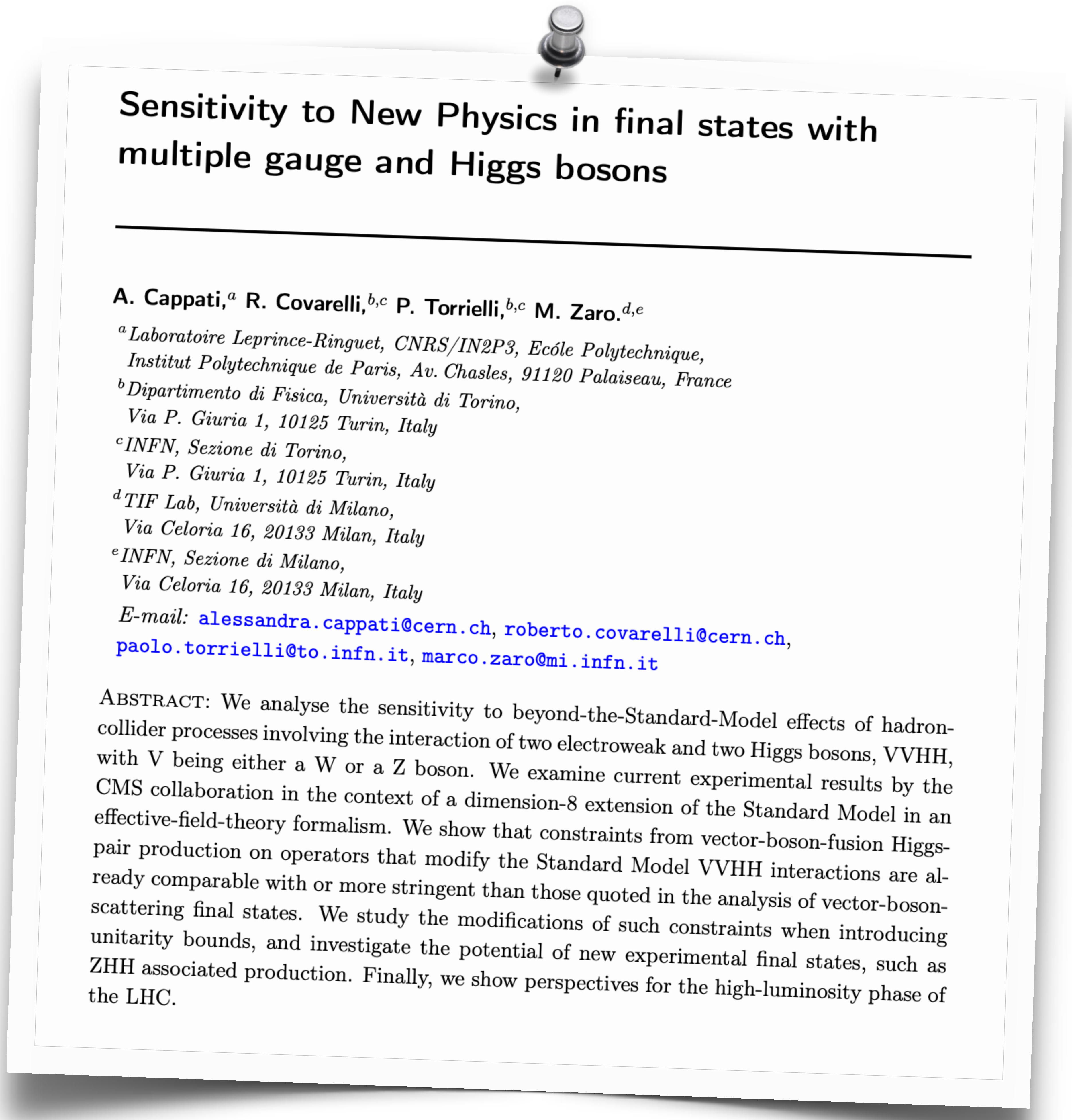


Unitarity in multiboson processes in EFT@dim8

Phenomenological study of Multiboson production (VVHH, ZHH) in **arXiv:2205.15959 [hep-ph]**:

- Derivation of constraints on dim8 Wilson coefficients in VVHH and ZHH processes;
- Application of unitarity bounds with a comparison between theoretical curves and measured CIs in different mass ranges;
- Truncation of range of the fit, not of the signal: **solves the problem of unphysical shape!**





Sensitivity to New Physics in final states with multiple gauge and Higgs bosons

A. Cappati,[a] R. Covarelli,[b,c] P. Torrielli,[b,c] M. Zaro.[d,e]

[a] *Laboratoire Leprince-Ringuet, CNRS/IN2P3, Ecóle Polytechnique, Institut Polytechnique de Paris, Av. Chasles, 91120 Palaiseau, France*
[b] *Dipartimento di Fisica, Università di Torino, Via P. Giuria 1, 10125 Turin, Italy*
[c] *INFN, Sezione di Torino, Via P. Giuria 1, 10125 Turin, Italy*
[d] *TIF Lab, Università di Milano, Via Celoria 16, 20133 Milan, Italy*
[e] *INFN, Sezione di Milano, Via Celoria 16, 20133 Milan, Italy*

E-mail: alessandra.cappati@cern.ch, roberto.covarelli@cern.ch, paolo.torrielli@to.infn.it, marco.zaro@mi.infn.it

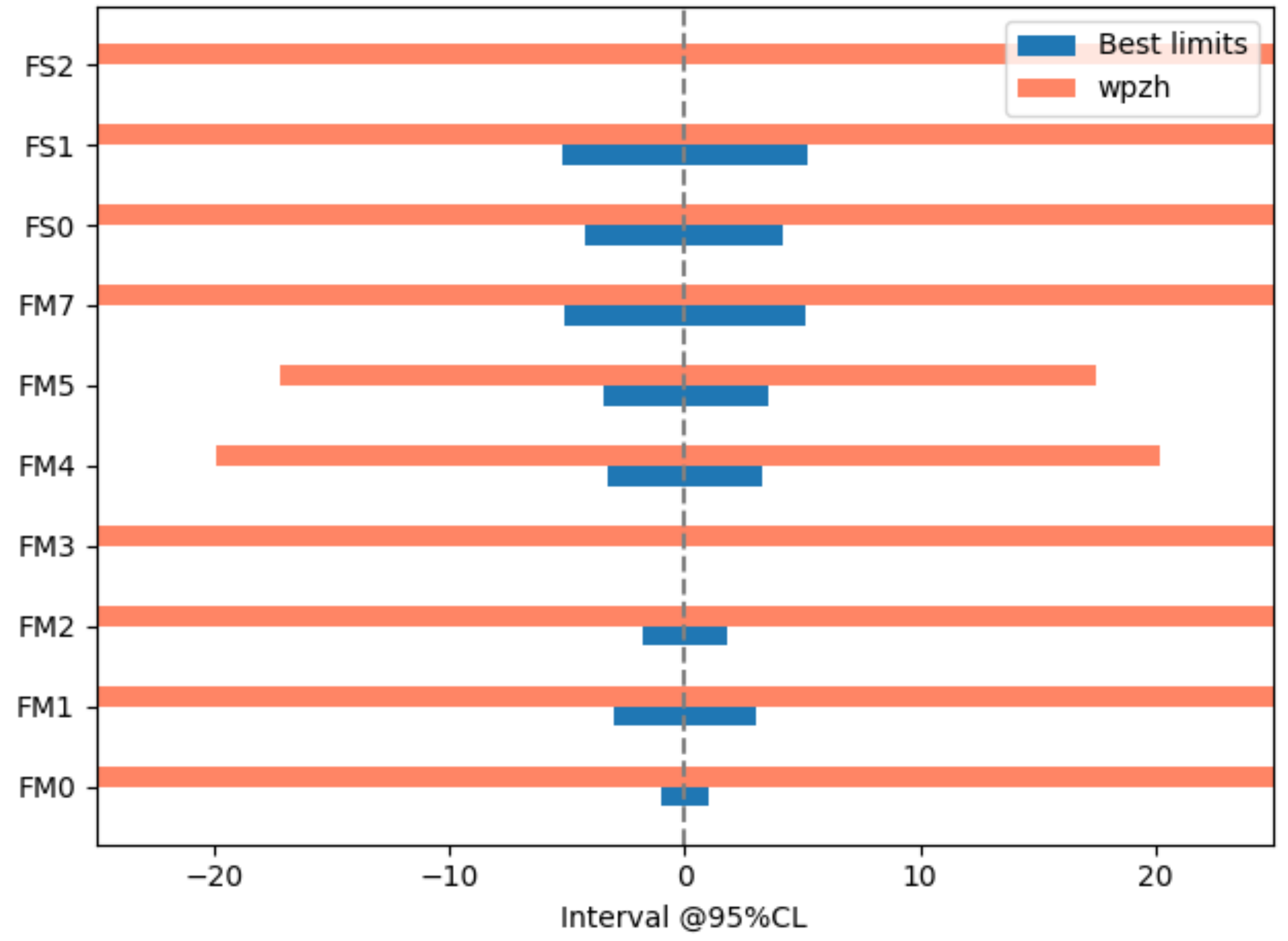
Abstract: We analyse the sensitivity to beyond-the-Standard-Model effects of hadron-collider processes involving the interaction of two electroweak and two Higgs bosons, VVHH, with V being either a W or a Z boson. We examine current experimental results by the CMS collaboration in the context of a dimension-8 extension of the Standard Model in an effective-field-theory formalism. We show that constraints from vector-boson-fusion Higgs-pair production on operators that modify the Standard Model VVHH interactions are already comparable with or more stringent than those quoted in the analysis of vector-boson-scattering final states. We study the modifications of such constraints when introducing unitarity bounds, and investigate the potential of new experimental final states, such as ZHH associated production. Finally, we show perspectives for the high-luminosity phase of the LHC.

Unitarity in multiboson processes in EFT@dim8

Collaboration with paper's authors for optimization and upgrade of the machinery for limits extraction with unitarity bounds.

- Update of pre-existing machinery;
- Organization in a user-friendly framework that allows automatic plotting, extraction of limits w/ and w/o unitarity and comparison with best limits to date on dim8 EFT parameters (from CMS analyses).

The framework was tested on few processes (WZH, ZZHH, ZHjj, WHjj), but some issues arised.



Ongoing work

In each public analysis from CMS ad ATLAS, theoretical curves come from Eboli's paper, derived using optical theorem for 2 to 2 scattering and are process independent.

✦ How could we expand those results to multiboson processes with 3+ **bosons in the final state**?

✦ Some (Q,J) bases include different final states, risk to reject relevant events due to **too tight constraints**.

Experimental analyses could benefit of an answer to those questions, providing physically meaningful EFT interpretations.

Goals:

Finding a unique prescription for unitarity implementation in experimental analyses;

Providing a user-friendly framework to easily compare theoretical and experimental curves and to estimate the sensitivity to EFT contributions for a given process.

Summary

- Unitarity problem is intrinsic in Effective Field Theories studies and it should be addressed in experimental analyses to mantain physical sense of the approach;
- Both ATLAS and CMS collaborations are interested in finding a common prescription for unitarity implementation;
- There's a debate on the validity of clipping method, since it uses an unphysical shape for the EFT signal;
- Work is ongoing to determine the limits of application of Eboli's paper results, to improve the implementation of clipping method and to make sensitivity studies more immediate before performing full-sim analysis.

Backup

Clipping method in LHC analyses

(Q, J)	States	Total
$(2,0)$	$W^+_\pm W^+_\pm$ $\quad W^+_0 W^+_0$	3
$(2,1)$	$W^+_\pm W^+_0$ $\quad W^+_0 W^+_\pm$	4
$(1,0)$	$W^+_\pm Z_\pm$ $\quad W^+_0 Z_0$ $\quad W^+_\pm \gamma_\pm$ $\quad W^+_0 H$	6
$(1,1)$	$W^+_0 Z_0$ $\quad W^+_\pm Z_0$ $\quad W^+_0 Z_\pm$ $\quad W^+_\pm Z_\pm$ $\quad W^+_0 \gamma_\pm$ $\quad W^+_\pm \gamma_\pm$ $\quad W^+_0 H$ $\quad W^+_\pm H$	14
$(0,0)$	$W^+_\pm W^-_\pm$ $\quad W^+_0 W^-_0$ $\quad Z_\pm Z_\pm$ $\quad Z_0 Z_0$ $\quad Z_\pm \gamma_\pm$ $\quad \gamma_\pm \gamma_\pm$ $\quad Z_0 H$ $\quad H H$	12
$(0,1)$	$W^+_0 W^-_0$ $\quad W^+_\pm W^-_0$ $\quad W^+_0 W^-_\pm$ $\quad W^+_\pm W^-_\pm$ $\quad Z_\pm Z_0$ $\quad Z_0 Z_\pm$ $\quad Z_\pm \gamma_\pm$ $\quad Z_0 \gamma_\pm$ $\quad Z_0 H$ $\quad Z_\pm H$ $\quad \gamma_\pm H$	20