Overview on CKM matrix status and on current anomalies in flavour data

Ludovico Vittorio (Sapienza Università di Roma & INFN, Sezione di Roma) ENP collaboration meeting – 19 May 2025







Istituto Nazionale di Fisica Nucleare



Introduction

The Standard Model (SM) describes very well many microscopic phenomena that we observe in Nature: plenty of theoretical predictions of SM parameters and observables have been found to agree with measurements !

•

Introduction

The Standard Model (SM) describes very well many microscopic phenomena that we observe in Nature: plenty of theoretical predictions of SM parameters and observables have been found to agree with measurements !

We can consider the SM as an Effective Field Theory (EFT) valid at low energies!



Which is the impact of the Higgs on flavor ?

Flavour blindness of the SM gauge sector:

 $\mathscr{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_\ell$

through the Lagr. $YH\bar\psi\psi$ term



Effect of the Yukawas: $\mathscr{G}_F(SM) = U(1)_B \times U(1)_L$

Which is the impact of the Higgs on flavor ?

Flavour blindness of the SM gauge sector:

 $\mathcal{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$



Effect of the Yukawas: $\mathscr{G}_F(SM) = U(1)_B \times U(1)_L$

Some important questions to be answered:

 Why three generations?
What determines the observed pattern of quark and lepton masses?
Why this *hierarchical* structure?



2

Which is the impact of the Higgs on flavor ?

Flavour blindness of the SM gauge sector:

 $\mathscr{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_\ell$



Effect of the Yukawas: $\mathscr{G}_F(SM) = U(1)_B \times U(1)_L$

Some important questions to be answered:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The CKM matrix describes the quark mixing, its elements can be determined <u>only</u> through a direct comparison with experimental data

Which is the impact of the Higgs on flavor?

Flavour blindness of the SM gauge sector:

 $\mathscr{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3$



Effect of the Yukawas:

Some important questions to be answered:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The CKM matrix describes the quark mixing, its elements can be determined only through a direct comparison with experimental data

 $\left.\begin{array}{c} 0.003715(93)\,e^{-i(65.1(1.3))^o}\\ 0.0420(5)\end{array}\right)$ $V_{\rm CKM} = \begin{pmatrix} 0.97431(19) & 0.22517(81) & 0.0\\ -0.22503(83) e^{+i(0.0351(1))^{\circ}} & 0.97345(20) e^{-i(0.00187(5))^{\circ}} \\ 0.00859(11) e^{-i(22.4(7))^{\circ}} & -0.04128(46) e^{+i(1.05(3))^{\circ}} \end{pmatrix}$ 0.999111(20)

UTfit Collaboration, Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894]

Which is the impact of the Higgs on flavor?

Flavour blindness of the SM gauge sector:

 $\mathscr{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_\ell$



Effect of the Yukawas:

 $\mathscr{G}_F(\mathrm{SM}) = U(1)_B \times U(1)_L$

Some important questions to be answered:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The CKM matrix describes the quark mixing, its elements can be determined <u>only</u> through a direct comparison with experimental data

 $V_{\rm CKM} = \begin{pmatrix} 0.97431(19) & 0.22517(81) & 0.003715(93) e^{-i(65.1(1.3))^{\circ}} \\ -0.22503(83) e^{+i(0.0351(1))^{\circ}} & 0.97345(20) e^{-i(0.00187(5))^{\circ}} & 0.0420(5) \\ 0.00859(11) e^{-i(22.4(7))^{\circ}} & -0.04128(46) e^{+i(1.05(3))^{\circ}} & 0.999111(20) \end{pmatrix}$

UTfit Collaboration, Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894]

4. Very close to the unit matrix: where does this hierarchical structure (again) come from?

3

Which is the impact of the Higgs on flavor?



Wolfenstein parametrization (L. Wolfenstein, PRL 51 (1983) 1945-1947):

$$V_{\rm CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parametrization (L. Wolfenstein, PRL 51 (1983) 1945-1947):

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

We can write the unitarity of the CKM matrix as
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

Triangle in the
 $(\bar{\rho}, \bar{\eta})$ plane !

Wolfenstein parametrization (L. Wolfenstein, PRL 51 (1983) 1945-1947):

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

We can write the unitarity of the CKM matrix as
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

Triangle in the
 $(\bar{\rho}, \bar{\eta})$ plane !

$$-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} - \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = R_b e^{i\gamma} + R_t e^{-i\beta} = 1 \simeq (\bar{\rho} + i\bar{\eta}) + (1 - \bar{\rho} - i\bar{\eta})$$



$$-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} - \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = R_b e^{i\gamma} + R_t e^{-i\beta} = 1 \simeq (\bar{\rho} + i\bar{\eta}) + (1 - \bar{\rho} - i\bar{\eta})$$



<u>KEY INFORMATION</u>:

the sides, the angles and the area of the triangle correspond to physical quantities ! E.g.:



$$-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} - \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = R_b e^{i\gamma} + R_t e^{-i\beta} = 1 \simeq (\bar{\rho} + i\bar{\eta}) + (1 - \bar{\rho} - i\bar{\eta})$$

Rare decays as a (second) guide

The most apppealing places to look for possible NP effects are rare decays, since <u>rare decays</u> are a <u>manifestation of broken (accidental) symmetries</u>, *e.g.* of physics beyond the Standard Model

Some general examples:

1. Proton decay



Test of baryon and lepton number conservation

Rare decays as a (second) guide

The most apppealing places to look for possible NP effects are rare decays, since <u>rare decays</u> are a <u>manifestation of broken (accidental) symmetries</u>, *e.g.* of physics beyond the Standard Model

Some general examples:

1. Proton decay

Test of baryon and lepton number conservation

2. Flavour Changing Neutral Currents (FCNCs): absent @ tree level & CKM-suppressed

$$\begin{array}{c|cccc} \textit{Of interest} & q_i & \to & q_j \ell^+ \ell^-, \\ \textit{for this} & q_i & \to & q_j \nu \bar{\nu}, \\ \textit{talk !} & q_i & \to & q_j \gamma \end{array}$$

Rare decays as a (second) guide

The most apppealing places to look for possible NP effects are rare decays, since <u>rare decays</u> are a <u>manifestation of broken (accidental) symmetries</u>, *e.g.* of physics beyond the Standard Model

Some general examples:

1. Proton decay

Test of baryon and lepton number conservation

2. Flavour Changing Neutral Currents (FCNCs): absent @ tree level & CKM-suppressed



Flavour physics within and beyond the SM **Tree-level FCNCs** processes

Flavour physics within and beyond the SM



Tree-level processes

$b \rightarrow c$ transitions



Many challenges in $b \rightarrow c$ decays at present

Although there is no direct evidence for New Physics from experiments, many phenomenological puzzles need a solution:

1. |Vcb| (e |Vub|) puzzle FLAG2024 4.5 $B \rightarrow D^* l v$ $B \rightarrow D \ell v$ inclusive $B \rightarrow \tau \nu$ 4 $4 \\ |V_{ub}| \times 10^{3}$ Tension: $-B_{5} + Klv$ $B_{5} + D_{5}lv$ high $-q^{2}$ 3σ \sim $B \rightarrow \pi \ell \nu$ 3 Nelv $B_s \rightarrow D_s^{(*)} \ell v$ 110-71 36 38 40 42 44 $|V_{cb}| \times 10^{3}$ FLAG Review 2024 [2411.04268]

Many challenges in $b \rightarrow c$ decays at present

Although there is no direct evidence for New Physics from experiments, many phenomenological puzzles need a solution:



New experimental data — $B \rightarrow D^* Iv$



A lot of new experimental data:

- four (normalized) differential decay rates
 - $\alpha = w, \cos \theta_{\ell}, \cos \theta_{v}, \chi$
- three experimental datasets by Belle/BelleII



 $\begin{aligned} \frac{d^4\Gamma}{dwd\cos\theta_v d\cos\theta_\ell d\chi} &= \frac{3}{16\pi} \Gamma_0 \sqrt{w^2 - 1} (1 - 2rw + r^2) \Big\{ H_+^2(w) \, \sin^2\theta_v \, (1 - \cos\theta_\ell)^2 \\ &+ H_-^2(w) \, \sin^2\theta_v \, (1 + \cos\theta_\ell)^2 + 4 \, H_0^2(w) \, \cos^2\theta_v \, \sin^2\theta_\ell \\ &- 2 \, H_-(w) H_+(w) \, \sin^2\theta_v \, \sin^2\theta_\ell \, \cos^2\chi \\ &- 2 \, H_+(w) H_0(w) \, \sin^2\theta_v \, \sin^2\theta_\ell \, (1 - \cos\theta_\ell) \, \cos\chi \\ &+ 2 \, H_-(w) H_0(w) \, \sin^2\theta_v \, \sin\theta_\ell \, (1 + \cos\theta_\ell) \, \cos\chi \Big\} , \\ &- \Big\{ \Gamma_0 \equiv \frac{\eta_{EW}^2 m_B m_{D^*}^2}{(4\pi)^3} G_F^2 |V_{cb}|^2 \Big\} \end{aligned}$

New experimental data — $B \rightarrow D^* Iv$



A lot of new experimental data:

- four (normalized) differential decay rates
 - $\alpha = w, \cos \theta_{\ell}, \cos \theta_{v}, \chi$
- three experimental datasets by Belle/BelleII



$$\begin{aligned} \frac{d^4\Gamma}{\cos\theta_v d\cos\theta_\ell d\chi} &= \frac{3}{16\pi} \Gamma_0 \sqrt{w^2 - 1} (1 - 2rw + r^2) \Big\{ H_+^2(w) \, \sin^2\theta_v \, (1 - \cos\theta_\ell)^2 \\ &+ H_-^2(w) \, \sin^2\theta_v \, (1 + \cos\theta_\ell)^2 + 4 \, H_0^2(w) \, \cos^2\theta_v \, \sin^2\theta_\ell \\ &- 2 \, H_-(w) H_+(w) \, \sin^2\theta_v \, \sin^2\theta_\ell \, \cos^2\chi \\ &- 2 \, H_+(w) H_0(w) \, \sin^2\theta_v \, \sin^2\theta_\ell \, (1 - \cos\theta_\ell) \, \cos\chi \\ &+ 2 \, H_-(w) H_0(w) \, \sin^2\theta_v \, \sin\theta_\ell \, (1 + \cos\theta_\ell) \, \cos\chi \Big\} , \\ &- \Big\{ \Gamma_0 \equiv \frac{\eta_{EW}^2 m_B m_{D^*}^2}{(4\pi)^3} G_F^2 |V_{cb}|^2 \Big\} \end{aligned}$$

How to best analyze this new quality of data as part of a precision test of the SM ?

Shapes of the FFs starting from lattice data



Shapes of the FFs starting from lattice data



2. Some differences exist among the extrapolated values of F_{1.2}(w) from JLQCD and from FNAL/MILC L. Vittorio (Sapienza U. of Rome & INFN)

These bands alone (w/out the usage of exp. data) are sufficient for some phenomenological applications:

• R(D*)

Lattice FFs	$R(D^*)$
FNAL/MILC [15]	0.275(8)
HPQCD[16]	0.266(12)
JLQCD [17]	0.247(8)
Average [15]-[17]	0.262(9)
(PDG scale factor)	(1.8)
Combined [15]-[17]	0.259(5)
Experimental value	0.284(12) [36]

These bands alone (w/out the usage of exp. data) are sufficient for some phenomenological applications:

• R(D*)

Lattice FFs	$R(D^*)$	
FNAL/MILC [15]	0.275(8)	
HPQCD [16]	0.266(12)	
JLQCD [17]	0.247(8)	
Average [15]-[17]	0.262(9)	
(PDG scale factor)	(1.8)	
Combined [15]-[17]	0.259(5)	
Experimental value	0.284(12) [36]	
1.5σ compatibility		

These bands alone (w/out the usage of exp. data) are sufficient for some phenomenological applications:

• R(D*)

Lattice FFs	$R(D^*)$	
FNAL/MILC [15]	0.275(8)	
HPQCD [16]	0.266(12)	
JLQCD [17]	0.247(8)	
Average [15]-[17]	0.262(9)	
(PDG scale factor)	(1.8)	
Combined [15]-[17]	0.259(5)	
Experimental value	0.284(12) [36]	
1.5 σ compatibility		

• Asymmetries and polarizations :



These bands alone (w/out the usage of exp. data) are sufficient for some phenomenological applications:

• R(D*)

Lattice FFs	$R(D^*)$	
$\mathrm{FNAL}/\mathrm{MILC}\left[15\right]$	0.275(8)	
HPQCD[16]	0.266(12)	
JLQCD [17]	0.247(8)	
Average [15]-[17]	0.262(9)	
(PDG scale factor)	(1.8)	
Combined [15]-[17]	0.259(5)	
Experimental value	0.284(12) [36]	
1.5 σ compatibility		

• Asymmetries and polarizations :



Martinelli, Simula, LV, PRD 111 (2025) 1, 013005

Take-home message: there are tensions amongst results from different lattice collaborations, but there are non-negligible tensions amongst experiments as well !

$|V_{cb}|$ determination

Several strategies:

A) Bin-per-bin analysis $|V_{cb}|_{\alpha,i} = \left(\Gamma_{exp} \left[\frac{1}{\Gamma} \frac{d\Gamma}{d\alpha}\right]_{exp}^{(i)} / \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]_{lat}^{(i)}\right)^{1/2}$

$|V_{cb}|$ determination

Several strategies:

A) Bin-per-bin analysis $|V_{cb}|_{\alpha,i} = \left(\Gamma_{exp} \left[\frac{1}{\Gamma} \frac{d\Gamma}{d\alpha}\right]_{exp}^{(i)} / \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]_{lat}^{(i)}\right)^{1/2}$

- B) Latt.+exp. fits
- C) Exp. fit only

(determine |V_{cb}| a posteriori from the total decay width)

$|V_{cb}|$ determination



|V_{cb}| determination



Summary of the results obtained so far: |Vcb| (and |Vub|)



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}



Summary of the results obtained so far: |Vcb| (and |Vub|)



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}



Summary of the results obtained so far: |Vcb| (and |Vub|)


Summary of the results obtained so far: |Vcb| (and |Vub|)



Inclusive values: **Vub**: **PDG** '24 Vcb : Finauri, Gambino, JHEP '24 [2310.20324]

Summary of the results obtained so far: |Vcb| (and |Vub|)



Flavour physics within and beyond the SM



Tree-level processes

$b \rightarrow c$ transitions



Flavour physics within and beyond the SM



Tree-level

processes

$b \rightarrow c$ transitions

Vud e Vus



Violation of unitarity in the 1st row ?



M. Gorshteyn, talk @ CKM23 conference

Violation of unitarity in the 1st row ?



$$|V_{ud}|^{2} + |V_{us}|^{2} + |V_{ub}|^{2} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 ~ 10⁻⁵

Violation of unitarity in the 1st row ?



Vud e Vus determinations show that: 1) there are mutual inconsistencies 2) there is an important tension with what would be the outcome of the Unitarity **Triangle analysis**

$$|V_{ud}|^{2} + |V_{us}|^{2} + |V_{ub}|^{2} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 ~ 10⁻⁵

For what concerns | Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0⁺-0⁺ decays (es.: $^{14}O \rightarrow ^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

For what concerns | Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0^+-0^+ decays (es.: ${}^{14}O \rightarrow {}^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

- Neutron β -decay:

 $|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$

For what concerns | Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0⁺-0⁺ decays (es.: $^{14}O \rightarrow ^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

- Neutron β -decay:

 $|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$

- $\pi^+ \rightarrow \pi^0 e^+ v$ decay:

 $|V_{ud}^{\pi\ell3}| = 0.9739 \, (27)_{exp} \, (1)_{RC}$

For what concerns |Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0⁺-0⁺ decays (es.: $^{14}O \rightarrow ^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

- Neutron β -decay:

 $|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$

- $\pi^+ \rightarrow \pi^0 e^+ v$ decay:

$$|V_{ud}^{\pi\ell^3}| = 0.9739 (27)_{exp} (1)_{RC}$$

It is the cleanest determination (talking about theory) ! An important improvement of exp. precision is expected at PIONEER (**2203.01981**)

For what concerns | Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0^+-0^+ decays (es.: ${}^{14}O \rightarrow {}^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

- Neutron β -decay:

 $|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$

- $\pi^+ \rightarrow \pi^0 e^+ v$ decay:

$$|V_{ud}^{\pi\ell^3}| = 0.9739 (27)_{exp} (1)_{RC}$$

It is the cleanest determination (talking about theory) ! An important improvement of exp. precision is expected at PIONEER (**2203.01981**) For what concerns |Vus|:



ETMC Collaboration, PRL '24 [arXiv:2403.05404]

L. Vittorio (Sapienza U. of Rome & INFN)

For what concerns | Vud |:

M. Gorshteyn, talk @ CKM23 conference

- Super allowed 0^+-0^+ decays (es.: ${}^{14}O \rightarrow {}^{14}N$)

 $|V_{ud}^{0^+-0^+}| = 0.97370 (1)_{exp, nucl} (3)_{NS} (1)_{RC} [3]_{total}$

- Neutron β -decay:

 $|V_{ud}^{\text{free n}}| = 0.9733 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$

- $\pi^+ \rightarrow \pi^0 e^+ v$ decay:

$$|V_{ud}^{\pi\ell^3}| = 0.9739 (27)_{exp} (1)_{RC}$$

It is the cleanest determination (talking about theory) ! An important improvement of exp. precision is expected at PIONEER (**2203.01981**) For what concerns |Vus|:



ETMC Collaboration, PRL '24 [arXiv:2403.05404]

Considering as many channels as possible will be very helpful in the future!

L. Vittorio (Sapienza U. of Rome & INFN)

Flavour physics within and beyond the SM



Tree-level

processes

$b \rightarrow c$ transitions

Vud e Vus











L. Vittorio (Sapienza U. of Rome & INFN) M. Reboud's talk @ LHC Impilication Workshop 2022 @ CERN



L. Vittorio (Sapienza U. of Rome & INFN) M. Reboud's talk @ LHC Impilication Workshop 2022 @ CERN

No general consensus on the answer to this question:



No general consensus on the answer to this question:



The «hypothetical» presence of NP seems to depend on the assumptions made on hadronic effects !

No general consensus on the answer to this question:



The «hypothetical» presence of NP seems to depend on the assumptions made on hadronic effects !

Data-driven: naïve expansion in q²:

$$\begin{split} H_V^- \propto & \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\rm SM} + h_-^{(0)} \right) \widetilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4 \right] \\ &+ \left(C_9^{\rm SM} + h_-^{(1)} \right) \widetilde{V}_{L-} \,, \end{split}$$

Advantage: clear interplay among hadronic contributions and NP !

M. Ciuchini et al, JHEP '16 [1512.07157], EPJC '17 [1704.05447], EPJC '19 [1903.09632], PRD '21 [2011.01212], EPJC '23 [2110.10126], PRD '23 [2212.10516]

L. Vittorio (Sapienza U. of Rome & INFN)

No general consensus on the answer to this question:



The «hypothetical» presence of NP seems to depend on the assumptions made on hadronic effects !

Data-driven: naïve expansion in q²:

$$\begin{split} H_V^- \propto & \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\rm SM} + h_-^{(0)} \right) \widetilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4 \right] \\ &+ \left(C_9^{\rm SM} + h_-^{(1)} \right) \widetilde{V}_{L-} \,, \end{split}$$

Advantage: clear interplay among hadronic contributions and NP !

M. Ciuchini et al, JHEP '16 [1512.07157], EPJC '17 [1704.05447], EPJC '19 [1903.09632], PRD '21 [2011.01212], EPJC '23 [2110.10126], PRD '23 [2212.10516]

L. Vittorio (Sapienza U. of Rome & INFN)

<u>Model-depedent</u>: the *h*-terms $[h_{(0)}, h_{(1)}, h_{(2)}]$ are set to zero.

Motivation: this assumption is supported by the results of the application of dispersion relations, analiticity and unitarity (together with LCSR data) to the description of non-local FFs!

C. Bobeth et al, EPJC '18 [1707.07305] M. Chrzaszcz et al, JHEP '19 [1805.06378] N. Gubernari et al, JHEP '21 [2011.09813], JHEP '22 [2206.03797], 2305.06301

Possible prospects for the future

i) Developments of new strategies to apply unitarity constraints on re-scattering diagrams (absent in present dispersive studies):



Possible prospects for the future

i) Developments of new strategies to apply unitarity constraints on re-scattering diagrams (absent in present dispersive studies):



Gopal, Gubernari, PRD '25 [2412.04388] (See also Mutke et al., JHEP '24 [2406.14608])

Possible prospects for the future

i) Developments of new strategies to apply unitarity constraints on re-scattering diagrams (absent in present dispersive studies):



Gopal, Gubernari, PRD '25 [2412.04388] (See also Mutke et al., JHEP '24 [2406.14608])

ii) Look at novel channels which should be affected by the same (hypothetical) NP effects: Bs $\rightarrow \mu\mu\gamma$ @ high-q2

Guadagnoli, et al., JHEP '23 [2303.02174, 2308.00034] ETMC Collaboration, PRD '24 [2402.03262] LHCb Collaboration, JHEP '24 [2404.03375]

L. Vittorio (Sapienza U. of Rome & INFN)





Rare semileptonic decays with di-neutrino final states

i) $B \rightarrow K(*)vv$: long-distance (LD) effects are much smaller than in $B \rightarrow K(*)\ell^+\ell^-$!





Rare semileptonic decays with di-neutrino final states i) $B \rightarrow K(*)vv$: long-distance (LD) effects are much smaller than in $B \rightarrow K(*)\ell^+\ell^-$!

Final prediction in the SM:

Major sources of uncertainty:

1. |Vcb| value (related to the |Vcb| puzzle)

2. Hadronic effects (i.e. FFs)

$$\mathcal{B}\left(B^{\pm} \to K^{\pm} \nu \bar{\nu}\right) = (4.44 \pm 0.30) \times 10^{-6}$$

D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]

to be compared with

$$\mathscr{B}\left(B^+ \to K^+ \nu \bar{\nu}\right)\Big|_{\text{Belle}-\text{II}} = (2.4 \pm 0.7) \times 10^{-5}$$

Belle-II Collaboration, PRD '24 [2311.14647]

Rare semileptonic decays with di-neutrino final states i) $B \rightarrow K(*)vv$: long-distance (LD) effects are much smaller than in $B \rightarrow K(*)\ell^+\ell^-$!

Final prediction in the SM:

1. |Vcb| value (related to the |Vcb| puzzle)

2. Hadronic effects (i.e. FFs)

$$\mathcal{B}\left(B^{\pm} \to K^{\pm} \nu \bar{\nu}\right) = (4.44 \pm 0.30) \times 10^{-6}$$

D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]

to be compared with

$$\mathscr{B}\left(B^+ \to K^+ \nu \bar{\nu}\right)\Big|_{\text{Belle}-\text{II}} = (2.4 \pm 0.7) \times 10^{-5}$$

Tension in $B \rightarrow Kvv$: 2.8 σ

Belle-II Collaboration, PRD '24 [2311.14647]

Several NP studies:

Berezhnoy, Melikhov, EPL '24 [2309.17191] Marzocca et al, EPJC '24 [2404.06533] Altmannshofer, Roy, PRD '25 [2411.06592] Rare semileptonic decays with di-neutrino final states i) $B \rightarrow K(*)vv$: long-distance (LD) effects are much smaller than in $B \rightarrow K(*)\ell^+\ell^-$!

Final prediction in the SM:

Altmannshofer, Roy, PRD '25 [2411.06592]

Major sources of uncertainty:
1. |Vcb| value (related to the |Vcb| puzzle)
2. Hadronic effects (i.e. FFs)

$$\begin{aligned}
\mathscr{B} \left(B^{\pm} \to K^{\pm} \nu \bar{\nu} \right) &= (4.44 \pm 0.30) \times 10^{-6} \\
D. Becirevic, G. Piazza & O. Sumensari, EPJC (23 [arXiv:2301.06990] \\
to be compared with \\
\mathscr{B} \left(B^{+} \to K^{+} \nu \bar{\nu} \right) &= (2.4 \pm 0.7) \times 10^{-5} \\
Belle-II & Collaboration, PRD (24 [2311.14647] \\
\end{aligned}$$
Tension in $B \to K \nu \nu$:

$$\begin{aligned}
\mathsf{B} \left(B^{+} \to K^{+} \nu \bar{\nu} \right) &= (2.4 \pm 0.7) \times 10^{-5} \\
Belle-II & Collaboration, PRD (24 [2311.14647] \\
\end{aligned}$$

ii) $\mathbf{K} \rightarrow \pi v v$: see Giancarlo's presentation for all the details

L. Vittorio (Sapienza U. of Rome & INFN)

 2.8σ



Flavour physics within and beyond the SM



 $b \rightarrow c$ transitions

|Vud| e |Vus|



$b \rightarrow s \ell^+ \ell^ q_i \rightarrow q_j v v$...



Unitarity triangle as a way to improve precision



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}



$$\overline{\rho}$$
 = 0.158 ± 0.009
 $\overline{\eta}$ = 0.352 ± 0.010

$$\lambda = 0.2250 \pm 0.0007$$

A= 0.826 ± 0.009
Unitarity triangle as a way to improve precision



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}



$$\overline{\rho}$$
 = 0.158 ± 0.009
 $\overline{\eta}$ = 0.352 ± 0.010

$$\lambda = 0.2250 \pm 0.0007$$

A= 0.826 ± 0.009

Status of the UT analysis in the SM :

1. Overall consistency of the SM fit

2. Reached precision of 5% (3%) on $\overline{\rho}$ ($\overline{\eta}$)

Unitarity triangle as a way to improve precision



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}







Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

A way to "measure" the agreement of a single measurement with the indirect determination from the fit (using the other inputs):



- Colour code: agreement between the predicted values and the measurements at better than 1, 2, ... no

- The crosses have the coordinates (x,y)=(central value, error) of the direct measurements

L. Vittorio (Sapienza U. of Rome & INFN)

x = exclusive * = inclusive



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

A way to "measure" the agreement of a single measurement with the indirect determination from the fit (using the other inputs):





Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

A way to "measure" the agreement of a single measurement with the indirect determination from the fit (using the other inputs):



Novel analysis of beauty decays and of D-meson mixing data in 2409.06449 L. Vittorio (Sapienza U. of Rome & INFN)

Still some tensions in these two cases ...

x = exclusive * = inclusive



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

Observables	Measurement	Prediction	rediction Pull (#o)	
sin2β	0.692 ± 0.019	0.763 ± 0.030	~ 2	
У	67.2 ± 2.9	65.6 ± 1.4	< 1	
α	95 ± 8	91.4 ± 1.4	< 1	
V _{cb} · 10 ³	41.20 ± 0.74	42.19 ± 0.48	~ 1.1	
$ V_{cb} \cdot 10^3$ (excl)	40.13 ± 0.55		~ 2.8	
 V_{cb} · 10³ (incl)	41.97 ± 0.48		< 1	
V _{ub} • 10 ³	3.84 ± 0.35	3.72 ± 0.10	< 1	
 V _{ub} • 10 ³ (excl)	3.57 ± 0.23	-	< 1	
V ub • 10 ³ (incl)	4.13 ± 0.26	-	~ 1.4	
$BR(B \rightarrow \tau v)[10^4]$	1.09 ± 0.24	0.88 ± 0.05	< 1	

L. Vittorio (Sapienza U. of Rome & INFN)

Unitarity triangle beyond the SM



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq}$$

ΔF=2

$$\begin{aligned} Q_{1} &= (\bar{q}_{Li}\gamma^{\mu}q_{Lj})(\bar{q}_{Li}\gamma^{\mu}q_{Lj}), & Q_{1}' &= (\bar{q}_{Ri}\gamma^{\mu}q_{Rj})(\bar{q}_{Ri}\gamma^{\mu}q_{Rj}) \\ Q_{2} &= (\bar{q}_{Ri}q_{Lj})(\bar{q}_{Ri}q_{Lj}), & Q_{2}' &= (\bar{q}_{Li}q_{Rj})(\bar{q}_{Li}q_{Rj}) \\ Q_{3} &= (\bar{q}_{Ri}^{\alpha}q_{Lj}^{\beta})(\bar{q}_{Ri}^{\beta}q_{Lj}^{\alpha}), & Q_{3}' &= (\bar{q}_{Li}^{\alpha}q_{Rj}^{\beta})(\bar{q}_{Li}^{\beta}q_{Rj}^{\alpha}) \\ Q_{4} &= (\bar{q}_{Ri}q_{Lj})(\bar{q}_{Li}q_{Rj}), & Q_{5} &= (\bar{q}_{Ri}^{\alpha}q_{Lj}^{\beta})(\bar{q}_{Li}^{\beta}q_{Rj}^{\alpha}). \end{aligned}$$

L. Vittorio (Sapienza U. of Rome & INFN)

Unitarity triangle beyond the SM



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq}$$



NP couplings Loop factors NP scale

ΔF=2

$$Q_{1} = (\bar{q}_{Li}\gamma^{\mu}q_{Lj})(\bar{q}_{Li}\gamma^{\mu}q_{Lj}), \qquad Q_{1}' = (\bar{q}_{Ri}\gamma^{\mu}q_{Rj})(\bar{q}_{Ri}\gamma^{\mu}q_{Rj})$$

$$Q_{2} = (\bar{q}_{Ri}q_{Lj})(\bar{q}_{Ri}q_{Lj}), \qquad Q_{2}' = (\bar{q}_{Li}q_{Rj})(\bar{q}_{Li}q_{Rj})$$

$$Q_{3} = (\bar{q}_{Ri}^{\alpha}q_{Lj}^{\beta})(\bar{q}_{Ri}^{\beta}q_{Lj}^{\alpha}), \qquad Q_{3}' = (\bar{q}_{Li}^{\alpha}q_{Rj}^{\beta})(\bar{q}_{Li}^{\beta}q_{Rj}^{\alpha})$$

$$Q_{4} = (\bar{q}_{Ri}q_{Lj})(\bar{q}_{Li}q_{Rj}), \qquad Q_{5} = (\bar{q}_{Ri}^{\alpha}q_{Lj}^{\beta})(\bar{q}_{Li}^{\beta}q_{Rj}^{\alpha}).$$

Unitarity triangle beyond the SM



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8} Davide Morgante^{9,10} Maurizio Pierini¹¹ Luca Silvestrini⁶ Silvano Simula² Achille Stocchi¹² Cecilia Tarantino^{2,7} Vincenzo Vagnoni⁴ Mauro Valli⁶ and Ludovico Vittorio^{6,8}



for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_w (~ 0.03). L. Vittorio (Sapienza U. of Rome & INFN)

Incredible moment for flavour physics: at present, a lot of new data are available and, in the next years, several new ones are going to come out !

 Pheno studies within the SM require precision techniques to study new data: comparison among different techniques, fundamental inputs from lattice QCD → effort to corroborate our comprehension of EW physics

Incredible moment for flavour physics: at present, a lot of new data are available and, in the next years, several new ones are going to come out !

- Pheno studies within the SM require precision techniques to study new data: comparison among different techniques, fundamental inputs from lattice QCD → effort to corroborate our comprehension of EW physics
- Global analyses are a fundamental tool to put together all the information we have at present: in this context, the Unitarity Triangle Analysis (within the SM) allows to determine precisely the SM parameters of the flavour sector, to test the compatibility of the experimental results with the theoretical calculations and to predict yet unmeasured flavour SM observables.

Incredible moment for flavour physics: at present, a lot of new data are available and, in the next years, several new ones are going to come out !

- Pheno studies within the SM require precision techniques to study new data: comparison among different techniques, fundamental inputs from lattice QCD → effort to corroborate our comprehension of EW physics
- Global analyses are a fundamental tool to put together all the information we have at present: in this context, the Unitarity Triangle Analysis (within the SM) allows to determine precisely the SM parameters of the flavour sector, to test the compatibility of the experimental results with the theoretical calculations and to predict yet unmeasured flavour SM observables.
- The evaluation of the degree of discrepancy between measurements and theoretical predictions offers the
 possibility of discovering New Physics effects at still unexplored energy scales. In this respect, the Unitarity
 Triangle Analysis (beyond the SM) is complementary to the search of new particles at colliders working at
 multi-TeV energies.

Incredible moment for flavour physics: at present, a lot of new data are available and, in the next years, several new ones are going to come out !

- Pheno studies within the SM require precision techniques to study new data: comparison among different techniques, fundamental inputs from lattice QCD → effort to corroborate our comprehension of EW physics
- Global analyses are a fundamental tool to put together all the information we have at present: in this context, the Unitarity Triangle Analysis (within the SM) allows to determine precisely the SM parameters of the flavour sector, to test the compatibility of the experimental results with the theoretical calculations and to predict yet unmeasured flavour SM observables.
- The evaluation of the degree of discrepancy between measurements and theoretical predictions offers the
 possibility of discovering New Physics effects at still unexplored energy scales. In this respect, the Unitarity
 Triangle Analysis (beyond the SM) is complementary to the search of new particles at colliders working at
 multi-TeV energies.

Flavour continues to be the best framework to test indirectly very high energy scales (much higher than the one presently investigated at colliders) ③

GRAZIE PER LA VOSTRA ATTENZIONE !

BACK-UP SLIDES

Methods for data analysis – QFT constraints



Okubo, PRD 3, 2807 (1971), PRD 4, 725 (1971) Okubo, Shih, PRD 4, 2020 (1971) Boyd, Grinstein, Lebed, PLB 353, 306 (1995), NPB461, 493 (1996), PRD 56, 6895 (1997)

HVP tensor:
$$\Pi_{V}^{\mu\nu}(q) = i \int d^{4}x e^{iqx} \langle 0 | T\{V^{\mu}(x)V^{\nu\dagger}(0) | 0 \rangle = \frac{1}{q^{2}} (q^{\mu}q^{\nu} - q^{2}g^{\mu\nu}) \Pi_{1^{-}}(q^{2}) + \frac{q^{\mu}q^{\nu}}{q^{2}} \Pi_{0^{+}}(q^{2})$$

Disp. relations:

$$\chi_{1-}(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{t \text{Im}\Pi_{1-}(t)}{(t-q^2)^3}$$

$$\frac{t \text{Im}\Pi_{1^-}(t)}{(t-q^2)^3} \qquad \qquad \chi_{0^+}(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{t \text{Im}\Pi_{0^+}(t)}{(t-q^2)^2}$$

Spectral sum:

$$\operatorname{Im}\Pi_{V}(q^{2}) \sim \frac{1}{2} \sum_{X} (2\pi)^{4} \delta^{4}(q - p_{X}) \left| \left\langle 0 \mid V \mid X \right\rangle \right|^{2}$$

e.g. $X = BD^{*}: \left| \left\langle 0 \mid V \mid BD^{*} \right\rangle \right| \propto \left| \left\langle D^{*} \mid V \mid B \right\rangle \right|$

Unitarity constraints:

$$\frac{1}{\pi \chi_X(q^2)} \int_{t_+}^{\infty} dt \frac{W(t) |f_X(t)|^2}{(t-q^2)^n} \le 1$$

- dispersion relation leads to constraint on form factors in each symmetry channel
- χ can be evaluated in perturbation theory, e.g. at q² = 0, or <u>lattice at whatever q²</u>

Martinelli, Simula, LV PRD 104 (2021) [2105.07851]

L. Vittorio (Sapienza U. of Rome & INFN)

Methods for data analysis – Dispersive Matrix method

By introducing an *auxiliary function* $g_t(z) \equiv \frac{1}{1 - \overline{z}(t)z}$,

we can finally build up the **dispersive matrix** :

Okubo, PRD 3 (1971) 2807, PRD 4 (1971) 725 Okubo and Shih, PRD 4 (1971) 2020 Bourrely et al. NPB 189 (1981) 157 Lellouch, NPB 479 (1996) 353 Di Carlo et al., LV, PRD 104 (2021) 5, 054502



All the inner products are explicitly computable:

Cauchy's theorem: $\langle g_{t_i} | B_X \phi_X f_X \rangle = B_X(t_i) \phi_X(t_i) f_X(t_i)$ $\langle g_{t_i} | g_{t_i} \rangle = \frac{1}{1 - |z(t_i)|^2} \quad \langle g_{t_i} | g_{t_j} \rangle = \frac{1}{1 - \bar{z}(t_i) z_i}$

Unitaritu constraint: $\langle B_X \phi_X f_X | B_X \phi_X f_X \rangle \leq \chi_X$

L. Vittorio (Sapienza U. of Rome & INFN)

Thanks to the positivity of the inner products:

 $\det M \ge 0$

 $f_{X,\text{lo}}(z) \le f_X(z) \le f_{X,\text{up}}(z)$

Model-independence: the functional form of f_x (as a function of z) is never specified !

A brief parenthesis on unitarity: the role of the susceptibilities $\pmb{\chi}$

They can be computed on the lattice! Focusing again for simplicity on the 0⁺, 1⁻ channels we have that

$$\chi_{0^{+}}(Q^{2}) \equiv \frac{\partial}{\partial Q^{2}} \left[Q^{2} \Pi_{0^{+}}(Q^{2}) \right] = \int_{0}^{\infty} dt \ t^{2} j_{0}(Qt) \ C_{0^{+}}(t) \ , \qquad \underbrace{W. \ l.}_{4} \longrightarrow \frac{1}{4} \int_{0}^{\infty} dt' \ t'^{4} \ \frac{j_{1}(Qt')}{Qt'} \left[(m_{b} - m_{c})^{2} C_{S}(t') + Q^{2} C_{0^{+}}(t') \right] \\ \chi_{1^{-}}(Q^{2}) \equiv -\frac{1}{2} \frac{\partial^{2}}{\partial^{2} Q^{2}} \left[Q^{2} \Pi_{1^{-}}(Q^{2}) \right] = \frac{1}{4} \int_{0}^{\infty} dt \ t^{4} \frac{j_{1}(Qt)}{Qt} \ C_{1^{-}}(t) \qquad C_{1^{-}}(t) = \frac{1}{3} \sum_{j=1}^{3} \int d^{3}x \langle 0|T \left[\bar{b}(x)\gamma_{j}c(x) \ \bar{c}(0)\gamma_{j}b(0) \right] |0\rangle \ , \qquad$$

Fundamental advantage on the lattice:

We can choose *whatever value of Q*² (i.e. near the region of production of the resonances)

NOT POSSIBLE IN PERTURBATION THEORY !

$$(m_b + m_c)\Lambda_{QCD} << (m_b + m_c)^2 - q^2$$

POSSIBLE IMPROVEMENT IN THE STUDY OF THE FFs through unitarity!

The first lattice QCD determination of susceptibilities of heavy-to-heavy transition current densities has been completed in **PRD '21 [arXiv:2105.07851]**, using the $N_f=2+1+1$ gauge ensembles generated by ETM Collaboration ($Q^2 = 0$):

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}[10^{-3}]$	6.204(81)		7.58(59)	
$\chi_{A_L}[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T} [10^{-4} \text{ GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}[10^{-4} \text{ GeV}^{-2}]$	3.894		4.69(30)	

Differences with PT: ~4% for 1⁻, ~7% for 0⁻, ~20 % for 0⁺ and 1⁺

For PT: Bigi, Gambino PRD '16; Bigi, Gambino, Schacht PLB '17; Bigi, Gambino, Schacht JHEP '17 L. Vittorio (LAPTh & CNRS, Annecy)

Our proposal: bin-per-bin exclusive Vcb determination through unitarity





Fig. 6. The FFs from Refs. [15–17] together with the bands (at 1σ level) obtained by using simultaneously all the lattice inputs within the DM_{IS} method (red bands) or the BGL approach supplemented by the unitary and kinematical constraints (green bands). The BGL z-expansions are truncated after the quartic (quintic) term for the FFs g and f (F₁ and F₂). The DM bands are rigorously truncation independent.

$|V_{cb}|$ — Strategy A: bin-by-bin analysis

Originally proposed in Martinelli, Simula, LV, PRD 105 (2022) 3, 034503:

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\exp} \left[\frac{1}{\Gamma} \frac{d\Gamma}{d\alpha} \right]_{\exp}^{(i)} / \left[\frac{d\Gamma_0}{d\alpha} (\mathbf{a}) \right]_{lat}^{(i)} \right)^{1/2}, \quad \text{where} \quad \Gamma_{\exp} = \frac{\mathscr{B}(B^0 \to D^{*,-} \mathscr{C}^+ \nu_{\mathscr{C}})}{\tau(B^0)}$$

- Each bin is per se a measurement of |V_{cb}|
- Per channel correlated average $\{w, \cos \theta_{\ell}, \cos \theta_{v}, \chi\}$
- Akaike-Information-Criterion analysis



Bordone, Jüttner, EPJC 85 (2025)

Strategy B – fit to lattice and exp. data



- **BGL fit to only lattice data (strategy A)** misses experimental points for two of the lattice coll.
- BGL fit to experimental and lattice data (strategy B) (good fit quality good): some BGL coefficients shift between strategy A) and B) by up to a few σ

Strategy C – fit to exp. data



and some experimental points

$B \rightarrow Kvv$ as the fundamental link among $b \rightarrow c$ and $b \rightarrow s$

b \rightarrow svv is theoretically cleaner than b \rightarrow sµµ: it is not affected by charm-loop effects !!

Major sources of uncertainty:

1. Value of |Vcb| (due to CKM suppression) 2. Determination of hadronic FFs

Final prediction

$$\mathscr{B}\left(B^{\pm} \to K^{\pm}\nu\bar{\nu}\right) = (4.44 \pm 0.30) \times 10^{-6}$$

Final prediction

$$\mathcal{B}\left(B^{\pm} \to K^{\pm^*} \nu \bar{\nu}\right) = (9.8 \pm 1.4) \times 10^{-6}$$

D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]

to be compared with

$$\mathscr{B}\left(B^+ \to K^+ \nu \bar{\nu}\right)\Big|_{\text{Belle-II}} = (2.4 \pm 0.7) \times 10^{-5}$$

A. Glazov, plenary talk EPS-HEP2023 Conference, Aug 20-25, 2023

L. Vittorio (Sapienza U. of Rome &



Tree-level contribution

 $B^{\pm} \to K^{\pm(*)} \nu \bar{\nu}$





Charged meson decay modes have a tree-level contribution from the annihilation to an intermediate τ

Using the narrow width approximation

$$\mathscr{B}\left(B^{+} \to K^{(*)+} \nu \bar{\nu}\right) \sim \mathscr{B}\left(B^{+} \to \tau^{+} \nu\right) \mathscr{B}\left(\tau^{+} \to K^{(*)+} \bar{\nu}\right)$$

$$\frac{\mathscr{B}\left(B \to K^{(*)}\nu\bar{\nu}\right)_{\text{tree}}}{\mathscr{B}\left(B \to K^{(*)}\nu\bar{\nu}\right)_{\text{loop}}} \simeq 14\%(11\%)$$

Non negligible contribution!

Belle-II can in principle disentangle these two contributions

Salvador Rosauro-Alcaraz @ GdR Annual Workshop 2023



Jason Aebischer et al., EPJC '20 [arXiv:1903.10434 [hep-ph]]

Legame tra $b \rightarrow s \in b \rightarrow c: SU(2)_L \times U(1)_Y$



Another possible way to describe the CKM matrix is offered by the so-called *Wolfenstein parametriza* tion 24. The physical hint at the basis of this alternative formulation is that, since $S_{13} \ll S_{23} \ll$ $S_{12} \ll 1$, it is reasonable to develop a perturbative expansion in powers of S_{12} . To be more specific, we introduce four new parameters λ , A, η and ρ , defined as

$$egin{array}{rcl} \lambda &\equiv S_{12}\simeq V_{us},\ A\lambda^2 &\equiv S_{23}\simeq V_{cb},\ A\lambda^3(
ho-i\eta) &\equiv S_{13}e^{-i\delta}\simeq V_{ub}, \end{array}$$

where the last relation holds for small δ . By putting all the ingredients together, we have the new form

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4),$$
(2.18)

from which it is now evident that CP non-conservation is condensed in the terms of order λ^3 or higher. In conclusion, we can also consider higher order corrections in Eq. (2.18). By way of example, including terms $O(\lambda^5)$ we obtain the expression

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 - \lambda^4/8 & \lambda + O(\lambda^7) & A\lambda^3(\rho - i\eta) \\ -\lambda + A^2\lambda^5[1 - 2(\rho + i\eta)]/2 & 1 - \lambda^2/2 - \lambda^4(1 + 4A^2)/8 & A\lambda^2 + O(\lambda^8) \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + A\lambda^4[1 - 2(\rho + i\eta)]/2 & 1 - A^2\lambda^4/2 \end{pmatrix}$$
(2.19)

where $\bar{\rho}$ and $\bar{\eta}$ are slightly modified versions of the Wolfenstein parameters ρ and η and are related to them through the relations

$$\bar{\rho} = \left(1 - \frac{1}{2}\lambda^2\right)\rho, \quad \bar{\eta} = \left(1 - \frac{1}{2}\lambda^2\right)\eta. \tag{2.20}$$