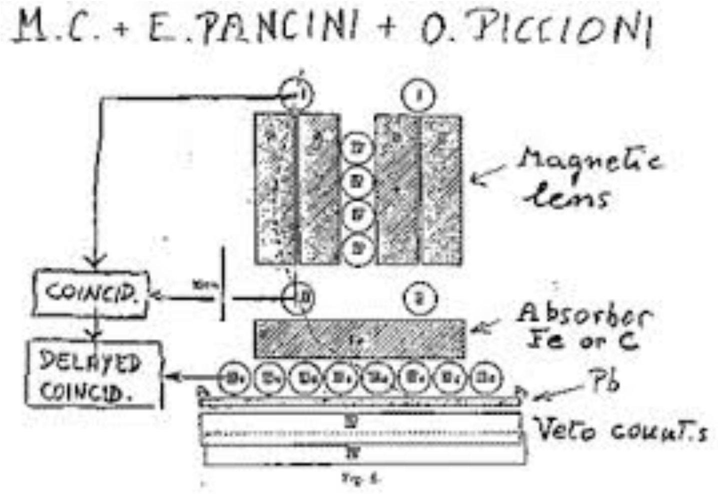
Istituto Nazionale di Fisica Nucleare

Roma, 19th May 2025

ENP collaboration meeting



Large N_c QCD predictions for Kaon physics

Giancarlo D’Ambrosio

INFN Sezione di Napoli

Predictions for the rare kaon decays
from QCD in the limit of a large number of colour
With M. Knecht and S.Neshatpour

e-Print: 2409.08568 [hep-ph] MDPI

Also Abhishek M. Iyer,Farvah Mahmoudi, S.Neshatpour,

Global Fits in LFUV in Kaons 2206.14748 ...2505.02568

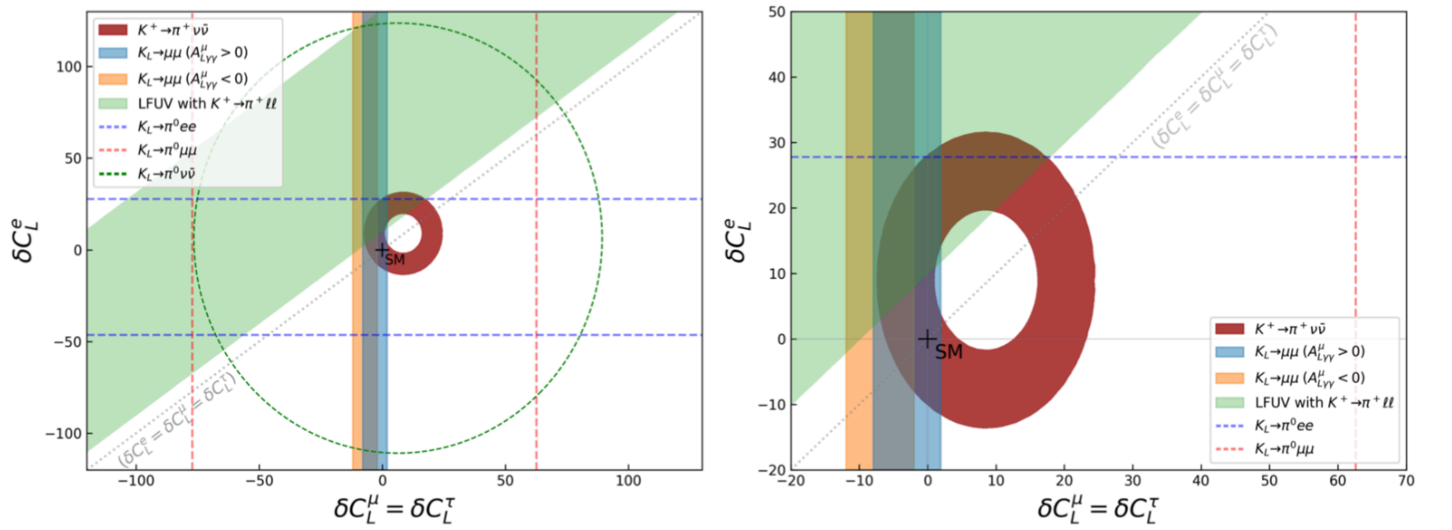


Figure 7: The bounds from individual observables. The right panel is the zoomed version of the left panel. The coloured regions correspond to 68% CL when there is a measurement and the dashed ones to upper limits at 90% CL. $K_L \to \mu\bar{\mu}$ has been shown for both signs of the long-distance contribution. For $K_L \to \pi^0 e\bar{e}$ and $K_L \to \pi^0\mu\bar{\mu}$, constructive interference between direct and indirect CP-violating contributions has been assumed.

Outline

- Motivations for K->πll $K^{\pm} \to \pi^{\pm} l^{+} l^{-} \quad K_S \to \pi^{0} l^{+} l^{-}$
- K->πll phenomenology
- Large N QCD: the dream '70s '80s '90s 't Hooft, Witten, Coleman
- Ideas, Successes, some undelivered messages $K \to \pi\pi : \ \Delta I = \frac{1}{2}$ rule $\quad \epsilon'$
- K->πll phenomenology and large N
- conclusions

European Strategy for Particle Physics 2026: Kaon physics input document

The kaon community requests to

- protect and amplify the European kaon-physics programme, explori[ng] opportunities for
  - $K^+ \to \pi^+ \nu\bar{\nu}$
  - $K_{S,L} \to \mu^+\mu^-$ decay and interference,
- enable European contributions for KOTO II for $K_L \to \pi^0 \nu\bar{\nu}$ and $K_L \to \pi^0 \ell^+\ell^-$, spanning both analysis and hardware development,
- maintain the European leadership on theory computations for kaon physics (phenomenology, dispersion theory, effective theory and lattice QCD, including high-performance computing).

EU strategy group

Lazzeroni

The contribution "Kaon Physics: A Cornerstone for Future Discoveries" has been drafted by:
M. Bordone, A. Ceccucci, A. Dery, M. Gorbahn, E. Goudzovski, M. Hoferichter, A. Jüttner, C. Lazzeroni, Z. Ligeti, D. Martinez, F. Mahmoudi, R. Marchevski, M. Moulson, G. Ruggiero.

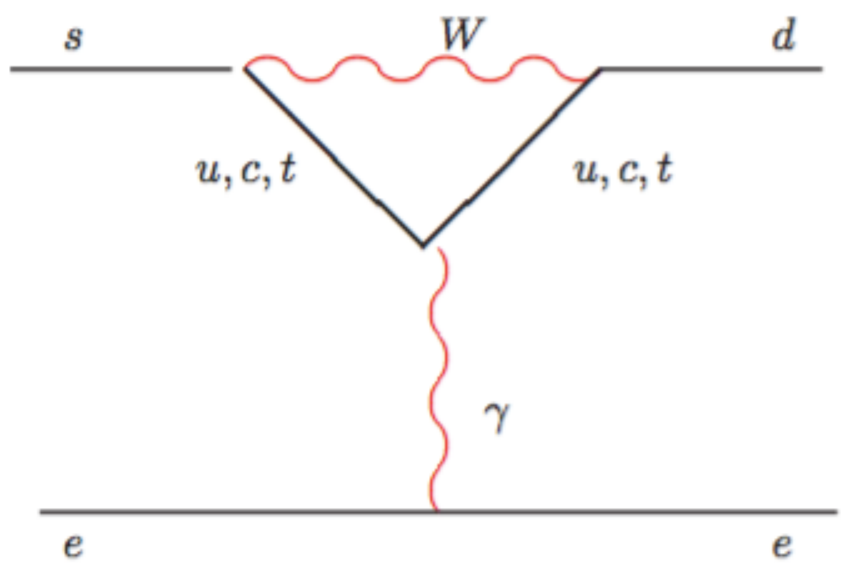
Year	Main object
1	Beam line survey
2	Construction of the rest of the detector
3-6	Phase I: Physics run for mainly $K_L \to \pi^0 \nu\bar{\nu}$
7	Single event sensitivity will reach 8.5×10^{-13} for the $K_L \to \pi^0 \nu\bar{\nu}$ search
8	Detector upgrade
9-12	Phase II: Physics run mainly for $K_L \to \pi^0 \ell^+\ell^-$ with an optimized setup
13	End of Phase II

Moulson

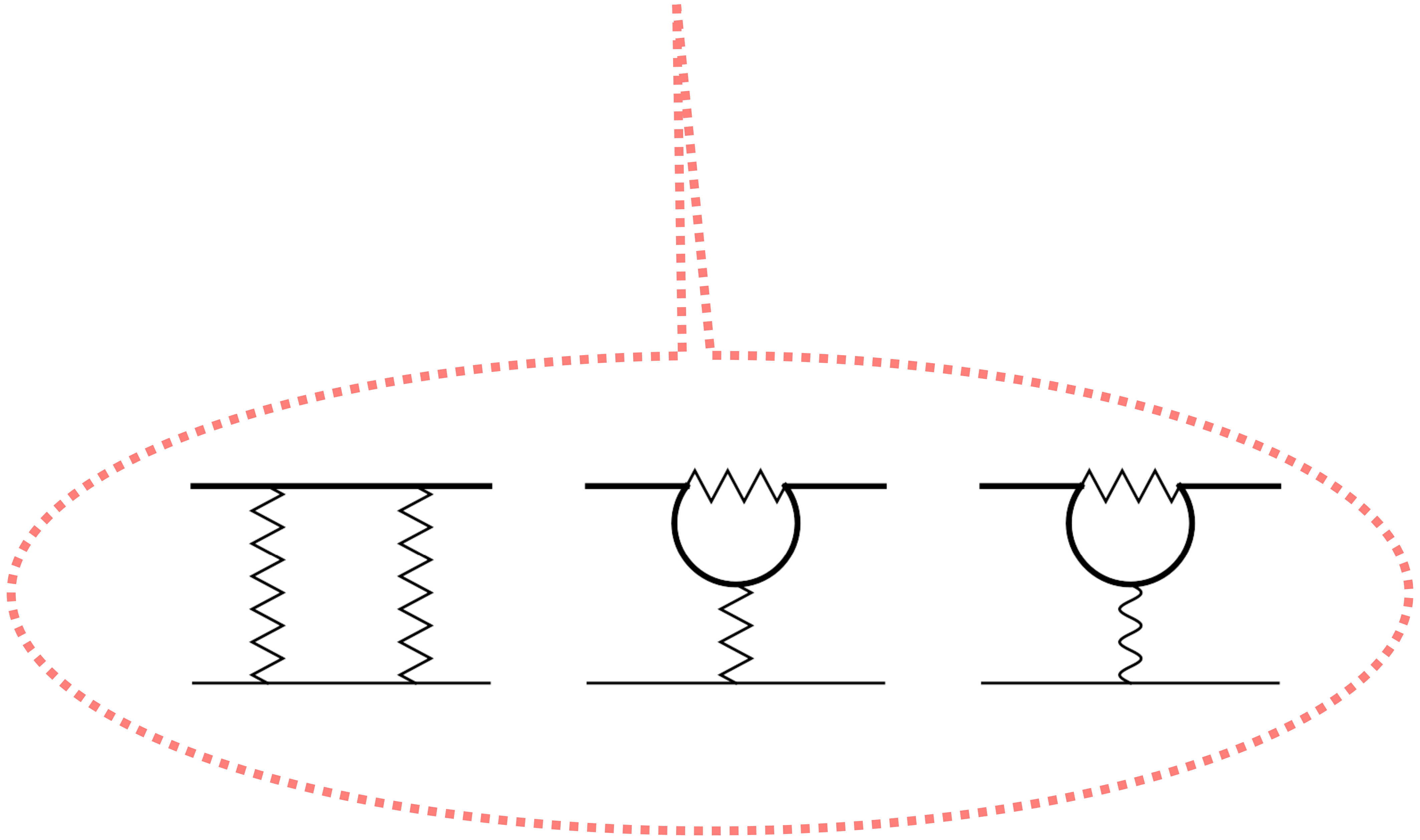
Long Distance enhancement

$$K^{\pm} \to \pi^{\pm} l^{+} l^{-} \quad K_S \to \pi^{0} l^{+} l^{-}$$

Gilman Wise 1980



Short distance



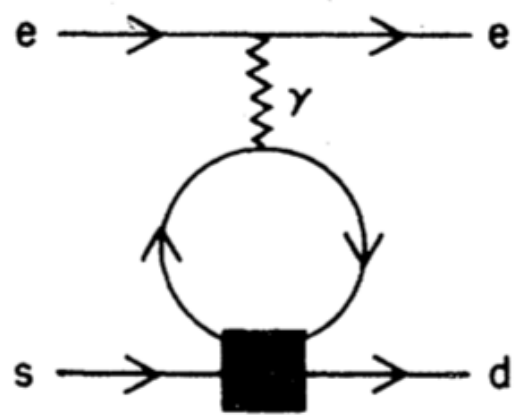


FIG. 1. Diagram contributing to C_7. The black box represents W exchange plus all strong-interaction corrections.

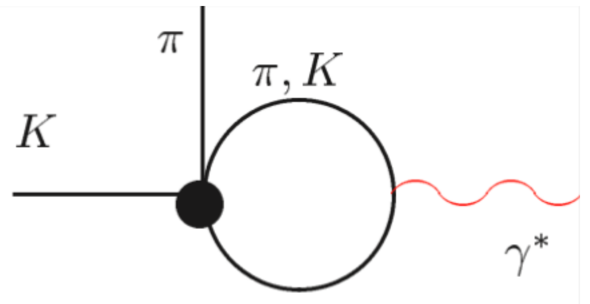
$$\begin{aligned}\mathcal{H}_{eff} = & -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \\ & -\frac{G_F}{\sqrt{2}} \frac{2}{9\pi} \frac{e^2}{4\pi} \left[A_c \ln\left(\frac{m_c^2}{\mu^2}\right) + A_t \ln\left(\frac{m_t^2}{\mu^2}\right) \right] \\ & \times (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{e} e)_V \\ & + \text{H.c.}\end{aligned}$$

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$K^+ \to \pi^+ l^+ l^-$

$k^2 = M_K^2\,, \quad p^2 = M_\pi^2\,, \quad q = k - p\,, \quad z = q^2/M_K^2\,, \quad r_\pi = M_\pi/M_K$

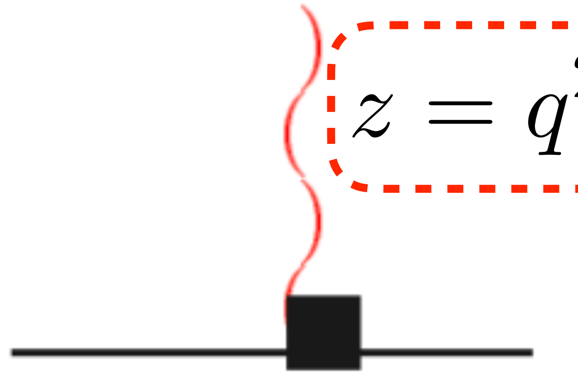


Chiral loop

$W_{\pi\pi}(z)$

Known from K->3pi

+



$z = q^2/M_K^2$

CT

a_+

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)|W(z)|^2\,,$$

KS No pion loop

$\mathcal{O}(p^4)$ a_+

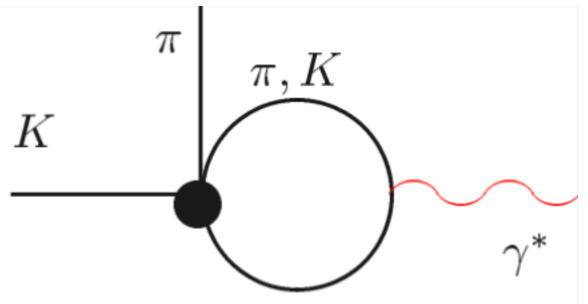
Data: the rate and spectrum not consistent with pheno

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$K_S \to \pi^0 l^+ l^-$

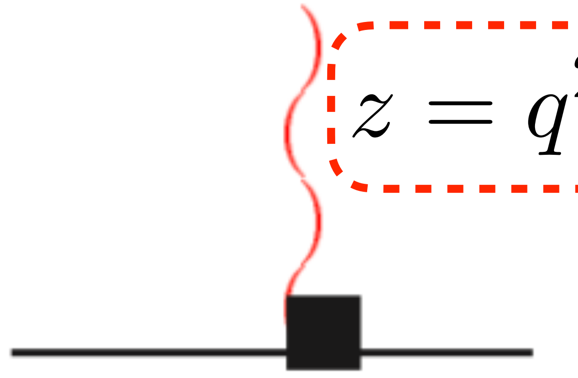
$k^2 = M_K^2\,,\quad p^2 = M_\pi^2\,,\quad q = k - p\,,\quad z = q^2/M_K^2\,,\quad r_\pi = M_\pi/M_K$



Chiral loop

No pion loop

+



CT

a_S

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)|W(z)|^2\,,$$

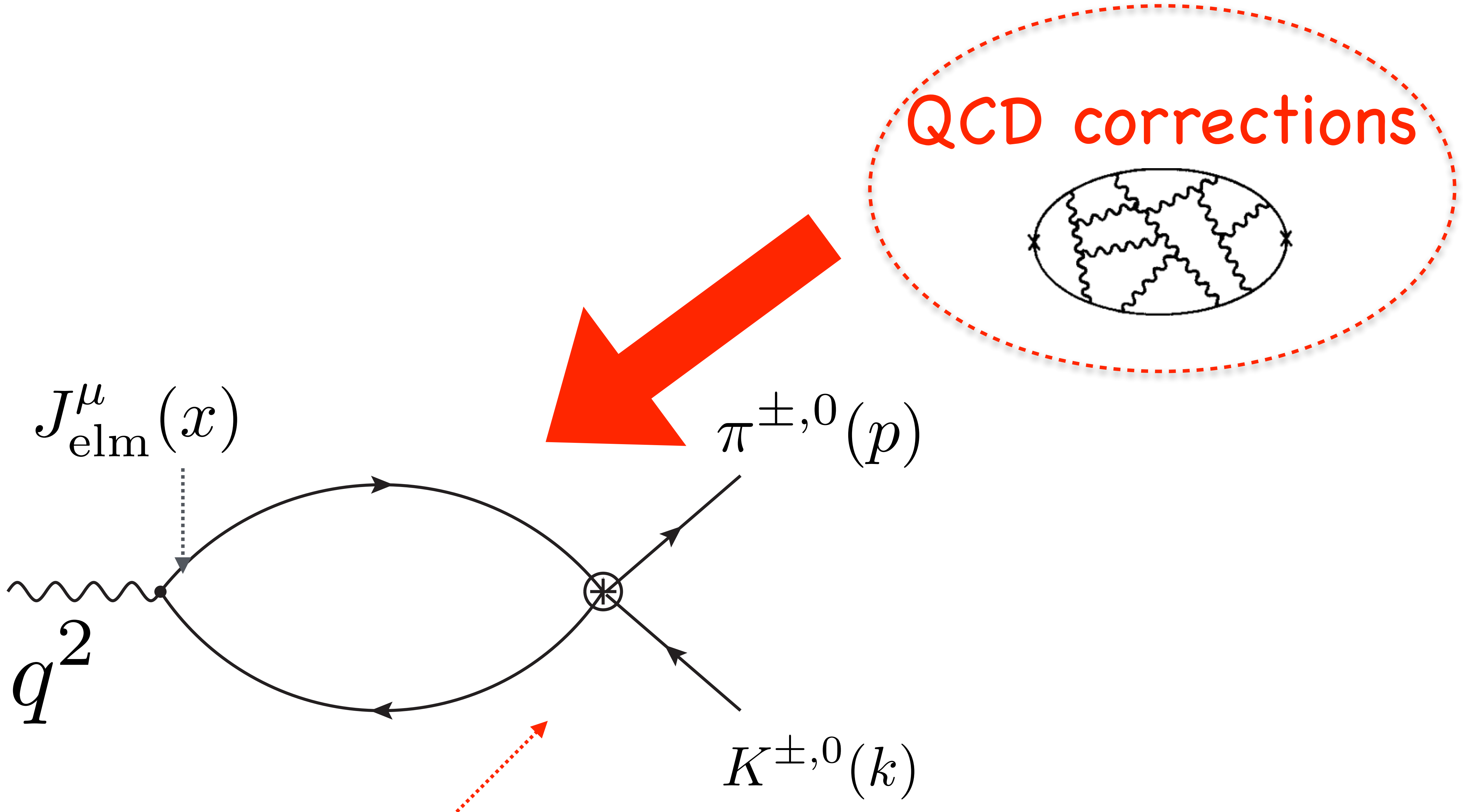
KS No pion loop for KS, CT

$\mathcal{O}(p^4)$ a_S

GD,Ecker,Isidori,Portoles '98

General consideration on the form factor

Lorentz and gauge invariance tell us on the structure of the amplitude and ff



$$z = \frac{q^2}{M_K^2}$$

$$i\int d^4x e^{iqx}\langle\pi(p)|T\left\{J^{\mu}_{\rm elm}(x)\mathcal{L}_{\Delta S=1}(0)\right\}|K(k)\rangle = \frac{W(z)}{(4\pi)^2}\left[z(k+p)^{\mu}-(1-r_{\pi}^2)q^{\mu}\right]$$

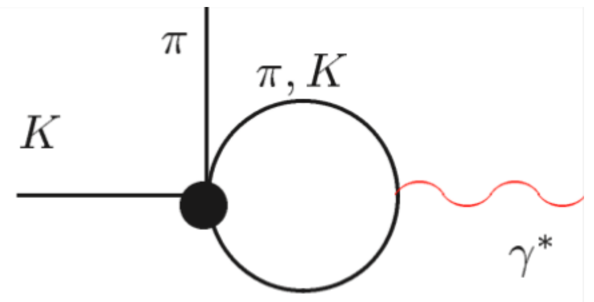
The integral at very short distances QCD quark loop

Pragmatic decision for O(p⁶) ff

GD,Ecker,Isidori,Portoles 98

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

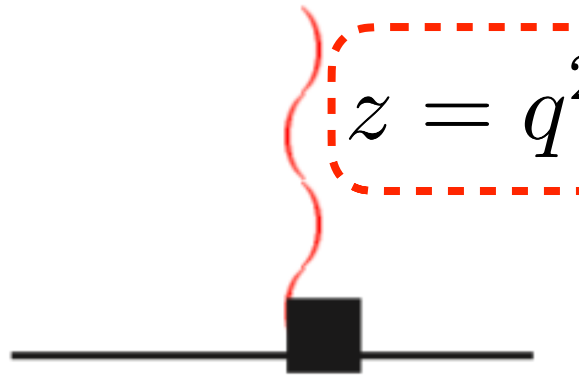
$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$



Chiral loop

$$W_{\pi\pi}(z)$$

Known from K->3pi



$z = q^2/M_K^2$

CT

$$a_i + b_i z$$

$$a_i + b_i z$$ Determined expt.

Extremely good agreement with data few %

$$\text{LFUV test } a_+^{\mu\mu} - a_+^{ee}$$

Crivellin et al '16, Nazila et al '22

$K \to \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \to \pi^+ e^+ e^-$	$10\,300$	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \to \pi^\pm e^+ e^-$	$7\,253$	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \to \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	$K_S \to \pi^0 \mu^+ \mu^-$	6

$$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$$

or

$$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$$

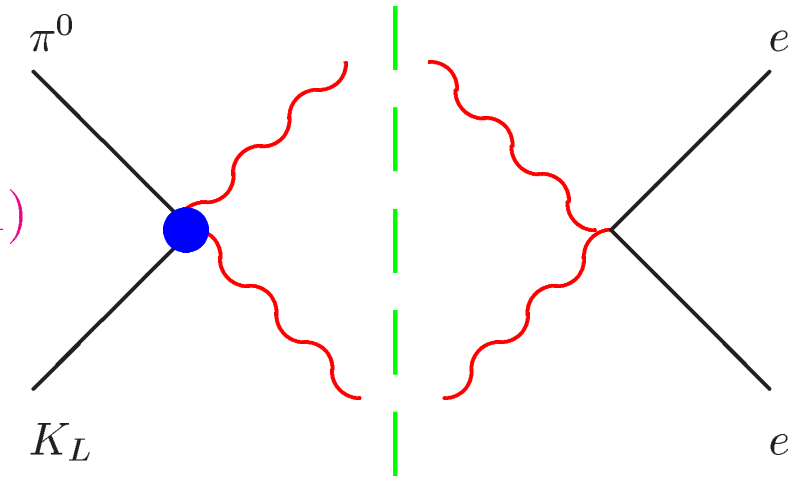
G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data

Why interesting $K_L \to \pi^0 l^+ l^-$?

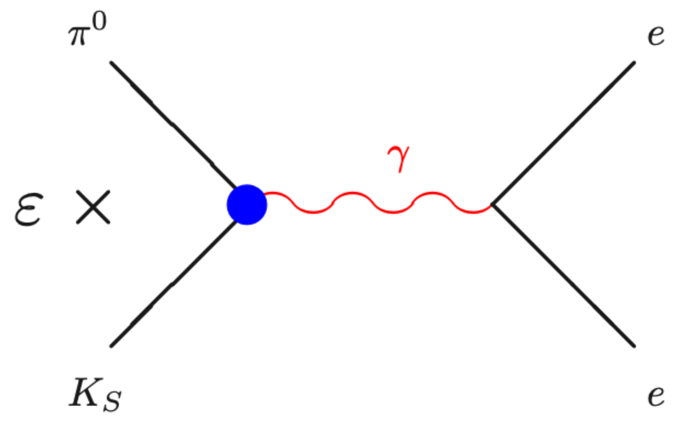
KOTO II , european strategy group

1)



Negligible for e, calculable for μ

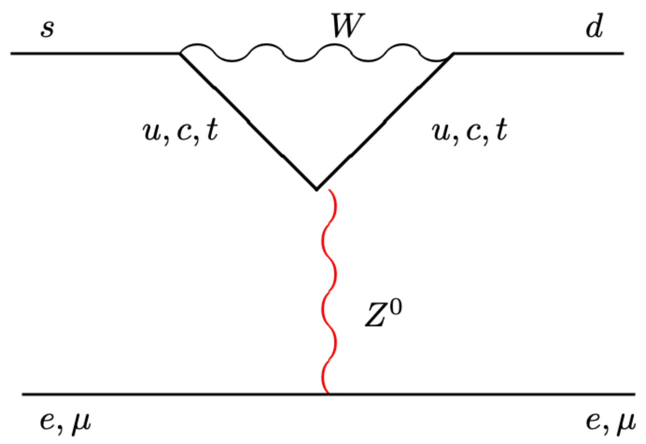
2)

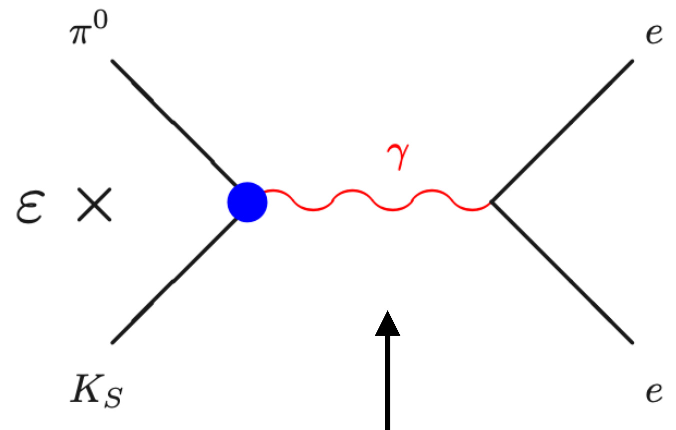


$\sim \epsilon(a_S + b_S z)$

form factor

3)





$$\mathrm{BR}(K_L \to \pi^0 \ell \bar{\ell}) = \left(C^\ell_{\rm dir} \pm C^\ell_{\rm int}|a_S| + C^\ell_{\rm mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \cdot 10^{-12}$$

$$|a_S| = 1.20 \pm 0.20$$

	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\,(1.55^{+0.60}_{-0.48}) \times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\,(0.94^{+0.21}_{-0.20}) \times 10^{-11}$$

$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11}$ at 90% CL

$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11}$ at 90% CL

Buchalla et al 03, Isidori et al 06

Nazila et al 22, Knecht GD 24 , Hoferichter et al.24

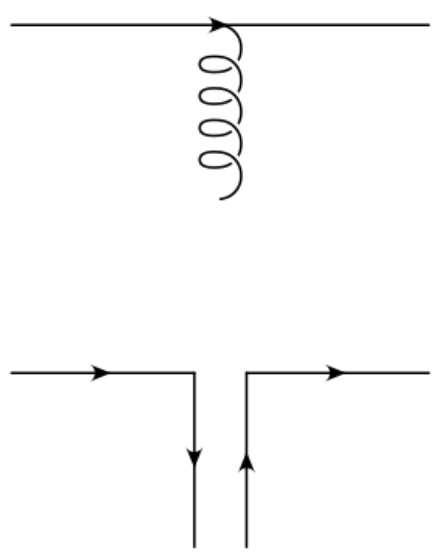
$$w_{7A,7V} = \mathrm{Im}\,(\lambda_t y_{7A,7V}) \,/\, \mathrm{Im}\,\lambda_t$$

LARGE N QCD

't Hooft, Witten, Coleman

$$\widetilde{\mathcal{L}}_{QCD} = \sum_{q=u,d,s} \bar{q}\gamma^{\mu}\left(i\partial_{\mu} - g_s \frac{\lambda_a}{2} G^a_{\mu}\right) q - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

QCD properties OK: asympt. Freedom $\mu \frac{dg}{d\mu} = -b_0 \frac{g^3}{16\pi^2} + \mathcal{O}\left(g^5\right), \quad b_0 = \frac{11}{3} N - \frac{2}{3} N_F$ Confinement assumed



$$8 + 1 = 3 + \bar{3}$$

Large N => correct expansion parameter? Geometric interpretation of this expansion: planar diagrams

Question in Large N: The leading order term in N of your amplitude

't Hooft model QCD_2	Exactly solved
Gross Neveu model	SSB+ mass Gap

Successes:

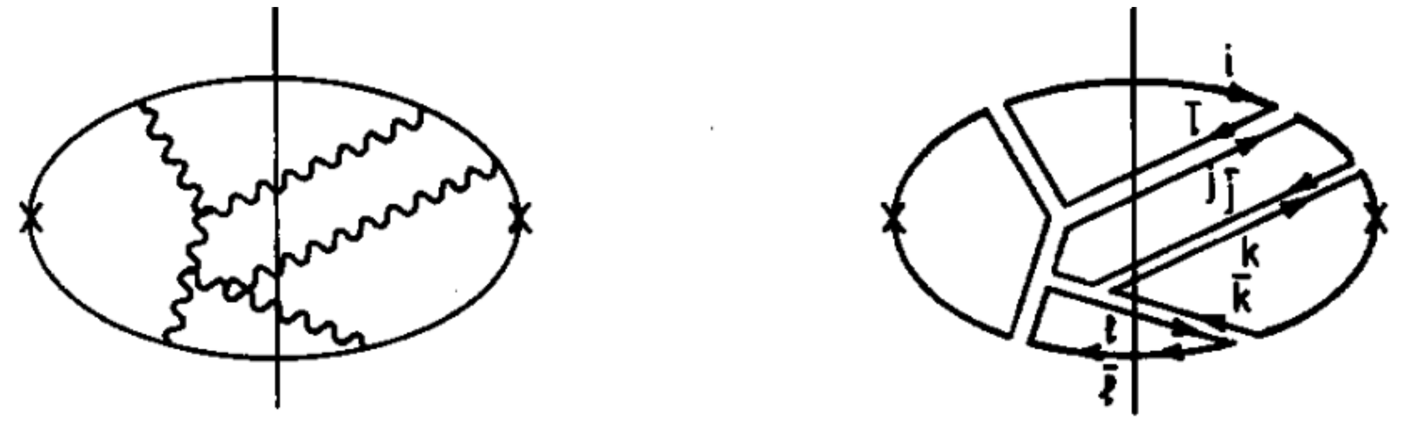
Zweig's rule (suppression of gluon exchange decays)

VMD, computability..

Witten '79 Large N QCD

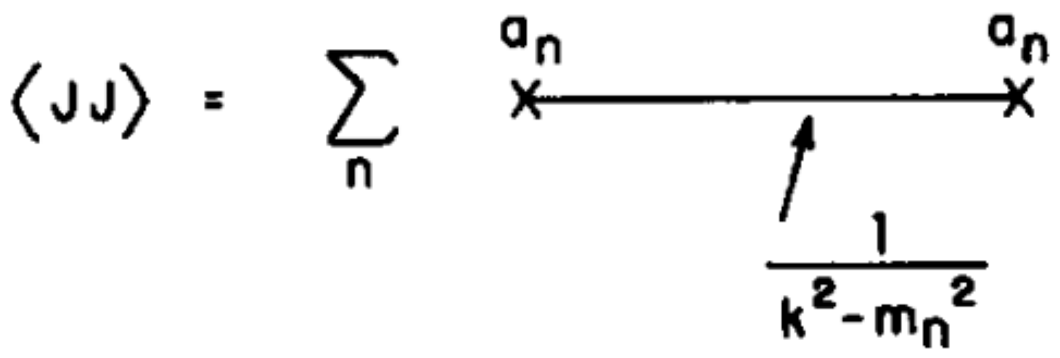
Section 3

- Meson are free, stable, non-interacting, the number of states infinite
- Meson decay amplitudes $\mathcal{O}(1/\sqrt{N})$
- One meson exchange leading (see two point function with J bilinear)



$$\langle J(q)J(-q)\rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2}\,.$$

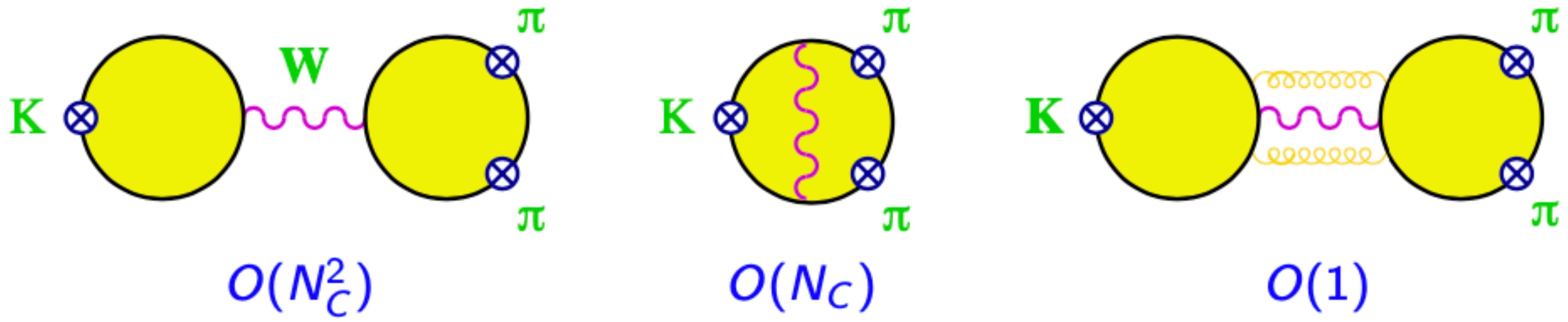
$$\langle J(q)J(-q)\rangle \underset{q_2\to\infty}{\sim} \log q^2$$



ig. 21. ⟨JJ⟩ represented as a sum of one-meson poles.

$K \to 2\pi$

Weak Currents Factorize at Large N_C



- Large sub
- Failing understanding $\Delta I = 1/2$ rule
- Lattice

$K_S \to \pi^0 l^+ l^-$: determination of the form factor

$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right]$$

$$k^2 = M_K^2 \;, \quad p^2 = M_\pi^2 \;, \quad q = k - p \;, \quad z = q^2/M_K^2 \;, \quad r_\pi = M_\pi/M_K \;,$$

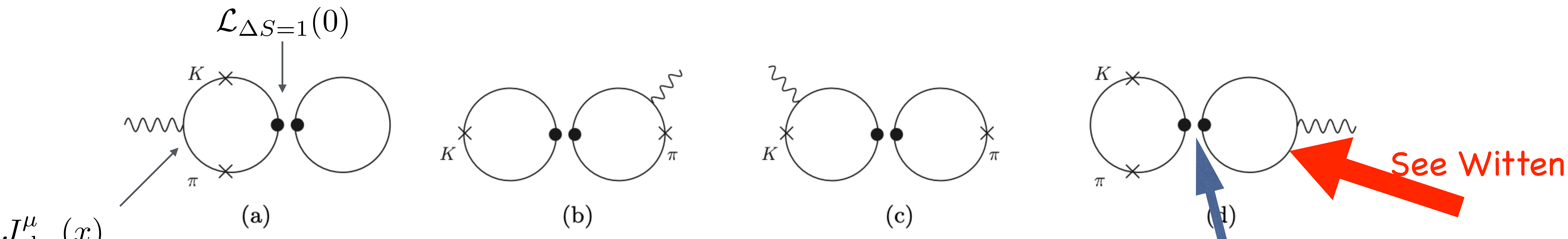


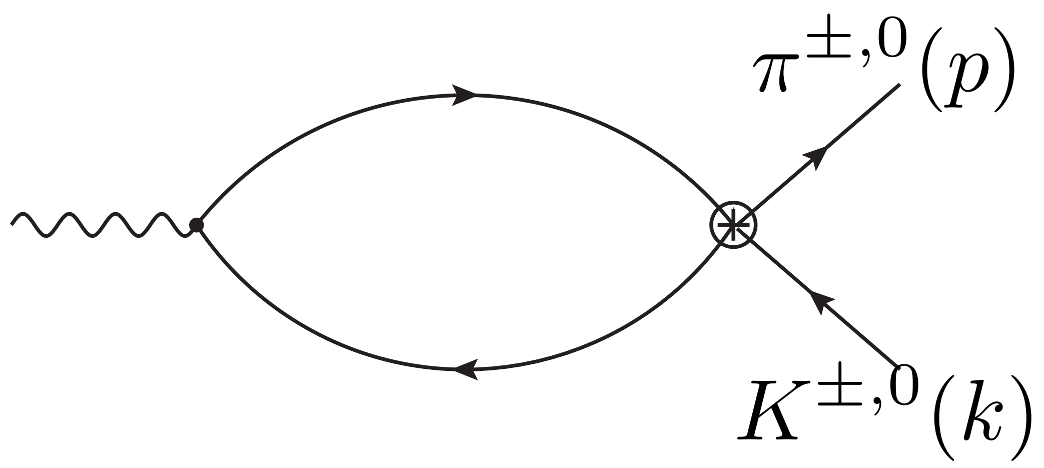
Figure 1: Potential diagrams contributing at leading order in large N_c (all at the order $(N_c)^2/(\sqrt{N_c})^2 = N_c$). The dots represent the effective $\Delta S = 1$ operators.

$$\mathcal{L}^{|\Delta S|=1}_{\rm non-lept} = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(\nu) + {\rm H.c}$$

$$Q_1 = (\bar{s}^i u_j)_{V-A} (\bar{u}^j d_i)_{V-A}, \; Q_2 = (\bar{s}^i u_i)_{V-A} (\bar{u}^j d_j)_{V-A}$$

Large Nc results for $K_S \to \pi^0 l^+ l^-$

GD Knecht, Neshatpour '24



$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$

Large Nc predictions =>Wilson coefficients +hadr. parameters

$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 5.1(1.7) \cdot 10^{-9}$

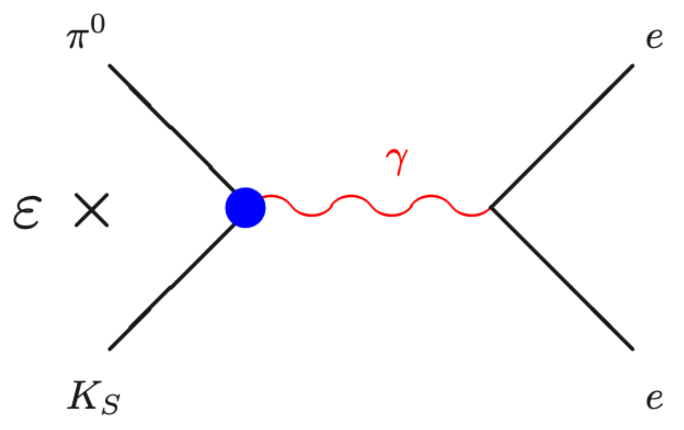
TH $\quad \mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165\ \mathrm{MeV}} = 2.9(1.0) \cdot 10^{-9}$

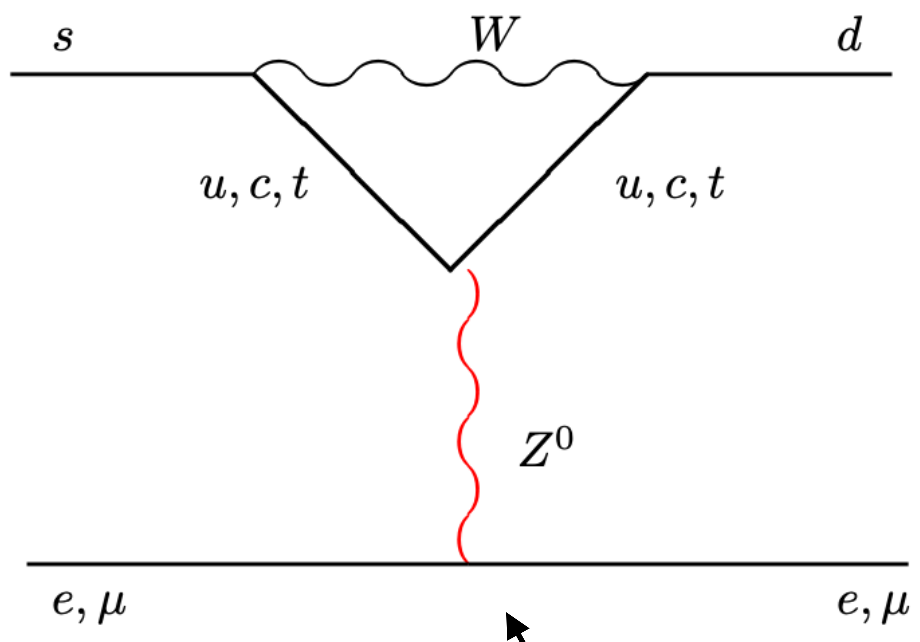
NA48/1 $\quad \mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165\ \mathrm{MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$

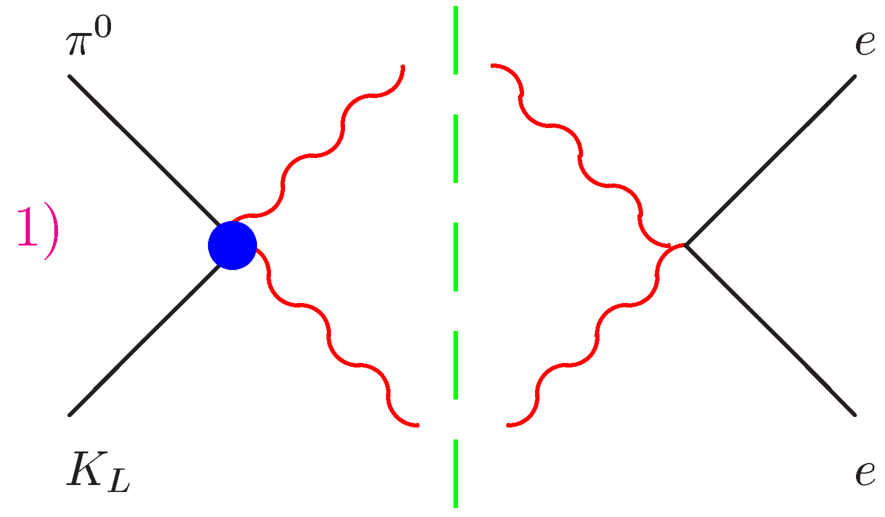
TH

$$\mathrm{Br}(K_S \to \pi^0 \mu^+ \mu^-) = 1.3(0.4) \cdot 10^{-9}$$

GD Knecht, Neshatpour '24







$$\mathrm{Br}(K_L \to \pi^0 \ell^+ \ell^-) = 10^{-12}\left[C_{\mathrm{mix}}^{(\ell)} + C_{\mathrm{int}}^{(\ell)} \frac{\mathrm{Im}\,\lambda_t}{10^{-4}} + C_{\mathrm{dir}}^{(\ell)} \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 + C_{\gamma^*\gamma^*}^{(\ell)}\right]$$

$$C_{\mathrm{int}}^{(e)} = +7.8(2.6)\,\frac{y_{7V}}{\alpha},\; C_{\mathrm{int}}^{(\mu)} = +1.9(0.6)\,\frac{y_{7V}}{\alpha}$$

Conclusions

Interesting TH CP violating and CP conserving

$$K_{S,L} \to \pi^0 l^+ l^-$$

Crucial to study

$K^+ \to \pi^+ l^+ l^-$

3-point function

$K_{S,L} \to \mu^+ \mu^- (e^+ e^-)$

EU strategy Group '25, Nazila et al '22

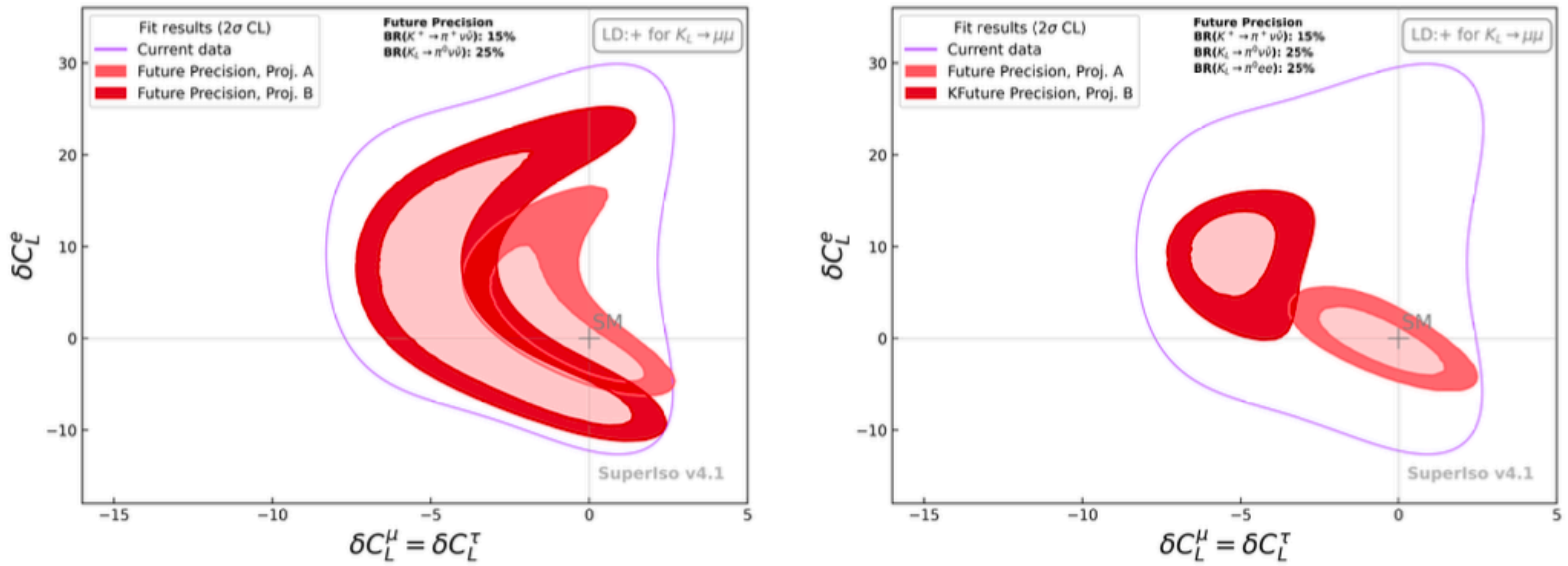


Figure 5: BSM parameter space for Wilson coefficients in scenarios with LFU violation where the NP effects for electrons are different from the those for muons and taus [18, 19]. Left: Impact on allowed parameter space from measurements of the golden channels $K \to \pi \nu \bar{\nu}$ from NA62 and KOTO II with the expected final precision. Right: Impact on the parameter space by the inclusion in the fits of a measurement of the $K_L \to \pi^0 e^+ e^-$ branching ratio with 25% precision.