# A Bayesian bridge between INFN LNF and INFN Roma 1

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18 May 2025

*ENP Collaboration meeting*

U. Roma 1

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# The common (back)ground – precision measurements at the LHC

**The context**

- The physics program of the LHC is being more and more characterized as a measurement one (indirect probe of new physics)
- Nothing excludes that the direct detection of new states may show up in the data at HL-LHC, but clearly this possibility is not assured
- To properly estimate the significant of a deviation in a precise measurement (or the constraint to a BSM model), the estimation of the theory uncertainty is also required

**The team**

- Both Marco and me have experience on different aspects of this program
- We have also both worked, at different times, on a Bayesian approach to Missing Higher Order Uncertainties (MHOUs)
- Lorenzo Paparella, a master student at Sapienza, worked on this during his thesis

# An example: the W mass measurement

- $M_W$ determination at hadron collider is performed indirectly by measuring observables that are strongly sensitive to W mass
- That makes it heavily dependent on having a refined theory framework
- This fact is reflected in the uncertainty budget of the currently available determination
- A huge effort from the theory community contributes directly to the exp. effort.

- **ATLAS** $\rightarrow m_W = 80366.5 \pm 9.8_{\mathrm{stat}} \pm 12.5_{\mathrm{exp}}$ MeV
- **CMS** $\rightarrow m_W = 80360 \pm 2.4_{\mathrm{stat}} \pm 9.6_{\mathrm{syst.}}$ MeV
- **LHCb** $\rightarrow m_W = 80354 \pm 23_{\mathrm{stat}} \pm 10_{\mathrm{exp}} \pm 17_{\mathrm{theory}} \pm 9_{\mathrm{PDF}}$ MeV
- **CDF II** $\rightarrow m_W = 80433.5 \pm 6.4_{\mathrm{stat}} \pm 6.9_{\mathrm{exp+mod.\ syst.}}$ MeV
- **D0** $\rightarrow m_W = 80375.5 \pm 11_{\mathrm{stat}} \pm 20_{\mathrm{exp+mod.\ syst.}}$ MeV

# The importance of an accurate theory framework

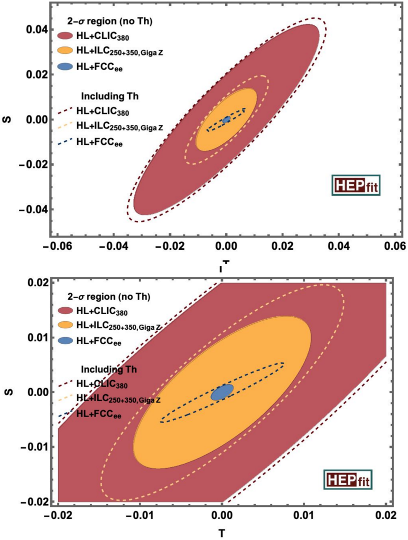
- CMS $\rightarrow m_W = 80360 \pm 2.4_{\text{stat}} \pm 9.6_{\text{syst.}}$ MeV

<table>
<tr><th>Systematic uncertainties</th><th>W-like $m_Z$</th><th>$m_W$</th></tr>
<tr><td>Muon efficiency</td><td>3127</td><td>3658</td></tr>
<tr><td>Muon eff. veto</td><td>–</td><td>531</td></tr>
<tr><td>Muon eff. syst.</td><td colspan="2">343</td></tr>
<tr><td>Muon eff. stat.</td><td colspan="2">2784</td></tr>
<tr><td>Nonprompt background</td><td>–</td><td>387</td></tr>
<tr><td>Prompt background</td><td>2</td><td>3</td></tr>
<tr><td>Muon momentum scale</td><td colspan="2">338</td></tr>
<tr><td>L1 prefire</td><td colspan="2">14</td></tr>
<tr><td>Luminosity</td><td colspan="2">1</td></tr>
<tr><td>PDF (CT18Z)</td><td colspan="2">60</td></tr>
<tr><td>Angular coefficients</td><td>177</td><td>353</td></tr>
<tr><td>W MiNNLO$_{\text{PS}}$ $\mu_F$, $\mu_R$</td><td>–</td><td>176</td></tr>
<tr><td>Z MiNNLO$_{\text{PS}}$ $\mu_F$, $\mu_R$</td><td colspan="2">176</td></tr>
<tr><td>PYTHIA shower $k_T$</td><td colspan="2">1</td></tr>
<tr><td>$p_T^V$ modeling</td><td>22</td><td>32</td></tr>
<tr><td>Nonperturbative</td><td>4</td><td>10</td></tr>
<tr><td>Perturbative</td><td>4</td><td>8</td></tr>
<tr><td>Theory nuisance parameters</td><td colspan="2">10</td></tr>
<tr><td>c, b quark mass</td><td colspan="2">4</td></tr>
<tr><td>Higher-order EW</td><td>6</td><td>7</td></tr>
<tr><td>Z width</td><td colspan="2">1</td></tr>
<tr><td>Z mass</td><td colspan="2">1</td></tr>
<tr><td>W width</td><td>–</td><td>1</td></tr>
<tr><td>W mass</td><td>–</td><td>1</td></tr>
<tr><td>$\sin^2\theta_W$</td><td colspan="2">1</td></tr>
<tr><td>Total</td><td>3750</td><td>4859</td></tr>
</table>

# The future – the next collider

**Future colliders**

- The past strategy update indicated a $e^+e^-$ Higgs factory the one priority for accelerator based physics
- Aside from the characterization of the Higgs, these machines will perform precise EW measurements, some of which may be theory limited
- More refined theory predictions will be required, along with an estimation of the remaining uncertainties




[Courtesy of J. de Blas] PRELIMINARY

# What is a theory uncertainty?

The so-called theory uncertainties affecting a measurement at a collider are of different kinds:

**Theory uncertainties**

- Missing Higher Order Uncertainties (MHOUs) due to the limited order of the perturbation expansion
- Parametric uncertainties due to the uncertainty in the physical parameter determination (e.g. *alpha*, $\alpha_S$)
- PDF uncertainties (exp.; model; MHOUs)
- Perturbative modelling uncertainties: e.g. different parton shower algorithm
- Non-perturbative uncertainties, such as intrinsic $k_T$, hadronisation, underlying event

# Missing higher order uncertainties

**MHOUs**

- The perturbative expansion of an observable known up to order $k$ is

$$O_k(Q,\mu) = \sum_{n=l}^{k} \alpha_s^n(\mu) c_n(Q,\mu) \qquad \text{(known)}$$

- $Q$ is the hard scale of the process, $\mu$ represents the unphysical scale(s) (e.g. the renormalization scale) from which the truncated perturbative expansion depends. We assume it set at the value $Q$
- The remainder of the series expansion is unknown and it is our MHOU

$$\Delta_k = \sum_{n=k+1}^{\infty} \alpha_s^n(Q) c_n(Q) \simeq \alpha_s^{k+1} c_{k+1} = ?$$

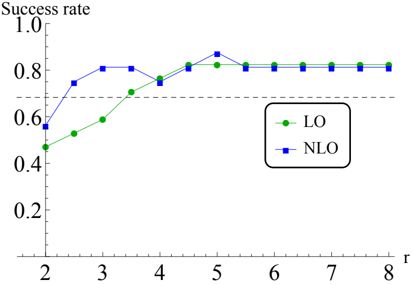
# Scale Variation

**Scale variation**

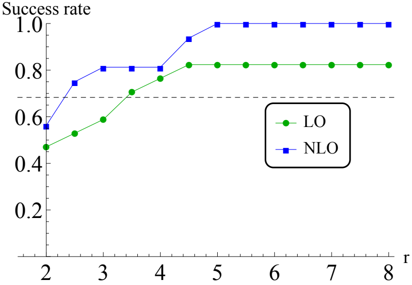
- Vary the unphysical scale(s) $\mu$ around the central scale $Q$ by an **arbitrary** factor $r$
- Different prescriptions used in the literature.
  1. **Scan**: vary $\mu$ between $Q/r$ and $r \times Q$ and use the maximum/minimum value of the observable to define the uncertainty
  2. **Extrema**: Use the maximum/minimum of the value of the observable obtained for $\mu = r \times Q, Q/r$
- Issue: the factor $r$ is arbitrary and the interval obtained has **no** statistical meaning

# Does it work?

Benchmark performed by considering a wide set of observables and check whether the next (know in this case) order is inside the band obtained with the scale variation (SV) prescription



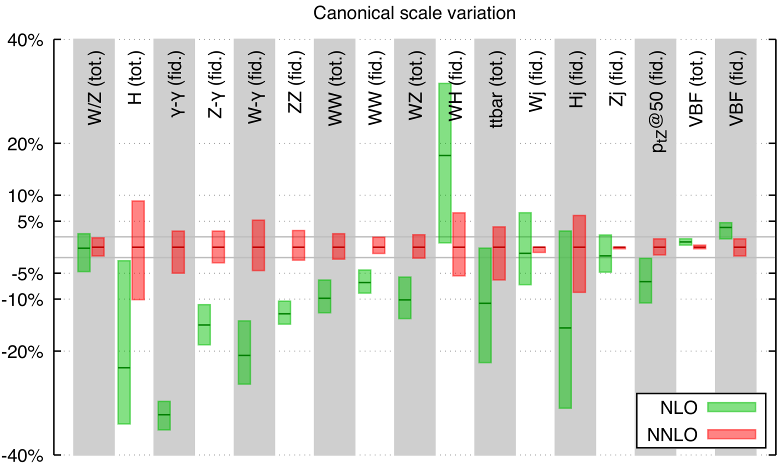

SV with just the three finite choices $\{Q/r, Q, r \times Q\}$




SV with the full scan in the interval $[Q/r, r \times Q]$

[EB, M. Cacciari, A. Guffanti, L. Jenniches '14]

# Does it work?




[G. Salam]

# The Bayesian Approach

## Scale variation

- Suppose there is an upper bound on the coefficients magnitude and call it $\bar{c}$
- The priors for the model are then given by

$$f_\epsilon(\ln \bar{c}) = \frac{1}{2|\ln \epsilon|}\chi_{|\ln \bar{c}| \leq |\ln \epsilon|}$$

$$f(c_n|\bar{c}) = \frac{1}{2\bar{c}}\begin{cases} 1 & \text{if } |c_n| \leq \bar{c} \\ 0 & \text{if } |c_n| > \bar{c} \end{cases}$$

$$f(\{c_i, i \in I\}|\bar{c}) = \prod_{i \in I} f(c_i|\bar{c})$$

- Bayesian inference gives then the uncertainty interval posterior

$$f(\Delta_k|c_l, \ldots, c_k) \simeq \left(\frac{n_c}{n_c+1}\right)\frac{1}{2\alpha_s^{k+1}\bar{c}_k}\begin{cases} 1 & \text{if } |\Delta_k| \leq \alpha_s^{k+1}\bar{c}_k \\ \frac{1}{(|\Delta_k|/(\alpha_s^{k+1}\bar{c}_k)^{n_c+1}} & \text{if } |\Delta_k| > \alpha_s^{k+1}\bar{c}_k \end{cases}$$

  where $n_c = k - l + 1$ and $\bar{c}_k = \max(c_l, \ldots, c_k)$
- Intervals have a statistical meaning in term of Degree of Belief (DoB)

# A family of different models

## Quick recap: Cacciari-Houdeau, geometric and $abc$ models

$$\Sigma = \Sigma_{\text{LO}}(\mu) \sum_{k\geq 0} \delta_k(\mu) \qquad \Sigma_{\text{LO}}(\mu)\delta_k(\mu) = c_k(\mu)\alpha_s^k(\mu)$$

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CH model assumes that $\delta_k$ behave as $\alpha_s^k \eta^k$ [Cacciari,Houdeau 1105.5152]

$$\Sigma_{\text{LO}}(\mu)\,|\delta_k(\mu)| \leq \bar{c}\,\alpha_s^k \eta^k, \qquad \eta = 1$$

with one hidden parameter $\bar{c}$, and $\eta$ describing a possible power growth of the coefficients $c_k$

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BCGJ adds a factorial growth [Bagnaschi,Cacciari,Guffanti,Jenniches 1409.5036]

$$\Sigma_{\text{LO}}(\mu)\,|\delta_k(\mu)| \leq \bar{c}\,\alpha_s^k \eta^k k!$$

with $\eta$ is determined from a survey over various observables

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My proposal: geometric behaviour model [Bonvini 2006.16293]

$$|\delta_k(\mu)| \leq c\,a^k$$

depends on two hidden parameters $c, a$, it accounts for a possible power growth of the coefficients within the model

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Asymmetric variant, called $abc$ model [Duhr,Huss,Mazeliauskas,Szafron 2106.04585]

$$\left|\delta_k(\mu) - b\,a^k\right| \leq c\,a^k$$

depends on three hidden parameters $a, b, c$, it also accounts for a possible sign pattern

# A family of different models

## Comparison among approaches to MHOU

| | CSV | CH | BCGJ | B geo | DHMS *abc* | B sca | B constr-sca |
|---|---|---|---|---|---|---|---|
| power series | | ✓ | ✓ | ✓ | ✓ | | |
| orders get smaller | | ✓ | | ✓ | ✓ | | |
| scale dep $\sim$ MHO | ✓ | | | | | ✓ | ✓ |
| scale dep reduce | | | | | | | ✓ |
| reliable | | | | ✓ | ✓ | ✓ | ✓ |
| not so arbitrary | | (✓) | (✓) | ✓ | ✓ | ✓ | ✓ |
| probabilistic | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

CSV: Canonical Scale Variation
CH: [Cacciari,Houdeau 1105.5152]
BCGJ: [Bagnaschi,Cacciari,Guffanti,Jenniches 1409.5036]
B: [Bonvini 2006.16293]
DHMS: [Duhr,Huss,Mazeliauskas,Szafron 2106.04585]

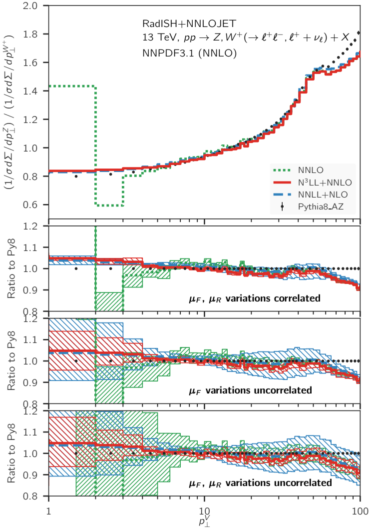
See also [David,Passarino 1307.1843] [Forte,Isgrò,Vita 1312.6688] [Ghosh,Nachman,Plehn,Shire,Tait,Whiteson 2210.15167] [McGowan,Cridge,Harland-Lang,Thorne 2207.04739] [Tackmann 2411.18606] [Lim,Poncelet 2412.14910]

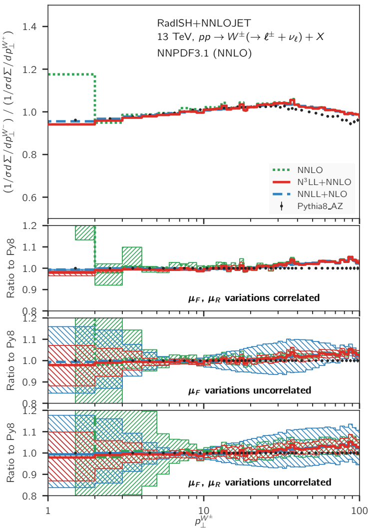
# Correlations in theory uncertainties

**Scale variation**

- Theory correlations may arise between different bins of the same observables; between observables of the same process; between different processes.
- The understanding is that these correlations are due to same physics being at play
- In these case also the theory uncertainties will be correlated – but how to estimate the correlation?
- One possibility is to use the unphysical scale(s) as the correlation parameter – but does it make sense?

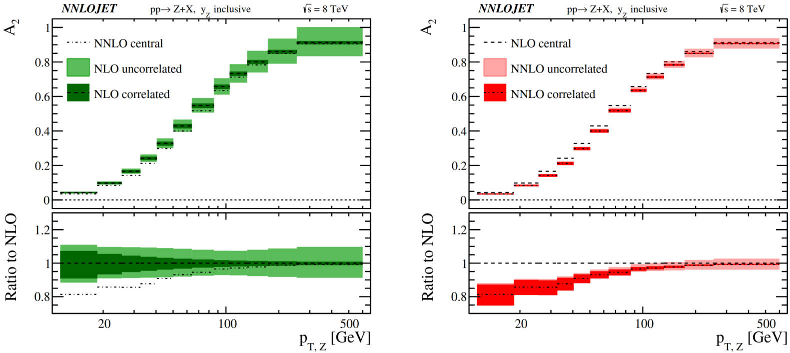
# Correlations matter: the $p_T^Z/p_T^W$ ratio in QCD




[Bizon et al. '19, 1905.05171]

# Correlations matter: Z angular coefficients




[Gauld et al. '17, 1708.00008]

## Application to $p_T$ Spectrum.

Step 2: Use $p_T$ factorization to organize (resum) the double series for $f_{nm}$

$$\frac{d\sigma^{(0)}}{dp_T} = \Big[H \times B_a \otimes B_b \otimes S\Big](\alpha_s; L \equiv \ln p_T/m_Z)$$

- Each function $F \equiv \{H, B, S\}$ has exponential form (solution of its RGE)

$$F(\alpha_s, L) = F(\alpha_s)\, \exp \int_0^L dL' \Big\{\Gamma[\alpha_s(L')]\, L' + \gamma_F[\alpha_s(L')]\Big\}$$

  - *Boundary conditions*

$$F(\alpha_s) = F_0 + \alpha_s\, F_1 + \alpha_s^2\, F_2 + \mathcal{O}(\alpha_s^3)$$

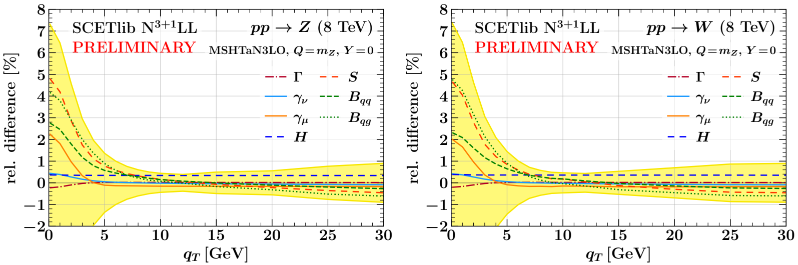
  - *Anomalous dimensions*

$$\Gamma(\alpha_s) = \alpha_s\big[\Gamma_0 + \alpha_s\,\Gamma_1 + \alpha_s^2\,\Gamma_2 + \mathcal{O}(\alpha_s^3)\big]$$
$$\gamma_F(\alpha_s) = \alpha_s\big[\gamma_0 + \alpha_s\,\gamma_1 + \alpha_s^2\,\gamma_2 + \mathcal{O}(\alpha_s^3)\big]$$

$\Rightarrow$ Entire problem reduces to several scalar series $F(\alpha_s)$, $\Gamma(\alpha_s)$, $\gamma_F(\alpha_s)$

# Theory nuisance parameters

## TNP Uncertainties in Drell-Yan $p_T$ Spectrum.




- N$^{3+1}$LL: Full N$^4$LL resummation with highest-order boundary conditions and anomalous dimensions as TNPs
- Important caveats:
  - Beam boundary conditions $B_{qj}$: Using $f_n = (0 \pm 2) \times f_n^{\text{true}}$
  - Hard boundary conditions $H$: No singlet corrections (enter only $Z$ not $W$)
  - DGLAP splitting functions are noncusp anom. dimensions, not varied here

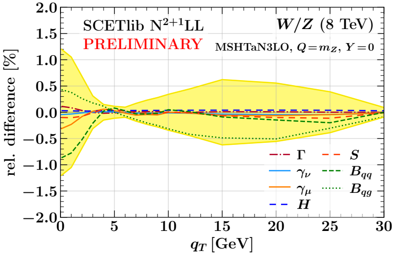
✓ Correlations across $p_T$ and between $W$ and $Z$ are correctly captured

2024-02-26 | Frank Tackmann 27/29.

# Theory nuisance parameters

## TNP Uncertainties in Drell-Yan $p_T$ Spectrum.

relative impact for $W/Z$

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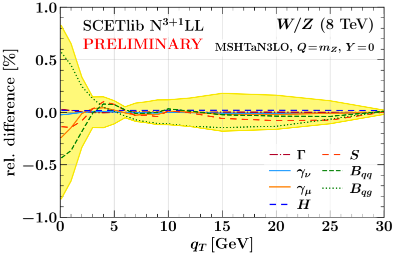


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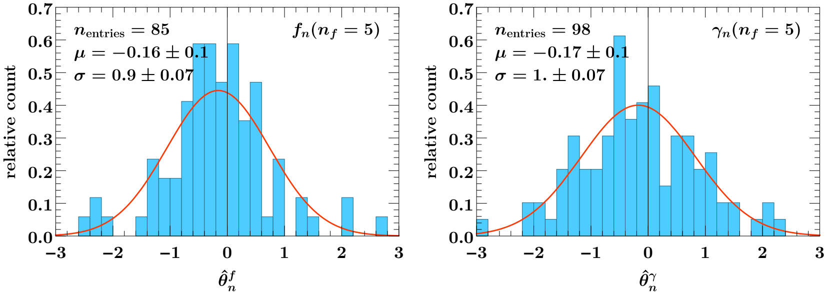
28/29.

# Issues with Tackmann’s approach




[Tackmann ’24]

- Tackmann’s defines in arbitrary way the probability distributions of the theory nuisance parameters
- However these are perturbative objects $\rightarrow$ use the Bayesian approach to determine these distributions!

# Our correlation setup

Consider the threshold resummation for a color singlet, i.e Higgs or Drell-Yan

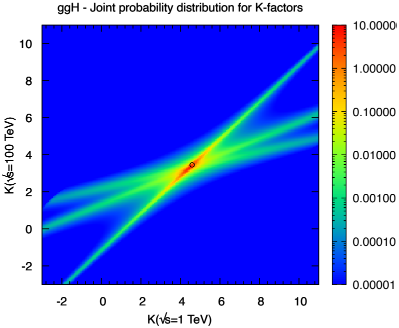
$$\int_0^1 d\tau \tau^N \Sigma(\tau) = PDFs(N) \times g_0(\alpha_S) \exp^{S(\alpha_S,N)} \quad \text{with} \quad \tau = \frac{M^2}{s}$$

and where

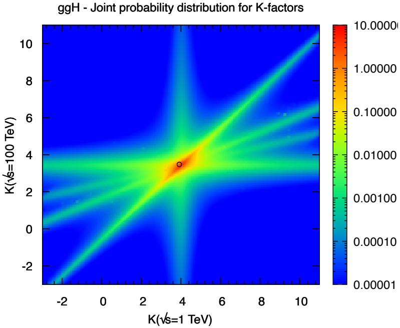
$$S(\alpha_S, N) = \int_0^1 dz \frac{z^N - 1}{1-z} \left( \int_{\mu_F^2}^{M^2(1-z)^2} \frac{d\mu^2}{\mu^2} 2A(\alpha_S(\mu^2)) + D(\alpha_S((1-z)^2 M^2)) \right)$$
$$A(\alpha_S) = \alpha_S A_1 + \alpha_S^2 A_2 + \alpha_S^3 A_3 + \alpha_S^4 + \dots$$
$$D(\alpha_S) = \alpha_S D_1 + \alpha_S^2 D_2 + \alpha_S^3 D_3 + \dots$$
$$g_0(\alpha_S) = \alpha_S g_{01} + \alpha_S^2 g_{02} + \alpha_S^3 g_{03} + \dots$$

- Apply the Bayesian model to perturbative elements $A$,$D$ and $g_0$ to obtain $P(A, D, g_0 | mathrmknownorders)$
- Add a totally uncorrelated piece for the non-resummed part

# Joint distributions between K-factors for $gg \to H$ at different energies

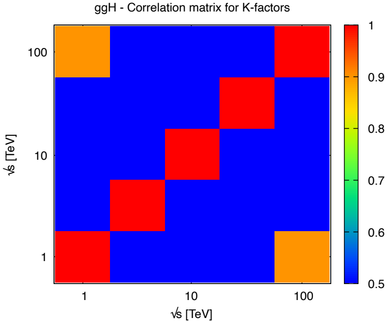



without the uncorrelated fixed order piece

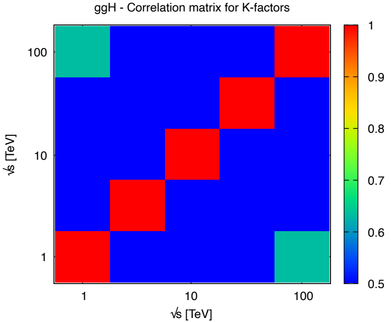



with the uncorrelated fixed order piece

# Correlation matrix of K-factors for $gg \to H$ at different energies

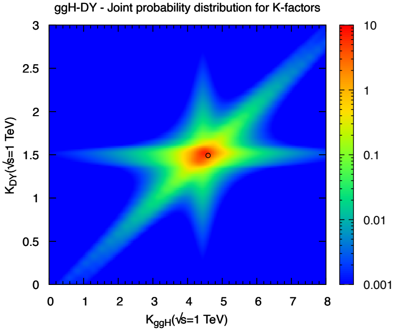



without the uncorrelated fixed order piece




with the uncorrelated fixed order piece

# Joint probability between DY and ggH




with the uncorrelated fixed order piece

# Outlook

## Outlook

- Built a collaborative relationship between the LNF and Roma 1 nodes focused on SM precision measurements
- The work in progress is focused on building a Bayesian model for theoretical uncertainty correlations
- The consistent modelling of these correlations is important for the precision physics program at the LHC, and future colliders