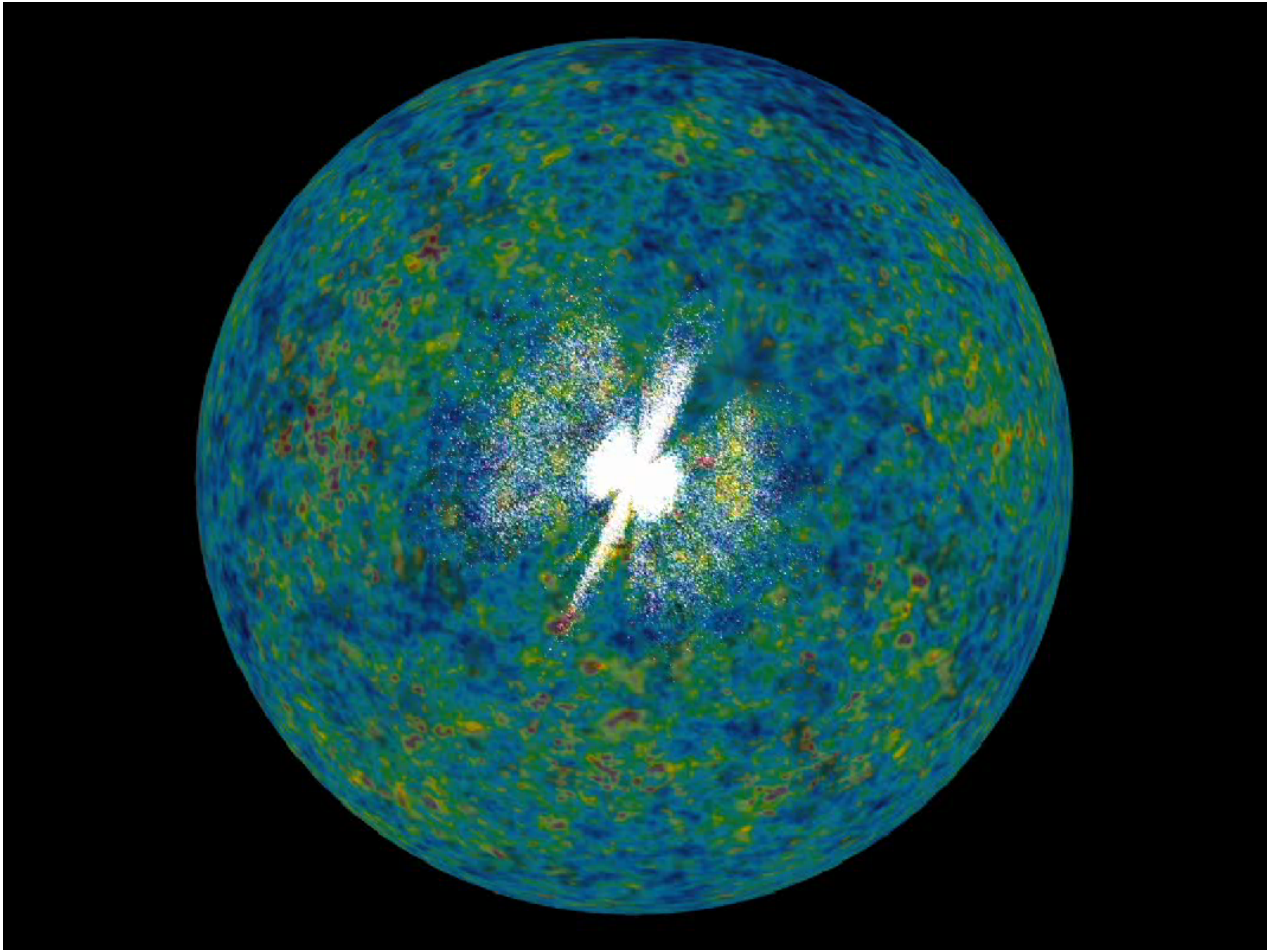
Paolo Creminelli, ICTP Trieste

Symmetries of the primordial perturbations

PC, 1108.0874 (PRD)
with J. Noreña and M. Simonović, 1203.4595

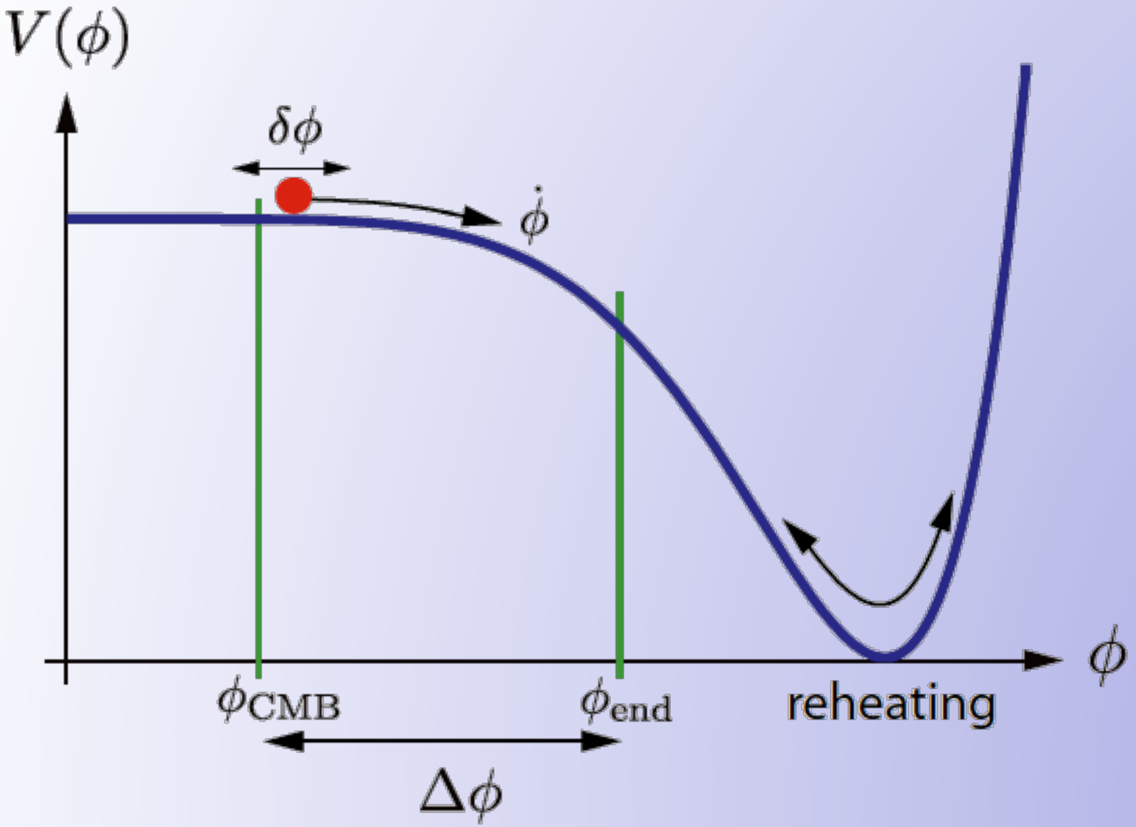
(with G. D'Amico, M. Musso and J. Noreña, 1106.1462 (JCAP)
with A. Nicolis and E. Trincherini, 1007.0027 (JCAP)
+ in progress with A. Joyce, J. Khoury and M. Simonović)



(Slow-roll) inflation

FRW metric: $$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \qquad H \equiv \frac{\dot{a}}{a}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0 \qquad S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$



$$\ddot{\phi} + \underline{3H\dot{\phi}} + V'(\phi) = 0$$

Friction is dominant

$$\epsilon = \frac{1}{2}M_P^2\left(\frac{V'}{V}\right)^2$$

$$\eta = M_P^2\frac{V''}{V}$$

$$\epsilon, \eta, \ldots \ll 1$$

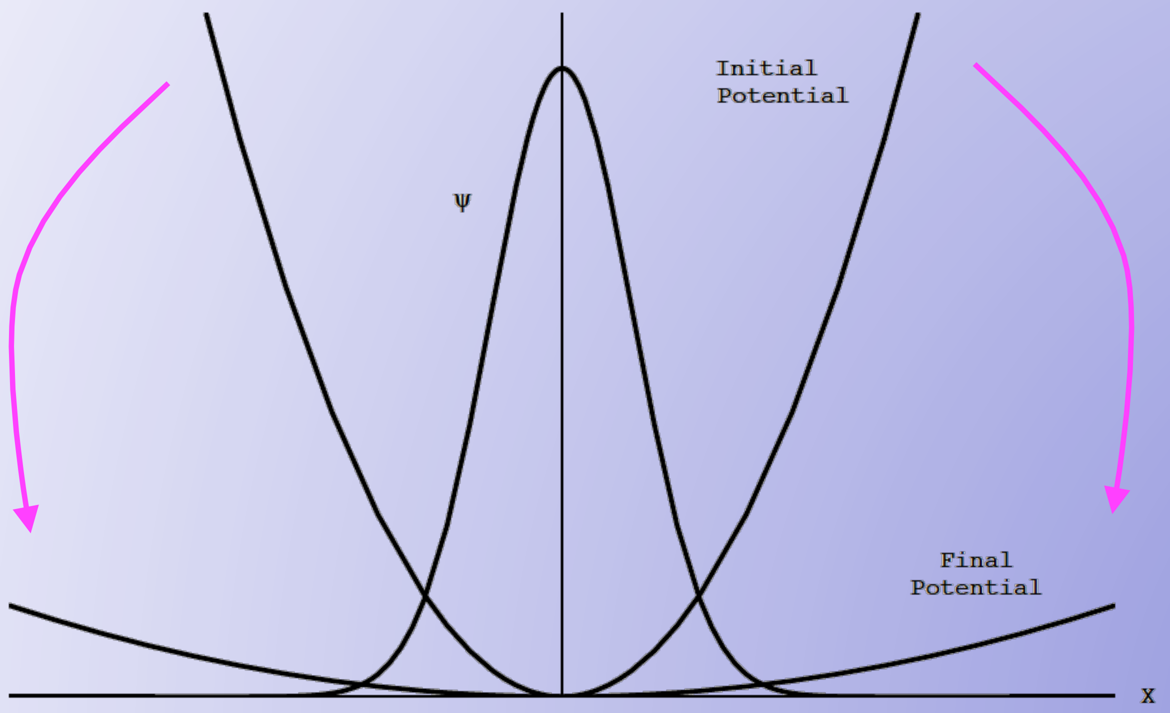
$\hbar \neq 0$

Each inflaton Fourier mode behaves as a harmonic oscillator
with time dependent parameters

$$S = \frac{1}{2} \int dt d^3x \frac{\dot{\phi}^2}{H^2} \left[a^3 \dot{\zeta}^2 - a(\partial\zeta)^2 \right]$$

$$\delta\phi = 0$$

$$h_{ij} = a^2(t) \left[e^{2\zeta} \delta_{ij} + \gamma_{ij} \right]$$



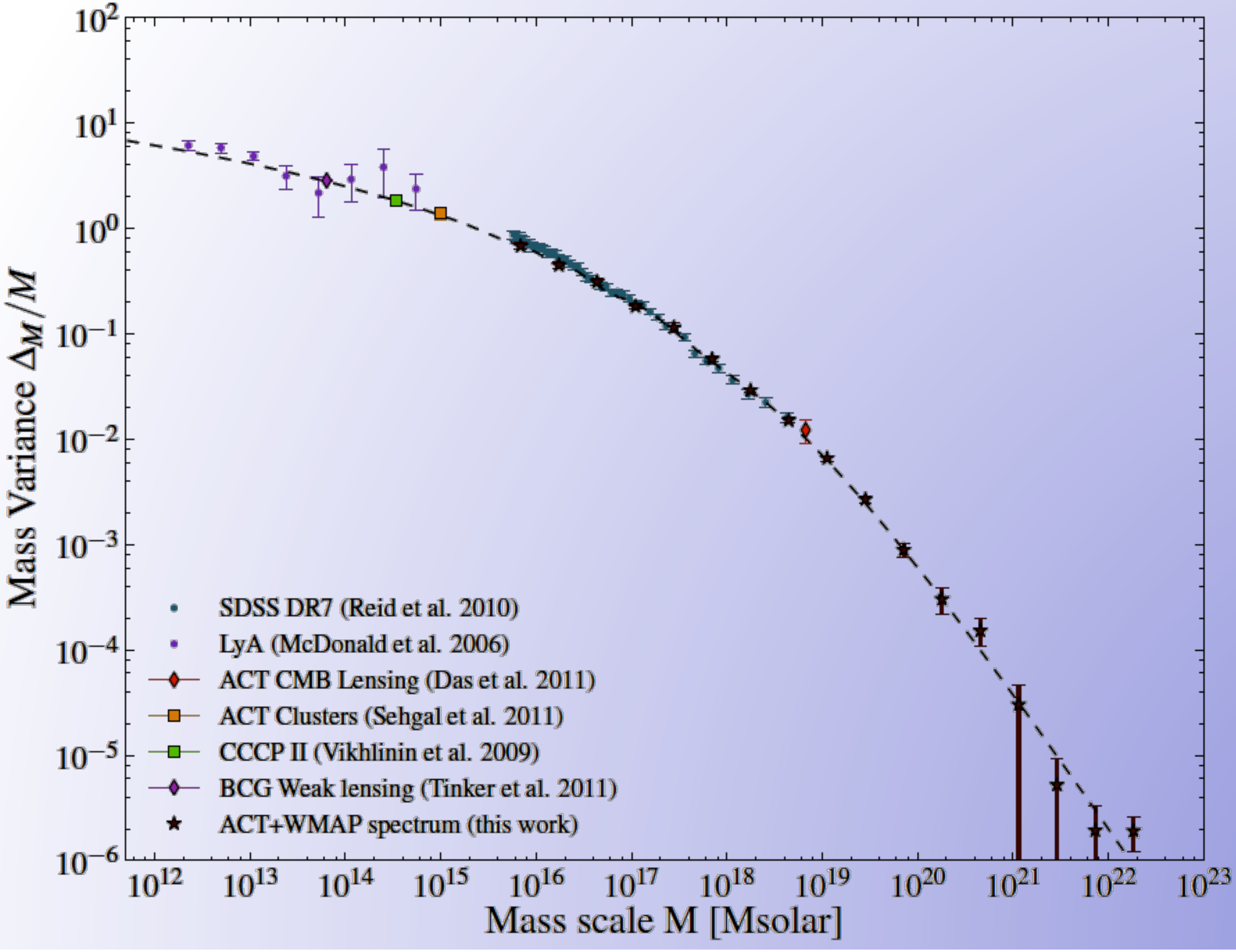
An old idea…

"With the new cosmology the universe must have started off in some very simple way. What, then, becomes of **the initial conditions required by dynamical theory**? Plainly there cannot be any, or they must be trivial. We are left in a situation which would be untenable with the old mechanics. If the universe were simply the motion which follow from a given scheme of equations of motion with trivial initial conditions, it could not contain the complexity we observe. **Quantum mechanics provides an escape from the difficulty. It enables us to ascribe the complexity to the quantum jumps, lying outside the scheme of equations of motion**."

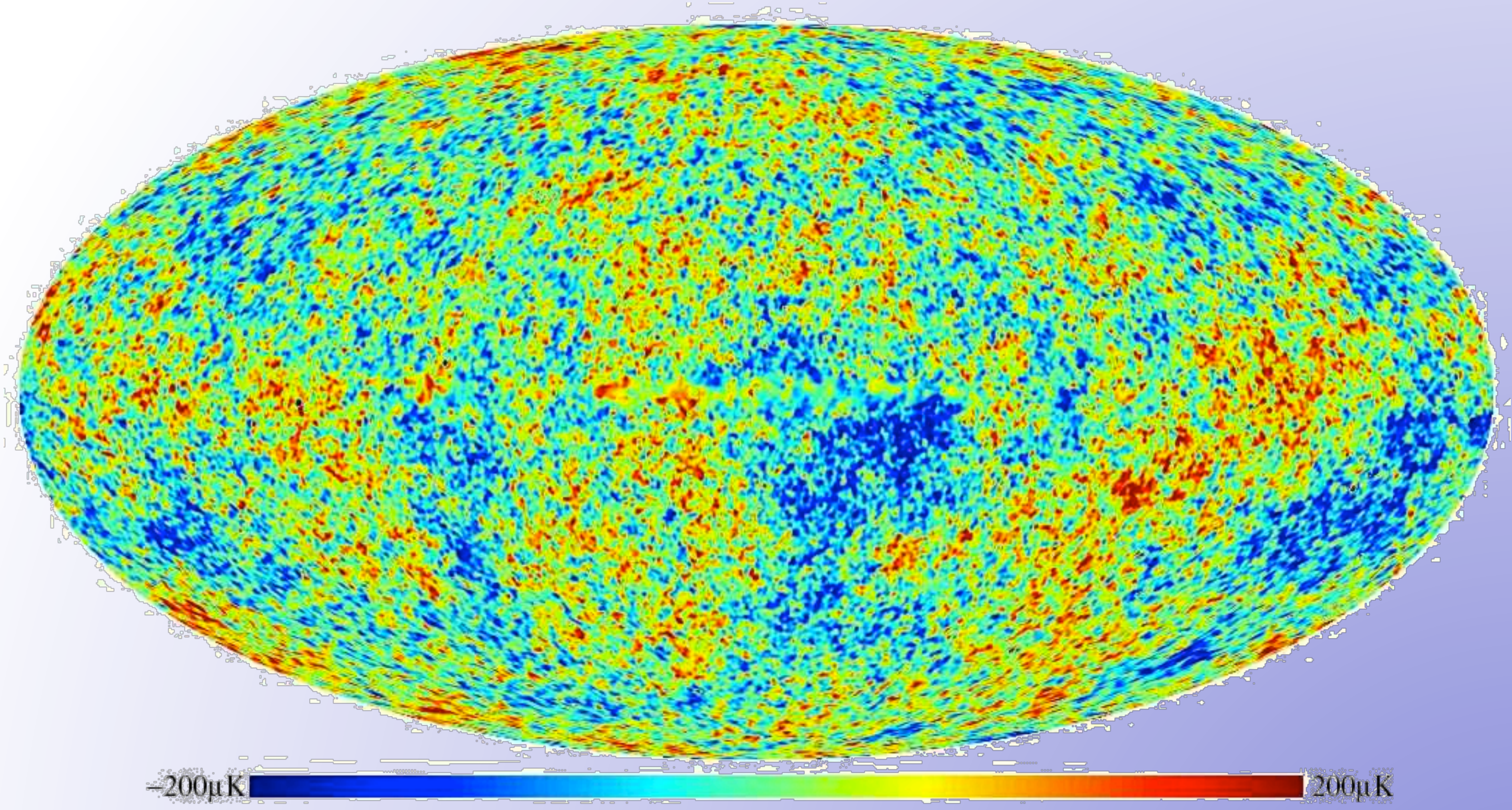
P.A.M. Dirac 1939

Power spectrum

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{1}{2k^3} \frac{H^4}{\dot{\phi}^2} \sim (10^{-5})^2$$



Is there any correlation among modes?



$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \cdots \zeta_{\vec{k}_n} \rangle \qquad \langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \cdots \gamma_{\vec{k}_n} \rangle$$

The most known thing

CMB:

WMAP 7:

$$f_{NL}^{\rm local} = 32 \pm 21 \ (68\% \ {\rm CL})$$

$$f_{NL}^{\rm equil} = 26 \pm 140 \ (68\% \ {\rm CL})$$

$\times 10^{-5}$!!

Planck: a factor of 4-6 improvement

Cosmic variance: an additional factor of 2 (?)

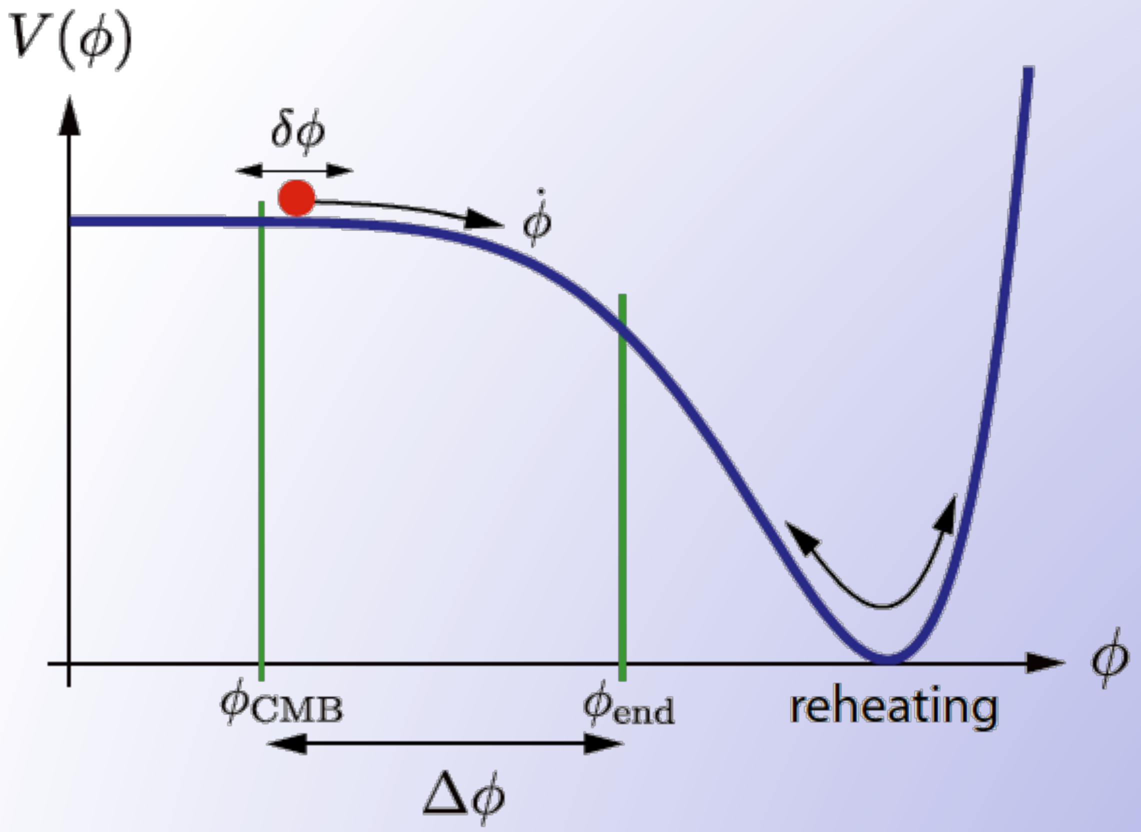
LSS:

SDSS: $f_{\rm NL}^{\rm local} = 20 \pm 25 \ (68\% {\rm CL})$

Future: $f_{NL}^{local} \sim 1$ (?)

21cm: $f_{NL} \sim \varepsilon, \eta$ (???)

Slow-roll = weak coupling = Gaussianity



$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2$$

$$\eta = M_P^2 \frac{V''}{V}$$

$$\epsilon, \eta, \ldots \ll 1$$

$$\lambda \equiv V^{(4)} \lesssim \mathcal{O}(\epsilon^3, \eta^3)(10^{-5})^2$$

Quantitative NG

Maldacena 02

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$\delta\phi = 0$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t) \left[e^{2\zeta} \delta_{ij} + \gamma_{ij} \right]$$

Solve for N, N^i ↓

$$\int d^4x\, a^3 \frac{\dot\phi^2}{2H^2} \left[\dot\zeta^2 - (\partial\zeta)^2/a^2 + \frac{2}{H} \frac{\partial_i}{\partial^2} \dot\zeta \partial_i \zeta \frac{\partial^2}{a^2} \zeta + \ldots \right]$$

$$\langle Q(t) \rangle = \left\langle \left[\bar{T} \exp\left(i \int_{-\infty}^{t} H_I(t)\, dt \right) \right] Q^I(t) \left[T \exp\left(-i \int_{-\infty}^{t} H_I(t)\, dt \right) \right] \right\rangle$$

$$\frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle}{\langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle^{3/2}} \sim \epsilon \frac{H}{\sqrt{\epsilon} M_P} \ll 10^{-5}$$

$$f_{\rm NL} \sim \epsilon$$

Smoking gun for "new physics"

Any signal would be a clear signal of something non-minimal

- Any modification enhances NG

1. Modify inflaton Lagrangian. Higher derivative terms (ghost inflation, DBI inflation), features in potential
2. Additional light fields during inflation. Curvaton, variable decay width…
3. Alternatives to inflation

- Potential wealth of information

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

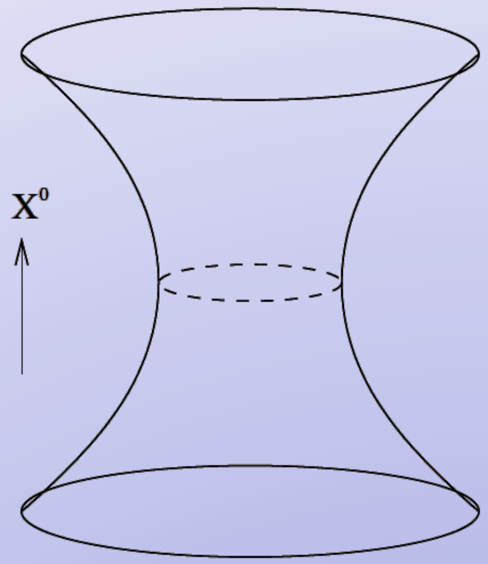
F contains information about the source of NG

Outline

- Scale-invariance → Conformal invariance ?
- Perturbations decoupled from the inflaton are SO(4,1) invariant
- Single field models: non-linear realization of conformal invariance

  → Generalization of the squeezed limit consistency relations
- Non-inflationary models (eg Galilean Genesis). SO(4,2) → SO(4,1)

de Sitter: SO(4,1)



Inflation takes place in ~ dS

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

- **Translations, rotations**: ok
- **Dilations** (if slow-roll) $\eta \rightarrow \lambda\eta,\ \vec{x} \rightarrow \lambda\vec{x}$

→ scale-invariance

$$\varphi_{\vec{k}} \rightarrow \lambda^3 \varphi_{\vec{k}/\lambda} \qquad \langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\rangle = (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2)\frac{1}{k_1^3}F(k_1\eta)$$

In general:

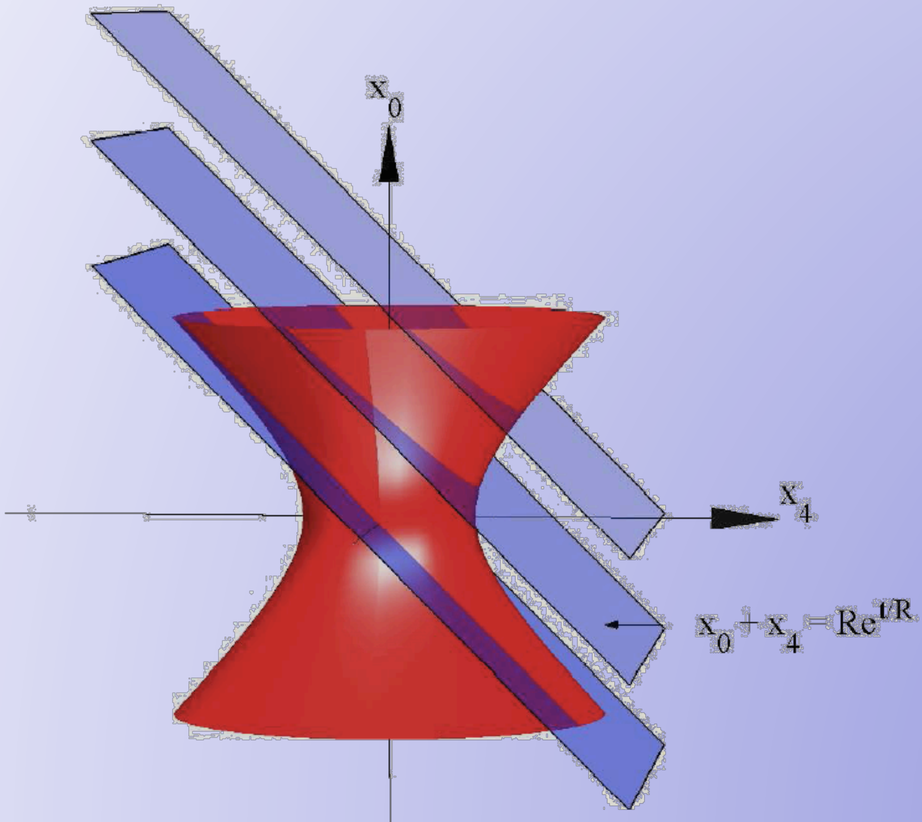
$$\langle \varphi_{\vec{k}_1} \dots \varphi_{\vec{k}_n}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)F(\vec{k}_1, \dots, \vec{k}_n)$$

with F homogeneous of degree -3(n-1)

Special conformal

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x}) \ , \quad x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$$

The inflaton background breaks these symmetries



Hierarchy of breakings

1. **Deviation of metric from dS**. One can consider the $\epsilon \to 0$ limit at fixed $H/(M_P\sqrt{\epsilon})$

2. **Breaking due to scalar background**. $\phi(\vec{x},t) = t + \pi(\vec{x},t)$

- Dilations. If we have (approximate) shift symmetry in ϕ, dilations are not broken

- Special conformal. I would need a Galilean invariance $\phi \to \phi + \vec{b}\cdot\vec{x}$

But this cannot be defined in the presence of gravity

Cheung etal 07

$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$

$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu\delta K^\nu{}_\mu + \ldots\Big]\,.$$

Time indep. coefficients

Scale → Conformal invariance

Antoniadis, Mazur and Mottola, 11   Maldacena and Pimental, 11

Curvaton, modulated reheating…

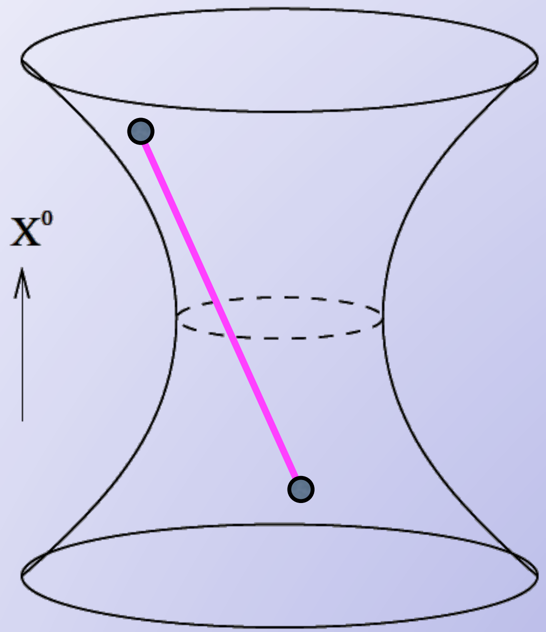
If perturbations are created by a sector with negligible interactions with the inflaton, correlation functions have the full SO(4,1) symmetry

They are conformal invariant

Independently of any details about this sector, even at strong coupling

Same as AdS/CFT

dS-invariant distance



$$\frac{|\vec{x}_i - \vec{x}_j|^2}{\eta_i \eta_j} - \left(\frac{\eta_i}{\eta_j} + \frac{\eta_j}{\eta_i} \right)$$

Scale → Conformal invariance

We are interested in correlators at late times

$$x^i \to x^i + b^i \vec{x}^2 - 2x^i(\vec{b}\cdot\vec{x}) \qquad \eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})$$

$$\varphi \sim \eta^\Delta\,, \quad \Delta = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right) \ll 1$$

This is the transformation of the a primary of conformal dim Δ

Example: $\quad m = \sqrt{2}H \qquad \Delta = 1$

$$\int d^4x\sqrt{-g}\frac{M}{6}\varphi^3 \longrightarrow \langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{\pi}{16}MH^2\eta_*^3 \cdot \frac{1}{k_1k_2k_3}$$

$$\langle\varphi(\vec{x}_1)\varphi(\vec{x}_2)\varphi(\vec{x}_3)\rangle = \frac{MH^2\eta_*^3}{128\pi^2}\cdot\frac{1}{|\vec{x}_1-\vec{x}_2||\vec{x}_1-\vec{x}_3||\vec{x}_2-\vec{x}_3|}$$

Massless scalars

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{H^2}{\prod_i 2k_i^3}\frac{2M}{3}\left[\sum_i k_i^3(-1+\gamma+\log(-k_t\eta_*)) + k_1k_2k_3 - \sum_{i\neq j} k_i^2 k_j\right]$$

$$k_t \equiv \sum_i k_i$$

Zaldarriaga 03
Seery, Malik,Lyth 08

$$\langle \varphi_1(\vec{x}_1)\varphi_2(\vec{x}_2)\varphi_3(\vec{x}_3)\rangle = \frac{MH^2}{48\pi^2}\log\frac{|\vec{x}_1-\vec{x}_2|}{A\eta_*}\log\frac{|\vec{x}_1-\vec{x}_3|}{A\eta_*}\log\frac{|\vec{x}_2-\vec{x}_3|}{A\eta_*}$$

Everything determined up to two constants

Independently of the interactions!

$$\frac{1}{M}\int d^4x\sqrt{-g}\nabla_\mu\varphi_1\nabla^\mu\varphi_2\varphi_3 \longrightarrow \frac{1}{M}\int d^4x\sqrt{-g}\frac{1}{2}(\Box\varphi_3\varphi_1\varphi_2 - \Box\varphi_1\varphi_2\varphi_3 - \Box\varphi_2\varphi_1\varphi_3)$$

The conversion to ζ will add a local contribution: $\zeta(\vec{x}) = A_I\varphi^I(\vec{x}) + B_{IJ}\varphi^I(\vec{x})\varphi^J(\vec{x})$

4-point function

$$\int d^4x \frac{1}{8M^4}(\partial_\mu \varphi)^2(\partial_\nu \varphi)^2$$

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{1}{M^4}\frac{H^8}{\prod_i 2k_i^3}\left[-\frac{144k_1^2k_2^2k_3^2k_4^2}{k_t^5} - 4\left(\frac{12k_1k_2k_3k_4}{k_t^5} + \frac{3\prod_{i<j<l}k_ik_jk_l}{k_t^4} + \frac{\prod_{i<j}k_ik_j}{k_t^3} + \frac{1}{k_t}\right)\right.$$
$$\left.((\vec{k}_1\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_3)(\vec{k}_2\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_4)(\vec{k}_2\cdot\vec{k}_3))+(\vec{k}_1\cdot\vec{k}_2)\left(\frac{4k_3^2k_4^2}{k_t^3} + \frac{12(k_1+k_2)k_3^2k_4^2}{k_t^4} + \frac{48k_1k_2k_3^2k_4^2}{k_t^5}\right) + 6\text{perm.}\right],$$

Not so obvious it is conformal invariant…

I can check it in **Fourier space**

Maldacena and Pimental, '11

$$\sum_{a=1,2,3,4}\left[6\vec{b}\cdot\vec{\partial}_{k_a} - \vec{b}\cdot\vec{k}_a\vec{\partial}^2_{k_a} + 2\vec{k}_i\cdot\vec{\partial}_{k_a}(\vec{b}\cdot\vec{\partial}_{k_a})\right]\langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle' = 0$$

In general: $$F\left(\frac{r_{13}r_{24}}{r_{12}r_{34}}, \frac{r_{23}r_{41}}{r_{12}r_{34}}\right)\prod_{i<j} r_{ij}^{-2\Delta/3}$$

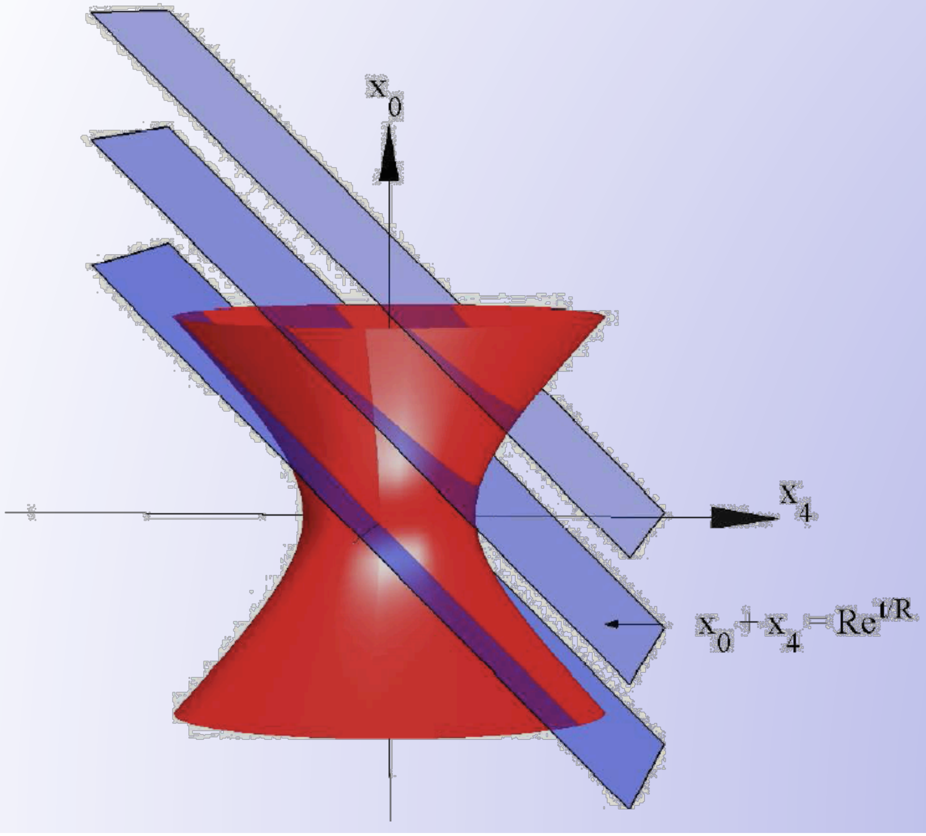
2 parameters instead of 5

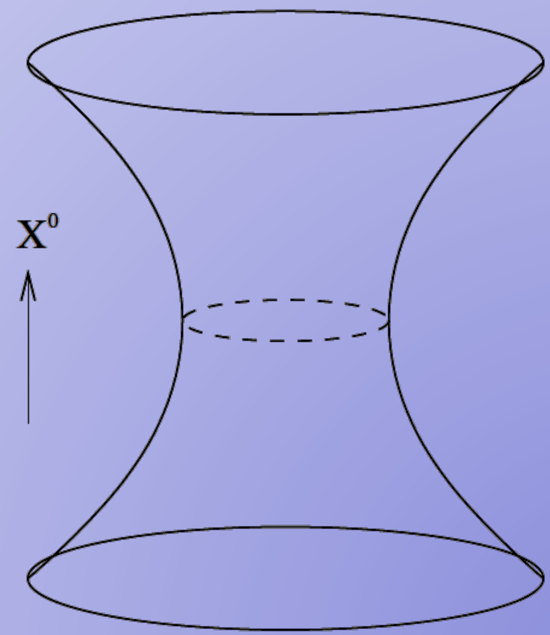
Therefore

If we see something beyond the spectrum

- Something not conformal would be a probe of a "sliced" de Sitter

- Something conformal would be a probe of pure de Sitter





Non-linearly realized symmetries

The inflaton background breaks the symmetry. **Spontaneously**.

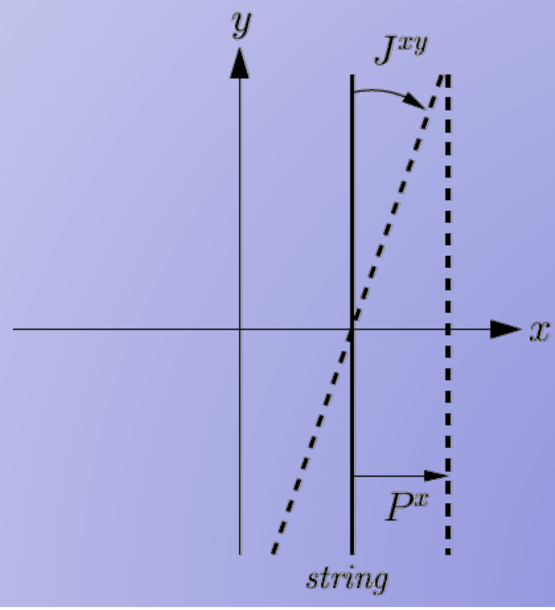
We expect the symmetry to be still there to regulate soft limit (q → 0) of correlation functions (Ward identities)

$$\lim_{q\to 0} q^\mu \Gamma^{(n)}_{J_\mu}(q;p_1,\ldots,p_n) = \delta\Gamma^{(n)}(p_1,\ldots,p_n)$$

For example. Soft emission of π's

For space time symmetries:
number of Goldstones ≠ broken generators

Manohar Low 01

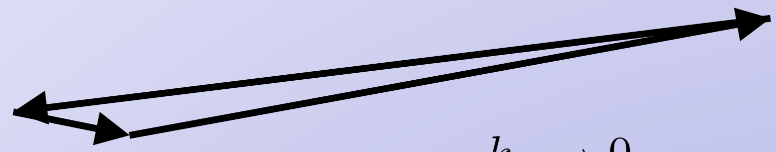


We expect Ward identities to say something about higher powers of q

3pf consistency relation

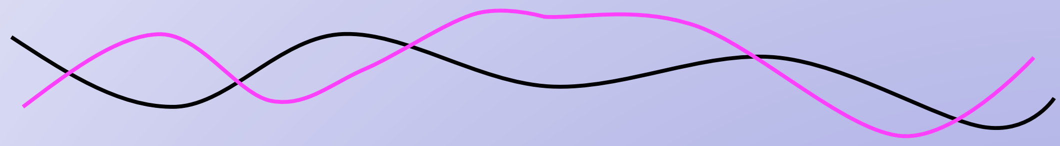
Maldacena 02
PC, Zaldarriaga 04
Cheung etal. 07

Squeezed limit of the 3-point function in single-field models



$k_1 \to 0$

Similar to the absence of isocurvature



$$\frac{\delta\rho_m}{\rho_m} = \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}$$

$$\phi(t,\vec{x}) = \phi_0(t) \qquad h_{ij} = e^{2\zeta(t,\vec{x})}\delta_{ij}$$

The long mode is already classical when the other freeze and acts just as a rescaling of the coordinates

$$\langle\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle|_{\bar{\zeta}(x)} \simeq \xi(\vec{x}_3-\vec{x}_2) + \bar{\zeta}(\vec{x}_+)[(\vec{x}_3-\vec{x}_2)\cdot\vec{\nabla}\xi(|\vec{x}_3-\vec{x}_2|)]$$

3pf consistency relation

$$\langle \bar{\zeta}(\vec{x}_1)\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle \simeq \langle \bar{\zeta}(\vec{x}_1)\bar{\zeta}(\vec{x}_+)\rangle[(\vec{x}_3-\vec{x}_2)\cdot\vec{\nabla}\xi(|\vec{x}_3-\vec{x}_2|)]$$

$$\simeq \int \frac{\mathrm{d}^3k_L}{(2\pi)^3}\int \frac{\mathrm{d}^3k_S}{(2\pi)^3} e^{i\vec{k}_L\cdot(\vec{x}_1-\vec{x}_+)} P(k_L)P(k_S)\left[\vec{k}_S\cdot\frac{\partial}{\partial\vec{k}_S}\right]e^{i\vec{k}_S\cdot\vec{x}_-}$$

$$= -\int \frac{\mathrm{d}^3k_1\,\mathrm{d}^3k_L\,\mathrm{d}^3k_S}{(2\pi)^9} e^{-i\vec{k}_1\cdot\vec{x}_1-i\vec{k}_L\cdot\vec{x}_++i\vec{k}_S\cdot\vec{x}_-}\left[(2\pi)^3\delta(\vec{k}_1+\vec{k}_L)P(k_1)P(k_S)\frac{\mathrm{d}\ln k_S^3P(k_S)}{\mathrm{d}\ln k_S}\right]$$

$\downarrow$

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)P(k_1)P(k_S)\frac{\mathrm{d}\ln(k_S^3P(k_S))}{\mathrm{d}\ln k_S}$$

$k_1 \to 0$

$$\vec{k}_S = (\vec{k}_2-\vec{k}_3)/2$$

Single-field 3pf is very suppressed in the squeezed limit

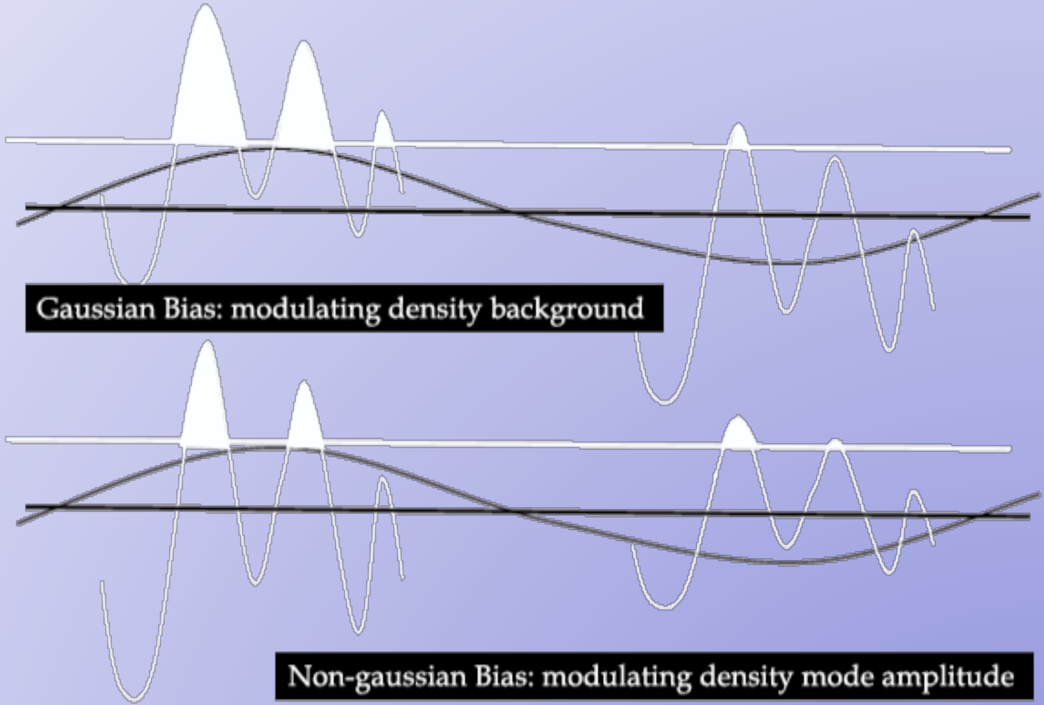
Phenomenologically relevant

1. A detection of a local f_{NL} would rule out any single-field model

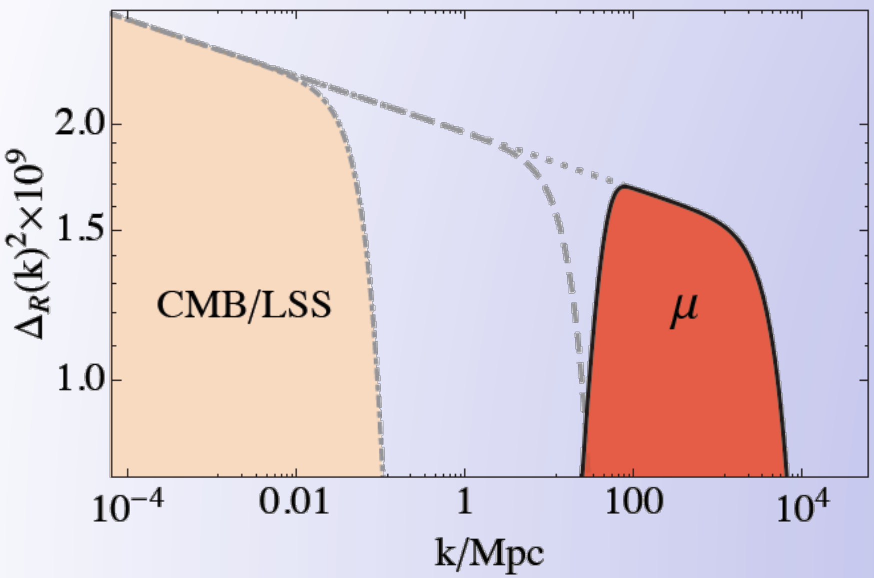
2. Some of the experimental probes are sensitive only to squeezed limits

- Scale dependent bias

Dalal etal 07



- CMB μ distorsion

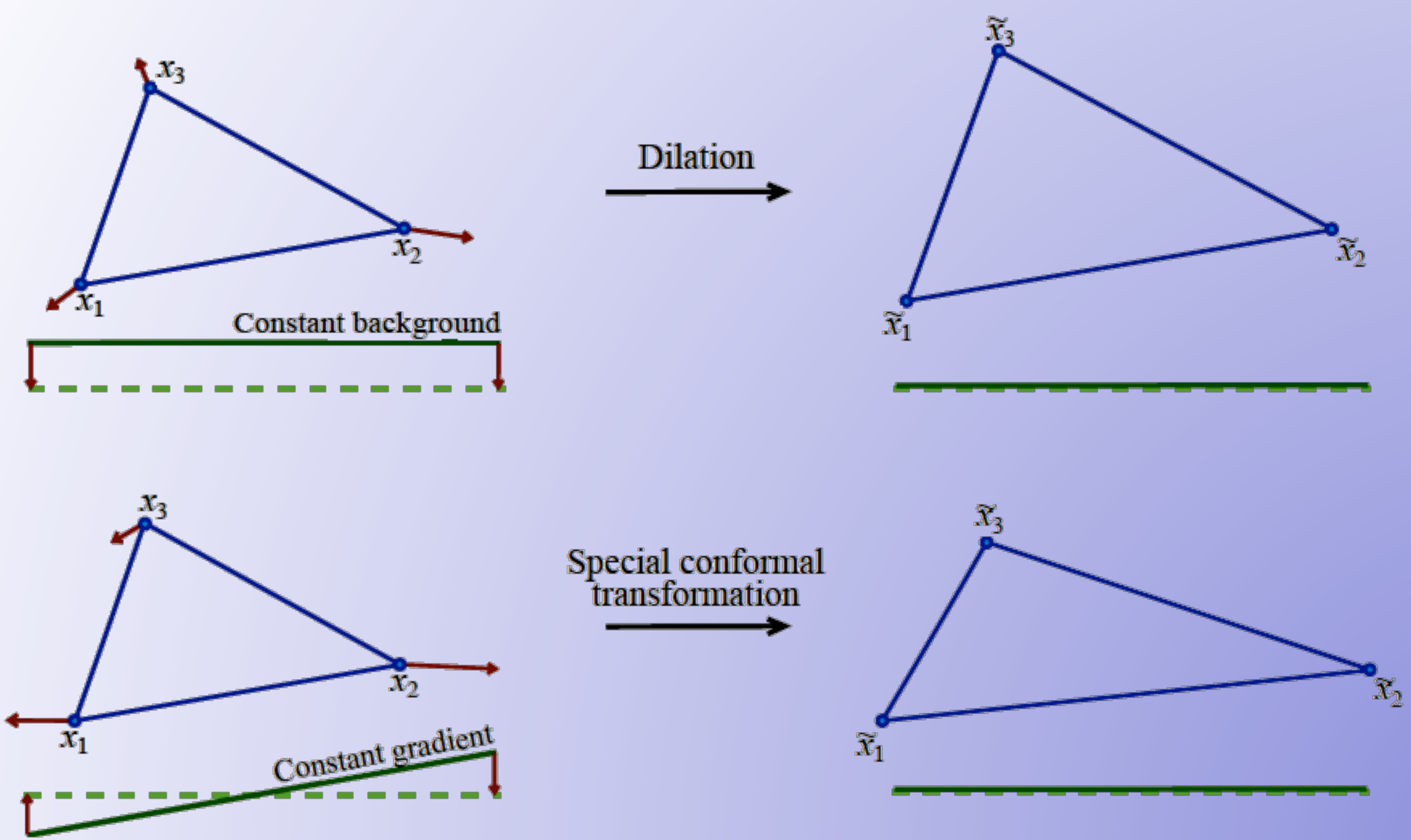


Pajer and Zaldarriaga 12

Extension to the full SO(4,1)

A special conformal transformation induces a conformal factor linear in x

$$\zeta = 2\vec{b} \cdot \vec{x} + \lambda$$



Conformal consistency relations

(Assuming zero tilt for simplicity)

$$\langle \zeta_{\vec{q}}\, \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \dots \zeta_{\vec{k}_n} \rangle' \overset{q\to 0}{=} -\frac{1}{2} P(q) q^i D_i \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \dots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6\vec{q}\cdot\vec{\partial}_{k_a} - \vec{q}\cdot\vec{k}_a \vec{\partial}^2_{k_a} + 2\vec{k}_a\cdot\vec{\partial}_{k_a}(\vec{q}\cdot\vec{\partial}_{k_a}) \right]$$

2- and 3-pf only depends on moduli and $q^i D_i$ reduces to: $\sum_{a=1}^{n} \vec{q}\cdot\vec{k}_a \left[\frac{4}{k_a}\frac{\partial}{\partial k_a} + \frac{\partial^2}{\partial k_a^2} \right]$

The variation of the 2-point function is zero: no linear term in the 3 pf

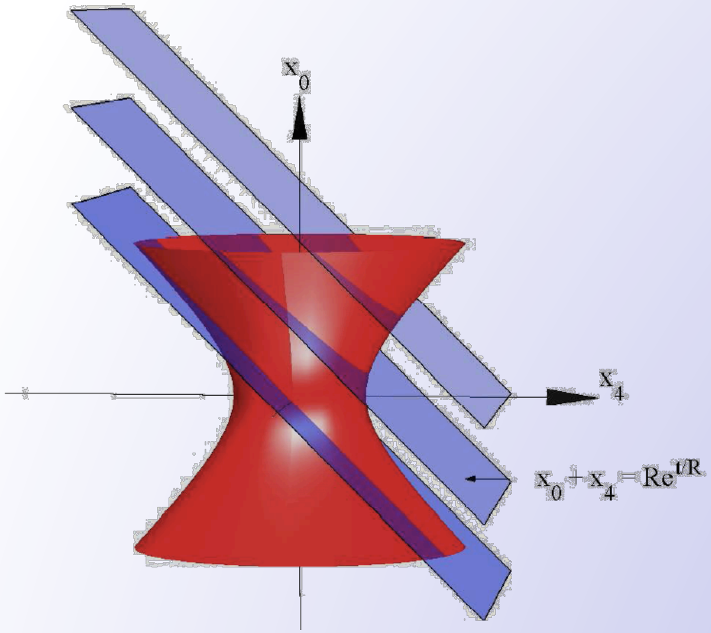
with D'Amico, Musso and Noreña, 11

Conformal consistency relations as Ward identities and with OPE methods

Hinterbichler, Hui and Khoury 12
Kheagias, Riotto 12
in progress by Goldberger, Hinterbichler, Hui, Khoury, Nicolis

Non-linear realization of dS isometries

In decoupling + dS limit: the inflaton breaks spontaneoulsy SO(4,1).
It is still non-linearly realized



$$\phi(\vec{x}, t) = t + \pi(\vec{x}, t)$$

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})$$

$$x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$$

$$\pi \to \pi + 2H^{-1}(\vec{b}\cdot\vec{x})$$

Notice the two meanings of SO(4,1):

- Isometry group of de Sitter
- Conformal group of 3d Euclidean

Adiabatic mode including gradients

Adiabatic modes can be constructed from unfixed gauge transformations (k=0)

Weinberg 03

In ζ gauge: $\phi(t,\vec{x}) = \phi_0(t) \qquad h_{ij} = e^{2\zeta(t,\vec{x})}\delta_{ij}$

- Cannot touch t
- Conformal transformation of the spatial coordinates: $\zeta = 2\vec{b}(t)\cdot\vec{x} + \lambda(t)$
- Impose it is the k → 0 limit of a physical solution

$$\partial_j(H\delta N - \dot{\zeta}) = 0 \qquad (3H^2 + \dot{H})\delta N + H\partial_i N^i = -\frac{\nabla^2}{a^2}\zeta + 3H\dot{\zeta}$$

- b and λ are time-independent + need a time-dep translation to induce the N^i

Long wavelength approx of an adiabatic mode up to $O(k^2)$

3pf - 4pf in slow-roll inflation

Maldacena 02

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta^3 (\sum \vec{k}_i) \frac{\dot{\rho}_*^4}{\dot{\phi}_*^4} \frac{H_*^4}{M_{pl}^4} \frac{1}{\prod_i (2k_i^3)} \mathcal{A}_*$$

$$\mathcal{A} = 2 \frac{\ddot{\phi}_*}{\dot{\phi}_* \dot{\rho}_*} \sum_i k_i^3 + \frac{\dot{\phi}_*^2}{\dot{\rho}_*^2} \left[\frac{1}{2} \sum_i k_i^3 + \frac{1}{2} \sum_{i \neq j} k_i k_j^2 + 4 \frac{\sum_{i>j} k_i^2 k_j^2}{k_t} \right]$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle^{\mathrm{CI}} = (2\pi)^3 \delta (\sum_a \mathbf{k}_a) \frac{H_*^6}{4\epsilon^2 \prod_a (2k_a^3)} \sum_{\mathrm{perms}} \mathcal{M}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

$$\mathcal{M}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = -2 \frac{k_1^2 k_3^2}{k_{12}^2 k_{34}^2} \frac{W_{24}}{k_t} \left(\frac{\mathbf{Z}_{12} \cdot \mathbf{Z}_{34}}{k_{34}^2} + 2\mathbf{k}_2 \cdot \mathbf{Z}_{34} + \frac{3}{4} \sigma_{12} \sigma_{34} \right) - \frac{1}{2} \frac{k_3^2}{k_{34}^2} \sigma_{34} \left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_t} W_{124} + 2 \frac{k_1^2 k_2^2}{k_t^3} + 6 \frac{k_1^2 k_2^2 k_4}{k_t^4} \right),$$

$$\begin{aligned} \sigma_{ab} &= \mathbf{k}_a \cdot \mathbf{k}_b + k_b^2 , \\ \mathbf{Z}_{ab} &= \sigma_{ab} \mathbf{k}_a - \sigma_{ba} \mathbf{k}_b , \\ W_{ab} &= 1 + \frac{k_a + k_b}{k_t} + \frac{2 k_a k_b}{k_t^2} , \\ W_{abc} &= 1 + \frac{k_a + k_b + k_c}{k_t} + \frac{2(k_a k_b + k_b k_c + k_a k_c)}{k_t^2} + \frac{6 k_a k_b k_c}{k_t^3} \end{aligned}$$

Lidsey, Seery, Sloth 06
Seery, Sloth, Vernizzi 09

Small speed of sound

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(M_{\rm Pl}^2 R + 2P(X,\phi)\right) \qquad X = -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$c_s^2 \equiv \frac{P_{,X}}{P_{,X}+2XP_{,XX}}$$

$$\lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}$$

$$\Sigma = X^2P_{,X} + 2X^2P_{,XX}\,.$$

$$P_\zeta = \frac{1}{2M_{\rm Pl}^2}\frac{H^2}{c_s\epsilon}$$

$$\mu = \frac{1}{2}X^2P_{,XX} + 2X^3P_{,XXX} + \frac{2}{3}X^4P_{,XXXX}$$

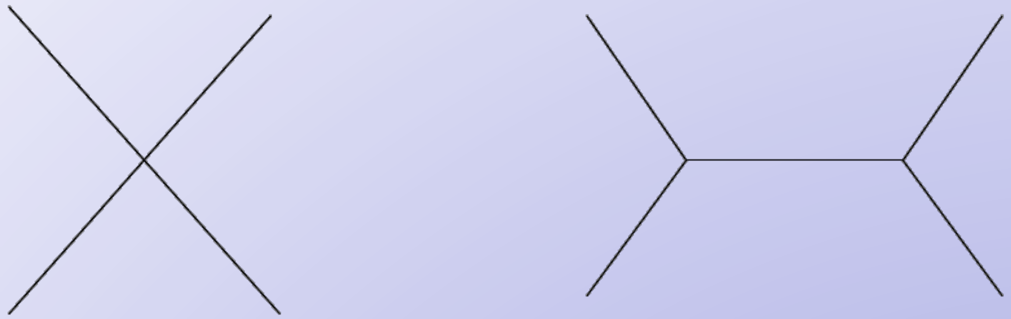
$$\langle\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_n}\rangle = (2\pi)^3\delta(\vec{k}_1+\cdots+\vec{k}_n)P_\zeta^{n-1}\prod_{i=1}^{n}\frac{1}{k_i^3}\mathcal{M}^{(n)}(\vec{k}_1,...,\vec{k}_n)$$

$$\mathcal{M}^{(3)} = \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2k_t^3} + \left(\frac{1}{c_s^2}-1\right)\left(-\frac{1}{k_t}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2k_t^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{8}\sum_i k_i^3\right)$$

E.g. X. Chen etal 09

Small speed of sound

4pf: scalar exchange diag.s
do not contribute to squeezed limit



$$\mathcal{M}^{(4)}_{cont} = \left[\frac{3}{2}\left(\frac{\mu}{\Sigma} - \frac{9\lambda^2}{\Sigma^2}\right)\frac{\prod_{i=1}^4 k_i^2}{k_t^5} - \frac{1}{8}\left(\frac{3\lambda}{\Sigma} - \frac{1}{c_s^2} + 1\right)\frac{k_1^2 k_2^2 (\vec{k}_3 \cdot \vec{k}_4)}{k_t^3}\left(1 + \frac{3(k_3+k_4)}{k_t} + \frac{12 k_3 k_4}{k_t^2}\right) \right.$$
$$\left. + \frac{1}{32}\left(\frac{1}{c_s^2} - 1\right)\frac{(\vec{k}_1 \cdot \vec{k}_2)(\vec{k}_3 \cdot \vec{k}_4)}{k_t}\left(1 + \frac{\sum_{i<j} k_i k_j}{k_t^2} + \frac{3 k_1 k_2 k_3 k_4}{k_t^3}\sum_{i=1}^{4}\frac{1}{k_i} + \frac{12 k_1 k_2 k_3 k_4}{k_t^4}\right)\right] + 23 \text{ perm.}$$

- At the level of observables, the non-linear relation among operators in the Lagrangian
- Squeezed limit is $1/c_s^2$ while the full 4pf is $1/c_s^4$

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \simeq -\frac{1}{2} P(q) q^i D_i \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \propto \frac{1}{c_s^2}$$

- A large 4pf cannot have a squeezed limit

Conformal consistency relations with tilt

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \dots \zeta_{\vec{k}_n} \rangle' \stackrel{q\to 0}{=} -P(q) \left[-3(n-1) - \sum_a \vec{k}_a \cdot \vec{\partial}_{k_a} + \frac{1}{2} q^i D_i \right] \langle \zeta_{\vec{k}_1} \dots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6\vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2\vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right] .$$

- Dilation part evaluated on a non-closed polygon

- Verified in modes with oscillations in the inflaton potential

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \dots \zeta_{\vec{k}_n} \rangle = (2\pi)^3 \delta(\vec{q} + \vec{k}_1 + \dots + \vec{k}_n) \left(-\frac{H}{\dot{\phi}} \right)^{n+1} \frac{H^{2n-2}}{2q^3 \prod_{i=1}^{n} 2k_i^3} I_{n+1}$$

$$I_{n+1} = -2\,\mathrm{Im} \int_{-\infty - i\epsilon}^{0} \frac{\mathrm{d}\eta}{\eta^4} V^{(n+1)}(\phi(\eta))(1 - iq\eta)(1 - ik_1\eta) \dots (1 - ik_n\eta) e^{ik_t\eta}$$

Generalizations

- Graviton correlation functions:

$$x_i \to x_i + A_{ij}x_j + B_{ijk}x_jx_k$$

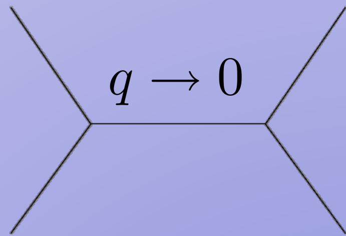
Induce long graviton with

$$A_{ij} = \frac{1}{2}\gamma_{ij} \ , \quad B_{ijk} = \frac{1}{4}(\partial_k\gamma_{ij} - \partial_i\gamma_{jk} + \partial_j\gamma_{ik})$$

$$\langle \gamma^s_{\vec{q}} \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle'_{q\to 0} = -\frac{1}{2}P_\gamma(q)\sum_a \epsilon^s_{ij}(\vec{q})k_{ai}\partial_{k_{aj}}\langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n}\rangle'$$
$$-\frac{1}{4}P_\gamma(q)\sum_a \epsilon^s_{ij}(\vec{q})\left(2k_{ai}(\vec{q}\cdot\vec{\partial}_{k_a}) - (\vec{q}\cdot\vec{k}_a)\partial_{k_{ai}}\right)\partial_{k_{aj}}\langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n}\rangle'$$

Not more than one…

- Soft internal lines



$$\langle \zeta_{\vec{k}_1} \cdots \zeta_{\vec{k}_n}\rangle'_{\vec{q}\to 0} = P_\zeta(q)\langle \zeta_{-\vec{q}}\zeta_{\vec{k}_1} \cdots \zeta_{\vec{k}_m}\rangle^*_{\vec{q}\to 0}\langle \zeta_{\vec{q}}\zeta_{\vec{k}_{m+1}} \cdots \zeta_{\vec{k}_n}\rangle^*_{\vec{q}\to 0} +$$
$$+ P_\gamma(q)\sum_s \langle \gamma^s_{-\vec{q}}\zeta_{\vec{k}_1} \cdots \zeta_{\vec{k}_m}\rangle^*_{\vec{q}\to 0}\langle \gamma^s_{\vec{q}}\zeta_{\vec{k}_{m+1}} \cdots \zeta_{\vec{k}_n}\rangle^*_{\vec{q}\to 0}$$

- More than one q going to zero together

SO(4,2) → SO(4,1)

Rubakov 09
Nicolis, Rattazzi, Trincherini 09
PC, Nicolis and Trincherini 10
Hinterbichler Khoury 11
Hinterbichler Joyce Khoury 12

Non linearly realized conformal symmetry. The time dependent solution is SO(4,1)

$$\mathcal{S}_\pi = \int d^4x\,\sqrt{-g}\left[f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(\partial\pi)^4\right]$$

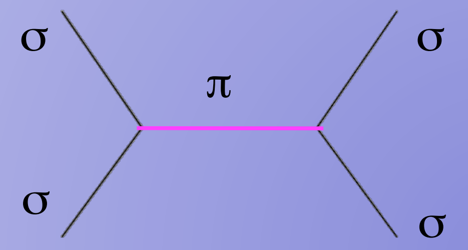
E.g. Galilean Genesis

$$e^{\pi_{\rm dS}} = -\frac{1}{H_0 t} \qquad -\infty < t < 0$$

$$g^{(\pi)}_{\mu\nu} \equiv e^{2\pi(x)}\eta_{\mu\nu}$$ is "de Sitter"

A test scalar σ must couple to the π "metric": correlation functions are SO(4,1)

But we also have the non-linear realization of SO(4,2): e.g. the mass of π is fixed and the soft emission of π



with Joyce, Khoury and Simonović in progress

Conclusions

- Linear realization of SO(4,1) for mechanisms decoupled from inflaton

- Non-linear realization for single-field models

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' \overset{q\to 0}{=} -P(q) \left[-3(n-1) - \sum_a \vec{k}_a \cdot \vec{\partial}_{k_a} + \frac{1}{2} q^i D_i \right] \langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6\vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2\vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right] .$$

- Fake SO(4,1) symmetry from 4d conformal

- Future directions:

  - Relation with Ward identities for spontaneously broken symmetries

  - Extension to models with SO(4,2)

  - Extension to late cosmology

The not-so-squeezed limit

P.C., G D'Amico, Musso, Noreña 11

At lowest order in derivatives

$$S_2 + S_3 = M_{\rm Pl}^2 \int d^4x \, \epsilon a^3 \left[(1 + 3\zeta_B)\dot{\zeta}^2 - (1 + \zeta_B)\frac{(\partial_i \zeta)^2}{a^2} \right]$$

Long mode reabsorbed by coordinate rescaling $\vec{x} \to (1 + \zeta_B)\vec{x}$

Corrections:

- Time evolution of ζ is of order k^2
- Spatial derivatives will be symmetrized with the short modes, giving k^2
- Constraint equations give order k^2 corrections

Final result: in the not-so-squeezed limit we have

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) P(k_1) P(k_S) \left[\frac{\mathrm{d} \ln (k_S^3 P(k_S))}{\mathrm{d} \ln k_S} + \mathcal{O}\left(\frac{k_1^2}{k_S^2}\right) \right]$$

Why do we care?

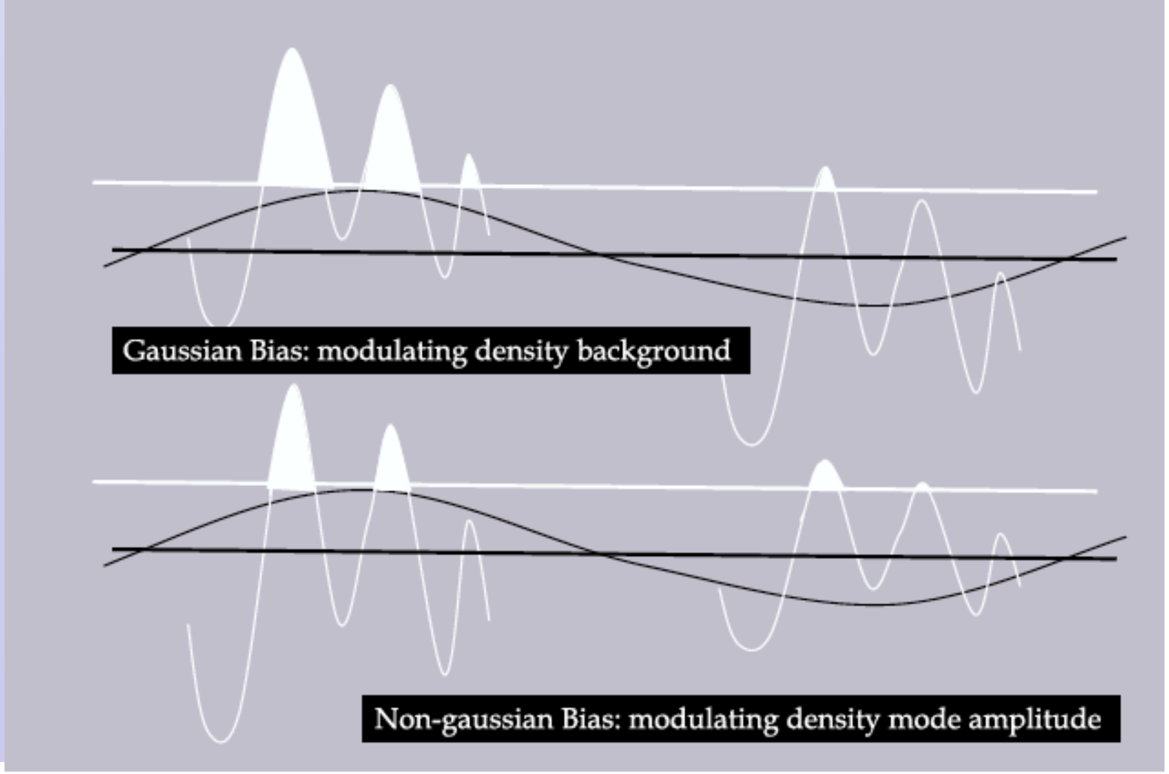
Dalal etal 07

LSS is a powerful probe of NG: **scale-dependent bias**

$$b_h(k) \sim \frac{\delta_h(k)}{\delta_m(k)}$$

Local NG induces a correlation between large scale and small scale power.
Modifies the relation among halo and matter perturbations.

Afshordi, Tolley 08



No scale dependent bias

Scale-dependent bias is sensitive to the squeezed limit

Probability of crossing: $\mathrm{Exp}\left(-\frac{\delta^2}{2\sigma^2}\right)$

Long mode changes the variance $\propto \Phi_L$

$\nabla^2\Phi_L \propto \delta_L$

Therefore, on large scales, for local NG, $\frac{\Delta b_h}{b_h} \sim \frac{f_{\rm NL}}{k^2}$

(Bias on large scales goes to a constant)

A detection of bias going as k^{-1} would **rule out all single field models**

How squeezed is squeezed?

Matarrese and Verde 08

$$\frac{\Delta b_h(k,R)}{b_h} = \frac{\delta_c}{D(z)\mathcal{M}_R(k)} \frac{1}{8\pi^2\sigma_R^2} \int_0^\infty \mathrm{d}k_1\, k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^{1} \mathrm{d}\mu\, \mathcal{M}_R\left(\sqrt{k^2+k_1^2+2kk_1\mu}\right) \frac{F\left(k_1, \sqrt{k^2+k_1^2+2kk_1\mu}, k\right)}{P_\zeta(k)}$$

$$\mathcal{M}_R(k) \equiv \frac{2k^2}{5\Omega_m H_0^2} T(k) W_R(k)$$

Local template cut at a squeezing ratio α