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RUB

Chiral dynamics: from hadrons to nuclei

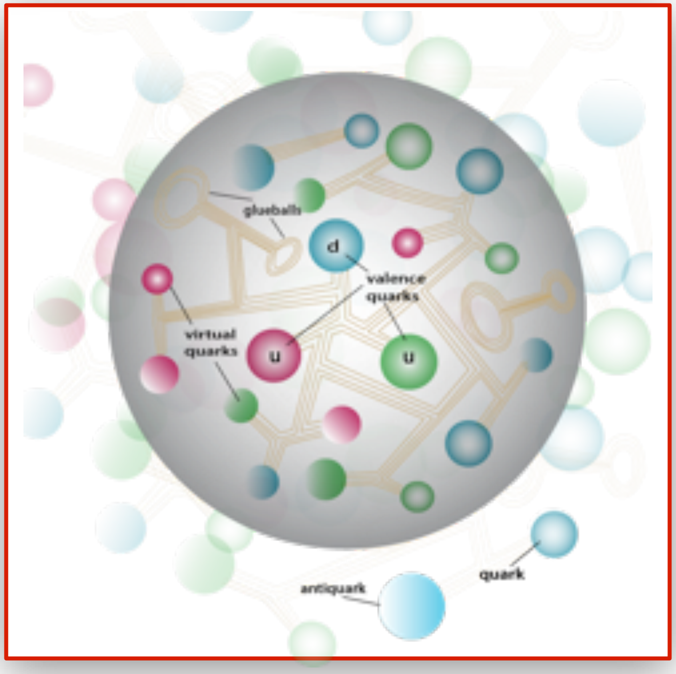
Outline

- Introduction
- Nuclear forces from χEFT: State of the art
- Three-nucleon force & πN dynamics
- Light nuclei on the lattice
- Summary & outlook

Quantum Chromodynamics (QCD)

The roadmap:

QCD ⟶ Chiral EFT ⟶ hadron dynamics

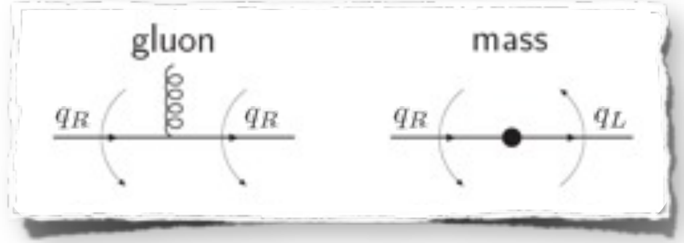


nucleon

nucleus

Chiral perturbation theory

- ## QCD and chiral symmetry



$$\mathcal{L}_{\rm QCD} = -\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \underbrace{\bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R}_{SU(2)_L \times SU(2)_R \text{ invariant}} - \underbrace{\bar{q}_L M q_R - \bar{q}_R M q_L}_{\text{break chiral symmetry}}$$

Light quark masses ($\overline{\rm MS},\ \mu = 2$ GeV): $\begin{array}{lcl} m_u &=& 1.5\ldots3.3 \text{ MeV} \\ m_d &=& 3.5\ldots6.0 \text{ MeV} \end{array} \ll \Lambda_{QCD} \sim 220 \text{ MeV}$

→ $\mathcal{L}_{\rm QCD}$ is approx. $SU(2)_L \times SU(2)_R$ invariant

spontaneous breakdown to $SU(2)_V \subset SU(2)_L \times SU(2)_R$ → Goldston Bosons (pions)

- ## Chiral perturbation theory

  - Ideal world [$m_u = m_d = 0$], zero-energy limit: non-interacting massless GBs (+ strongly interacting massive hadrons)

  - Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy: weakly interacting light GBs (+ strongly interacting massive hadrons)

→ expand about the ideal world (ChPT)

Chiral perturbation theory

Effective Lagrangian for hadronic DOF (π, N, ...) Chiral symmetry!

pion decay constant (in the chiral limit)
F_π ~ 94 MeV

pion fields

quark mass matrix

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mathrm{Tr}(U\chi + U^\dagger\chi)\right], \quad \text{where } \chi = 2B\mathcal{M}.$$

$$\mathcal{L}_\pi^{(4)} = L_1[\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)]^2 + L_2 \mathrm{Tr}(\partial_\mu U^\dagger \partial_\nu U)\mathrm{Tr}(\partial^\mu U^\dagger \partial^\nu U) + \ldots + L_8 \mathrm{Tr}(\chi U \chi U + \chi U^\dagger \chi U^\dagger)$$

low-energy constants

Gasser, Leutwyler '84

- Low-energy observables computable via a perturbative expansion in $Q = \frac{p \sim M_\pi}{\Lambda_\chi}$
  Weinberg '79
  hard scale that enters L_i

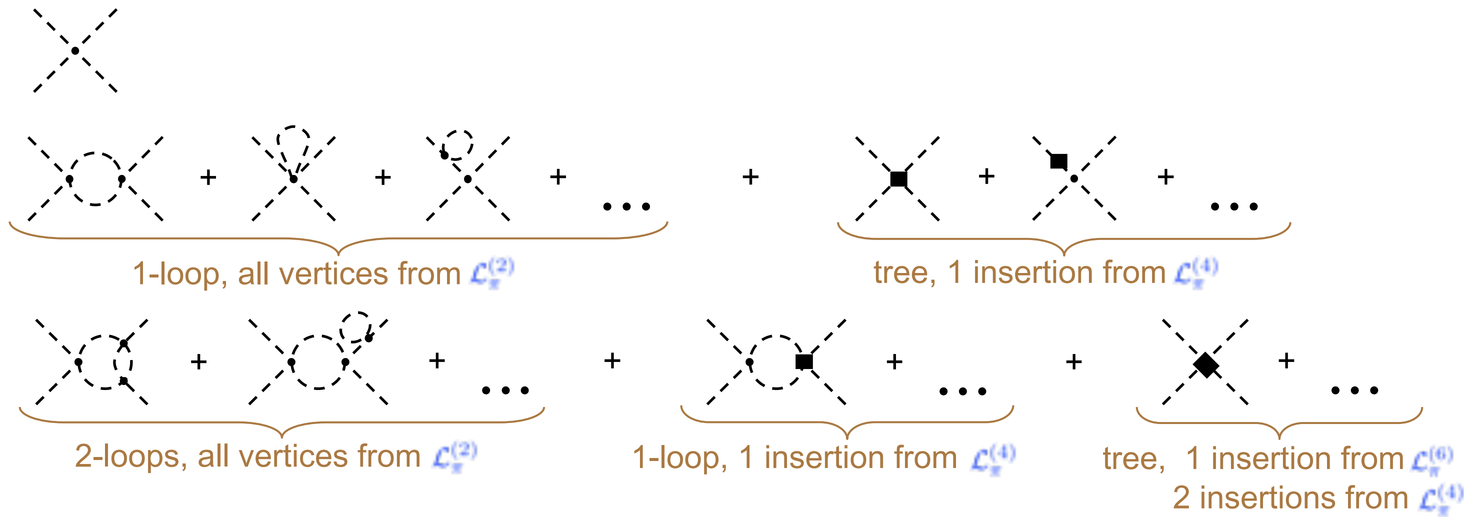
- At any order Q^n, a finite number of (unknown) LECs contribute

Pion scattering lengths in ChPT

Q^2 :

Q^4 :

Q^6 :



Predictive power?

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \dots$$

# of LECs increasing…

S-wave ππ scattering length

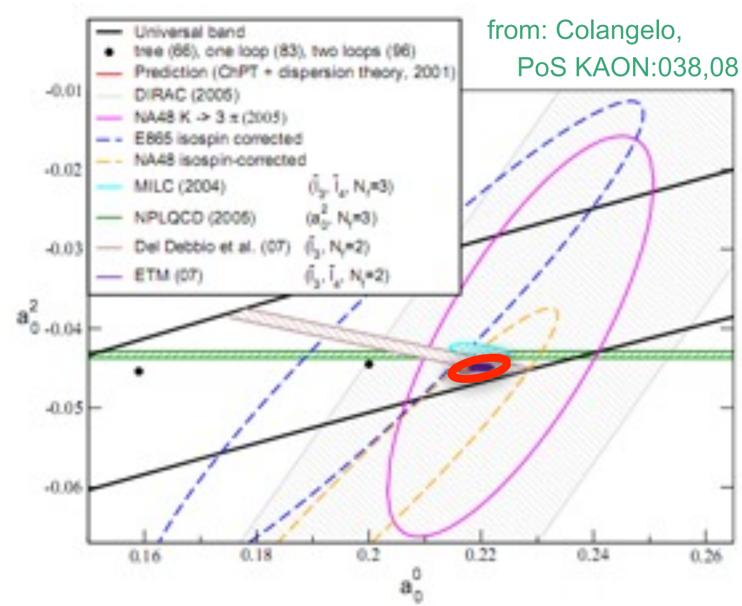
LO: $a_0^0 = 0.16$ (Weinberg '66)

NLO: $a_0^0 = 0.20$ (Gasser, Leutwyler '83)

NNLO: $a_0^0 = 0.217$ (Bijnens et al. '95)

NNLO + disp. relations: (Colangelo et al.)

$$a_0^0 = 0.217 \pm 0.008\,(\rm exp) \pm 0.006\,(\rm th)$$



from: Colangelo, PoS KAON:038,08

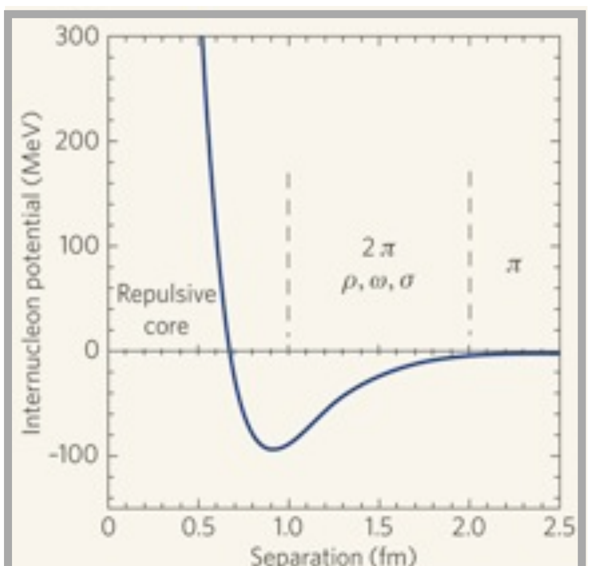
Two and more nucleons

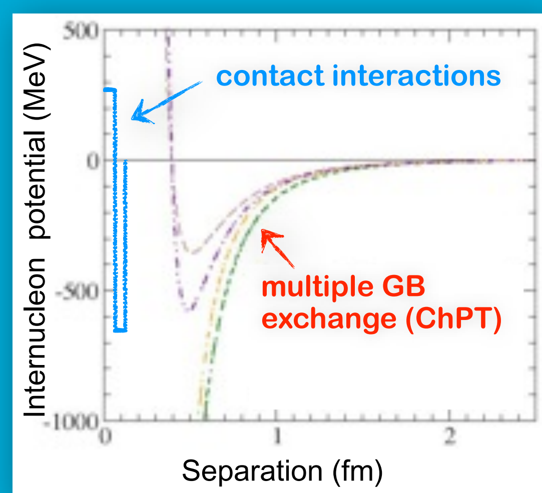
The roadmap: QCD ⟶ Chiral Perturbation Theory ⟶ hadron dynamics

NN interaction is strong, resummations/nonperturbative methods needed...

Simplification: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟶ **the QM A-body problem** Weinberg '91

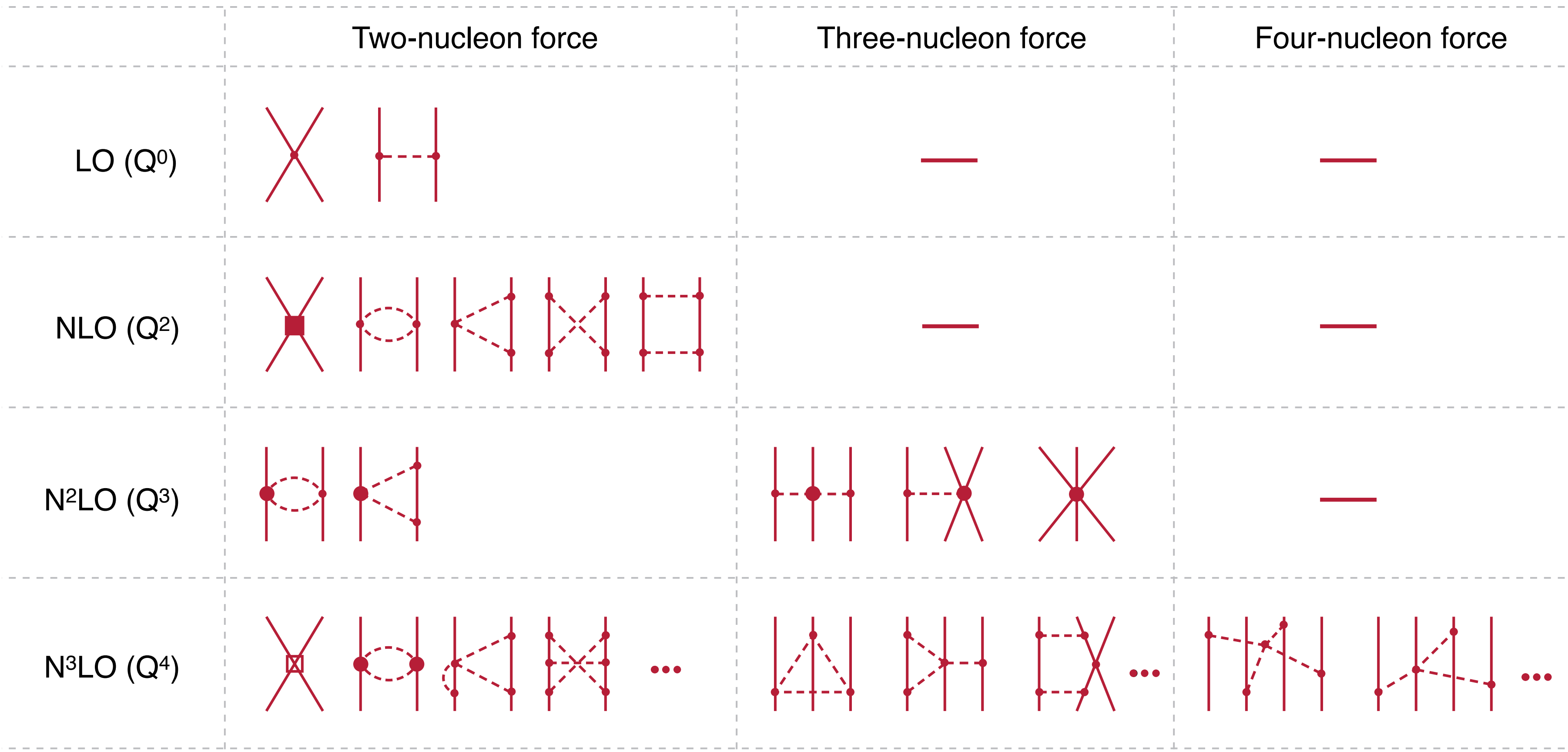
$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Nuclear forces: State of the art

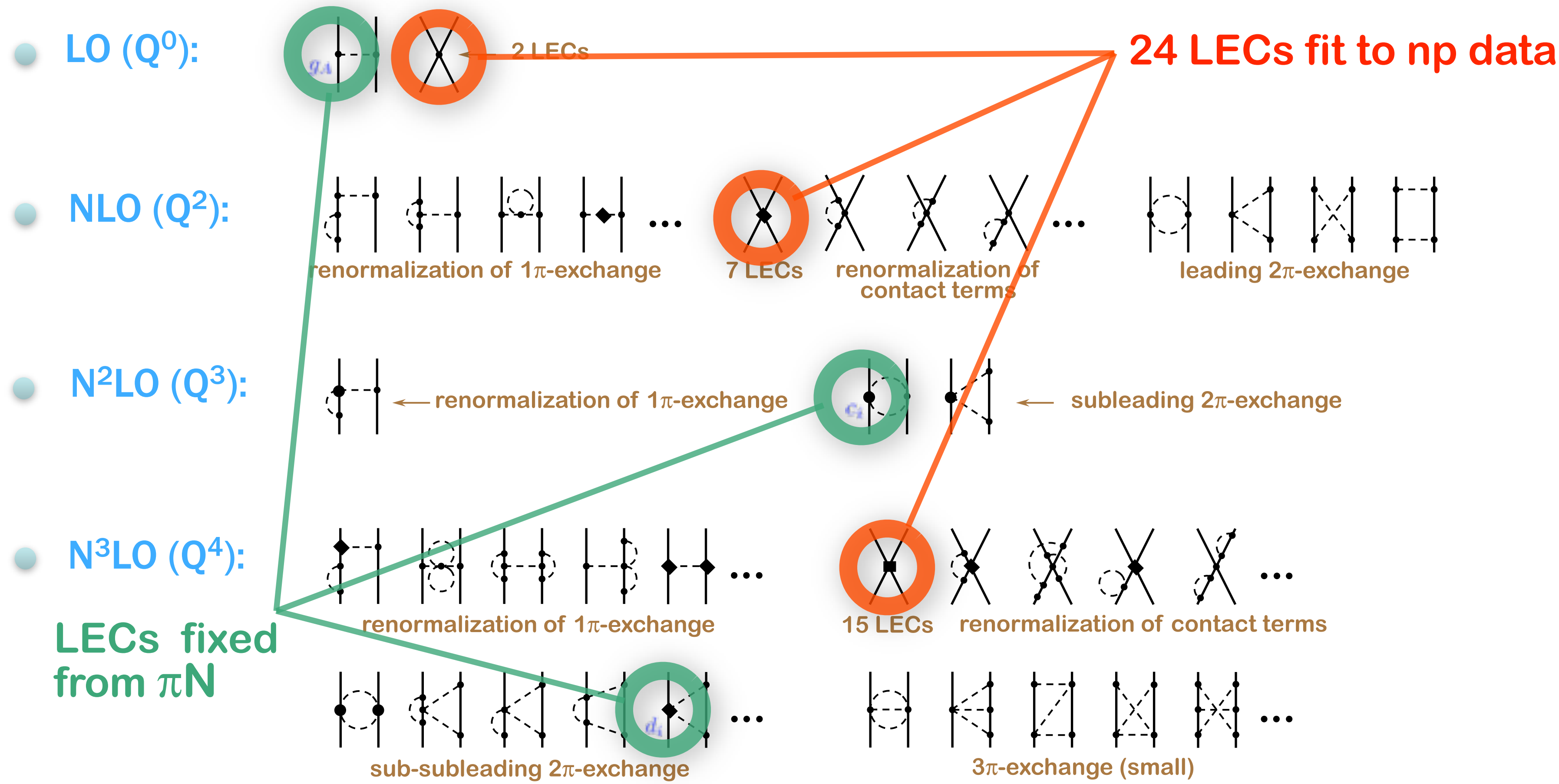


$\langle V_{2N} \rangle \sim 20$ MeV/pair $\quad \langle V_{3N} \rangle \sim 1$ MeV/triplet $\quad \langle V_{4N} \rangle \sim 0.1$ MeV/quartet

(numerical estimations based on Pudliner et al. PRL 74 (95) 4396)

Chiral expansion of the NN force

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,’03; Kaiser ’99-’01; Higa, Robilotta ’03; …

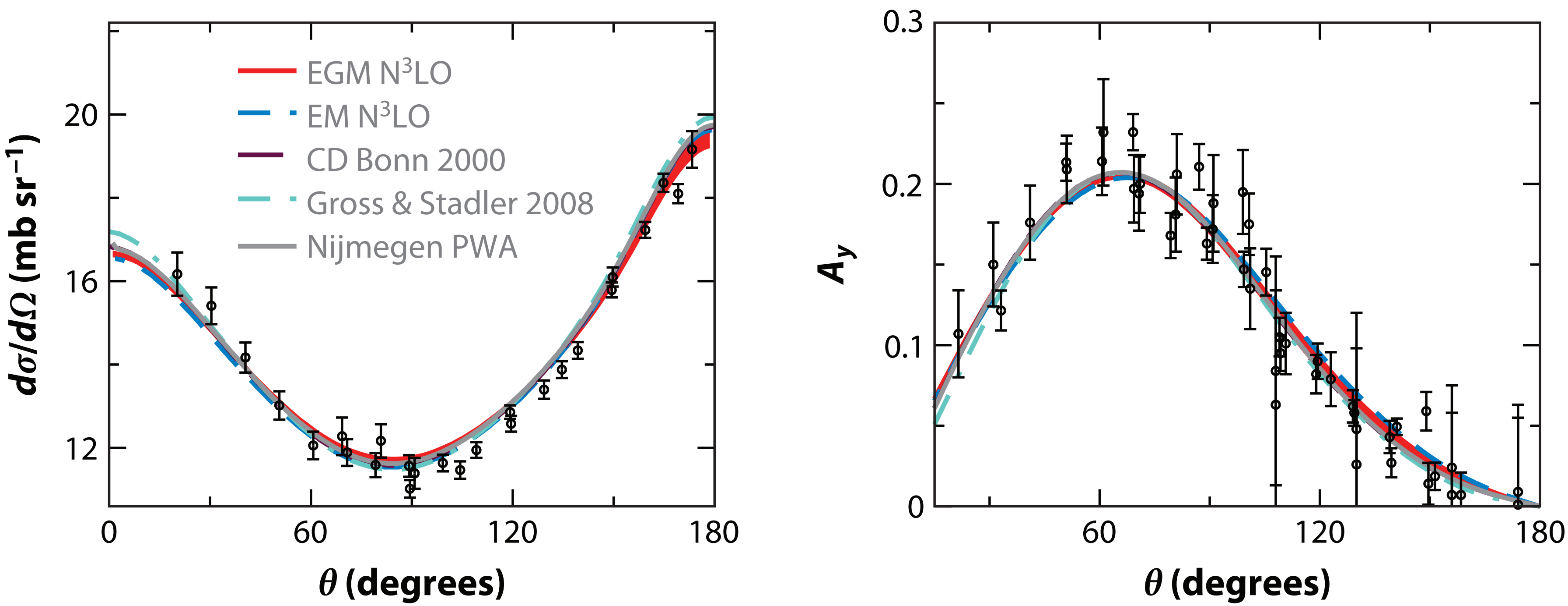


\+ isospin-breaking corrections…

van Kolck et al. ’93,’96; Friar et al. ’99,’03,’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

Nucleon-nucleon scattering at N^3LO

Neutron-proton differential cross section and analyzing power at E_{lab} = 50 MeV



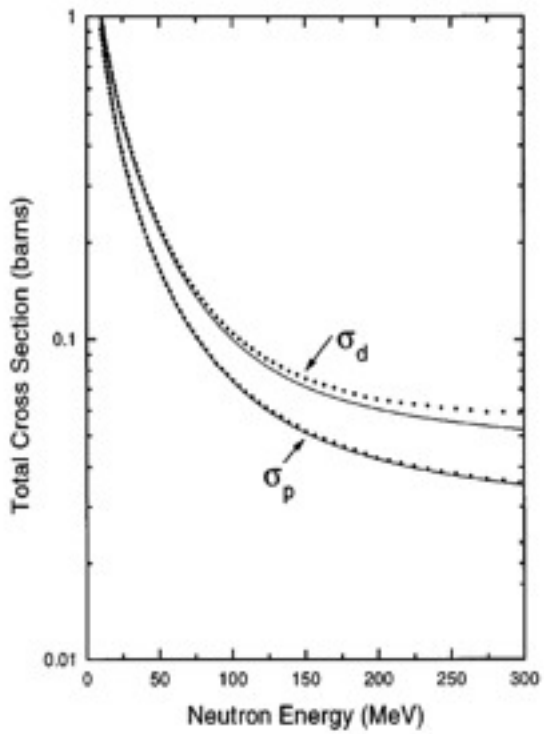
Deuteron observables

	N^3LO	Exp
E_d [MeV]	$-2.216 \ldots -2.223$	$-2.224575(9)$
A_S [$fm^{-1/2}$]	$0.882 \ldots 0.883$	$0.8846(9)$
η_d	$0.0254 \ldots 0.0255$	$0.0256(4)$

Three-nucleon force

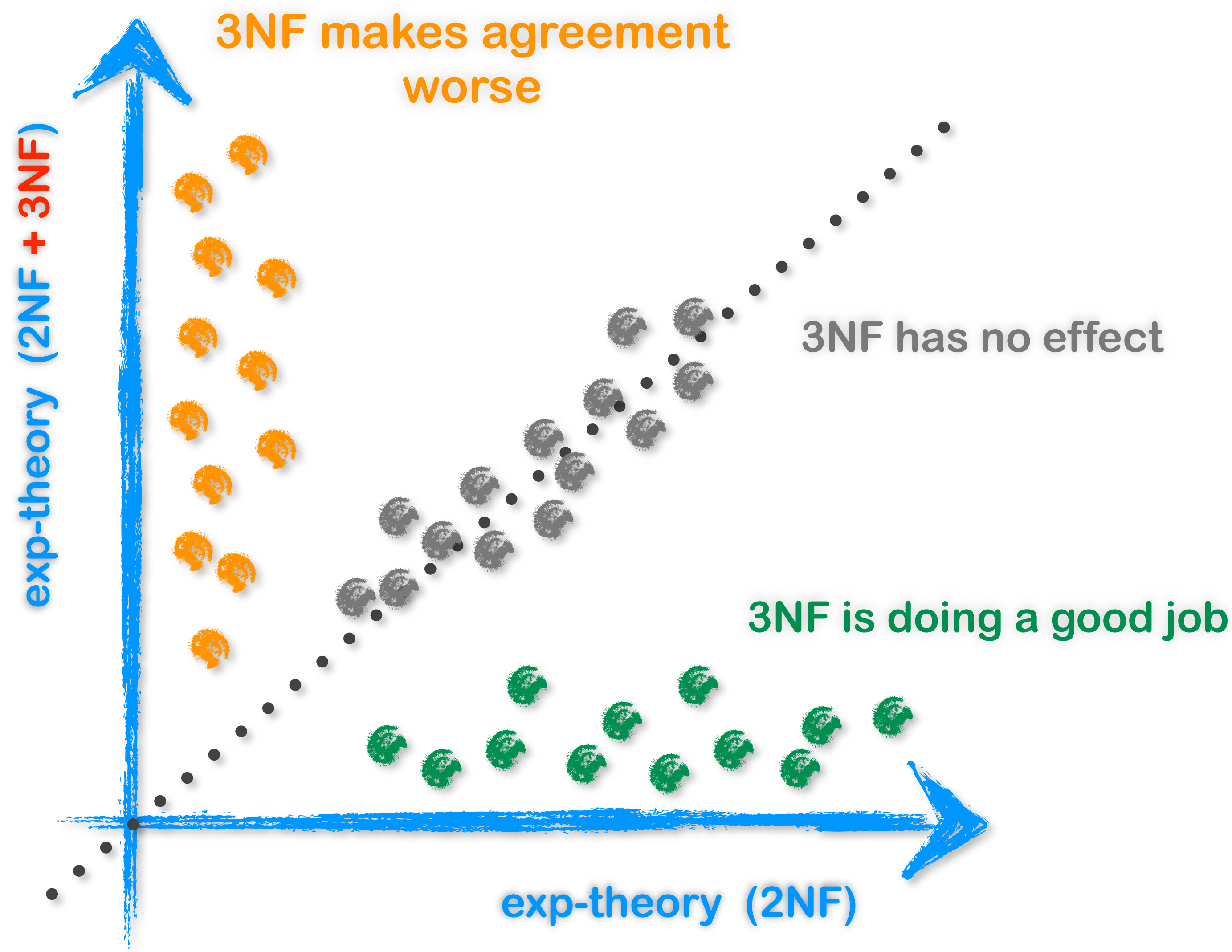
Some indications of the 3NF

- ^3H binding energy calculated based on V_{NN} is typically underbound by ~ 1 MeV
- Three-nucleon continuum...



Phenomenological 3NF models

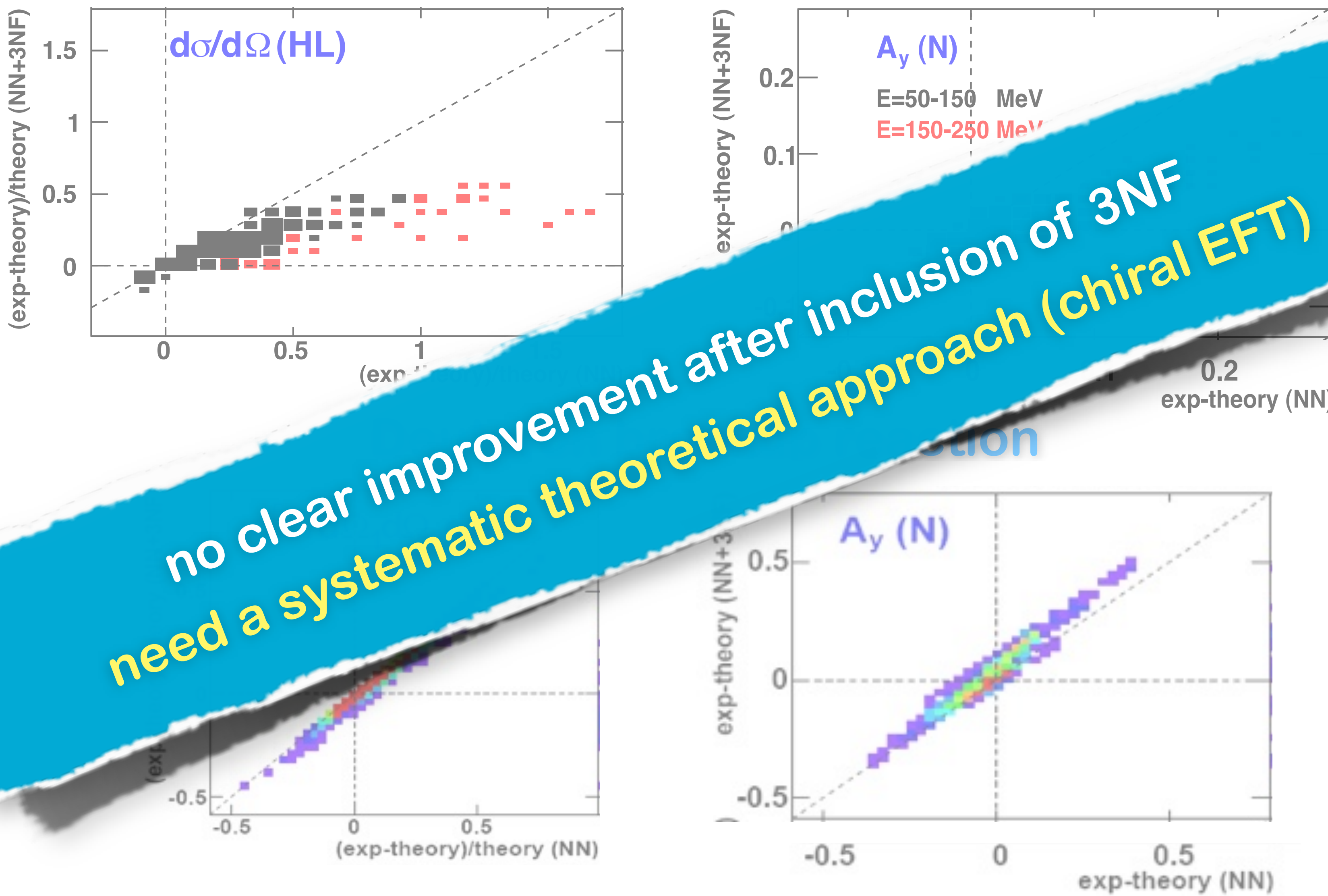
Fugita-Miyazawa,
Tucson-Melbourne,
Brasil, Urbana IX, Illinois, ...



Phenomenological 3NF models

Fugita-Miyazawa,
Tucson-Melbourne,
Brasil, Urbana IX, Illinois, ...

Elastic nucleon-deuteron scattering



no clear improvement after inclusion of 3NF

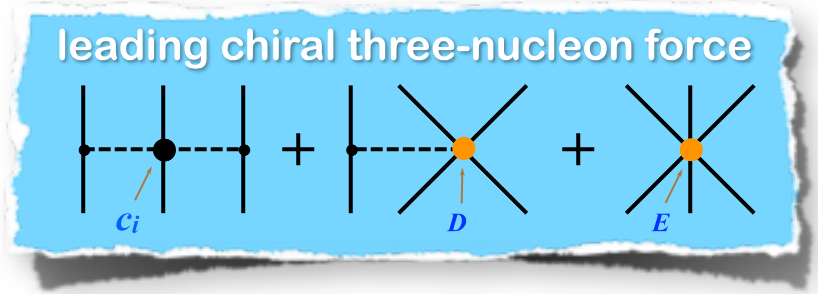
need a systematic theoretical approach (chiral EFT)

from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (2012) 016301

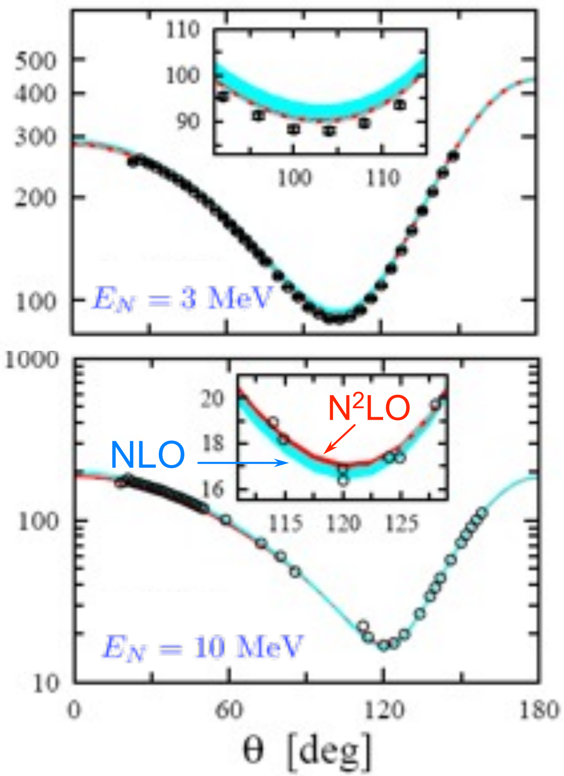
Chiral 3NF & nd elastic scattering

3NF first appears ar N^{2}LO

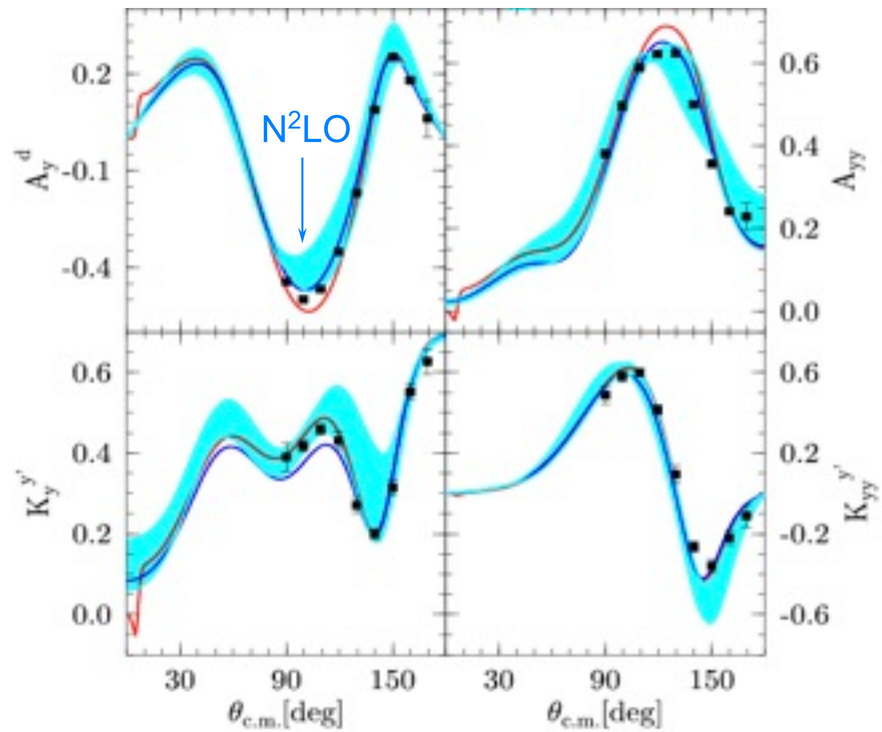
The LECs D,E can be fixed e.g. from ^{3}H BE
and nd doublet scattering length EE, Nogga et al.



Nd elastic cross sections at low energies



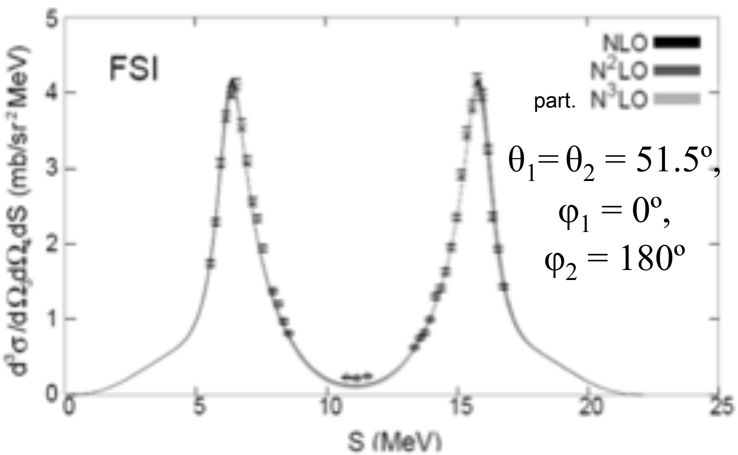
Nd elastic scattering at E$_N$=90 MeV



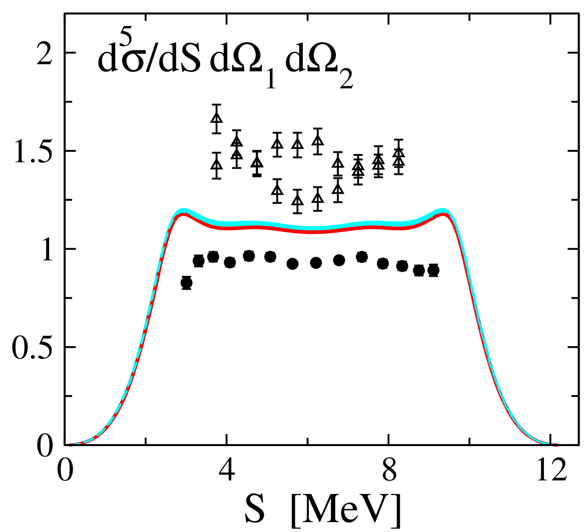
Chiral 3NF and deuteron breakup

FSI configuration at E_p=16 MeV

Düwecke et al., PRC 72 (2006) 044001

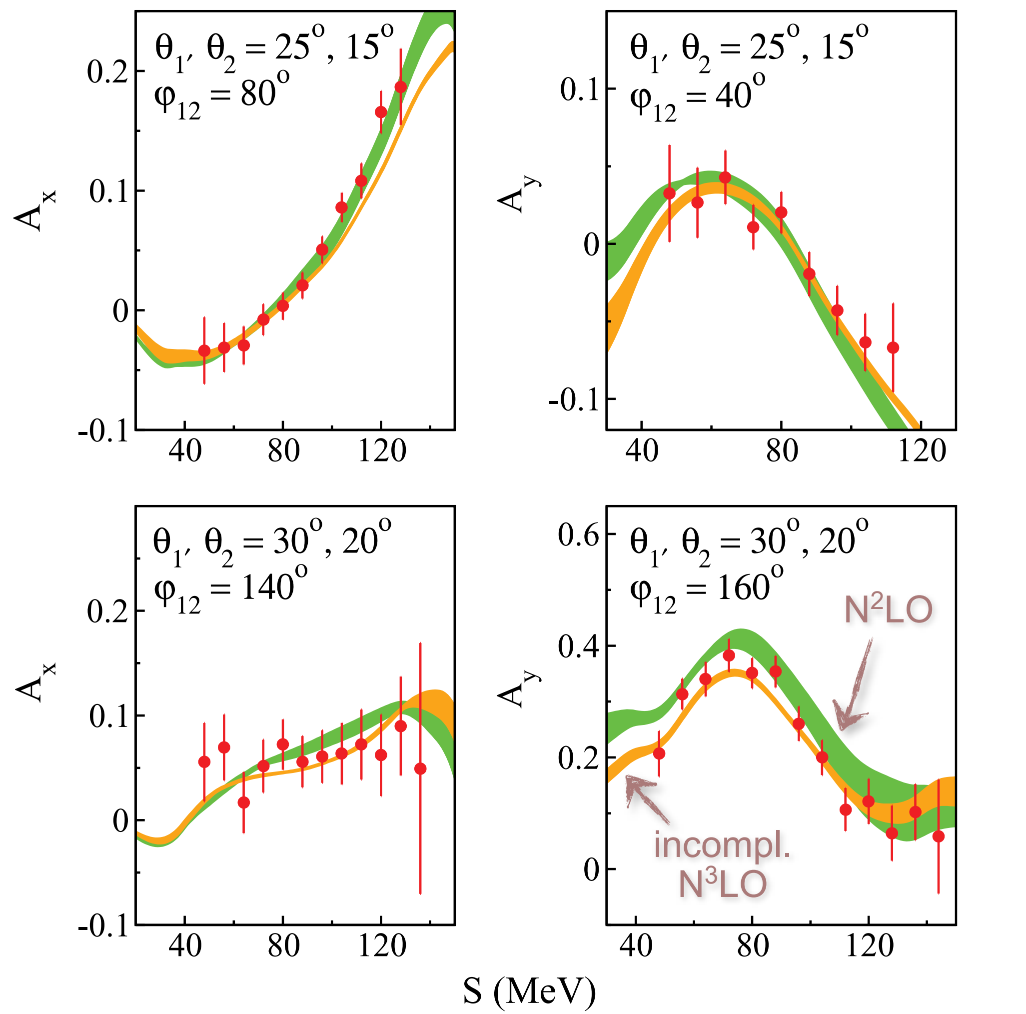


SST configuration at E_N=13 MeV



Nd breakup at E_d=130 MeV

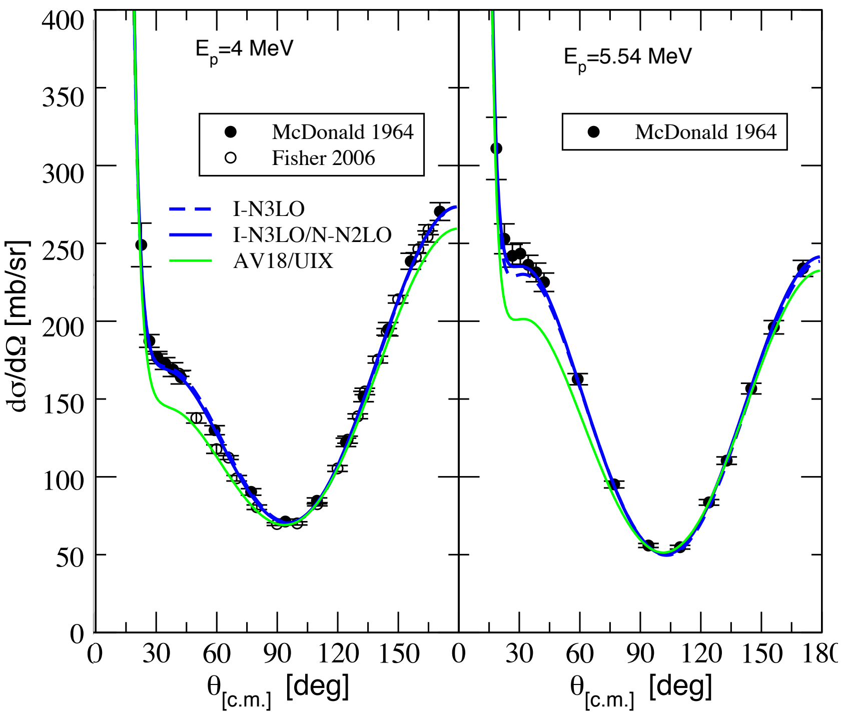
Stephan et al., PRC 82 (2010) 014003



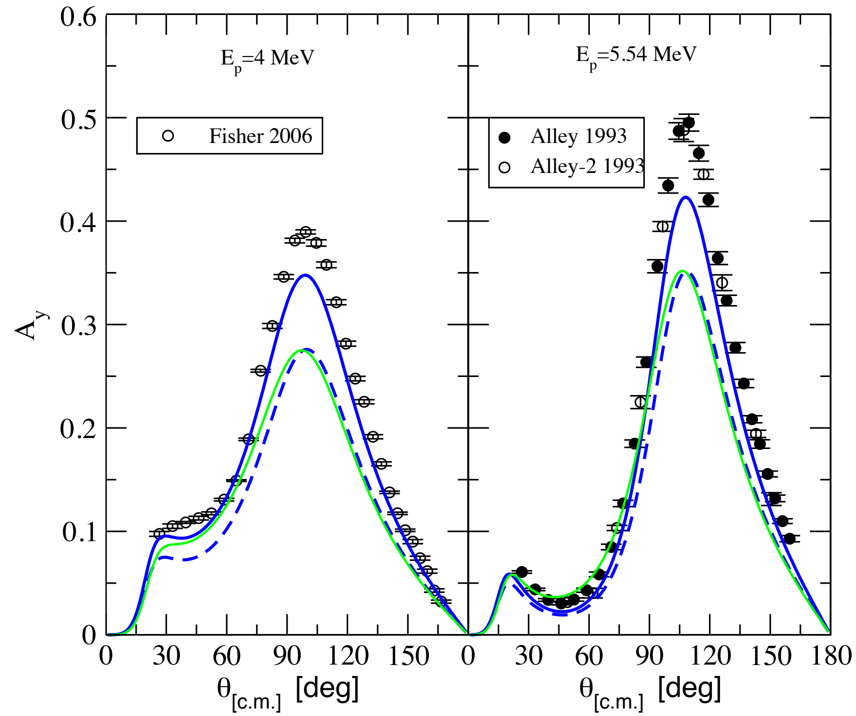
Chiral 3NF and 4N scattering

Viviani, Girlanda, Kievsky, Marcucci, Rosatti arXiv:1004.1306

p-^{3}He differential cross section



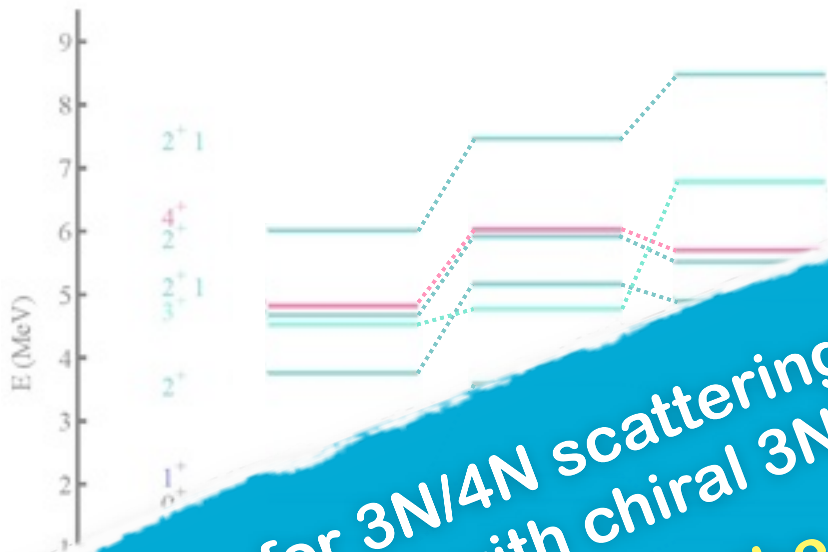
A_y-puzzle in p-^{3}He elastic scattering



(the LECs D,E are tuned to the ^{3}H and ^{4}He binding energies)

Chiral 3NF and nuclear structure

^{10}B



Navratil et al., PRL ...
(2007) ...

promising results for 3N/4N scattering, light nuclei and nuclear matter with chiral 3NF at N^2LO

need to go to higher orders in the chiral expansion

2NF (N^3LO)
+ 3NF (N^2LO)

...ons of chiral 3NF at N^2LO:

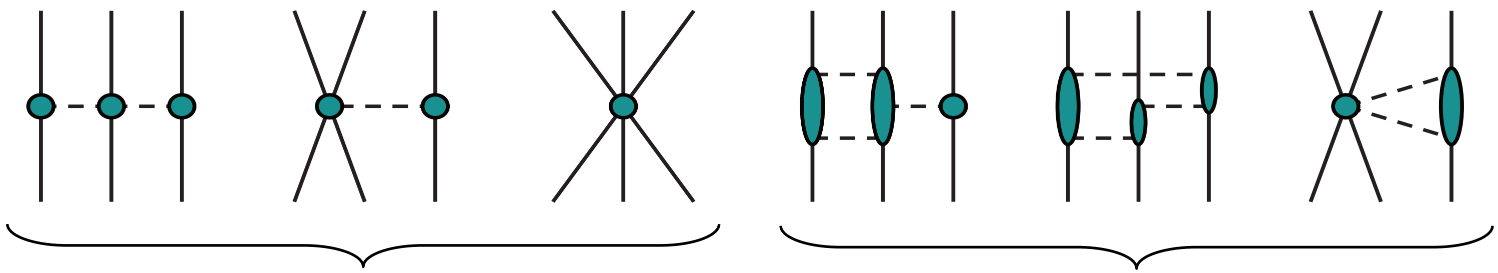
– ex...ing the long lifetime of ^{14}C Holt, Kaiser, Weise ’10

– constraining the properties of neutron-rich matter & neutron star radii Hebeler et al.’10

– explaining the structure of Ca isotopes Holt, Otsuka, Schwenk, Suzuki ’10

Chiral 3NF beyond N^{2}LO (work in progress)

- 3NF topologies up to N^{4}LO (subleading one-loop order)



- start contributing at N^{2}LO
- fairly restricted operator structure
- also included in various models

- first appear at N^{3}LO
- rich operator structure
- parameter-free
- not converged (Δ-effects are missing)

→ need to go to higher orders and/or Δ-full EFT...

Gasparyan, EE, Krebs, work in progress...

- First corrections to the leading 3NF (at N^{3}LO) are worked out

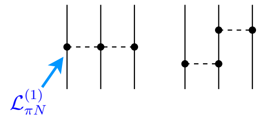
Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); PRC84 (11)

- Implementation in progress (numerical partial wave decomposition: JUROPA@FZJ)

Skibinski et al., PRC84 (11); work in progress...

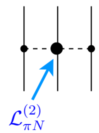
Two-pion exchange 3NF up to N^{4}LO

Next-to-leading order



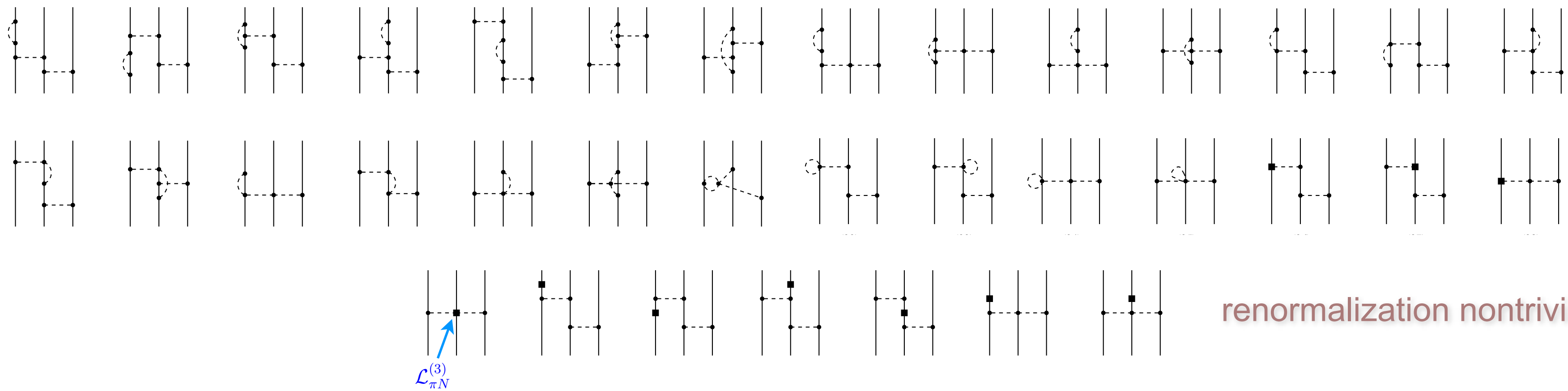
← yield vanishing 3NF contributions

Next-to-next-to-leading order



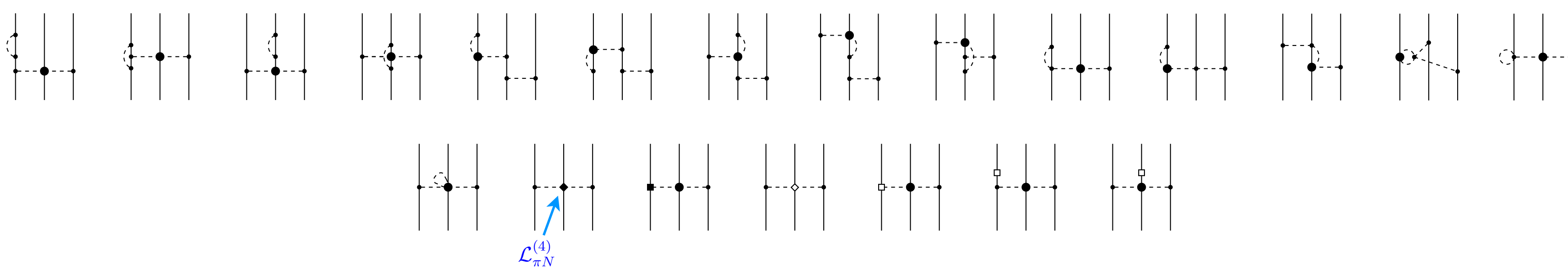
← first nonvanishing 3NF,
encodes information about the Δ:

Next-to-next-to-next-to-leading order (leading 1 loop)



renormalization nontrivial !

Next-to-next-to-next-to-next-to-leading order (subleading 1 loop)



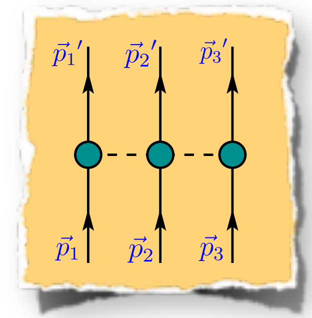
Two-pion exchange 3NF up to N^{4}LO

Effective Lagrangian needed to compute 3NF at N^{4}LO

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= N_v^\dagger \Big[iv\cdot\partial - \frac{1}{4F^2}\boldsymbol{\tau}\times\boldsymbol{\pi}\cdot(v\cdot\partial\boldsymbol{\pi}) + \frac{8\alpha-1}{16F^4}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\,\boldsymbol{\tau}\times\boldsymbol{\pi}\cdot(v\cdot\partial\boldsymbol{\pi}) \\
&\quad - \frac{\mathring{g}_A}{F}\boldsymbol{\tau}\cdot(S\cdot\partial\boldsymbol{\pi}) + \frac{\mathring{g}_A}{2F^3}\left((4\alpha-1)\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\pi}\cdot(S\cdot\partial\boldsymbol{\pi}) + 2\alpha\boldsymbol{\pi}^2\boldsymbol{\tau}\cdot(S\cdot\partial\boldsymbol{\pi})\right)\Big] N_v + \ldots, \\
\mathcal{L}_{\pi N}^{(2)} &= N_v^\dagger \Big[4M^2c_1 - \frac{2}{F^2}c_1M^2\boldsymbol{\pi}^2 + \frac{1}{F^2}\left(c_2 - \frac{g_A^2}{8m}\right)(v\cdot\partial\boldsymbol{\pi})\cdot(v\cdot\partial\boldsymbol{\pi}) + \frac{1}{F^2}c_3(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) \\
&\quad - \frac{i}{F^2}\left(c_4 + \frac{1}{4m}\right)\Big[S_\mu,\, S_\nu\Big]\boldsymbol{\tau}\times(\partial^\nu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) + \frac{M^2c_1}{2F^4}(8\alpha-1)(\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2 + \frac{c_3}{F^4}\big((1-4\alpha)\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\,\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} \\
&\quad -2\,\alpha\,\boldsymbol{\pi}\cdot\boldsymbol{\pi}\,\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\big) - \frac{i\,c_4}{2F^4}\left(2\,(1-4\alpha)\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi} - 4\,\alpha\,\boldsymbol{\pi}\cdot\boldsymbol{\pi}\,\partial_\mu\boldsymbol{\pi}\cdot(\boldsymbol{\tau}\times\partial_\nu\boldsymbol{\pi})\right)[S^\mu,S^\nu] + \frac{\vec{\nabla}^2}{2m} \\
&\quad + \frac{i\mathring{g}_A}{2Fm}\left(\boldsymbol{\tau}\cdot(v\cdot\partial\,S\cdot\partial\boldsymbol{\pi}) + 2\boldsymbol{\tau}\cdot(v\cdot\partial\boldsymbol{\pi})S\cdot\partial\right) + \frac{i}{8F^2m}\left(\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times(\vec{\nabla}^2\boldsymbol{\pi})) + \boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\vec{\nabla}\boldsymbol{\pi})\vec{\nabla}\right)\Big] N_v + \ldots, \\
\mathcal{L}_{\pi N}^{(3)} &= N_v^\dagger \Big[\frac{2}{F^2}\left(d_1 + d_2 - \frac{c_4}{4m}\right)\boldsymbol{\tau}\times(\partial_\mu v\cdot\partial\boldsymbol{\pi}))\cdot(\partial^\mu\boldsymbol{\pi}) + \frac{2}{F^2}d_3\boldsymbol{\tau}\times((v\cdot\partial)^2\boldsymbol{\pi})\cdot(v\cdot\partial\boldsymbol{\pi}) - \frac{4}{F^2}d_5M^2\boldsymbol{\tau}\times\boldsymbol{\pi}\cdot(v\cdot\partial\boldsymbol{\pi}) \\
&\quad - \frac{2i}{F^2}(d_{14}-d_{15})\Big[(S\cdot\partial\,v\cdot\partial\boldsymbol{\pi}),\ (S\cdot\partial\boldsymbol{\pi})\Big] - \frac{2}{F}(2d_{16}-d_{18})M^2\boldsymbol{\tau}\cdot(S\cdot\partial\,\boldsymbol{\pi}) \\
&\quad + \frac{i\,c_2}{F^2m}\left((v\cdot\partial\boldsymbol{\pi})\cdot(\partial_\mu\boldsymbol{\pi})\overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu(v\cdot\partial\boldsymbol{\pi})\cdot(\partial_\mu\boldsymbol{\pi})\right) - \frac{c_4}{2F^2m}\Big(\boldsymbol{\tau}\cdot((v\cdot\partial\boldsymbol{\pi})\times(\partial^2\boldsymbol{\pi})) \\
&\quad - 2\boldsymbol{\tau}\cdot((v\cdot\partial\boldsymbol{\pi})\times(\partial_\mu\boldsymbol{\pi}))[S^\mu,S^\nu]\overrightarrow{\partial}_\nu + 2\overleftarrow{\partial}_\nu\boldsymbol{\tau}\cdot((v\cdot\partial\boldsymbol{\pi})\times(\partial_\mu\boldsymbol{\pi}))[S^\mu,S^\nu]\Big)\Big] N_v + \ldots, \\
\mathcal{L}_{\pi N}^{(4)} &= N_v^\dagger \Big[2(8e_{38}+e_{115}+e_{116})M^4 - \frac{i}{F^2}\Big[S_\mu,\, S_\nu\Big]\Big(-18e_{17}\boldsymbol{\tau}\times(\partial^\rho\partial^\mu\boldsymbol{\pi})\cdot(\partial_\rho\partial^\nu\boldsymbol{\pi}) + e_{18}\boldsymbol{\tau}\times(v\cdot\partial\partial^\mu\boldsymbol{\pi})\cdot(v\cdot\partial\partial^\nu\boldsymbol{\pi}) \\
&\quad + 4(2e_{21}-e_{37})M^2\boldsymbol{\tau}\times(\partial^\nu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi})\Big) + 8e_{14}(\partial_\mu\partial_\nu\boldsymbol{\pi})\cdot(\partial^\mu\partial^\nu\boldsymbol{\pi}) + 8e_{15}(v\cdot\partial\partial_\mu\boldsymbol{\pi})\cdot(v\cdot\partial\partial^\mu\boldsymbol{\pi}) \\
&\quad + 8e_{16}((v\cdot\partial)^2\boldsymbol{\pi})\cdot((v\cdot\partial)^2\boldsymbol{\pi}) + 4M^2(2e_{19}-e_{22}-e_{36})(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) + 8e_{20}M^2(v\cdot\partial\boldsymbol{\pi})\cdot(v\cdot\partial\boldsymbol{\pi}) \\
&\quad - 4e_{22}M^2\boldsymbol{\pi}\cdot(\partial_\mu\partial^\mu\boldsymbol{\pi}) - 8e_{35}M^2\boldsymbol{\pi}\cdot((v\cdot\partial)^2\boldsymbol{\pi}) - 16e_{38}M^4\boldsymbol{\pi}^2\Big] N_v + \ldots,
\end{aligned}
$$

Two-pion exchange 3NF up to N^{4}LO

The TPE 3NF has the form (modulo 1/m-terms):



$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

- leading-order:
  van Kolck '94

$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

- subleading:

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right) M_\pi^3 + 2\left(g_A^2 + 1\right) M_\pi q_2^2\Big]\,,$$

$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2 + 1)M_\pi\Big]$$

Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

- sub-subleading:
  Gasparyan, EE, Krebs '12

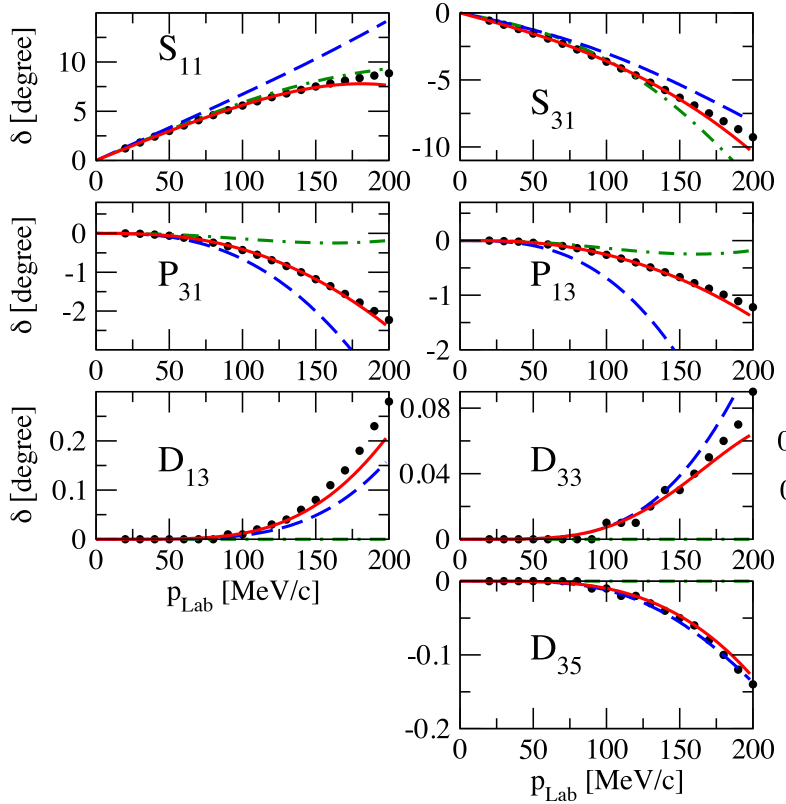
$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2 (F_\pi^2 \left(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\right) \\
&+ g_A(144 c_1 - 53 c_2 - 90 c_3)) + M_\pi^4 \left(F_\pi^2 \left(4608\pi^2 \bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\right)\right. \\
&+ \left. g_A \left(72\left(64\pi^2 \bar{l}_3 + 1\right) c_1 - 24 c_2 - 36 c_3\right)\right) + q_2^4 \left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\right)\Big] \\
&- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2\right)\left(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right)\,, \\
\mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2 \left(F_\pi^2 \left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\right) + 108 g_A^3 c_4 + 24 g_A c_4\right) \\
&+ q_2^2 \left(5 g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2)\left(4M_\pi^2 + q_2^2\right)
\end{aligned}$$

Two-pion exchange 3NF up to N^{4}LO

Gasparyan, EE, Krebs '12

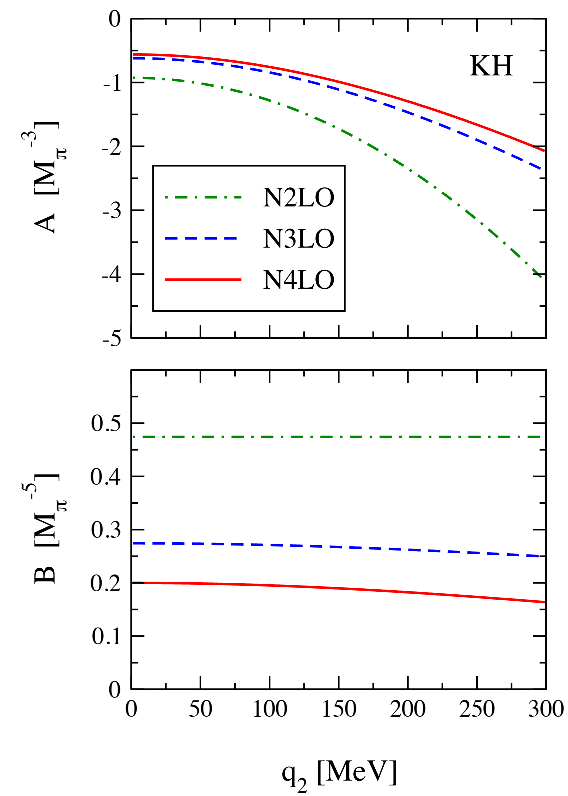
πN phase shifts in HB ChPT up to Q^4

(fit to Karlsruhe-Helsinki PWA)



3NF „structure functions"

(fit to Karlsruhe-Helsinki PWA)



- all relevant (combinations of) LECs are determined from πN scattering
- extracted LECs are fairly stable (KH vs GW PWA) and of a reasonably natural size
- reasonably good convergence of HB ChPT (consistent with Fettes, Meißner '00)

Most general structure of a local 3NF

Gasparyan, EE, Krebs, in preparation

What is the relative size of the TPE topology compared to other long-range terms?

→ need to work with a complete set of independent operators

Generators $\mathcal{G}$ of 89 independent operators	S	A	G_1	G_2	$G_1(12)$	$G_2(12)$
1	$\mathcal{O}_1$	-	-	-	-	-
$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$	$\mathcal{O}_2$	-	$\mathcal{O}_3$	$\mathcal{O}_4$	-	-
$\vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\mathcal{O}_5$	-	$\mathcal{O}_6$	$\mathcal{O}_7$	-	-
$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\mathcal{O}_8$	-	$\mathcal{O}_9$	$\mathcal{O}_{10}$	-	-
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\mathcal{O}_{11}$	$\mathcal{O}_{12}$	$\mathcal{O}_{13}$	$\mathcal{O}_{14}$	$\mathcal{O}_{15}$	$\mathcal{O}_{16}$
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\mathcal{O}_{17}$	-	-	-	-	-
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\mathcal{O}_{18}$	-	$\mathcal{O}_{19}$	$\mathcal{O}_{20}$	-	-
$\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\mathcal{O}_{21}$	$\mathcal{O}_{22}$	$\mathcal{O}_{23}$	$\mathcal{O}_{24}$	$\mathcal{O}_{25}$	$\mathcal{O}_{26}$
$\vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\mathcal{O}_{27}$	-	$\mathcal{O}_{28}$	$\mathcal{O}_{29}$	-	-
$\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\mathcal{O}_{30}$	-	$\mathcal{O}_{31}$	$\mathcal{O}_{32}$	-	-
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\mathcal{O}_{33}$	$\mathcal{O}_{34}$	$\mathcal{O}_{35}$	$\mathcal{O}_{36}$	$\mathcal{O}_{37}$	$\mathcal{O}_{38}$
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\mathcal{O}_{39}$	$\mathcal{O}_{40}$	$\mathcal{O}_{41}$	$\mathcal{O}_{42}$	$\mathcal{O}_{43}$	$\mathcal{O}_{44}$
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\mathcal{O}_{45}$	$\mathcal{O}_{46}$	$\mathcal{O}_{47}$	$\mathcal{O}_{48}$	$\mathcal{O}_{49}$	$\mathcal{O}_{50}$
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\mathcal{O}_{51}$	$\mathcal{O}_{52}$	$\mathcal{O}_{53}$	$\mathcal{O}_{54}$	$\mathcal{O}_{55}$	$\mathcal{O}_{56}$
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_2 \vec{q}_1 \cdot \vec{\sigma}_3$	$\mathcal{O}_{57}$	-	$\mathcal{O}_{58}$	$\mathcal{O}_{59}$	-	-
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\mathcal{O}_{60}$	$\mathcal{O}_{61}$	$\mathcal{O}_{62}$	$\mathcal{O}_{63}$	$\mathcal{O}_{64}$	$\mathcal{O}_{65}$
$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\mathcal{O}_{66}$	-	$\mathcal{O}_{67}$	$\mathcal{O}_{68}$	-	-
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\mathcal{O}_{69}$	-	$\mathcal{O}_{70}$	$\mathcal{O}_{71}$	-	-
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\mathcal{O}_{72}$	$\mathcal{O}_{73}$	$\mathcal{O}_{74}$	$\mathcal{O}_{75}$	$\mathcal{O}_{76}$	$\mathcal{O}_{77}$
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\mathcal{O}_{78}$	$\mathcal{O}_{79}$	$\mathcal{O}_{80}$	$\mathcal{O}_{81}$	$\mathcal{O}_{82}$	$\mathcal{O}_{83}$
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\mathcal{O}_{84}$	-	$\mathcal{O}_{85}$	$\mathcal{O}_{86}$	-	-
$\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\mathcal{O}_{87}$	-	$\mathcal{O}_{88}$	$\mathcal{O}_{89}$	-	-

Most general, local 3NF involves 89 operators, can be generated (by permutations) from 22 structures:

$$V_{\rm 3N}^{\rm loc.} = \sum_{i=1}^{22} \mathcal{G}_i \; + 5\,\text{perm.}$$

The generators $\mathcal{G}_i$ are defined as:

$$S(\mathcal{O}) := \frac{1}{6} \sum_{P \in S_3} P\mathcal{O}$$

$$A(\mathcal{O}) := \frac{1}{6} \sum_{P \in S_3} (-1)^P P\mathcal{O}$$

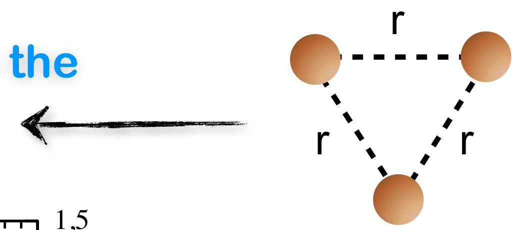
$$G_2(\mathcal{O}) := \frac{\sqrt{3}}{2} \left[S_{23} S_{13} - S_{12} S_{13} \right] (\mathcal{O})$$

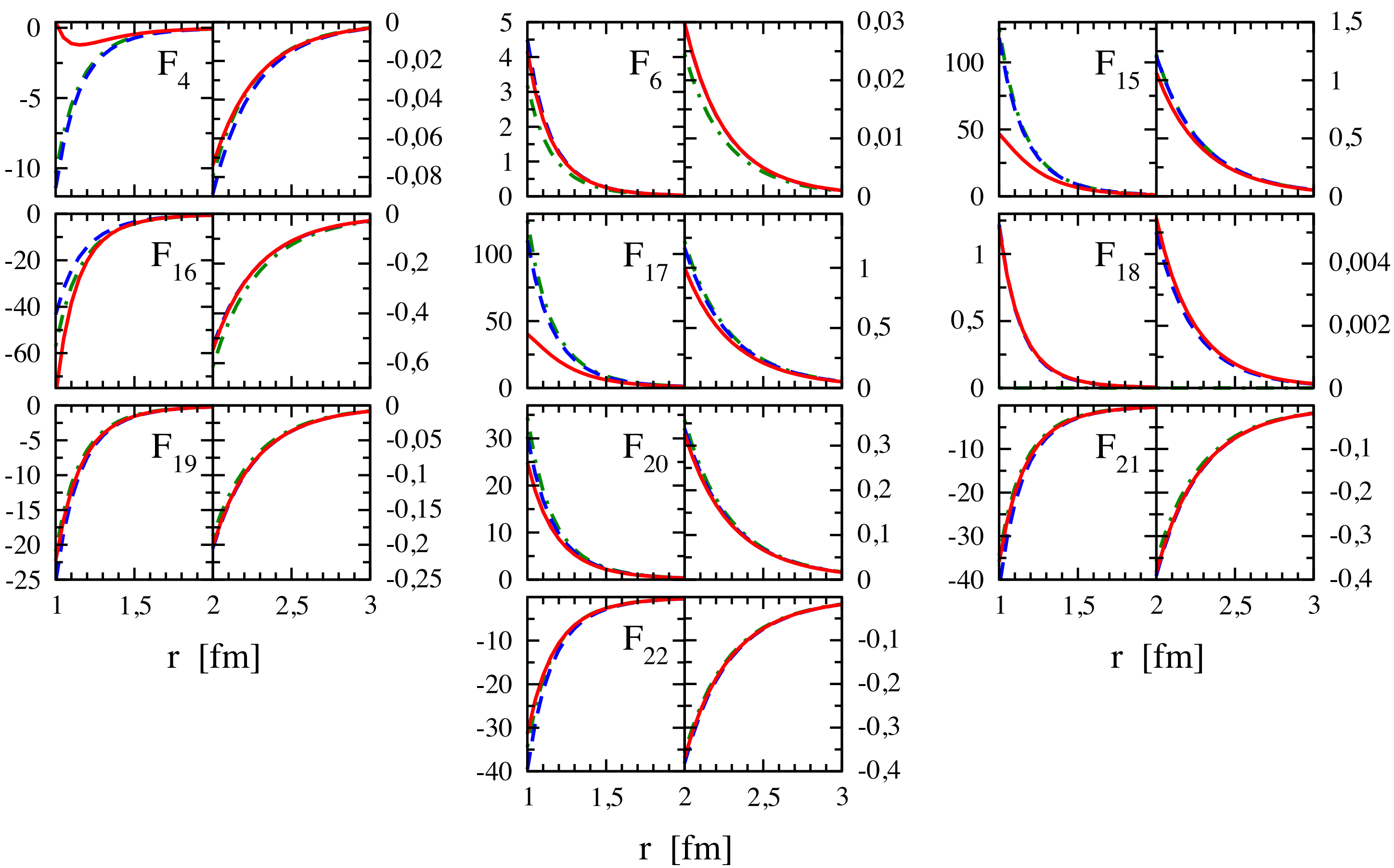
$$G_1(\mathcal{O}) := \left[S_{13} - \frac{1}{2} \left(S_{23} S_{13} + S_{12} S_{13} \right) \right] (\mathcal{O})$$

Two-pion exchange 3NF up to N^{4}LO

Gasparyan, EE, Krebs, in preparation

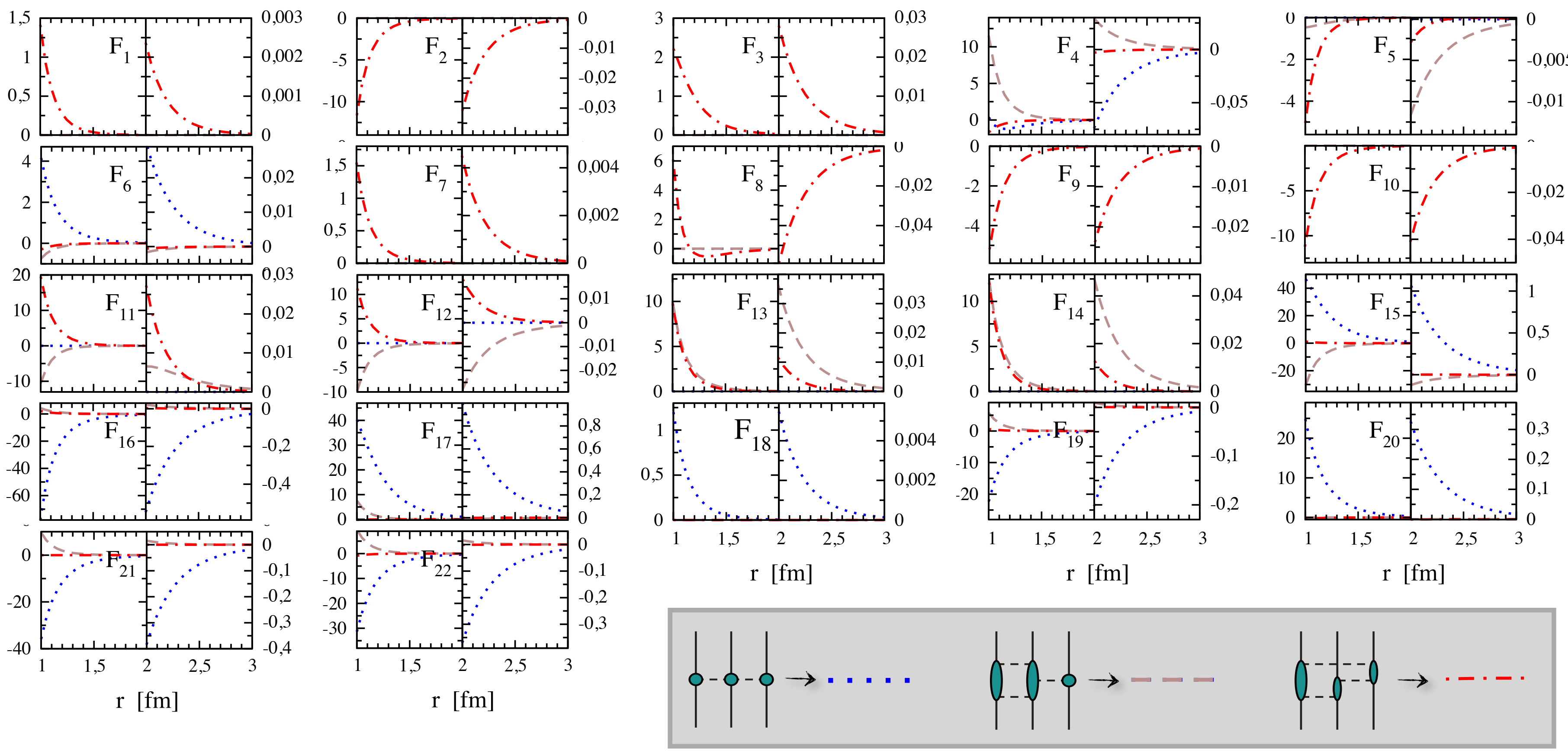
Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration





Complete long-range 3NF at N^4LO

Gasparyan, EE, Krebs, in preparation



- predictions based entirely on chiral symmetry + input from πN, benchmarks for lattice-QCD
- implications for Nd, light nuclei & nuclear matter? (work in progress...)
- 2π-1π and ring topology: already converged? ChPT with explicit Δ's (work in progress...)

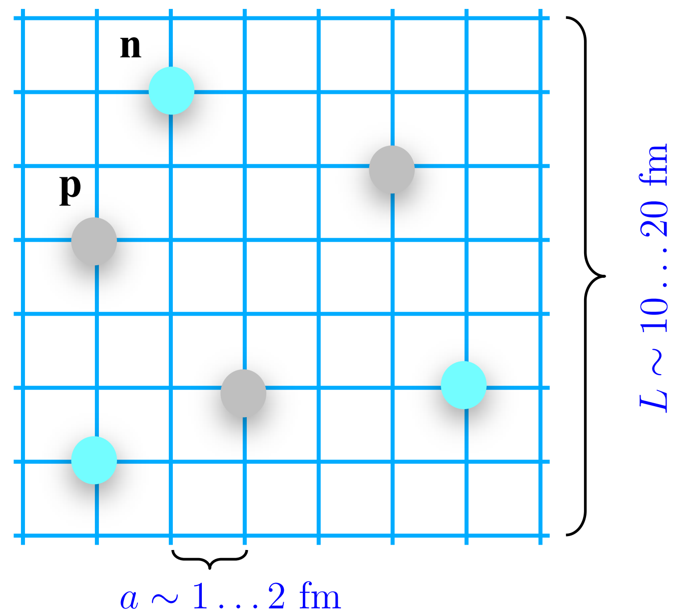
Nuclear Lattice Effective Field Theory collaboration

E.E. (Bochum), Hermann Krebs (Bochum), Timo Lahde (Jülich), Dean Lee (NC State) and Ulf-G. Meißner (Bonn/Jülich)

Discretized version of chiral EFT for nuclear dynamics

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,
Eur. Phys. J. A34 (07) 185,
Eur. Phys. J. A35 (08) 343,
Eur. Phys. J. A35 (08) 357,
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,
Eur. Phys. J A41 (09) 125,
Phys. Rev. Lett 104 (10) 142501,
Eur. Phys. J. 45 (10) 335,
Phys. Rev. Lett. 106 (11) 192501

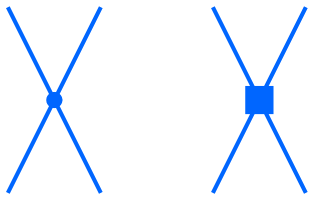


Calculation strategy

Lattice action (improved to minimize discr. errors, accurate to Q^3)

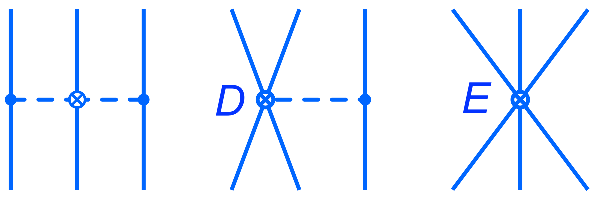
⇓ Lanczos method

Solve 2N Schröd. Eq. with the spherical wall boundary cond. → phase shifts → **fix** the LO and NLO (perturbatively) **contact terms**



⇓ projection Monte Carlo (with auxiliary fields)

Determine the LECs D, E from ^{3}H and ^{4}He BEs → the nuclear Hamiltonian completely fixed up to NNLO (Q^3)

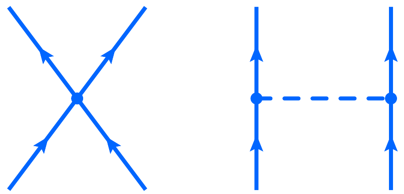


⇓ (Multi-channel) projection Monte Carlo with auxiliary fields

Simulate the ground (and excited) states of **light nuclei**

Lattice actions

Q^0

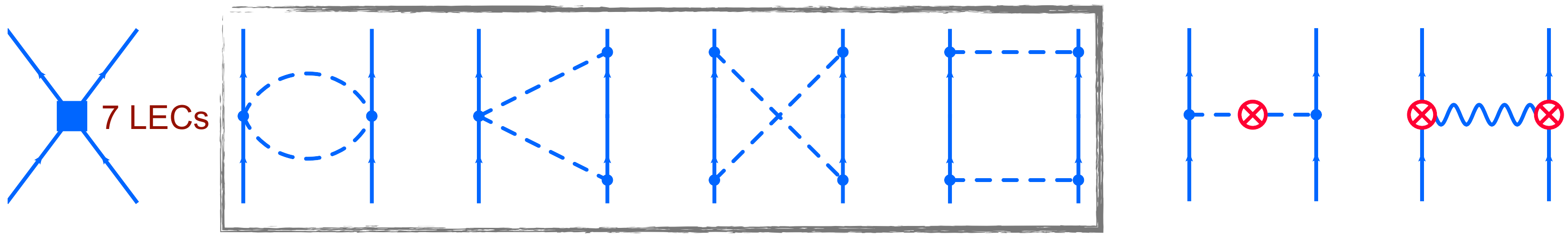


Different actions employed
- LO1: no smearing,
- LO2: smearing in all waves,
- LO3: smearing in even-l waves

used in the simulation

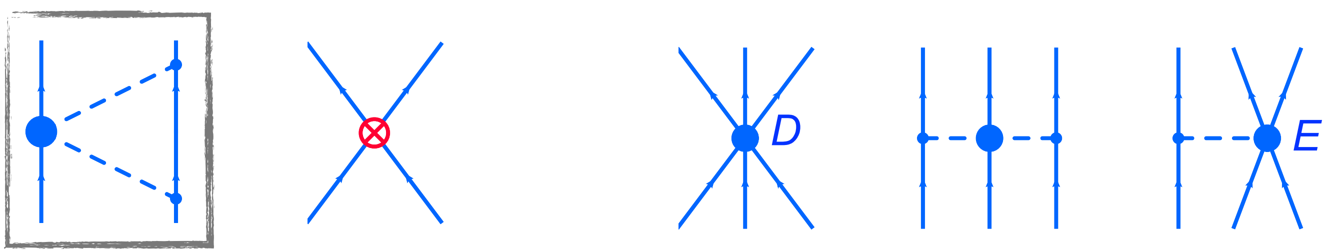
inserted perturbatively

Q^2



for a ~ 2 fm (Λ ~ 314 MeV) can well be represented by contact terms

Q^3



Calculation strategy

Slater determinant of 1N states

- $|\Psi(t')\rangle = \left(M_{\mathrm{SU(4)}}\right)^{L_{t_0}} |\Psi^{\mathrm{init}}\rangle$ - cheap, no sign problem (even A) Lee'05,'07; Chen, Lee, Schäfer '04

  $L_{t_0}\alpha_t$ transfer matrix (pion-less, SU(4)-inv.): $M_{\mathrm{SU(4)}} =: \exp(-H_{\mathrm{SU(4)}}\alpha_t)$:

- Transition amplitude: $Z_{\mathrm{LO}}(t) = \langle\Psi(t')|\,(M_{\mathrm{LO}})^{L_t}\,|\Psi(t')\rangle$ with $M_{\mathrm{LO}} =: \exp\left(-H_{\mathrm{LO}}\alpha_t\right)$:

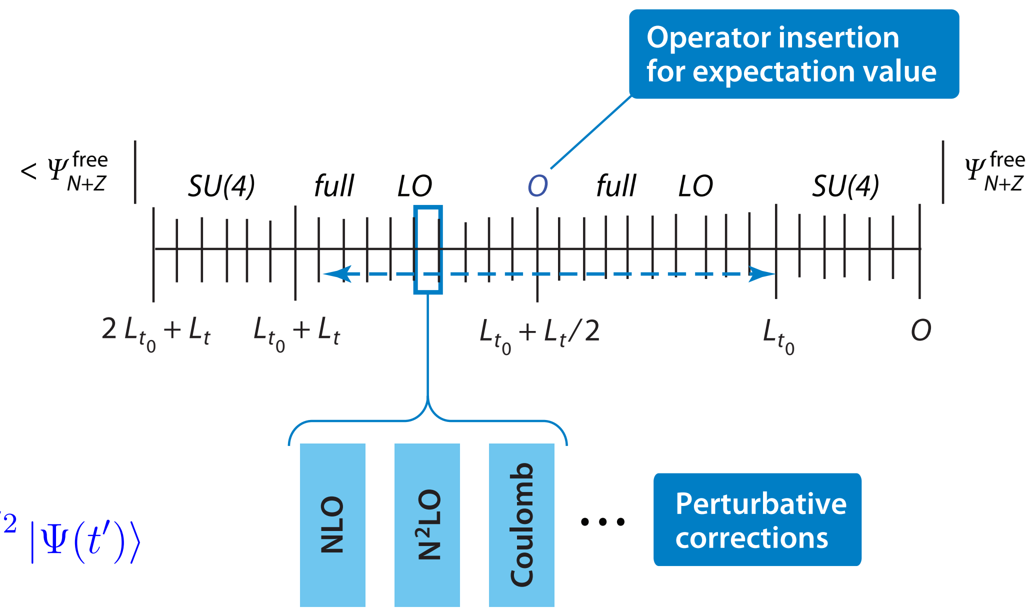
- Ground state energy: $\exp\left(-E_0^{\mathrm{LO}}\alpha_t\right) = \lim_{t\to\infty} Z(t+\alpha_t)/Z(t)$

- Higher-order corrections perturbatively, expectation value for a general operator:

$$\langle\Psi_0|O|\Psi_0\rangle = \lim_{t\to\infty}\frac{Z_O(t)}{Z(t)}|\Psi(t')\rangle$$

with

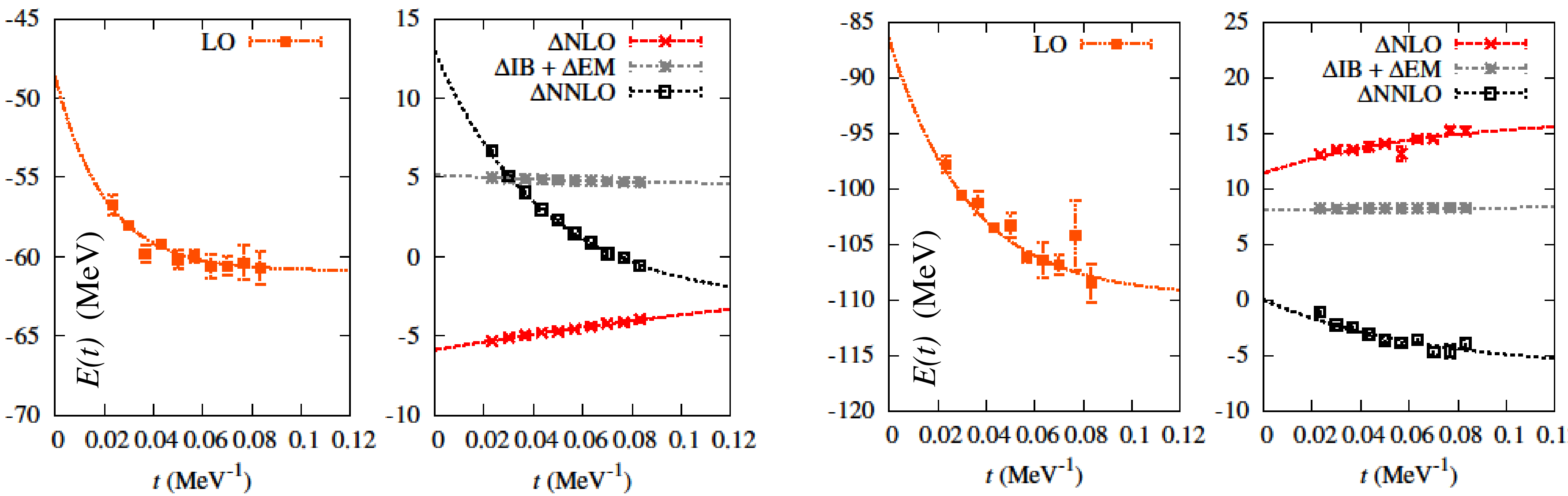
$$Z_O(t) = \langle\Psi(t')|\,(M_{\mathrm{LO}})^{L_t/2}\;O\;(M_{\mathrm{LO}})^{L_t/2}\,|\Psi(t')\rangle$$



Ground states of ^{8}Be and ^{12}C

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations for ^{8}Be and ^{12}C, L=11.8 fm

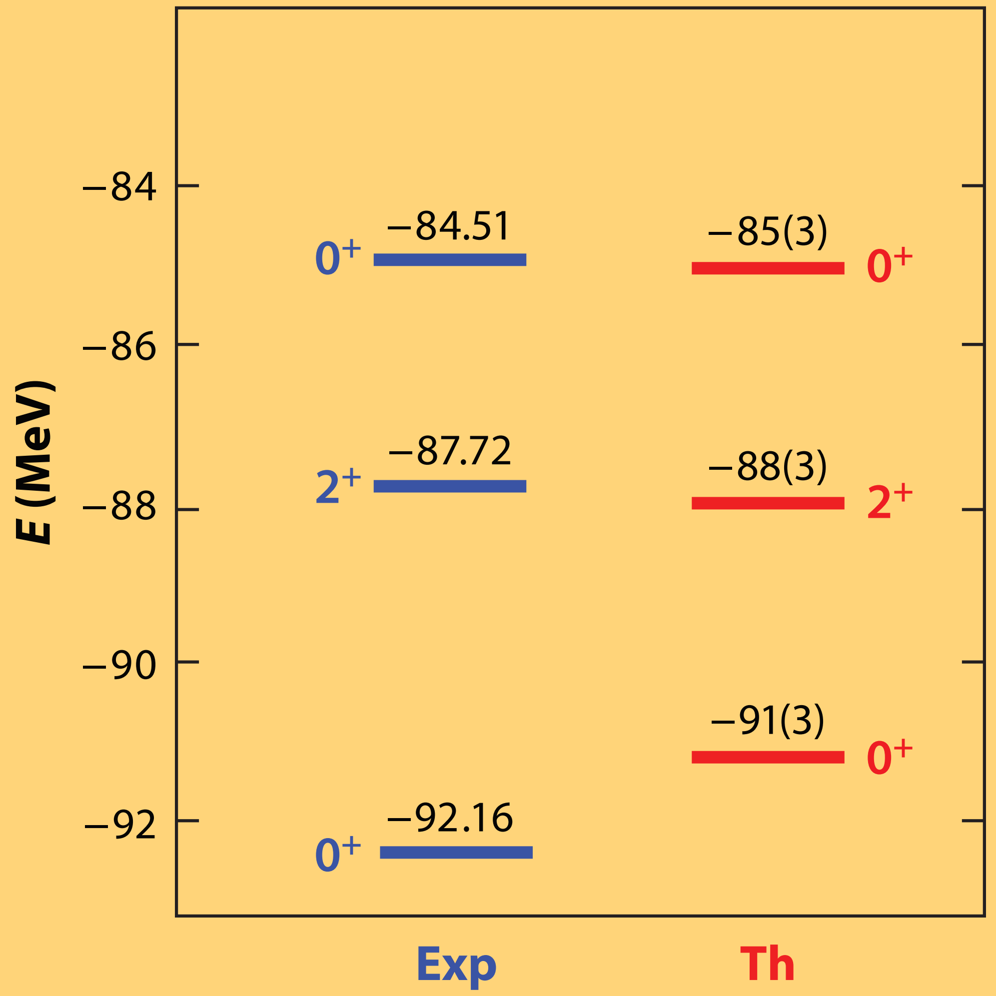


Various contributions to ^{4}He, ^{8}Be and ^{12}C

	^{4}He	^{8}Be	^{12}C
LO $[O(Q^0)]$	−24.8(2)	−60.9(7)	−110(2)
NLO $[O(Q^2)]$	−24.7(2)	−60(2)	−93(3)
IB + EM $[O(Q^2)]$	−23.8(2)	−55(2)	−85(3)
NNLO $[O(Q^3)]$	−28.4(3)	−58(2)	−91(3)
Experiment	−28.30	−56.50	−92.16

The Hoyl

E.E., Krebs, Lee, Meißner, PRL 106 (



	0_2^+	$2_1^+, J_z = 0$	$2_1^+, J_z = 2$
LO $[O(Q^0)]$	−94(2)	−92(2)	−89(2)
NLO $[O(Q^2)]$	−82(3)	−87(3)	−85(3)
IB + EM $[O(Q^2)]$	−74(3)	−80(3)	−78(3)
NNLO $[O(Q^3)]$	−85(3)	−88(3)	−90(4)
Experiment	−84.51	−87.72	

Summary & outlook

Nuclear chiral EFT enters precision era:

- accurate nuclear potentials at N^3LO
- detailed analyses of electroweak currents
- high-precision few-N calculations (πN scatt. lengths, radiative/muon capture...)

Time to tackle unsolved problems:

- chiral symmetry + πN = predictions for the long-range structure of the 3NF
  (work in progress...)

Nuclear lattice simulations:

- combining EFT with lattice simulations ⟶ access to light nuclei
- exciting results for the ^{12}C spectrum (Hoyle state)

  Work in progress: structure & quark mass dependence of the Hoyle state, ^{16}O, volume dependence, ...