5-th INTERNATIONAL CONFERENCE
“CHANNELING 2012” SEPTEMBER 23 - 28 2012
ALGHERO, ITALY

Radiation from Channeling Electrons, Stimulated by Laser Beam

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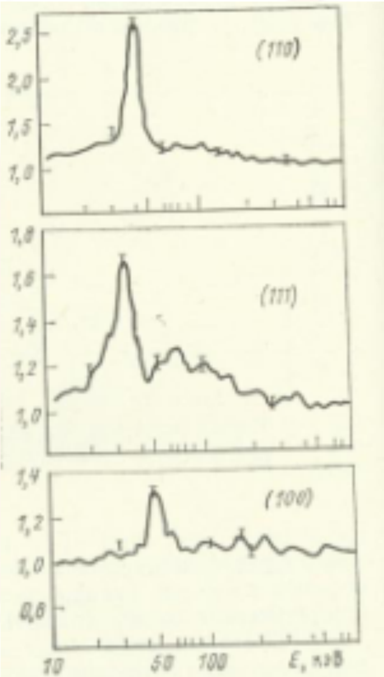
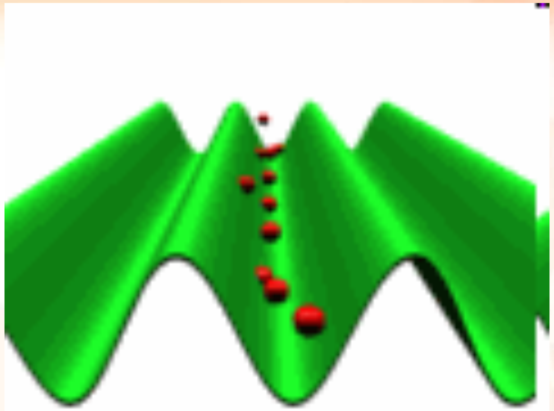
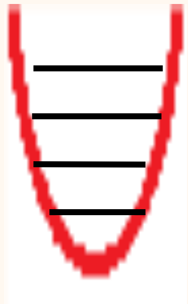
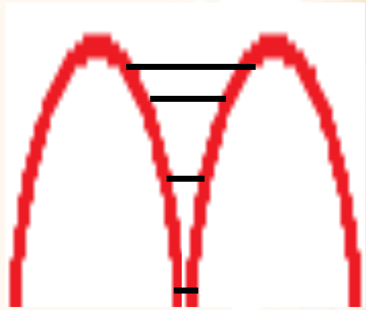
Standard approach









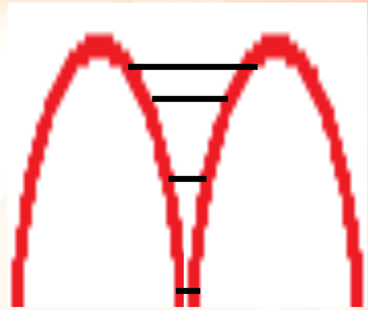
$\Delta E \sim (U/E)^{1/2}(hc/d) \sim (10\text{-}100eV)(mc^2/E)^{1/2};$

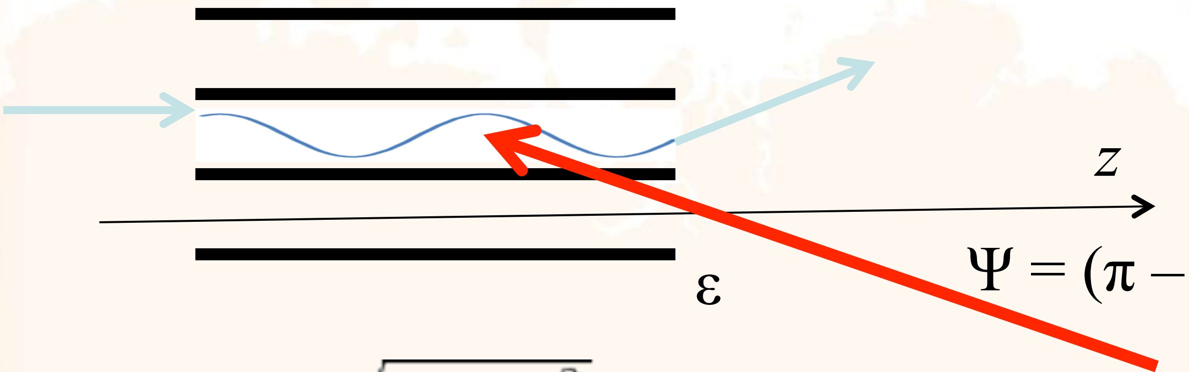
$h\nu = \Delta E(E/mc^2)^2 \sim (10\text{-}100eV)(E/mc^2)^{3/2}$

$U' = U_0 E/mc^2 \quad N \sim (E/mc^2)^{1/2} \qquad \Delta E \sim U'/N \sim U_0 (E/mc^2)^{1/2}$

Channeling radiation

Radiation, stimulated by laser beam





$$\omega = \omega_0 \cdot \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cdot\cos\theta} = \sim 2\omega_0/(mc^2/E + E\psi^2/mc^2) < 2\omega_0 E/mc^2$$

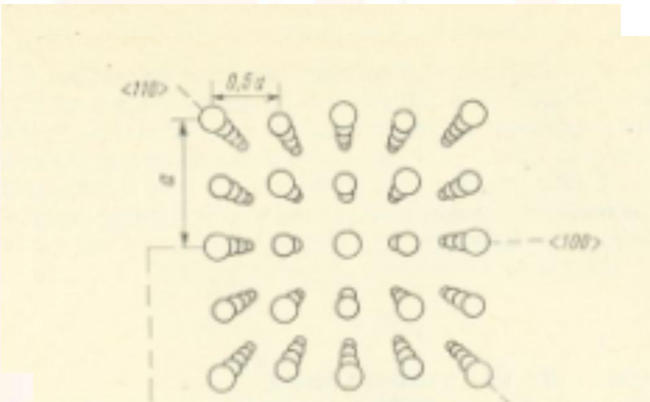
In the *accompanying system*, moving parallel with the electron beam

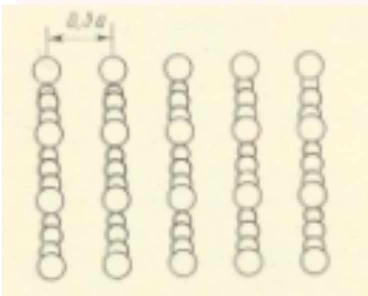
$\omega = (2\pi\Delta E/h)(E/mc^2) \sim (2\pi c/d)(EU)^{1/2}/mc^2 < 2\omega_0 E/mc^2$

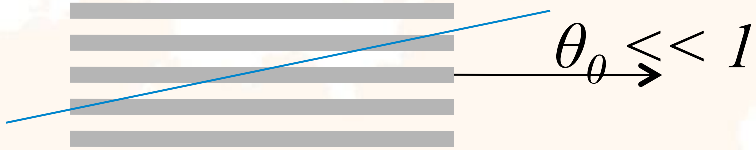
$\Delta E \sim (10\text{-}100eV)(mc^2/E)^{1/2} < h\omega_0/\pi \sim 5eV$

Channeling radiation

Radiation, stimulated by periodically inhomogeneous planar channeling potential







Electron in planar channeling.
Plane consisting of axises

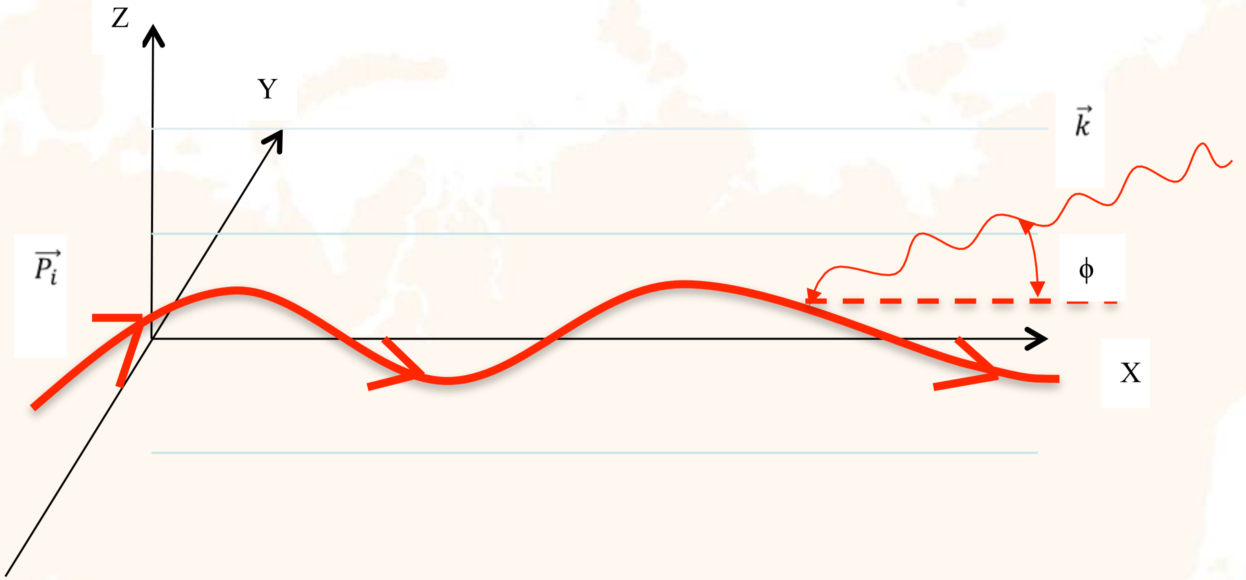
$$\Delta E \sim (U/E)^{1/2}(hc/d) \sim (hc/d)\theta_L \,(planar)$$

$$\theta_0 \;>\; \theta_L\,(axial) \;>>\; \theta_L\,(planar)$$

$$h\nu = hc(\theta_0/d)(E/mc^2)^2 \;>>\; \Delta E(E/mc^2)^2 \sim$$
$$\sim hc(\theta_L/d)(E/mc^2)^2$$

Kinematics

$$\vec{P_i} = \vec{P_i}(x) + \vec{P}_{i\perp}(\theta) = P_0\left(1 - \frac{\theta_0^2}{2}\right)\vec{\imath} + P_0\theta_0\vec{\jmath}$$



$$\frac{P_{i\perp}^2}{2m} \lessapprox < U >, \qquad \frac{P_0^2\theta_0^2}{2m} \lessapprox < U >, \qquad \theta_0 \leq \frac{2m < U >}{P_o^2} = \theta_L^2$$

"Accompanying" reference (coordinate) system

$$P'_x = \frac{P_x - \frac{V}{c^2}}{\sqrt{1 - \frac{V^2}{C^2}}} = 0 \qquad \Rightarrow V = \frac{P_x}{E} c^2$$

$$P'_y = P_y$$
$$P'_z = P_z \qquad \Rightarrow P'_\perp = P_\perp$$

$$E' = \frac{E - VP_x}{\sqrt{1 - \frac{V^2}{C^2}}} \Rightarrow m_0 c^2 + \frac{P_\perp^2}{2m_0}$$

In “accompanying” system

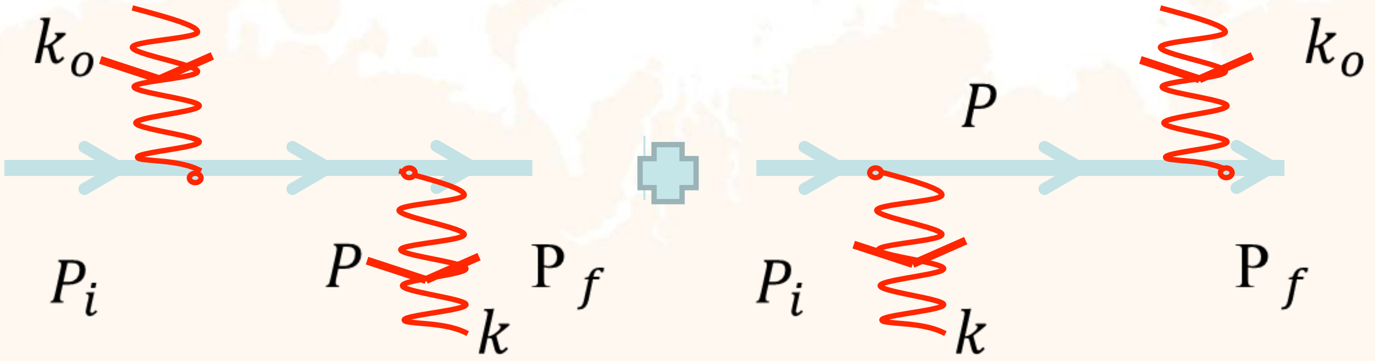
$$\omega = \omega_0 \frac{\sqrt{1-\frac{V^2}{c^2}}}{1+\frac{\vec{V}\vec{k}}{\omega_0}} = \omega_0 \frac{\sqrt{1-\frac{V^2}{c^2}}}{1+\frac{V}{c}\cos\psi}$$

$\psi = \pi - \varepsilon$

$$\omega = \omega_0 \frac{\sqrt{1-\frac{V^2}{c^2}}}{1-\frac{V}{c}+\frac{V}{c}\frac{\varepsilon^2}{2}} \Rightarrow \omega = \omega_0 \frac{1}{\gamma^{-1}+\frac{\varepsilon^2}{2}\gamma}$$

Induce scattering laser photon by "bound" electron

Richard P. FEYMAN diagrams



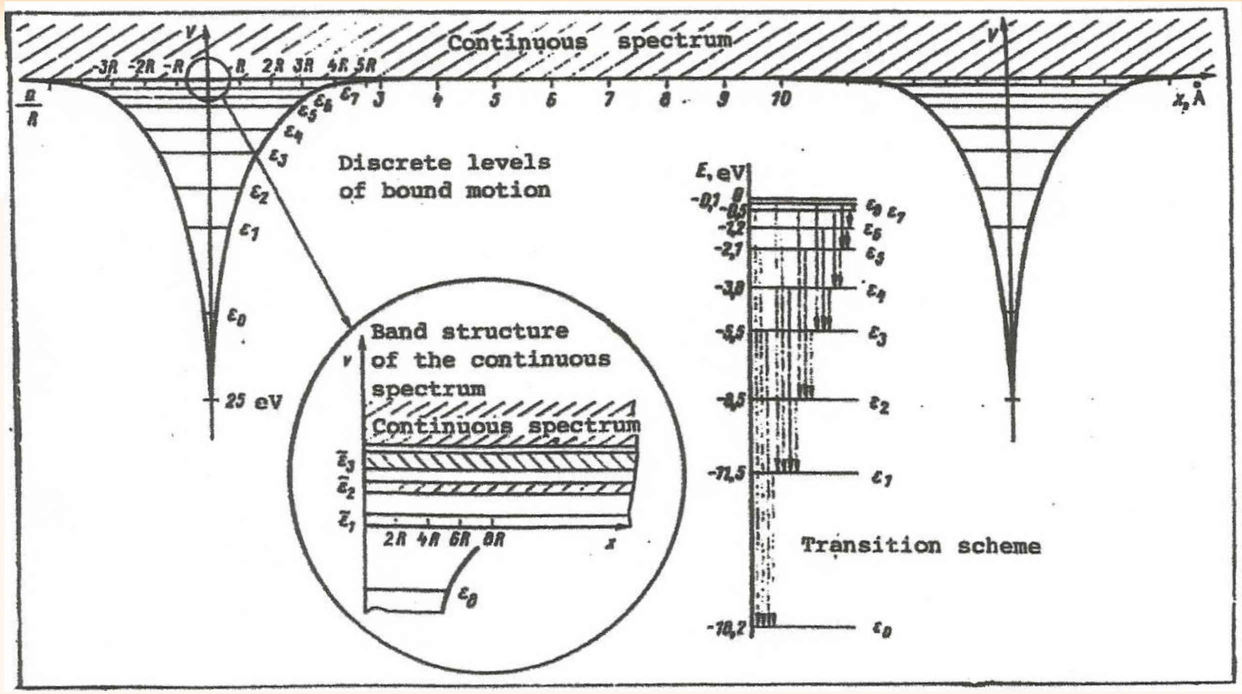
$$P_i + k_0 = P_f + k$$

$$P_i \cos\theta_0 + \frac{\hbar\omega_0}{c}\cos\psi = P_f\cos\theta_f + \frac{\hbar\omega}{c}\cos\psi_f$$

$$E_i + \hbar\omega_0 = E_f + \hbar\omega$$

$$\hbar\omega = E_i - E_f + \hbar\omega_0$$

Structure of energy bands and radiative transitions of 56-MeV electrons channeled along the (110) plane in Si



$$\overline{\Delta E} \sim \frac{U_{acc}}{N} \sim \frac{Uo\,\gamma}{\sqrt{\gamma}} \sim U_0\sqrt{\gamma}$$

$$U_0(Si) \approx 25eV \qquad R \approx 0{,}2\ \dot{A}$$

$$\overline{\omega_{12}}(lab) \sim \gamma^{3/2}$$

$$\omega_{max}(lab) \sim U_0\gamma^2$$

Scattering of a photon by a “bound” electron

$$\overrightarrow{A_i}(x)=\overrightarrow{A_i}(r)e^{-i\omega_i t} = \frac{\overrightarrow{e_i}}{\sqrt{2\omega_i}} e^{i(\overrightarrow{k_i}\vec{r}-\omega_i t)}$$

$$\overrightarrow{A_f}(x)=\overrightarrow{A_f}(r)e^{i\omega_f t} = \frac{\overrightarrow{e_f}}{\sqrt{2\omega_f}} e^{-i(\overrightarrow{k_f}\vec{r}-\omega_f t)}$$

$$\psi_i(x) = \psi_i(\vec{r})e^{-iE_i t}$$

$$\psi_f(x) = \psi_f(\vec{r})e^{-iE_f t}$$

Matrix element

$$U_{i\to f} = \frac{2\pi\alpha}{\sqrt{\omega_i\omega_f}}\sum_n\left\{\frac{\left(f\middle|\widehat{e_f}e^{-i\overrightarrow{k_f}\vec{r}}\middle|n\right)\left(n\middle|\widehat{e_i}e^{i\overrightarrow{k_i}\vec{r}}\middle|i\right)}{E_n - E_i - \omega_i} + \frac{\left(f\middle|\widehat{e_i}e^{i\overrightarrow{k_i}\vec{r}}\middle|n\right)\left(n\middle|\widehat{e_f}e^{-i\overrightarrow{k_f}\vec{r}}\middle|i\right)}{E_n - E_f - \omega_f}\right\}$$

Probability of photon scattering by a "bound" electron

$$S_{i\to f} = -(2\pi)iU_{i\to f}\,\delta(E_i + \omega_i - E_f\text{-}\omega_f),\ \text{where}$$

$$U_{i\to f} = 2\pi\alpha e^{i(\overrightarrow{k_i}-\overrightarrow{k_j})\vec{R}}\sqrt{\omega_i\omega_f}\sum_n\left\{\frac{(\vec{r}\overrightarrow{e_f})_{fn}(\vec{r}\overrightarrow{e_i})_{ni}}{E_i - E_n + \omega_i} + \frac{(\vec{r}\overrightarrow{e_i})_{fn}(\vec{r}\overrightarrow{e_f})_{ni}}{E_i - E_n - \omega_f}\right\}$$

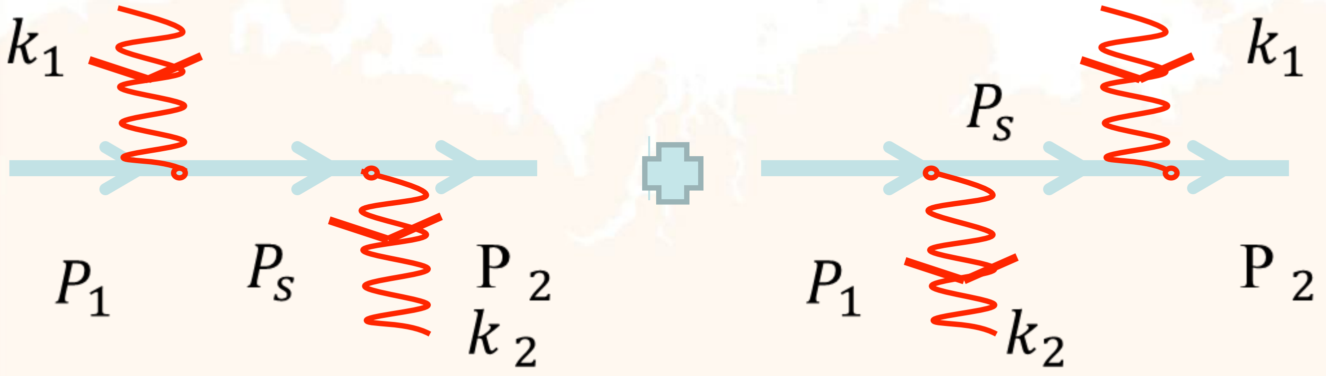
$$d\sigma = 2\pi\left|U_{if}\right|^2\delta(E_i + \omega_i - E_f - \omega_f)\frac{d^3\overrightarrow{k_f}}{(2\pi)^3}$$

$$d\sigma = \left|\sum_n\left[\frac{(\vec{Q}\vec{e}_f)_{fn}(\vec{Q}\overrightarrow{e_i})_{ni}}{E_i - E_n + \omega_i} + \frac{(\vec{Q}\vec{e}_i)_{fn}(\vec{Q}\overrightarrow{e_f})_{ni}}{E_i - E_n - \omega_f}\right]\right|^2\omega_i{\omega_f}^2 d\Omega_f$$

Resonance scattering laser photons

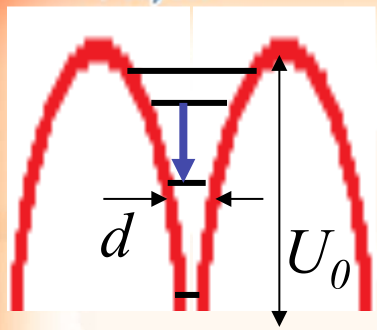
$$\omega_1 = E_s - E_1 \qquad (\omega_1 = \omega_2)$$

$$\psi_s \sim e^{-i(E_s - \frac{i}{2} r_s)t}$$

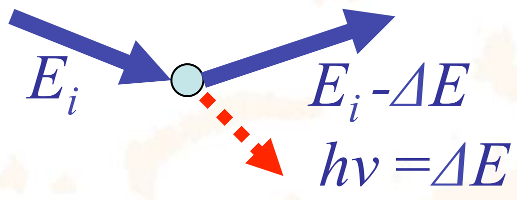


$$d\sigma = \omega_1 \omega_2{}^3 d\Omega_2 \sum \frac{\langle 2|\vec{Q}\widehat{e_2}|s\rangle\langle s|\vec{Q}\widehat{e_1}|1\rangle}{(E_s - E_1 - \omega_1)^2 + \frac{1}{4} r_s{}^2)}$$

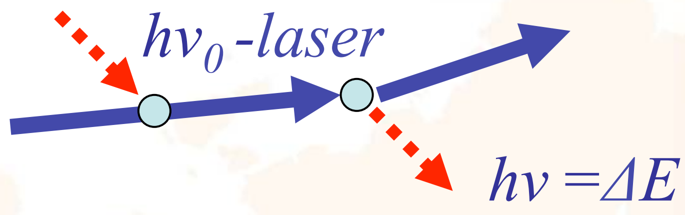
Channeling radiation



Spontaneous radiation



Stimulated radiation



1. Potential in lab. system: $U_0 \sim 20eV;\ d \sim 0{,}2\text{-}0{,}3\ A\ (Si);$
   in accompanying system (v_x = 0): $U = U_0\ (E/mc^2)$
2. Number of levels: $N \sim P_{x\,max} d\,/\,h \sim (EU_0)^{1/2} d\,/\,hc$
3. Distance between levels in acc. system: $\Delta E \sim U/N \sim (EU_0)^{1/2}(h/mcd)$
4. Laser photon energy in acc. system:

$$h\nu = 2h\nu_0/(mc^2/E + E\psi^2/mc^2) < 2h\nu_0 E/mc^2$$

5. In resonance conditions stimulated radiation can be very effective:

$$\Delta E \sim (EU_0)^{1/2}(h/mcd) = h\nu < 2h\nu_0 E/mc^2 = 2hE/mc\lambda_0$$

6. Resonance can be reached by correctly orienting laser beam, if:

$$E/U_0 > (\lambda_0/2d)^2 \sim 10^{7\text{-}8}$$

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Thank You for Your Attention!